# **A Bayesian Approach to Modelling Subnational Spatial Dynamics of Worldwide Non-State Terrorism, 2010 - 2016**

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**Summary**. Terrorism persists as a worldwide threat, as exemplified by the ongoing lethal attacks perpetrated by ISIS in Iraq, Syria, Al Qaeda in Yemen, and Boko Haram in Nigeria. In response, states deploy various counterterrorism policies, the costs of which could be reduced through efficient preventive measures. Statistical models able to account for complex spatio-temporal dependencies have not yet been applied, despite their potential for providing guidance to explain and prevent terrorism. In an effort to address this shortcoming, we employ hierarchical models in a Bayesian context, where the spatial random field is represented by a stochastic partial differential equation. Our main findings suggest that lethal terrorist attacks tend to generate more deaths in ethnically polarised areas and in locations within democratic countries. Furthermore, the number of lethal attacks increases close to large cities and in locations with higher levels of population density and human activity.

*Keywords*: Bayesian hierarchical models; GMRF; Space-time models; SPDE; **Terrorism** 

## **1. Introduction**

Terrorism represents a worldwide threat, illustrated by the ongoing deadly attacks perpetrated by the Islamic State (also called ISIS, ISIL, or Daesh) in Iraq and Syria, Boko Haram in Nigeria, and the al-Nusra Front (also called Jabhat al-Nusra) in Syria for example. In response to this threat, states may use a combination of counterterrorism policies, which include the use of criminal justice, military power, intelligence, psychological operations, and preventives measures (Crelinsten, 2009, p. 45). In particular, following the tragic terrorist attacks on September 11, 2001 (9/11) in New York, states have tended to increase their spending to counter terrorism. From 2001 to 2008, the expenditure of worldwide homeland security increased by US\$ 70 billion

(NATO, 2008). In the US alone, the 2013 federal budget devoted to combat terrorism reached around US\$ 17.2 billion (The Washington Post, 2013).

Theoretical work and empirical studies at country-level pointed out that the causes of terrorism are complex and multidimensional and include economic, political, social, cultural, and environmental factors (Brynjar and Skjølberg, 2000; Richardson, 2006, p. 60; Gassebner and Luechinger, 2011; Krieger and Meierrieks, 2011; Hsiang et al., 2013). Moreover, the relationship between specific factors and terrorism is often not straightforward. For example, Hoffman (2006, p. 194) described the role of media in covering terrorism as a "double-edged sword". Publicity promotes terrorist groups, which facilitates the recruitment of new members and strengthens the cohesion of groups, but in turn, encourages society to marshal resources to combat terrorism (Rapoport, 1996). At the individual level, the actions and beliefs of each member of terrorist groups are important drivers of terrorism as well (Crenshaw, 1983, p.29; Wilkinson, 1990, p.141; Richardson, 2006, pp.92-93).

The activity of terrorism may also vary over time and often exhibits temporally clustered patterns like crime or insurgencies (Anselin et al., 2000; Eck et al., 2005; Zammit-Mangion et al., 2012). Similar to contagious diseases (Jacquez, 1996; Mantel, 1967; Loftin, 1986) or seismic activity (Mohler et al., 2011; Crough and Jurdy, 1980; Courtillot et al., 2003), terrorist events are rarely homogeneously distributed in space. In contrast, they tend to exhibit high concentration levels in specific locations (socalled hot-spots) (LaFree et al., 2009, 2010, 2012; Nacos, 2010; Steen et al., 2006; Piegorsch et al., 2007).

Despite successful applications of space-time Bayesian models in similar fields of research, such as crime and conflict (Zammit-Mangion et al., 2012, 2013; Mohler, 2013; Lewis et al., 2012; Rodrigues et al., 2010), these models have not yet been applied in terrorism research. Most empirical research in terrorism has focused on its temporal dimension (Hamilton and Hamilton, 1983; Porter and White, 2012; Brandt and Sandler, 2012; Barros, 2003; Suleman, 2012; Bilal et al., 2012; Enders and Sandler, 1993, 2005; Enders et al., 2011; Raghavan et al., 2013), or has considered purely spatial models only (Braithwaite and Li, 2007; Savitch and Ardashev, 2001; Brown et al., 2004). Moreover, studies that have explicitly combined both space and time dimensions have been carried out at country level, on larger geographical areas (LaFree et al., 2010; Midlarsky et al., 1980; Neumayer and Plümper, 2010; Enders and Sandler, 2006; Gao et al., 2013; LaFree et al., 2017), or at subnational level of analysis but within specific study areas (LaFree et al., 2012; Behlendorf et al., 2012; Nunn, 2007; Piegorsch et al., 2007; Ocal and Yildirim, 2010; Medina et al., 2011; Siebeneck et al., 2009; Mohler, 2013).

As a result, scholars have failed to systematically capture the subnational spatial dynamics of terrorism. Local drivers of terrorism have not been identified and their effects have not been systematically assessed. In an effort to address these shortcomings, we use space-time Bayesian models based on the stochastic partial differential equation (SPDE) approach implemented through computationally efficient integrated nested Laplace approximation (INLA) techniques (Rue et al., 2009; Lindgren et al., 2011). Our approach, which combines spatially explicit covariates and data on terrorist events, allows us to capture local-scale spatial patterns of: (i) lethality: the propensity of terrorist attack to be lethal; (ii) severity: the number of deaths given that the attack is lethal; (iii) frequency: the total number of lethal attacks per location.

By specifically accounting for the local-scale dependence structure of the data, the effects of potential drivers of the lethality of terrorism, severity, and frequency of lethal terrorist attacks across the world from 2010 to 2015 are assessed at a subnational level. The results of this study could benefit policy makers needing a systematic and spatially accurate assessment of the security threat posed by deadly terrorist activity. The paper is structured as follows. Section 2 briefly introduces the data used for the analysis. The statistical models are described in Section 2.2 and the results are provided in Section 3. Finally, conclusions and recommendations for further research are discussed in Section 4. The computer code used in this paper is available in supplementary material.

## **2. Data Selection**

### *2.1. Terrorism database*

In order to build valuable, empirically-based models, it is crucial to base an analysis on a data source that is as suitable as possible for a given study (Zammit-Mangion et al., 2012). There are currently four major databases that provide data on worldwide non-state terrorism (terrorism perpetrated by non-state actors): the Global Terrorism Database (GTD), the RAND Database of Worldwide Terrorism Incidents (RDWTI), the International Terrorism: Attributes of Terrorist Events (ITERATE), and the Global Database of Events, Language, and Tone (GDELT). ITERATE has been extensively referred to in terrorism research (Enders et al., 2011), however, events are geolocalised at the country level, which does not allow to capture subnational processes. GDELT is not suitable for our purpose since it does not provide information on the lethality of terrorist events. Equally problematic, GDELT uses a fully automated coding system based on Conflict and Mediation Event Observations (CAMEO) (for further information on CAMEO, see: http://data.gdeltproject. org/documentation/CAMEO.Manual.1.1b3.pdf), which may lead to a strong geographic bias, as mentioned by Hammond and Weidmann (2014).

Hence, RDWTI and GTD are the only potentially relevant databases that provide geolocalised terrorist events across the world. Drawing from Sheehan's approach to compare terrorism databases (2012), we defined four criteria to select the one which will be used in our study: *conceptual clarity, scope, coding method,* and *spatial ac*curacy. Given that the concept of terrorism is intrinsically ambiguous and being debated to this day (Beck and Miner, 2013), conceptual clarity in both the definition of terrorist events and the coding method used to gather data are crucial. In both GTD and RDWTI, the definition used to class an event as terrorism is clearly specified. The coding method of GTD appears more rigorous, since events are gathered

from numerous sources and articles. On a monthly average, out of 400,000 articles detected as potential sources of information, 16,000 (4%) are considered relevant (GTD, 2017). The thorough review and analysis of various independent sources performed by GTD experts limits the risk of bias resulting from possible inconsistencies in the way terrorist events are reported by the media, as pointed out by Drakos and Gofas (2006) and Drakos (2007). The data collection methodology used in RDWTI is less reliable since some events are gathered from two sources only. Moreover, the scope of GTD is wider than RDWTI. GTD is updated annually and includes more than 170,000 events from 1970 until 2016 (START, 2017), whereas RDWTI was not updated after 2009 and includes 40,129 events from 1969 to 2009 only (RAND, 2011).

Since this research investigates subnational spatial phenomena, we put particular emphasis on the spatial accuracy of the data. GTD is the only database that includes a variable assigning the spatial accuracy of each individual observation. Spatial accuracy is represented by an ordinal variable called specificity, with 5 possible levels of spatial accuracy (for further information on specificity, see GTD codebook: https://www.start.umd.edu/gtd/downloads/Codebook.pdf). Based on all these considerations we have chosen GTD as the appropriate data source for this study. The dataset contains 57,323 spatially accurate events (events corresponding to the highest levels of spatial accuracy, with specificity  $= \{1, 2\}$ , occurring between 2010 and 2016.

## *2.2. Covariates*

A thorough review of 43 studies carried out at country level by Gassebner and Luechinger (2011), along with global studies carried out at local level (Python et al., 2017; LaFree et al., 2017; Nemeth, 2010), suggested a wide range of covariates relevant to the occurrence of terrorist events. Among those, we consider covariates that satisfy two essential characteristics: (i) potential relationship with the lethality of terrorism and/or severity and/or frequency of lethal terrorist attacks; (ii) availability at high spatial resolution, in order to model subnational spatial dynamics of terrorism worldwide. Seven spatial and space-time covariates met these criteria; their potential association with terrorism is described in more detail below: satellite night light  $(lum)$ , population density  $(pop)$ , ethnic polarisation  $(ethpol)$ , political regime (pol), altitude (alt), slope (slo), travel time to the nearest large city (tt), and distance to the nearest national border *(distb)*.

