+91 newseqs10 Bill Gosper 6577

Marc Le Brun −6591

```
cd hisdir
gauss$ cat newseqs10
From fernwood.mpk.ca.us!autodesk!desk!mlb Tue Jul  2 20:00 PDT 1991
Received: by gauss; Wed Jul  3 02:12:09 EDT 1991
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        id AA02517; Tue, 2 Jul 91 20:00:33 PDT
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        id AA06530; Tue, 2 Jul 91 19:57:01 PDT
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        id AA02514; Tue, 2 Jul 91 20:00:30 PDT
Received: from desk.autodesk.com by air.acad (4.0/SMI-4.0)
        id AA06527; Tue, 2 Jul 91 19:56:58 PDT
Date: Tue, 2 Jul 91 20:00 PDT
From: Marc Le Brun <autodesk!desk!mlb@fernwood.mpk.ca.us>
Subject: HIS II
To: air!"njas@research.att.com"@fernwood.mpk.ca.us
Cc: air!"rwg@RUSSIAN.SPA.Symbolics.COM"@fernwood.mpk.ca.us
In-Reply-To: <19910701072504.6.RWG@SWEATHOUSE.SPA.Symbolics.COM>,
             The message of 30 Jun 91 08:13 PDT from njas@research.att.com
Supersedes: <19910703005502.5.MLB@desk.autodesk.com>
Message-Id: <19910703030049.6.MLB@desk.autodesk.com>
Status: R
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Bill Gosper kindly forwarded me the welcome news that you are updating HIS!

While I'm merely an "ardent amateur", I can't resist offering you a few ideas that might be of use or interest:

First, I combed through my copy, looking for marginalia (some of what I found was written by RWG, when we shared an office several years ago).

Next to #1174, #1454 and #1780 he's written "see 1646", and there he has placed a footnote that starts with "(5n)!/n!(4n+1)!..." Bill, do you remember what this was about? If not I'll FAX a copy of the full scribble to you.

Two miscellaneous contributions of mine were:

```
  1 1 3 14 147 3462 ... = partitions of 1 into ordered parts of form 1/n     new 6584
  1 0 1 6 72 2320 245765 ... = partitions of 1 into unique 1/n               new 6585
```

I've also flirted a bit with the bivariate generating functions g(x,y) for various binary operators. The degenerate cases f(x)=g(x,x) of course have a sequence of coefficients which are the sums of the minor diagonals for the full operator. Sometimes these appear in HIS, sometimes not; here's some basic ones:

```
 +       2 6 12 20 30 42 56 72 90 110 132 156 182 210 240 272 306 342 380 420...   n(n+1)
 *       1 4 10 20 35 56 84 120 165 220 286 364 455 560 680 816 969 1140 1330...   (n 3)
 MIN     1 2 4 6 9 12 16 20 25 30 36 42 49 56 64 72 81 90 100 110...   A2620
 MAX     1 4 8 14 21 30 40 52 65 80 96 114 133 154 176 200 225 252 280 310...   6578 ✓
 GCD     1 2 4 4 9 6 12 12 17 10 28 12 25 30 32 16 45 18 52 44...   new   6579 ✓   (1+x)(1-x)^3
 LCM     1 4 8 20 21 56 60 96 105 220 152 364 301 360 464 816 549 1140 760...   6580
 LOGAND  1 0 4 4 5 0 12 16 21 16 24 20 17 0 32 48 65 64 84 84...   6581
 LOGXOR  0 6 4 12 20 42 32 40 48 78 84 116 148 210 176 176 176 214 212 252...   6582 new
 LOGIOR  1 6 8 16 25 42 44 56 69 94 108 136 165 210 208 224 241 278 296 336...   6583
```

Where the first three are #s 616, 1363, and 374, but the rest are absent (note that the last one is necessarily the sum of the previous two).

