A “table of Mendeleev” for physical quantities?

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Outline

1. Introduction
   - Introductory quote
   - Research questions

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   - The properties of the *d*-dimensional integer lattice
   - Isomorphism between classes of physical quantities and the 7D integer lattice
   - Metalanguage of physics
   - Orthogonal decomposition of an integer lattice point
   - Gödel encoding of physical quantities up to a signed permutation

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Introductory quote from R.P. Feynman

R.P. Feynman [Feynman, 1985] remarks in "The Character of Physical Law" that:

> *"There is a very strong tendency, when someone comes up with an idea and says,*
> *"Let's suppose that the world is this way",*
> *for people to say to him,*
> *"What would you get for answer to such and such a problem?"*
> *And he says,*
> *"I haven't developed it far enough".*
> *And they say,*
> *"Well we have already developed it much further, and we can get the answers very accurately".*
> *So it is a problem whether or not to worry about philosophies behind ideas."*

Examples of the physical language

$$E = mc_0^2$$

$$Q = \frac{B_0^2 d^2}{\mu_0 \rho \nu \lambda}$$

$$i\hbar \frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

$$\hbar\omega(a^\dagger a + \frac{1}{2})\psi(q) = E\psi(q)$$

$$\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\psi - \nabla^2\psi + \frac{m^2 c_0^2}{\hbar^2}\psi = 0$$

$$i\hbar\gamma^\mu \partial_\mu \psi - mc_0\psi = 0$$

$$\langle \mathbf{p} \rangle = \langle \psi | \frac{\hbar}{i}\nabla|\psi\rangle = \hbar\mathbf{k}$$

$$\langle E \rangle = \langle \psi | \frac{\hbar}{i}\frac{\partial}{\partial t}|\psi\rangle = \hbar\omega$$

$$\Delta r \Delta k \geq \hbar$$

$$G^{\mu\nu} \doteq R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = -\frac{8\pi G}{c_0^4}T^{\mu\nu}$$

$$\frac{\mathrm{d}^2 x^\delta}{\mathrm{d}\tau^2} + \Gamma^\delta_{\beta\gamma}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}\frac{\mathrm{d}x^\gamma}{\mathrm{d}\tau} = 0$$

$$v = H_0 D$$

$$H^2 \doteq (\frac{\dot{a}}{a})^2$$

$$\Box\phi_\nu = -\frac{4\pi}{c_0}\mu J_\nu$$

$$\sum_\nu \phi_\nu^2 = |\mathbf{A}|^2 - \epsilon\mu V^2$$

J.C. Maxwell I

Maxwell [Maxwell, 1874] remarked 140 years ago in "On the Mathematical Classification of Physical Quantities" that:

> *"The first part of the growth of a physical science consists in the discovery of a system of quantities on which its phenomena may be conceived to depend. The next stage is the discovery of the mathematical form of the relations between these quantities. After this, the science may be treated as a mathematical science, and the verification of the laws is effected by a theoretical investigation of the conditions under which certain quantities can be most accurately measured, followed by an experimental realisation of these conditions, and actual measurement of the quantities."*

.

R.P. Feynman

Feynman [Feynman, 1985] remarked that:

> *"One of the most important things in this 'guess-compute consequences-compare with experiment' business is to know when you are right... You can recognize truth by its beauty and simplicity....Your guess is, in fact, that something is very simple. If you cannot see immediately that it is wrong, and it is simpler than it was before, then it is right.... We have to find a new view of the world that has to agree with everything that is known, but disagree in its predictions somewhere, otherwise it is not interesting. And in that disagreement it must agree with nature. If you can find any other view of the world which agrees over the entire range where things have already been observed, but disagrees somewhere else, you have made a great discovery."*

.

Description of the problem

The facts about the SI physical quantities are:

- The SI is adopted worldwide by convention for the semantics and syntax in the domains of science and technology [BIPM, 2006].
- The algebraic structure for quantity calculus is a multiplicative group [Fleischmann, 1951].
- What are the symmetries in the set of physical quantites?
- The number of physical quantities is infinite [BIPM, 2006].
- The number of physical relations is infinite [BIPM, 2006].
- The number of fundamental laws of physics is unknown [Feynman, 1966].

R.P. Feynman

R.P. Feynman [Feynman, 1966] remarks that:

> *"the fundamental laws of physics, when discovered, can appear in so many different forms that are not apparently identical at first, but with a little mathematical fiddling you can show the relationship"*

.

Research questions

We consider the following research questions:

- Is there a "table of Mendeleev" for physical quantities, that organizes the physical quantities?
- What selection rules apply to constellations of physical quantities?
- What type of form equation is realizable?
- Are there unique form equations?
- Are the selection rules falsifiable?

What are the axioms of the 8^{th} edition of the SI? I

We posit from the 8^{th} edition of the SI [BIPM, 2006] a set of axioms that are forming the algebra of quantity calculus [de Boer, 1994] .

Axiom (1)

The physical quantities are organized according to a system of dimensions.

Axiom (2)

The base quantities are length, mass, time, electric current, thermodynamic temperature, amount of substance and luminous intensity.

Axiom (3)

The base quantities are independent.

What are the axioms of the 8th edition of the SI? II

Axiom (4)

For each base quantity of the SI, there exists one and only one dimension.

Axiom (5)

For each base quantity of the SI, there exists one and only one base unit.

Axiom (6)

The product of two quantities is the product of their numerical values and units.

Axiom (7)

The quotient of two quantities is the quotient of their numerical values and units.

What are the base quantities and base dimensions used in the SI?

base quantity	quantity symbol	dimension symbol
length	$l, x, r, \ldots$	L
mass	m	M
time	t	T
electric current	I, i	I
thermodynamic temperature	T	Θ
amount of substance	n	N
luminous intensity	I_ν	J

What is the SI dimension of a physical quantity?

Definition

The SI dimension of a physical quantity q is expressed as a dimensional product:

$$\dim(q) = \mathrm{L}^{\alpha}\mathrm{M}^{\beta}\mathrm{T}^{\gamma}\mathrm{I}^{\delta}\Theta^{\epsilon}\mathrm{N}^{\zeta}\mathrm{J}^{\eta}\,; \tag{1}$$

where the exponents $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta \in \mathbb{Z}$ are called dimensional exponents.

The base quantities have no unique symbols. Only the SI dimensions of the base quantities have a unique symbol that is written in roman font.

The properties of the d-dimensional integer lattice

The properties of the d-dimensional integer lattice are described in the literature [Conway, 1999c] and [Birkhoff, 1979].

- Every lattice point is expressed in an unique way as the linear combination $\mathbf{f} = f_1\mathbf{e}_1 + \ldots + f_d\mathbf{e}_d$ where the coefficients f_i are called the coordinates of $\mathbf{f}$.
- The basis $\mathbf{e}_1, \ldots, \mathbf{e}_d$, that generates the integer lattice $\mathbb{Z}^d$ is orthonormal.
- The automorphism group of $\mathbb{Z}^d$ consists of all signed permutation matrices acting on the integer lattice points, and has order $2^d d!$ and is the Weyl group of root system $\mathrm{B_d}$ [Conway, 1999c, Carter, 2005].
- The set of integer lattice points $\mathbb{Z}^d$ is unordered.
- $\mathbb{Z}^d$ is a subgroup of $\mathbb{R}^d$ and is closed for addition, substraction and mutiplication by $k \in \mathbb{Z}$.

The valuations in the d-dimensional integer lattice

Definition

The ℓ_1-norm is represented by the expression

$$N(\mathbf{f}) \doteq \|\mathbf{f}\|_1 = \sum_{i=1}^{d} \sum_{k=1}^{d} a_{ik} f_i f_k$$

Definition

The ℓ_2-norm is represented by the expression

$$\|\mathbf{f}\|_2 \doteq \sqrt{\sum_{i=1}^{d} \sum_{k=1}^{d} a_{ik} f_i f_k}$$

Definition

The ℓ_∞-norm of $\mathbf{f}$ is represented by the expression
$\|\mathbf{f}\|_\infty \doteq \max\{|f_1|, \ldots, |f_d|\}$

Isomorphism between classes of physical quantities and $\mathbb{Z}^7$ I

- Let the set of physical quantities be denoted by $\mathcal{Q}$.
- Consider the physical quantities $a, b \in \mathcal{Q}$ and assume the following equivalence relation a is dimensionally equivalent with b with notation $a \sim b$.
- As physical quantities can be represented by tensors, we can without loss of generality consider a component of a tensor and denote it as q.
- The set of all equivalence classes in $\mathcal{Q}$ given the equivalence relation $\sim$ is the quotient set $\mathcal{Q}/\sim$.
- We define the surjective function $\dim(q)$ from $\mathcal{Q}$ to $\mathcal{Q}/\sim$ given by $\mathrm{dim(q)} = [\mathrm{q}]_\sim$.
- We represent the equivalence classes as $[q]_\sim = \{p \in \mathcal{Q} : p \sim q\}$.

Isomorphism between classes of physical quantities and $\mathbb{Z}^7$ II

- Consider the set of integer septuples $\mathbb{Z}^7 \doteq \{(f_1, \ldots, f_7) : f_i \in \mathbb{Z}\}$, that is a special case of the *n*-dimensional integer lattice [Conway, 1999c].
- We know that $\mathbb{Z}^7, +$ is an additive group and that from the algebra of the quantity calculus we have that $\mathcal{Q}/\sim, \cdot$ is a multiplicative group.
- We define the group isomorphism dex formally as $\mathrm{dex} : \mathcal{Q}/\sim \to \mathbb{Z}^7 : \mathrm{dex}\left([q]_\sim\right) \doteq \mathbf{f} = (f_1, \ldots, f_7)$ where $f_i \in \mathbb{Z}$.
- We will in the sequel make the abuse of notation and write $[q] = [q]_\sim$.
- The dimensional exponents of a physical quantity, taken in the correct order, form the coordinates of an integer lattice point in $\mathbb{Z}^7$.

