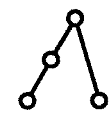
7
Hasse Diagrams (n ≤ 6)

A Hasse diagram is an artful rendering of a blank poset or covering set. The partial order is given by vertical levels, in which two nodes are comparable if there is an edge or series of edges connecting them monotonically — without a change of direction. This is the essence of the visual analog. In particular note there are no triangles — the third edge is redundant — as transitivity of three monotonic nodes is assumed. And no loops since reflexivity is assumed or unnecessary. Antisymmetry is just uni-directionality: if A is above B, it is not below B.

Nodes on the same level are always incomparable. The number of levels is known as the *depth* of the poset. Note that for this poset the level of the node at extreme right is ambiguous — it could be on the first or second level — but that, even so, the number of levels is well-defined. This is because every poset has at least one *spine* (or *maximum chain*) — a series of edges spanning all levels monotonically, a well-ordered series within the partial order. Nodes on the top level are unambiguous.

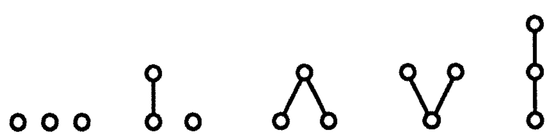


Though the question of who is on what level is not entirely answered in a poset, the governing convention in the drawings is this: For a node to be on level **k** it must have an immediate predecessor on level **k–1**. If not, then it is on a lower level. A node with no subordinates must be on level **1**. This makes the structures suitable as chains of command.