$\left[\frac{i}{j}\right]$ 2541

Similarly, in the following (where the function named is applied to the ratio)

$\left[\frac{i}{j}\right]$ 6218

$f(n)=\sum_{i=1}^{n-1} \mathrm{lcm}\{i,n-i\}$ etc

$\left\{\frac{i}{j}\right\}$ 6586

Scan ~~A57~~ A6577-
-A6591

Marc Le Brun
email to NJAS
July 3 1991

add to many

FLOOR 1 2 4 5 8 9 12 14 17 18 23 24 27 30 34 35 40 41 46 49... - have but obscure 2541
CEILING 1 3 5 8 10 14 16 20 23 27 29 35 37 41 45 50 52 58 60 66... 6218 have
ROUND 1 2 4 ~~7~~ 6 8 10 14 16 19 22 24 28 31 34 37 41 45 47 52 54... 6586

$\sum_{k=1}^{n} \left[\frac{n}{k}\right]$

the first is #362 but the others are unlisted.

Since "monic" integer power series form a group under composition, the inverse functions for all such functions also generate sequences suitable for HIS. Maybe these are even interesting, or worth cataloging. For example, the inverse of the polylogarithm Li[-1](x)=x/(1-x^2) (which Gosper calls a "negapolylog") gives Catalan numbers (so the "parentheses number" generating function might rightly be called a "negapolyexponential"!) Some further members of this family might be good to have in HIS II.

Gosper once gave me the motto "denominators are usually easier than numerators" to follow when trying to understand rational power series. In fact, as you're no doubt aware, very often the denominator sequence is based on something simple, but has been subject to sporadic cancellation due to accidentally sharing factors with the numerator. That is, a rational series that at first looks like "very-hard over pretty-easy" will turn out to be just "hard over easy". As a trivial example, if the denominators are O(n!) often you can turn the rationals into a sequence of integers by multiplying by n!. Therefore I have wished, for rational series that fall into this category, that HIS would not only list the numerators and denominators as independent entries, but also the "completed" numerators that result from multiplying out the "easy" form of the denominator. (I hope this makes sense, if not, I'll be happy to try to clarify).

It would seem that a vast variety of integer sequences could be generated in a "natural" way by considering successive configurations in the evolution of a one-dimensional cellular automata as integers expressed in the base of the number of cell states (as long as the activity never invaded "the right side of the decimal point"). I have none of these to offer, but perhaps someone like Steve Wolfram would know all about them.

No doubt you have heard criticism of HIS's practice of suppressing or inserting spurious 1s at the heads of the sequences. This can become very distracting when dealing with sequences like those from my two "miscellaneous" kinds of partitions of 1 given earlier. I hope that HIS II is cured of this somehow.

Finally, would you be willing to share with me a little about the more mundane aspects of this revision? For example, with computers much more prevalent nowadays, the data in HIS really ought to be in a machinable form. If it was, it could be supplemented in ways impractical for print media, such as including some actual generating expressions, either in some standard computing language, like C, Lisp or one of the symbolic math systems, or in a presentation language, such as TeX or postscript.

You might also look into the rapid advances being made in CD ROM and similar media. For an ambitious example, you could potentially replace the opaque references in HIS with hypertext links to images of the actual source documents! It would be interesting to try to get a fix on what that would entail -- my first guess (which may be wildly off) is that it would require under $1M to do all the library searches, although arranging the "reprint rights" for 4K+ documents might be formidable!

Also, a machinable database of the HIS sequences could be used by an AI system to do some automated searching in conjunction with a suitable math program. Order of growth and parity considerations could be used, for example, to prune

or guide a heuristic search. As an arbitrary example, even a brute force search to see if an unknown sequence might be f(g(x)) where f and g were in HIS would only require checking 16M cases.

Many other possibilities come to mind. Here at Autodesk we are persuing these sorts of things in various ways. Have you made formal arrangements yet for the publication of HIS II? Would you be interested in exploring some of these ideas further?

In any case, best wishes in your worthy endeavor!

-- Marc Le Brun
Director, Advanced Technology
Autodesk, Inc

From RUSSIAN.SPA.Symbolics.COM!rwg Wed Jul 3 02:25 0700 1991
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Date: Wed, 3 Jul 1991 02:25-0700
From: Bill Gosper <rwg@RUSSIAN.SPA.Symbolics.COM>
Subject: HIS II
To: "mlb@autodesk.com"@ELEPHANT-BUTTE.SCRC.Symbolics.COM
cc: "njas@research.att.com"@ELEPHANT-BUTTE.SCRC.Symbolics.COM
Message-ID: <19910703092528.4.RWG@SWEATHOUSE.SPA.Symbolics.COM>
Status: R

> Next to #1174, #1454 and #1780 he's written "see 1646", and there he has placed a footnote that starts with "(5n)!/n!(4n+1)!..." Bill, do you remember what this was about?