First, we assess the role of economic factors, whose possible effects are still under debate. Most country-level empirical studies have not provided any evidence of a linear relationship between terrorism and gross domestic product (GDP) (Abadie, 2006; Drakos and Gofas, 2006; Gassebner and Luechinger, 2011; Krueger and Laitin, 2008; Krueger and Maleckova, 2003; Piazza, 2006), without excluding possible nonlinear relationship (Enders and Hoover, 2012). Case studies focused in the Middle East, including Israel and Palestine, showed that GDP is not significantly related to the number of suicide terrorist attacks (Berman and Laitin, 2008). Few studies, however, found that countries with high per capita GDP may encounter high levels of terrorist attacks (Tavares, 2004; Blomberg and Rosendorff, 2009). In line with the subnational nature of our study, we use NOAA satellite lights at night (Version 4 DMSP-OLS) as a covariate, which provides information about worldwide human activities on a yearly basis and at a high spatial resolution (30 arc-second grid or  $\approx 1$  |km| at the Equator) (Chen and Nordhaus, 2011; NOAA, 2014). This variable has been used as a proxy for socio-economic development measures such as per capita GDP estimation (Sutton and Costanza, 2002; Sutton et al., 2007; Elvidge et al., 2007; Ebener et al., 2005; Henderson et al., 2009).

Second, we assess the role of demography. Cities may provide more human mobility, anonymity, audiences and a larger recruitment pool in comparison to rural areas (Crenshaw, 1990, p. 115; Savitch and Ardashev, 2001). Large cities, in particular, offer a high degree of anonymity for terrorists to operate (Laqueur, 1999, p. 41). More specifically, densely populated areas appear vulnerable and are usually more prone to terrorism than sparsely populated areas (Ross, 1993; Savitch and Ardashev, 2001; Crenshaw, 1981; Swanstrom, 2002; Coaffee, 2010). In addition, locations that shelter high-value symbolic targets (buildings or installations), human targets (government officials, mayors, etc.), and public targets (public transports, shopping centres, cinemas, sport arenas, public venues, etc.) are particularly vulnerable to suicide terrorism (Hoffman, 2006, p. 167). We use the  $(v4)$  Gridded Population of the World (CIESIN, 2016), which provides population density on a five-year basis and at high-resolution (30 arc-second grid). Moreover, terrorists usually require free and rapid movement by rail or road in order to move from and to target points (Heyman and Mickolus, 1980; Wilkinson, 1979, p. 189). We compute the travel time from each terrorist event to the nearest large city (more than 50,000 inhabitants) based on Travel Time to Major Cities (Nelson, 2008) at a high spatial resolution (30 arc-second grid).

Third, we assess the role of geographical variables: altitude, surface topography (slope), and distance to the nearest national border. Although the relationship between altitude, slope, and terrorism is not straightforward, both variables provide geographical information on the location of the events, which could be a determining factor for terrorists regarding their choice of target (Ross, 1993). Moreover, Nemeth et al. (2014) suggested that distance to the nearest national border and altitude or slope might have an impact on terrorist activity. We extract both variables from NOAA Global Relief Model (ETOPO1), which provides altitude values at high spatial resolution (1 arc-minute grid) (Amante and Eakins, 2009).

Fourth, we assess the role of democracy. Under-reporting biases may occur especially in non-democratic countries where the press is often not free (Drakos and Gofas, 2006; Drakos, 2007). We extract the level of democracy from Polity IV Project, Political Regime Characteristics and Transitions, 1800-2016 (Polity IV) (Marshall and Elzinga-Marshall, 2014). Polity IV informs about the level of freedom of press, and captures the level of democracy from  $-10$  (hereditary monarchy) to  $+10$  (consolidated democracy) for most independent countries from 1800 to 2016. It has been

commonly referred as proxy for measuring the type of regime or the extent of constraints in democratic institutions (Gleditsch, 2007; Li, 2005; Piazza, 2006).

Fifth, we assess the role of ethnic division, proxied by ethnic polarisation gathered from the data provided in Python et al. (2017), who computed several indices at subnational level, based on "Georeferencing of ethnic groups" (GREG) ethnic database—the digitalised version of the Soviet Atlas Narodov Mira (ANM) which counts 1,276 ethnic groups around the world (Weidmann et al., 2010). Ethnic division is commonly measured through *ethnic fractionalisation* or *ethnic polarisation*. The former represents the probability that two individuals randomly chosen within an area belong to two different ethnic groups. The maximum occurs when each individual belongs to a different ethnic group. The latter highlights the ability of ethnic groups to engage in a conflict and reaches a maximum in areas with two confronting ethnic groups of equal size (Esteban et al., 2012; Alesina et al., 2003; Montalvo and Reynal-Querol, 2005).

In the context of ethnic conflict, terrorists may deliberately target ethnically polarised societies in order to rise ethnic tensions, which may lead to ethnic conflict. Consequently, government may carry out disproportionate and violent repressive measures towards the warring parties and innocent people, who in turn might end up supporting terrorist organisations to fight government policies (Posen, 1993; Kalyvas, 2006; Kalyvas and Kocher, 2007). Therefore, we expect that ethnically polarised areas should be more targeted by terrorist groups, who may use terrorism as a "provocation" strategy (Fromkin, 1975; Kydd and Walter, 2006).

#### *2.3. SPDE Framework*

We assume that the characteristics of terrorism under consideration—lethality of terrorism, severity, and frequency of lethal terrorist attacks—are continuous phenomena with Gaussian properties, which exhibit dependencies in both space and time. We suggest modelling their spatial dynamics through the SPDE approach introduced by Lindgren et al. (2011). The solution of the SPDE given in Equation (1) is a Gaussian field (GF) (Lindgren et al., 2011), whose approximation represents a Gaussian Markov random field (GMRF) used herein to model the spatio-temporal dependencies inherent in the data. The linear SPDE can be formulated as:

$$
(\kappa^2 - \Delta)^{\gamma/2}(\tau\zeta(s)) = \epsilon(s), \quad s \in \mathcal{D} , \tag{1}
$$

with the Laplacian  $\Delta$ , smoothness parameter  $\gamma = \lambda + 1$  (for two-dimensional processes), scale parameter  $\kappa > 0$ , variance parameter  $\tau$ , domain  $\mathcal{D}$ , and Gaussian spatial white noise  $\epsilon(s)$  (Blangiardo and Cameletti, 2015, chapter 6). The stationary solution of Equation (1) is the GF  $(\zeta(s))$  with Matérn correlation function:

$$
Cov(\zeta(\mathbf{s}_i), \zeta(\mathbf{s}_j)) = \sigma_{\zeta}^2 \frac{1}{\Gamma(\lambda)2^{\lambda-1}} \left( \kappa \left\| \mathbf{s}_i - \mathbf{s}_j \right\| \right)^{\lambda} K_{\lambda} \left( \kappa \left\| \mathbf{s}_i - \mathbf{s}_j \right\| \right), \tag{2}
$$

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where  $\|\bm{s_i} - \bm{s_j}\|$  is the Euclidean distance between two locations,  $\sigma_{\zeta}^2$  is the marginal variance, and  $K_{\lambda}$  is the modified Bessel function of the second kind and order  $\lambda > 0$ . The distance from which the spatial correlation becomes negligible (for  $\lambda > 0.5$ ) is given by the range r (vertical dotted line in Figure 1, *left, centre* and *right*), which can be empirically derived from the scale parameter  $r = \sqrt{8\lambda}/\kappa$  to be estimated. The GF  $(\zeta(s))$  is approximated as a GMRF through a finite element method using basis functions defined on a constrained refined Delaunay triangulation (mesh) over the earth, modelled as a sphere (Figure 2) (Lindgren, 2015; Lindgren et al., 2011). Here, we use a Bayesian hierarchical modelling framework (Banerjee et al., 2014) to model (i) the lethality ( $L$ -model) of terrorism, (ii) the severity ( $S$ -model) of lethal terrorist attacks, and (iii) the frequency of lethal terrorist attacks (F-model).



Fig. 1: Matérn correlation function. Matérn correlation function (solid line) with parameters  $\lambda = 1$  and its  $CI_{95\%}$  (dashed lines) illustrated for the (left): lethality of terrorism (L-model), (centre): severity of lethal attacks (S-model), and (right): frequency of lethal attacks (F-model). The corresponding posterior mean range of the Matérn correlation function parameters are:  $r_L \approx 192$  [km] (left),  $r_S \approx 245$  [km] (centre), and  $r_F \approx 192$  [km] (right). The vertical dotted lines (left, centre, right) correspond to spatial correlation  $\approx 0.1$  (*horizontal dotted line*)

#### *2.4. Modelling the Lethality, Severity and Frequency of Lethal Terrorism*

Terrorist attacks occur at specific locations on earth and are therefore inherently discrete. However, we consider three characteristics of terrorism: potential to kill (lethality  $L$ ), number of deadly victims (severity  $S$ ), and the number of lethal attacks (frequency  $F$ ) as continuous phenomena (in the sense of geostatistics (Cressie, 1991)), which are observed at s locations (attacks) over the surface of the earth  $\mathbb{S}^2$  at time  $t \in \mathbb{R}$ . A better understanding of these three facets of lethal terrorism by city



Fig. 2: Constrained refined Delaunay triangulation (mesh). Illustrations of the meshes on top of which the SPDE and its approximate solutions (GMRF) are constructed (Lindgren, 2012). We used mesh with  $5,706$  vertices (*left*) to fit the models and rougher meshes for robustness tests, with  $2,157$  (centre) and  $1,341$  (right) vertices, respectively

planners, emergency managers, insurance companies, and property administrators may allow them to better allocate resources used to prevent and counter terrorism (Piegorsch et al., 2007; Nunn, 2007).

