The Ghost in the Quantum Turing Machine

Scott Aaronson*

Abstract

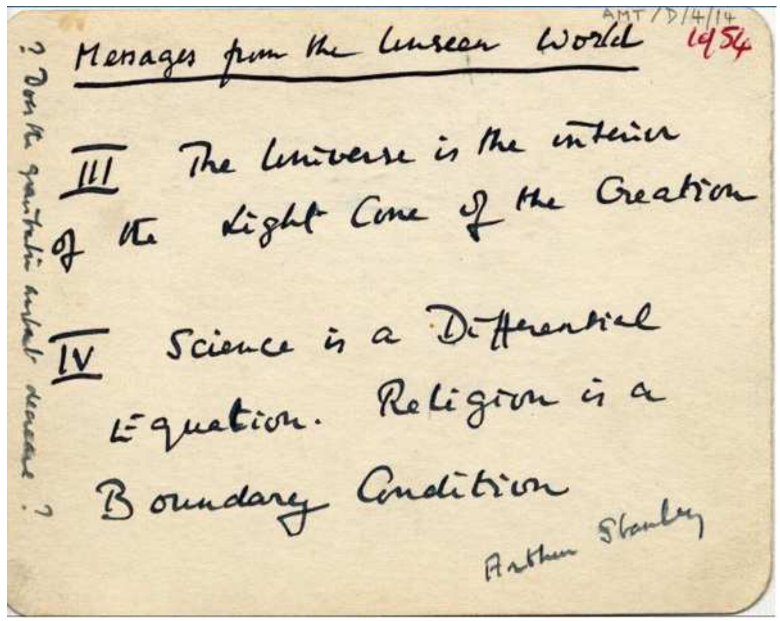
In honor of Alan Turing's hundredth birthday, I unwisely set out some thoughts about one of Turing's obsessions throughout his life, the question of physics and free will. I focus relatively narrowly on a notion that I call "Knightian freedom": a certain kind of in-principle physical unpredictability that goes beyond probabilistic unpredictability. Other, more metaphysical aspects of free will I regard as possibly outside the scope of science.

I examine a viewpoint, suggested independently by Carl Hoefer, Cristi Stoica, and even Turing himself, that tries to find scope for "freedom" in the universe's boundary conditions rather than in the dynamical laws. Taking this viewpoint seriously leads to many interesting conceptual problems. I investigate how far one can go toward solving those problems, and along the way, encounter (among other things) the No-Cloning Theorem, the measurement problem, decoherence, chaos, the arrow of time, the holographic principle, Newcomb's paradox, Boltzmann brains, algorithmic information theory, and the Common Prior Assumption. I also compare the viewpoint explored here to the more radical speculations of Roger Penrose.

The result of all this is an unusual perspective on time, quantum mechanics, and causation, of which I myself remain skeptical, but which has several appealing features. Among other things, it suggests interesting empirical questions in neuroscience, physics, and cosmology; and takes a millennia-old philosophical debate into some underexplored territory.

Contents

*MIT. Email: aaronson@csail.mit.edu. This material is based upon work supported by the National Science Foundation under Grant No. 0844626, as well as by an NSF STC grant, a TIBCO Chair, a Sloan Fellowship, and an Alan T. Waterman Award.



Postcard from Alan M. Turing to Robin Gandy, March 1954 (reprinted in Hodges [43])
It reads, in part:

> Messages from the Unseen World
> The Universe is the interior of the Light Cone of the Creation
> Science is a Differential Equation. Religion is a Boundary Condition

"Arthur Stanley" refers to Arthur Stanley Eddington, whose books were a major early influence on Turing.

1 Introduction

When I was a teenager, Alan Turing was at the top of my pantheon of scientific heroes, above even Darwin, Ramanujan, Einstein, and Feynman. Some of the reasons were obvious: the founding of computer science, the proof of the unsolvability of the Entscheidungsproblem, the breaking of the Nazi Enigma code, the unapologetic nerdiness and the near-martyrdom for human rights. But beyond the facts of his biography, I idolized Turing as an "über-reductionist": the scientist who had gone further than anyone before him to reveal the mechanistic nature of reality. Through his discovery of computational universality, as well as the Turing Test criterion for intelligence, Turing had finally unmasked the pretensions of anyone who claimed there was anything more to mind, brain, or the physical world than the unfolding of an immense computation. After Turing, it seemed to me, one could assert with confidence that all our hopes, fears, sensations, and choices were just evanescent patterns in some sort of *cellular automaton*: that is, a huge array of bits, different in detail but not in essence from Conway's famous Game of Life,[1] getting updated in time by simple, local, mechanistic rules.