Isomorphism between classes of physical quantities and $\mathbb{Z}^7$ III

- Each lattice point is the image of one and only one class of physical quantities that are dimensionally equivalent and so the mapping dex is bijective from $\mathcal{Q}/\sim$ on $\mathbb{Z}^7$.
- We select seven basis lattice points of $\mathbb{Z}^7$ and define:
  - $\mathbf{e}_1 = \mathrm{dex}\left([\textit{length}]\right) = (1,0,0,0,0,0,0)$
  - $\mathbf{e}_2 = \mathrm{dex}\left([\textit{mass}]\right) = (0,1,0,0,0,0,0)$
  - $\mathbf{e}_3 = \mathrm{dex}\left([\textit{time}]\right) = (0,0,1,0,0,0,0)$
  - $\mathbf{e}_4 = \mathrm{dex}\left([\textit{electric current}]\right) = (0,0,0,1,0,0,0)$
  - $\mathbf{e}_5 = \mathrm{dex}\left([\textit{thermodynamic temperature}]\right) = (0,0,0,0,1,0,0)$
  - $\mathbf{e}_6 = \mathrm{dex}\left([\textit{amount of substance}]\right) = (0,0,0,0,0,1,0)$
  - $\mathbf{e}_7 = \mathrm{dex}\left([\textit{luminous intensity}]\right) = (0,0,0,0,0,0,1)$

  with $\mathbf{e}_i \in \mathbb{Z}^7$.

Isomorphism between classes of physical quantities and $\mathbb{Z}^7$ IV

- We will adopt the Conway abbreviation [Conway, 1999c] for the components of lattice points and write

Example

$\mathbf{e}_3 \doteq \mathrm{dex}\left([\mathit{time}]\right) = (0,0,1,0,0,0,0)$ as $(0^2,1,0^4)$.

- One could object that some derived physical quantities (rms of a quantity, noise spectral density, specific detectivity, thermal inertia, thermal effusivity, ...) are defined as the square root of some product or quotient of other physical quantities.
- We define these derived physical quantities as fractional physical quantities where the coordinates $f_i \in \mathbb{Q}$ do not comply with the SI definition for a physical quantity (i.e. the determinant of the change of base transformation matrix A has to be $\det(\mathrm{A}) = \pm 1$ [Fleischmann, 1951].

Isomorphism between classes of physical quantities and $\mathbb{Z}^7$ V

- Each of these fractional physical quantities are, by a proper exponentiation, transformed to a physical quantity having integer exponents for the base quantities b_i .
- We know that the concept energy occurs in different forms in physics and that we use dedicated words as: energy, potential energy, kinetic energy, work, Lagrange function, Hamilton function, Hartree energy, ionization energy, electron affinity, electro-negativity, dissociation energy. . . in our formulations of physical relations.
- The equivalence class that we denote $[E]_\sim$ represents all these quantities.
- Consider the ordered septuple $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta) \in \mathbb{Z}^7$ formed by the dimensional exponents of a physical quantity q.

Isomorphism between classes of physical quantities and $\mathbb{Z}^7$ VI

- We rename the ordered septuple so that $f_1 = \alpha$, $f_2 = \beta$, $f_3 = \gamma$, . . . $f_7 = \eta$.
- We associate the ordered septuple $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta)$ to an integer lattice point $\mathbf{f} = (f_1, \ldots, f_7)$.
- The properties of $\mathbb{Z}^7$ are derived from the general case $\mathbb{Z}^d$ where $d = 7$ and are found in several publications [Conway, 1999c].

Absolute leader classes of a lattice I

- The representative lattice point, called in signal processing an absolute leader, has only coordinates that are non-negative integers.
- A leader class is the set of lattice points of $\mathbb{Z}^d$ that are connected through a signed permutation.
- Let $A = \{0, 1, 2, \ldots, s\}$ be a totally ordered alphabet.
- The representative of a leader class is a word w constructed from the alphabet A.
- The words w have a length d that corresponds to the dimension of $\mathbb{Z}^d$.
- Let d_i be the number of characters of type i of the alphabet A.

Absolute leader classes of a lattice II

- We denote a leader class of $\mathbb{Z}^d$ as $[w] = [(f_1, \ldots, f_d)]$, where $(f_1, \ldots, f_d)$ are the coordinates of the representative lattice point.
- We write the characters in graded reverse lex order [Cox, 2007].
- Each leader class forms a set of lattice points that are centro-symmetric about the origin $\mathbf{o}$ [Coxeter,1973].
- Suppose that the characters of w are subjected to a signed permutation, then the cardinality of the leader class is given by the equation:

$$\#([w]) = 2^{d-d_0} \frac{d!}{d_0!d_1!d_2!\ldots d_s!} .$$

- The set of the cardinality values of the leader classes is finite.

Absolute leader classes of a lattice III

- This set represents the number of vertices of 7-polytopes generated by the Coxeter group B_7 .

$$\{\#([w])\} = \{1, 14, 84, 128, 168, 280, 560, 840, 896, 1\,680, 2\,240, 2\,688, 3\,360, 4\,480, 5\,376, 6\,720, 13\,440, 17\,920, 20\,160, 26\,880, 40\,320, 53\,760, 80\,640, 161\,280, 107\,520, 322\,560, 645\,120\}$$

- The first leader class having all the possible signed permutations is the leader class: $[(7, 6, 5, 4, 3, 2, 1)]$ with $\#([w]) = 645\,120$ and Gödel number $= 2.67728\mathrm{E}{+}18$.
- The union of leader classes is called a lattice codebook [Vasilache, 2003].

Theta series in $\mathbb{Z}^7$ I

The OEIS sequence A008451 [OEIS, 2011] represents the number of ways of writing a positive integer N as a sum of 7 integral squares and is defined by:

$$\Theta_{\mathbb{Z}^7}(z) = \sum_{N=0}^{\infty} r_7(N) q^N ,$$

where $q = e^{\pi i z}$ and N is the norm of the lattice point [Conway, 1999a].

Lattice shells in $\mathbb{Z}^7$ I

The terms $r_7(N)$ in the theta series of the integer lattice $\mathbb{Z}^7$ are the sum of cardinalities of absolute leader classes.
We find in the OEIS [OEIS, 2011] the sequence A008451 given by

$$r_7(N) = 1, 14, 84, 280, 574, 840, 1\,288, 2\,368, 3\,444, 3\,542, 4\,424, 7\,560, 9\,240, 8\,456, 11\,088, 16\,576, 18\,494, 17\,808, 19\,740, 27\,720, 34\,440, 29\,456, 31\,304, 49\,728, 52\,808, 43\,414, 52\,248, 68\,320, 74\,048, 68\,376, 71\,120, 99\,456, 110\,964, 89\,936, 94\,864, 136\,080, \ldots$$

Partitioning of the lattice shells in $\mathbb{Z}^7$ I

The union of absolute leader classes with norm $N(\mathbf{f})$ is forming a lattice shell [Vasilache, 2002, Vasilache, 2003, Bruhn, 2008].
The enumeration table gives the relation between the sequence A008451 and the partitioning of lattice shells in absolute leader classes for $N \leq 35$.

N	disjunct union of absolute leader classes	$r_7(N)$
0	$[0^7]$	1
1	$[10^6]$	14
2	$[1^2 0^5]$	84
3	$[1^3 0^4]$	280
4	$[1^4 0^3] \cup [20^6]$	574
5	$[1^5 0^2] \cup [210^5]$	840
6	$[1^6 0] \cup [21^2 0^4]$	1288
7	$[1^7] \cup [21^3 0^3]$	2368
8	$[2^2 0^5] \cup [21^4 0^2]$	3444
9	$[2^2 10^4] \cup [21^5 0] \cup [30^6]$	3542
10	$[2^2 1^2 0^3] \cup [21^6] \cup [310^5]$	4424
11	$[2^2 1^3 0^2] \cup [31^2 0^4]$	7560
12	$[2^3 0^4] \cup [2^2 1^4 0] \cup [31^3 0^3]$	9240
13	$[2^3 10^3] \cup [2^2 1^5] \cup [320^5] \cup [31^4 0^2]$	8456
14	$[2^3 1^2 0^2] \cup [3210^4] \cup [31^5 0]$	11088