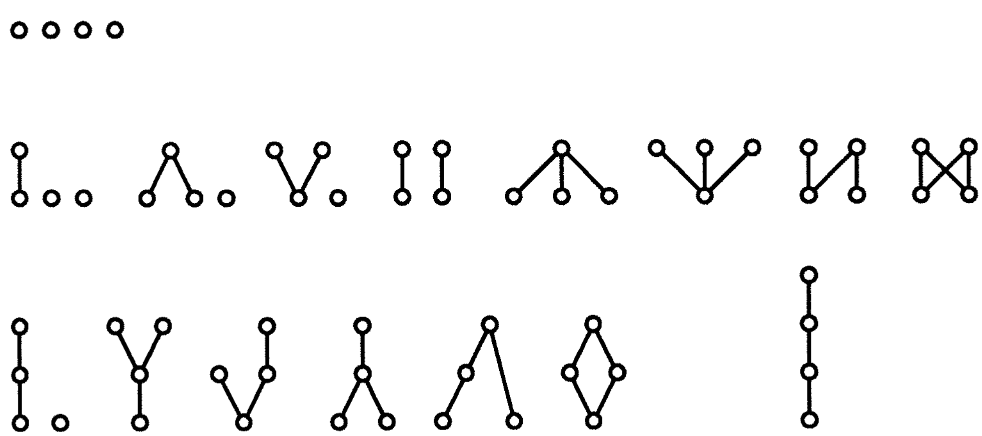
n = 1

n = 2

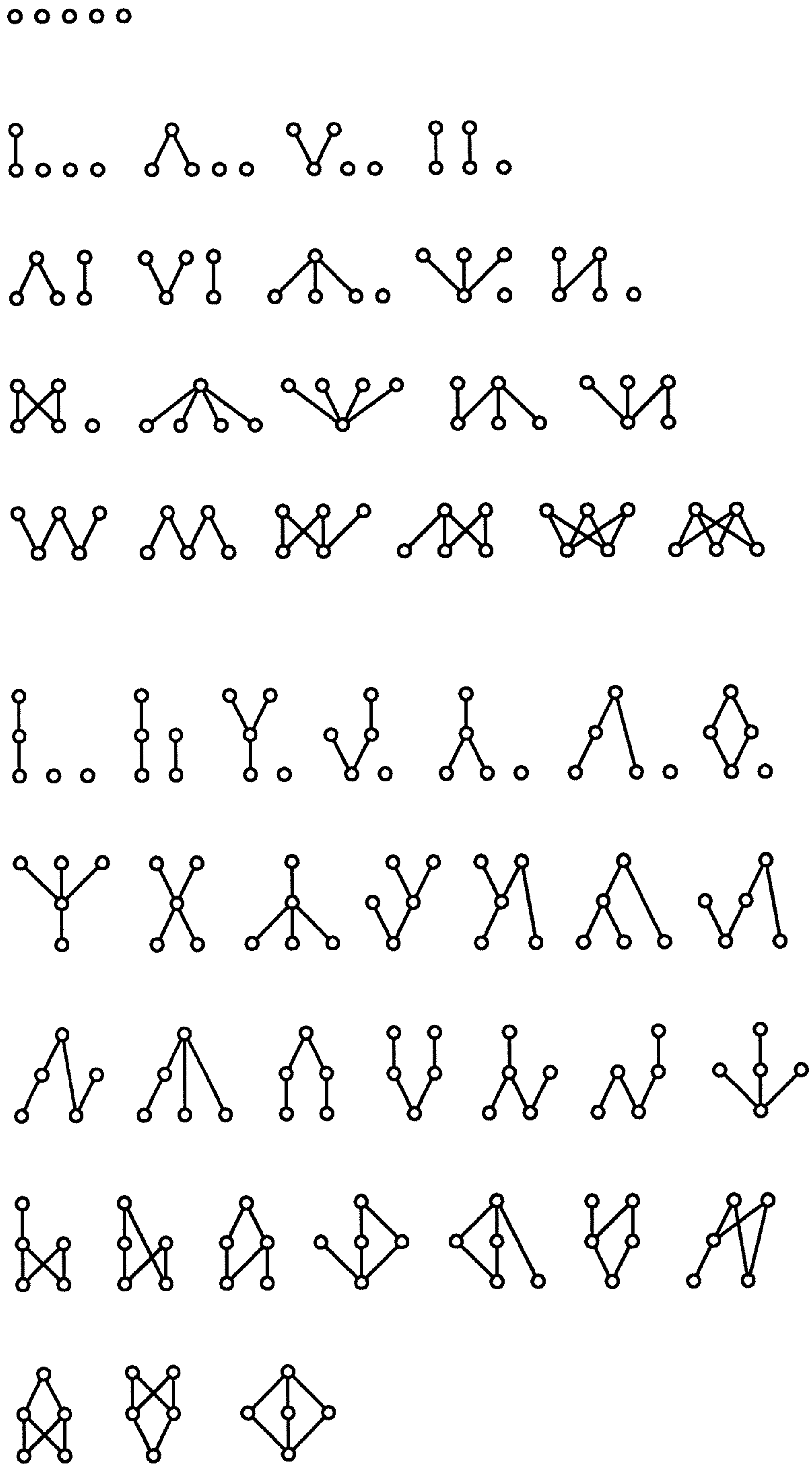
