

AMAT November 2021

Instructions

1. You are not expected to answer all questions, unanswered questions are not penalised. Each incorrect answer deducts half a mark from your result.
2. The test consists of 30 questions — 20 are worth one mark, 10 are worth two marks. You will be able to see how many marks each question is worth during the test.
3. All questions are multiple choice. Each question presents four choices, only one is correct.
4. You are allowed to use A4 papers for working out your answers.
5. This is a closed-book examination, any access to resources and calculators are not allowed.

0.1 Question 1

What is the value of the following expression?

$$\frac{\log_5(64)}{\log_5(2)}$$

- (A) 32
- (B) 3!
- (C) 2^3
- (D) $\log_5(62)$

0.2 Question 2

$$\frac{2x}{(x^2 - 1)(x + 2)} \equiv \frac{A}{x - 1} + \frac{B}{x + 1} + \frac{C}{x + 2}$$

What is $A + B + C$?

- (A) -2
- (B) 0
- (C) $\frac{8}{3}$
- (D) None of the above

0.3 Question 3

Evaluate the following:

$$\log_{0.5} 32 - \log_{0.5} 16 + \log_{0.5} 4$$

- (A) $\frac{1}{3}$
- (B) -3
- (C) $\log_0 .520$
- (D) $2\sqrt{2}$

0.4 Question 4

Find all real solutions to this equation:

$$\binom{4}{0}x^4 - \binom{4}{1}x^3 + \binom{4}{2}x^2 - \binom{4}{3}x + \binom{4}{4} = 1$$

- (A) $x = 0$
- (B) $x = 1$
- (C) $x = 1, x = -1$
- (D) $x = 0, x = 2$

0.5 Question 5

Find the gradient of the following function at $x = -2$:

$$f(x) = |x^2 + 5x + 4|$$

- (A) 1
- (B) -1
- (C) -2
- (D) None of the above

0.6 Question 6

Evaluate the following:

$$\int_a^{2a} \ln(e^{x^2}) + \ln(x^2) + \ln(1) dx$$

- (A) $3a^3 + 6a + a \ln(16a^6)$
- (B) $\frac{7}{3}a^3 - 2a + a \ln(16a^2)$
- (C) $\frac{7}{3}a^3 - 2a + 2a \ln(4a)$
- (D) $3a^3 + 6a + 2a \ln(4a^3)$

0.7 Question 7

A curve is defined by the parametric equations:

$$x = 2 \arcsin(t), y = \tan(2t), -\pi \leq t \leq \pi$$

What is $\frac{dx}{dy}$?

- (A) $\sec^2(2t)\sqrt{1-t^2}$
- (B) $\frac{4}{\sec^2(2t)\sqrt{1-t^2}}$
- (C) $\frac{4 \cos^2(2t)}{\sqrt{1-t^2}}$
- (D) $\frac{\cos(4t)+1}{2\sqrt{1-t^2}}$

0.8 Question 8

Compute the following integral:

$$\int_{e^3}^{e^5} \frac{1}{x \ln x} dx$$

- (A) $\ln \frac{5}{3}$
- (B) $\ln 15$
- (C) $\ln \frac{3}{5}$
- (D) $\frac{5}{3}$

0.9 Question 9

Consider the following vector equations and evaluate \vec{F} at $t = 0$ and $m = 2$:

$$\vec{F} = m \frac{d\vec{v}}{dt}$$

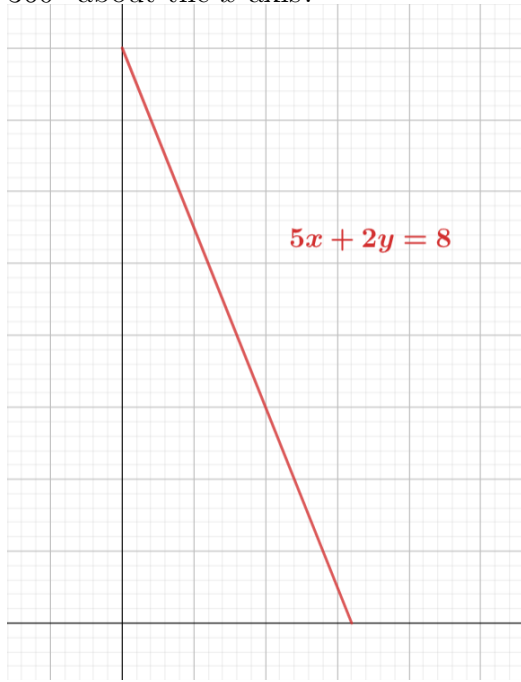
$$\vec{v} = m \frac{d\vec{s}}{dt}$$

$$\vec{s} = \begin{pmatrix} e^{\sin(t)} \\ 2^t \\ t^{\frac{3}{2}} \end{pmatrix}$$

- (A) $\begin{pmatrix} 2 \\ 2 \ln 2 \\ 0 \end{pmatrix}$
- (B) $\begin{pmatrix} 1 \\ 2(\ln 2)^2 \\ 0 \end{pmatrix}$
- (C) $\begin{pmatrix} 2 \\ 2(\ln 2)^2 \\ 0 \end{pmatrix}$
- (D) None of the above

0.10 Question 10

What is the volume of revolution when the graph below has been rotated 360° about the x -axis?