Partitioning of the lattice shells in $\mathbb{Z}^7$ II

15	$[2^3 1^3 0] \cup [321^2 0^3] \cup [31^6]$	16576
16	$[2^4 0^3] \cup [2^3 1^4] \cup [321^3 0^2] \cup [40^6]$	18494
17	$[2^4 10^2] \cup [32^2 0^4] \cup [321^4 0] \cup [410^5]$	17808
18	$[2^4 1^2 0] \cup [3^2 0^5] \cup [32^2 10^3] \cup [321^5] \cup [41^2 0^4]$	19740
19	$[2^4 1^3] \cup [3^2 10^4] \cup [32^2 1^2 0^2] \cup [41^3 0^3]$	27720
20	$[2^5 0^2] \cup [3^2 1^2 0^3] \cup [32^2 1^3 0] \cup [41^4 0^2] \cup [420^5]$	34440
21	$[2^5 10] \cup [32^3 0^3] \cup [3^2 1^3 0^2] \cup [32^2 1^4] \cup [41^5 0] \cup [4210^4]$	29456
22	$[2^5 1^2] \cup [3^2 20^4] \cup [32^3 10^2] \cup [3^2 1^4 0] \cup [41^6] \cup [421^2 0^3]$	31304
23	$[3^2 210^3] \cup [32^3 1^2 0] \cup [3^2 1^5] \cup [421^3 0^2]$	49728
24	$[2^6 0] \cup [3^2 21^2 0^2] \cup [32^3 1^3] \cup [421^4 0] \cup [42^2 0^4]$	52808
25	$[2^6 1] \cup [32^4 0^2] \cup [3^2 21^3 0] \cup [421^5] \cup [42^2 10^3] \cup [430^5] \cup [50^6]$	43414
26	$[3^2 2^2 0^3] \cup [32^4 10] \cup [3^2 21^4] \cup [42^2 1^2 0^2] \cup [4310^4] \cup [510^5]$	52248
27	$[3^3 0^4] \cup [3^2 2^2 10^2] \cup [32^4 1^2] \cup [42^2 1^3 0] \cup [431^2 0^3] \cup [51^2 0^4]$	68320
28	$[2^7] \cup [3^3 10^3] \cup [3^2 2^2 1^2 0] \cup [42^2 1^4] \cup [42^3 0^3] \cup [431^3 0^2] \cup [51^3 0^3]$	74048
29	$[3^3 1^2 0^2] \cup [32^5 0] \cup [3^2 2^2 1^3] \cup [42^3 10^2] \cup [431^4 0] \cup [4320^4] \cup [51^4 0^2] \cup [520^5]$	68376
30	$[3^2 2^3 0^2] \cup [3^3 1^3 0] \cup [32^5 1] \cup [42^3 1^2 0] \cup [431^5] \cup [43210^3] \cup [51^5 0] \cup [5210^4]$	71120
31	$[3^3 20^3] \cup [3^2 2^3 10] \cup [3^3 1^4] \cup [42^3 1^3] \cup [4321^2 0^2] \cup [51^6] \cup [521^2 0^3]$	99456
32	$[3^3 210^2] \cup [3^2 2^3 1^2] \cup [42^4 0^2] \cup [4^2 0^5] \cup [4321^3 0] \cup [521^3 0^2]$	110964
33	$[3^3 21^2 0] \cup [32^6] \cup [42^4 10] \cup [4^2 10^4] \cup [4321^4] \cup [432^2 0^3] \cup [521^4 0] \cup [52^2 0^4]$	89936
34	$[3^2 2^4 0] \cup [3^3 21^3] \cup [42^4 1^2] \cup [432^2 10^2] \cup [43^2 0^4] \cup [4^2 1^2 0^3] \cup [521^5] \cup [52^2 10^3] \cup [530^5]$	94864
35	$[3^3 2^2 0^2] \cup [3^2 2^4 1] \cup [43^2 10^3] \cup [4^2 1^3 0^2] \cup [432^2 1^2 0] \cup [52^2 1^2 0^2] \cup [5310^4]$	136080

The parity of the sum of the coordinates in the d-dimensional integer lattice I

Definition

Let the surjective function "psc", represent the parity of the sum of coordinates of a lattice point of $\mathbb{Z}^d$ and define:

$$\mathrm{psc} : \mathbb{Z}^{\mathrm{d}} \to \{0, 1\} \mid \mathrm{psc}(\mathbf{f}) = |\sum_{i=1}^{d} f_i| \pmod 2, f_i \in \mathbb{Z} .$$

- The "psc" function is a 2-colouring function.
- We write $\mathbb{Z}^d = \mathbb{Z}_e^d \oplus \mathbb{Z}_o^d$ so that it splits as the sum of even and odd parts with the origin $\mathbf{o}$ in $\mathbb{Z}_e^d$.
- We have an evensum lattice point when $\mathrm{psc}(\mathbf{f}) = 0$ and an oddsum lattice point when $\mathrm{psc}(\mathbf{f}) = 1$ where $\mathbf{f} \in \mathbb{Z}^d$.

The parity of the sum of the coordinates in the d-dimensional integer lattice II

- Observe that the lattice points $\mathbf{f}$ for which $\mathrm{psc}\left(\mathbf{f}\right) = 0$ are elements of D_7 that is an indecomposable root lattice [Coppel, 2009] defined as
  $\mathrm{D}_7 = \{(\mathrm{f}_1, \ldots, \mathrm{f}_7) \in \mathbb{Z}^7 \mid \sum_{\mathrm{i}=1}^{7} \mathrm{f}_\mathrm{i} \text{ is even}\}$.
- The lattice D_7 has 84 minimal points, that are $\pm\mathbf{e}_j \pm \mathbf{e}_k$ where $(1 \leq j < k \leq 7)$.
- These 84 points form a simple basis derived from the canonical basis $\mathbf{e}_1, \ldots, \mathbf{e}_7$ of $\mathbb{Z}^7$.

Metalanguage of physics

The hypothesis of the existence of selection rules that have to be respected by the laws of physics, has been proposed by Wigner and Feynman, see Lange [Lange, 2009].
We elaborate on this problem by proving two of these selection rules applicable for binary form equations $[z] = [f(\Pi)][x][y]$ between the distinct physical quantities $[f(\Pi)], [x], [y], [z]$:

- 4-point theorem of physical binary form equations
- Bicoloring patterns of a 4-cycle

Geometric representation of form equations between physical quantities

Theorem (4-point theorem of physical binary form equations)

The binary form equation $[z] = [f(\Pi)][x][y]$ *is physically valid, with* $[f(\Pi)], [x], [y], [z]$ *distinct classes of physical quantities obeying the properties:*

$$\mathrm{dex}^{-1}([z]) \circ \mathrm{dex}([z]) = [z], \quad \mathrm{dex}^{-1}([f(\Pi)]) \circ \mathrm{dex}([f(\Pi)]) = [f(\Pi)],$$
$$\mathrm{dex}^{-1}([x]) \circ \mathrm{dex}([x]) = [x], \quad \mathrm{dex}^{-1}([y]) \circ \mathrm{dex}([y]) = [y],$$

if and only if, the 4-cycle **oyzxo** *is a parallelogram in the integer lattice* $\mathbb{Z}^7$ *and* $\mathrm{dex}([x]) = \mathbf{x}$, $\mathrm{dex}([y]) = \mathbf{y}$, $\mathrm{dex}([z]) = \mathbf{z}$, $\mathrm{dex}([f(\Pi)]) = \mathbf{o}$ *are distinct integer lattice points with* **o** *being the origin of the integer lattice* $\mathbb{Z}^7$ *.*

Parallelogram **oyzxo** representing the binary form equation $[z] = [f(\Pi)][x][y]$ in $\mathbb{Z}^7$.

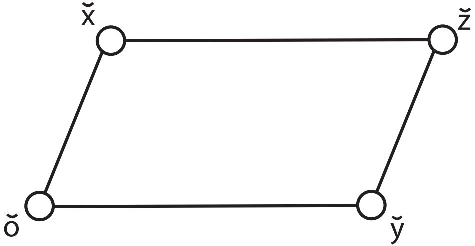

Figure: Parallelogram **oyzxo** representing the binary form equation $[z] = [f(\Pi)][x][y]$ in $\mathbb{Z}^7$.

Cardinality of isoperimetric parallelograms I

- We explore the integer lattice and search for a constellation where the elements of the constellation are forming a parallelogram **oyzxo** .
- The followed approach was to select two fixed points $\mathbf{o}, \mathbf{z}$ and to vary the point **x** and derive the coordinates of the lattice point **y** .
- For ease of calculation perimeters of triangles p_t instead of parallelograms p_p were calculated and then converted.
- The fixed point to start the survey through the integer lattice was selected to be $\mathbf{z} = (2, 1, -2, 0, 0, 0, 0)$ representing energy.
- The questions became now more specific: Which lattice points are generating triangles representing an energy constellation between physical quantities and how many of these triangles have the same perimeter?

Cardinality of isoperimetric parallelograms II

- Two polygons are called isoperimetric [Audet, 2007] if they have the same perimeter.
- The absolute frequency of occurrence of these parallelogram perimeters p_p are tabulated as a sequence of non-negative integers and represented graphically for $\mathbf{z} = \mathbf{E}$, as a discrete value distribution [Barford, 1990] in a logarithmic scatter plot.
- We observed that the constellations representing energy are connected through the discrete value distribution in such a way that the frequency f is identical to the order n of a graph G of vertices representing relations between physical quantities and edges representing a connection between relations of physical quantities.
- This approach is similar to the one followed by Wigner where the laws of nature are the entities to which the symmetry laws apply [Wigner, 1964].

Logarithmic scatter plot of parallelogram perimeters p_p in $\mathbb{Z}^7$ resulting in the physical quantity energy.

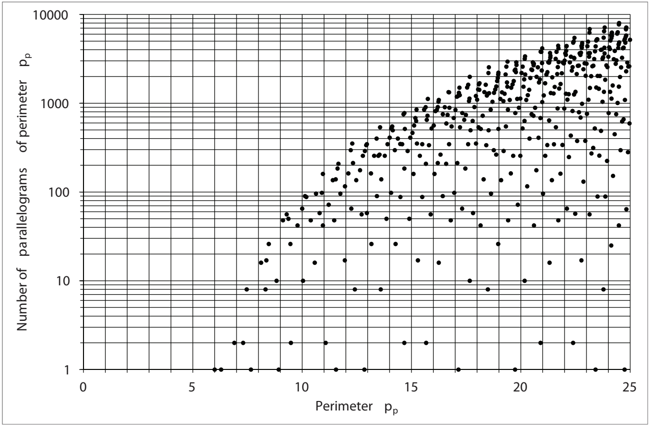

Figure: Logarithmic scatter plot of parallelogram perimeters p_p in $\mathbb{Z}^7$ resulting in the physical quantity energy.

