Content Specifications
for the Summative Assessment of the
Common Core State Standards for Mathematics

REVISED DRAFT

July 2015

Developed with input from content experts and Smarter Balanced Assessment Consortium Staff, Work Group Members, and Technical Advisory Committee

Acknowledgements

More than 200 individuals and organizations offered feedback on one or more drafts of the content specifications. The organizations included the State Departments of Education from California, Colorado, Connecticut, Delaware, Hawaii, Idaho, Kansas, Maine, Michigan, Missouri, North Carolina, Oregon, Utah, Vermont, Washington, West Virginia, and Wisconsin, as well as:

ACT, Inc.
Addison Central Supervisory Unit
Aiea High School
Altus Network of Charter Schools
Asheboro City Schools
Asheville City Schools
Asheville Middle School
Association of California School Administrators
Beaufort County Public Schools
Berlin Area School District
Bridgeport Public Schools
Brien McMahon HS
Cabarrus County Schools
California Office to Reform Education
California Teachers Association
Californians Together
Camden Hills Regional High School
Cascade MCS
Catholic Diocese of Wichita, Kansas
Central Connecticut State University
Chippewa Falls Area Schools
Clinton City Schools
College Board
Connecticut Education Resource Center
Connecticut Technical High School
Council of the Great City Schools
Craven County Schools
Delavan-Darien School District
Discovery Charter School
East Lyme Public Schools
East Oakland Leadership
Edith Bowen Laboratory School
Elk Grove Unified School District
Envision Schools/3CS
Federal Way Public Schools
Freedom Area School District
Golden Valley HS
Granite School District
Hayward High School
Heritage Academies
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International Reading Association
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Junction City High School, Geary Co. Schools
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Monterey County Office of Education
National Council of La Raza
National Writing Project
Nebo School District
New Hope Elementary School District
Newhall Middle School
Northeast Elementary School
Northside High School
Odessa R-VII School District
Old Saybrook High School
Orange Unified School District
Partnership for 21st Century Skills
Pearson
Pewaukee School District
Pymatuning Valley High School
Randolph School District
Riverside Unified School District
San Bernardino City Unified School District
San Bernardino County Superintendent of Schools
San Diego Unified School District
San Luis Obispo County Office of Education
Santa Clara County Office of Education
Santa Monica–Malibu USD
SERC
Southington High School
Spring Creek Middle School, Cache County School District
Sundale Union Elementary School District
UC Riverside
University of Bridgeport
Vallejo City USD
Wagner Community School
Washoe County School District
WestEd
Westerly Public Schools
Western Connecticut State University
Winston-Salem Forsyth County Schools
Wiseburn School District
Woodburn School District
Zanesville High School

Table of Contents

Introduction and Background

Using This Document: This version of the Smarter Balanced Assessment Consortium's work on Content Specifications and Content Mapping consists of several sets of materials. It includes changes based on the two rounds of extensive and productive feedback provided to the Consortium.

Pages 1–68 represent the core of this document, and should be read carefully for comment and feedback. Appendices are intended to provide further elaboration of our work so far. All are embedded in this document, as it might be most useful for a reader to have them ready at hand.

This document follows an earlier release by the Consortium of a companion document covering specifications for English language arts and literacy. Both of these sets of documents have been developed in collaboration with comments from Consortium members and other stakeholders. The table below outlines the schedule that was used for the two rounds of public review for the content specifications of mathematics and English language arts/literacy when they were first developed.

Smarter Balanced Content Specifications Development Timelines and Activities

Review Steps	Date
Internal Review Start: ELA/Literacy - ELA/Literacy content specifications distributed to specific Smarter Balanced work groups for initial review and feedback	07/05/11
Internal Review Due: ELA/Literacy - Emailed to Smarter Balanced	07/15/11
Technical Advisory Committee (TAC) Review: ELA/Literacy - Draft submitted to TAC for review, comment, and feedback	07/27/11
Webinar: ELA/Literacy (including Evidence-Based Design Orientation) - Orientation for Smarter Balanced members to Evidence-Based Design and walkthrough of draft ELA/Literacy specifications document	08/08/11
Release for Review: ELA/Literacy (Round 1) - ELA/Literacy specifications documents posted on Smarter Balanced website and emailed to stakeholder groups	08/09/11
Internal Review Start: Mathematics - Mathematics content specifications distributed to specific Smarter Balanced work groups for preliminary review and feedback	08/10/11
Technical Advisory Committee (TAC) Review: Mathematics - Draft submitted to TAC for review, comment, and feedback	08/10/11
Internal Review Due: Mathematics - Emailed to Smarter Balanced	08/15/11
Release to Item Specifications to Bidders: ELA/Literacy - Current drafts of ELA/Literacy content specifications posted to OSPI website to support Item Specifications RFP process	08/15/11
Webinar: Mathematics - Walkthrough for Smarter Balanced members of the draft Mathematics specifications document	08/29/11
Release for Review: Mathematics (Round 1) - Mathematics content specifications posted on Smarter Balanced website and emailed to stakeholder groups	08/29 /11

Release of Specifications to Bidders: Mathematics - Current drafts of Mathematics content specifications posted to OSPI website to support Item Specifications RFP process	08/29/11
Feedback Surveys Due: ELA/Literacy (Round 1) - Emailed to Smarter Balanced	08/29/11
Feedback Surveys Due: Mathematics (Round 1) - Emailed to Smarter Balanced	09/19/11
Release for Review: ELA/Literacy (Round 2) - ELA/Literacy content specifications posted on Smarter Balanced website and emailed to stakeholder groups	09/19/11
Feedback Surveys Due: ELA/Literacy (Round 2) - Emailed to Smarter Balanced	09/27/11
Release for Review: Mathematics (Round 2) - Mathematics content specifications posted on Smarter Balanced website; email notification sent to stakeholder groups	12/09/11
Feedback Surveys Due: Mathematics (Round 2) - Emailed to Smarter Balanced	01/03/12
ELA/Literacy Claims Webinar Discussion - Summative assessment claims are discussed in preparation for subsequent vote by Governing states. Voting will be open 1/22/12 through 1/29/12.	01/29/12
Mathematics Claims Webinar Discussion - Summative assessment claims are discussed in preparation for subsequent vote by Governing states. Voting will be open 3/19/12 through 3/26/12	3/13/12
ELA/Literacy Claims adopted by Governing States - Summative assessment claims are established as policy for the Consortium through email voting of Governing State leads	03/01/12
Final Content Specifications and Content Mapping Released: ELA/Literacy - Final ELA/Literacy content specifications posted to Smarter Balanced website; email notification sent to member states and partner organizations	Early Apr 2012
Mathematics Claims adopted by Governing States - Summative assessment claims are established as policy for the Consortium through email voting of Governing State leads	Early Apr 2012
Final Content Specifications and Content Mapping Released: Mathematics - Final Mathematics content specifications posted to Smarter Balanced website; email notification sent to member states and partner organizations	Late Apr 2012

The contents of this document describe the Consortium's specification of critically important claims about student learning in mathematics that are derived from the Common Core State Standards. These claims will serve as the basis for the Consortium's system of summative and interim assessments and its formative assessment support for teachers. Open and transparent decision-making is one of the Consortium's central principles. A series of draft of the mathematics content specifications has been made available for comment consistent with that principle, and all responses to this work have been considered as the document has been refined.

Purpose of the Content Specifications: The Smarter Balanced Assessment Consortium is developing a comprehensive assessment system for mathematics and English language arts/literacy— aligned to the Common Core State Standards—with the goal of preparing all students for success in college and the workforce. Developed in partnership with member states, leading researchers, content expert experts,

and the authors of the Common Core, content specifications are intended to ensure that the assessment system accurately assesses the full range the standards.

This content specification of the Common Core mathematics standards provides clear and rigorous focused assessment targets that will be used to translate the grade-level Common Core standards into content frameworks along a learning continuum, from which specifications for items, tasks, and test blueprints will be established. Assessment evidence at each grade level provides item and task specificity and clarifies the connections between instructional processes and assessment outcomes.

Smarter Balanced Summative Assessment Development Overview

Released June 2010	Released for Review August 2011	Begins December 2011	Begins March 2012	2014-2015 School Year
Common Core State Standards Define the knowledge and skills students need for college and the workplace	**Content Specifications in ELA and math** / Prototypes, items/tasks will be developed to inform test design, item specification and test specifications	**Item Specifications** / **Test Design and Test Specifications**	**Item Writing** / Item writing materials will be developed using the item specification and content specifications	**Smarter Balanced Assessment**

The Consortium Theory of Action for Assessment Systems: As stated in the Smarter Balanced Assessment Consortium's (Smarter Balanced) Race to the Top proposal, "the Consortium's Theory of Action calls for full integration of the learning and assessment systems, leading to more informed decision-making and higher-quality instruction, and ultimately to increased numbers of students who are well prepared for college and careers" (p. 31). To that end, the Smarter Balanced proposed system features rigorous content standards; common adaptive summative assessments that make use of technology-enhanced item types, extended performance tasks that provide students the opportunities to demonstrate proficiency both with content and in the mathematical practices described in the Common Core State Standards; computer adaptive interim assessments that provide mid-course information about what students know and can do; instructionally sensitive formative tools, processes, and practices that can be accessed on-demand; focused ongoing support to teachers through professional development opportunities and exemplary instructional materials; and an online, tailored, reporting and tracking system that allows teachers, administrators, and students to access information about progress towards achieving college- and career-readiness as well as to identify specific strengths and weaknesses along the way. Each of these components serve to support the Consortium's overarching goal: *to ensure that*

all students leave high school prepared for post-secondary success in college or a career through increased student learning and improved teaching. Meeting this goal will require the coordination of many elements across the educational system, including but not limited to a high quality assessment system that strategically "balances" summative, interim, and formative components (Darling-Hammond & Pecheone, 2010; Smarter Balanced, 2010).

The proposed Smarter Balanced mathematics assessments and the assessment system are shaped by a set of characteristics shared by the systems of high-achieving nations and states, and include the following principles:[1]

1) **Assessments are grounded in a thoughtful, standards-based curriculum and are managed as part of an <u>integrated system</u>** of standards, curriculum, assessment, instruction, and teacher development. Curriculum and assessments are organized around a set of learning progressions[2] along multiple dimensions within subject areas. These guide teaching decisions, classroom-based assessment, and external assessment.

2) **Assessments include <u>evidence of student performance</u>** on challenging tasks that evaluate Common Core standards of 21st century learning. Instruction and assessments seek to teach and evaluate knowledge and skills that generalize and can transfer to higher education and multiple work domains. They emphasize deep knowledge of core concepts and ideas within and across the disciplines, along with analysis, synthesis, problem solving, communication, and critical thinking. This kind of learning and teaching requires a focus on complex performances as well as the testing of specific concepts, facts, and skills.

3) **<u>Teachers are integrally involved</u> in the development and scoring of assessments.** While many assessment components can and will be efficiently and effectively scored with computer assistance, teachers will also be involved in the interim/benchmark, formative, and summative assessment systems so that they deeply understand and can teach to the standards.

4) **Assessments are structured to <u>continuously improve teaching and learning.</u>** Assessment *as*, *of*, and *for* learning is designed to develop understanding of what learning standards are, what high-quality work looks like, what growth is occurring, and what is needed for student learning. This includes:

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[1] Darling-Hammond, L. (2010) <u>Performance counts</u>. Washington, DC: Council of Chief State School Officers.
[2] Empirically-based learning progressions visually and verbally articulate a hypothesis, or an anticipated path, of how student learning will typically move toward increased understanding over time with good instruction (Hess, Kurizaki, & Holt, 2009). The major concept of learning progressions is that students should progress through mathematics by building on what they know, moving toward some defined goals. While the structure of the mathematics shapes the pathways, there is *not* one prescribed or optimal pathway through the content.

- developing assessments around learning progressions that allow teachers to see what students know and can do on multiple dimensions of learning and to strategically support their progress;
- using computer-based technologies to adapt assessments to student levels to more effectively measure what they know, so that teachers can target instruction more carefully and can evaluate growth over time;
- creating opportunities for students and teachers to get feedback on student learning throughout the school year, in forms that are actionable for improving success;
- providing curriculum-embedded assessments that offer models of good curriculum and assessment practice, enhance curriculum equity within and across schools, and allow teachers to see and evaluate student learning in ways that can feed back into instructional and curriculum decisions; and
- allowing close examination of student work and moderated teacher scoring as sources of ongoing professional development.

5) **Assessment, reporting, and accountability systems provide useful information on multiple measures that is educative for all stakeholders.** Reporting of assessment results is timely, specific, and vivid—offering specific information about areas of performance and examples of student responses along with illustrative benchmarks, so that teachers and students can follow up with targeted instruction. Multiple assessment opportunities (formative and interim/benchmark, as well as summative) offer ongoing information about learning and improvement. Reports to stakeholders beyond the school provide specific data, examples, and illustrations so that administrators and policymakers can more fully understand what students know in order to guide curriculum and professional development decisions.

Accessibility to Content Standards and Assessments: In addition to these five principles, Smarter Balanced is committed to ensuring that the content standards, summative assessments, teacher-developed performance tasks, and interim assessments adhere to the principles of accessibility for students with disabilities and English Language Learners.[3] It is important to understand that the purpose of accessibility is *not* to reduce the rigor of the Common Core State Standards, but rather to avoid the creation of barriers for students who may need to demonstrate their knowledge and skills at the same level of rigor in different ways. Toward this end, each of the claims for the CCSS in Mathematics is

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[3] Accessibility in assessments refers to moving "beyond merely providing a way for students to participate in assessments. Accessible assessments provide a means for determining whether the knowledge and skills of each student meet standards-based criteria. This is not to say that accessible assessments are designed to measure whatever knowledge and skills a student happens to have. Rather, they measure the same knowledge and skills at the same level as traditional … assessments. Accessibility does not entail measuring different knowledge and skills for students with disabilities [or English Language Learners] from what would be measured for peers without disabilities" (Thurlow, Laitusis, Dillon, Cook, Moen, Abedi, & O'Brien, 2009, p. 2).

briefly clarified in terms of accessibility considerations. Information on what this means for content specifications and mapping will be developed further during the test and item development phases.

