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# Simple Graphs by Number of Edges ( $e \leq 8$ )

The number of edges in a graph can be significant regardless of the number of vertices. The number of reactions among a group of molecules may be more important than the molecules themselves. In a transportation system or communication network one might disregard the stations and highlight the conduits. Properties of networks may depend solely on the number of edges.

The graphs below (which for simplicity have no isolated vertices) are grouped according to number of edges,  $e$ , and within each group the graphs are classified by the number of connected components,  $\omega$  (omega). Note that this does not thereby group graphs by number of vertices.

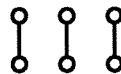
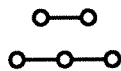
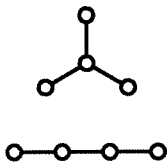
$e = 1$



$e = 2$



$e = 3$

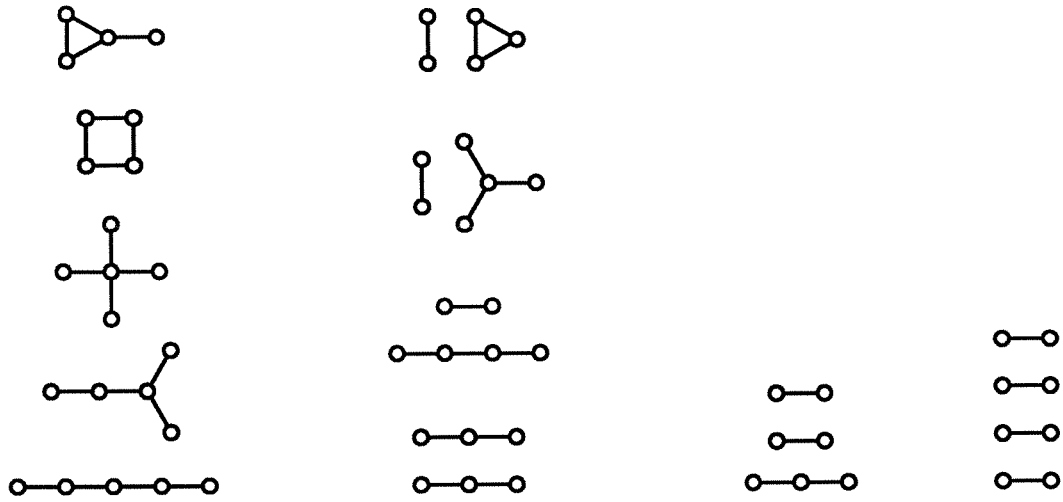


$\omega = 1$

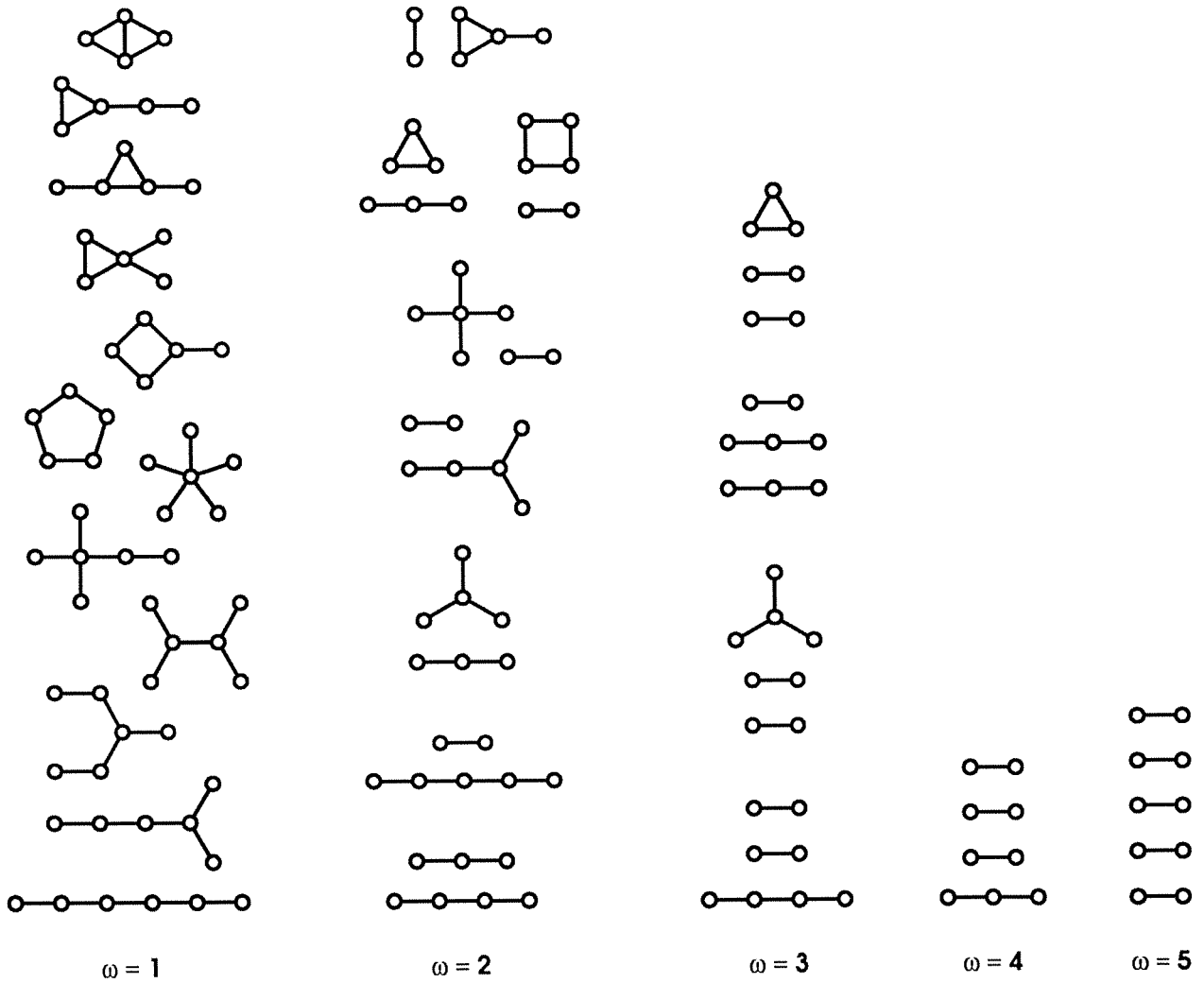
$\omega = 2$

$\omega = 3$

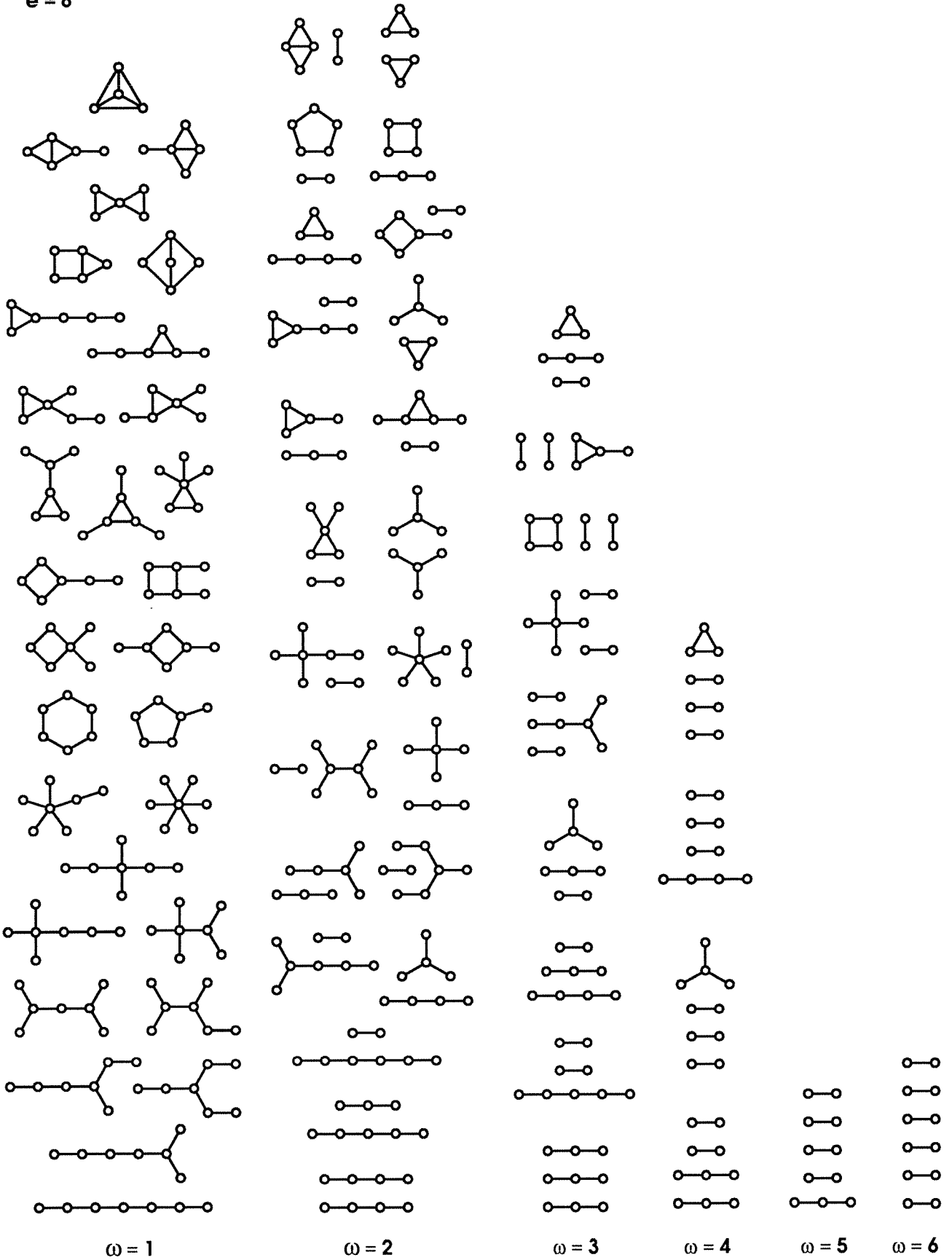
$e = 4$



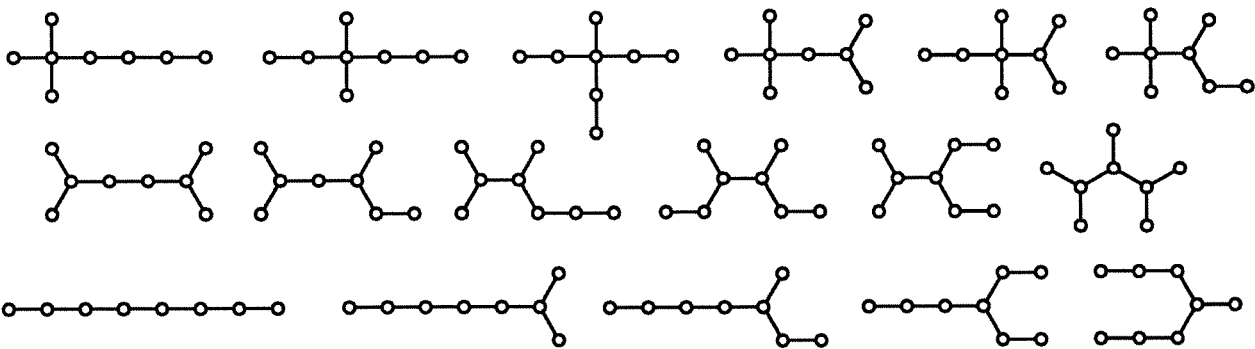
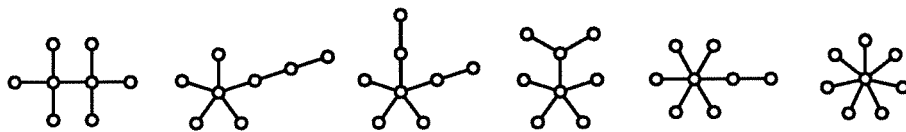
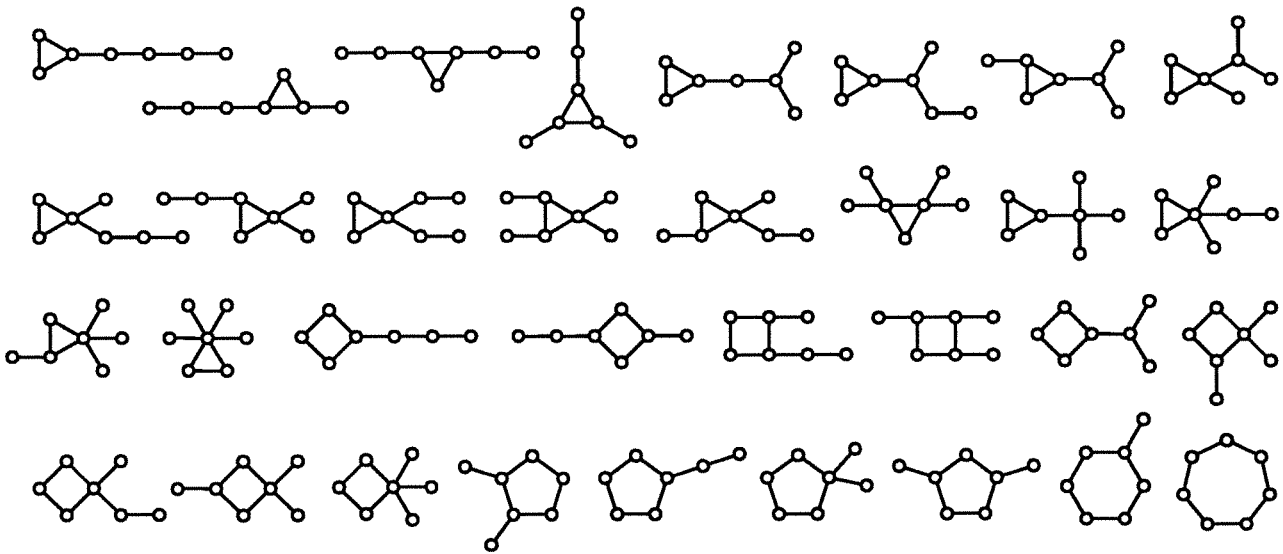
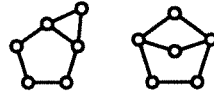
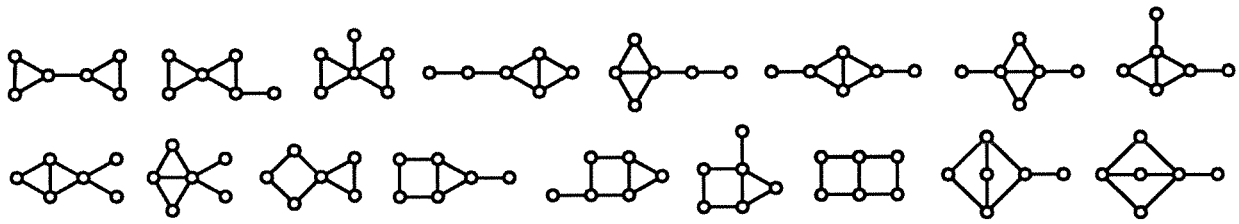
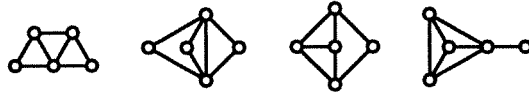
$e = 5$