Vaguely. Those are the coeffs for a root of a nice quintic equation. I sent email to math fun about it. In fact, it may have recurred in the REVERT paper I sent you.

> If not I'll FAX a copy of the full scribble to you.

Please do.

> Here at Autodesk we are persuing these sorts of things in various ways.

Perusing or pursuing?

I regret to admit that I didn't buy myself an HIS when you moved out with yours, and I worry that I will suggest old data. I wish I had collected all the mysterious sequences I've encountered over the years. I'll do a massive email archive string search for "sequence".

6577

How about "number of 3n+1 steps to 1"? I can supply continued fraction terms for constants, e.g. Khinchin's and Euler's, if needed.

3*square U square+square = the known right-triangular rep-n-tiles, I believe.

gauss$

A2541	A6218
$\sum \left\lfloor \frac{n-i}{i} \right\rfloor$ ✓	$\sum_{1}^{n} \left\lfloor \frac{n}{i} \right\rfloor$ ✓
$\sum \left\lceil \frac{n-i}{i} \right\rceil$	$\sum \left\lceil \frac{n}{i} \right\rceil$
A6218 ✓	6590 ✓
$\sum \left\{ \frac{n-i}{i} \right\}$	$\sum \left\{ \frac{n}{i} \right\}$
A6586 ✓	6591
f( )	h( )

For the circle, the ratio of peak to average power is 2. It now follows from (25) that the ratio of peak to average power for the non-uniform signaling scheme is

$$\left(\frac{2}{3-2f_0}\right) \times \frac{P_{\max}(S^1)}{P_{\mathrm{ave}}(S^1)} = 1.38 \times 2 = 2.76. \tag{28}$$

Forney [12] has calculated the ratio of peak to average power for a signal constellation bounded by the Voronoi region of the Gosset lattice E_8. The ratio is 6.98 or 4.65 depending on whether E_8 is realized as a binary lattice or as a ternary lattice.

2. Non-Equiprobable Signaling with Two-Dimensional Constellations

A. Theoretical Analysis

Constellation shaping requires T scaled copies $(i+1)^{1/2}\mathcal{R}$, $i = 0, 1, \ldots, T-1$, of a basic region $\mathcal{R}$. The signal constellation Ω is a union of subconstellations Ω_i, $i = 0, 1, \ldots, T-1$, where $\Omega_i = \Omega \cap \left((i+1)^{1/2}\mathcal{R} \setminus i^{1/2}\mathcal{R}\right)$. The subconstellation Ω_i is used with frequency f_i and signals within Ω_i are equiprobable. The maximum information rate arising from the choice of subconstellation Ω_i is $H(f_0, f_1, \ldots, f_{T-1})$. From (24), the maximum biasing gain $\gamma_2(f_0, f_1, \ldots, f_{T-1})$, subject to choosing Ω_i with frequency f_i, is given by

$$\gamma_2(f_0, \ldots, f_{T-1}) = \frac{2^{H(f_0, f_1, \ldots, f_{T-1})}}{\sum_{i=0}^{T-1} (2i+1) f_i}. \tag{29}$$

Non-equiprobable signaling expands the signal constellation. From (25) the minimum constellation expansion ratio $\mathrm{CER}_2(f_0, \ldots, f_{T-1})$ occurs with the maximum information rate $H(f_0, \ldots, f_{T-1})$ and is given by

$$\mathrm{CER}_2(f_0, \ldots, f_{T-1}) = \frac{T}{2^{H(f_0, \ldots, f_{T-1})}}. \tag{30}$$

From (25), the factor $\mathrm{PAR}_2(f_0, \ldots, f_{T-1})$ by which the ratio of peak to average power has been increased, for the information rate $H(f_0, \ldots, f_{T-1})$, is given by

$$\mathrm{PAR}_2(f_0, \ldots, f_{T-1}) = \mathrm{CER}_2(f_0, \ldots, f_{T-1})\, \gamma_2(f_0, \ldots, f_{T-1}). \tag{31}$$