- (A) $\frac{256\pi}{75}$
- (B) $\frac{64\pi}{3}$
- (C) $\frac{32\pi}{15}$
- (D) $\frac{128\pi}{15}$

0.11 Question 11

Given that:

$$\cot(x) = \frac{1}{\tan(x)}$$

What is the exact value of:

$$\cot\left(\frac{3\pi}{4}\right)$$

- (A) 1
- (B) -1

(C) $\frac{1}{\sqrt{3}}$

(D) $\frac{-1}{\sqrt{3}}$

0.12 Question 12

What is the exact value of $\tan 15^\circ$?

(A) $2 - \sqrt{3}$

(B) $2 + \sqrt{3}$

(C) $-2 - \sqrt{3}$

(D) $-\sqrt{3} + \sqrt{2}$

0.13 Question 13

What are the first 3 terms of the Taylor series of the following function when $x = 0$?

$$f(x) = \cot(x)$$

(A) $x - \frac{x^3}{3} + \frac{x^5}{5}$

(B) $\frac{1}{x} + \frac{3}{x} + \frac{5}{x}$

(C) $\frac{1}{x} - \frac{x}{3} - \frac{x^3}{45}$

(D) $x + \frac{x^3}{3} + \frac{2x^5}{15}$

0.14 Question 14

Given that $z = \sqrt{3} - i$, which of the following is equal to z^4 ?

(A) $16 (\cos(120^\circ) + i \sin(120^\circ))$

(B) $16 (\cos(120^\circ) - i \sin(120^\circ))$

(C) $16 (\cos(-240^\circ) + i \sin(-240^\circ))$

(D) $16 (\cos(240^\circ) - i \sin(240^\circ))$

0.15 Question 15

Given that $\mathbf{a} = 7t\mathbf{i} - 4t\mathbf{j} + 5t\mathbf{k}$ and $\hat{\mathbf{a}} = \frac{7\sqrt{5}t}{15}\mathbf{i} - \frac{4t}{3\sqrt{5}}\mathbf{j} + \frac{5t}{\sqrt{45}}\mathbf{k}$, what are the possible values of t ?

- (A) $\pm\frac{\sqrt{2}}{2}$
- (B) $\pm\sqrt{\frac{45}{58}}$
- (C) $\pm\sqrt{2}$
- (D) $\pm\frac{1}{2}$

0.16 Question 16

Given that $3\mathbf{a} - 2\mathbf{b}$ is parallel to \mathbf{c} , what is k ?

$$\mathbf{a} = k\mathbf{i} - 2k\mathbf{j}$$

$$\mathbf{b} = 3\mathbf{i} + 2\mathbf{j}$$

$$\mathbf{c} = \mathbf{i} + 2\mathbf{j}$$

- (A) $\frac{10}{3}$
- (B) $\frac{7}{3}$
- (C) -1
- (D) $\frac{2}{3}$

0.17 Question 17

Two sides of triangle ABC are given as:

$$\overrightarrow{AB} = \mathbf{i} - 2\mathbf{j}$$

$$\overrightarrow{BC} = 3\mathbf{i} + 4\mathbf{j}$$

Point D is such that $\overrightarrow{AD} = 2\overrightarrow{AB}$ and point E is such that $\overrightarrow{AE} = 2\overrightarrow{AC}$. What is the area of ADE?

- (A) 5units^2
- (B) 10units^2
- (C) 15units^2
- (D) 20units^2

0.18 Question 18

Matrix M is defined as:

$$\begin{bmatrix} 1 & x \\ -1 & 2x \end{bmatrix}$$

What value of x satisfies $\det(M^2) = \det(I)$?

- (A) $\pm\frac{1}{9}$
- (B) $\pm\frac{1}{3}$
- (C) ± 1
- (D) $\pm\frac{1}{2}$

0.19 Question 19

$$A = \begin{bmatrix} x^2 + 7x + 2 & e^x \\ \sin^{-1} x & x! \end{bmatrix}$$

Note: The inverse sine function produces an output in radians.