Too often, individuals knowledgeable about students with disabilities and English learners are not included at the beginning of the process of thinking about standards and assessments, with the result being that artificial barriers are set up in the definition of the content domain and the specification of how the content maps onto the assessment. These barriers can prevent these students from showing their knowledge and skills via assessments. The focus on "accessibility," as well as the five principles shared by systems of high-achieving nations and states, underlies the Consortium's approach to content mapping and the development of content specifications for the Smarter Balanced assessment system.

Accessibility is a broad term that covers both instruction (including access to the general education curriculum) and assessment (including summative, interim, and formative assessment tools). *Universal design* is another term that has been used to convey this approach to instruction and assessment (Johnstone, Thompson, Miller, & Thurlow, 2008; Rose, Meyer, & Hitchcock, 2005; Thompson, Thurlow, & Malouf, 2004; Thurlow, Johnstone, & Ketterline Geller, 2008; Thurlow, Johnstone, Thompson, & Case, 2008). The primary goal is to move beyond merely *including* students in instruction or assessment, but (a) to ensure that students learn what other students learn, and (b) to determine whether the knowledge and skills of each student meet standards-based criteria.

Several approaches have been developed to meet the two major goals of accessibility and universal design. They include a focus on multiple means of representation, multiple means of expression, and multiple means of engagement for instruction. Use of multiple media is also a key feature of accessibility. Elements of universally designed assessments and considerations for item and test review are a focus for developing accessible assessments. Increased attention has been given to computer-based assessments (Thurlow, Lazarus, Albus, & Hodgson, 2010) and the need to establish common protocols for item and test development, such as those described by Mattson and Russell (2010).

For assessments, the goal for all students with disabilities (except those students with significant cognitive disabilities who participate in an alternate assessment based on alternate achievement standards) is to measure the same knowledge and skills at the same level as traditional assessments, be they summative, interim, or formative assessments. Accessibility does not entail measuring different knowledge and skills for students with disabilities from what would be measured for peers without disabilities (Thurlow, Laitusis, Dillon, Cook, Moen, Abedi, & O'Brien, 2009; Thurlow, Quenemoen, Lazarus, Moen, Johnstone, Liu, Christensen, Albus, & Altman, 2008). It does entail understanding the characteristics and needs of students with disabilities and addressing ways to design assessments and provide accommodations to get around the barriers created by their disabilities.

Similarly, the goal for students who are English language learners is to ensure that performance is not impeded by the use of language that creates barriers that are unrelated to the construct being measured.

Unnecessary linguistic complexity may affect the accessibility of assessments for all students, particularly for those who are non-native speakers of English (Abedi, in press; Abedi, 2010; Solano-Flores, 2008). Several studies have shown how the performance of ELL students can be confounded during mathematics assessments as a function of unfamiliar cultural referents and unnecessary linguistic complexities (see for example, Abedi, 2010; Abedi & Lord, 2001; Solano-Flores, 2008).

In particular, research has demonstrated that several linguistic features unrelated to mathematics content could slow the reader down, increase the possibility of misinterpretation of mathematics items, and add to the ELL student's cognitive load, thus interfering with understanding the assessment questions and explaining the outcomes of assessments. Indices of language difficulty that may be unrelated to the mathematics content include unfamiliar (or less commonly used) vocabulary, complex grammatical structures, and styles of discourse that include extra material, conditional clauses, abstractions, and passive voice construction (Abedi, 2010).

A distinction has been made between language that is relevant to the focal construct (mathematics in this case) and language that is irrelevant to the content (construct-irrelevant). Smarter Balanced intends to address issues concerning the impact of unnecessary linguistic complexity of mathematics items as a source of construct-irrelevant factor for ELL students, and provide guidelines on how to control for such sources of threat to the reliability and validity of mathematics assessments for these students. Studies on the impact of language factors on the assessment outcomes have also demonstrated that they impact performance of students with learning and reading disabilities. Thus, controlling for such sources of impact will also help students with learning/reading disabilities (Abedi, 2010).

In addition, ELL students' abilities to communicate could substantially confound their level of proficiency in mathematics, as it is required for many of the mathematical tasks. For example, a major requirement for a successful performance in mathematics as outlined in the CCSSM is a high level of verbal and written communication skills. Each of the four claims indicates that successful completion of mathematics operations may not be sufficient to claim success in the tasks and that students should also be able to clearly and fluently communicate their reasoning. This could be a major obstacle for ELL students who are highly proficient in mathematical concepts and mathematical operations but not at the level of proficiency in English to provide clear explanation of the operations in words alone. Allowing students to show their reasoning using mathematical models, equations, diagrams, and drawings as well as written text will provide more complete access to students' thinking and understanding.

In the case of English learners (EL), ensuring appropriate assessment will require a reliable and valid measure of EL students' level of proficiency in their native language (L1) and in English (L2). In general, if students are not proficient in English but are proficient in L1 and have been instructed in L1, then a native language version of the assessment should be considered, since an English version of the assessment will not provide a reliable and valid measure of students' abilities to read, write, listen, and speak. If students are at the level of proficiency in reading in English to meaningfully participate in an English-only assessment (based, for example, on a screening test or the Title III ELP assessment), then it will be appropriate to provide access in a computer adaptive mode to items that are consistent with their

level of English proficiency but measure the same construct as other items in the pool. (See Abedi et al., 2011, for a computer adaptive system based on students' level of English language proficiency.)

As issues of accessibility are being considered, attention first should be given to ensuring that the design of the assessment itself does not create barriers that interfere with students showing what they know and can do in relation to the content standards. Several approaches to doing this were used in the development of alternate assessments based on modified achievement standards and could be brought into regular assessments to meet the needs of all students, not just those with disabilities, once the content is more carefully defined. To determine whether a complex linguistic structure in the assessment is a necessary part of the construct (i.e., construct-relevant), a group of experts (including content and linguistic experts and teachers) should convene at the test development phase and determine all the construct-relevant language in the assessments. This analysis is part of the universal design process.

Accommodations then should be identified that will provide access for students who still need assistance getting around the barriers created by their disabilities or their level of English language proficiency after the assessments themselves are as accessible as possible. For example, where it is appropriate, items may be prepared at different levels of linguistic complexity so that students can have the opportunity to respond to the items that are more relevant for them based on their needs, ensuring that the focal constructs are not altered when making assessments more linguistically accessible. Both approaches (designing accessible assessments and identifying appropriate accommodations) require careful definition of the content to be assessed.

Careful definitions of the content are being created by Smarter Balanced. These definitions involve identifying the Smarter Balanced assessment claims, the rationale for them, what sufficient evidence looks like, and possible reporting categories for each claim. Further explication of these claims provides the basis for ensuring the accessibility of the content – accessibility that does not compromise the intended content for instruction and assessment – as well as accommodations that might be used without changing the content. Sample explications are provided under each of the claims.

Further Readings: Each of the Smarter Balanced assessment system principles is interwoven throughout this document in describing the content mapping and content specifications. Readers may want to engage in additional background reading to better understand how the concepts below have influenced the development of the Smarter Balanced mathematics assessment design.

- **Principles of evidence-based design (EBD); The Assessment Triangle (see next page); Cognition and transfer; Performances of novices/experts**
  (see Pellegrino, Chudowsky, & Glaser, 2001; Pellegrino, 2002)
- **Enduring understandings, transfer**
  (see Wiggins & McTighe, 2001)
- **Principles of evidence-centered design (ECD) for assessment**
  (see Mislevy, 1993, 1995)
- **Learning progressions/learning progressions frameworks**
  (see Hess, 2008, 2010, 2011; National Assessment Governing Board, 2007; Popham, 2011; Wilson, 2009)
- **Universal Design for Learning (UDL); Increased accessibility of test items**
  (see Abedi, 2010; Bechard, Russell, Camacho, Thurlow, Ketterlin Geller, Godin, McDivitt, Hess, & Cameto, 2009; Hess, McDivitt, & Fincher, 2008).
- **Cognitive rigor, Depth of Knowledge; Deep learning**
  (see Alliance for Excellence in Education, 2011; Hess, Carlock, Jones, & Walkup, 2009; Webb, 1999)
- **Interim assessment; Formative Assessment**
  (see Perie, Marion, & Gong, 2007; Heritage, 2010; Popham, 2011; Wiliam, 2011)
- **Constructing Questions and Tasks for Technology Platforms**
  (see Scalise & Gifford, 2006)

Content Mapping and Content Specifications for Assessment Design: The Assessment Triangle, illustrated on the following page, was first presented by Pellegrino, Chudowsky, and Glaser in *Knowing What Students Know/*KWSK (NRC, 2001). "[T]he corners of the triangle represent the three key elements underlying any assessment…a model of student *cognition* and learning in the domain, a set of beliefs about the kinds of *observations* that will provide evidence of students' competencies, and an *interpretation* process for making sense of the evidence" (NRC, 2001, p. 44). KWSK uses the heuristic of this 'assessment triangle' to illustrate the fundamental components of evidence-based design (EBD), which articulates the relationships among learning models (Cognition), assessment methods (Observation), and inferences one can draw from the observations made about what students truly know and can do (Interpretation) (Hess, Burdge, & Clayton, 2011).

Application of the assessment triangle not only contributes to better test design; the interconnections among Cognition, Observation, and Interpretation can be used to gain insights into student learning. For example, learning progressions offer a coherent starting point for thinking about how students develop competence in an academic domain and how to observe and interpret the learning as it unfolds over

time; they reflect appropriate content emphases at different times as the curriculum advances, and as students' understandings grow. Hypotheses about typical pathways of learning can be validated, in part, through systematic (empirical) observation methods and analyses of evidence produced in student work samples from a range of assessments.

Observation: A set of specifications for assessment tasks that will elicit illuminating responses from students

Interpretation: The methods and analytic tools used to make sense of and reason from the assessment observations/evidence

Cognition: Beliefs about how humans represent information and develop competence in a particular academic domain
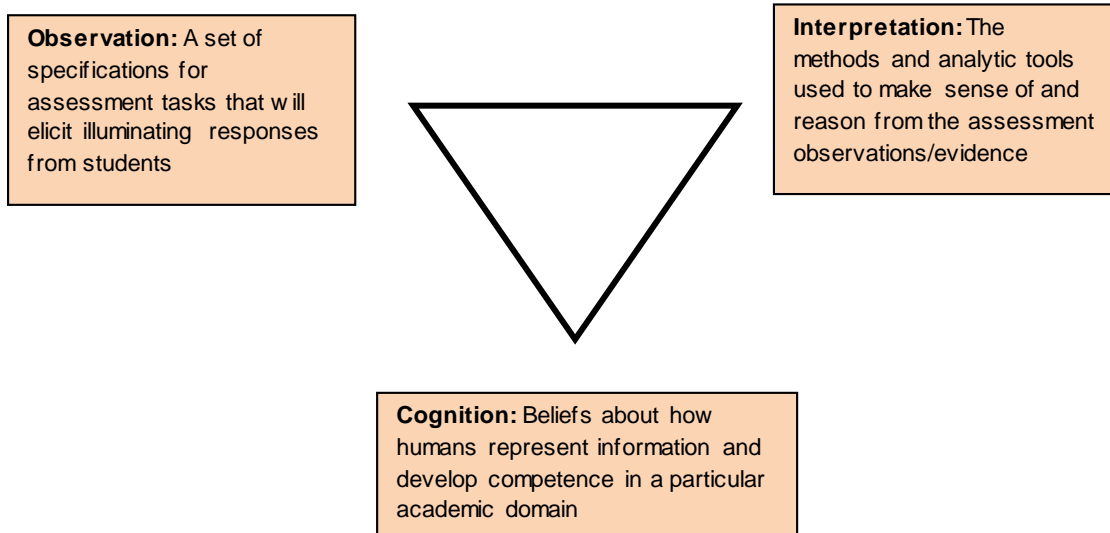
The Assessment Triangle (NRC, 2001, p. 44)

Evidence-based Design: Smarter Balanced is committed to using evidence-based design in its development of assessments in the Consortium's system. The Smarter Balanced approach is detailed in the following section, but a brief explanation is as follows. In this document, four "claims" are set forth regarding what students should know and be able to do in the domain of mathematics. Each claim is accompanied by a "Rationale" that provides the basis for establishing the claim as central to mathematics. The claims and Rationales represent the "cognition" part of the assessment triangle. For each claim and Rationale there is a section representing the "observation" corner of the triangle. Here, a narrative description lays out the kinds of evidence that would be sufficient to support the claim, which is followed by tables with "Assessment Targets" linked to the Common Core standards. Finally, the "interpretation" corner of the triangle is represented by a section for each claim that lists the "Proposed Reporting Categories" that the assessment would provide.

Part I – General Considerations for the Use of Items and Tasks to Assess Mathematics Content and Practices

Assessing Mathematics: The Common Core State Standards for mathematics require that mathematical content and mathematical practices be *connected* (CCSSM, p. 8). In addition, two of the major design principles of the standards are *focus* and *coherence* (CCSSM, p. 3). Together, these features of the standards have important implications for the design of the Smarter Balanced assessment system.

Using Various Types of Items and Tasks to Connect Content and Practice: There are multiple dimensions to mathematical proficiency, ranging from knowing important mathematical facts and procedures to being able to use that knowledge in the solution of complex problems. Smarter Balanced intends to use a variety of types of assessment items and tasks to assess student mathematical proficiency. The type of assessment item or task that is called upon will be aligned to the type of mathematical learning that is being assessed.

For example, knowledge of mathematics content and procedures such as how to add fractions, or how to solve two linear equations with two unknowns can usually be assessed with single-point ("correct/incorrect") problems like: selected response, multiple choice, completion, and technology-enhanced (click-and-drop, etc.) items. On the other hand, demonstrating the skill to model a mathematical situation or to explain the rationale for a given approach to solving a problem typically requires using assessment "tasks" that are scored using more nuanced scoring guides, or "rubrics", usually on a multiple-point scale (zero-to-two points, zero-to-three points, etc.) Finally, the assessment of student capacity to apply several mathematical principles and practices to solve real-world problems will require the development of more complex "performance events" that use a combination of single-point items and multiple-point tasks around a central theme or scenario.

Sometimes this distinction between items, tasks, and performance events is confused with how easy or difficult the problem is for the student. Care should be taken not to confuse overall difficulty with the assessment type. Some single-point multiple choice items can be quite difficult, and some complex performance events can contain fairly simple and straightforward items and tasks. So, the more complex tasks and performance events are not used as a means to develop more challenging problems; rather, they are used because they are a more direct means of assessing the application of skills such as problem solving, reasoning.