$e = 6$

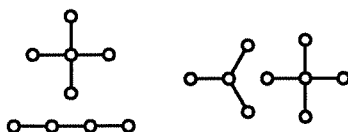
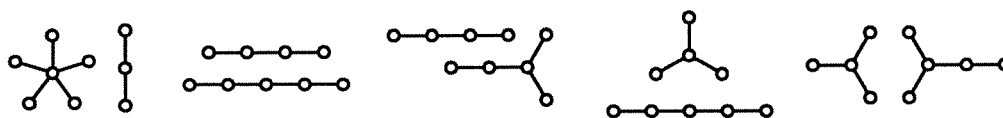
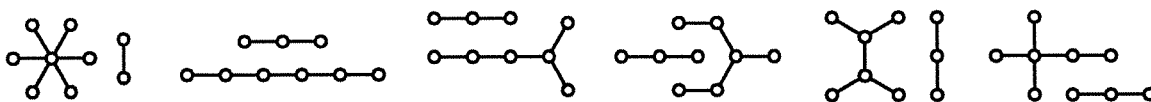
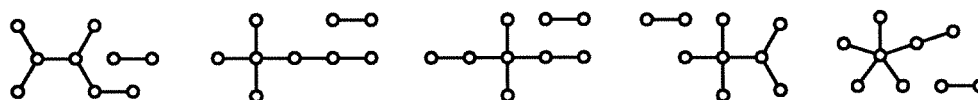
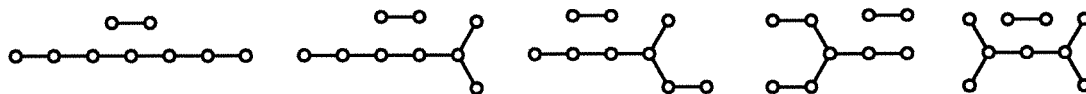
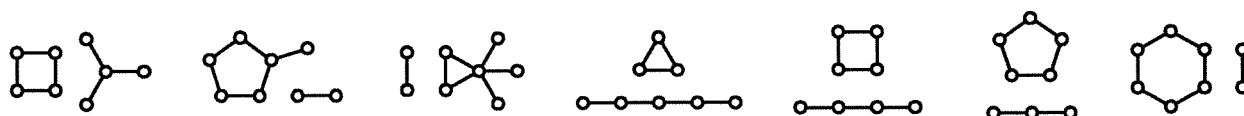
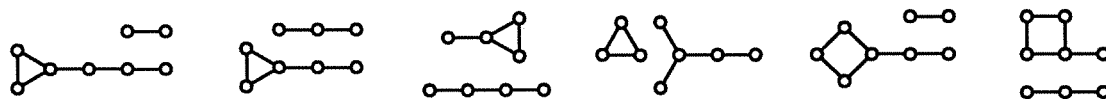
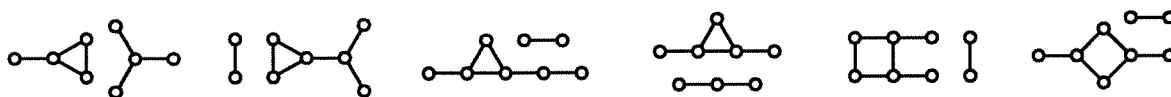
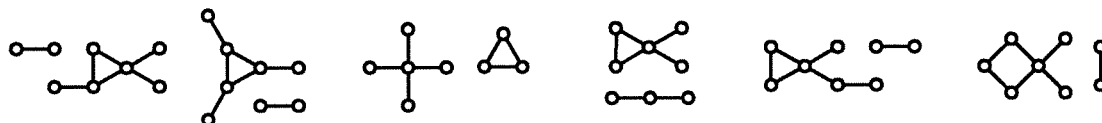
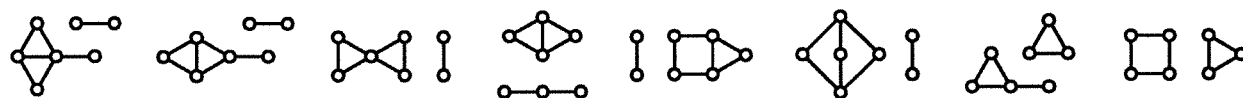
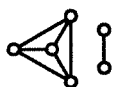