Find the determinant(s) of matrix A where

$$x^3 - 2x + x = 0$$

- (A) $1, \frac{e\pi}{2} - 10$
- (B) $2, 10 - \frac{e\pi}{2}$
- (C) $2, 10 + \frac{e\pi}{2}$
- (D) None of the above

0.20 Question 20

Find the range of values of k for which the following function has no real roots.

$$f(x) = 2x^2 + 3kx - 6k$$

- (A) $0 < k < \frac{16}{3}$
- (B) $0 \leq k \leq \frac{16}{3}$
- (C) $\frac{-16}{3} < k < 0$
- (D) $\frac{-16}{3} \leq k \leq 0$

0.21 Question 21

Two functions are defined as:

$$f(x) = |x^2 - 4|, x \in R$$

$$g(x) = x + 2, x \in R$$

Which range of values satisfy the inequality $f(x) < g(x)$?

- (A) $-2 < x < 3$
- (B) $-2 < x < 1$
- (C) $1 < x < 3$
- (D) $x < 1$ and $x > 3$

0.22 Question 22

Calculate the gradient of the line perpendicular to:

$$y = \left(\frac{{}^7C_3}{{}^5C_3}\right)^2 x$$

- (A) $\frac{-4}{49}$
- (B) $\frac{49}{16}$
- (C) $\frac{-4}{25}$
- (D) $\frac{-36}{49}$

0.23 Question 23

Find the value of x that satisfies :

$$\sum_{r=1}^{\infty} (\sqrt{3}x)^{r-1} = 2$$

- (A) $\frac{\sqrt{3}}{6}$
- (B) $\frac{1}{\sqrt{3}}$
- (C) $\frac{\sqrt{3}}{2}$
- (D) $\frac{2}{\sqrt{3}}$

0.24 Question 24

The third term of an arithmetic series is $\frac{9}{10}x$ and the sixth term is $\frac{3}{2}x$. What is the difference between the sum of the first 50 terms and the sum of the first 10 terms?

- (A) $14x$
- (B) $256x$
- (C) $270x$
- (D) $284x$

0.25 Question 25

$$1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} \dots \infty = c$$

Determine $\ln c$?

- (A) 1
- (B) e
- (C) 0
- (D) None of the above

0.26 Question 26

Estimate the value of 0.8^6 using the first four terms of the binomial expansion of $(1 - \frac{x}{2})^6$.

- (A) $\frac{6}{250}$
- (B) $\frac{24}{100}$
- (C) 0.0024
- (D) 0.026

0.27 Question 27

In the expansion of $(4 + kx)^{-2}$, the coefficient of the x term is $\frac{1}{16}$. What is the coefficient of the x^3 term?

- (A) $\frac{1}{2}$
- (B) $\frac{1}{32}$
- (C) 2
- (D) $\frac{1}{8}$

0.28 Question 28

$$f(n) = \frac{(2n)!}{2^n n!}$$

Which of the following is $f(n)$ equivalent to?

- (A) $f(n) = (2n - 1)(2n - 3)(2n - 5)\dots 1$
- (B) $f(n) = (2n + 1)(2n - 1)(2n - 3)\dots 1$
- (C) $f(n) = (2n)(2n - 1)(2n - 2)\dots 1$
- (D) $f(n) = (2n)(2n - 2)(2n - 4)\dots 1$

0.29 Question 29

Evaluate the following expression:

$$\ln 2(1 + \ln 2(1 + \ln 2(1 + \ln 2(1 + \dots_\infty))))$$

- (A) $\frac{1}{1 - \ln 2}$
- (B) $\frac{\ln 2}{1 - \ln 2}$
- (C) Does not converge
- (D) None of the above

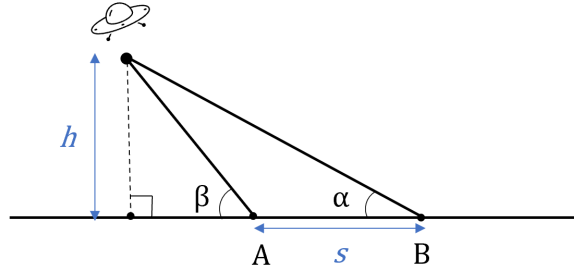


Figure 1: Question 30

0.30 Question 30

At places A and B, the elevation angles of a flying object are measured as α and β . S is the distance between A and B. Find the altitude h .

Ignore the curvature of the Earth and take the drone as a point mass.

- (A) $\frac{3\sqrt{3}}{2}S$
- (B) $\sqrt{3}S$
- (C) $\frac{\sqrt{3}}{2}S$
- (D) $\frac{\sqrt{3}}{3}S$

Question	Score	Choice
1	10	B
2	10	B
3	10	B
4	20	D
5	10	B
6	10	C
7	10	D
8	20	A
9	20	C
10	10	D
11	10	B
12	20	A
13	10	C
14	10	B
15	10	A
16	10	D
17	20	D
18	20	B
19	20	D
20	10	C
21	10	C
22	10	A
23	10	A
24	10	B
25	10	A
26	10	B
27	10	B
28	20	A
29	20	A
30	20	C

Table 1: Solutions