

TABLE 2.1 Number of graphs with v vertices and ω components

v	$\omega = 1$	2	3	4	5	6	7	8	9	10	totals	mean ω
1	1										1	1
2	1	1									2	1.5
3	2	1	1								4	1.750
4	6	3	1	1							11	1.727
5	21	8	3	1	1						34	1.618
6	112	30	9	3	1	1					156	1.423
7	853	145	32	9	3	1	1				1044	1.248
8	11 117	1028	154	33	9	3	1	1			12 346	1.121
9	261 080	12 320	1065	156	33	9	3	1	1		274 668	1.055
10	11 716 571	274 806	12 513	1074	157	33	9	3	1	1	12 005 168	1.025

The last column, mean number of components, illustrates the notion that, for an arbitrary number of vertices, the random graph is connected.

TABLE 2.2 Number of graphs with v vertices, e edges, ω components, and aggregant α (number of vertices in largest component)

$v = 2$

e	$\omega = 1$	2	totals
0		1	1
1	1		1

$v = 3$

e	$\omega = 1$	2	3	totals
0			1	1
1		1		1
2	1			1
3	1			1

$v = 4$

e	$\omega = 1$	2	3	4	totals	mean ω	random mean ω
0				1	1	4	4
1			1		1	3	3
2		2			2	2	2
3	2	1			3	1.333	1.2
4	2				2	1	1
5	1				1	1	1
6	1				1	1	1

$v = 4$

e	$\alpha = 4$	3	2	1	totals	mean α	random mean α
0				1	1	1	1
1			1		1	2	2
2		1	1		2	2.5	2.8
3	2	1			3	3.667	3.8
4	2				2	4	4
5	1				1	4	4
6	1				1	4	4

Random mean α and random mean ω are weighted according to the graphs' probabilities.

TABLE 2.2b Number of graphs with
 v vertices, e edges, ω components, and
 aggrent α (number of vertices in largest component)

$v = 5$

e	$\omega = 1$	2	3	4	5	totals	mean ω	random mean ω
0					1	1	5	5
1				1		1	4	4
2			2			2	3	3
3		3	1			4	2.250	2.083
4	3	3				6	1.500	1.405
5	5	1				6	1.167	1.119
6	5	1				6	1.167	1.024
7	4					4	1	1
8	2					2	1	1
9	1					1	1	1
10	1					1	1	1

$v = 5$

e	$\alpha = 5$	4	3	2	1	totals	mean α	random mean α
0					1	1	1	1
1				1		1	2	2
2			1	1		2	2.5	2.667
3		2	2			4	3.5	3.667
4	3	2	1			6	4.333	4.548
5	5	1				6	4.833	4.881
6	5	1				6	4.833	4.976
7	4					4	5	5
8	2					2	5	5
9	1					1	5	5
10	1					1	5	5

TABLE 2.2c Number of graphs with
 v vertices, e edges, ω components, and
 aggregant α (number of vertices in largest component)

$v = 6$

e	$\omega = 1$	2	3	4	5	6	totals	mean ω	random mean ω
0						1	1	6	6
1					1		1	5	5
2				2			2	4	4
3			4	1			5	3.200	3.045
4		6	3				9	2.333	2.209
5	6	8	1				15	1.667	1.598
6	13	7	1				21	1.300	1.272
7	19	5					24	1.208	1.114
8	22	2					24	1.083	1.042
9	20	1					21	1.048	1.012
10	14	1					15	1.067	1.002
11	9						9	1	1
12	5						5	1	1
13	2						2	1	1
14	1						1	1	1
15	1						1	1	1

$v = 6$

e	$\alpha = 6$	5	4	3	2	1	totals	mean α	random mean α
0						1	1	1	1
1					1		1	2	2
2				1	1		2	2.5	2.571
3			2	2	1		5	3.2	3.495
4		3	4	2			9	4.111	4.440
5	6	5	3	1			15	5.061	5.287
6	13	5	2	1			21	5.429	5.706
7	19	4	1				24	5.750	5.883
8	22	2					24	5.917	5.958
9	20	1					21	5.950	5.988
10	14	1					15	5.933	5.998
11	9						9	6	6
12	5						5	6	6
13	2						2	6	6
14	1						1	6	6
15	1						1	6	6