So it's striking that Turing's own views about these issues, as revealed in his lectures as well as private correspondence, were much more complicated than my early caricature. As a teenager, Turing devoured the popular books of Sir Arthur Eddington, who was one of the first (though not, of course, the last!) to speculate about the implications of the then-ongoing quantum revolution in physics for ancient questions about mind and free will. Later, as a prize from his high school in 1932, Turing selected John von Neumann's just-published *Mathematische Grundlagen der Quantenmechanik* [65]: a treatise on quantum mechanics famous for its mathematical rigor, but also for its perspective that the collapse of the wavefunction ultimately involves the experimenter's mental state. As detailed by Turing biographer Andrew Hodges [43], these early readings had a major impact on Turing's intellectual preoccupations throughout his life, and probably even influenced his 1936 work on the theory of computing.

Turing also had a more personal reason for worrying about these "deep" questions. In 1930, Christopher Morcom—Turing's teenage best friend, scientific peer, and (probably) unrequited love—died from tuberculosis, sending a grief-stricken Turing into long ruminations about the nature of personal identity and consciousness. Let me quote from a remarkable disquisition, entitled "Nature of Spirit," that the 19-year-old Turing sent in 1932 to Christopher Morcom's mother.

> It used to be supposed in Science that if everything was known about the Universe at any particular moment then we can predict what it will be through all the future. This idea was really due to the great success of astronomical prediction. More modern science however has come to the conclusion that when we are dealing with atoms and electrons we are quite unable to know the exact state of them; our instruments being made of atoms and electrons themselves. The conception then of being able to know

[1] Invented by the mathematician John Conway in 1970, the Game of Life involves a large two-dimensional array of pixels, with each pixel either "live" or "dead." At each (discrete) time step, the pixels get updated via a deterministic rule: each live pixel "dies" if less than 2 or more than 3 of its 8 neighbors were alive, and each dead pixel "comes alive" if exactly 3 of its 8 neighbors were alive. 'Life' is famous for the complicated, unpredictable patterns that typically arise from a simple starting configuration and repeated application of the rules. Conway (see [53]) has expressed certainty that, on a large enough Life board, living beings would arise, who would then start squabbling over territory and writing learned PhD theses! Note that, with an *exponentially*-large Life board (and, say, a uniformly-random initial configuration), Conway's claim is vacuously true, in the sense that one could find essentially any regularity one wanted just by chance. But one assumes that Conway meant something stronger.

> the exact state of the universe then really must break down on the small scale. This means then that the theory which held that as eclipses etc. are predestined so were all our actions breaks down too. We have a will which is able to determine the action of the atoms probably in a small portion of the brain, or possibly all over it. The rest of the body acts so as to amplify this. (Quoted in Hodges [43])

The rest of Turing's letter discusses the prospects for the survival of the "spirit" after death, a topic with obvious relevance to Turing at that time. In later years, Turing would eschew that sort of mysticism. Yet even in a 1951 radio address *defending* the possibility of human-level artificial intelligence, Turing still brought up Eddington, and the possible limits on prediction of human brains imposed by the uncertainty principle:

> If it is accepted that real brains, as found in animals, and in particular in men, are a sort of machine it will follow that our digital computer suitably programmed, will behave like a brain. [But the argument for this conclusion] involves several assumptions which can quite reasonably be challenged. [It is] necessary that this machine should be of the sort whose behaviour is in principle predictable by calculation. We certainly do not know how any such calculation should be done, and it was even argued by Sir Arthur Eddington that on account of the indeterminacy principle in quantum mechanics no such prediction is even theoretically possible.[2] (Reprinted in Shieber [80])

Finally, two years after his sentencing for "homosexual indecency," and a few months before his tragic death by self-poisoning, Turing wrote the striking aphorisms that I quoted earlier: "The universe is the interior of the light-cone of the Creation. Science is a differential equation. Religion is a boundary condition."

The reason I'm writing this essay is that I *think* I now understand what Turing could have meant by these remarks. Building on ideas of Hoefer [45], Stoica [85], and others, I'll examine a perspective—which I call the "freebit perspective," for reasons to be explained later—that locates a nontrivial sort of freedom in the universe's boundary conditions, even while embracing the mechanical nature of the time-evolution laws. We'll find that a central question, for this perspective, is how well complicated biological systems like human brains can *actually* be predicted: not by hypothetical Laplace demons, but by prediction devices compatible with the laws of physics. It's in the discussion of this predictability question (and *only* there) that quantum mechanics enters the story.

