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RESEARCH PROBLEMS

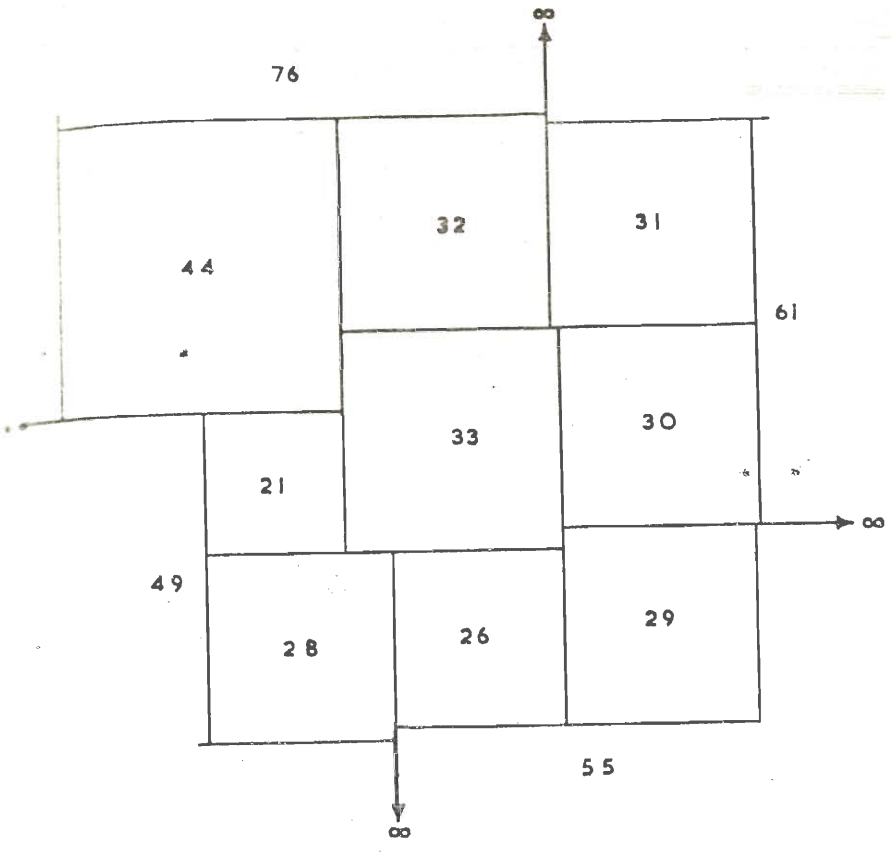


FIGURE 3

5. Robert R. Korfhage: *On a sequence of prime numbers.*

In the research problem entitled *Recursive function theory* (Bull. Amer. Math. Soc. 69 (1963), 737), Mullin raises a series of questions concerning prime sequences generated by following Euclid's scheme for proving the infinitude of the primes. We address ourselves to the second question, namely, whether or not the sequence generated in this manner, choosing at each step the highest prime factor, is monotone increasing. A short calculation on our IBM 7090 has shown that the sequence in question is 2, 3, 7, 43, 139, 50207, 340999, 3202139, 410353, . . . , and hence is not monotone. In fact, an examination of the table of prime factors given below shows that there is no way to choose the prime factors to form a monotone sequence, since at each

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stage there is at most one possible choice, namely the highest prime factor.

n	P_n	Prime Factors of $\prod_{i=1}^n P_i + 1$
1	2	3
2	3	7
3	7	43
4	43	13, 139
5	139	5, 50207
6	50207	23, 1607, 340999
7	340999	5521, 3202139
8	3202139	5, 53, 199, 410353
9	410353	...

In view of this result, it seems natural to add the following questions to those proposed by Mullin. (i) Are any, or all, of the sets generated in this manner and choosing the prime factor at each stage in any way recursive? (ii) Do any, or all, of these sets contain all of the prime numbers?

(Received December 20, 1963.)

11. Solomon W. Golomb: *Random permutations.*

Let L_N be the expected length of the longest cycle in a random permutation on N letters, and let $\lambda_N = L_N/N$. (Thus, $\lambda_1 = 1$, $\lambda_2 = 3/4$, $\lambda_3 = 13/18$, $\lambda_4 = 67/96$, etc.) It is easily shown that the sequence $\{\lambda_N\}$ is monotonically decreasing, and hence a limit λ exists. Computation has shown $\lambda = .62432965 \dots$, but nothing is known of the relationship of λ to other constants. What can be proved about the irrationality or transcendence of λ , and its relationship to classical mathematical constants? (Some nearby values unequal to λ include $5/8$, $1 - e^{-1}$, $(5^{1/2} - 1)/2$, and $\pi/5$.) (Received June 8, 1964.)

ERRATA

Robert R. Korfhage: *Correction to 'On a sequence of prime numbers.'*

It has been brought to my attention that because of the lack of an overflow check in the programming system used the factors listed for $n = 7$ are in error. Thus the value of P_8 is also wrong. Present knowledge indicates that probably $P_9 > P_8$, and thus Mullin's problem is still open. (Received July 16, 1964.)

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