TABLE 2.2d Number of graphs with
 v vertices, e edges, ω components, and
 aggreant α (number of vertices in largest component)

$v = 7$

e	$\omega = 1$	2	3	4	5	6	7	totals	mean ω	random mean ω
0							1	1	7	7
1						1		1	6	6
2					2			2	5	5
3				4	1			5	4.200	4.026
4			7	3				10	3.300	3.123
5		11	9	1				21	2.524	2.353
6	11	22	7	1				41	1.951	1.784
7	33	27	5					65	1.569	1.435
8	67	28	2					97	1.330	1.235
9	107	23	1					131	1.191	1.1225
10	132	15	1					148	1.115	1.0602
11	138	10						148	1.068	1.0271
12	126	5						131	1.038	1.0108
13	95	2						97	1.021	1.0036
14	64	1						65	1.015	1.0009
15	40	1						41	1.024	1.0001
16	21							21	1	1
17	10							10	1	1
18	5							5	1	1
19	2							2	1	1
20	1							1	1	1
21	1							1	1	1

TABLE 2.2d Number of graphs with
 v vertices, e edges, ω components, and
 aggregant α (number of vertices in largest component)

$v = 7$

e	$\alpha = 7$	6	5	4	3	2	1	totals	mean α	random mean α
0							1	1	1	1
1						1		1	2	2
2					1	1		2	2.5	2.500
3				2	2	1		5	3.2	3.342
4			3	4	3			10	4	4.246
5		6	8	5	2			21	4.857	5.224
6	11	13	10	6	1			41	5.659	6.037
7	33	19	9	4				65	6.246	6.507
8	67	22	6	2				97	6.588	6.749
9	107	20	3	1				131	6.779	6.874
10	132	14	2					148	6.878	6.9365
11	138	9	1					148	6.926	6.9728
12	126	5						131	6.962	6.9892
13	95	2						97	6.979	6.9964
14	64	1						65	6.985	6.9991
15	40	1						41	6.976	6.9999
16	21							21	7	7
17	10							10	7	7
18	5							5	7	7
19	2							2	7	7
20	1							1	7	7
21	1							1	7	7

TABLE 2.2e Number of graphs with
 v vertices, e edges, ω components, and
 aggregant α (number of vertices in largest component)

$v = 8$

e	$\omega = 1$	2	3	4	5	6	7	8	totals	mean ω	random mean ω
0								1	1	8	8
1							1		1	7	7
2						2			2	6	6
3					4	1			5	5.2	5.017
4				8	3				11	4.273	4.079
5			14	9	1				24	3.458	3.225
6		23	25	7	1				56	2.750	2.506
7	23	58	29	5					115	2.139	1.964
8	89	101	29	2					221	1.747	1.6023
9	236	142	23	1					402	1.475	1.3708
10	486	161	15	1					663	1.293	1.2270
11	814	156	10						980	1.180	1.1355
12	1169	138	5						1312	1.113	1.0786
13	1454	101	2						1557	1.067	1.0438
14	1579	66	1						1646	1.041	1.0233
15	1515	41	1						1557	1.028	1.0138
16	1290	22							1312	1.015	1.00535
17	970	10							980	1.010	1.00223
18	658	5							663	1.0075	1.00081
19	400	2							402	1.0050	1.00024
20	220	1							221	1.0045	1.00005
21	114	1							115	1.0087	1.000007
22	56								56	1	1
23	24								24	1	1
24	11								11	1	1
25	5								5	1	1
26	2								2	1	1
27	1								1	1	1
28	1								1	1	1

As we move through the poset, adding edges to a fixed set of vertices, how quickly do those vertices become connected? How soon does a precipitate congeal or crystallize out of the set? In a sense this entire chapter is about the question: How connected is a random graph? The last columns on pages 40 & 41, random mean α and random mean ω , as well as the dot plot opposite, illustrate the notion that when a graph has as many edges as vertices, it is *almost connected*. The aggregant in particular (plot on facing page) rises rapidly in a sigmoid curve as e approaches v , indicating a change of phase. The change is sharper for larger v .