Of course, the idea that quantum mechanics might have *something* to do with free will is not new; neither are the problems with that idea or the controversy surrounding it. While I chose Turing's postcard for the opening text of this essay, I also could have chosen a striking claim by Niels Bohr, from a 1932 lecture about the implications of Heisenberg's uncertainty principle:

> [W]e should doubtless kill an animal if we tried to carry the investigation of its organs so far that we could tell the part played by the single atoms in vital functions. In

[2] As Hodges (personal communication) points out, it's interesting to contrast these remarks with a view Turing had expressed just a year earlier, in "Computing Machinery and Intelligence" [89]: "It is true that a discrete-state machine must be different from a continuous machine. But if we adhere to the conditions of the imitation game, the interrogator will not be able to take any advantage of this difference." Note that there's no actual contradiction between this statement and the one about the uncertainty principle, especially if we distinguish (as I will) between simulating a *particular* brain and simulating *some* brain-like entity able to pass the Turing test. However, I'm not aware of any place where Turing explicitly makes that distinction.

> every experiment on living organisms there must remain some uncertainty as regards the physical conditions to which they are subjected, and the idea suggests itself that the minimal freedom we must allow the organism will be just large enough to permit it, so to say, to hide its ultimate secrets from us. (Reprinted in [17])

Or this, from the physicist Arthur Compton:

> A set of known physical conditions is not adequate to specify precisely what a forthcoming event will be. These conditions, insofar as they can be known, define instead a range of possible events from among which some particular event will occur. When one exercises freedom, by his act of choice he is himself adding a factor not supplied by the physical conditions and is thus himself determining what will occur. That he does so is known only to the person himself. From the outside one can see in his act only the working of physical law. [24]

I want to know:

> *Were Bohr and Compton right or weren't they? Does quantum mechanics (specifically, say, the No-Cloning Theorem or the uncertainty principle) put interesting limits on an external agent's ability to scan, copy, and predict human brains and other complicated biological systems, or doesn't it?*

Of course, one needs to spell out carefully what one means by "interesting limits," an "external agent," the "ability to scan, copy, and predict," and so forth.[3] But once that's done, I regard the above as an unsolved scientific question, and a big one. Many people seem to think the answer is obvious (though they disagree on what it is!), or else they reject the question as meaningless, unanswerable, or irrelevant. In this essay I'll argue strongly for a different perspective: that we can easily imagine worlds consistent with quantum mechanics (and all other known physics and biology) where the answer to the question is yes, and other such worlds where the answer is no. And we don't yet know which kind we live in. The most we can say is that, like P versus NP or the nature of quantum gravity, the question is well beyond our *current* ability to answer.

Furthermore, the two kinds of world lead, not merely to different philosophical stances, but to different visions of the remote future. Will our descendants all choose to upload themselves into a digital hive-mind, after a "technological singularity" that makes such things possible? Will they then multiply themselves into trillions of perfect computer-simulated replicas, living in various simulated worlds of their own invention, inside of which there might be further simulated worlds with still more replicated minds? What will it be *like* to exist in so many manifestations: will each copy have its own awareness, or will they comprise a single awareness that experiences trillions of times more than we do? Supposing all this to be possible, is there any reason why our descendants might want to hold back on it?

Now, if it turned out that Bohr and Compton were wrong—that human brains were as probabilistically predictable by external agents as ordinary digital computers equipped with random-number generators—then the freebit picture that I explore in this essay would be falsified, to whatever extent it says anything interesting. It should go without saying that I see the freebit picture's vulnerability to future empirical findings as a feature rather than a bug.

[3] My own attempt to do so is in Appendix 12.

In summary, I'll make no claim to show here that the freebit picture is *true*. I'll confine myself to two weaker claims:

(1) That the picture is *sensible* (or rather, not obviously much crazier than the alternatives): many considerations one might think would immediately make a hash of this picture, fail to do so for interesting reasons.

(2) That the picture is *falsifiable*: there are open empirical questions that need to turn out one way rather than another, for this picture to stand even a *chance* of working.

I ask others to take this essay as I do: as an exercise in what physicists call model-building. I want to see *how far I can get* in thinking about time, causation, predictability, and quantum mechanics in a certain unusual but apparently-consistent way. "Resolving" the millennia-old free will debate isn't even on the table! The most I can hope for, if I'm lucky, is to construct a model whose strengths and weaknesses help to move the debate slightly forward.

1.1 "Free Will" Versus "Freedom"

There's one terminological issue that experience has shown I need to dispense with before anything else. In this essay, I'll sharply distinguish between "free will" and another concept that I'll call "freedom," and will mostly concentrate on the latter.