The lethality, frequency, and severity are assumed to be the realisations of continuously indexed space-time processes  $L(s, t), S(s, t), F(s, t) \equiv \{l(s, t), s(s, t), f(s, t):$  $(s, t) \in \mathcal{D} \subseteq \mathbb{S}^2 \times \mathbb{R}$ , from which inference can be made about the processes at any desired locations in the space-time domain D (Cameletti et al., 2013). Hence, we define three corresponding hierarchical models:

(a) L-model (lethality of terrorism):

$$
L(\boldsymbol{s}_i, t) | \boldsymbol{\theta_L}, \tilde{\zeta}_L(\boldsymbol{s}_i, t) \sim \text{Bernoulli}(\pi(\boldsymbol{s}_i, t))
$$
 (3a)

$$
logit(\pi(s_i, t)) = \beta_{L0} + \boldsymbol{x}_L(s_i, t)\boldsymbol{\beta}_L + \tilde{\zeta}_L(s_i, t) + \epsilon_L(s_i, t)
$$
(3b)

 $\boldsymbol{\theta}_L \sim p(\boldsymbol{\theta}_L).$  (3c)

(b) S-model (severity of lethal terrorism):

$$
S(\boldsymbol{s}_j, t) | \boldsymbol{\theta_S}, \tilde{\zeta}_S(\boldsymbol{s}_i, t) \sim \text{Poisson}(\mu_S(\boldsymbol{s}_j, t))
$$
(4a)

$$
\log(\mu_S(\mathbf{s}_j, t)) = \beta_{S0} + \mathbf{x}_S(\mathbf{s}_j, t)\beta_S + \tilde{\zeta}_S(\mathbf{s}_j, t) + \epsilon_S(\mathbf{s}_j, t)
$$
(4b)

 $\theta_S \sim p(\theta_S)$ . (4c)

(c) F-model (frequency of lethal terrorism):

$$
F(\boldsymbol{s}_k, t) | \boldsymbol{\theta}_F, \tilde{\zeta}_F(\boldsymbol{s}_k, t) \sim \text{Poisson}(\mu_F(\boldsymbol{s}_k, t))
$$
 (5a)

$$
\log(\mu_F(\mathbf{s}_k, t)) = \beta_{F0} + \mathbf{x}_F(\mathbf{s}_k, t)\boldsymbol{\beta}_F + \tilde{\zeta}_F(\mathbf{s}_k, t) + \epsilon_F(\mathbf{s}_k, t)
$$
(5b)

$$
\boldsymbol{\theta}_F \sim p(\boldsymbol{\theta}_F). \tag{5c}
$$

The lethality of terrorism  $L(s_i, t)$  is a dichotomous variable that takes the value 1 if the attack generated one or more deaths, and 0 if not. The random variable is assumed to come from a Bernoulli distribution (Equation (3a)). From GTD (2017), information on lethality is gathered from all observed space-time locations  $s_i =$  $\{1, \ldots, 45, 555\}$  of terrorist attacks that occurred worldwide between year  $t = 2010$ and  $t = 2015$  (we kept 2016 for predictions). The linear predictor  $\logit(\pi(\mathbf{s}_i, t))$  of the L-model includes a vector of m covariates  $\boldsymbol{x}_L(s_i,t) = (x_{L1}(s_i,t), \ldots, x_{Lm}(s_i,t))$ with coefficient vector  $\beta_L = (\beta_{L1}, \ldots, \beta_{Lm})'$ , a GMRF  $\tilde{\zeta}_L(\mathbf{s}_i, t)$ , and Gaussian white noise  $\epsilon_L(s_i, t) \sim \mathcal{N}(0, \sigma_{\epsilon_L}^2)$ , with measurement error variance  $\sigma_{\epsilon_L}^2$  (Equation (3b)).

The severity  $S(s_i, t)$  counts the number of deaths given that the attack is lethal and follows a Poisson distribution (Equation (4a)). Here, the observed number of deaths represents a subset consisting of  $s_j = \{1, \ldots, 24, 049\}$  space-time locations. The linear predictor  $log(\mu_S(s_j,t))$  (Equation (4b)) of the S-model, includes a vector of l covariates  $x_S(s_j,t) = (x_{S1}(s_j,t), \ldots, x_{S1}(s_j,t))$  with coefficient vector  $\beta_S =$  $(\beta_{S1},\ldots,\beta_{Sm})'$ , a GMRF  $\tilde{\zeta}_S(s_j,t)$ , and Gaussian white noise  $\epsilon_S(s_j,t) \sim \mathcal{N}(0,\sigma_{\epsilon_S}^2)$ , with measurement error variance  $\sigma_{\epsilon_S}^2$ .

The frequency  $F(\mathbf{s}_k, t)$  counts the number of lethal attacks that occurred between  $t = 2010$  and  $t = 2015$  within a 0.5° radius ( $\approx 55$  [km] at the Equator) of cities' centroids, which is consistent with spatial uncertainty in the data (see Section 2.1). We assume that  $F(s_k, t)$  follows a Poisson distribution. Since we model a "count" variable, it is convenient to aggregate events that occurred in very close locations within identical municipality areas for example. This has resulted in spatial aggregation reducing the number of observations from 24,049 to  $s_k = \{1, \ldots, 6, 948\}$ locations. The linear predictor  $log(\mu_F(\mathbf{s}_k t))$  (Equation (5b)) of the F-model, includes a vector of q covariates  $\mathbf{x}_F(s_k, t) = (x_{F1}(s_k, t), \ldots, x_{Fq}(s_k, t))$  with coefficient vector  $\beta_F = (\beta_{F1}, \ldots, \beta_{Fq})'$ , a GMRF  $\tilde{\zeta}_F(s_k, t)$ , and Gaussian white noise  $\epsilon_F(\mathbf{s}_k,t) \sim \mathcal{N}(0,\sigma_{\epsilon_F}^2)$ , with measurement error variance  $\sigma_{\epsilon_F}^2$ .

In order to minimise the complexity of the models, and consequently, reduce the computing time required to fit them, we assume a separable space-time covariance (Blangiardo and Cameletti, 2015, chapter 7). The results of a preliminary analysis using a first-order autoregressive model  $(AR(1))$  provided a posterior mean value of the autoregressive parameter of the  $AR(1)$  very close to 1 (results not reported). Hence, we let the GMRF  $(\zeta_L)$  follow a first-order random walk  $(RW(1))$ , so that:  $\tilde{\zeta}_L(\boldsymbol{s}_i,t)|\tilde{\zeta}_L(\boldsymbol{s}_{i,t-1})\sim \mathcal{N}(\tilde{\zeta}_L(\boldsymbol{s}_{i,t-1}),\sigma_{rw}^2),$  with  $Cov(\tilde{\zeta}_L(\boldsymbol{s}_i,t),\tilde{\zeta}_L(\boldsymbol{s}_l,u))=0$  if  $t\neq u$ , and  $Cov(\tilde{\zeta}_L(\mathbf{s}_i), \tilde{\zeta}_L(\mathbf{s}_l))$  if  $t = u, \forall t, u \in \{2010, \ldots, 2015\}$ . Likewise,  $\tilde{\zeta}_S$  and  $\tilde{\zeta}_F$  follow a RW(1) with a separable space-time covariance.

Drawing from recent development of complex spatio-temporal latent Gaussian models (Jones-Todd et al., 2017), interactions between the lethality of terrorism and severity of lethal terrorist events might be represented through a joint latent modelling approach. However, we did not opt for this approach, since it resulted in a non-significant multiplier parameter of the shared GMRF (zero included within the 95% credible interval  $(CI_{95\%})$ , which considerably decreased the predictive performance of the model (results not reported).

Prior distributions are set for the parameters to be estimated  $(\theta_L, \theta_S, \theta_F)$  (Equations (3c, 4c, and 5c)). We use default priors provided by R-INLA, which consist in i.i.d. zero-mean Normal distributions, with precision equals to 0.001 for the fixed effects  $(\beta_L, \beta_S, \beta_F)$  and unstructured random effects  $(\epsilon_L, \epsilon_S, \epsilon_F)$ . An improper uniform prior between  $-\infty$  and  $\infty$  is set on the intercepts  $(\beta_{L0}, \beta_{S0}, \beta_{F0})$ .

With regard to the parameters of the Matérn correlation function  $\tilde{\zeta}_L, \tilde{\zeta}_S$ , and  $\tilde{\zeta}_F$ , we use Penalised complexity (PC) priors—informative priors invariant to reparameterisation which favour parsimonious model formulations based on the Kullback-Leibler divergence (Kullback and Leibler, 1951) and allow the user to set sensible scale and values for the investigated parameter (Simpson et al., 2017; Fuglstad et al., 2017). We set a 50% probability that: (i) the standard deviation is above 1 and (ii) the range is above 0.3 ( $\approx 1,911$  [km]).

These weakly informative PC priors assume: (i) a relative high variation in the spatial correlation of terrorist events, and (ii) terrorist events should not substantially influence each other beyond relatively large distances ( $\approx 1,911$  [km]) e.g. through demonstration and imitation processes promoted by the media (Brosius and Weimann, 1991; Enders et al., 1992; Brynjar and Skjølberg, 2000). This threshold seems reasonable for most attacks, apart from those highly publicised (e.g. masscasualty attacks perpetrated by ISIS), which may have a wider (sometimes worldwide) influence on other terrorist groups or individuals to commit copycat attacks (Nacos, 2016, p. 112).

