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Subject: A Handbook of Integer Sequences

Dear Dr. Sloane,

Please consider the following sequences for inclusion in your forthcoming second edition of the above. An appropriate title for this sequence might be the Joseph Louis Francois Bertrand sequence. "Each of the primes ... is less than twice its predecessor in the list." [Hardy] It is as follows: 2, 3, 5, 7, 13, 23, 43, 83, 163, 317, 631, 1259, 2503, 5003, 9973, 19937, 39869, 79699, 159389, 318751, 637499, 1274989, 2549951, 5099893, 10199767, 20399531, 40799041, 81598067, 163196129, 326392249, 652784471, 1305568919, 2611137817, 5222275627, 10444551233, 20889102457, 41778204911, 83556409789, 167112819547, 334225639093, 668451278147, 1336902556279, 2673805112521, 5347610225021, 10695220450027, 21390440900033, 42780881800057, 85561763600057, 171123527200081, 342247054400159, 684494108800091, 1368988217600167, 2737976435200319, 5475952870400627, 10951905740801243, 21903811481602373, 43807622963204729, 87615245926409407, 175230491852818793, 350460983705637557, 700921967411275081, 1401843934822550129, 2803687869645100253, 5607375739290200429, ...

"... in 1932, Breusch showed that if  $n \geq 48$ , then there exists a prime between  $n$  and  $\frac{9}{8}n$ .

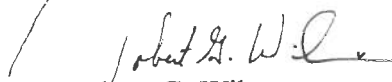
Thus, if  $0 < k \leq 7$  and  $n \geq 48$ , then there exists a prime  $p$  such that  $kn + 1 \leq p \leq \frac{9}{8}(kn + 1) \leq (k + 1)n$ . ... By the prime number theorem, for every  $h \geq 1$  there exists  $n_0 = n_0(h) > h$  such that, if  $n \geq n_0$ , then there exists a prime  $p$  such that  $n < p < (1 + \frac{1}{h})n$ . [Ribenboim]

For incremented Egyptian Fractions, beginning with  $h = 1$ , the series is as follows: 2, 8, 9, 24, 25, 32, 33, 48, 115, 116, 117, 118, 118, 140, 140, 141, 200, 212, 212, 213, 294, 294, 318,

318, 319, 319, 320, 320, 320, 524, 525, 525, 526, 526, 526, 527, 527, 528, 528, 1328, 1329, 1330, 1331, 1331, 1332, 1333, 1333, 1334, 1334, 1335, 1335, 1336, 1336, 1337, 1337, 1338, 1338, 1338, 1339, 1339, 1340, 1340, 1340, 1341, 1341, 1341, 1341, 1342, 1342, 1670, 1670, 1670, 1671, 1671, 1671, 1672, 1672, 1672, 1672, 1673, 1673, 1673, 1673, 1674, 1674, 1674, 1674, 1674, 1952, 1952, 2180, 2180, 2180, 2180, 2181, 2478, 2478, 2478, 2478, 2479, 2479, 2479, 2479, 2980, .... As this series increases, the number of repeats does also. Therefore; the table on the following page represents the above series less the repeated entries. An appropriate title might be the J. L. F. Bertrand Reciprocal Postulate Extension. However; to keep track of the growth of this series, the first column is the "h" and the second column is the "p" from the equation cited above.

- Reference: William and Fern Ellison, Prime Numbers, pg.2, John Wiley&Sons, NY, 1985.  
 Godfrey Harold Hardy & Edward Maitland Wright, An Introduction To The Theory of Numbers, 5<sup>th</sup> Edition, Sec. 22.3, pgs. 343-5, Clarendon Press, Oxford, 1979.  
 Donald Ervin Knuth, The Art Of Computer Programming, 2<sup>th</sup> Edition, Vol. 1, "Fundamental Algorithms," p506, Addison-Wesley Publ. Co., Reading, MS, 1973.  
 Ivan Morton Niven & Herbert Samuel Zuckerman, An Introduction To The Theory Of Numbers, 4<sup>th</sup> Edition, Sec. 8.3, pg 224-6, John Wiley&Sons, NY, 1980.  
 Paulo Ribenboim, The Little Book of BIG Primes, pgs. 126, 140, 185-6, Springer-Verlag, New York, 1991.  
 Joe Roberts, Lure of the Integers, p101, The Math. Assoc. of Am., Spectrum Series, Vol. ,  
 SI2: Waclaw Sierpinski, A Selection of Problems in the Theory of Numbers, pgs. 27-8&106, Pergamon Press, Macmillan, NY, 1964.