$e = 7$



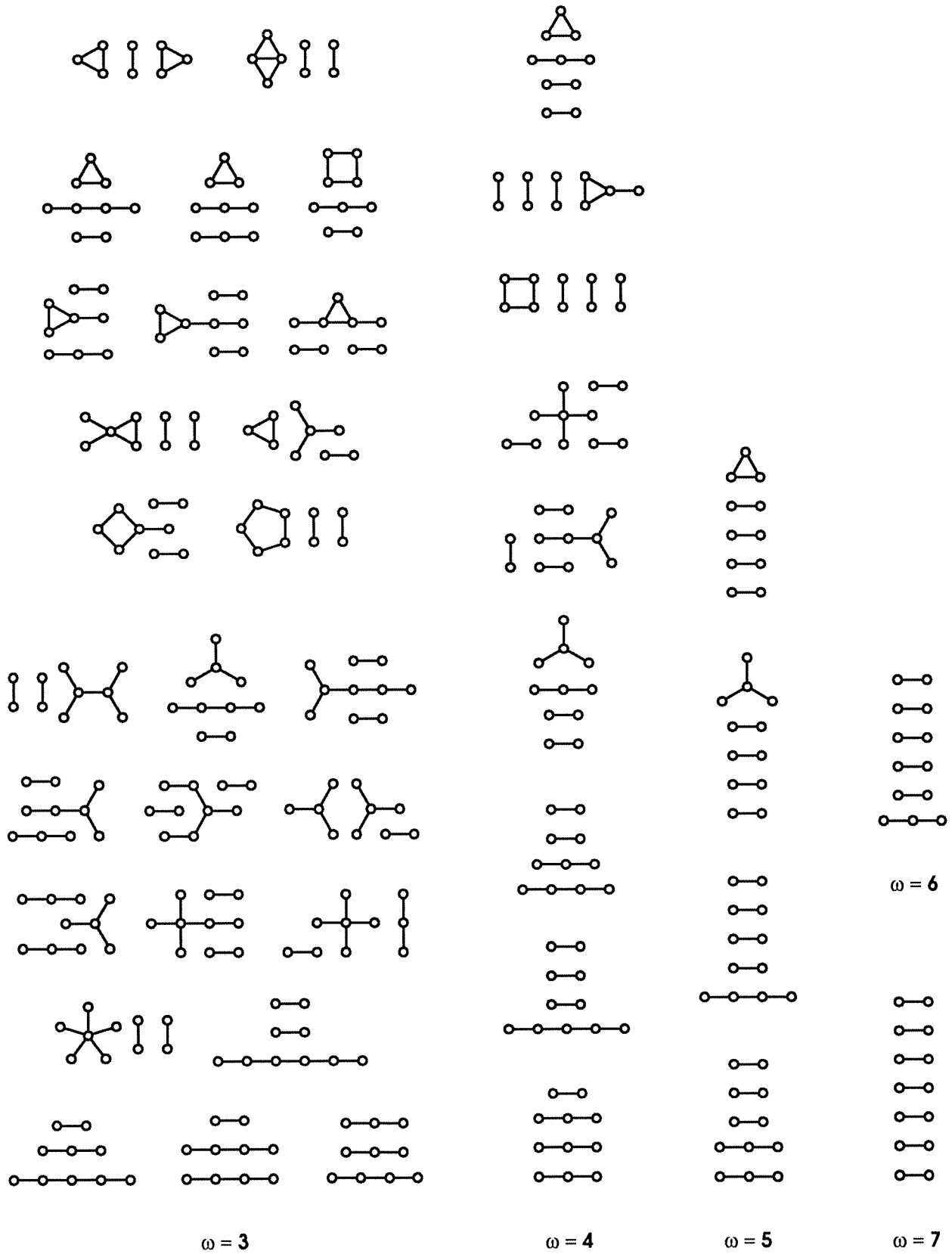
$\omega = 1$

e = 7

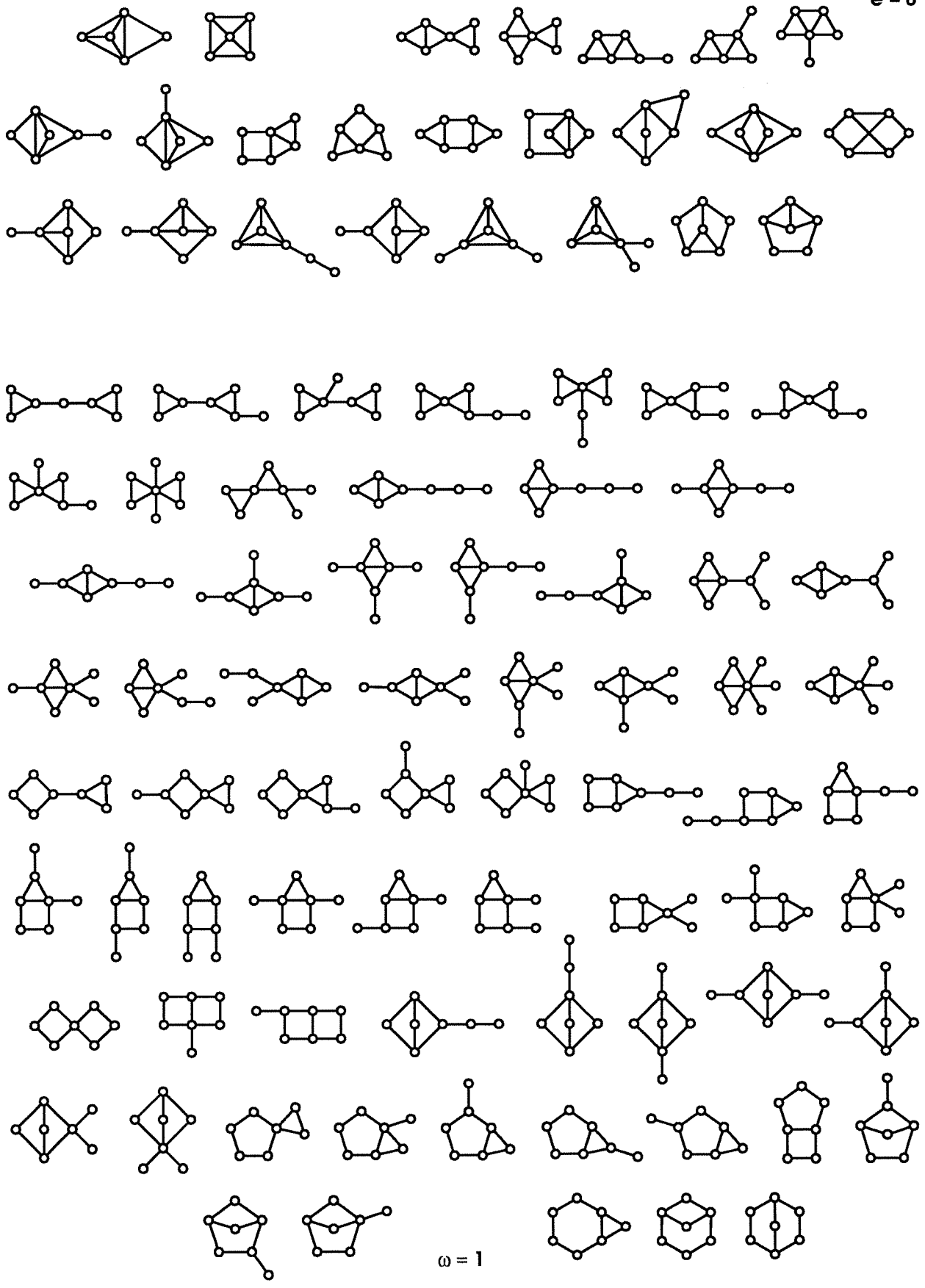


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$e = 7$

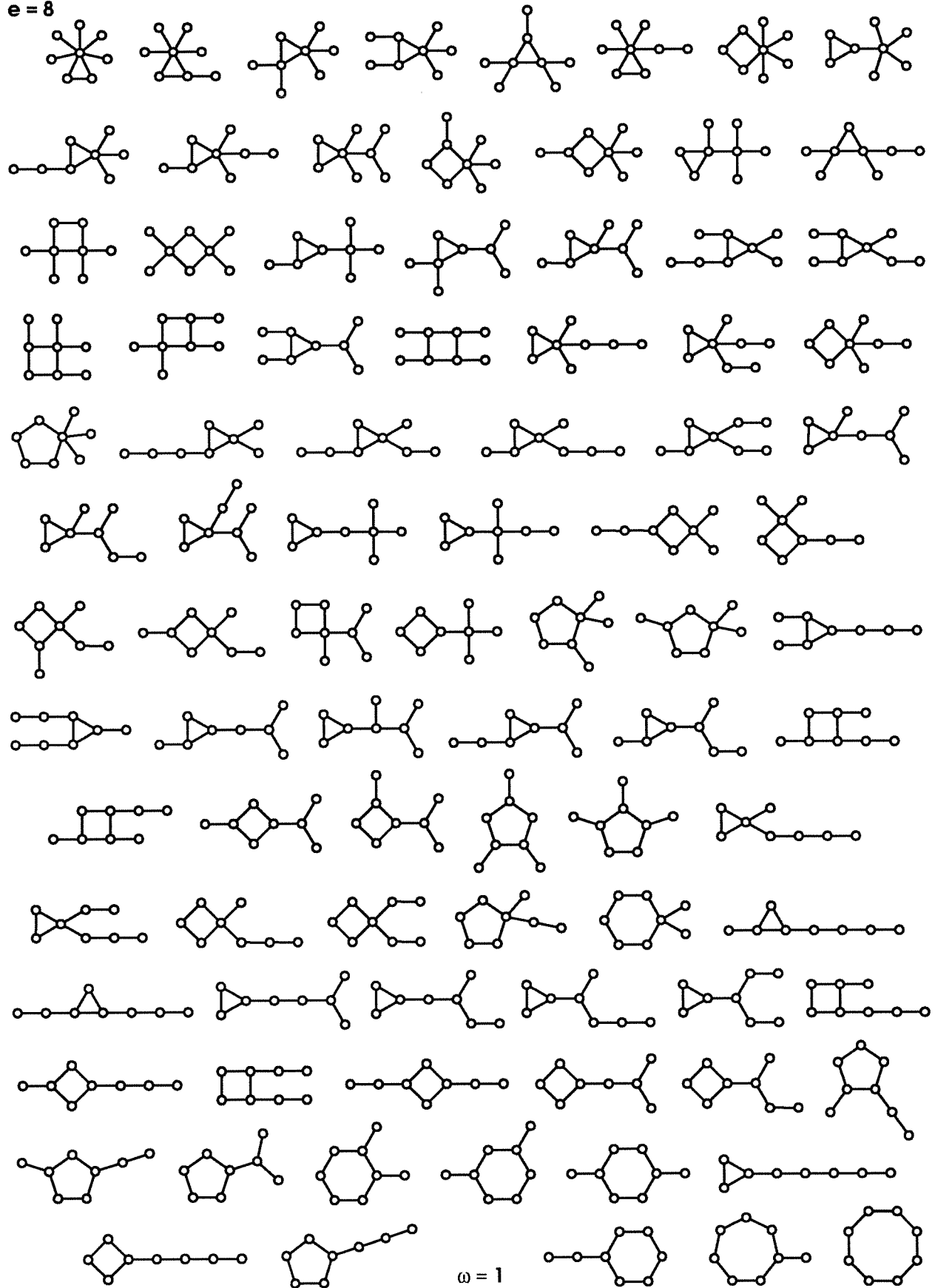


$e = 8$



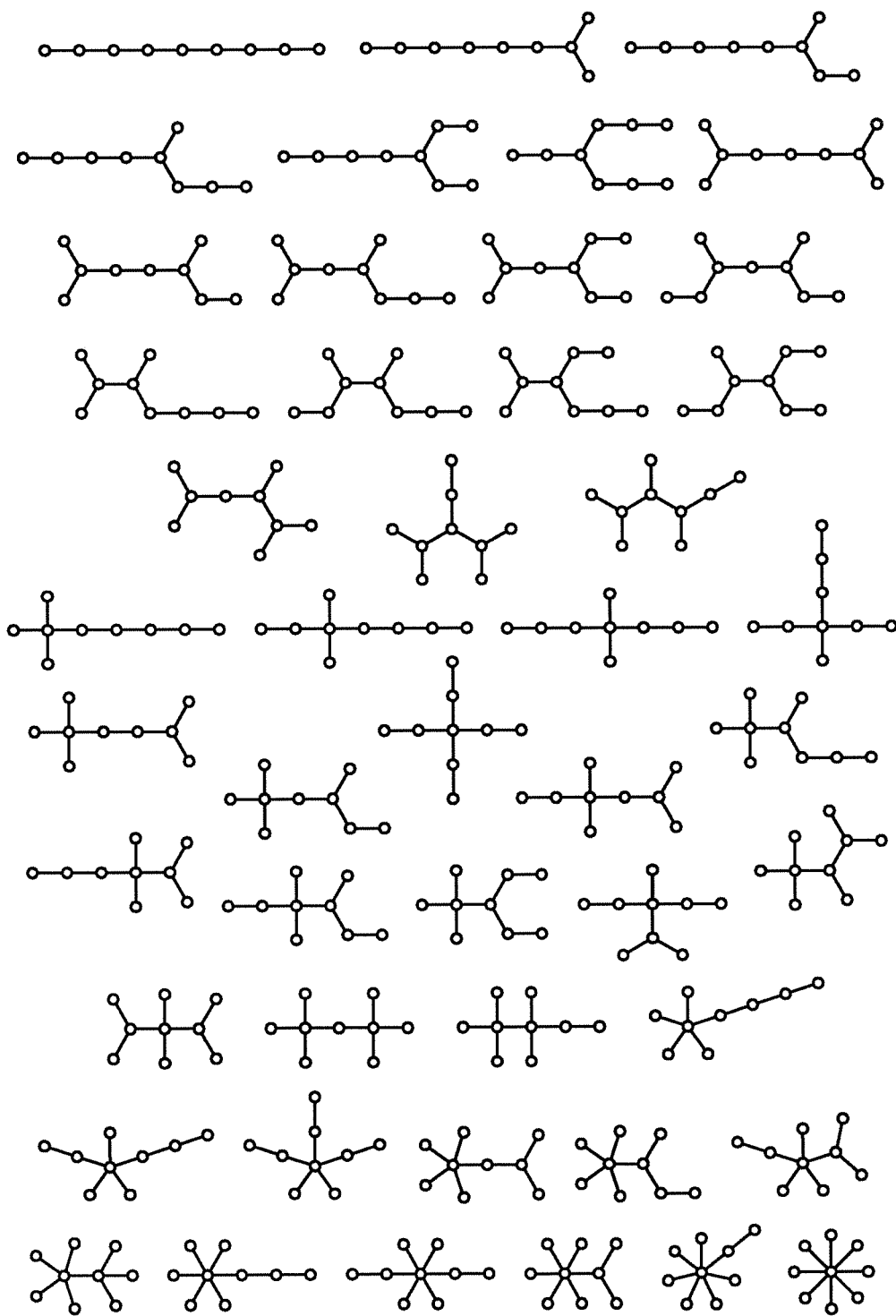