Demonstrating the skills to model a mathematical situation and explain the rationale for the approach depends on deciding what is mathematically important in that situation, representing it with mathematical symbolism, operating on the symbols appropriately, and then interpreting the results in meaningful ways. Assessing this deeper understanding of mathematics can best be accomplished through the use of more complex assessment tasks. As demonstrated throughout this document, Smarter Balanced is committed to the notion that a balanced and meaningful assessment that assesses the full

range of the CCSS in mathematics needs to draw upon a spectrum of item types -- ranging from brief items targeting particular concepts or skills through more elaborate constructed response tasks and performance events that call upon the application of mathematical concepts.

Focus and Coherence: The principles of focus and coherence on which the CCSSM are based have additional implications for mathematics assessment and instruction. Coherence implies that the standards are more than a mere checklist of disconnected statements; the cluster headings, domains, and other text in the standards all organize the content in ways that highlight the unity of the subject. The standards' focus is meant to allow time for students and teachers to master the intricate, challenging, and necessary things in each grade that open the way to a variety of applications even as they form the prerequisite study for future grades' learning. The Smarter Balanced assessment will strive to reinforce focus and coherence at each grade level by testing for proficiency with central and pivotal mathematics rather than covering too many ideas superficially – a key point of the Common Core State Standards. It will, as well, reflect changes in curricular emphases as students move toward engagement with new content (e.g., specific aspects of arithmetic will be emphasized and de-emphasized as students make the transition from reasoning with numbers to reasoning algebraically.)

An emphasis on focus and coherence in assessment rests on the prioritization of content for purposes of sampling – it is simply not feasible to thoroughly assess every student on all topics, but it *is* essential to provide information regarding student understanding and facility with centrally important topics. Thus, for purposes of focused and coherent coverage, this document identifies a subset of the content clusters that are identified as high-priority assessment clusters. The sampling of content within the assessment will emphasize content in the high-priority clusters, with content that is not in high-priority clusters being sampled with less frequency. The overall ratio on the assessment of content in high-priority clusters to other content should be about 3:1. Thus any particular student's assessment will sample in greatest proportion from content clusters representing the major work of that grade, but, over the whole population, all content will be assessed.

Part II – Overview of Claims and Evidence for CCSS Mathematics Assessment

Assessment Claims

The theory of action articulated in the Consortium's proposal to the U.S. Department of Education (http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/02/Smarter-Balanced-Theory-of-Action1.pdf) illustrates the vision for an assessment system supporting inferences that ensure all students are well-prepared for college and careers after high school. "Inference is reasoning from what one knows and what one observes, to explanations, conclusions, or predictions. One attempts to establish the weight and coverage of evidence in what is observed" (Mislevy, 1995, p 2). *Claims* are the broad statements of the assessment system's learning outcomes, each of which requires *evidence* that articulates the types of data/observations that will support interpretations of competence towards achievement of the claims. A first purpose of this document is to identify the critical and relevant claims that will "identify the set of knowledge and skills that is important to measure for the task at hand" (Pellegrino, Chudowsky, and Glaser, 2001), which in this case are the learning outcomes for the CCSS for mathematics.

This document has now been subject to two extensive field reviews, and revised accordingly. Initial reviews were quite favorable, the second set even more so. This document, revised in light of the second set of reviews, presents the resulting claims for the mathematics assessment. The assessment claims described below will be presented to the Smarter Balanced governing states for approval as Consortium policy. Governing state approval of the claims will ensure that all governing states have full endorsement of the major components of the summative assessments, and will establish those statements as the fundamental drivers for the design of the Consortium's summative assessments.

For this reason, within this document the claims stand out as being of particular significance. In fact, the other material presented here (in particular the Assessment Targets and the commentaries related to them) is meant to serve as general guidance and support for further development of the summative assessments. However, this additional material will not be subjected to endorsement by the governing states, and should not be viewed as Consortium policy. A more useful interpretation would be to view the Assessment Targets and commentaries as the "best thinking" of those who have contributed to this document, and should be considered as guidance for the further specifications of items and tasks and for the overall test design.

Five claims are proposed for the summative mathematics assessment – one overall composite claim associated with the entire assessment, and four separate domain claims which each address a sub-component of the overall composite. A detailed treatment of each claim follows in Part III, below. Each claim is a summary statement about the knowledge and skill students will be expected to demonstrate on the assessment related to a particular aspect of the CCSS for mathematics. The level of the knowledge

and skill necessary for a student to be proclaimed "Proficient" will be established through the development of Achievement Level Descriptors and during the setting of performance standards on the assessments.

Claims for Mathematics Summative Assessment

Overall Claim for Grades 3-8	"Students can demonstrate progress toward college and career readiness in mathematics."
Overall Claim for Grade 11	"Students can demonstrate college and career readiness in mathematics."
Claim #1	**Concepts & Procedures** "Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency."
Claim #2	**Problem Solving** "Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies."
Claim #3	**Communicating Reasoning** "Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."
Claim #4	**Modeling and Data Analysis** "Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."

Presentation of the Claims in Part III

Rationale for Claims: In Part III of this document, each claim is followed by a section describing what it is about this particular aspect of what students should know and be able to do that warrants a claim. The Rationale presents both the scope of the claim and its connection and alignment to the CCSS. In addition the claim is described in further detail than could be expected from the claim's single-sentence statement, and this description is provided in terms of what would be expected of a student who would demonstrate proficiency. In this way, the Rationale should be viewed as a starting point for the development of Achievement Level Descriptors.

Sufficient Evidence: Accompanying each claim in Part III is a description of the sufficient relevant evidence from which to draw inferences or conclusions about student attainment of the claim. Relevant and sufficient evidence needs to be collected in order to support each claim. The assessment system will provide the opportunity to use a variety of assessment items and tasks applied in different contexts. It is important that the Smarter Balanced pool of items and tasks for each claim be designed so the summative assessment can measure and be used to make interpretations about year-to-year student progress.

The sufficient evidence section for each claim includes a brief analysis of the assessment issues to be addressed to ensure accessibility to the assessment for all students, with particular attention to students with disabilities and English learners.

Assessment Targets: Finally, each claim is accompanied by a set of assessment targets that provide more detail about the range of content and Depth of Knowledge levels. The targets are intended to support the development of high-quality items and tasks that contribute evidence to the claims. We use the cluster level headings of the standards in the CCSSM, in order to allow for the creation and use of assessment tasks that require proficiency in a broad range of content and practices. Use of more fine-grained descriptions would risk a tendency to atomize the content, which might lead to assessments that would not meet the intent of the standards. It is important to keep in mind the importance of developing items and tasks that reflect the richness of the mathematics in the CCSSM.

Reporting Categories

As used here, "Reporting Categories" define the levels of aggregation of score points on the assessment that will be reported *at the individual student level.*

First and foremost, because the summative assessment will be used for school, district, and state accountability consistent with current ESEA requirements, there needs to be a composite **"Total Mathematics"** score at the individual student level. Also, consistent with the Smarter Balanced proposal and with requirements in the USED Notice Inviting Applications, the composite mathematics score will

need to have scaling properties that allow for the valid determination of student growth over time. This score will be a weighted composite from the four claims, with Claim #1 (Concepts and Procedures) contributing roughly 50%, claim 3 (Communicating Reasoning) contributing roughly 25%; and combined claims #2 and #4 (Problem Solving and Modeling and Data Analysis) contributing about 25%.

Will there be subscores below the claim level?

In its 2000 *Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics describes the Connections standard:

> Mathematics is not a collection of separate strands or standards, though it is often partitioned and presented in this manner. Rather, mathematics is an integrated field of study. Viewing mathematics as a whole highlights the need for studying and thinking about the connections within the discipline, as reflected both within the curriculum of a particular grade and between grade levels. (p. 64)

Large-scale assessments have contributed to the partitioning of mathematics into discrete topics by reporting scores on separate areas of mathematics (e.g., Algebra or Geometry), or in some cases even finer-grained detail (e.g., Computations with Fractions or Place Value). The implications of this approach to assessment on curriculum have been fairly evident in classrooms across the United States. The reporting of scores should not contribute to or exacerbate this problem. At the same time, as discussed in the principles, the sampling of items within each category should reflect the focus, coherence, and prioritization of core mathematics, as discussed in Part I.

Evidence-centered design provides a framework for re-thinking the reporting structure of summative assessments. If we agree that connections in mathematics are a critical component of curriculum, instruction, and assessment; then the potential for invalid inferences based on a reporting structure that partitions the content into separate areas of mathematics is quite high. Take the following Common Core Measurement & Data standard as an example:

> 3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units – whole numbers, halves or quarters.

Traditionally, an item developed for this standard would fall into the "Measurement and Data" reporting category and be consumed in a subscore for that category. A student answering an item based on this standard incorrectly may be just as likely to have a weak foundation in Fractions as in Measurement and Data. The focus and coherence of the Common Core State Standards at each grade level maximize the connections within and across domains, an approach that is consistent with that of several high-achieving countries. Therefore, a traditional content-based approach to summative assessment reporting would not support the "interpretation" vertex of the evidence-centered design framework described earlier.

Based on Smarter Balanced's commitment to providing student-level data from which valid inferences can be made, the reporting categories for the summative mathematics assessment include four scores: a Total Mathematics composite score and a subscore for each claim, with claims 2 and 4 combined for the purposed of reporting. The table below provides a summary of these reporting categories.

Reporting Categories for Summative Mathematics Assessment

Total Mathematics Composite Score		
Claim 1: Concepts and Procedures Score	Claim 3: Communicating Reasoning Score	Claims 2 & 4: Problem Solving/ Modeling and Data Analysis Score

Part III – Detailed Rationale and Evidence for Each Claim

Overall Mathematics Claim
<u>For Grades 3-8</u> **Students can demonstrate progress toward college and career readiness in mathematics.**
<u>For High School</u> **Students can demonstrate college and career readiness in mathematics.**

Rationale for Overall Mathematics Claim

Part of the rationale for an overall claim is simply in response to the ways in which scores on this assessment are likely to be used by educators and policy makers. Results of the summative assessment will be used to inform a number of important decisions about students, educators, and schools. In some instances the assessment results may be the sole source of data used for a decision (e.g., for calculation of Adequate Yearly Progress under current NCLB requirements, or for declaring that a high school student may enter into credit-bearing Math courses in college or university), and in some instances the assessment results may be but one part of a larger collection of information (e.g., for the evaluation of the effectiveness of certain instructional or intervention programs, or for the determination of whether or not a teacher or a principal is in need of improvement.) Regardless of the particular use, however, each of these examples will draw inferences about the knowledge and skills of individual students and of groups of students based on performance on the total test, as aligned to the Common Core of State Standards.

A second rationale is no less important, but is perhaps less immediately evident. The examples listed above, in many cases, can be characterized as having relatively high stakes for those affected by the outcome. Schools and districts are dramatically impacted by AYP results; students determined not to be ready for credit-bearing courses must spend additional time (and finances) on their post-secondary education; personnel decisions are obviously high-stakes decisions. Principles of fairness dictate that those who use assessment results for high stakes decisions should use the most reliable and accurate information available. Scores derived from the total test, based on performance across all of the assessed domains, will be more accurate and will lead to fewer incorrect inferences than will scores on individual domains.

What sufficient evidence looks like for the Overall Mathematics Claim

The evidence to support student progress toward, or attainment of college and career readiness will be provided by student performance on the items and tasks for the four domain claims. This claim represents a weighted composite of all evidence gathered across the four domain-specific claims. That is, the contributions to the overall claim provided by each of the domain-specific claims will be need to be weighted through an analytic and judgmental process. It would be unreasonable to make the a priori assumption that the contribution to a claim about overall college/career readiness of, for example, mathematical operations and procedures carries the same weight the contribution of each of the remaining domains. Determining the weighting of the domain-specific claims is a decision that will need to be made based on the psychometric characteristics of the evidence from the four domain claims and on empirical data and policy direction provided by member states. This work will need to be carried out during the standard setting phase of the project.

Proposed Reporting Categories for the Overall Mathematics Claim

There will be a <u>Total Mathematics</u> score, which will be a weighted composite based on the student's performance across the four domain-specific claims. The Total mathematics scores will be vertically scaled across grades.

Mathematics Claim #1
CONCEPTS AND PROCEDURES

Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Rationale for Claim #1

This claim addresses procedural skills and the conceptual understanding on which developing skills depend. It is important to assess how aware students are of how concepts link together, and why mathematical procedures work in the way that they do. This relates to the structural nature of mathematics:

> Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. (Practice 7, CCSSM)

> They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. (Practice 7, CCSSM)

> Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. (Practice 8, CCSSM)

Assessments should include items/tasks that test the precision with which students are able to carry out procedures, describe concepts and communicate results.

> Mathematically proficient students … state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of

measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. (Practice 6, CCSSM)

Items/tasks should also assess how well students are able to use appropriate tools strategically.

> Students are able to use technological tools to explore and deepen their understanding of concepts. (Practice 5; CCSSM)

Many individual content standards in CCSSM set an expectation that students can *explain why* given procedures work.

> One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness. (CCSSM, p.4).

Finally, throughout the K-6 standards in CCSSM there are also individual content standards that set expectations for fluency in computation (e.g., fluent multiplication and division within the times tables in Grade 3). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding, thoughtful practice, and extra support where necessary. Technology may offer the promise of assessing fluency more thoughtfully than has been done in the past. This, too, is part of 'measuring the full range of the standards.'

Following our discussion of the types of evidence appropriate for contributing to assessment of Claim #1, we describe specific grade-level content emphases.

What sufficient evidence looks like for Claim #1

Evidence on each student's progress along the progressions of mathematical content is the focus of attention in assessing this claim.

Essential properties of items and tasks that assess this claim: Items and tasks that could provide evidence for this claim include brief items – selected response and short constructed response items – that focus on a particular procedural skill or concept. Brief items could also include items that require students to translate between or among representations of concepts (words, diagrams, symbols) and items that require students to identify an underlying structure. Brief constructed response items can include items that provide scaffolded support for the student; it is probably possible for a Computer Adaptive environment to adjust the level of scaffolding that is provided depending on the student's performance level.