n = 3



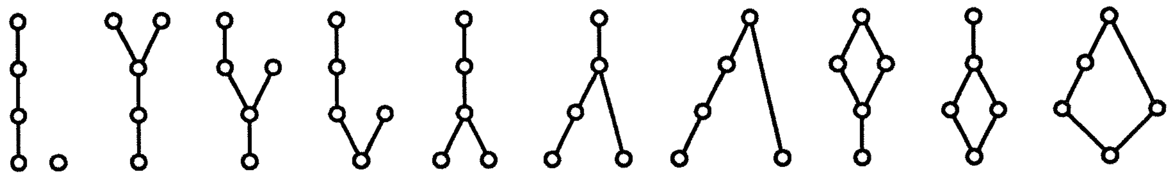
n = 4



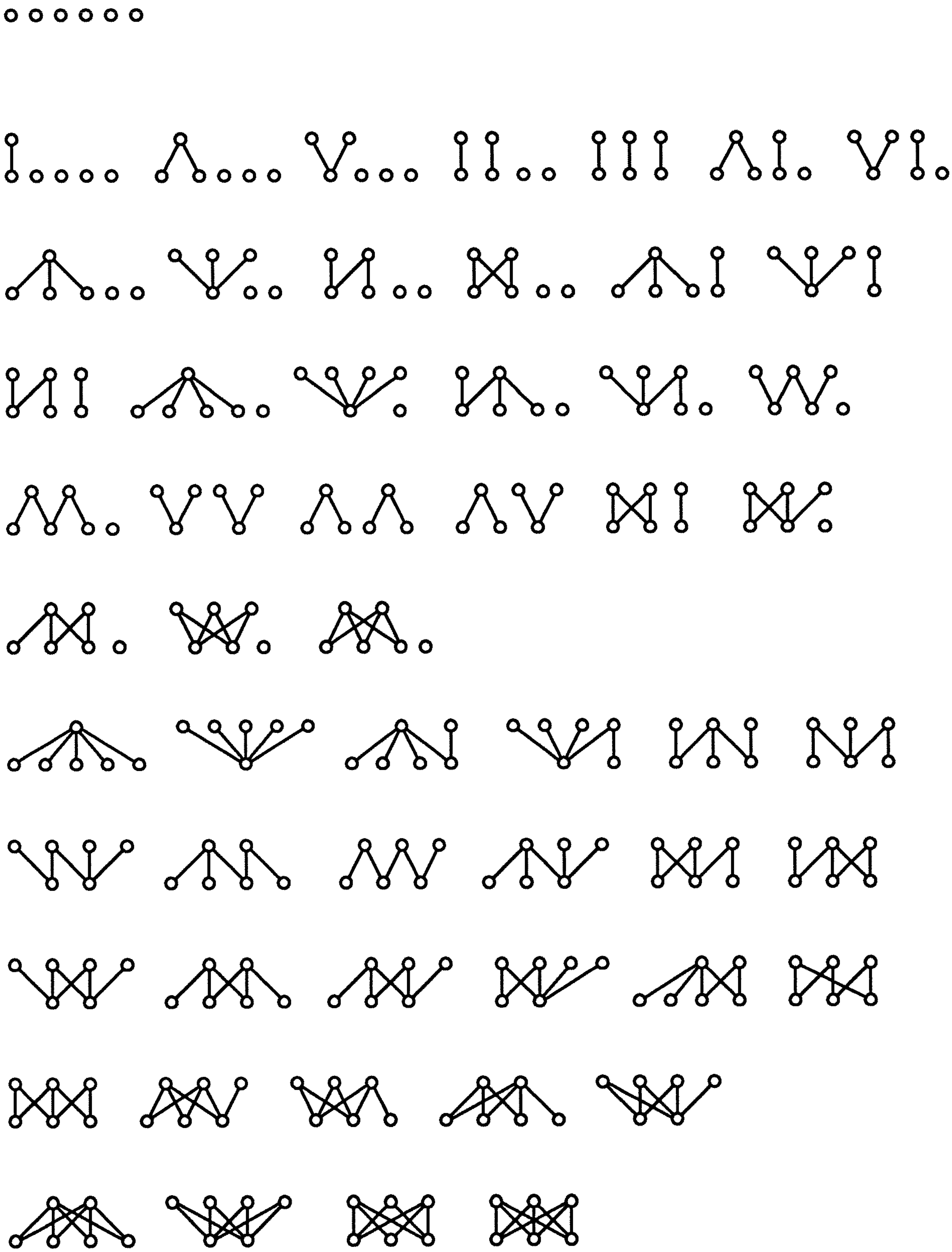