$\omega = 1$

$e = 8$



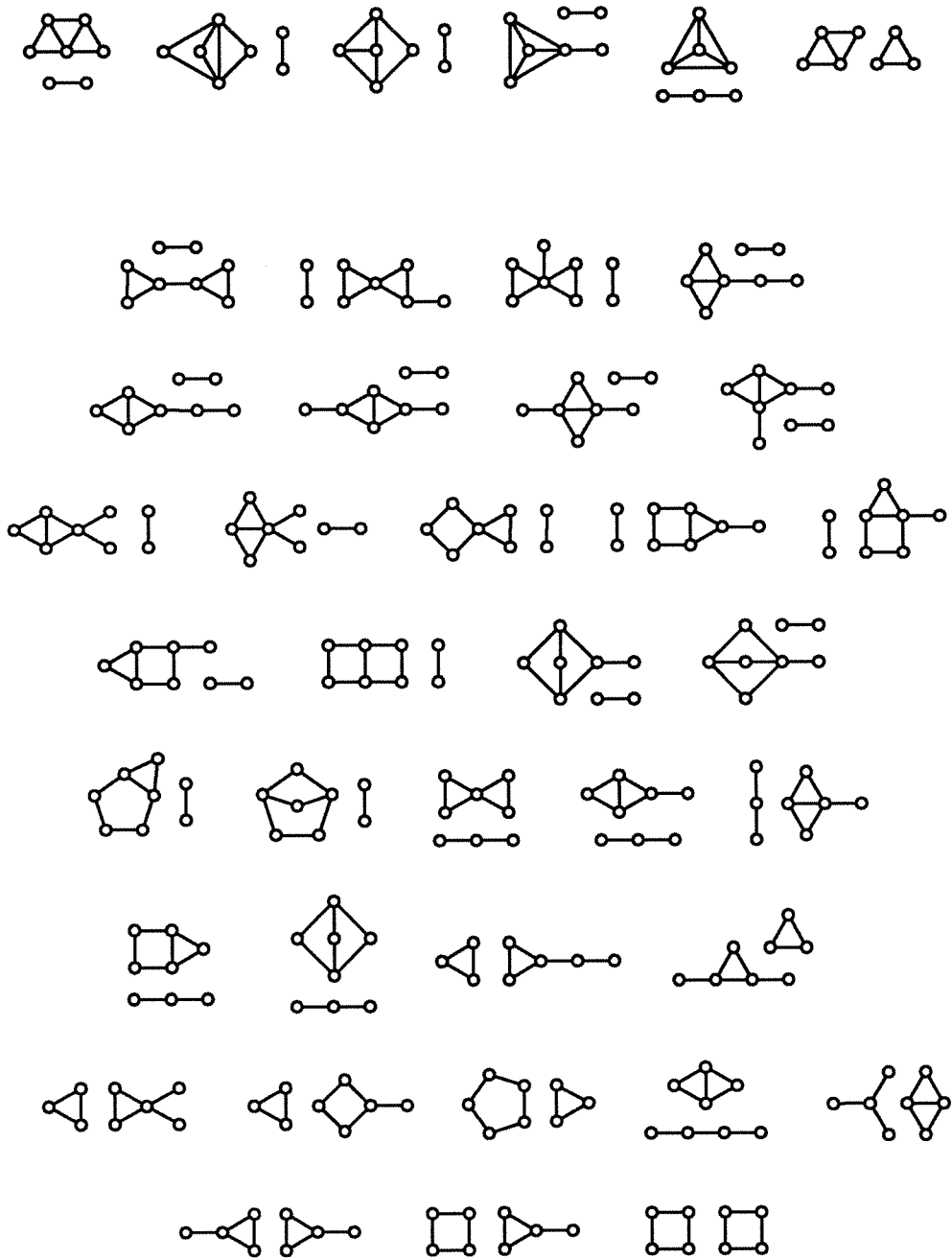
$\omega = 1$





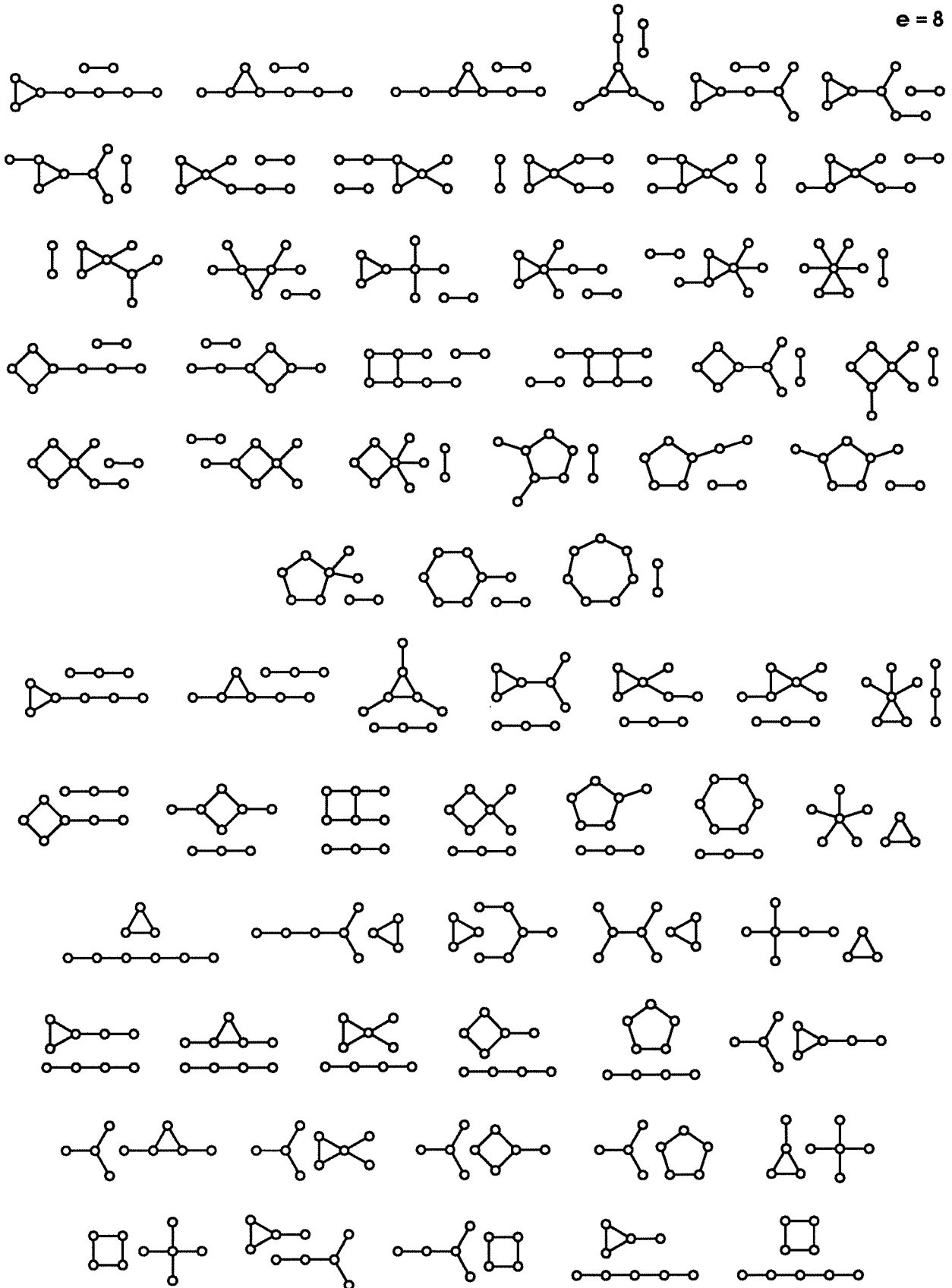
$\omega = 1$

$e = 8$



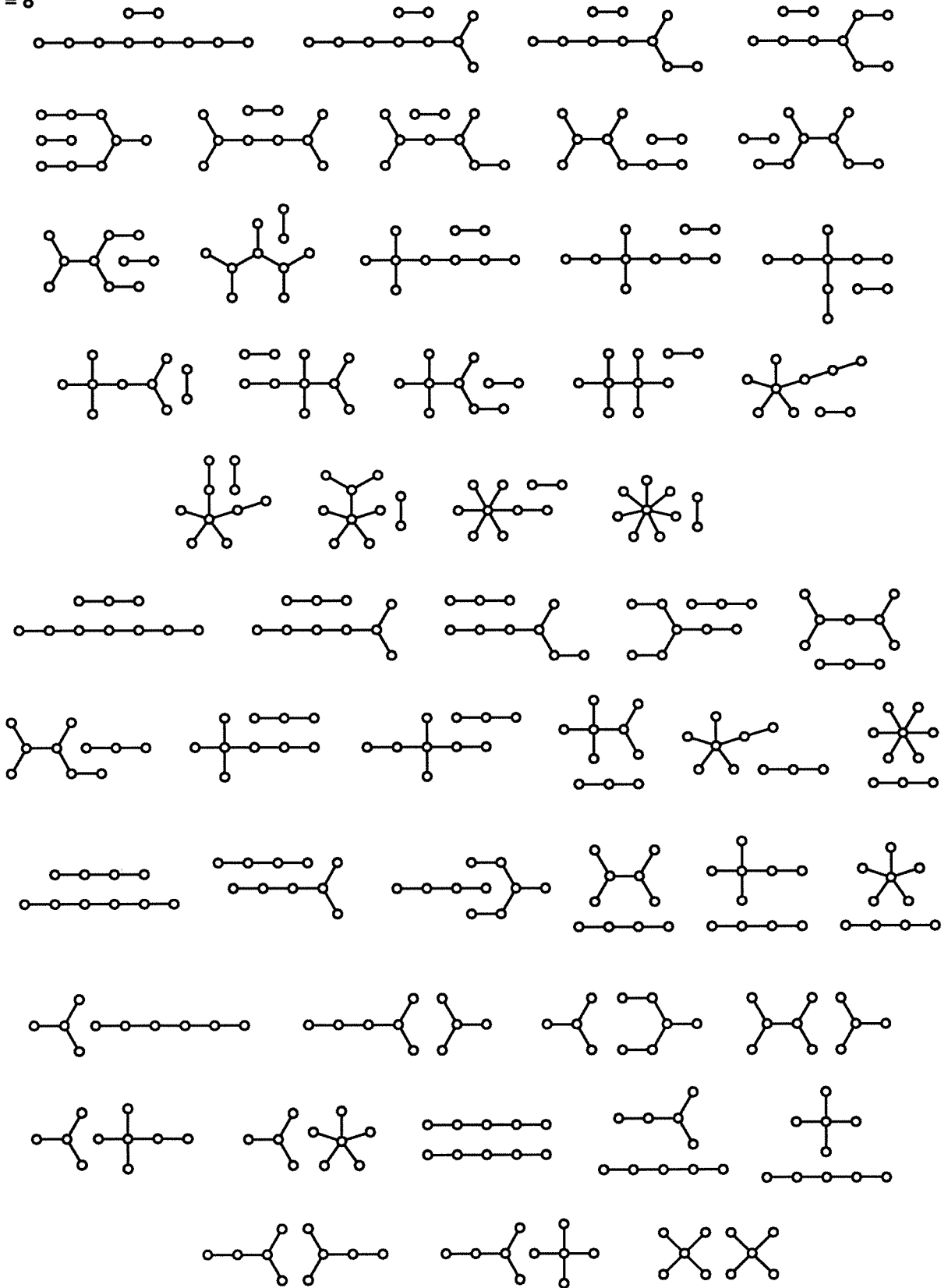
$\omega = 2$

$e = 8$

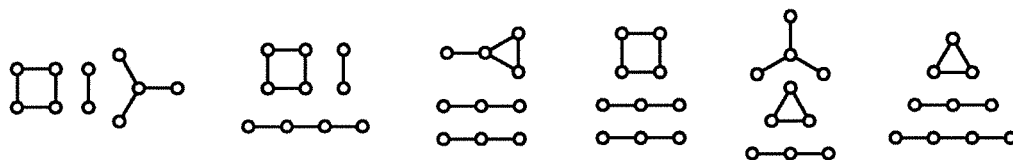