Unique parallelograms resulting in the physical quantity energy.

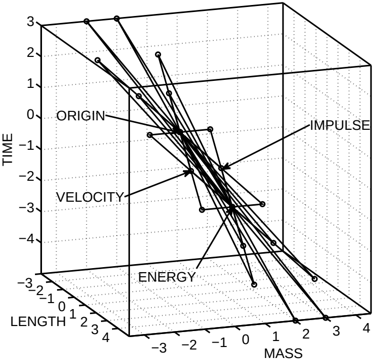

Figure: Unique parallelograms resulting in the physical quantity energy.

Theorem

The isoperimetric distribution, for parallelograms containing the integer lattice points **o** *and* **z**, *is invariant when the coordinates of the integer lattice point* **z** *are subjected to a signed permutation.*

Proof.

The isometric property of the above mapping and mapping combinations is the origin of the invariance in the isoperimetric distribution [Conway, 1999b]. The perimeter of the parallelogram is based on the Euclidean distance (ℓ_2-distance) between the lattice points and so neither a permutation of the coordinates nor a change in the sign of the coordinates will modify the value of the distance between the lattice points. □

Invariance of the isoperimetric distribution II

Each signed permutation matrix is an orthogonal matrix [Conway, 1999b].

Example

The components of the physical quantity force, represented by (1, 1, -2, 0, 0, 0, 0), and the components of the physical quantity angular momentum, represented by (2, 1, -1, 0, 0, 0, 0), have the same isoperimetric distribution.

Example

The components of the physical quantity mass, represented by (0, 1, 0, 0, 0, 0, 0), and the components of the physical quantity frequency, represented by (0, 0, -1, 0, 0, 0, 0), have the same isoperimetric distribution.

Bicoloring patterns of a 4-cycle representing a binary form equation between physical quantities

Theorem (Bicoloring of physical form equations)

Any binary equation $[z] = [f(\Pi)][x][y]$ *between distinct physical quantities* $[f(\Pi)], [x], [y], [z]$ *represents a distinct ordered coloring pattern* $(\mathrm{psc}(\mathbf{o}), \mathrm{psc}(\mathbf{x}), \mathrm{psc}(\mathbf{y}), \mathrm{psc}(\mathbf{z}))$ *that is an element of the set of ordered coloring patterns*
$\{(0,0,0,0),(0,0,1,1),(0,1,0,1),(0,1,1,0)\}$.

Binary form equations of physics have to comply with the bicoloring of 4-cycles
[Quattrocchi, 2001, Gionfriddo, 2010a, Gionfriddo, 2010b] in $\mathbb{Z}^7$.

Linear independence and orthogonality between classes of physical quantities I

We underline the difference between linearly independent physical quantities and orthogonal physical quantities [Rodgers, 1984].
From these properties we define 6 types of pairwise combinations of $[x]$ and $[y]$ that classify a binary form equation $[z] = [f(\Pi)][x][y]$:

1. $\mathbf{x} \cdot \mathbf{y} > 0$ and 2×7 matrix rank $= 2$ (not orthogonal with positive inner product, linearly independent)
2. $\mathbf{x} \cdot \mathbf{y} = 0$ and 2×7 matrix rank $= 2$ (**orthogonal, linearly independent**)
3. $\mathbf{x} \cdot \mathbf{y} < 0$ and 2×7 matrix rank $= 2$ (not orthogonal with negative inner product, linearly independent)
4. $\mathbf{x} \cdot \mathbf{y} > 0$ and 2×7 matrix rank < 2 (not orthogonal with positive inner product, linearly dependent)

Linear independence and orthogonality between classes of physical quantities II

5. $\mathbf{x} \cdot \mathbf{y} = 0$ and 2×7 matrix rank < 2 (orthogonal, linearly dependent)
6. $\mathbf{x} \cdot \mathbf{y} < 0$ and 2×7 matrix rank < 2 (not orthogonal with negative inner product, linearly dependent)

The class of orthogonal and linearly independent pairwise physical quantities resulting in the binary form equation $[z] = [f(\Pi)][x][y]$ is finite!

Orthogonal decomposition of an integer lattice point I

The decomposition of a vertex $\mathbf{z}$ in pairwise orthogonal vertices $\mathbf{x}$ and $\mathbf{y}$ assumes the existence of a system of Diophantine equations:

$$\mathbf{x} + \mathbf{y} - \mathbf{z} = \mathbf{0}\ , \tag{2a}$$

$$\mathbf{x} \cdot \mathbf{y} = 0\ , \tag{2b}$$

where $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{Z}^7$. We eliminate $\mathbf{y}$ from the equation (2b) and find:

$$\mathbf{x} \cdot \mathbf{x} - \mathbf{x} \cdot \mathbf{z} = 0\ . \tag{3}$$

We apply the method of "completing the square" and write equation (3) as:

$$(\mathbf{x} - \frac{\mathbf{z}}{2})^2 = (\frac{\mathbf{z}}{2})^2\ , \tag{4}$$

that represents a 7D-hypersphere with center at $\dfrac{\mathbf{z}}{2}$ and radius $\|\dfrac{\mathbf{z}}{2}\|_2$.

Distribution of rectangles in the $\mathbb{Z}_+^7$ I

We determine the distribution of non-degenerated unique rectangles formed by 4 lattice points $\mathbf{o}, \mathbf{x}, \mathbf{y}, \mathbf{z}$ in $\mathbb{Z}_+^7$ as function of the infinity norm $\|\mathbf{z}\|_\infty = s$ where $\mathbf{z} = \mathbf{x} + \mathbf{y}$.

We define a sample space Ω consisting of 7D-hyperspheres with infinity norm $\|\mathbf{z}\|_\infty = s$, with $s \in \{1, \ldots, 10\}$ and search for the event of an unique perimeter p .

Table 3 gives the result of the search for unique rectangles.

We find in $\mathbb{Z}_+^7$ where $\|\mathbf{z}\|_\infty \leq 10$, a total of 7 747 unique rectangles out of 6 510 466 998 rectangles.

The unique rectangles represent unique realizable binary form equations of the type $[z] = f(\Pi)[x][y]$ for the selected physical quantity $[z]$.

Distribution of rectangles in the $\mathbb{Z}_+^7$ II

Table: Distribution of rectangles in $\mathbb{Z}_+^7$ as function of the infinity norm $\|\mathbf{z}\|_\infty = s$.

infinity norm $\|\mathbf{z}\|_\infty = s$	$UR =$ # unique rectangles	$R =$ # rectangles	$\frac{UR}{R}$
1	1	120	8.33E-03
2	7	7 196	9.73E-04
3	26	162 554	1.60E-04
4	79	1 341 957	5.89E-05
5	182	9 255 603	1.97E-05
6	333	40 532 530	8.22E-06
7	693	168 302 117	4.12E-06
8	1 180	523 421 602	2.25E-06
9	1 999	1 637 895 896	1.22E-06
10	3 247	4 129 547 423	7.86E-07
Total	7 747	6 510 466 998	1.19E-06

These sequences of integers are not listed in the OEIS [OEIS, 2014] and we suggest further research on it.

$\frac{UR}{R}$ as function of the infinity norm I

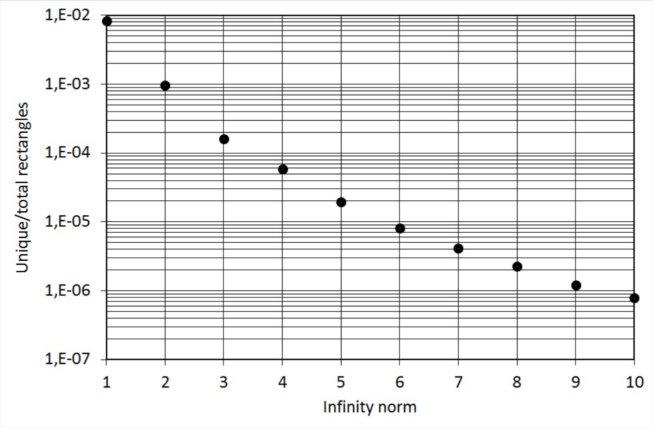

Figure: $\frac{UR}{R}$ for infinity norm $\|\mathbf{z}\|_\infty \leq 10$.

Sources of unique binary form equations I

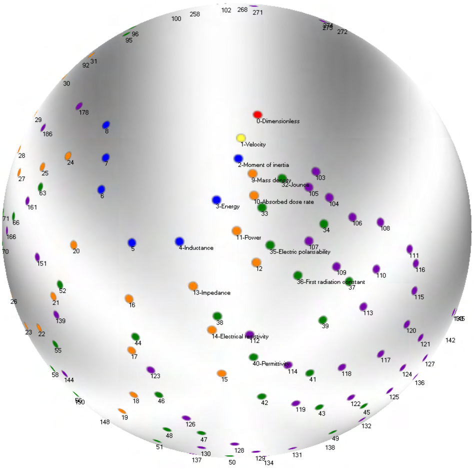

Figure: Sources of unique binary form equations for $\|\mathbf{z}\|_\infty \leq 5$.