Selected response items, including computer-enhanced items, can probe conceptual understanding, particularly when the distractors are chosen to embody common misconceptions. In designing such items, it is essential to try to make sure that students do not obtain correct answers because of "test taking skills" rather than understanding of the mathematical content. Computer administration of the assessment affords the possibility of assessing student fluency with mathematical operations by means of monitoring the response time.

Short Constructed response items can assess mathematical thinking directly; short items of this kind can provide direct evidence of students' mastery of standard procedures. Among items/tasks that require students to produce a response, short constructed response items are the most likely to be able to be machine scored.

Highly scaffolded tasks, where the student is guided through a series of short steps set in a common problem context, offer another approach to the design of short constructed response items.

Extended Response items, requiring a more solid demonstration of conceptual understanding and procedural skills that students may be expected to have learned and practiced, may also provide evidence for this claim. These can include the following task types**:**

- **Application tasks** using exercises to assess relatively standard applications of mathematical principles. Here, students can be expected to use important concepts and skills to tackle problem situations that should be in the learned part of the curriculum.

- **Translation tasks**, where students are asked to represent concepts in different ways and translate between representations (words, numbers, tables, graphs, symbolic algebra).

- **Explanation tasks,** where students are asked to explain why a given standard procedure works. This may involve the straightforward adaptation of a standard procedure.

Accessibility & Claim #1: This claim clarifies the importance of conceptual understanding and procedural knowledge underlying the important core content in CCSSM. The standards refer to the ability to carry out procedures, describe concepts, communicate results, use appropriate tools strategically, and explain why specific procedures make sense. Neither the claim itself nor the CCSSM explicitly addresses the challenges that some students with disabilities face in the area of mathematical calculations. Because of the importance of building skills in computation in early schooling, the explication of the content may be different in early school grades compared to later school grades. Providing assistive technologies such as an abacus or calculator may not be considered appropriate up through about grade 4. At some point during intermediate grades, however, the use of these tools is considered an appropriate avenue of access to allow students to demonstrate that they are able to "calculate accurately and efficiently."

It is also important to address access to mathematics via decoding text and written expression. The uses of alternative means of access and expression are ones used by successful individuals (Reitz, 2011) to demonstrate high levels of success, and thus are an appropriate avenue of access to the content for students with disabilities in the areas of reading decoding and fluency as well as for those with blindness

or visual impairments. Likewise, allowing students alternative ways to express their understanding of mathematics content is important. Students who are unable to explain mathematical processes via writing or computer entry might instead provide their explanation via speech to text technology (or a scribe) or via manipulation of physical objects.

A major aspect of all the claims, including Claim #1, is communication, especially students' ability to explain *why or how* given procedures or approaches work. To maximize access to English learners who are at a lower proficiency in writing and speaking, it is important for Smarter Balanced to explore allowing ELL students to use diagrams, drawings, equations, and mathematical models, as well as words. It will also be useful to provide opportunities for ELL students to communicate their understanding through performance tasks or other approaches where multiple domain input can be provided. Furthermore, when a major performance difference exists between tasks such as expanding and explaining, it will be important to allow students to express their views through the use of native language, where that is appropriate.

Assessment Targets for Claim #1

Cluster headings as assessment targets: In the CCSSM the cluster headings usually serve to communicate the larger intent of a group of standards. For example, a cluster heading in Grade 4 reads: "Generalize understanding of place value for multi-digit numbers." Individual standards in this cluster pinpoint some signs of success in the endeavor, but the important endeavor itself is stated directly in the cluster heading. In addition, the word "generalize" signals that there is a multi-grade progression in grades K-3 leading up to this group of standards. With this in mind, the cluster headings can be viewed as the most effective means of communicating the focus and coherence of the standards. Therefore, this content specifications document *uses the cluster headings as the targets of assessment* for generating evidence for Claim #1. For each cluster, guidance is provided that gives item developers important information about item/task considerations for the cluster. Sample items are also provided that illustrate the content scope and range of difficulty appropriate to assess a cluster. Claim #1 assessment targets are shown below for Grades 3 through 8 and Grade 11. Content emphases for all grades are shown in the tables for Claim 1, which are based on the cluster level of the Common Core State Standards for Mathematics.

Content emphases in the standards: In keeping with the design principles of focus and coherence in the standards as a whole, not all content is emphasized equally in the Standards for Mathematical Content.

- The standards communicate emphases in many ways, including by the use of domain names that vary across the grades, and that are sometimes much more fine-grained than the top-level

organizers in previous state standards (e.g., Ratios and Proportional Relationships). These and other features of the standards and their progressions point to the <u>major</u> work of each grade.[4]

- Standards for topics that are not major emphases in themselves are generally written in such a way as to <u>support</u> and strengthen the areas of major emphases. This promotes valuable connections that add coherence to the grade. Still other topics that may not connect tightly or explicitly to the major work of the grade would fairly be called <u>additional</u>.

In the tables that follow, these designations—"<u>major</u>" and "<u>additional/supporting</u>" — are provided at the cluster level.

Working at the cluster level helps to avoid obscuring the big ideas and getting lost in the details of specific standards (which are individually important, but impossible to measure in their entirety within the bounds of reasonable testing time). Clusters provide an appropriate grain size for following the contours of important progressions in the standards across grades, for example: the integration of place value understanding and the meanings and properties of operations that must happen as students develop computation strategies and algorithms for multi-digit numbers during grades K-6; or the appropriate development of functional thinking in middle school leading to the emergence of functions as a content domain in Grade 8.

Identifying some standards within "major" clusters and others within "additional/supporting" clusters is not to say that anything in the standards can be neglected. To do so would leave gaps in student preparation for later mathematics. In other words, all content is eligible for and should be encompassed in the assessment. However, evidence for Claim #1 will strongly focus on the major clusters and take into account ways in which the standards tie supporting clusters to the major work of each grade, such that the items/tasks seen by every student will sample in much greater proportion from clusters representing the major work of each grade. Appendix A provides a sampling scheme for the CAT engine that reflects the structure of the standards and captures emphases appropriately at each grade.

In what follows, Claim #1 Assessment Targets are provided for grades 3 through 8 and high school.

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[4] Further detail on emphases can be seen in the *Progressions* documents drafted by members of the Common Core State Standards Working Group, and published through the Institute for Mathematics and Education of the University of Arizona: http://ime.math.arizona.edu/progressions/. More information is also available in the *K-8 Publishers' Criteria*, developed by the CCSSM authors, available at www.corestandards.org.

GRADE 3 Summative Assessment Targets
Providing Evidence Supporting Claim #1

Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content for this claim may be drawn from any of the Grade 3 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4. Detailed information about how each Claim 1 assessment target is measured can be found in the Item Specifications "Mathematics Grades 3-5" zip folder available at http://www.smarterbalanced.org/smarter-balanced-assessments/.

Operations and Algebraic Thinking

Target A [m]: Represent and solve problems involving multiplication and division.[5] (DOK 1)

Target B [m]: Understand properties of multiplication and the relationship between multiplication and division. (DOK 1)

Target C [m]: Multiply and divide within 100. (DOK 1)

Target D [m]: Solve problems involving the four operations, and identify and explain patterns in arithmetic. (DOK 2)

Number and Operations—Base Ten

Target E [a/s]: Use place value understanding and properties of arithmetic to perform multi-digit arithmetic. (DOK 1)

Number and Operations—Fractions

Target F [m]: Develop understanding of fractions as numbers. (DOK 1, 2)

Measurement and Data

Target G [m]: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. (DOK 1, 2)

Target H [a/s]: Represent and interpret data. (DOK 2)

Target I [m]: Geometric measurement: understand concepts of area and relate area to multiplication and to addition. (DOK 2)

Target J [a/s]: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. (DOK 1)

Geometry

Target K [a/s]: Reason with shapes and their attributes. (DOK 1, 2)

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[5] See CCSSM, Table 2, p. 89 for additional information.

Grade 4 SUMMATIVE ASSESSMENT TARGETS
Providing Evidence Supporting Claim #1

Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content for this claim may be drawn from any of the Grade 4 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4. Detailed information about how each Claim 1 assessment target is measured can be found in the Item Specifications "Mathematics Grades 3-5" zip folder available at http://www.smarterbalanced.org/smarter-balanced-assessments/.

Operations and Algebraic Thinking (4.OA)

Target A [m]: Use the four operations with whole numbers to solve problems. (DOK 1, 2)

Target B [a/s]: Gain familiarity with factors and multiples. (DOK 1)

Target C [a/s]: Generate and analyze patterns. (DOK 2, 3)

Number and Operations in Base Ten (4.NBT)

Target D [m]: Generalize place value understanding for multi-digit whole numbers. (DOK 1, 2)

Target E [m]: Use place value understanding and properties of operations to perform multi-digit arithmetic. (DOK 1, 2)

Number and Operations – Fractions (4.NF)

Target F [m]: Extend understanding of fraction equivalence and ordering. (DOK 1, 2)

Target G [m]: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers (DOK 1,2)

Target H [m]: Understand decimal notation for fractions, and compare decimal fractions. (DOK 1, 2)

Measurement and Data (4.MD)

Target I [a/s]: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (DOK 1, 2)

Target J [a/s]: Represent and interpret data. (DOK 1, 2)
Target K [a/s]: Geometric measurement: understand concepts of angle and measure angles. (DOK 1, 2)

Geometry (4.G)

Target L [a/s]: Draw and identify lines and angles, and classify shapes by properties of their lines and angles. (DOK 1, 2)

Grade 5 SUMMATIVE ASSESSMENT TARGETS
Providing Evidence Supporting Claim #1

Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content for this claim may be drawn from any of the Grade 5 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4. Detailed information about how each Claim 1 assessment target is measured can be found in the Item Specifications "Mathematics Grades 3-5" zip folder available at http://www.smarterbalanced.org/smarter-balanced-assessments/.

Operations and Algebraic Thinking

Target A [a/s]: Write and interpret numerical expressions. (DOK 1)

Target B [a/s]: Analyze patterns and relationships. (DOK 2)

Number and Operations—Base Ten

Target C [m]: Understand the place value system. (DOK 1, 2)

Target D [m]: Perform operations with multi-digit whole numbers and with decimals to hundredths. (DOK 1, 2)

Number and Operations—Fractions

Target E [m]: Use equivalent fractions as a strategy to add and subtract fractions. (DOK 1, 2)

Target F [m]: Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (DOK 1, 2)

Measurement and Data

Target G [a/s]: Convert like measurement units within a given measurement system. (DOK 1)

Target H [a/s]: Represent and interpret data. (DOK 2)

Target I [m]: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. (DOK 1, 2)

Geometry

Target J [a/s]: Graph points on the coordinate plane to solve real-world and mathematical problems. (DOK 1)

Target K [a/s]: Classify two-dimensional figures into categories based on their properties. (DOK 2)

Grade 6 SUMMATIVE ASSESSMENT TARGETS
Providing Evidence Supporting Claim #1

Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content for this claim may be drawn from any of the Grade 6 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4. Detailed information about how each Claim 1 assessment target is measured can be found in the Item Specifications "Mathematics Grades 6-8" zip folder available at http://www.smarterbalanced.org/smarter-balanced-assessments/.

Ratios and Proportional Relationships (6.RP)

Target A [m]: Understand ratio concepts and use ratio reasoning to solve problems. (DOK 1, 2)

The Number System (6.NS)

Target B [m]: Apply and extend previous understandings of multiplication and division to divide fractions by fractions. (DOK 1, 2)

Target C [a/s]: Compute fluently with multi-digit numbers and find common factors and multiples. (DOK 1, 2)

Target D [m]: Apply and extend previous understandings of numbers to the system of rational numbers. (DOK 1, 2)

Expressions and Equations (6.EE)

Target E [m]: Apply and extend previous understandings of arithmetic to algebraic expressions. (DOK 1, 2)

Target F [m]: Reason about and solve one-variable equations and inequalities. (DOK 1, 2)

Target G [m]: Represent and analyze quantitative relationships between dependent and independent variables. (DOK 2)

Geometry (6.G)

Target H [a/s]: Solve real-world and mathematical problems involving area, surface area, and volume. (DOK 1, 2)

Statistics and Probability (6.SP)

Target I [a/s]: Develop understanding of statistical variability. (DOK 2)

Target J [a/s]: Summarize and describe distributions. (DOK 1, 2)

Grade 7 SUMMATIVE ASSESSMENT TARGETS
Providing Evidence Supporting Claim #1

Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content for this claim may be drawn from any of the Grade 7 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4. Detailed information about how each Claim 1 assessment target is measured can be found in the Item Specifications "Mathematics Grades 6-8" zip folder available at http://www.smarterbalanced.org/smarter-balanced-assessments/.

Ratios and Proportional Relationships (7.RP)

Target A [m]: Analyze proportional relationships and use them to solve real-world and mathematical problems. (DOK 2)

The Number System (7.NS)

Target B [m]: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. (DOK 1, 2)

Expressions and Equations (7.EE)

Target C [m]: Use properties of operations to generate equivalent expressions. (DOK 1, 2)

Target D [m]: Solve real-life and mathematical problems using numerical and algebraic expressions and equations. (DOK 1, 2)

Geometry (7.G)

Target E [a/s]: Draw, construct and describe geometrical figures and describe the relationships between them. (DOK 1, 2)

Target F [a/s]: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. (DOK 1, 2)

Statistics and Probability (7.SP)

Target G [a/s]: Use random sampling to draw inferences about a population. (DOK 1, 2)

Target H [a/s]: Draw informal comparative inferences about two populations. (DOK 2)

Target I [a/s]: Investigate chance processes and develop, use, and evaluate probability models. (DOK 1, 2)

Grade 8 SUMMATIVE ASSESSMENT TARGETS
Providing Evidence Supporting Claim #1

Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content for this claim may be drawn from any of the Grade 8 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4. Detailed information about how each Claim 1 assessment target is measured can be found in the Item Specifications "Mathematics Grades 6-8" zip folder available at http://www.smarterbalanced.org/smarter-balanced-assessments/.