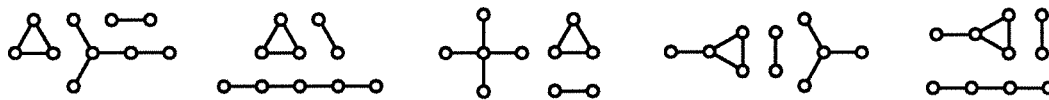
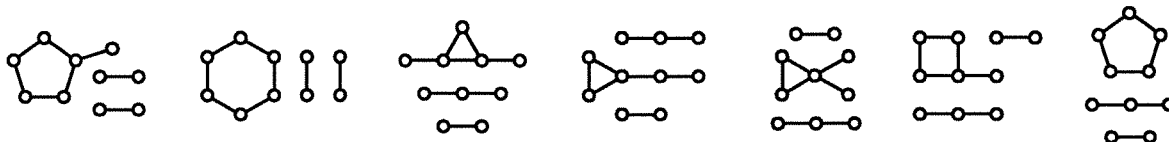
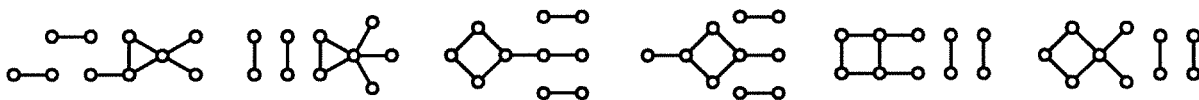
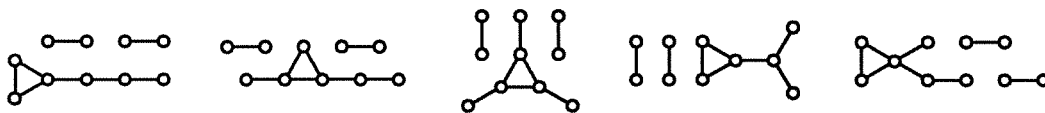
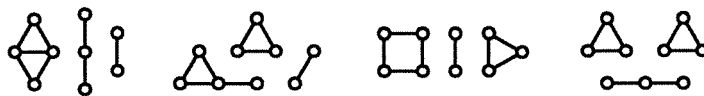
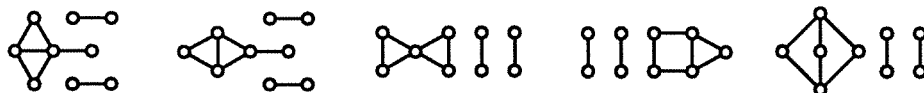