The first 41 sources I

Id	leader class	physical quantity	$N(\mathbf{z})$	number of vertices	Gödel number
0	$[0^7]$	Dimensionless	0	1	1
1	$[1^2 0^5]$	Velocity	2	84	6
2	$[210^5]$	Moment of inertia	5	168	12
3	$[2^2 10^4]$	Energy	9	840	180
4	$[2^3 10^3]$	Inductance	13	2 240	6 300
5	$[2^4 10^2]$	Unknown	17	3 360	485 100
6	$[2^5 10]$	Unknown	21	2 688	69 369 300
7	$[2^5 1^2]$	Unknown	22	2 688	1 179 278 100
8	$[2^6 1]$	Unknown	25	896	15 330 615 300
9	$[310^5]$	Mass density	10	168	24
10	$[320^5]$	Absorbed dose rate	13	168	72
11	$[3210^4]$	Power	14	1 680	360
12	$[3^2 10^4]$	Unknown	19	840	1 080
13	$[32^2 10^3]$	Impedance	18	6 720	12 600
14	$[3^2 210^3]$	Electrical resistivity	23	6 720	37 800
15	$[3^3 10^3]$	Unknown	28	2 240	189 000
16	$[32^3 10^2]$	Unknown	22	13 440	970 200
17	$[3^2 2^2 10^2]$	Unknown	27	20 160	2 910 600
18	$[3^3 210^2]$	Unknown	32	13 440	14 553 000
19	$[3^4 10^2]$	Unknown	37	3 360	101 871 000
20	$[32^4 10]$	Unknown	26	13 440	138 738 600
21	$[3^2 2^3 10]$	Unknown	31	26 880	416 215 800
22	$[3^3 2^2 10]$	Unknown	36	26 880	2 081 079 000
23	$[3^4 210]$	Unknown	41	13 440	14 567 553 000

The first 41 sources II

24	$[32^5 1]$	Unknown	30	5 376	30 661 260 600
25	$[3^2 2^4 1]$	Unknown	35	13 440	91 983 691 800
26	$[3^5 10]$	Unknown	46	2 688	160 243 083 000
27	$[3^3 2^3 1]$	Unknown	40	17 920	459 918 459 000
28	$[3^4 2^2 1]$	Unknown	45	13 440	3 219 429 213 000
29	$[3^5 21]$	Unknown	50	5 376	35 413 721 343 000
30	$[3^6 1]$	Unknown	55	896	460 378 377 459 000
31	$[3^6 2]$	Unknown	58	896	7 826 432 416 803 000
32	$[410^5]$	Jounce	17	168	48
33	$[41^2 0^4]$	Unknown	18	840	240
34	$[430^5]$	Unknown	25	168	432
35	$[4210^4]$	Electric polarizability	21	1 680	720
36	$[4310^4]$	First radiation constant	26	1680	2 160
37	$[4^2 10^4]$	Unknown	33	840	6 480
38	$[42^2 10^3]$	Unknown	25	6 720	25 200
39	$[4^2 1^2 0^3]$	Unknown	34	3 360	45 360
40	$[43210^3]$	Permittivity	30	13 440	75 600
41	$[4^2 210^3]$	Unknown	37	6 720	226 800

The first 41 sources III

Only 13 of the 275 sources for $\|\mathbf{z}\|_\infty \leq 5$ are presently known!

Equivalence classes of the d-dimensional integer lattice based on the infinity norm ℓ_∞ I

We calculate the number of equivalence classes that can be formed in a d-dimensional hypercube P_d^s [Coxeter,1973] when the infinity norm $\ell_\infty = s$ and $s \in \mathbb{N}$.
The result is known as the multiset number and given by:

$$\#\left(P_d^s\right) = \binom{d+s-1}{s}$$

For $d = 7$ we find the integer sequence A000579 [OEIS, 2014]:
$\#\left(P_7^s\right) = 1,\ 7,\ 28,\ 84,\ 210,\ 462,\ 924,\ 1\,716,\ 3\,003,\ 5\,005,\ 8\,008\ \ldots$
where $s \in \{0, \ldots, 10\}$.
The value of $s = 10$ is relevant when considering the second hyper-polarizability that has the largest coordinate value (-2,-3,10,4,0,0,0) of the tabulated physical quantities.

Equivalence classes of the d-dimensional integer lattice based on the infinity norm ℓ_∞ II

$\|\mathbf{z}\|_\infty = s$	sum(# ([w]))	sum(# ([w])) cumul	# (P_7^s)	# (P_7^s) cumul
0	1	1	1	1
1	2 186	2 187	7	8
2	75 938	78 125	28	36
3	745 418	823 543	84	120
4	3 959 426	4 782 969	210	330
5	14 704 202	19 487 171	462	792
6	43 261 346	62 748 517	924	1 716
7	108 110 858	170 859 375	1 716	3 432
8	239 479 298	410 338 673	3 003	6 435
9	483 533 066	893 871 739	5 005	11 440
10	907 216 802	1 801 088 541	8 008	19 448

Gödel encoding of physical quantities up to a signed permutation

We encode each integer lattice point of $\mathbb{Z}_+^7$ by using a similar scheme to the Gödel encoding [Feferman, 1986].

Definition

$$\phi_d(f_1, \ldots, f_d) = \prod_{i=1}^{d} p_i^{f_i} \ , \qquad (5)$$

where p_i is the i-th prime number, $\mathbf{f} = (f_1, \ldots, f_d)$ and $f_i \in \mathbb{Z}_+$.

Observe that we are forming a monomial $\phi_d(f_1, \ldots, f_d)$.

Example

$\phi_7(2, 2, 1, 0, 0, 0, 0) = 2^2 \cdot 3^2 \cdot 5^1 \cdot 7^0 \cdot 11^0 \cdot 13^0 \cdot 17^0 = 180$

Factorization of Gödel number in distinct factors

- The encoding of the leader classes with a Gödel number allows the factorization of the Gödel number in distinct factors.
- Richard J. Mathar (http://home.strw.leidenuniv.nl/ mathar/) has listed in the OEIS [OEIS, 2011] the integer series A045778 that gives the factorization F_n of non-negative integers i up to $i = 1500$ and David W. Wilson gives for A045778 a table of the number of factorizations $\sum_{n=1}^{nmax} F_n$ of i into distinct factors greater than 1 for $i \leq 10000$.
- A signed permutation matrix can be found that maps the form equations of the leader class in $\mathbb{Z}_+^7$ to the form equations of the desired physical quantity.

4-factorization of the leader class $(2^2, 1, 0^4)$

We show the method for the physical quantity energy.
The absolute leader class $(2^2, 1, 0^4)$ is representative for the physical quantity energy.
It has Gödel number $\phi_7(2, 2, 1, 0, 0, 0, 0) = 180$.
From the OEIS [OEIS, 2011] A045778 series we find as factorizations: F_2=8, $F_3 = 8$ and $F_4 = 1$.
The 4-factoring results in 1 form equation that represents a quaternary form equation.

$$180 = 2 \times 3 \times 5 \times 6$$

1. $(2, 2, 1, 0, 0, 0, 0) =$
$(0, 0, 0, 0, 0, 0, 0) + (1, 0, 0, 0, 0, 0, 0) + (0, 1, 0, 0, 0, 0, 0) + (0, 0, 1, 0, 0, 0, 0) + (1, 1, 0, 0, 0, 0, 0)$;

3-factorization of the leader class $(2^2, 1, 0^4)$

The 3-factoring results in 8 form equations that represent each a ternary form equation.

$$180 = 2 \times 3 \times 30 = 2 \times 5 \times 18 = 2 \times 6 \times 15 = 2 \times 9 \times 10 = 3 \times 4 \times 15 = 3 \times 5 \times 12 = 3 \times 6 \times 10 = 4 \times 5 \times 9$$

1. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (1,0,0,0,0,0,0) + (0,1,0,0,0,0,0) + (1,1,1,0,0,0,0)$
2. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (1,0,0,0,0,0,0) + (0,0,1,0,0,0,0) + (1,2,0,0,0,0,0)$
3. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (1,0,0,0,0,0,0) + (1,1,0,0,0,0,0) + (0,1,1,0,0,0,0)$
4. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (1,0,0,0,0,0,0) + (0,2,0,0,0,0,0) + (1,0,1,0,0,0,0)$
5. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (0,1,0,0,0,0,0) + (2,0,0,0,0,0,0) + (0,1,1,0,0,0,0)$
6. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (0,1,0,0,0,0,0) + (0,0,1,0,0,0,0) + (2,1,0,0,0,0,0)$
7. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (0,1,0,0,0,0,0) + (1,1,0,0,0,0,0) + (1,0,1,0,0,0,0)$
8. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (2,0,0,0,0,0,0) + (0,0,1,0,0,0,0) + (0,2,0,0,0,0,0)$ (orthogonal)

2-factorization of the leader class $(2^2, 1, 0^4)$

The 2-factoring results in 8 binary form equations.

$$180 = 2 \times 90 = 3 \times 60 = 4 \times 45 = 5 \times 36 = 6 \times 30 = 9 \times 20 = 10 \times 18 = 12 \times 15$$

1. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (1,0,0,0,0,0,0) + (1,2,1,0,0,0,0)$
2. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (0,1,0,0,0,0,0) + (2,1,1,0,0,0,0)$
3. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (2,0,0,0,0,0,0) + (0,2,1,0,0,0,0)$ (orthogonal)
4. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (0,0,1,0,0,0,0) + (2,2,0,0,0,0,0)$ (orthogonal)
5. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (1,1,0,0,0,0,0) + (1,1,1,0,0,0,0)$
6. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (0,2,0,0,0,0,0) + (2,0,1,0,0,0,0)$ (orthogonal)
7. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (1,0,1,0,0,0,0) + (1,2,0,0,0,0,0)$
8. $(2,2,1,0,0,0,0) = (0,0,0,0,0,0,0) + (2,1,0,0,0,0,0) + (0,1,1,0,0,0,0)$

Factorization based on divisibility

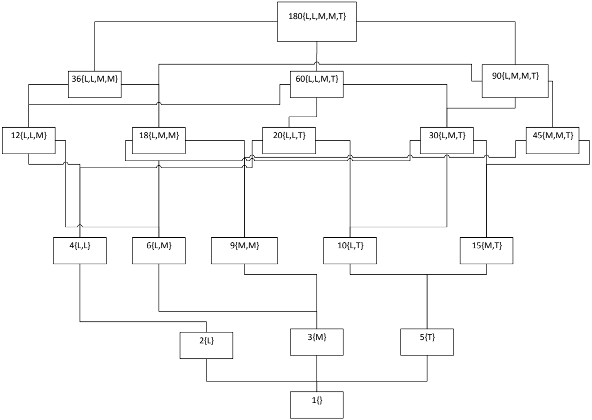

Figure: Factorization of energy based on the divisibility relation | between the natural numbers induce a lattice structure [Birkhoff, 1979].