n = 5



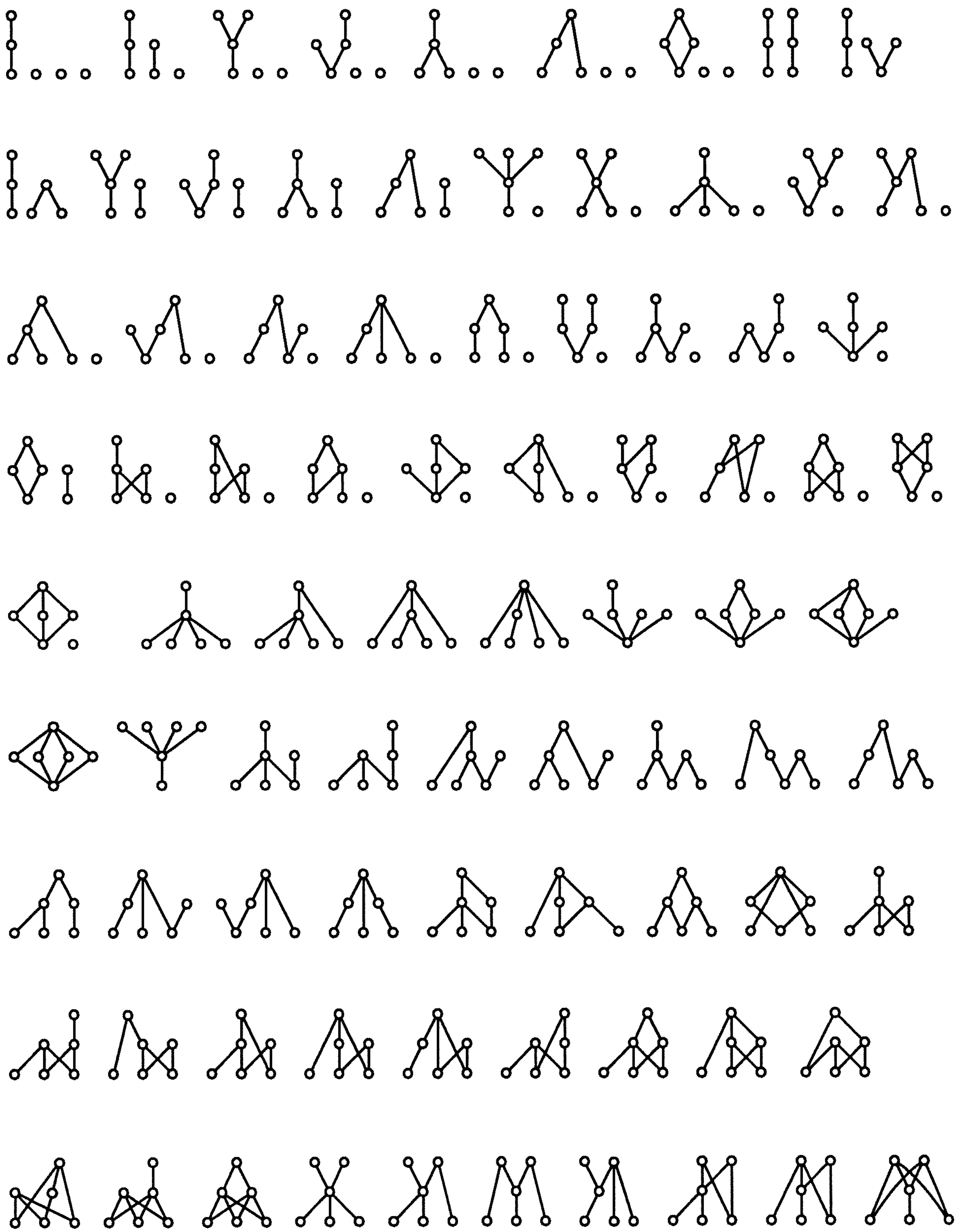
n = 5 (continued)