$\omega = 2$

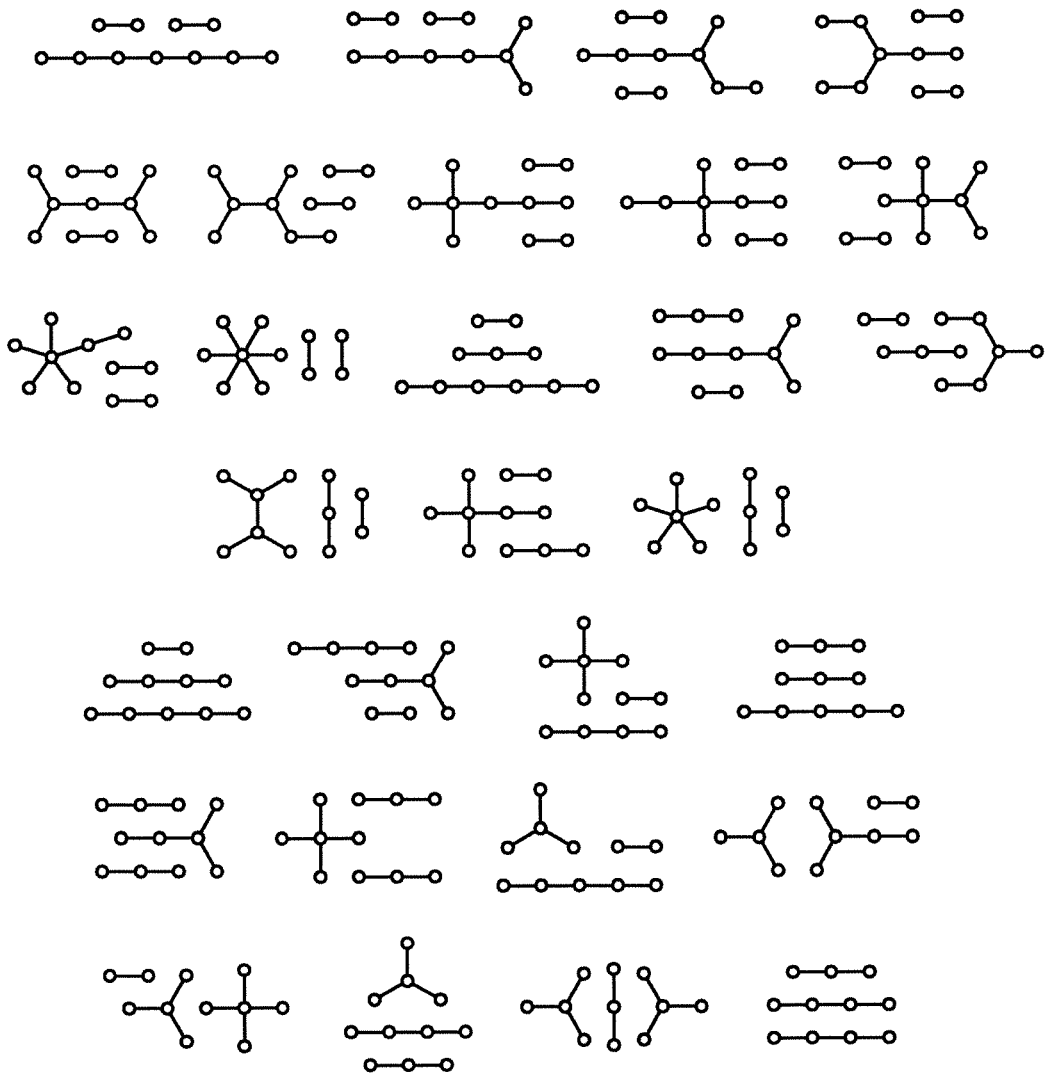
$e = 8$



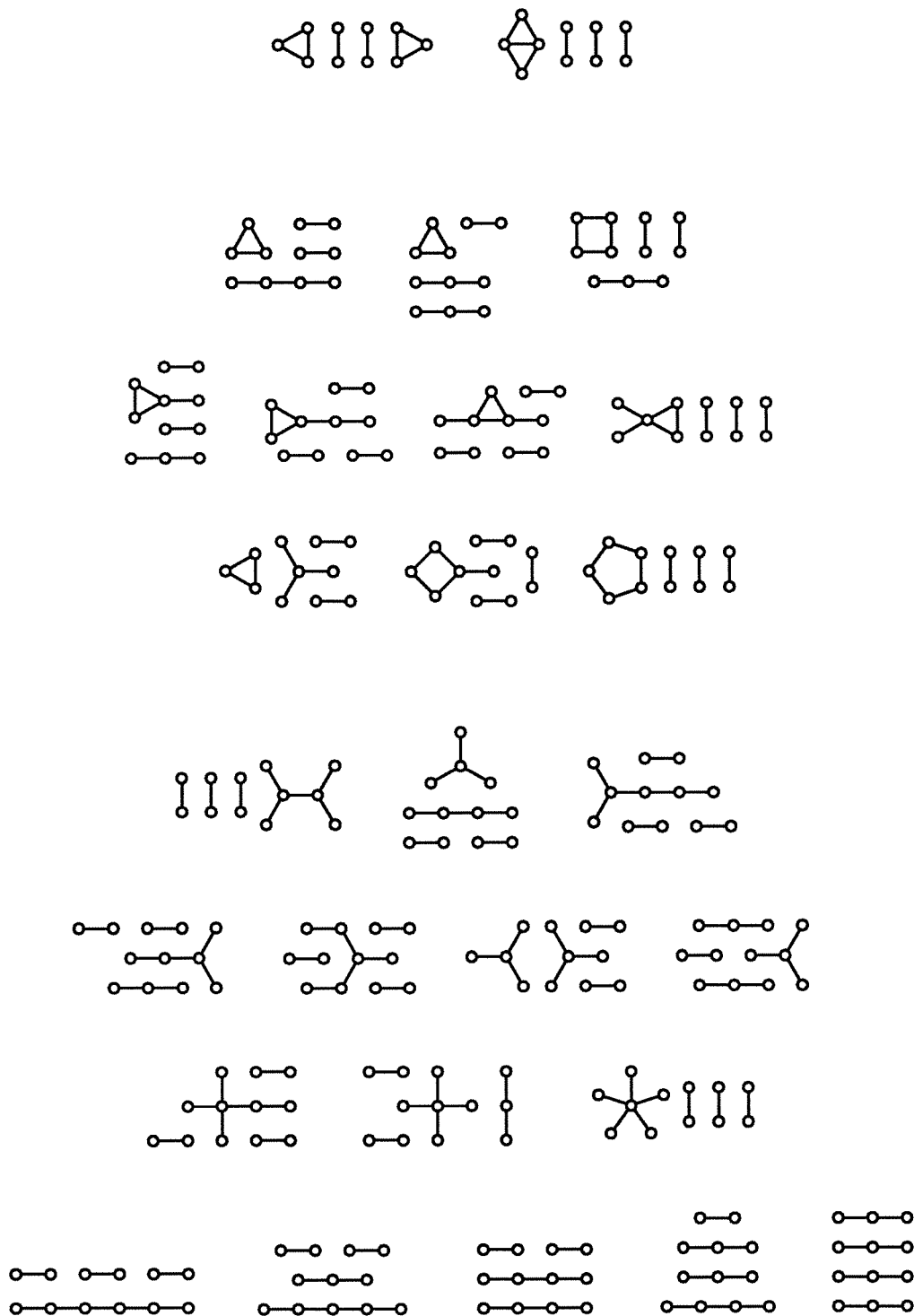
$\omega = 2$



$e = 8$

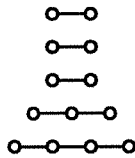
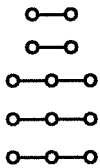
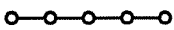
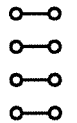
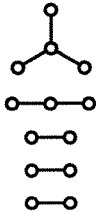
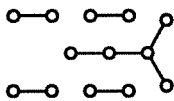
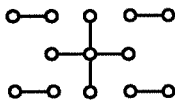
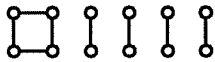
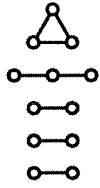