The Number System

Target A [a/s]: Know that there are numbers that are not rational, and approximate them by rational numbers. (DOK 1, 2)

Expressions and Equations

Target B [m]: Work with radicals and integer exponents. (DOK 1)

Target C [m] Understand the connections between proportional relationships, lines, and linear equations. (DOK 1, 2)

Target D [m]: Analyze and solve linear equations and pairs of simultaneous linear equations. (DOK 1, 2)

Functions

Target E [m]: Define, evaluate, and compare functions. (DOK 1, 2)

Target F [m]: Use functions to model relationships between quantities. (DOK 1, 2)

Geometry

Target G [m]: Understand congruence and similarity using physical models, transparencies, or geometry software. (DOK 1, 2)

Target H [m]: Understand and apply the Pythagorean theorem. (DOK 2)

Target I [a/s]: Solve real-world and mathematical problems involving volume of cylinders, cones and spheres. (DOK 2)

Statistics and Probability

Target J [a/s]: Investigate patterns of association in bivariate data. (DOK 1, 2)

Grade 11 SUMMATIVE ASSESSMENT TARGETS **Providing Evidence Supporting Claim #1**
Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Content for this claim may be drawn from any of the high school clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4. Detailed information about how each Claim 1 assessment target is measured can be found in the Item Specifications "Mathematics High School" zip folder available at http://www.smarterbalanced.org/smarter-balanced-assessments/.

<div align="center">Number and Quantity (9-12.N)</div>

Target A [a/s]: Extend the properties of exponents to rational exponents. (DOK 1, 2)

Target B [a/s]: Use properties of rational and irrational numbers. (DOK 1, 2)

Target C [m]: Reason quantitatively and use units to solve problems. (DOK 1, 2)

<div align="center">Algebra (9-12.A)</div>

Target D [m]: Interpret the structure of expressions. (DOK 1)

Target E [m]: Write expressions in equivalent forms to solve problems. (DOK 1, 2)

Target F [a/s]: Perform arithmetic operations on polynomials. (DOK 1)

Target G [a/s]: Create equations that describe numbers or relationships. (DOK 1, 2)

Target H [m]: Understand solving equations as a process of reasoning and explain the reasoning. (DOK 1, 2)

Target I [m]: Solve equations and inequalities in one variable. (DOK 1, 2)

Target J [m]: Represent and solve equations and inequalities graphically. (DOK 1, 2)

<div align="center">Functions (9-12.F)</div>

Target K [m]: Understand the concept of a function and use function notation. (DOK 1)

Target L [m]: Interpret functions that arise in applications in terms of a context. (DOK 1, 2)

Target M [m]: Analyze functions using different representations. (DOK 1, 2, 3)

Target N [m]: Build a function that models a relationship between two quantities. (DOK 1, 2)

<div align="center">Geometry (9-12.G)</div>

Target O: Define trigonometric ratios and solve problems involving right triangles (DOK 1, 2)

<div align="center">Statistics and Probability (9-12.SP)</div>

Target P [m]: Summarize, represent and interpret data on a single count or measurement variable. (DOK 2)

Notes on Grades 9-12 Content Clusters Not Identified as Assessment Targets for Claim 1

Algebra

Content from the remaining Algebra clusters will also provide content and context for tasks in Claims 2-4, though these will be sampled in lesser proportion than those explicitly listed as targets for Claim 1. Clusters not explicitly identified as targets for Claim 1 are the following:

- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions
- Solve systems of equations*

*Content from this cluster may be sampled in greater proportion due to its interconnectivity to some of the targets listed under Claim 1.

Functions

Content from the remaining Functions clusters will also provide content and context for tasks in Claims 2-4, though these will be sampled in lesser proportion than those explicitly listed as targets for Claim 1. Clusters not explicitly identified as targets for Claim 1 are the following:

- Build new functions from existing functions
- Construct and compare linear, quadratic, and exponential models and solve problems*
- Interpret expressions for functions in terms of the situation they model*
- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

*Content from these clusters may be sampled in greater proportion due to its interconnectivity to some of the targets listed under Claim 1.

Geometry

While only one content cluster from the Geometry domain[6] is highlighted for task development under Claim 1, the remaining clusters will be used to build tasks for Claims 2-4. In general, the clusters listed below provide natural and productive opportunities to connect the work of algebra, functions and geometry in the context of problems for Claims 2-4:

- Use coordinates to prove simple geometric theorems algebraically
- Explain volume formulas and use them to solve problems
- Apply geometric concepts in modeling situations

Content from the remaining Geometry clusters will also provide content and context for tasks in Claims 2-4, though these will be sampled in lesser proportion than those listed above and that explicitly listed as a target for Claim 1.

- Experiment with transformations in the plane

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[6] The phrase "Conceptual Category" is used in place of domain in the CCSS document. "Domain" is used here to maintain consistency with Grades 3-8 for the purposes of task development and item tagging.

- Understand congruence in terms of rigid motions
- Make geometric constructions
- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Prove geometric theorems
- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles
- Translate between the geometric description and the equation for a conic section
- Visualize relationships between two-dimensional and three-dimensional objects

Statistics and Probability

While only one content cluster from the Statistics and Probability domain[7] is highlighted for task development under Claim 1, the remaining clusters will be used to build tasks for Claims 2-4. In general, the clusters listed below provide productive opportunities to connect the work of algebra, functions and statistics and probability in the context of problems for Claims 2-4:

- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Content from the remaining Statistics and Probability clusters will also provide content and context for tasks in Claims 2-4, though these will be sampled in lesser proportion than those listed above and that explicitly listed as a target for Claim 1.

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies
- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

Understanding Assessment Targets in an Adaptive Framework: In building an adaptive test, it is essential to understand how content gets "adapted." In a computer adaptive summative assessment, it doesn't make much sense to repeatedly offer formulaic multiplication and division items to a highly fluent Grade 3 student, making the Grade 3 Target OA.C [m] less relevant for this student than it may be for another. The higher-achieving student could be challenged further, while a student who is struggling could be given less complex items to ascertain how much each understands within the domain. The table below illustrates several items for the Grade 3 Operations and Algebraic Thinking domain that would likely span the difficulty spectrum for this grade. The items generally get more difficult with each row (an important feature of adaptive test item banks). (Pilot data will be used to determine more precisely the levels of difficulty associated with each kind of task.)

Sample for Grade 3, Claim #1 – Operations and Algebraic Thinking

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[7] The phrase "Conceptual Category" is used in place of domain in the CCSS document. "Domain" is used here to maintain consistency with Grades 3-8 for the purposes of task development and item tagging.

Adapting Items within a Claim & Domain	Claim #1 – Operations and Algebraic Thinking
8 x 5 = □	Target C [m]: Multiply and divide within 100.
6 x □ = 30	Target A [m]: Represent and solve problems involving multiplication and division.
9 x 4 = □ x 9	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.
6 x 2 x □ = 60	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.
4 x 2 x □ = 5 x 2 x 2 x 2	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.
9 x 4 = 4 x □ x □ (May appear as a drag and drop TE item where "1" is not one of the choices for dragging.)	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.
8 x □ = 4 x □ Give two different pairs of numbers that could fill the boxes to make a true equation (selected response, drag and drop, or fill-in would work).	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.

Some of the more difficult items in the table incorporate several elements of this potential Grade 3 progression (fluency with multiplication → understanding the "unknown whole number" in a multiplication problem → applying properties of operations). Thus, a student who is consistently successful with items like the one in the final rows would not necessarily be assessed on items in previous rows within an adaptive test. In this way adaptive testing has the benefit of reduced test length while providing coverage of a broad scope of knowledge and skills. Adapting to greater and lesser difficulty levels than those illustrated in the table may require the use of items from other grades.

The relative impact of a student's ability or inability to "multiply and divide within 100" (Target C) would likely affect his/her performance on other clusters in the domain of Operations and Algebraic Thinking, thus serving as a baseline for much of the other content in this domain.

The sample items in the table illustrate another point – that the cluster level of the CCSS provides a suitable grain size for the development of a well-supplied item bank for computer adaptive testing. Item quality should not be compromised, particularly in an adaptive framework, by unnecessarily writing items at too fine a grain size. Since efficiency and appropriate item selection are optimized by

minimizing constraints on the adaptive test (Thompson & Weiss, 2011), it is critical to ensure that items provide an appropriate range of difficulty within each domain for Claim #1.

Again, CAT sampling proportions for Claim 1 are given in Appendix A.

Mathematics Claim #2
PROBLEM SOLVING

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

Rationale for Claim #2

Assessment items and tasks focused on this claim include well-posed problems in pure mathematics and problems set in context. *Problems* are presented as items and tasks that are well posed (that is, problem formulation is not necessary) and for which a solution path is not immediately obvious.[8] These problems require students to construct their own solution pathway, rather than to follow a provided one. Such problems will therefore be less structured than items and tasks presented under Claim #1, and will require students to select appropriate conceptual and physical tools to use.

At the heart of doing mathematics is making sense of problems and persevering in solving them[9]. This claim addresses the core of mathematical expertise – the set of competences that students can use when they are confronted with challenging tasks.

"Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary." (Practice 1, CCSSM)

Problem solving, which, of course, builds on a foundation of knowledge and procedural proficiency, sits at the core of *doing* mathematics. Proficiency at problem solving requires students to choose to use concepts and procedures from across the content domains and check their work using alternative methods. As problem-solving skills develop, student understanding of and access to mathematical concepts becomes more deeply established.

For example, "older students might, depending on the context of the problem, transform algebraic

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[8] Schoenfeld, A. H. (1985). *Mathematical problem solving.* Orlando, FL: Academic Press.
[9] See, e.g., Halmos, P. (1980). The heart of mathematics. *American Mathematical Monthly*, 87, 519-524

expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can approach and solve a problem by drawing upon different mathematical characteristics, such as: correspondences among equations, verbal descriptions of mathematical properties, tables graphs and diagrams of important features and relationships, graphical representations of data, and regularity or irregularity of trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches." (Practice 1, CCSSM)

Development of the capacity to solve problems also corresponds to the development of important meta-cognitive skills such as oversight of a problem-solving process while attending to the details. Mathematically proficient students continually evaluate the reasonableness of their intermediate results, and can step back for an overview and shift perspective. (Practice 7, Practice 8, CCSM)

Problem solving also requires students to identify and select the tools that are necessary to apply to the problem. The development of this capacity – to frame a problem in terms of the steps that need to be completed and to review the appropriateness of various tools that are available – are critical to further learning in mathematics, and generalize to real-life situations. This includes both mathematical tools and physical ones:

"Tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge." (Practice 5, CCSSM)

What sufficient evidence looks like for Claim #2

Although items and tasks designed to provide evidence for this claim must primarily assess the student's ability to identify the problem and to arrive at an acceptable solution, mathematical problems nevertheless require students to apply mathematical concepts and procedures. Thus, though the primary purpose of items/tasks associated with this claim is to assess problem-solving skills, these items/tasks might also contribute to evidence that is gathered for Claim #1.

Properties of items/tasks that assess this claim: The assessment of many relatively discrete and/or single-step problems can be accomplished using short constructed response items, or even computer-enhanced or selected response items.

Additionally, more extensive constructed response items can effectively assess multi-stage problem solving and can also indicate unique and elegant strategies used by some students to solve a given problem, and can illuminate flaws in student's approach to solving a problem. These tasks could:

- Present non-routine[10] problems where a substantial part of the challenge is in deciding what to do, and which mathematical tools to use; and
- Involve chains of autonomous[11] reasoning, in which some tasks may take a successful student 5 to 10 minutes, depending on the age of student and complexity of the task.

A distinctive feature of both single-step and multi-step items and tasks for Claim #2 is that they are "well-posed". That is, whether the problem deals with pure or applied contexts, the problem itself is completely formulated; the challenge is in identifying or using an appropriate solution path. Two examples of well-posed problems are provided below, following the Assessment Targets for Claim #2.

Because problems like these might be new to many students, especially on a state-level assessment, it will be worthwhile to explore developing scaffolded supports within the assessment to facilitate entry and assess student progress towards expertise. The degree of scaffolding for individual students could be determined as part of the adaptability of the computer-administered test. Even for such "scaffolded tasks," part of the task will involve a chain of autonomous reasoning. Additionally, because some multi-stage problem-solving tasks might present significant cognitive complexity, consideration should be given to framing more complex problem solving tasks with mathematical concepts and procedures that have been mastered in an earlier grade.

Problems in pure mathematics: These are well-posed problems within mathematics where the student must find an approach, choose which mathematical tools to use, carry the solution through, and explain the results.

Design problems: These problems have much the same properties but within a design context from the real, or a fantasy, world. See, for example, "sports bag" from the assessment sampler.

Planning problems: Planning problems involve the coordinated analysis of time, space, cost – and people. They are design tasks with a time dimension added. Well-posed problems of this kind assess the student's ability to make the connections needed between different parts of mathematics.

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[10] As noted earlier, by "non-routine" we mean that the student will not have been taught a closely similar problem, so will not be expected to *remember* a solution path but will have to *adapt* or *extend* their earlier knowledge to find one.
[11] By "autonomous" we mean that the student responds to a single prompt, without further guidance within the task.

This is not a complete list; other types of tasks that fit the criteria above may well be included. But a balanced mixture of these types will provide enough evidence for Claim #2, as well as contributing evidence with regard to Claim #1. Illustrative examples of each type are shown in the item and task specifications as well as in the publicly available practice tests available online.

Scoring rubrics for extended response items and tasks should be consistent with the expectations of this claim, giving substantial credit to the choice of appropriate methods of tackling the problem, to reliable skills in carrying it through, and to explanations of what has been found. Scoring for Claims 2, 3, & 4 is anchored to the general [rubrics](#).

Accessibility and Claim #2: This claim about mathematical problem solving focuses on the student's ability to make sense of problems, construct pathways to solving them, persevering in solving them, and the selection and use of appropriate tools. This claim includes student use of appropriate tools for solving mathematical problems, which for some students may extend to tools that provide full access to the item or task and to the development of reasonable solutions. For example, students who are blind and use Braille or assistive technology such as text readers to access written materials, may demonstrate their modeling of physical objects with geometric shapes using alternate formats. Students who have physical disabilities that preclude movement of arms and hands should not be precluded from demonstrating with assistive technology their use of tools for constructing shapes. As with Claim #1, access via text to speech and expression via scribe, computer, or speech to text technology will be important avenues for enabling many students with disabilities to show what they know and can do in relation to framing and solving complex mathematical problems.