Without any specific prior knowledge on the temporal structure, we use the default PC prior settings suggested in R-INLA for the RW(1) temporal structure of the GMRF with variance  $\sigma_{rw}^2$ , where  $P(\sigma_{rw}^2 > 1 = 0.01)$ , which assumes a variance very likely lower than 1. For prior sensitivity analysis, the results are compared with alternative priors on the parameters of the Matérn covariance function, which is further discussed in Section 3.

## **3. Results**

## *3.1. Explaining the Spatial Dynamics of Terrorism*

We use INLA as an accurate and computationally effective model fitting alternative to MCMC (Rue et al., 2009; Held et al., 2010; Simpson et al., 2016; Martino and Rue, 2010). With a 12-core Linux machine (Intel Xeon<sup>®</sup> cpu, 99 GB RAM, Open Multi-Processing parallel computing),  $R$ -INLA (coded in C) requires between 9 to 14 hours to fit each model using high resolution mesh (5,706 vertices, see Figure 2). We select the most parsimonious models through backward elimination, which initiates with all available covariates and, at each step, removes one by one the variable, whose loss minimises potential increase of the Watanabe-Akaike Information Criterion (WAIC). Thus, we select the models with the lowest WAIC (Watanabe, 2010). One may reasonably assume that that differences across countries have some influence on some characteristics of terrorism. The S and F models include a country random-effect. However it is not included in the L-model since it decreases considerably the fitting performance (results not reported).

In line with most country-level studies (Ross, 1993; Savitch and Ardashev, 2001; Crenshaw, 1981; Swanstrom, 2002; Coaffee, 2010), the F-model (Table 1) suggests that a higher number of lethal terrorist attacks are expected in areas with higher levels of population density and human activity  $(CI_{95\%} \; \beta_{pop}, \beta_{lum} > 0)$  and close to large cities  $(CI_{95\%} \beta_{tt} < 0)$ . While fewer deaths are expected close to large cities  $(CI_{95\%} \beta_{tt} > 0)$ , the S-model shows that the number of deaths tends to increase in locations within democratic countries  $(CI_{95\%}$   $\beta_{pol} > 0)$ . As pointed out by Li (2005), the presence of freedom of speech, movement, and association in democratic countries might reduce the costs to conduct terrorist activities compared to those in autocratic countries. More specifically, the results provide support to Pape (2003, 2006), who expects democracies to be more heavily targeted by high-casualty attacks (e.g. suicide terrorism), which often aim at coercing democratic governments to withdraw their presence from areas considered by terrorists as their motherland.

Consistent with country-level (Gassebner and Luechinger, 2011; Kurrild-Klitgaard et al., 2006) and subnational-level (Nemeth et al., 2014; Python et al., 2017) findings, we found that ethnic polarisation is positively associated with the severity of the attacks (S-model) ( $CI_{95\%}$   $\beta_{ethpol} > 0$ ), which supports the theory that terrorism is used as a strategy of provocation (Section 2.2). These results are further discussed in Section 4. Despite potential effects  $(CI_{95\%} \, \beta_{tt}, \beta_{border} > 0)$  observed in the L-model, they are not further analysed. The effects of  $\beta_{border}$  and  $\beta_{tt}$  are very weak—a 10<sup>-3</sup> order of magnitude at the limits of their 95% CI.

As an illustration, we compare the effect of an hypothetical 50% increase in luminosity (lum) on the expected number of lethal attacks ( $\mu_F$  in the F-model), which corresponds to an increase from 0 to  $\approx 0.23$  on a standardised scale. With all predictors held equal to 0, the linear predictor (Equation (5b)) equals 0 and  $\mu_F =$  $\exp(\eta) = \exp(0) = 1$ . With 50% more luminosity,  $\eta = \beta_{lum} \times$ lum ≅  $0.59 \times 0.23 \approx 0.14$  $(\beta_{lum} \cong 0.59)$ , see Table 1, columns "F-model"). Hence,  $\mu_F \cong \exp(0.14) \cong 1.15$ . It results that 50% more luminosity increases the expected number of lethal attacks by approximately 15%.

Furthermore, we compare the *in-sample* predictive performance between the selected models through the WAIC (see Table 1, column 'GoF'). All selected models show better performance than the zero-covariate models, indicated by lower WAIC values. We further investigate the *out-of-sample* predictive performance of the models, based on the conditional predictive ordinate (CPO) (column 'CPO' in Table 1) a "leave-one-out" cross-validation approach, where  $CPO_i = P(y_i|y_{-i})$  predicts the

value of observation  $y_i$  given the values observed elsewhere  $y_{-i}$  (Gilks et al., 1996; Gelfand et al., 1992; Gilks et al., 1996; Pettit, 1990). In addition, we use the rootmean square error (RMSE) to compare the ability of the models to predict 2016 values using data from five previous years (2010-2015) exclusively.

With regard to the *out-of-sample* predictive performance, the performance of the L, S and F models compared with their respective zero-covariate models appears very similar, which indicates that the GMRF is the main driver and the effects of the covariates are of less importance for the purpose of out-of-sample predictions (see Table 1, column 'GoF').



Table 1: Lethality of terrorism (L-model), and severity (S-model) and frequency (F-model) of lethal F-model) of lethal terrorist attacks: summary. Posterior mean, standard deviation, and  $95\%$  credible intervals (CI) of the intercepts, terrorist attacks: summary. Posterior mean, standard deviation, and 95% credible intervals (CI) of the intercepts, coefficients of the standardised covariates, parameters of the GMRFs, and predictive performance of the models. coefficients of the standardised covariates, parameters of the GMRFs, and predictive performance of the models.S-model) and frequency (1)  $L$ -model), and severity ( $\lambda$ Table 1: Lethality of terrorism (

### *3.2. Quantifying the Uncertainty*

As a further step, we explore the spatial dynamics of terrorism by visualising relevant parameters that vary in both space and time. For this purpose, the posterior mean and standard deviation of the GMRFs provide valuable insight into the understanding of the spatial dynamics of terrorism and more particularly, of the uncertainty in the predictions of the lethality  $(\pi(s,t))$  of terrorism, and severity  $(\mu_S(s,t))$  and frequency  $(\mu_F(\mathbf{s}, t))$  of lethal terrorist attacks. High values of the posterior standard deviation signify that there is high uncertainty with regard to the estimated values of the posterior mean of the GMRFs, mainly due to the scarcity or absence of data. Some areas have not encountered any terrorist attack during the entire period and therefore exhibit persistently high values, such as Siberia, the Amazonian region, Central Australia, or Greenland. In contrast, several regions strongly affected by terrorism in South America, Africa, Gulf Peninsula, India and Pakistan exhibit less uncertainty and therefore lower values of the posterior standard deviation in 2010 (Figures 3b, 4b, and 5b) and 2015 (Figures 3e, 4e, and 5e).

#### *3.3. Hot-spots*

The identification of hot-spots is highly valuable since it highlights areas more vulnerable to terrorism, which call for increased vigilance. We identify regions of abnormally high values (hot-spots) of the lethality of terrorism, and the severity and frequency of lethal attacks from the estimated posterior distribution of the variable of interest, interpolated in all space-time locations  $s, t \in \mathcal{D}$ . For each year  $(t = 2010, \ldots, 2015)$ , we identify locations (using a 1, 440  $\times$  720 grid) where the 95% credible interval (CI) for the corresponding variable is above a threshold  $\alpha$ :

$$
H_{CI_{95\%}}\nu(\boldsymbol{s},t) > \alpha, \quad \boldsymbol{s}, t \in \mathcal{D},\tag{6}
$$

where  $\alpha$  is the threshold,  $\mathcal{D}$  is the domain, and  $H_{CI_{95\%}}\nu(\mathbf{s},t)$  is the lower bound of the 95% CI of the variable under consideration (lethality, severity, and frequency). We consider the group of contiguous locations  $\nu(s, t)$ —we used a first-order neighbourhood to define contiguity—that satisfy Equation (6); for lethality, we use  $\alpha = 0.5$ , which means that we define hot-spot areas where it is more likely to have lethal attacks than non-lethal attacks. In other terms, we are 95% confident that the true value of the probability of lethal attack is greater than 50%. For severity and frequency hot-spots, we use a threshold corresponding to the  $90<sup>th</sup>$  percentile of the observed corresponding variable in the sample; namely 9 deaths and 11 lethal attacks, respectively.

Important changes are observed in the lethality of terrorism (Figures 6a, 6b), and severity (Figures 6c, 6d) and frequency of lethal terrorist attacks (Figures 6e, 6f) in various locations in Afghanistan from 2010 to 2015.