For further information see the work of Erdős & Renyi on random graphs.

v = 8

e	$\alpha = 8$	7	6	5	4	3	2	1	totals	mean α	random mean α
0								1	1	1	1
1							1		1	2	2
2						1	1		2	2.5	2.444
3					2	2	1		5	3.2	3.214
4				3	4	3	1		11	3.818	4.058
5			6	8	7	3			24	4.708	5.027
6		11	19	13	11	2			56	5.464	5.987
7	23	33	32	17	9	1			115	6.357	6.785
8	89	67	41	16	8				221	6.964	7.2938
9	236	107	42	12	5				402	7.386	7.5907
10	486	132	34	8	3				663	7.644	7.7597
11	814	138	23	4	1				980	7.796	7.8601
12	1169	126	14	2	1				1312	7.875	7.9201
13	1454	95	7	1					1557	7.928	7.9558
14	1579	64	3						1646	7.957	7.9767
15	1515	40	2						1557	7.972	7.9861
16	1290	21	1						1312	7.982	7.99465
17	970	10							980	7.990	7.99777
18	658	5							663	7.992	7.99919
19	400	2							402	7.995	7.99976
20	220	1							221	7.995	7.99995
21	114	1							115	7.991	7.999993
22	56								56	8	8
23	24								24	8	8
24	11								11	8	8
25	5								5	8	8
26	2								2	8	8
27	1								1	8	8
28	1								1	8	8

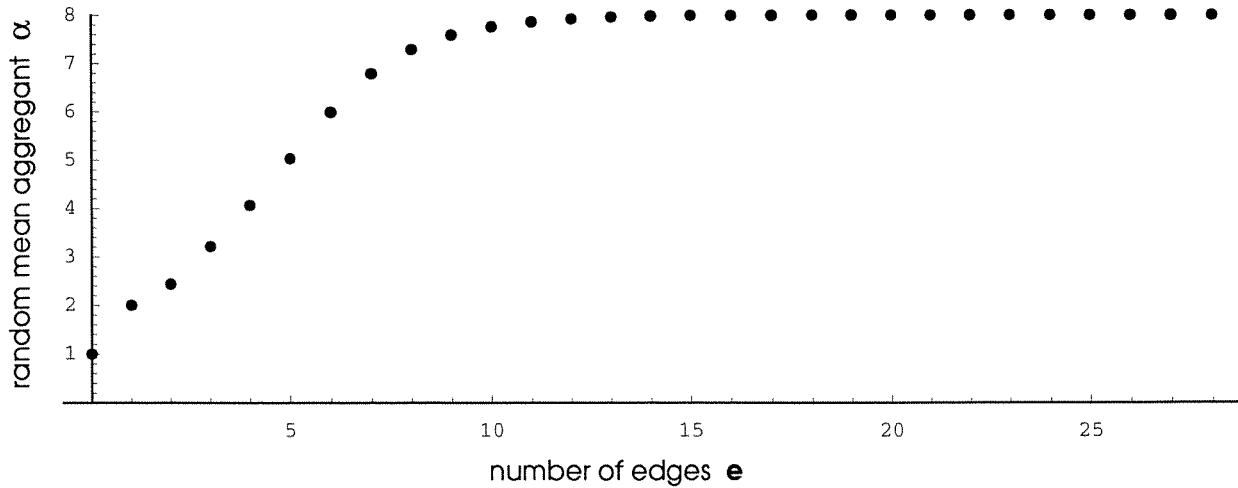


TABLE 2.2f

 $v = 9$

e	$\omega = 1$	2	3	4	5	6	7	8	9	totals	mean ω	random mean ω
0									1	1	9	9
1								1		1	8	8
2							2			2	7	7
3						4	1			5	6.200	6.012
4					8	3				11	5.273	5.053
5				15	9	1				25	4.440	4.152
6			29	26	7	1				63	3.683	* 3.35
7		46	68	29	5					148	2.953	* 2.65
8	47	157	110	29	2					345	2.368	* 2.12
9	240	358	149	23	1					771	1.946	* 1.78
10	797	660	164	15	1					1637	1.633	* 1.51
11	2075	1010	157	10						3252	1.416	* 1.34
12	4495	1356	139	5						5995	1.275	* 1.23
13	8404	1613	101	2						10 120	1.180	* 1.16
14	13 855	1693	66	1						15 615	1.117	* 1.09
15	20 303	1588	41	1						21 933	1.076	* 1.06
16	26 631	1334	22							27 987	1.049	* 1.03
17	31 400	993	10							32 403	1.031	* 1.02
18	33 366	669	5							34 040	1.020	* 1.01
19	31 996	405	2							32 403	1.013	< 1.01
20	27 764	222	1							27 987	1.008	< 1.01
21	21 817	115	1							21 933	1.0053	< 1.01
22	15 558	57								15 615	1.0037	< 1.01
23	10 096	24								10 120	1.0024	< 1.01
24	5984	11								5995	1.0018	< 1.01
25	3247	5								3252	1.0015	< 1.01
26	1635	2								1637	1.0012	< 1.01
27	770	1								771	1.0013	< 1.01
28	344	1								345	1.0029	< 1.01
29	148									148	1	1
30	63									63	1	1
31	25									25	1	1
32	11									11	1	1
33	5									5	1	1
34	2									2	1	1
35	1									1	1	1
36	1									1	1	1