With respect to English learners, the expectation for verbal explanations of problems will be more achievable if formative materials and interim assessments provide illustrative examples of the communication required for this claim, so that ELL students have a better understanding of what they are required to do. In addition, formative tools can help teachers teach ELL students ways to communicate their ideas through simple language structures in different language modalities such as speaking and writing. Finally, attention to English proficiency in shaping the delivery of items (e.g. native language or linguistically modified, where appropriate) and the expectations for scoring will be important.

Assessment Targets for Claim #2

Claim #2 is aligned to the mathematical practices from the MCCSS. For this reason, the Assessment Targets are all *acts of problem solving* that are consistent across grades and also evolve across grades. Consistent with the above discussion, these acts of problem solving are also tied to content (CCSSM, p. 8).

SUMMATIVE ASSESSMENT TARGETS
Providing Evidence Supporting Claim #2
Claim #2: Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.
To preserve the focus and coherence of the standards as a whole, tasks must draw clearly on knowledge and skills that are articulated in the content standards. At each grade level, the content standards offer natural and productive settings for generating evidence for Claim #2. These connections are specified below. Tasks generating evidence for Claim #2 in a given grade will draw upon knowledge and skills articulated in the progression of standards up through that grade, though more complex problem-solving tasks may draw upon knowledge and skills from lower grade levels. Any given task will provide evidence for several of the following assessment targets. Each of the following targets should not lead to a separate task: it is in *using* content from different areas, including work studied in earlier grades, that students demonstrate their problem-solving proficiency.
Content clusters and domains recommended for the majority of Claim 2 item development are given below. Tasks can center on a single cluster or standard listed, or synthesize across listed clusters or standards.

Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	HS
3.OA.A	4.OA.A	5.NBT.B	6.RP.A	7.RP.A	8.EE.B	N-Q.A
3.OA.D	4.NBT.B	5.NF.A	6.NS.A	7.NS.A	8.EE.C	A-SSE.A
3.NBT.A*	4.NF.A	5.NF.B	6.NS.C	7.EE.A	8.F.A	A-SSE.B
3.MD.A	4.NF.B	5.MD.A*	6.EE.A	7.EE.B	8.F.B*	A-CED.A
3.MD.B*	4.NF.C	5.MD.C	6.EE.B	7.G.A*	8.G.A	A-REI.2
3.MD.C	4.MD.A*	5.G.A*	6.EE.C	7.G.B*	8.G.B	A-REI.B
3.MD.D*	4.MD.C*		6.G.A*		8.G.C*	A-REI.C
						A-REI.D
						F-IF.A
						F-IF.B
						F-IF.C
						F-BF.A
						G-SRT.C
						S-ID.C
						S-CP.A

* Denotes additional and supporting clusters

Target A: Apply mathematics to solve well-posed problems in pure mathematics and those arising in everyday life, society, and the workplace. (DOK 2, 3)

Target B: Select and use appropriate tools strategically. (DOK 1, 2)

Target C: Interpret results in the context of a situation. (DOK 2)

Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)

Example of a short answer task for Claim #2

"Toys for Charity" **(First-year Algebra)**

Phil and Cathy want to raise money for charity. They decide to make and sell wooden toys. They could make them in two sizes: small and large.

Phil will carve them from wood. A small toy takes 2 hours to carve and a large toy takes 3 hours to carve. Phil only has a total of 24 hours available for carving.

Cath will decorate them. She only has time to decorate 10 toys.

The small toy will make $8 for charity.
The large toy will make $10 for charity.

They want to make as much money for charity as they can.

How many small and large toys should they make?

How much money will they then make for charity?

For the above example, scaffolding could prompt the student to think about questions like:

1.   If they were to make only small toys, how much money would they make for charity?
2.   If they were to make 2 small toys, how many large ones could they also make?

Example of an extended response task for Claim #2

Making a Water Tank **(Grade 11 – students provided graphing calculator as a tool)**

A square metal sheet (6 feet x 6 feet) is to be made into an open-topped water tank by cutting squares from the four corners of the sheet, and bending the four remaining rectangular pieces up, to form the sides of the tank. These edges will then be welded together.

$\longleftarrow$ 6 ft $\longrightarrow$

6 ft
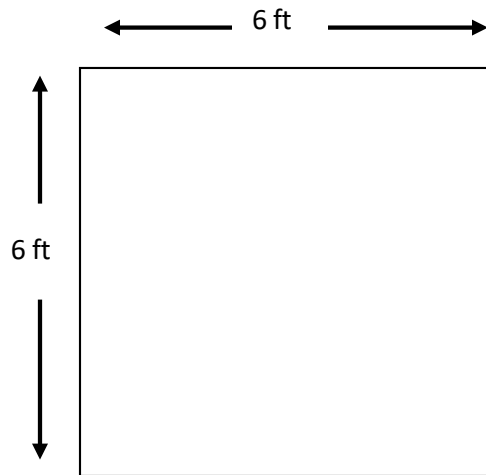
A. How will the final volume of the tank depend upon the size of the squares cut from the corners?

    Describe your answer by:

      i) Sketching a rough graph

      ii) explaining the shape of your graph in words

      iii) writing an algebraic formula for the volume

B. How large should the four corners be cut, so that the resulting volume of the tank is as large as possible?

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Rationale for Claim #3

This claim refers to a recurring theme in the CCSSM content and practice standards: the ability to construct and present a clear, logical, convincing argument. For older students this may take the form of a rigorous deductive proof based on clearly stated axioms. For younger students this will involve more informal justifications. Assessment tasks that address this claim will typically present a claim or a proposed solution to a problem and will ask students to provide, for example, a justification, and explanation, or counter-example.

Rigor in reasoning is about the precision and logical progression of an argument: first avoiding making false statements, then saying more precisely what one assumes, and providing the sequence of deductions one makes on this basis. Assessments for this claim should use tasks that examine a student's ability to analyze a provided explanation, to identify flaws, to present a logical sequence, and to arrive at a correct argument.

> "Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and— if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments." (Practice 3, CCSSM)

Items and tasks supporting this claim should also assess a student's proficiency in using concepts and definitions in their explanations:

"Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions." (Practice 6, CCSSM)

What sufficient evidence looks like for Claim #3

Assessment of this claim can be accomplished with a variety of item/task types, including selected response and short constructed response items, and with extended constructed response tasks. Sufficient evidence would be unlikely to be produced if students were not expected to produce communications about their own reasoning and the reasoning of others. That said, students are likely to be unfamiliar with assessment tasks asking them to explain their reasoning. In order to develop items/tasks that capture student reasoning, it will be important for early piloting and cognitive labs to explore and understand how students express their explanations of reasoning. As students (and teachers) become more familiar with the expectations of the assessment, and as instruction in the Common Core takes hold, students will become more and more successful on tasks aligned to Claim #3 with increasing frequency.

Items and tasks aligned to this claim should reflect the values set out for this claim, giving substantial weight to the quality and precision of the reasoning reflected in at least one, or several of the manners listed below. Options for selected response items and scoring guides for constructed response tasks should be developed to provide credit for demonstration of reasoning and to capture and identify flaws in student logic or reasoning. Features of options and scoring guides include:

- Assuring an explanation of the assumptions made;
- Asking for or recognizing the construction of conjectures that appear plausible, where appropriate;
- Having the student construct examples (or asking the student to distinguish among appropriate and inappropriate examples) in order to evaluate the proposition or conjecture;
- Requiring the student to describe or identify flaws or gaps in an argument;
- Evaluating the clarity and precision with which the student constructs a logical sequence of steps to show how the assumptions lead to the acceptance or refutation of a proposition or conjecture;
- Rating the precision with which the student describes the domain of validity of the proposition or conjecture.

As noted above, communicating mathematical reasoning is not just a requirement of the Standards for Mathematical Practice—it is also a recurrent theme in the Standards for Mathematical Content. For example, many content standards call for students to explain, justify, or illustrate. Below is content standard 4.NBT.5—note the highlighted words:

> Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. ==Illustrate and explain== the calculation by using equations, rectangular arrays, and/or area models.

The Smarter/Balanced assessments will attend thoroughly to those places in the content standards that call explicitly for communicating mathematical reasoning. This is important so that the system captures the Standards' evident design for "doing content differently" at these important junctures. Students are not asked to "Reason" in the abstract—rather, they are asked to reason about the central ideas in mathematics that they are studying. This is an important element of making mathematics education coherent for students. Clearly, the reasoning elements of the content standards cannot be thoroughly assessed under Claim #1 alone. Therefore, in order to measure the full range of the Standards, Claim #3 tasks must be used to assess those parts of the content standards that call for communicating reasoning. In practice, this implies that the large majority of Claim #3 tasks, at least 70%, will be written at small grain size, keyed primarily to a single content standard or part thereof which concerns communicating mathematical reasoning. Targeted content standards for Claim #3 will always belong to the major work of the grade (as in the 4.NBT.5 example shown above). These features justify the weight of Claim #3 in the summative score even as they ensure that Claim #3 actively promotes both the focus and coherence of the Standards.

Occasionally, Claim #3 items/tasks may involve the application of concepts and procedures across more than one content domain. Because of the high strategic demand that such substantial non-routine tasks present, the technical demand for these items/tasks will be lower – typically met by content first taught in earlier grades, consistent with the emphases described under Claim #1.

Accessibility and Claim #3: Successful performance under Claim #3 requires a high level of linguistic proficiency. Many students with disabilities have difficulty with written expression, whether via putting pencil to paper or fingers to computer. The claim does not suggest that correct spelling or punctuation is a critical part of the construction of a viable argument, nor does it suggest that the argument has to be in words. Thus, for those students whose disabilities create barriers to development of text for demonstrating reasoning and formation of an argument, it is appropriate to model an argument via symbols, geometric shapes, or calculator or computer graphic programs. As for Claims #1 and #2, access via text to speech and expression via scribe, computer, or speech to text technology will be important avenues for enabling many students with disabilities to construct viable arguments.

The extensive communication skills anticipated by this claim may also be challenging for many ELL students who nonetheless have mastered the content. Thus it will be important to provide multiple opportunities to ELL students for explaining their ideas through different methods and at different levels of linguistic complexity. Based on the data on ELL students' level of proficiency in L1 and L2, it will be useful to provide opportunities as appropriate for bilingual explanations of the outcomes. Furthermore, students' engagement in critique and debate should not be limited to oral or written words, but can be demonstrated through diagrams, tables, and structured mathematical responses where students provide examples or counter-examples of additional problems.

Assessment Targets for Claim #3

Claim #3 is aligned to the mathematical practices from the MCCSS. For this reason, the Assessment Targets are all *acts of reasoning* that are consistent across grades and also evolve across grades. Consistent with the above discussion, these acts of reasoning are also tied to content (CCSSM, p. 8).

SUMMATIVE ASSESSMENT TARGETS **Providing Evidence Supporting Claim #3**
Claim #3: Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
To preserve the focus and coherence of the standards as a whole, tasks must draw clearly on knowledge and skills that are articulated in the content standards. At each grade level, the content standards offer natural and productive settings for generating evidence for Claim #3. Tasks generating evidence for Claim #3 in a given grade will draw upon knowledge and skills articulated in the standards in that same grade, with strong emphasis on the major work of the grade. Any given task will provide evidence for several of the following assessment targets; each of the following targets should not lead to a separate task.

Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	High School	
3.OA.B	4.OA.3	5.NBT.2	6.RP.A	7.RP.2	8.EE.1	N-RN.A	G-CO.A
3.NF.A	4.NBT.A	5.NBT.6	6.RP.3	7.NS.A	8.EE.5	N-RN.B	G-CO.B
3.NF.1	4.NBT.5	5.NBT.7	6.NS.A	7.NS.1	8.EE.6	N-RN.3	G-CO.C
3.NF.2	4.NBT.6	5.NF.1	6.NS.1	7.NS.2	8.EE.7a	A-SSE.2	G-CO.9
3.NF.3	4.NF.A	5.NF.2	6.NS.C	7.EE.1	8.EE.7b	A-APR.1	G-CO.10
3.MD.A	4.NF.1	5.NF.B	6.NS.5	7.EE.2	8.EE.8a	A-APR.B	G-CO.11
3.MD.7	4.NF.2	5.NF.3	6.NS.6		8.F.1	A-APR.4	G.SRT.A
	4.NF.3a	5.NF.4	6.NS.7		8.F.2	A-APR.6	G.SRT.B
	4.NF.3b	5.NF.7a	6.EE.A		8.F.3	A-REI.A	F-TF.1
	4.NF.3c	5.NF.7b	6.EE.3		8.G.1	A-REI.1	F-TF.2
	4.NF.4a	5.MD.C	6.EE.4		8.G.2	A-REI.2	F-TF.8
	4.NF.4b	5.MD.5a	6.EE.B		8.G.4	A-REI.C	
	4.NF.C	5.MD.5b	6.EE.6		8.G.5	A-REI.10	
	4.NF.7	5.G.B*	6.EE.9		8.G.6	A-REI.11	
		5.G.4*			8.G.8	F-IF.1	
						F-IF.5	
						F-IF.9	
						F-BF.3	
						F-BF.4a	

*Denotes additional and supporting clusters

Target A: Test propositions or conjectures with specific examples. (DOK 2)

Target B: Construct, autonomously,[12] chains of reasoning that will justify or refute propositions or conjectures. (DOK 3, 4).[13]

Target C: State logical assumptions being used. (DOK 2, 3)

Target D: Use the technique of breaking an argument into cases. (DOK 2, 3)

Target E: Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is. (DOK 2, 3, 4)

Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (DOK 2, 3)

Target G: At later grades, determine conditions under which an argument does and does not apply. (For example, area increases with perimeter for squares, but not for all plane figures.) (DOK 3, 4)

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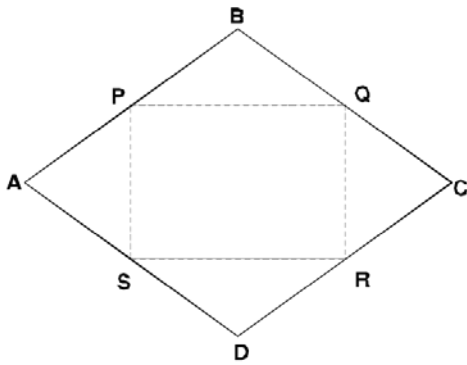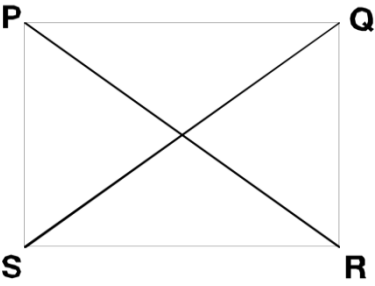
[12] By "autonomous" we mean that the student responds to a single prompt, without further guidance within the task.
[13] At the secondary level, these chains may take a successful student 10 minutes to construct and explain. Times will be somewhat shorter for younger students, but still giving them time to think and explain. For a minority of these tasks, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such "apprentice tasks" part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.
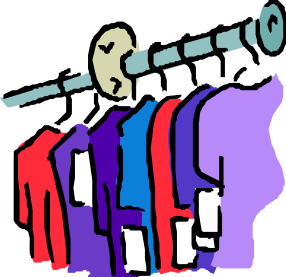
Types of Extended Response Tasks for Claim #3

Proof and justification tasks: These begin with a proposition and the task is to provide a reasoned argument why the proposition is or is not true. In other tasks, students may be asked to characterize the domain for which the proposition is true (see Assessment Target G).