Fig. 3: Bernoulli (L-model): illustrative maps for the lethality of terrorism. Bernoulli (L-model) model values of  $\sigma_{\zeta_L(s,t)}$  in e.g. Siberia or Amazonian areas due to the sparsity or absence of terrorist events (top centre and L-model) model  $\sigma_{\tilde{c}_L(s,t)}$  in e.g. Siberia or Amazonian areas due to the sparsity or absence of terrorist events (top centre and of the lethality of terrorist attacks with GMRF posterior mean  $\zeta_L(s,t)$  (*left*), posterior standard deviation  $\sigma_{\zeta_L(s,t)}^z$  (*centre*), and posterior mean probability of lethal attack  $\pi(s,t)$  (*right*) estimated in th  $\pi(s,t)$  (right) estimated in the 45,555 locations of mesh 2. Illustrative projected maps provide values on land surface for years  $t = 2010$  (top) and  $t = 2015$  (bottom). Note the presence of high uncertainty expressed through high  $L(\mathbf{s}, t)$  ( $left$ ), posterior standard deviation L-model): illustrative maps for the lethality of terrorism. Bernoulli ( of the lethality of terrorist attacks with GMRF posterior mean  $\tilde{\zeta}$ U. ∈s(centre), and posterior mean probability of lethal attack vertices and interpolated in all locations on land surface Fig. 3: Bernoulli (1 bottom centre).bottom centre)

(d) post. mean  $\tilde{\zeta}_L(s, 2015)$ 

(d) post. mean  $\zeta_L(s, 2015)$ 

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Fig. 4: lethal attacks with GMRF posterior mean scale), and posterior severity of lethal attacks log( $\mu_S(\boldsymbol{s},t)$ )) (right) (logarithmic scale) estimated in the 24,049 locations of mesh vertices and interpolated for all locations s ∈ S2. Illustrative projected maps provide values in land surface for years  $t = 2010$  ( top) and  $t = 2015$  ( bottom)

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Fig. 5:  $F$ -model: illustrative maps for the frequency of lethal terrorist attacks. Poisson model of the F-model: illustrative maps for the frequency of lethal terrorist attacks. Poisson model of the frequency of lethal attacks with GMRF posterior mean  $\tilde{\zeta}_F(s,t)$  (left), posterior standard deviation  $\log(\sigma_{\tilde{\zeta}_F(s,t)})$  (centre)  $F(s, t)$ ) (right) (logarithmic scale) estimated in the  $^2$  . Illustrative projected maps provide values frequency of lethal attacks with GMRF posterior mean  $\tilde{\zeta}_F(s,t)$  (left), posterior standard deviation log( U. ∈µ Fs6,948 locations of mesh vertices and interpolated for all locations (logarithmic scale), and posterior frequency of lethal attacks log( in land surface for years  $t = 2010(top)$  and  $t = 2015(tobton)$ in land surface for years  $t = 2010$  ( $top$ ) and  $t = 2015$  ( $bottom$ )

(d) post. mean  $\zeta_F(s, 2015)$ 

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Fig. 6: Lethality, severity and frequency hot-spots (2010 and 2015): illustrative maps. The maps highlights the main areas impacted by high levels of lethal terrorism from 2010 to 2015 and show hot-spots of lethality (2010: Figure 6a, 2015: Figure 6b), severity (2010: Figure 6c, 2015: Figure 6d), and frequency of lethal attacks (2010: Figure 6e, 2015: Figure 6f)). Hot-spots are identified if the lower bound of the 95% credible intervals is: (i) lethality:  $\pi(s) > 0.5$ ; (ii) severity  $log(\mu_S(s)) > 2.2 = log(9)$ ; (iii) frequency  $log(\mu_F(s)) > 2.4 = log(11)$ .

## *3.4. Robustness tests*

The choice of prior distribution remains challenging even for stationary GMRFs characterised only by range and marginal variance (Fuglstad et al., 2017). As a robustness test, we run a prior sensitivity analysis for the  $L, S$ , and the  $F$  models changing the PC prior distribution of the parameters of the Matérn correlation function (range and standard deviation). For all three models, we chose the typical standard deviation  $\sigma = 2$ , so that  $P(\sigma > 2 = 0.5)$ , which corresponds to twice the default value. In this framework, we assume a higher variation in the spatial structure. Furthermore, in order to assess potential effect with a more limited spatial autocorrelation a priori, we set the typical range  $r = 1,000$  [km] for the L, S, and F models, such that  $P(r > 1,000$  [km] = 0.5).

The mean and the credible intervals of the estimated coefficients  $\beta$  and the parameters of the Matérn covariance function  $(\tau, r)$  are illustrated for the L (Figure 7a), S (Figure 7b), and F (Figure 7c) models, where default priors  $\bullet$ ) and modified priors  $(\blacktriangle)$  are specified. For better interpretability, priors are set on the standard deviation  $\sigma$  instead of  $\tau$ , with  $P(\sigma > 1 = 0.5)$  and  $P(r > 1, 911$  [km])  $> 0.5$ . The models with alternative priors use a higher typical variance, with  $P(\sigma_{\zeta} > 2 = 0.5)$ , and a smaller typical range, with  $P(r > 1,000$  [km])  $> 0.5$ . Both mean and credible intervals of all parameters are not affected.

In addition, we assess potential effects of the mesh size on the results. Using two different meshes with number of vertices  $(n_1 = 2, 157, 157, n_1)$ , the results of the L-model are not affected except that  $\beta_{ethpol}$  is negative and marginally significant with mesh size  $n_2$ . In the S-model, all covariate coefficients are robust except that  $\beta_{lum}$  becomes negative and significant with mesh size  $n_1$  and  $n_2$ . The results of the F-model are not affected. As a complementary robustness test, we assess potential effects on the results of the  $F$ -model by using different levels of spatial aggregation of the data based on different radius values (rad =  $\{0.25^{\circ}, 0.75^{\circ}, 1^{\circ}, 1.5^{\circ}\}\)$ . Except for  $\beta_{lum}$ , which loses significance at a 1.5° level of aggregation, all other coefficients are robust to all four alternative levels of aggregation (results not reported).

## **4. Discussion**

This study proposes a Bayesian hierarchical framework to model the lethality of terrorism, severity, and frequency of lethal terrorist attacks across the world between 2010 and 2015. The statistical framework integrates spatial and temporal dependencies through a GMRF whose parameters have been estimated with R-INLA. The novelty of this study lies in its ability to systematically capture the effects of factors that explain the aforementioned three characteristics of lethal terrorism worldwide and at subnational levels. Moreover, the analysis of hot-spots at subnational level provides key insight into understanding the spatial dynamics of lethal terrorism that occurred worldwide from 2010 to 2015. In this Section, we highlight the main findings and limitations of this study, and suggest potential improvement, which could be carried out in future studies.



frequency (Figure  $7c$ ) of lethal terrorist attacks), the results are robust to the choice of priors.  $\tau$  of the Matérn covariance function). On all three models (lethality of terrorism (Figure 7a), severity (Figure 7b) and the parameters of the Matérn covariance function. Estimation of the mean and the  $95\%$  credible intervals (line frequency (Figure 7c) of lethal terrorist attacks), the results are robust to the choice of priors.segment) of the posterior distribution of the: intercept the parameters of the Matérn covariance function Fig. 7: Lethality of terrorism, and severity and frequency of lethal terrorist attacks: robustness test of of the Matérn covariance function). On all three models (lethality of terrorism (Figure 7a), severity (Figure 7b) and Lethality of terrorism, and severity and frequency of lethal terrorist attacks: robustness test of β0. Estimation of the mean and the 95% credible intervals (line , covariates β, and spatial parameters (range r and precision

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Piazza (2006) suggested that more economically developed and literate societies with high standards of living may exhibit more lucrative targets, and therefore are expected to be more targeted by terrorist attacks. While most country-level studies did not find a significant relationship between the number of terrorist attacks and economic variables (Krueger and Maleckova, 2003; Abadie, 2006; Drakos and Gofas, 2006; Krueger and Laitin, 2008; Gassebner and Luechinger, 2011), we showed that areas with higher human activity are more prone to encounter a higher number of lethal attacks, which brings support to Piazza's theory at subnational level. However, our study does not allow us to extend these findings to he lethality of terrorism and the number of deaths of lethal attacks since the estimated effects are not significant.

In comparison with rural areas, cities provide more human mobility, anonymity, and recruitment pool for terrorists (Crenshaw, 1990, p. 115; Savitch and Ardashev, 2001) and offer a higher number of vulnerable human targets, including: government officials, mayors and location with vulnerable civilian targets (e.g. inside bus, shopping malls, or theatres) (Hoffman, 2006, p. 167). Attacks in cities may provoke an impact on a larger audience, which is often a desired outcome (Laqueur, 1999, p. 41; Crenshaw, 1990, p. 115; Savitch and Ardashev, 2001). Not surprisingly, the frequency of lethal attacks is positively correlated with population density.

Furthermore, terrorists benefit from high density communication network (road and rail) in large cities to move freely and rapidly from and to target points (Heyman and Mickolus, 1980; Wilkinson, 1979, p. 189). It is also not uncommon that terrorists deliberately target communication network infrastructure, as exemplified by the March 11, 2004 simultaneous attacks on several commuter trains in Madrid, Spain, which killed 191 people (Los Angeles Times, 2014). As expected, the frequency of lethal attacks is negatively correlated with the distance to large cities.

The theory of "provocation" may explain the positive association observed between ethnic polarisation and the number of deaths per attack and the number of lethal attacks. As pointed out in Esteban et al. (2012) however, one should acknowledge that ethnic polarisation is only one possible measure of ethnic division among others, including ethnic fractionalisation. Further analysis using different approaches to measure ethnic division is required in order to assess the role of ethnic division in its wider sense.

The effect of democracy on terrorism is still under debate. However, it has been often acknowledge that democracies are more inclined to experience terrorism than autocracies (Chenoweth, 2013). Our results showed that areas with higher level of democracy—measured through the variable  $Polity$  IV—tend to encounter more deaths when attacks are lethal, which brings support to Pape (2003, 2006) who found a positive association between suicide terrorism—a type of terrorist attacks with a high potential to generate many casualties—and the level of democracy. Further studies allowing to distinguish suicide attacks from other types of terrorist events might complement our global analysis.