Sequentially yours,



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1.....	2	115.....	2974	449.....	31400	1111.....	58837
2.....	8	117.....	3272	456.....	31401	1132.....	58838
3.....	9	122.....	3273	462.....	31402	1141.....	59282
4.....	24	126.....	3274	469.....	31403	1163.....	59283
5.....	25	131.....	3275	476.....	31404	1186.....	59284
6.....	32	137.....	3276	484.....	31405	1210.....	59285
7.....	33	143.....	3277	491.....	31406	1211.....	60540
8.....	48	144.....	4298	499.....	31407	1236.....	60541
9.....	115	149.....	4299	507.....	31408	1262.....	60542
10.....	116	154.....	4300	515.....	31409	1289.....	60543
11.....	117	160.....	4301	524.....	31410	1317.....	60544
12.....	118	162.....	4832	533.....	31411	1346.....	60545
14.....	140	167.....	4833	542.....	31412	1375.....	79700
16.....	141	173.....	4834	550.....	34062	1399.....	79701
17.....	200	175.....	5592	559.....	34063	1402.....	89690
18.....	212	181.....	5593	568.....	34064	1424.....	89691
20.....	213	187.....	5594	578.....	34065	1447.....	89692
21.....	294	192.....	5750	588.....	34066	1471.....	89693
23.....	318	199.....	5751	598.....	34067		
25.....	319	206.....	5752	609.....	34068		
27.....	320	213.....	5754	620.....	34069		
30.....	524	217.....	6492	631.....	34070		
31.....	525	224.....	6493	643.....	34071		
33.....	526	231.....	6918	656.....	34072		
36.....	527	239.....	6919	660.....	35618		
38.....	528	242.....	7254	673.....	35619	1495.....	89694
40.....	1328	250.....	8468	685.....	35620	1521.....	89695
41.....	1329	257.....	8469	687.....	35678	1547.....	89696
42.....	1330	265.....	8470	700.....	35679	1574.....	89697
43.....	1331	266.....	9552	714.....	35680	1602.....	89698
45.....	1332	273.....	9553	723.....	43332	1631.....	89699
46.....	1333	281.....	9554	735.....	43333	1652.....	107377
48.....	1334	290.....	9555	748.....	43334	1678.....	107378
50.....	1335	294.....	9974	761.....	43335	1705.....	107379
52.....	1336	303.....	9975	764.....	44294	1732.....	107380
54.....	1337	312.....	9976	778.....	44295	1761.....	107381
56.....	1338	322.....	9977	791.....	44296	1790.....	107382
59.....	1339	333.....	9978	806.....	44297	1810.....	110359
61.....	1340	338.....	10800	821.....	44298	1840.....	110360
64.....	1341	346.....	11744	836.....	44299	1871.....	110361
68.....	1342	356.....	11745	852.....	44300	1903.....	110362
70.....	1670	357.....	15684	869.....	44301	1937.....	110363
73.....	1671	365.....	15685	887.....	44302	1950.....	155928
76.....	1672	374.....	15686	905.....	44303	1974.....	155929
80.....	1673	378.....	19610	918.....	45894	2000.....	155930
84.....	1674	385.....	19611	937.....	48680	2026.....	155931
89.....	1952	393.....	19612	955.....	48681	2052.....	155932
91.....	2180	401.....	19613	974.....	48682	2080.....	155933
95.....	2181	409.....	19614	994.....	48683	2108.....	155934
96.....	2478	418.....	19615	1015.....	58832	2137.....	155935
100.....	2479	427.....	19616	1033.....	58833	2166.....	155936
104.....	2480	436.....	19617	1051.....	58834	2197.....	155937
107.....	2972	437.....	31398	1070.....	58835	2228.....	155938
111.....	2973	443.....	31399	1090.....	58836	2260.....	155939