$\omega = 3$

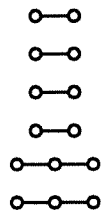
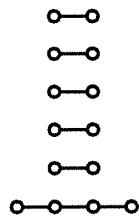
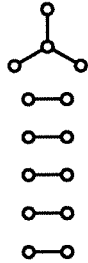
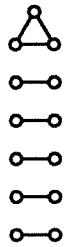


$\omega = 4$

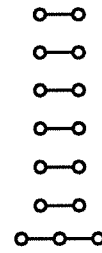
$e = 8$



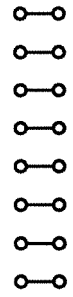
$\omega = 5$



$\omega = 6$



$\omega = 7$



$\omega = 8$



TABLE 1.1a Number of graphs with  $e$  edges,  $\omega$  components, and no isolated vertices

$e$	$\omega = 1$	2	3	4	5	6	7	8	9	10	11	12	totals	mean $\omega$
1	1												1	1
2	1	1											2	1.5
3	3	1	1										5	1.6
4	5	4	1	1									11	1.818
5	12	8	4	1	1								26	1.885
6	30	23	9	4	1	1							68	1.912
7	79	57	26	9	4	1	1						177	1.921
8	227	160	68	27	9	4	1	1					497	1.897
9	710	456	197	71	27	9	4	1	1				1476	1.850
10	2322	1402	567	208	72	27	9	4	1	1			4613	1.798
11	8071	4468	1748	604	211	72	27	9	4	1	1		15 216	1.740
12	29 503	15 071	5555	1874	615	212	72	27	9	4	1	1	52 944	1.681

TABLE 1.1b Number of graphs with  $e$  edges, aggregant  $\alpha$  (# of vertices in largest component), and no isolated vertices (except for  $e = 0$ )

$e$	$\alpha = 1$	2	3	4	5	6	7	8	9	10	11	totals	mean $\alpha$
0	1											1	1
1		1										1	2
2		1	1									2	2.5
3		1	2	2								5	3.2
4		1	3	4	3							11	3.818
5		1	4	7	8	6						26	4.538
6		1	6	15	16	19	11					68	5.162
7		1	7	24	34	44	44	23				177	5.904
8		1	9	38	71	97	122	112	47			497	6.622
9		1	11	61	133	211	295	371	287	106		1476	7.375
10		1	13	90	249	457	659	1015	1131	763	235	4613	8.132

TABLE 1.2a Number of graphs with  $e$  edges,  $\omega$  components, and no isolated vertices (expanded by number of vertices)

$e$	$\omega = 1$	2	3	4	5	6
1	2v 1					
2	3v 1	4v 1				
3	3v 1 <u>4v 2</u> 3	5v 1	6v 1			
4	4v 2 <u>5v 3</u> 5	5v 1 <u>6v 3</u> 4	7v 1	8v 1		
5	4v 1 5v 5 <u>6v 6</u> 12	6v 3 <u>7v 5</u> 8	7v 1 <u>8v 3</u> 4	9v 1	10v 1	
6	4v 1 5v 5 6v 13 <u>7v 11</u> 30	6v 2 7v 9 <u>8v 12</u> 23	8v 3 <u>9v 6</u> 9	9v 1 <u>10v 3</u> 4	11v 1	12v 1
7	5v 4 6v 19 7v 33 <u>8v 23</u> 79	6v 1 7v 8 8v 25 <u>9v 23</u> 57	8v 2 9v 10 <u>10v 14</u> 26	10v 3 <u>11v 6</u> 9	11v 1 <u>12v 3</u> 4	13v 1
8	5v 2 6v 22 7v 67 8v 89 <u>9v 47</u> 227	7v 6 8v 34 9v 68 <u>10v 52</u> 160	8v 1 9v 9 10v 29 <u>11v 29</u> 68	10v 2 11v 10 <u>12v 15</u> 27	12v 3 <u>13v 6</u> 9	13v 1 <u>14v 3</u> 4

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12

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14v 1

15v 1

16v 1

TABLE 1.2b Number of graphs with  
 $e$  edges,  $\omega$  components, and no isolated vertices  
 (expanded by number of vertices)

$e$	$\omega = 1$	2	3	4	5	6
9	5v 1					
	6v 20	7v 3				
	7v 107	8v 35	9v 7	10v 1		
	8v 236	9v 122	10v 39	11v 9	12v 2	
	9v 240	10v 186	11v 83	12v 30	13v 10	14v 3
	10v 106	11v 110	12v 68	13v 31	14v 15	15v 6
	<u>710</u>	<u>456</u>	<u>197</u>	<u>71</u>	<u>27</u>	<u>9</u>
10	5v 1					
	6v 14	7v 1				
	7v 132	8v 29	9v 3			
	8v 486	9v 174	10v 39	11v 7	12v 1	
	9v 797	10v 436	11v 144	12v 40	13v 9	14v 2
	10v 657	11v 509	12v 233	13v 87	14v 30	15v 10
	11v 235	12v 253	13v 148	14v 74	15v 32	16v 15
<u>2322</u>	<u>1402</u>	<u>567</u>	<u>208</u>	<u>72</u>	<u>27</u>	
11	6v 9	7v 1				
	7v 138	8v 18	9v 1			
	8v 814	9v 196	10v 31	11v 3		
	9v 2075	10v 798	11v 197	12v 40	13v 7	14v 1
	10v 2678	11v 1485	12v 522	13v 149	14v 40	15v 9
	11v 1806	12v 1400	13v 651	14v 248	15v 88	16v 30
	12v 551	13v 570	14v 346	15v 164	16v 76	17v 32
<u>8071</u>	<u>4468</u>	<u>1748</u>	<u>604</u>	<u>211</u>	<u>72</u>	
12	6v 5					
	7v 126	8v 12	9v 1			
	8v 1169	9v 187	10v 19	11v 1		
	9v 4495	10v 1210	11v 214	12v 32	13v 3	
	10v 8548	11v 3439	12v 912	13v 203	14v 40	15v 7
	11v 8833	12v 5014	13v 1799	14v 547	15v 150	16v 40
	12v 5026	13v 3866	14v 1814	15v 702	16v 252	17v 88
	13v 1301	14v 1343	15v 796	16v 389	17v 170	18v 77
<u>29 503</u>	<u>15 071</u>	<u>5555</u>	<u>1874</u>	<u>615</u>	<u>212</u>	

7

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11

12

15v 1  
16v 3  
 4

17v 1

18v 1

16v 3  
17v 6  
 9

17v 1  
18v 3  
 4

19v 1

20v 1

16v 2  
 17v 10  
18v 15  
 27

18v 3  
19v 6  
 9

19v 1  
20v 3  
 4

21v 1

22v 1

16v 1  
 17v 9  
 18v 30  
19v 32  
 72

18v 2  
 19v 10  
20v 15  
 27

20v 3  
21v 6  
 9

21v 1  
22v 3  
 4

23v 1

24v 1