*Estimated with *Mathematica* function ExactRandomGraph

TABLE 2.2f

 $v = 9$

e	$\alpha = 9$	8	7	6	5	4	3	2	1	totals	mean α	random mean α
0									1	1	1	1
1								1		1	2	2
2							1	1		2	2.5	2.4
3						2	2	1		5	3.2	3.106
4					3	4	3	1		11	3.818	3.893
5				6	8	7	4			25	4.640	4.809
6			11	19	16	13	4			63	5.317	* 5.79
7		23	44	38	28	13	2			148	6.203	* 6.76
8	47	89	100	60	37	11	1			345	7.035	* 7.55
9	240	236	174	74	39	7	1			771	7.699	* 8.08
10	797	486	239	75	36	4				1637	8.173	* 8.41
11	2075	814	270	65	27	1				3252	8.489	* 8.62
12	4495	1169	264	48	18	1				5995	8.680	* 8.76
13	8404	1454	221	30	11					10 120	8.799	* 8.85
14	13 855	1579	159	17	5					15 615	8.874	* 8.90
15	20 303	1515	104	9	2					21 933	8.920	* 8.94
16	26 631	1290	61	4	1					27 987	8.949	* 8.97
17	31 400	970	31	2						32 403	8.968	* 8.98
18	33 366	658	15	1						34 040	8.980	* 8.99
19	31 996	400	7							32 403	8.987	> 8.99
20	27 764	220	3							27 987	8.992	> 8.99
21	21 817	114	2							21 933	8.9946	> 8.99
22	15 558	56	1							15 615	8.9963	> 8.99
23	10 096	24								10 120	8.9976	> 8.99
24	5984	11								5995	8.9982	> 8.99
25	3247	5								3252	8.9985	> 8.99
26	1635	2								1637	8.9988	> 8.99
27	770	1								771	8.9987	> 8.99
28	344	1								345	8.9971	> 8.99
29	148									148	9	9
30	63									63	9	9
31	25									25	9	9
32	11									11	9	9
33	5									5	9	9
34	2									2	9	9
35	1									1	9	9
36	1									1	9	9