As with any statistical analyses of complex social phenomena, the outcome of this study should be taken with caution. First, since we aim to investigate terrorism

across the entire world and at high spatial resolution, the availability of suitable covariates is limited. As a result, our study has unavoidably omitted numerous relevant drivers of the three investigated facets of lethal terrorism, which include personal features (e.g. character traits, psychological processes) of each member of terrorist groups, ideology, beliefs, and cultural factors (Crenshaw, 1983, p. 29; Wilkinson, 1990, p. 151; Brynjar and Skjølberg, 2000; Richardson, 2006, pp. 92-93), or reciprocal interactions between counterterrorism and terrorism (English, 2010; Hoffman, 2002). We are unable to estimate the marginal effect of individual unobserved factors, their aggregated effects have been however taken into account through the space-time dependence structure represented by the GMRF.

Second, one could reasonably expect some spatial variability in the lethality of terrorism and frequency and severity of lethal terrorist attacks, especially within large cities that are regularly targeted by terrorists. However, since terrorist events from GTD are reported at the centroid of the nearest city in which they occurred, spatial variability of terrorism within cities cannot be captured. Also, we excluded spatially inaccurate events, which reduced the number of observations. However this operation did not affect the temporal trends in the data. The mean and standard deviation of the annual number of lethal attacks remain robust to this data selection process. The models discussed in this article assume both stationarity—invariance to translation—and isotropy—invariance to rotation. However, one might consider the processes driving terrorist attacks to be non-stationary (e.g. attacks may cluster in one region, but occur at random in other locations) or anisotropic (e.g. terrorist attacks increase in frequency systematically in some direction). The range might vary according to the the number of fatalities generated by the attacks and/or media outreach. Mass-casualty attacks might typically influence other terrorist groups on a wide scale through a broad media coverage. Further studies might investigate the use of non-stationary models, which have been recently investigated for the model class that may be fitted with INLA (Ingebrigtsen et al., 2015).

Third, the temporal unit exhibits limitations as well. Our study period is discretised into five years, which allows capturing variability on a relatively large temporal scale. The temporal accuracy of GTD allows the analysis of terrorism on a finer temporal granularity, which might find practical applications in e.g. counterterrorism operations which require daily analysis. For computational reasons, we assume no interaction between spatial and temporal dependencies of the lethality of terrorism, severity, and frequency of lethal terrorist attacks (i.e. *separable* space-time models), where the covariance structure can be written as the product of a purely spatial and a purely temporal covariance function for all space-time locations (Gneiting et al., 2006). In our models, the GMRFs follow a first-order random walk in time. In non-separable models, the dependencies structure in both space and time is usually highly complex (Harvill, 2010), and therefore more computationally demanding.

Fourth, future studies might combine each individual response (L-S-F) in a principled statistical framework in order to evaluate similarities and differences across each investigated process of lethal terrorism. Such an endeavour might face identifiability and computational challenges—a three-dimensional response would require four shared GMRFs: a shared field for each pair of responses (L-S, L-F, and S-F) and a field between all three fields and each individual response (L-S-F), along with a response specific field (L, S, and F). One way of preventing potential computational barriers would be to exclude the temporal dimension (i.e. a spatial instead of a spatio-temporal analysis) and/or to reduce the size of the area of interest (e.g. analysing the processes within a few countries only) and hence the mesh size.

Fifth, subjective choices have been made throughout the entire modelling process, which might affect both the internal and external validity of our results. A major concern is the absence of consensus on the definition of terrorism (Beck and Miner, 2013; Jackson, 2016), and subjectivity is therefore inevitable (Hoffman, 2006, p. 23). In line with English (2010, pp. 24-25), we agree that is all the more important that studies on terrorism must clearly state how terrorism is understood. Accordingly, we use data from GTD, which clearly states the definition used to classify acts as terrorist events. Moreover, as with any Bayesian analysis, our study involves a degree of subjectivity with regard to the choice of priors. Because of our relatively large dataset, we are confident that the choice of priors does not influence our results, as confirmed by our prior sensitivity analysis (Section 3). However, subjectivity remains in the definition of the threshold for hot-spots. We have chosen ones which ensures that the probability of lethal attacks is higher than non-lethal and that the expected number of deaths and lethal attacks correspond to high percentile. We recommend practitioners and researchers in the field of terrorism to take particular care in choosing cut-off values, which might vary according to the purpose of their study.

This study suggested a Bayesian framework to investigating the subnational spatial dynamics of the lethality of terrorism and the severity and frequency of lethal terrorist attacks that occurred across the world from 2010 to 2015. It identified, estimated, and subtly analysed distinct impacts of several important drivers involved in three different facets of lethal terrorism. Hence, the results of this work provided a systematic assessment of theories at a subnational level. Furthermore, the uncertainty of the predictions accounted in the modelling process remains crucial for policy-makers to make informed decisions (Zammit-Mangion et al., 2013, p. 64) or to evaluate the impact of counterterrorism policies (Perl, 2007). Ultimately, this research may provide complementary tools to enhance the efficacy of preventive counterterrorism policies.

## **References**

- Abadie, A. (2006). Poverty, political freedom and the roots of terrorism. The American Economic Review  $96(2)$ , 50–56.
- Alesina, A., A. Devleeschauwer, W. Easterly, S. Kurlat, and R. Wacziarg (2003). Fractionalization. Journal of Economic growth  $8(2)$ , 155-194.
- Amante, C. and B. Eakins (2009). ETOPO1 1 Arc-Minute Global Relief Model: Procedures, Data Sources and Analysis. NOAA Technical Memorandum NESDIS NGDC-24. National Geophysical Data Center, NOAA. http://www.ngdc.noaa.gov/mgg/global/ global.html. [Accessed 7 December 2014].
- Anselin, L., J. Cohen, D. Cook, W. Gorr, and G. Tita (2000). Spatial analyses of crime. Criminal justice  $\frac{1}{2}$  (2), 213–262.
- Banerjee, S., B. P. Carlin, and A. E. Gelfand (2014). Hierarchical modeling and analysis for spatial data. Crc Press.
- Barros, C. P. (2003). An intervention analysis of terrorism: The Spanish ETA case. Defence and Peace Economics 14, 401–412.
- Beck, C. J. and E. Miner (2013). Who gets designated a terrorist and why? Social Forces 91, 837–872.
- Behlendorf, B., G. LaFree, and R. Legault (2012). Microcycles of violence: Evidence from terrorist attacks by ETA and the FMLN. Journal of Quantitative Criminology 28, 49–75.
- Berman, E. and D. D. Laitin (2008). Religion, terrorism and public goods: Testing the club model. Journal of Public Economics 92 (10), 1942–1967.
- Bilal, A. R., N. B. A. Talib, I. U. Haq, M. N. A. A. Khan, and T. Islam (2012). How terrorism and macroeconomic factors impact on returns: A case study of Karachi stock exchange. World Applied Sciences Journal 19, 1575–1584.
- Blangiardo, M. and M. Cameletti (2015). Spatial and Spatio-temporal Bayesian Models with R-INLA. John Wiley & Sons.
- Blomberg, S. B. and P. B. Rosendorff (2009). A gravity model of globalization, democracy and transnational terrorism. In G. D. Hess (Ed.), Guns and Butter, pp. 25–156. Cambridge, MA, USA: Cambridge MIT Press.
- Braithwaite, A. and Q. Li (2007). Transnational terrorism hot spots: Identification and impact evaluation. Conflict Management and Peace Science 24, 281–296.
- Brandt, P. T. and T. Sandler (2012). A Bayesian Poisson vector autroregression model. Political Analysis 20, 292–315.
- Brosius, H.-B. and G. Weimann (1991). The contagiousness of mass-mediated terrorism. European Journal of Communication  $6(1)$ , 63-75.
- Brown, D., J. Dalton, and H. Hoyle (2004). Spatial forecast methods for terrorist events in urban environments. Intelligence and Security Informatics 3073, 426–435.
- Brynjar, L. and K. Skjølberg (2000). Why terrorism occurs: a survey of theories and hypotheses on the causes of terrorism.
- Cameletti, M., F. Lindgren, D. Simpson, and Håvard (2013). Spatio-temporal modeling of particulate matter concentration through the SPDE approach. Advances in Statistical Analysis 97, 109–131.
- Chen, X. and W. D. Nordhaus (2011). Using luminosity data as a proxy for economic statistics. Proceedings of the National Academy of Sciences 108 (21), 8589–8594.
- Chenoweth, E. (2013). Terrorism and democracy. Annual Review of Political Science 16, 355–378.
- CIESIN (2016). Gridded Population of the World, Version 4 (GPWv4): Population Density Grid. Data from Center for International Earth Science Information Network (CIESIN). Columbia University. http://dx.doi.org/10.7927/H4NP22DQ. [Accessed 26 October 2017].
- Coaffee, J. (2010). Protecting vulnerable cities: the UK's resilience response to defending everyday urban infrastructure. *International Affairs 86(4)*, 939–954.
- Courtillot, V., A. Davaille, J. Besse, and J. Stock (2003). Three distinct types of hotspots in the Earth's mantle. Earth and Planetary Science Letters 205(3), 295–308.
- Crelinsten, R. (2009). Counterterrorism. Cambridge, UK: Polity Press.
- Crenshaw, M. (1981). The causes of terrorism. Comparative Politics 13, 379–399.
- Crenshaw, M. (1983). Introduction: Reflections on the Effects of Terrorism, pp. 1–37. Middletown, CT, USA: Wesleyan University Press.
- Crenshaw, M. (1990). The Causes of Terrorism, pp. 113–126. New York, NY, USA: St. Martin's Press, Inc.
- Cressie, N. (1991). Statistics for Spatial Data (1st ed.). London, UK: John Wiley & Sons.
- Crough, S. T. and D. M. Jurdy (1980). Subducted lithosphere, hotspots, and the geoid. Earth and Planetary Science Letters  $48(1)$ , 15–22.
- Drakos, K. (2007). The size of under-reporting bias in recorded transnational terrorist activity. Journal of the Royal Statistical Society: Series A (Statistics in Society) 170 (4), 909–921.
- Drakos, K. and A. Gofas (2006). In search of the average transnational terrorist attack venue. Defence and Peace Economics 17, 73–93.
- Ebener, S., C. Murray, A. Tandon, and C. C. Elvidge (2005). From wealth to health: modelling the distribution of income per capita at the sub-national level using night-time light imagery. International Journal of Health Geographics  $\lambda(1)$ , 5.
- Eck, J., S. Chainey, J. Cameron, and R. Wilson (2005). Mapping crime: Understanding hotspots. chapter 1: Crime hot spots: What they are, why we have them, and how to map them. Technical report, U.S. National Institute of Justice.
- Elvidge, C. D., J. Safran, B. Tuttle, P. Sutton, P. Cinzano, D. Pettit, J. Arvesen, and C. Small (2007). Potential for global mapping of development via a nightsat mission.  $Geoburnal 69(1-2), 45-53.$
- Enders, W. and G. A. Hoover (2012). The nonlinear relationship between terrorism and poverty. American Economic Review 102, 267–272.
- Enders, W., G. F. Parise, and T. Sandler (1992). A time-series analysis of transnational terrorism: Trends and cycles. Defence and Peace Economics  $3(4)$ ,  $305-320$ .
- Enders, W. and T. Sandler (1993). The effectiveness of anti-terrorism policies: A vectorautogression-intervention analysis. American Political Science Review 87, 829–844.
- Enders, W. and T. Sandler (2005). Transnational terrorism 1968-2000: Thresholds, persistence and forecasts. Southern Economic Journal 71, 467–482.
- Enders, W. and T. Sandler (2006). Distribution of transnational terrorism among countries by income class and geography after  $9/11$ . International Studies Quarterly  $50(2)$ , 367–393.
- Enders, W., T. Sandler, and K. Gaibulloev (2011). Domestic versus transnational terrorism: Data, decomposition, and dynamics. *Journal of Peace Research 48*, 319–337.
- English, R. (2010). *Terrorism: How to respond*. Oxford University Press.
- Esteban, J., L. Mayoral, and D. Ray (2012). Ethnicity and conflict: Theory and facts. Science 336 (6083), 858–865.