We infer the existence of 18 canonical *n*-ary form equations representing the physical quantity energy.
The signed permutation matrix $\mathsf{P}_{\text{energy}}$ transforms the absolute leader class representative $(2, 2, 1, 0, 0, 0, 0)$ in the lattice point $(2, 1, -2, 0, 0, 0, 0)$ and is given below:

$$\mathsf{P}_{\text{energy}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

Energy constellation in $\mathbb{Z}^7$ I

We apply the matrix P_{energy} on the 18 equations.
The columns marked **i**, **j**, **k**, **l** and **m** contain the 18 lattice points in $\mathbb{Z}^7$ forming the energy constellation.

i	**j**	**k**	**l**	**m**	form equation
(0^7)	$(1, 0^6)$	$(0, 0, -1, 0^4)$	$(0, 1, 0^5)$	$(1, 0, -1, 0^4)$	$E_1 = f(\Pi) r\omega m v$
(0^7)	$(1, 0^6)$	$(0, 0, -1, 0^4)$	$(1, 1, -1, 0^4)$	(0^7)	$E_2 = f(\Pi) r\omega p$
(0^7)	$(1, 0^6)$	$(0, 1, 0^5)$	$(1, 0, -2, 0^4)$	(0^7)	$E_3 = f(\Pi) rma$
(0^7)	$(1, 0^6)$	$(1, 0, -1, 0^4)$	$(0, 1, -1, 0^4)$	(0^7)	$E_4 = f(\Pi) rv \frac{\partial m}{\partial t}$
(0^7)	$(1, 0^6)$	$(0, 0, -2, 0^4)$	$(1, 1, 0^5)$	(0^7)	$E_5 = f(\Pi) r\omega^2 \int m \, dr$
(0^7)	$(0, 0, -1, 0^4)$	$(2, 0^6)$	$(0, 1, -1, 0^4)$	(0^7)	$E_6 = f(\Pi) \omega r^2 \frac{\partial m}{\partial t}$
(0^7)	$(0, 0, -1, 0^4)$	$(0, 1, 0^5)$	$(2, 0, -1, 0^4)$	(0^7)	$E_7 = f(\Pi) \omega m \frac{\partial A}{\partial t}$
(0^7)	$(0, 0, -1, 0^4)$	$(1, 0, -1, 0^4)$	$(1, 1, 0^5)$	(0^7)	$E_8 = f(\Pi) \omega v \int m \, dr$
(0^7)	$(2, 0^6)$	$(0, 1, 0^5)$	$(0, 0, -2, 0^4)$	(0^7)	$E_9 = f(\Pi) r^2 m\omega^2$

Energy constellation in $\mathbb{Z}^7$ II

(0^7)	$(1, 0^6)$	$(1, 1, -2, 0^4)$	(0^7)	(0^7)	$E_{10} = f(\Pi) \int F \, \mathrm{d}r$
(0^7)	$(0, 0, -1, 0^4)$	$(2, 1, -1, 0^4)$	(0^7)	(0^7)	$E_{11} = f(\Pi) J\omega$
(0^7)	$(2, 0^6)$	$(0, 1, -2, 0^4)$	(0^7)	(0^7)	$E_{12} = f(\Pi) r^2 \frac{\partial^2 m}{\partial t^2}$
(0^7)	$(0, 1, 0^5)$	$(2, 0, -2, 0^4)$	(0^7)	(0^7)	$E_{13} = f(\Pi) m v^2$
(0^7)	$(1, 0, -1, 0^4)$	$(1, 1, -1, 0^4)$	(0^7)	(0^7)	$E_{14} = f(\Pi) v p$
(0^7)	$(0, 0, -2, 0^4)$	$(2, 1, 0^5)$	(0^7)	(0^7)	$E_{15} = f(\Pi) \omega^2 \iint m \, \mathrm{d}A$
(0^7)	$(1, 1, 0^5)$	$(1, 0, -2, 0^4)$	(0^7)	(0^7)	$E_{16} = f(\Pi) a \int m \, \mathrm{d}r$
(0^7)	$(2, 0, -1, 0^4)$	$(0, 1, -1, 0^4)$	(0^7)	(0^7)	$E_{17} = f(\Pi) \frac{\partial A}{\partial t} \frac{\partial m}{\partial t}$
(0^7)	$(2, 1, -2, 0^4)$	(0^7)	(0^7)	(0^7)	$E_{18} = f(\Pi) E_0$

Energy constellation in $\mathbb{Z}^7$ III

The symbols used in the column form equation have the following interpretation: E_i: energy; r: distance; t: time; ω: circular frequency; m: mass; A: area; v: speed; F: force; J: angular momentum; p: linear momentum; a: acceleration; $f(\Pi)$: dimensionless function.

An additive partition partitions an integer i into all possible sums without regard to order.

Observe that the 18 form equations are constructed from the elements $[1,1,1,4], [1,1,5], [1,6], [7]$ of the set

$$\begin{aligned} partition(7,7) = \{&[1,1,1,1,1,1,1], [1,1,1,1,1,2], [1,1,1,2,2], \\ &[1,2,2,2], [1,1,1,1,3], [1,1,2,3], [2,2,3], \\ &[1,3,3], [1,1,1,4], [1,2,4], [3,4], \\ &[1,1,5], [2,5], [1,6], [7]\} \end{aligned}$$

Canonical factorization of the Gödel number ≤ 1500 in n distinct factors I

leader class	source	deg	psc (z)	$N(\mathbf{z})$	#vertices	Gödel number	F_2	F_3	F_4	F_5
$[0^7]$	No	0	0	0	1	1	0	0	0	0
$[10^6]$	No	1	1	1	14	2	0	0	0	0
$[20^6]$	No	2	0	4	14	4	0	0	0	0
$[1^2 0^5]$	Yes	2	0	2	84	6	1	0	0	0
$[30^6]$	No	3	1	9	14	8	1	0	0	0
$[210^5]$	Yes	3	1	5	168	12	2	0	0	0
$[310^5]$	Yes	4	0	10	168	24	3	1	0	0
$[1^3 0^4]$	No	3	1	3	280	30	3	1	0	0
$[2^2 0^5]$	No	4	0	8	84	36	3	1	0	0
$[21^2 0^4]$	No	4	0	6	840	60	5	3	0	0
$[320^5]$	Yes	5	1	13	168	72	5	3	0	0
$[31^2 0^4]$	No	5	1	11	840	120	7	7	1	0
$[2^2 10^4]$	Yes	5	1	9	840	180	8	8	1	0
$[1^4 0^3]$	No	4	0	4	560	210	7	6	1	0
$[3^2 0^5]$	No	6	0	18	84	216	7	8	1	0
$[3210^4]$	Yes	6	0	14	1680	360	11	17	5	0
$[21^3 0^3]$	No	5	1	7	2240	420	11	15	4	0
$[31^3 0^3]$	No	6	0	12	2240	840	15	29	13	1
$[2^3 0^4]$	No	6	0	12	280	900	12	20	7	0
$[3^2 10^4]$	Yes	7	1	19	840	1080	15	33	17	1
$[2^2 1^2 0^3]$	No	6	0	10	3360	1260	17	35	16	1

Canonical factorization of the Gödel number ≤ 1500 in n distinct factors II

- The same methodology, as shown for the physical quantity energy, can be applied to any physical quantity.
- This will then generate for that physical quantity its complete set of canonical form equations.
- The enumeration for common leader classes with Gödel number ≤ 1500 of the factorization of the Gödel number in n distinct factors.
- The number of distinct factors is found in the respective columns F_n where $n \in [2, \ldots, 5]$.
- We conclude that there is a finite number of canonical form equations for each physical quantity.

Conclusion 1: "table of Mendeleev" for physical quantities I

1. We show that dimensionally equivalent physical SI quantities are mapped to integer lattice points of $\mathbb{Z}^7$.
2. We show that integer lattice points are partitioned, using signed permutations as equivalence relation, in absolute leader classes with representative lattice point in $\mathbb{Z}^7_+$.
3. We show that each absolute leader class has an unique Gödel number that organizes the physical quantities in a similar way as the atomic number organizes the chemical elements in the celebrated "table of Mendeleev".

Conclusion 2: Realizable unique binary form equations I

1. We show that each absolute leader has a unique 7D-hypersphere such that its lattice points are rectangles formed by 4 lattice points $\mathbf{o}, \mathbf{x}, \mathbf{y}, \mathbf{z}$ in $\mathbb{Z}_+^7$ where $\mathbf{z} = \mathbf{x} + \mathbf{y}$. The resulting rectangles are the geometric representation of the realizable binary form equations $[z] = f(\Pi)[x][y]$ for the selected physical quantity $[z]$.
2. We find in $\mathbb{Z}_+^7$ where $\|\mathbf{z}\|_\infty \leq 10$, a total of 7 747 unique rectangles out of 6 510 466 998 rectangles.