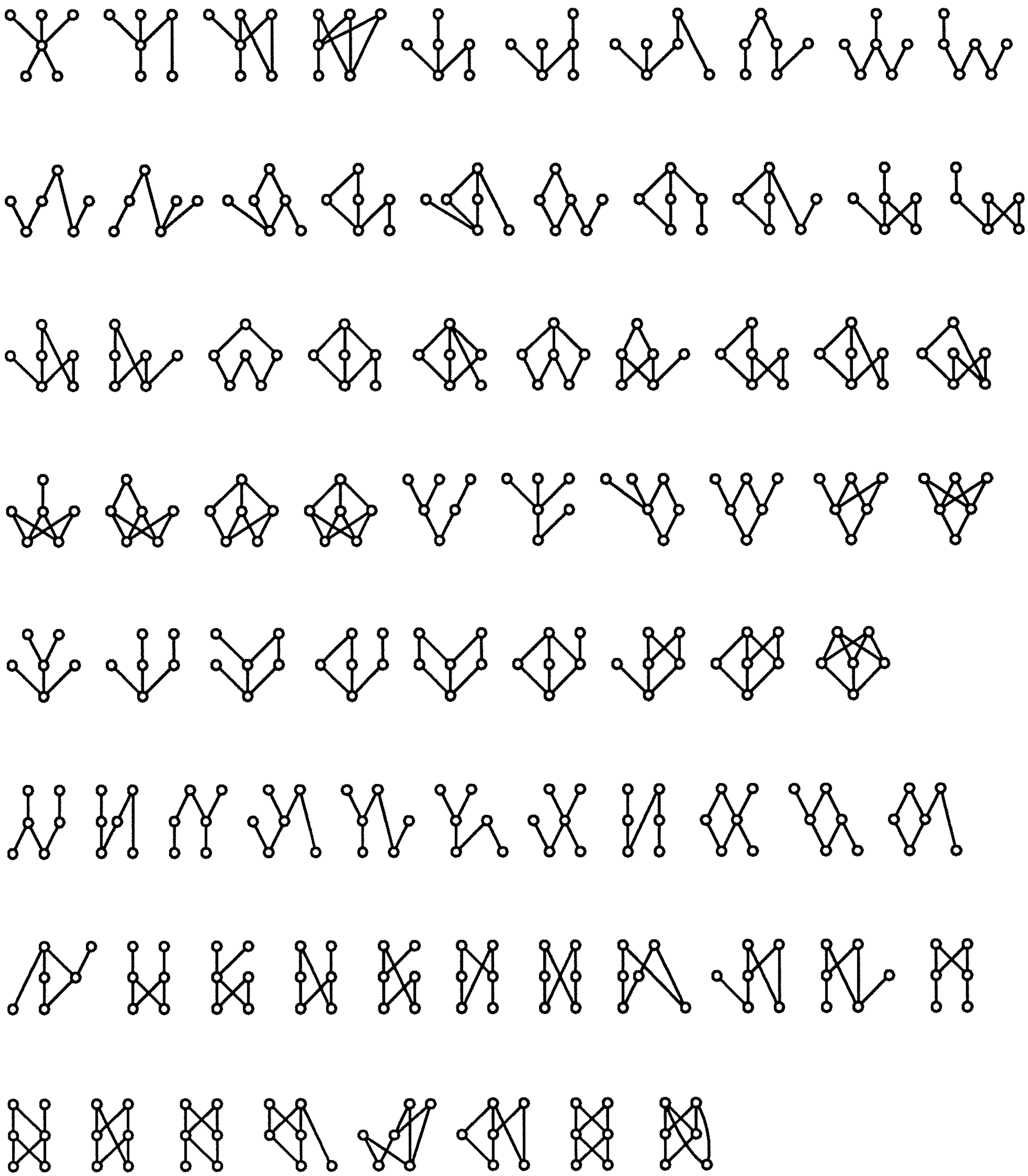
n = 6



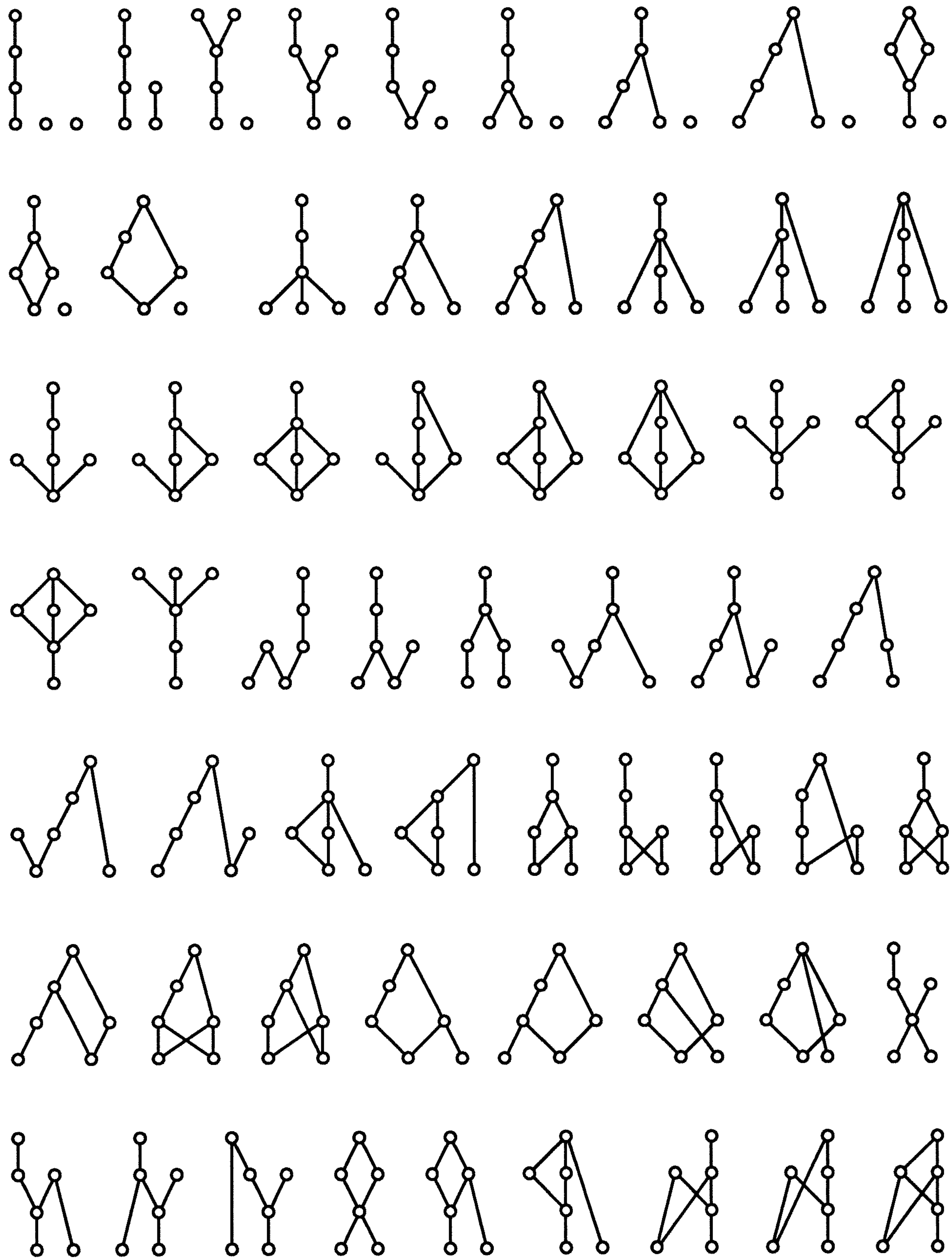
n = 6 (continued)



n = 6 (continued)



n = 6 (continued)



n = 6 (continued)

