CENTRAL POLYGONAL NUMBERS IN FINITE SEQUENCES

Enrique Navarrete[1]

Starting with a(1) = 1 and not allowing the digit 0, a(n) = smallest nonnegative integer not yet in the sequence such that the last digit of a(n-1) plus the first digit of a(n) is equal to k, k = 1,..., 9. This defines 9 finite sequences, each of length equal to the k^{th} polygonal number k = 1,..., 9 (OEIS A002061). For k = 10, the sequence is infinite.

Here are the sequences:

k = 1: 1. (length = 1)

k = 2: 1, 11, 12. (length = 3)

k = 3: 1, 2, 11, 21, 22, 12, 13. (length = 7)

k = 4: 1, 3, 11, 31, 32, 2, 21, 33, 12, 22, 23, 13, 14. (length = 13)

k = 5: 1, 4, 11, 41, 42, 3, 2, 31, 43, 21, 44, 12, 32, 33, 22, 34, 13, 23, 24, 14, 15. (length = 21)

k = 6: 1, 5, 11, 51, 52, 4, 2, 41, 53, 3, 31, 54, 21, 55, 12, 42, 43, 32, 44, 22, 45, 13, 33, 34, 23, 35, 14, 24, 25, 15, 16. (length = 31)

k = 7: 1, 6, 11, 61, 62, 5, 2, 51, 63, 4, 3, 41, 64, 31, 65, 21, 66, 12, 52, 53, 42, 54, 32, 55, 22, 56, 13, 43, 44, 33, 45, 23, 46, 14, 34, 35, 24, 36, 15, 25, 26, 16, 17. (length = 43)

k = 8: 1, 7, 11, 71, 72, 6, 2, 61, 73, 5, 3, 51, 74, 4, 41, 75, 31, 76, 21, 77, 12, 62, 63, 52, 64, 42, 65, 32, 66, 22, 67, 13, 53, 54, 43, 55, 33, 56, 23, 57, 14, 44, 45, 34, 46, 24, 47, 15, 35, 36, 25, 37, 16, 26, 27, 17, 18. (length = 57)

k = 9: 1, 8, 11, 81, 82, 7, 2, 71, 83, 6, 3, 61, 84, 5, 4, 51, 85, 41, 86, 31, 87, 21, 88, 12, 72, 73, 62, 74, 52, 75, 42, 76, 32, 77, 22, 78, 13, 63, 64, 53, 65, 43, 66, 33, 67, 23, 68, 14, 54, 55, 44, 56, 34, 57, 24, 58, 15, 45, 46, 35, 47, 25, 48, 16, 36, 37, 26, 38, 17, 27, 28, 18, 19. (length = 73)

Some terms for k = 10: 1, 9, 11, 91, 92, 8, 2, 81, 93, 7, 3, 71, 94, 6, 4, 61, 95, 5, 51, 96, 41, 97, 31, 98, 21, 99, 12, 82, 83, 72, 84, 62, 85, 52, 86, 42, 87, 32, 88, 22, 89, 13, 73, 74, 63, 75, 53, 76, 43, 77, 33, 78, 23, 79, 14, 64, 65, 54, 66, 44, 67, 34, 68, 24, 69, 15, 55, 56, 45, 57, 35, 58, 25, 59, 16, 46, 47, 36, 48, 26, 49, 17, 37, 38, 27, 39, 18, 28, 29, 19, 111, 911, 912, 811, 913, 711, 914, 611, 915, 511, 916, 411, 917, 311, 918, 211, 919, 112, 812, 813, 712, ...

---

[1] *Enrique Navarrete, Universidad de Antioquia, Universidad Pontificia Bolivariana*

Enrique.navarrete.p@gmail.com,

Proof for k = 1,..., 9: Note that for each k, the numbers in the sequence can be arranged in a rectangle consisting of k rows (the starting digits) and (k-1) columns (the ending digits). The sequence ends as soon as a number ending with the digit k appears. This happens at the term 1k, which lies out of the rectangle. Hence the total number of terms in the sequence is k(k-1)+1, which is the k^{th} polygonal number.

For example, for k = 5, the terms of the sequence in rectangular form (plus the additional term 1k that ends the sequence) are:

1	2	3	4	
21	22	23	24	
31	32	33	34	
41	42	43	44	
11	12	13	14	15

[1]*Enrique Navarrete, Universidad de Antioquia, Universidad Pontificia Bolivariana*

Enrique.navarrete.p@gmail.com,