Is the cardinality of unique rectangles in $\mathbb{Z}^7$ finite and if yes, confirm that Feynman was right?

Feynman [Feynman, 1985] remarks that:

> *"What of the future of this adventure? What will happen ultimately? We are going along guessing the laws; how many laws are we going to have to guess? I do not know. Some of my colleagues say that this fundamental aspect of our science will go on; but I think there will certainly not be perpetual novelty, say for a thousand years. This thing cannot keep on going so that we are always going to discover more and more new laws. If we do, it will become boring that there are so many levels one underneath the other."*

Q & A

Thank you for your kind attention!

Any questions ?

Leader class: $[0^7]$, $\#([w]) = 1$, Gödel number=1 I

physical quantity	vertex
plane angle	(0,0,0,0,0,0,0)
solid angle	(0,0,0,0,0,0,0)
linear strain	(0,0,0,0,0,0,0)
shear strain	(0,0,0,0,0,0,0)
bulk strain	(0,0,0,0,0,0,0)
relative elongation	(0,0,0,0,0,0,0)
refractive index	(0,0,0,0,0,0,0)
electric susceptibility	(0,0,0,0,0,0,0)
mass ratio	(0,0,0,0,0,0,0)
fine-structure constant	(0,0,0,0,0,0,0)
redshift	(0,0,0,0,0,0,0)
Poisson's ratio	(0,0,0,0,0,0,0)

Leader class: $[10^6]$, $\#([w]) = 14$,Gödel number=2 I

physical quantity	vertex
length	(1,0,0,0,0,0,0)
width	(1,0,0,0,0,0,0)
height	(1,0,0,0,0,0,0)
thickness	(1,0,0,0,0,0,0)
distance	(1,0,0,0,0,0,0)
radius	(1,0,0,0,0,0,0)
diameter	(1,0,0,0,0,0,0)
path length	(1,0,0,0,0,0,0)
persistence length	(1,0,0,0,0,0,0)
length of arc	(1,0,0,0,0,0,0)
Planck length	(1,0,0,0,0,0,0)
wavelength	(1,0,0,0,0,0,0)
Compton wavelength	(1,0,0,0,0,0,0)
relaxation length	(1,0,0,0,0,0,0)
luminosity distance	(1,0,0,0,0,0,0)

Leader class: $[10^6]$, $\#([w]) = 14$,Gödel number=2, Continued

physical quantity	vertex
mass	(0,1,0,0,0,0,0)
reduced mass	(0,1,0,0,0,0,0)
Planck mass	(0,1,0,0,0,0,0)
time	(0,0,1,0,0,0,0)
period	(0,0,1,0,0,0,0)
relaxation time	(0,0,1,0,0,0,0)
time constant	(0,0,1,0,0,0,0)
time interval	(0,0,1,0,0,0,0)
proper time	(0,0,1,0,0,0,0)
Planck time	(0,0,1,0,0,0,0)
half-life time	(0,0,1,0,0,0,0)
specific impulse	(0,0,1,0,0,0,0)
electric current	(0,0,0,1,0,0,0)
thermodynamic temperature	(0,0,0,0,1,0,0)
Planck temperature	(0,0,0,0,1,0,0)

Leader class: $[10^6]$, $\#([w]) = 14$,Gödel number=2, Continued

physical quantity	vertex
thermal expansion coefficient	(0,0,0,0,-1,0,0)
amount of substance	(0,0,0,0,0,1,0)
luminous intensity	(0,0,0,0,0,0,1)
luminous flux	(0,0,0,0,0,0,1)
wave number	(-1,0,0,0,0,0,0)
optical power	(-1,0,0,0,0,0,0)
spatial frequency	(-1,0,0,0,0,0,0)
absorption coefficient	(-1,0,0,0,0,0,0)
laser gain	(-1,0,0,0,0,0,0)
rotational constant	(-1,0,0,0,0,0,0)
Rydberg constant	(-1,0,0,0,0,0,0)
frequency	(0,0,-1,0,0,0,0)
angular frequency	(0,0,-1,0,0,0,0)
circular frequency	(0,0,-1,0,0,0,0)
activity	(0,0,-1,0,0,0,0)
specific material permeability	(0,0,-1,0,0,0,0)
angular velocity	(0,0,-1,0,0,0,0)
decay constant	(0,0,-1,0,0,0,0)
Avogadro constant	(0,0,0,0,0,-1,0)

Leader class: $[20^6]$, $\#([w]) = 14$,Gödel number=4

physical quantity	vertex
area	(2,0,0,0,0,0,0)
elastic modulus	(2,0,0,0,0,0,0)
Thomson cross section	(2,0,0,0,0,0,0)
spacetime curvature	(-2,0,0,0,0,0,0)
angular acceleration	(0,0,-2,0,0,0,0)

Leader class: $[1^2 0^5]$, $\#([w]) = 84$,Gödel number=6

physical quantity	vertex
velocity	(1,0,-1,0,0,0,0)
group velocity	(1,0,-1,0,0,0,0)
volumetric flux	(1,0,-1,0,0,0,0)
speed	(1,0,-1,0,0,0,0)
speed of light in vacuum	(1,0,-1,0,0,0,0)
magnetic field strength	(-1,0,0,1,0,0,0)
magnetisation	(-1,0,0,1,0,0,0)
temperature gradient	(-1,0,0,0,1,0,0)
electric charge	(0,0,1,1,0,0,0)
electric flux	(0,0,1,1,0,0,0)
catalytic activity	(0,0,-1,0,0,1,0)
molar mass	(0,1,0,0,0,-1,0)
second radiation constant	(1,0,0,0,1,0,0)
luminous energy	(0,0,1,0,0,0,1)
linear density	(-1,1,0,0,0,0,0)
mass flow rate	(0,1,-1,0,0,0,0)

Leader class: $[30^6]$, $\#([w]) = 14$, Gödel number=8

physical quantity	vertex
volume	(3,0,0,0,0,0,0)
Loschmidt constant	(-3,0,0,0,0,0,0)
number density	(-3,0,0,0,0,0,0)

Leader class: [210^5], $\#([w]) = 168$, Gödel number=12 I

physical quantity	vertex
acceleration	(1,0,-2,0,0,0,0)
areal velocity	(2,0,-1,0,0,0,0)
mass attenuation coefficient	(2,-1,0,0,0,0,0)
radiant exposure	(0,1,-2,0,0,0,0)
diffusion constant	(2,0,-1,0,0,0,0)
thermal diffusivity	(2,0,-1,0,0,0,0)
kinematic viscosity	(2,0,-1,0,0,0,0)
quantum of circulation	(2,0,-1,0,0,0,0)
electric current density	(-2,0,0,1,0,0,0)
luminance	(-2,0,0,0,0,0,1)
illuminance	(-2,0,0,0,0,0,1)
luminous emittance	(-2,0,0,0,0,0,1)
irradiance	(-2,0,0,0,0,0,1)
magnetic dipole moment	(2,0,0,1,0,0,0)
Bohr magneton	(2,0,0,1,0,0,0)
surface density	(-2,1,0,0,0,0,0)
surface tension	(0,1,-2,0,0,0,0)
stiffness	(0,1,-2,0,0,0,0)
compliance	(0,-1,2,0,0,0,0)
moment of inertia	(2,1,0,0,0,0,0)
accelerator luminosity	(-2,0,-1,0,0,0,0)

Leader class: $[40^6]$, $\#([w]) = 14$,Gödel number=16

physical quantity	vertex
second moment of area	(4,0,0,0,0,0,0)

Leader class: $[310^5]$, $\#([w]) = 168$, Gödel number=24

physical quantity	vertex
mass density	(-3,1,0,0,0,0,0)
specific volume	(3,-1,0,0,0,0,0)
amount of substance concentration	(-3,0,0,0,0,1,0)
molar volume	(3,0,0,0,0,-1,0)
heat flux density	(0,1,-3,0,0,0,0)
Poynting vector	(0,1,-3,0,0,0,0)
radiative flux	(0,1,-3,0,0,0,0)
thermal emittance	(0,1,-3,0,0,0,0)
sound intensity	(0,1,-3,0,0,0,0)
radiance	(0,1,-3,0,0,0,0)
irradiance	(0,1,-3,0,0,0,0)
radiant exitance	(0,1,-3,0,0,0,0)
radiant emittance	(0,1,-3,0,0,0,0)
radiosity	(0,1,-3,0,0,0,0)
volume rate of flow	(3,0,-1,0,0,0,0)
jerk	(1,0,-3,0,0,0,0)

Leader class: $[1^3 0^4]$, $\#([w]) = 280$, Gödel number=30

physical quantity	vertex
electric dipole moment	(1,0,1,1,0,0,0)
linear momentum	(1,1,-1,0,0,0,0)
Faraday constant	(0,0,1,1,0,-1,0)
dynamic viscosity	(-1,1,-1,0,0,0,0)
fluidity	(1,-1,1,0,0,0,0)
magnetogyric ratio	(0,-1,1,1,0,0,0)
vacuum condensate of Higgs field	(0,1,-1,-1,0,0,0)

Leader class: $[2^2 0^5]$, $\#([w]) = 84$, Gödel number=36 I

physical quantity	vertex
absorbed dose	(2,0,-2,0,0,0,0)
dose equivalent	(2,0,-2,0,0,0,0)
specific energy	(2,0,-2,0,0,0,0)
gravitational potential	(2,0,-2,0,0,0,0)