Fromkin, D. (1975, July). The strategy of terrorism. Foreign Affairs.

- Fuglstad, G.-A., D. Simpson, F. Lindgren, and H. Rue (2017). Constructing priors that penalize the complexity of Gaussian random fields. Journal of the American Statistical Association. In press.
- Gao, P., D. Guo, K. Liao, J. J. Webb, and S. L. Cutter (2013). Early Detection of Terrorism Outbreaks Using Prospective Space-Time Scan Statistics. The Professional Geographer 65, 676–691.
- Gassebner, M. and S. Luechinger (2011). Lock, stock, and barrel: A comprehensive assessment of the determinants of terror. *Public Choice 149*, 235–261.
- Gelfand, A. E., D. K. Dey, and H. Chang (1992). Model determination using predictive distributions with implementation via sampling-based methods. Technical report, DTIC Document.
- Gilks, W. R., S. Richardson, and D. J. Spiegelhalter (1996). *Markov chain Monte Carlo in* practice. Interdisciplinary statistics,. Boca Raton, FL: Chapman & Hall/CRC.
- Gleditsch, K. S. (2007). Transnational dimensions of civil war. Journal of Peace Research  $44(3)$ , 293-309.
- Gneiting, T., M. G. Genton, and P. Guttorp (2006). Geostatistical space-time models, stationarity, separability, and full symmetry. Monographs On Statistics and Applied Probability 107, 151.
- GTD (2017, June). Global Terrorism Database (GTD) Codebook: Inclusion Criteria and Variables. http://www.start.umd.edu/gtd/downloads/Codebook.pdf. [Accessed 26 October 2017].
- Hamilton, L. C. and J. D. Hamilton (1983). Dynamics of terrorism. International Studies Quarterly 27, 39–54.
- Hammond, J. and N. B. Weidmann (2014). Using machine-coded event data for the microlevel study of political violence. Research & Politics  $1(2)$ , 2053168014539924.
- Harvill, J. L. (2010). Spatio-temporal processes. Wiley interdisciplinary reviews: computational statistics  $2(3)$ , 375–382.
- Held, L., B. Schrödle, and H. Rue (2010). Posterior and cross-validatory predictive checks: a comparison of MCMC and INLA. In Statistical modelling and regression structures, pp. 91–110. Springer.
- Henderson, J. V., A. Storeygard, and D. N. Weil (2009). Measuring economic growth from outer space. no. w15199. In National Bureau of Economic Research.
- Heyman, E. and E. Mickolus (1980). Observations on "why violence spreads". International Studies Quarterly 24, 299–305.
- Hoffman, B. (2002). Rethinking terrorism and counterterrorism since 9/11. Studies in Conflict and Terrorism  $25(5)$ , 303-316.
- Hoffman, B. (2006). Inside Terrorism (Rev. and expanded ed.). New York, NY, USA: Columbia University Press.
- Hsiang, S. M., M. Burke, and E. Miguel (2013). Quantifying the influence of climate on human conflict. *Science* 341(6151), 1235367.
- Ingebrigtsen, R., F. Lindgren, I. Steinsland, and S. Martino (2015). Estimation of a nonstationary model for annual precipitation in southern Norway using replicates of the spatial field. Spatial Statistics  $14$  (Part C),  $338 - 364$ .
- Jackson, R. (2016). The Ashgate Research Companion to Political Violence, Chapter Critical Accounts of Terrorism, pp. 47–62. Routledge.
- Jacquez, G. M. (1996). A k nearest neighbour test for space-time interaction. Statistics in medicine 15 (18), 1935–1949.
- Jones-Todd, C. M., B. Swallow, J. B. Illian, and M. Toms (2017). A spatiotemporal multispecies model of a semicontinuous response. *Journal of the Royal Statistical Society:* Series C (Applied Statistics)  $67(3)$ ,  $705 - 722$ .
- Kalyvas, S. N. (2006). The logic of violence in civil war. Cambridge University Press.
- Kalyvas, S. N. and M. A. Kocher (2007). How Free is Free Riding in civil wars?: Violence, insurgency, and the collective action problem. World Politics  $59(02)$ , 177–216.
- Krieger, T. and D. Meierrieks (2011). What causes terrorism? Public Choice 147, 3-27.
- Krueger, A. B. and D. D. Laitin (2008). *Kto Kogo?: A Cross-Country Study of the Origins* and Targets of Terrorism, pp. 148–173. Cambridge: Cambridge University Press.
- Krueger, A. B. and J. Maleckova (2003). Education, poverty and terrorism: Is there a causal connection? The Journal of Economic Perpectives 17, 119–144.
- Kullback, S. and R. A. Leibler (1951). On information and sufficiency. The annals of mathematical statistics  $22(1)$ , 79–86.
- Kurrild-Klitgaard, P., M. Justesen, and R. Klemmensen (2006). The political economy of freedom, democracy and transnational terrorism. Public Choice 128, 289–315.
- Kydd, A. H. and B. F. Walter (2006). The strategies of terrorism. International Security  $31(1)$ , 49–80.
- LaFree, G., L. Dugan, M. Xie, and P. Singh (2012). Spatial and temporal patterns of terrorist attacks by ETA 1970 to 2007. Journal of Quantitative Criminology 28, 7–29.
- LaFree, G., N. A. Morris, and L. Dugan (2010). Cross-national patterns of terrorism comparing trajectories for total, attributed and fatal attacks, 1970–2006. British Journal of  $Criminology 50(4), 622–649.$
- LaFree, G., M. Xie, and A. M. Matanock (2017). The contagious diffusion of world-wide terrorism: is it less common than we might think? Studies in Conflict  $\mathcal C$  Terrorism  $(0),$ 1–20.
- LaFree, G., S.-M. Yang, and M. Crenshaw (2009). Trajectories of terrorism. Criminology  $\mathcal{B}$ Public Policy  $8(3)$ , 445–473.
- Laqueur, W. (1999). The New Terrorism: Fanaticism and the Arms of Mass Destruction (1st ed.). New York, NY, USA: Oxford University Press.
- Lewis, E., G. Moehler, P. J. Brantingham, and A. L. Bertozzi (2012). Self-exciting point process models of civilian deaths in Iraq. Security Journal 25, 244–264.
- Li, Q. (2005). Does democracy promote or reduce transnational terrorist incidents? Journal of Conflict resolution  $49(2)$ , 278-297.
- Lindgren, F. (2012). Continuous domain spatial models in R-INLA. The ISBA Bul*letin*  $19(4)$ , 14-20.
- Lindgren, F. (2015). Spatial Data Analysis with R-INLA with Some Extensions. *Journal of* Statistical Software 63 (20), 1–31.
- Lindgren, F., H. Rue, and J. Lindström (2011). An explicit link between Gaussian fields and Gaussian Markov random fields: the stochastic partial differential equation approach. Journal of the Royal Statistical Society: Series B (Statistical Methodology) 73 (4), 423–498.
- Loftin, C. (1986). Assaultive violence as a contagious social process. *Bulletin of the New* York Academy of Medicine 62(5), 550.
- Los Angeles Times (2014, March). Spain still healing 10 years after Madrid '11-M' train bombings. http://articles.latimes.com/2014/mar/09/world/

la-fg-spain-bombings-20140309. [Accessed 12 July 2014].