*Estimated with *Mathematica* function ExactRandomGraph

v = 10

TABLE 2.2g

e	$\omega = 1$	2	3	4	5	6	7	8	9	10	totals	mean ω	random mean ω
0										1	1	10	10
1									1		1	9	9
2								2			2	8	8
3							4	1			5	7.200	7.008
4						8	3				11	6.273	6.038
5					16	9	1				26	5.423	5.107
6				32	26	7	1				66	4.652	*4.24
7			60	71	29	5					165	3.873	*3.46
8		99	186	112	29	2					428	3.180	*2.83
9	106	426	397	150	23	1					1103	2.602	*2.30
10	657	1233	699	164	15	1					2769	2.151	*1.94
11	2678	2873	1041	157	10						6759	1.809	*1.66
12	8548	5705	1375	139	5						15 772	1.564	*1.48
13	22 950	9985	1625	101	2						34 663	1.391	*1.34
14	53 863	15 689	1699	66	1						71 318	1.270	*1.24
15	112 618	22 183	1590	41	1						136 433	1.187	*1.17
16	211 866	28 354	1335	22							241 577	1.129	*1.11
17	361 342	32 820	994	10							395 166	1.088	*1.08
18	561 106	34 411	669	5							596 191	1.060	*1.05
19	795 630	32 691	405	2							828 728	1.040	*1.03
20	1 032 754	28 182	222	1							1 061 159	1.027	*1.02
21	1 229 228	22 045	115	1							1 251 389	1.018	*1.01
22	1 343 120	15 675	57								1 358 852	1.012	<1.01
23	1 348 674	10 154	24								1 358 852	1.0075	<1.01
24	1 245 369	6009	11								1 251 389	1.0048	<1.01
25	1 057 896	3258	5								1 061 159	1.0031	<1.01
26	827 086	1640	2								828 728	1.0020	<1.01
27	595 418	772	1								596 191	1.0013	<1.01
28	394 820	345	1								395 166	1.0009	<1.01
29	241 428	149									241 577	1.0006	<1.01
30	136 370	63									136 433	1.0005	<1.01
31	71 293	25									71 318	1.0004	<1.01
32	34 652	11									34 663	1.0003	<1.01
33	15 767	5									15 772	1.0003	<1.01
34	6757	2									6759	1.0003	<1.01
35	2768	1									2769	1.0004	<1.01
36	1102	1									1103	1.0009	<1.01
37	428										428	1	1
38	165										165	1	1
39	66										66	1	1
40	26										26	1	1
41	11										11	1	1
42	5										5	1	1
43	2										2	1	1
44	1										1	1	1
45	1										1	1	1

*Estimated with
Mathematica function
 ExactRandomGraph

v = 10

TABLE 2.2g

e	$\alpha = 10$	9	8	7	6	5	4	3	2	1	totals	mean α	random mean α
0										1	1	1	1
1									1		1	2	2
2								1	1		2	2.5	2.364
3							2	2	1		5	3.2	3.015
4						3	4	3	1		11	3.818	3.751
5					6	8	7	4	1		26	4.538	4.603
6				11	19	16	15	5			66	5.242	* 5.54
7			23	44	44	31	20	3			165	6.061	* 6.56
8		47	112	111	85	51	20	2			428	6.886	* 7.51
9	106	240	325	218	131	64	18	1			1103	7.731	* 8.30
10	657	797	722	339	167	75	12				2769	8.421	* 8.85
11	2678	2075	1300	444	186	70	6				6759	8.944	* 9.21
12	8548	4495	1983	503	178	62	3				15 772	9.302	* 9.47
13	22 950	8404	2623	491	148	46	1				34 663	9.540	* 9.64
14	53 863	13 855	3033	423	110	34					71 318	9.694	* 9.75
15	112 618	20 303	3094	325	73	20					136 433	9.796	* 9.83
16	211 866	26 631	2805	220	42	13					241 577	9.863	* 9.88
17	361 342	31 400	2260	135	23	6					395 166	9.908	* 9.93
18	561 106	33 366	1628	76	12	3					596 191	9.938	* 9.95
19	795 630	31 996	1058	38	5	1					828 728	9.959	* 9.96
20	1 032 754	27 764	620	18	2	1					1 061 159	9.973	* 9.98
21	1 229 228	21 817	334	9	1						1 251 389	9.9820	> 9.99
22	1 343 120	15 558	170	4							1 358 852	9.9883	> 9.99
23	1 348 674	10 096	80	2							1 358 852	9.9924	> 9.99
24	1 245 369	5984	35	1							1 251 389	9.9952	> 9.99
25	1 057 896	3247	16								1 061 159	9.9969	> 9.99
26	827 086	1635	7								828 728	9.9980	> 9.99
27	595 418	770	3								596 191	9.9987	> 9.99
28	394 820	344	2								395 166	9.9991	> 9.99
29	241 428	148	1								241 577	9.9994	> 9.99
30	136 370	63									136 433	9.9995	> 9.99
31	71 293	25									71 318	9.9996	> 9.99
32	34 652	11									34 663	9.9997	> 9.99
33	15 767	5									15 772	9.9997	> 9.99
34	6757	2									6759	9.9997	> 9.99
35	2768	1									2769	9.9996	> 9.99
36	1102	1									1103	9.9991	> 9.99
37	428										428	10	10
38	165										165	10	10
39	66										66	10	10
40	26										26	10	10
41	11										11	10	10
42	5										5	10	10
43	2										2	10	10
44	1										1	10	10
45	1										1	10	10

*Estimated with
Mathematica function
ExactRandomGraph