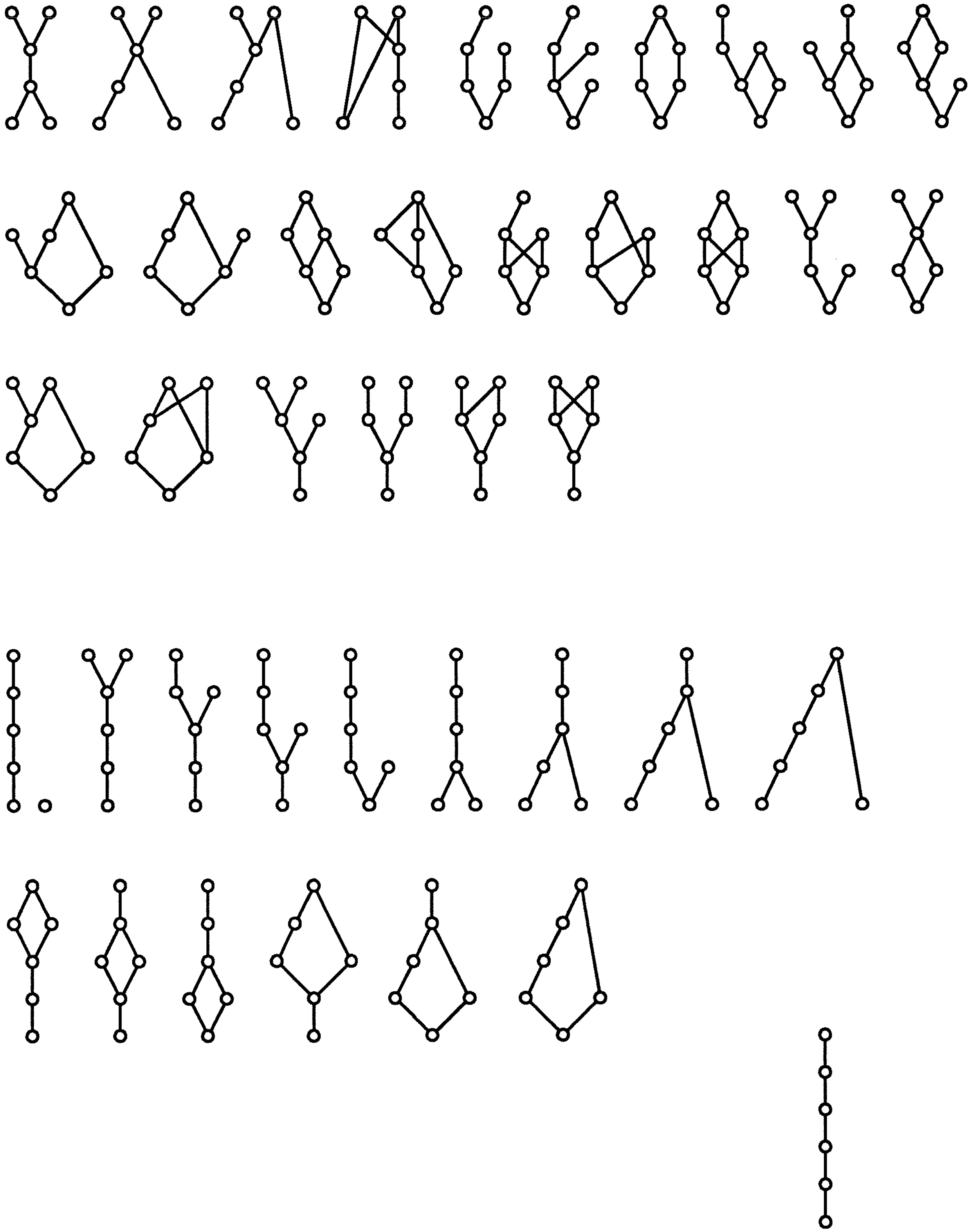


TABLE 7.1 Number of posets on **n** elements with depth **d**
(numbers from Brinkmann & McKay, *Posets on up to 16 Points.*)

n	**d = 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**all posets**	**connected posets**
1	1										1	1
2	1	1									2	1
3	1	3	1								5	3
4	1	8	6	1							16	10
5	1	20	31	10	1						63	44
6	1	55	162	84	15	1					318	238
7	1	163	940	734	185	21	1				2045	1650
8	1	556	6372	7305	2380	356	28	1			16 999	14 512
9	1	2222	52 336	86 683	35 070	6259	623	36	1		183 231	163 341
10	1	10 765	534 741	1 261 371	619 489	125 597	14 258	1016	45	1	2 567 284	2 360 719