Example of a standard proof task

Math – Grade 11	Item Type: CR	DOK: (Webb 1- 4) 3

Domain(s): Geometry

Content Cluster(s) and/or Standard(s):

G.CO **Prove** geometric theorems

G.CO.11 **Prove** theorems about parallelograms.

Claim #3 Assessment Targets

Target B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.

Target C: State logical assumptions being used.

Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.

The Envelope

Unfolded envelope   **Folded envelope**

Prove that when the rectangular envelope (PQRS) is unfolded, the shape obtained (ABCD) is a rhombus.

Critiquing tasks: Some flawed 'student' reasoning is presented and the task is to correct and improve it.

Math – Grade 7	Item Type: CR	DOK: (Webb 1- 4) 3
Domain(s): Ratios and Proportional Relationships **Content Cluster(s) and/or Standard(s)** 7.RP **Analyze** proportional relationships and use them to **solve** real-world and mathematical problems. 7.RP.3 Use proportional relationships to **solve** multistep ratio and percent problems.		
Claim #3 Assessment Targets Target A: Test propositions or conjectures with specific examples. Target B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Target D: Use the technique of breaking an argument into cases. Target E: Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument, explain what it is.		
Sale prices Max bought 2 items in a sale. One item was 10% off. One item was 20% off. Max says he saved 15% altogether. Is he right? Explain.		

Mathematical investigations: Students are presented with a phenomenon and are invited to formulate conjectures about it. They are then asked to go on and prove one of their conjectures. This kind of task benefits from a longer time scale, and might best be incorporated into items/tasks associated with the Performance Tasks that afford a longer period of time for students to complete their work.

Sums of Consecutive Numbers

Many whole numbers can be expressed as the sum of two or more positive <u>consecutive</u> whole numbers, some of them in more than one way.

For example, the number 5 can be written as

$5 = 2 + 3$

and that's the only way it can be written as a sum of consecutive whole numbers.

In contrast, the number 15 can be written as the sum of consecutive whole numbers in three different ways:

$15 = 7 + 8$

$15 = 4 + 5 + 6$

$15 = 1 + 2 + 3 + 4 + 5$

Now look at other numbers and find out all you can about writing them as sums of consecutive whole numbers.

Write an account of your investigation. If you find any patterns in your results, be sure to point them out, and also try to explain them fully.

This is not a complete list; other types of task that fit the criteria above may well be included. But a balanced mixture of these types will provide enough evidence for Claim #3.

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Mathematics Claim #4
MODELING AND DATA ANALYSIS

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Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Rationale for Claim #4

> "Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decision-making." (p.72, CCSSM)

As such, modeling is the bridge across the "school math"/"real world" divide that has been missing from many mathematics curricula and assessments[14]. It is the twin of *mathematical literacy*, the focus of the PISA international comparison tests in mathematics. CCSSM features modeling as both a mathematical practice at all grades and a content focus in high school.

> "Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose." (Practice 4; CCSSM)

In the real world, problems do not come neatly 'packaged'. Real world problems are complex, and often contain insufficient or superfluous data. Assessment tasks will involve *formulating* a problem that is tractable using mathematics - that is, formulating a model. This will usually involve making assumptions and simplifications. Students will need to select from the data at hand, or estimate data that are missing. (Such tasks are therefore distinct from the problem-solving tasks described in Claim #2, that are well-formulated). Students will identify variables in a situation, and construct relationships between these. When students have formulated the problem, they then tackle it, often in a decontextualized form, before

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[14] In their everyday life and work, most adults use none of the mathematics they are first taught after age 11. They often do not see the mathematics that they do use (in planning, personal accounting, design, thinking about political issues etc.) as mathematics.

interpreting their results and checking them for reasonableness.

> "Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects." (Practice 2; CCSSM)

Finally, students interpret, validate, and report their solutions through the successive phases of the modeling cycle, illustrated in the following diagram from CCSSM.
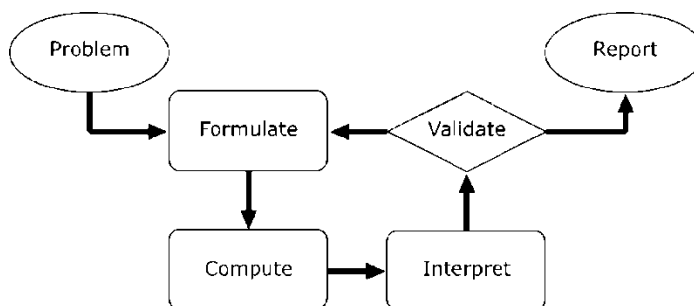
Assessment tasks will also test whether students are able to use technology in this process.

> "When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts." (Practice 5; CCSSM)

What sufficient evidence looks like for Claim #4

A key feature of items and tasks in Claim #4 is that the student is confronted with a contextualized, or "real world" situation and must decide which information is relevant and how to represent it. As some of the examples provided below illustrate, "real world" situations do not necessarily mean questions that a student might really face; it means that mathematical problems are embedded in a practical, application context. In this way, items and tasks in Claim #4 differ from those in Claim #2, because while the goal is clear, the problems themselves are not yet fully formulated (well-posed) in mathematical terms.

Items/tasks in Claim #4 assess student expertise in choosing appropriate content and using it effectively in formulating models of the situations presented and making appropriate inferences from them. Claim #4 items and tasks should sample across the content domains, with many of these involving more than one domain. Items and tasks of this sort require students to apply mathematical concepts at a significantly deeper level of understanding of mathematical content than is expected by Claim #1. Because of the high strategic demand that substantial non-routine tasks present, the technical demand will be lower – normally met by content first taught in earlier grades, consistent with the emphases described under Claim #1. Although most situations faced by students will be embedded in longer performance tasks, within those tasks, some selected-response and short constructed-response items will be appropriate to use.

Modeling and data analysis in the Common Core State Standards trace a visible arc of growing prominence across the grades, showing low prominence in grades K-5, higher prominence in grades 6-8 (which is when the Statistics and Probability domain first appears), and highest prominence in High School (which is when Modeling appears as a content category with the full modeling cycle). Therefore to align to the Standards, Claim #4 will be more important on the assessment in high school, less important in grades 6-8, and the least important in grades 3-5. Again, to align to the Standards, Claim #4 tasks will be most sophisticated and complete in high school (cf. the modeling cycle in CCSSM pp. 72, 73), less sophisticated/more tied to specific content in middle school, and least sophisticated/most tied to specific content in grades 3-5.

Accessibility and Claim #4: Many students with disabilities can analyze and create increasingly complex models of real world phenomena but have difficulty communicating their knowledge and skills in these areas. Successful adults with disabilities rely on alternative ways to express their knowledge and skills, including the use of assistive technology to construct shapes or develop explanations via speech to text. Others rely on calculators, physical objects, or tools for constructing shapes to work through their analysis and reasoning process.

For English learners, it will be important to recognize ELL students' linguistic background and level of proficiency in English in assigning tasks and to allow explanations that include diagrams, tables, graphic representations, and other mathematical representations in addition to text. It will also be important to include in the scoring process a discussion of ways to resolve issues concerning linguistic and cultural factors when interpreting responses.

Assessment Targets for Claim #4

Claim #4 is aligned to the mathematical practices from the MCCSS. For this reason, the Assessment Targets are all *acts of modeling* that are consistent across grades and also evolve across grades. Consistent with the above discussion, these acts of modeling are tied to content (CCSSM, p. 8).

	SUMMATIVE ASSESSMENT TARGETS	
	Providing Evidence Supporting Claim #4	

Claim #4 - Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

To preserve the focus and coherence of the standards as a whole, tasks must draw clearly on knowledge and skills that are articulated in the content standards. At each grade level, the content standards offer natural and productive settings for generating evidence for Claim #4. Tasks generating evidence for Claim #4 in a given grade will draw upon knowledge and skills articulated in the progression of standards up to that grade.

Content clusters and domains recommended for the majority of Claim 4 item development are given below. Tasks can center on a single cluster or standard listed, or synthesize across listed clusters or standards.

Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	HS
3.OA.A	4.OA.A	5.NBT.B	6.RP.A	7.RP.A	8.EE.3	N-Q.A
3.OA.D	4.NF.B	5.NF.A	6.NS.A	7.NS.A	8.EE.4	A-SSE.B
3.MD.A	4.MD.A*	5.NF.B	6.NS.C	7.EE.B	8.EE.B	A-CED.A
3.MD.C	4.MD.B*	5.MD.A*	6.EE.B	7.G.A*	8.EE.C	A-REI.A
3.MD.D*	4.MD.C*	5.MD.B*	6.EE.C	7.G.B*	8.F.B*	A-REI.B
		5.MD.C	6.G.A*	7.SP.A*	8.G.B	A-REI.C
		5.G.A*	6.SP.A*	7.SP.B*	8.G.C*	F-IF.B
			6.SP.B*	7.SP.C*	8.SP.A*	F-IF.C
						F-BF.A
						S-ID.A
						S-ID.B
						S-IC.1
						S-IC.B
						F-LE.A
						F-LE.B
						F-TF.5
						G-GMD.3
						G-MG

*Denotes additional and supporting clusters

REMINDER: Claim 4 tasks may also ask students to apply content from prior grades in sophisticated applications.

Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. (DOK 2, 3)

Target B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. (DOK 2, 3, 4).[15]

**Target C: State logical assumptions being used. (DOK 1, 2)**

Target D: Interpret results in the context of a situation. (DOK 2, 3)

Target E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. (DOK 3, 4)

Target F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)

Target G: Identify, analyze and synthesize relevant external resources to pose or solve problems. (DOK 3, 4)

Design a Tent (Grade 8)

Pole

These ends should zip together at night
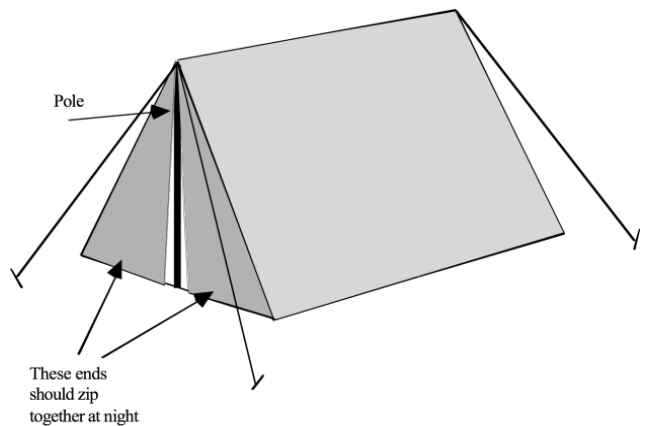
Your task is to design a 2-person tent like the one in the picture.

Your design must satisfy these conditions:

• It must be big enough for someone to move around in while kneeling down, and big enough for all their stuff.
• The bottom of the tent will be made from a thick rectangle of plastic.
• The sloping sides and the two ends will be made from a single, large sheet of material.
• Two vertical tent poles will hold the whole tent up.
Make drawings to show how you will cut the plastic and the material.

Make sure you show the measures of all relevant lengths and angles clearly on your drawings, and explain why you have made the choices you have made.

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[15] At the secondary level, these chains should typically take a successful student 10 minutes to complete. Times will be somewhat shorter for younger students, but still giving them time to think and explain. *For a minority of these tasks*, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such "apprentice tasks" part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

The Taxicab Problem (**Grade 9**)

You work for a business that has been using two taxicab companies, Company A and Company B.

Your boss gives you a list of (early and late) "Arrival times" for taxicabs from both companies over the past month.

Your job is to analyze those data using charts, diagrams, graphs, or whatever seems best. You are to:

1. Make the best argument that you can in favor of Company A;

2. Make the best argument that you can in favor of Company B;

3. Write a memorandum to your boss that makes a reasoned case for choosing one company or the other, using the relevant mathematical tools at your disposal.

Here are the data:

Company A		Company B	
3 min. 30 sec. EARLY	2 min. 15 sec. LATE	3 min. 45 sec. LATE	1 min. 30 sec. LATE
45 sec. LATE	9 min. 15 sec. LATE	4 min. 30 sec. LATE	3 min. 30 sec. LATE
1 min. 30 sec. LATE	3 min. 30 sec. LATE	3 min. LATE	6 min. LATE
4 min. 30 sec. LATE	1 min. 15 sec. LATE	5 min. LATE	4 min. 30 sec. LATE
45 sec. EARLY	30 sec. EARLY	2 min. 15 sec. LATE	5 min. 30 sec. LATE
2 min. 30 sec. EARLY	2 min. 30 sec. LATE	2 min. 30 sec. LATE	2 min. 30 sec. LATE
4 min. 45 sec. LATE	30 sec. LATE	1 min. 15 sec. LATE	4 min. 15 sec. LATE
3 min. 45 sec. LATE	7 min. 15 sec. LATE	45 sec. LATE	2 min. 45 sec. LATE
30 sec. LATE	5 min. 30 sec. LATE	3 min. LATE	3 min. 45 sec. LATE
1 min. 30 sec. EARLY	3 min. LATE	30 sec. EARLY	4 min. 45 sec. LATE

To work this problem the student needs to decide how to conceptualize the data, which computations to make, and how to represent the data from those computations. It turns out that Company A has a better mean arrival time than company B (this is the core of the argument they should make if they decide in favor of A - and for which they would receive credit), but it has a much greater spread of arrival times. The narrow spread is the compelling argument for B - you can't risk waiting for a cab that is extremely late, even if the company's average is good. Thus the best solution is to use company B, but to ask that they come a bit earlier than you actually need them - thus guaranteeing they arrive on time.[16]

With such problems, we see how students decide which information is a given problem context is important, and then how they use it. This is a dimension that is not found in Claim #2.

