

I know that you know that I know that you know...

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Philosophy Welcome Talk, Oxford
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A welcome via: epistemic Logic puzzles!

I welcome you all to Oxford University, especially those studying:

- Maths and Philosophy
- Physics and Philosophy
- Computer Science and Philosophy
- Psychology, Philosophy and Linguistics
- or any philosophy or logic-related course. . .

Let me show you some fun puzzles in the logic of knowledge.

Five logicians walk into the *Logic Café*

Waiter: *Do you all want beer?*

First logician: *I don't know.*

Second logician: *I don't know.*

Third logician: *I don't know.*

Fourth logician: *I don't know.*

Fifth logician: *No.*

Question

How many beers does the waiter bring?

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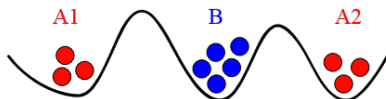
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Exactly four. The first four logicians definitely want beer, and the fifth definitely does not.

The two generals problem

Two red generals want to coordinate their attack on the blue army, but are separated by hills. If both attack, it will succeed. But if only one general attacks, they will fail. Communication is difficult.



The first red general dispatches a messenger through the dangerous blue valley: *We attack at dawn; agreed?*

The message gets through! The other red general replies: *Yes, at dawn! Please confirm.*

Confirming the message

- **A1: *We attack at dawn; agreed?***
- **A2: *Yes, at dawn! Please confirm.***
- *A1: Received your message! Ready to go at dawn...provided we know you get this message.*
- *A2: Got it! We're definitely on, once we know you have received this.*

How much confirmation suffices for the generals to have the information they need in order to count on the other general?

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How much confirmation suffices?

Obviously, the first message requires confirmation.

Let n be the smallest number of confirming messages that need to be sent.

But in this case, the protocol should work whether or not the last message is actually sent, since it might have been lost.

So $n - 1$ messages actually suffice, a contradiction.

Conclusion: no finite number of message can ensure that the generals have the information that is required to be confident in the plan.

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Another version: the two-parents problem

In younger days, my wife would pick up our daughter Hypatia at school, and I would get Horatio. One day, it was convenient to swap.

Me: I'd like to pick up Hypatia today, if you can get Horatio. Please confirm if this is alright.

Wife: Sure! But let me know, so I know the plan is on.

Me: OK, we're on! ...if I know you get this message.

Wife: Got the message. We're on! But let me know that you got this, so I can count on you.

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The two-parents problem

Just as with the two generals, at no stage could we know for certain that the other person had all the necessary confirmation to count on the other person making the swap.

Clearly the first message needed confirmation, and the reply needed its own confirmation, for otherwise she couldn't have known that I got the confirmation; and then that reply needed to be confirmed, and so on endlessly, just as before.

In the end, despite all the messages, we both did the logical thing: we abandoned the swap idea and just picked up the usual child at the usual school.

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This is not just a silly problem!

The issue arises in the design of internet protocols.

Two computers want to coordinate a data transfer—when will a video stream be sent? How will a financial transaction be completed? How can the two parties achieve the state of common knowledge that appears required?

General conclusion: they can't. protocols cannot be designed to achieve perfect coordination with common knowledge.

Cheryl's birthday

A few years ago a logic puzzle from the Singapore Math Olympiad went viral on the internet:

24. Albert and Bernard just become friends with Cheryl, and they want to know when her birthday is. Cheryl gives them a list of 10 possible dates.

May 15	May 16	May 19
June 17	June 18	
July 14	July 16	
August 14	August 15	August 17

Cheryl then tells Albert and Bernard separately the month and the day of her birthday respectively.

Albert: I don't know when Cheryl's birthday is, but I know that Bernard does not know too.
Bernard: At first I don't know when Cheryl's birthday is, but I know now.
Albert: Then I also know when Cheryl's birthday is.

So when is Cheryl's birthday?

The right image is a screenshot of a Facebook post by Kenneth Jameson. The post text reads: "This question causes a debate with my wife ... and is a fun 3 question." Below the text are several comments, including one from Kenneth Jameson himself stating: "Kenneth Jameson though I can't recall which answer is a possible one, but definitely July 14 is a highly possible one. But how to answer it, you may check out the post from my past." The screenshot also shows a "Study Room" link and a "Study Room" section with a video thumbnail.

The problem stumped millions of people all over the world!
Can we solve it?