- Mantel, N. (1967). The detection of disease clustering and a generalized regression approach. Cancer research 27(2 Part 1), 209–220.
- Marshall, M. G. and G. Elzinga-Marshall (2014). Global report 2017: Conflict, governance and state fragility. Technical report, Center for Systemic Peace.
- Martino, S. and H. Rue (2010). Case studies in Bayesian computation using INLA. In Complex data modeling and computationally intensive statistical methods, pp. 99–114. Springer.
- Medina, R. M., L. K. Siebeneck, and G. F. Hepner (2011). A geographic information systems (GIS) analysis of spatiotemporal patterns of terrorist incidents in Iraq 2004-2009. Studies in Conflict & Terrorism 34, 862–882.
- Midlarsky, M. I., M. Crenshaw, and F. Yoshida (1980). Why violence spreads: The contagion of international terrorism. International Studies Quarterly 24, 262–298.
- Mohler, G. (2013). Modeling and estimation of multi-source clustering in crime and security data. The Annals of Applied Statistics 7, 1525–1539.
- Mohler, G. O., M. B. Short, P. J. Brantingham, F. P. Schoenberg, and G. E. Tita (2011). Self-exciting point process modeling of crime. Journal of the American Statistical Association  $106(493)$ , 100-108.
- Montalvo, J. G. and M. Reynal-Querol (2005). Ethnic diversity and economic development. Journal of Development economics 76 (2), 293–323.
- Nacos, B. (2016). Mass-mediated Terrorism: Mainstream and Digital Media in Terrorism and Counterterrorism (third ed.). Plymouth, UK: Rowman & Littlefield.
- Nacos, B. L. (2010). Revisiting the contagion hypothesis: Terrorism, news coverage, and copycat attacks. Perspectives on Terrorism  $3(3)$ , 3-13.
- NATO (2008). The mechanics of terrorism. http://www.nato.int/docu/Review/2008/04/ AP\_COST/EN/index.htm. [Accessed 10 February 2015].
- Nelson, A. (2008). Estimated travel time to the nearest city of 50,000 or more people in year 2000. http://bioval.jrc.ec.europa.eu/products/gam/. [Accessed 10 June 2014].
- Nemeth, S. C. (2010). A Rationalist Explanation of Terrorist Targeting. Ph. D. thesis, University of Iowa, USA.
- Nemeth, S. C., J. A. Mauslein, and C. Stapley (2014, 4). The primacy of the local: Identifying terrorist hot spots using Geographic Information Systems. The Journal of Politics 76, 304– 317.
- Neumayer, E. and T. Plümper (2010). Galton's problem and contagion in international terrorism along civilizational lines. Conflict management and peace science  $27(4)$ , 308– 325.
- NOAA (2014). Version 4 DMSP-OLS Nighttime Lights Time Series. National Oceanic and Atmospheric Administration, National Geophysical Data Center.
- Nunn, S. (2007). Incidents of terrorism in the United States, 1997-2005. Geographical Review 97, 89–111.
- Ocal, N. and J. Yildirim (2010). Regional effects of terrorism on economic growth in Turkey: A geographically weighted regression approach. Journal of Peace Research  $\frac{1}{4}7(4)$ , 477–489.
- Pape, R. A. (2003). The strategic logic of suicide terrorism. American Political Science  $Review 97(3), 343361.$
- Pape, R. A. (2006). Dying to win: the strategic logic of suicide terrorism. New York, USA: Random House Trade Paperbacks.
- Perl, A. (2007). Combating terrorism: The challenge of measuring effectiveness. Technical report, Technical Re-port RL33160, Congressional Research Services. Available at http: //fpc.state.gov/documents/organization/57513.pdf.
- Pettit, L. (1990). The conditional predictive ordinate for the normal distribution. Journal of the Royal Statistical Society. Series B (Methodological) 52 (1), 175–184.
- Piazza, J. (2006). Rooted in poverty? terrorism, poor economic development, and social cleavages. Political Violence 18, 159–177.
- Piegorsch, W. W., S. L. Cutter, and F. Hardisty (2007). Benchmark analysis for quantifying urban vulnerability to terrorrst incidents. Risk Analysis 27, 1411–1425.
- Porter, M. D. and G. White (2012). Self-exciting hurdle models for terrorist activity. The Annals of Applied Statistics  $6(1)$ , 106-124.
- Posen, B. R. (1993). The security dilemma and ethnic conflict. Survival 35(1), 27–47.
- Python, A., J. Brandsch, and A. Tskhay (2017). Provoking local ethnic violence: A global study on ethnic polarization and terrorist targeting. Political Geography 58, 77–89.
- Raghavan, V., A. Galstyan, and A. G. Tartakovsky (2013). Hidden Markov models for the activity profile of terrorist groups. The Annals of Applied Statistics  $\gamma(4)$ , 2402–2430.
- RAND (2011). Database scope. http://www.rand.org/nsrd/projects/ terrorism-incidents/about/scope.html. [Accessed 12 June 2014].
- Rapoport, D. C. (1996). Editorial: The media and terrorism: Implications of the Unabomber case. Terrorism and Political Violence  $8(1)$ , 7–9.
- Richardson, L. (2006). What Terrorists Want (1st ed.). London, UK: John Murray.
- Rodrigues, A., P. Diggle, and R. Assuncao (2010). Semiparametric approach to point source modelling in epidemiology and criminology. Journal of the Royal Statistical Society - Series C Applied Statistics 59, 533–542.
- Ross, J. I. (1993). Structural causes of oppositional political terrorism: Towards a causal model. Journal of Peace Research 30, 317–329.
- Rue, H., S. Martino, and N. Chopin (2009). Approximate Bayesian inference for latent Gaussian models by using integrated nested Laplace approximations. Journal of the royal statistical society: Series b (statistical methodology) 71 (2), 319–392.
- Savitch, H. V. and G. Ardashev (2001). Does terror have an urban future? Urban Studies 38, 2515–2533.
- Sheehan, I. S. (2012). Assessing and Comparing Data Sources for Terrorism Research, pp. 13–40. New York, NY, USA: Springer.
- Siebeneck, L. K., R. M. Medina, I. Yamada, and G. F. Hepner (2009). Spatial and temporal analyses of terrorist incidents in Iraq, 2004-2006. Studies in Conflict  $\mathcal C$  Terrorism 32, 591–610.
- Simpson, D., J. Illian, F. Lindgren, S. Sørbye, and H. Rue (2016). Going off grid: computationally efficient inference for log-Gaussian Cox processes. Biometrika  $103(1)$ , 49–70.
- Simpson, D. P., H. Rue, A. Riebler, T. G. Martins, and S. H. Sørbye (2017). Penalising Model Component Complexity: A Principled, Practical Approach to Constructing Priors. Statistical Science 32 (1), 1–28.
- START (2017). Data collection methodology. http://www.start.umd.edu/gtd/. [Accessed 15 February 2017].
- Steen, J., P. W. Liesch, G. Knight, and M. R. Czinkota (2006). The contagion of international terrorism and its effects on the firm in an interconnected world. Public Money  $\mathcal{C}$ Management 26, 305–312.

- Suleman, M. T. (2012). Stock market reaction to terrorist attacks; empirical evidence from a front line state. Australasian Accounting Business and Finance Journal 6, 97–110.
- Sutton, P. C. and R. Costanza (2002). Global estimates of market and non-market values derived from nighttime satellite imagery, land cover, and ecosystem service valuation. Ecological Economics 41 (3), 509–527.
- Sutton, P. C., C. D. Elvidge, and T. Ghosh (2007). Estimation of gross domestic product at sub-national scales using nighttime satellite imagery. International Journal of Ecological Economics & Statistics  $8(S07)$ , 5-21.
- Swanstrom, T. (2002). Are fear and urbanism at war? Urban Affairs Review 38(1), 135–140.
- Tavares, J. (2004). The open society assesses its enemies: shocks, disasters and terrorist attacks. Journal of monetary economics 51 (5), 1039–1070.
- The Washington Post (2013). \$52.6 billion: The black budget. http://www. washingtonpost.com/wp-srv/special/national/black-budget. [Accessed 09 February 2015].
- Watanabe, S. (2010). Asymptotic equivalence of Bayes cross validation and widely applicable information criterion in singular learning theory. The Journal of Machine Learning Research 11, 3571–3594.
- Weidmann, N. B., J. K. Rød, and L.-E. Cederman (2010). Representing ethnic groups in space: A new dataset. Journal of Peace Research  $47(4)$ , 1–9.
- Wilkinson, P. (1979). *Terrorism and the Liberal State* (2nd ed.). Hong Kong, China: The MacMillan Press Ltd.
- Wilkinson, P. (1990). The Sources of Terrorism: Terrorists' Ideologies and Beliefs, pp. 139–145. New York, NY, USA: St. Martin's Press, Inc.
- Zammit-Mangion, A., M. Dewar, V. Kadirkamanathan, A. Flesken, and G. Sanguinetti (2013). Modeling Conflict Dynamics with Spatio-temporal Data (1st ed.). Heidelberg, Germany: SpringerBriefs in Applied Sciences and Technology.
- Zammit-Mangion, A., M. Dewar, V. Kadirkamanathand, and G. Sanguinetti (2012). Point Process Modelling of the Afghan war diary. *PNAS 109*, 12414–12419.