Leader class: $[410^5]$, $\#([w]) = 168$, Gödel number=48

physical quantity	vertex
jounce	(1,0,-4,0,0,0,0)

Leader class: $[21^2 0^4]$, $\#([w]) = 840$, Gödel number=60 I

physical quantity	vertex
force	(1,1,-2,0,0,0,0)
energy density	(-1,1,-2,0,0,0,0)
radiant energy density	(-1,1,-2,0,0,0,0)
sound energy density	(-1,1,-2,0,0,0,0)
toughness	(-1,1,-2,0,0,0,0)
pressure	(-1,1,-2,0,0,0,0)
modulus of elasticity	(-1,1,-2,0,0,0,0)
Young's modulus	(-1,1,-2,0,0,0,0)
shear modulus	(-1,1,-2,0,0,0,0)
compression modulus	(-1,1,-2,0,0,0,0)
normal stress	(-1,1,-2,0,0,0,0)
shear stress	(-1,1,-2,0,0,0,0)
energy momentum tensor	(-1,1,-2,0,0,0,0)
Planck constant	(2,1,-1,0,0,0,0)
angular momentum	(2,1,-1,0,0,0,0)
action	(2,1,-1,0,0,0,0)
spin	(2,1,-1,0,0,0,0)
acoustic impedance	(-2,1,-1,0,0,0,0)
mass flux	(-2,1,-1,0,0,0,0)
magnetic flux density	(0,1,-2,-1,0,0,0)
magnetic induction	(0,1,-2,-1,0,0,0)
surface charge density	(-2,0,1,1,0,0,0)
dielectric polarisation	(-2,0,1,1,0,0,0)
electrical displacement	(-2,0,1,1,0,0,0)
electrical quadrupole moment	(2,0,1,1,0,0,0)
luminous exposure	(-2,0,1,0,0,0,1)

Leader class: $[320^5]$, $\#([w]) = 168$,Gödel number=72

physical quantity	vertex
absorbed dose rate	(2,0,-3,0,0,0,0)

Leader class: $[31^2 0^4]$, $\#([w]) = 840$, Gödel number=120

physical quantity	vertex
electric field gradient	(0,1,-3,-1,0,0,0)
electric charge density	(-3,0,1,1,0,0,0)
heat transfer coefficient	(0,1,-3,0,-1,0,0)
thermal insulance	(0,-1,3,0,1,0,0)
spectral exitance	(-1,1,-3,0,0,0,0)
spectral radiance	(-1,1,-3,0,0,0,0)
spectral irradiance	(-1,1,-3,0,0,0,0)
spectral power	(1,1,-3,0,0,0,0)
spectral intensity	(1,1,-3,0,0,0,0)
luminous energy density	(-3,0,1,0,0,0,1)
catalytic activity concentration	(-3,0,-1,0,0,1,0)
reaction rate	(-3,0,-1,0,0,1,0)

Leader class: $[2^2 10^4]$, $\#([w]) = 840$, Gödel number=180 I

physical quantity	vertex
torque	(2,1,-2,0,0,0,0)
moment of a force	(2,1,-2,0,0,0,0)
specific heat capacity	(2,0,-2,0,-1,0,0)
energy	(2,1,-2,0,0,0,0)
potential energy	(2,1,-2,0,0,0,0)
kinetic energy	(2,1,-2,0,0,0,0)
work	(2,1,-2,0,0,0,0)
Lagrange function	(2,1,-2,0,0,0,0)
Hamilton function	(2,1,-2,0,0,0,0)
Hartree energy	(2,1,-2,0,0,0,0)
ionization energy	(2,1,-2,0,0,0,0)
electron affinity	(2,1,-2,0,0,0,0)
electronegativity	(2,1,-2,0,0,0,0)
dissociation energy	(2,1,-2,0,0,0,0)

Leader class: $[3210^4]$, $\#([w]) = 1680$, Gödel number=360

physical quantity	vertex
radiant intensity	(2,1,-3,0,0,0,0)
radiant flux	(2,1,-3,0,0,0,0)
Newton constant of gravitation	(3,-1,-2,0,0,0,0)
power	(2,1,-3,0,0,0,0)
sound energy flux	(2,1,-3,0,0,0,0)
bolometric luminosity	(2,1,-3,0,0,0,0)

Leader class: $[21^3 0^3]$, $\#([w]) = 2240$, Gödel number=420 I

physical quantity	vertex
molar Planck constant	(2,1,-1,0,0,-1,0)
magnetic vector potential	(1,1,-2,-1,0,0,0)
thermal conductivity	(1,1,-2,0,-1,0,0)
thermal resistivity	(-1,-1,2,0,1,0,0)

Leader class: $[4210^4]$, $\#([w]) = 1680$, Gödel number=720

physical quantity	vertex
electric polarisability	(0,-1,4,2,0,0,0)

Leader class: $[31^3 0^3]$, $\#([w]) = 2240$,Gödel number=840

physical quantity	vertex
thermal conductivity	(1,1,-3,0,-1,0,0)
first hyper-susceptibility	(-1,-1,3,1,0,0,0)
electric field	(1,1,-3,-1,0,0,0)

Leader class: $[2^2 1^2 0^3]$, $\#([w]) = 3360$, Gödel number=1260 I

physical quantity	vertex
magnetic constant	(1,1,-2,-2,0,0,0)
permeability	(1,1,-2,-2,0,0,0)
magnetic flux	(2,1,-2,-1,0,0,0)
magnetic moment	(2,1,-2,-1,0,0,0)
entropy	(2,1,-2,0,-1,0,0)
specific heat	(2,1,-2,0,-1,0,0)
Boltzmann constant	(2,1,-2,0,-1,0,0)
Josephson constant	(-2,-1,2,1,0,0,0)
magnetic flux quantum	(2,1,-2,-1,0,0,0)
chemical potential	(2,1,-2,0,0,-1,0)
molar energy	(2,1,-2,0,0,-1,0)

Leader class: $[4310^4]$, $\#([w]) = 1680$,Gödel number=2160

physical quantity	vertex
Stefan-Boltzmann constant	(0,1,-3,0,-4,0,0)
first radiation constant	(4,1,-3,0,0,0,0)

Leader class: $[321^2 0^3]$, $\#([w]) = 6720$,Gödel number=2520

physical quantity	vertex
responsivity	(-2,-1,3,1,0,0,0)
electric potential difference	(2,1,-3,-1,0,0,0)
electric potential	(2,1,-3,-1,0,0,0)
thermal conductance	(2,1,-3,0,-1,0,0)
thermal resistance	(-2,-1,3,0,1,0,0)
electromotive force	(2,1,-3,-1,0,0,0)
luminous efficacy	(-2,1,3,0,0,0,1)

Leader class: $[2^3 1 0^3]$, $\#([w]) = 2240$, Gödel number=6300

physical quantity	vertex
inductance	(2,1,-2,-2,0,0,0)
self-inductance	(2,1,-2,-2,0,0,0)
mutual inductance	(2,1,-2,-2,0,0,0)
magnetisability	(2,-1,2,2,0,0,0)

Leader class: $[3^2 1^2 0^3]$, $\#([w]) = 3360$, Gödel number=7560

physical quantity	vertex
specific resistance	(3,1,-3,-1,0,0,0)

Leader class: $[32^2 10^3]$, $\#([w]) = 6720$, Gödel number=12600

physical quantity	vertex
electrical resistance	(2,1,-3,-2,0,0,0)
reactance	(2,1,-3,-2,0,0,0)
impedance	(2,1,-3,-2,0,0,0)
conductance	(-2,-1,3,2,0,0,0)
admittance	(-2,-1,3,2,0,0,0)
susceptance	(-2,-1,3,2,0,0,0)
characteristic impedance of vacuum	(2,1,-3,-2,0,0,0)
von Klitzing constant	(2,1,-3,-2,0,0,0)

Leader class: $[2^2 1^3 0^2]$, $\#([w]) = 6720$, Gödel number=13860

physical quantity	vertex
molar heat capacity	(2,1,-2,0,-1,-1,0)
molar gas constant	(2,1,-2,0,-1,-1,0)
molar entropy	(2,1,-2,0,-1,-1,0)

Leader class: $[431^2 0^3]$, $\#([w]) = 6720$, Gödel number=15120

physical quantity	vertex
electrical mobility	(3,1,-4,-1,0,0,0)

Leader class: $[42^2 10^3]$, $\#([w]) = 6720$, Gödel number=25200

physical quantity	vertex
electric capacitance	(-2,-1,4,2,0,0,0)

Leader class: $[3^2 2 1 0^3]$, $\#([w]) = 6720$, Gödel number=37800

physical quantity	vertex
electrical resistivity	(3,1,-3,-2,0,0,0)
electrical conductivity	(-3,-1,3,2,0,0,0)

Leader class: $[43210^3]$, $\#([w]) = 13440$,Gödel number=75600

physical quantity	vertex
electric constant	(-3,-1,4,2,0,0,0)
permittivity	(-3,-1,4,2,0,0,0)

Leader class: [73210^3], $\#([w]) = 13440$,Gödel number=604800

physical quantity	vertex
first hyper-polarisability	(-1,-2,7,3,0,0,0)

Leader class: $[62^3 0^3]$, $\#([w]) = 2240$, Gödel number=705600

physical quantity	vertex
second hyper-susceptibility	(-2,-2,6,2,0,0,0)

Leader class: $[(10)4320^3]$, $\#([w]) = 13440$, Gödel number=508032000

physical quantity	vertex
second hyper-polarisability	(-2,-3,10,4,0,0,0)

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