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Solving Cheryl's birthday

Albert has only the month, and Bernard has only the day.
Both of them know this.

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Bernard: *At first I didn't know Cheryl's birthday, but now I know.
So the date is not 14.*

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So the month is not August. It must be July 16!

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Cheryl privately gives Albert and Bernard each a natural number.

Cheryl: You have different natural numbers. Whose is larger?

Albert: I don't know.

Bernard: I don't know either.

Albert: I still don't know whose number is larger.

Bernard: Alas, I remain in ignorance.

Albert: Ah, now that you say that, suddenly I know whose number is larger!

Bernard: Really? In that case, I know both numbers!

Albert: And now I also know both numbers.

Can we deduce the numbers?

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Albert can reason just as we did.

The blue-eyed islanders

There is a remote island with 100 perfectly logical inhabitants, all with blue-eyes.

In their cultural practice, they do not discuss eye color.

Indeed, if any of them should come to know they have blue eyes, they must leave the island the next dawn with a flashy display.

One day a trusted visitor arrives. Departing at the end of his visit, he says, *“At least one of you has blue eyes.”*

Exactly one hundred days later, all the islanders make a big flashy display at dawn and everyone leaves the island.

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Pirate-treasure division problem

A pirate ship has a crew of fearsome, perfectly logical pirates, with a treasure of 100 gold coins to be divided.

Long agreed-upon pirate-treasure division procedure:

- Pirates are ordered by rank.
- Lowest-rank pirate mounts plank, proposes division plan.
- Pirates vote. If approved, done. Otherwise, Walk the Plank!
- Continue as above until done.

Question

You are pirate five, the lowest rank. What is your plan?



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Pirate value system

The pirates are all perfectly logical, and they subscribe to the following value system:

- Most importantly, stay alive.
- Secondly, get gold, the more the better.
- After this, cause the death of other pirates, if possible.
- Finally, all other things being equal, arrange that gold goes to senior pirates, when possible.

And all the pirates know all this, and that they all know it, etc.

What happens?

The pirates naturally consider each proposal in comparison with the alternative, the next plan if that one walks the plank.

So we can work backwards, to figure out what those plans will be.

One pirate

Suppose there is only one pirate, the captain.

Following the procedure, she mounts the plank, and clearly she should propose “Pirate one gets all the gold,” and she should vote in favor of this plan, and so pirate one gets all the gold, as anyone would have expected.

Two pirates

Suppose there are exactly two pirates. Pirate two mounts the plank. What will she propose?

She needs a majority of two, which means she must get the captain also to vote for her plan.

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Pirate three needs only two votes. One of them is her own.

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So pirate three proposes: Pirate three gets ALL the gold!
Approved with two votes.

Four pirates

Pirate four needs three votes.

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Pirates 1 and 2 can be bought off with a gold coin, since otherwise they will get none.

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Pirate four needs three votes.

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So the proposal is: one coin each for Pirates 1 and 2, and 98 for pirate four.

Approved with three votes.

Five pirates

Pirate five needs three votes, including her own.

She can buy the vote of pirate three with one coin, since otherwise three gets nothing.

It takes two coins for another vote, and this should go to the captain.

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Proposal: 2 0 1 0 97

Approved with three votes.

More than five

	1	2	3	4	5	6	7	8	9	10
One pirate	100									
Two pirates	*	X								
Three pirates	0	0	100							
Four pirates	1	1	0	98						
Five pirates	2	0	1	0	97					
Six pirates	0	1	2	1	0	96				
Seven pirates	1	2	0	0	1	0	96			
Eight pirates	2	0	1	1	0	1	0	95		
Nine pirates	0	1	2	0	1	0	1	0	95	
Ten pirates	1	2	0	1	0	1	0	1	0	94

Variations

The pirate puzzle has innumerable variations.

- Consider the case of very few coins. Just two or three.
- Or one. It's quite interesting, since you can't buy enough votes.
- Or none: the Lives/Dies popularity game.
- Change the rules on tie-breaks.
- Don't allow proposer to vote.
- Investigate asymptotic behavior.

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Thank you.

Find more epistemic logic puzzles on my blog:

jdh.hamkins.org.

My challenge problem: Cheryl's rational gifts, available at:

<http://jdh.hamkins.org/cheryls-rational-gifts>.

Follow me on Twitter, if you're inclined:

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