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[16] This problem has been used with thousands of students, and is well within their capacity. It is very different from a problem that gives the students the same numbers and asks them to calculate the mean times, ranges, etc.

Types of Extended Response Tasks for Claim #4

The following types of tasks, when well-designed and developed through piloting, naturally produce evidence on the aspects of a student's performance relevant to this claim. Some examples of are given below, with an analysis of what they assess.

Making decisions from data: These tasks require students to select from a data source, analyze the data and draw reasonable conclusions from it. This will often result in an *evaluation or recommendation*. The purpose of these tasks is not to provide a setting for the student to demonstrate a particular data analysis skill (e.g. box-and-whisker plots)—rather, the purpose is the drawing of conclusions in a realistic setting, using a range of techniques.

Making reasoned estimates: These tasks require students to make reasonable estimates of things they do know, so that they can then build a chain of reasoning that gives them an estimate of something they *do not know*.

Math – Grade 7	Item Type: CR	DOK: (Webb 1- 4) 3
Domain(s): Geometry		

Content Cluster(s) and/or Standard(s)

7.G **Solve** real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.SP **Investigate** patterns of association in bivariate data.

Claim #4 Assessment Targets

Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.
Target C: State logical assumptions being used.
Target D: Interpret results in the context of a situation.

Wrap the Mummy

Pam is thirteen today.
She is holding a party at which she plans to play the game 'Wrap the mummy'.
In this game, players try to completely cover themselves with toilet paper.

A roll of toilet paper contains 100 feet of paper, 4 inches wide.

Will one toilet roll be enough to wrap a person?

Describe your reasoning as fully as possible.
(You will need to estimate the average size of an adult person)

Plan and design tasks: Students recognize that this is a problem situation that arises in life and work. Well-posed planning tasks involving the coordinated analysis of time, space, and cost have already been commended for assessing Claim #2. For Claim #4, the problem will be presented in a more open form, asking the student to identify the variables that need to be taken into account, and the information they will need to find. An example of a relatively complex plan and design task is:

Planning a Class Trip

You and your friends on the Class Activities Committee are charged with deciding where this year's class trip will be. You have a fixed budget for the class and you need to figure out what will be the most fun and affordable option. Your committee members have collected a bunch of brochures from various parks - e.g., Marine World, Great Adventure, and others (see inbox of materials) - which have different admissions costs and are different distances from school. You have also collected information about the costs of meals and buses. Your job is to plan and justify a trip that includes bus fare, admission and possibly rides, as well as lunch, within the fixed budget the class has.

Evaluate and recommend tasks: These tasks involve understanding a model of a situation and/or some data about it and making a recommendation. For example:

Safe driving distances

A car with good brakes can stop in a distance "*D*" feet that is related to its speed "*v*" miles per hour by the model:
$$D = 1.5vt + v^2/20$$
where "*t*" is the driver's reaction time in seconds.

Using this model, you have been asked to recommend how close behind the car ahead it is safe to drive (in feet) for various speeds of *v* miles per hour.

Interpret and critique tasks: These tasks involve interpreting some data and critiquing an argument based on it. Again the purpose of these tasks is not to provide a setting for the student to demonstrate a particular data analysis skill, but to draw conclusions in a realistic setting, using a range of techniques. For example:

Choosing for the Regionals

Our school has to select a girl for the long jump at the regional championship. Three girls are in contention. We have a school jump-off. Their results, in meters, are given below:

Elsa	Ilse	Olga
3.25	3.55	3.67
3.95	3.88	3.78
4.28	3.61	3.92
2.95	3.97	3.62
3.66	3.75	3.85
3.81	3.59	3.73

Hans says, "Olga has the longest average. She should go to the championship."

Do you think Hans is right? Is Olga the best choice? Explain your reasoning.

This is not a complete list; other types of task that fit the criteria above may well be included. A balanced mixture of these types will provide enough evidence for Claim #4.

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Appendix A: CAT Sampling Proportions for Claim 1

The Content Specifications suggest that the computer-adaptive selection of items and tasks for Claim #1 be divided according to those clusters identified as "major" and those identified as "additional/supporting." This breakdown of clusters for each grade level was conducted in close collaboration with lead authors of CCSSM and members of the CCSSM validation committee.

The tables below show the categorization for each cluster in CCSSM, and also show "internal relative weights" suggested by the Content Specification authors. The Consortium is encouraged to investigate the feasibility of incorporating internal relative weights into the computer adaptive administration of Smarter Balanced.

The two components envisioned for Smarter Balanced assessment of CCSSM are:

> *High-intensity assessed clusters, about 75%-80% of the item level scores*
> - o Also high-adaptivity: 3 or more questions, and can cross into neighboring grades
> - o Consists of the major clusters (generally the progress to algebra continuum)
> - o Internal relative weights used for content balancing

> *Low-intensity assessed clusters, about 20%-25% of the item level scores*
> - o Consists of the additional and supporting clusters
> - o Internal relative weights used in a pure sampling approach

On the following pages are grade content tables, each with the following five columns:

Component (high / low intensity)	Percent of Claim 1 Points	Cluster Code	Cluster Text	Approx. Internal Weight (within component)

Notes on the tables:

- The percent of Claim 1 points adds to 100% across the high and low intensity components combined.
- The approximate internal weight within each component adds to 100% across all of the clusters in that component. The approximate internal weight values are meant to inform content balancing in the CAT so that it reflects - as well as possible given psychometric constraints - the structure and emphases of the standards at each grade level.
- When a single internal weight value W refers to $N \geq 2$ clusters, it means the clusters are thought of as equally weighted (i.e., cluster weights are W/N). These groupings are made for the sake of simplicity in numbers and do not indicate mathematical or conceptual affinities between clusters. Groups are sorted in decreasing order of W.

GRADE 3

Hi	**75%**	**3.OA.B**	**Understand properties of multiplication and the relationship between multiplication and division**	**75 %**
		3.OA.C	**Multiply and divide within 100**	
		3.MD.C	**Geometric measurement: understand concepts of area and relate area to multiplication and to addition**	
		3.MD.A	**Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects**	
		3.OA.D	**Solve problems involving the four operations, and identify and explain patterns in arithmetic[1]**	
		3.NF.A	**Develop understanding of fractions as numbers**	
		3.OA.A	**Represent and solve problems involving multiplication and division**	**25 %**

Lo	**25%**	**3.NBT.A**	**Use place value understanding and properties of operations to perform multi-digit arithmetic**	**60 %**
		3.G.A	**Reason with shapes and their attributes**	
		3.MD.B	**Represent and interpret data**	**40 %**
		3.MD.D	**Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures**	

GRADE 4

Hi	**75%**	**4.OA.A**	**Use the four operations with whole numbers to solve problems**	**60 %**
		4.NBT.B	**Use place value understanding and properties of operations to perform multi-digit arithmetic**	
		4.NF.A	**Extend understanding of fraction equivalence and ordering**	
		4.NF.B	**Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers**	**25 %**
		4.NBT. A	**Generalize place value understanding for multi-digit whole numbers**	**10 %**
		4.NF.C	**Understand decimal notation for fractions, and compare decimal fractions**	**5%**

Lo	**25%**	**4.MD.A**	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit**	**50 %**
		4.MD.C	**Geometric measurement: understand concepts of angle and measure angles**	
		4.OA.B	**Gain familiarity with factors and multiples**	**30 %**
		4.OA.C	**Generate and analyze patterns**	
		4.MD.B	**Represent and interpret data**	
		4.G.A	**Draw and identify lines and angles, and classify shapes by properties of their lines and angles**	**20 %**

GRADE 5

Hi	**75%**	**5.NF.A**	Use equivalent fractions as a strategy to add and subtract fractions	**40 %**
		5.MD.C	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	
		5.NF.B	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	**30 %**
		5.NBT.B	Perform operations with multi-digit whole numbers and with decimals to hundredths	**30 %**
		5.NBT. A	Understand the place value system	

Lo	**25%**	**5.G.A**	Graph points on the coordinate plane to solve real-world and mathematical problems	**60 %**
		5.G.B	Classify two-dimensional figures into categories based on their properties	
		5.OA.A	Write and interpret numerical expressions	**40 %**
		5.OA.B	Analyze patterns and relationships	
		5.MD.A	Convert like measurement units within a given measurement system	
		5.MD.B	Represent and interpret data	

GRADE 6

Hi	**75%**	**6.EE.A**	Apply and extend previous understandings of arithmetic to algebraic expressions	**40 %**
		6.EE.B	Reason about and solve one-variable equations and inequalities	
		6.RP.A	Understand ratio concepts and use ratio reasoning to solve problems	**25 %**
		6.EE.C	Represent and analyze quantitative relationships between dependent and independent variables	**20 %**
		6.NS.A	Apply and extend previous understandings of multiplication and division to divide fractions by fractions	
		6.NS.C	Apply and extend previous understandings of numbers to the system of rational numbers	**15 %**

Lo	**25%**	**6.NS.B**	Compute fluently with multi-digit numbers and find common factors and multiples	**100%**
		6.G.A	Solve real-world and mathematical problems involving area, surface area and volume	
		6.SP.A	Develop understanding of statistical variability	
		6.SP.B	Summarize and describe distributions	

GRADE 7

Hi	75%	7.RP.A	Analyze proportional relationships and use them to solve real-world and mathematical problems	60%
		7.EE.B	Solve real-life and mathematical problems using numerical and algebraic expressions and equations	
		7.NS.A	Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers	40%
		7.EE.A	Use properties of operations to generate equivalent expressions	

Lo	25%	7.G.A	Draw, construct and describe geometrical figures and describe the relationships between them	70%
		7.G.B	Solve real-life and mathematical problems involving angle measure, area, surface area and volume	
		7.SP.A	Use random sampling to draw inferences about a population	30%
		7.SP.B	Draw informal comparative inferences about two populations	
		7.SP.C	Investigate chance processes and develop, use, and evaluate probability models	

GRADE 8

Hi	75%	8.EE.B	Understand the connections between proportional relationships, lines and linear equations	40%
		8.EE.C	Analyze and solve linear equations and pairs of simultaneous linear equations	
		8.EE.A	Work with radicals and integer exponents	40%
		8.F.A	Define, evaluate and compare functions	
		8.G.A	Understand congruence and similarity using physical models, transparencies or geometry software	
		8.F.B	Use functions to model relationships between quantities	20%
		8.G.B	Understand and apply the Pythagorean Theorem	

Lo	25%	8.NS.A	Know that there are numbers that are not rational, and approximate them by rational numbers	100%
		8.G.C	Solve real-world and mathematical problems involving volume of cylinders, cones and spheres	
		8.SP.A	Investigate patterns of association in bivariate data	

Appendix B – Cognitive Rigor Matrix/Depth of Knowledge (DOK)

The Common Core State Standards require high-level cognitive demand, such as asking students to apply content knowledge and skills to new situations and sustained tasks. For each assessment target, the depth(s) of knowledge (DOK) that the student needs to bring to the item/task has been identified. The Cognitive Rigor Matrix integrates two widely accepted measures to describe cognitive rigor: Bloom's (revised) Taxonomy of Educational Objectives and Webb's Depth-of-Knowledge Levels. Smarter Balanced items are written to specific DOK levels, but use the matrix to ensure coverage across the range within a DOK level.

A "Snapshot" of the Cognitive Rigor Matrix (Hess, Carlock, Jones, & Walkup, 2009)[17]

Depth of Thinking (Webb) + Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	- Recall conversions, terms, facts			
Understand	-Evaluate an expression -Locate points on a grid or number on number line -Solve a one-step problem -Represent math relationships in words, pictures, or symbols	- Specify, explain relationships -Make basic inferences or logical predictions from data/observations -Use models /diagrams to explain concepts -Make and explain estimates	-Use concepts to solve non-routine problems -Use supporting evidence to justify conjectures, generalize, or connect ideas -Explain reasoning when more than one response is possible -Explain phenomena in terms of concepts	-Relate mathematical concepts to other content areas, other domains -Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	-Follow simple procedures -Calculate, measure, apply a rule (e.g., rounding) -Apply algorithm or formula -Solve linear equations -Make conversions	-Select a procedure and perform it -Solve routine problem applying multiple concepts or decision points -Retrieve information to solve a problem -Translate between representations	-Design investigation for a specific purpose or research question - Use reasoning, planning, and supporting evidence -Translate between problem & symbolic notation when not a direct translation	-Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	-Retrieve information from a table or graph to answer a question -Identify a pattern/trend	-Categorize data, figures -Organize, order data -Select appropriate graph and organize & display data -Interpret data from a simple graph -Extend a pattern	-Compare information within or across data sets or texts -Analyze and draw conclusions from data, citing evidence -Generalize a pattern -Interpret data from complex graph	-Analyze multiple sources of evidence or data sets
Evaluate			-Cite evidence and develop a logical argument -Compare/contrast solution methods -Verify reasonableness	-Apply understanding in a novel way, provide argument or justification for the new application
Create	- Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	-Generate conjectures or hypotheses based on observations or prior knowledge and experience	-Develop an alternative solution -Synthesize information within one data set	-Synthesize information across multiple sources or data sets -Design a model to inform and solve a practical or abstract situation

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[17] To download full article describing the development and uses of the Cognitive Rigor Matrix and other support CRM materials, go to: http://www.nciea.org/publications/cognitiverigorpaper_KH11.pdf

Appendix C: Core Content from Grades 6-8 that Remains Widely Applicable in High School

Solving problems at a level of sophistication appropriate to high school by:

- Applying ratios and proportional relationships.

- Applying percentages and unit conversions, e.g., in the context of complicated measurement problems involving quantities with derived or compound units (such as mg/mL, kg/m3, acre-feet, etc.).

- Applying basic function concepts, e.g., by interpreting the features of a graph in the context of an applied problem.

- Applying concepts and skills of geometric measurement e.g., when analyzing a diagram or schematic.

- Applying concepts and skills of basic statistics and probability (see 6-8.SP).

- Performing rational number arithmetic fluently.