By "free will," I'll mean a metaphysical attribute that I hold to be largely outside the scope of science—and which I can't even *define* clearly, except to say that, if there's an otherwise-undefinable thing that people have tried to get at for centuries with the phrase "free will," then free will is that thing! More seriously, as many philosophers have pointed out, "free will" seems to combine two distinct ideas: first, that your choices are "free" from any kind of external constraint; and second, that your choices are not arbitrary or capricious, but are "willed by you." The second idea—that of being "willed by you"—is the one I consider outside the scope of science, for the simple reason that no matter what the empirical facts were, a skeptic could always deny that a given decision was "really" yours, and hold the true decider to have been God, the universe, an impersonating demon, etc. I see no way to formulate, in terms of observable concepts, what it would even mean for such a skeptic to be right or wrong.

But crucially, the situation seems different if we set aside the "will" part of free will, and consider only the "free" part. Throughout, I'll use the term *freedom*, or *Knightian freedom*, to mean a certain strong kind of physical unpredictability: a lack of determination, even probabilistic determination, by knowable external factors. That is, a physical system will be "free" if and only if it's unpredictable in a sufficiently strong sense, and "freedom" will simply be that property possessed by free systems. A system that's not "free" will be considered "mechanistic."

Many issues arise when we try to make the above notions more precise. For one thing, we need a definition of unpredictability that does *not* encompass the "merely probabilistic" unpredictability of (say) a photon or a radioactive atom—since, as I'll discuss in Section 3, I accept the often-made point that *that* kind of unpredictability has nothing to do with what most people would call "freedom," and is fully compatible with a system's being "mechanistic." Instead, we'll want what economists call "Knightian" unpredictability, meaning unpredictability that we lack a reliable way even to quantify using probability distributions. Ideally, our criteria for Knightian unpredictability will be so stringent that they won't encompass systems like the Earth's weather—for which, despite the presence of chaos, we arguably *can* give very well-calibrated probabilistic forecasts.

A second issue is that unpredictability seems observer-relative: a system that's unpredictable to one observer might be perfectly predictable to another. This is the reason why, throughout this essay, I'll be interested less in particular methods of prediction than in the *best* predictions that could ever be made, consistent both with the laws of physics and with the need not to destroy the system being studied.

This brings us to a third issue: it's not obvious what should count as "destroying" the system, or which interventions a would-be predictor should or shouldn't be allowed to perform. For example, in order to ease the prediction of a human brain, should a predictor first be allowed to replace each neuron by a "functionally-equivalent" microchip? How would we decide whether the microchip *was* functionally equivalent to the neuron?

I'll offer detailed thoughts about these issues in Appendix 12. For now, though, the point I want to make is that, once we *do* address these issues, it seems to me that "freedom"—in the sense of Knightian unpredictability by any external physical observer—is perfectly within the scope of science. We're no longer talking about ethics, metaphysics, or the use of language: only about whether such-and-such a system is or isn't physically predictable in the relevant way! A similar point was recently made forcefully by the philosopher Mark Balaguer [9], in his interesting book *Free Will as an Open Scientific Problem.* (However, while I strongly agree with Balaguer's basic thesis, as discussed above I reject any connection between freedom and "merely probabilistic" unpredictability, whereas Balaguer seems to accept such a connection.)

Surprisingly, my experience has been that many scientifically-minded people will happily accept that humans plausibly *are* physically unpredictable in the relevant sense. Or at least, they'll accept my own position, that whether humans *are or aren't* so predictable is an empirical question whose answer is neither obvious nor known. Again and again, I've found, people will concede that chaos, the No-Cloning Theorem, or some other phenomenon might make human brains physically unpredictable—indeed, they'll seem oddly indifferent to the question of whether they do or don't! But they'll never fail to add: "even if so, who cares? we're just talking about unpredictability! that obviously has nothing to do with *free will*!"

For my part, I grant that free will can't be *identified* with unpredictability, without doing violence to the usual meanings of those concepts. Indeed, it's precisely because I grant this that I write, throughout the essay, about "freedom" (or "Knightian freedom") rather than "free will." I insist, however, that unpredictability has *something* to do with free will—in roughly the same sense that verbal intelligence has *something* to do with consciousness, or optical physics has *something* to do with subjectively-perceived colors. That is, some people might see unpredictability as a pale empirical shadow of the "true" metaphysical quality, free will, that we really want to understand. But the great lesson of the scientific revolution, going back to Galileo, is that understanding the "empirical shadow" of something is *vastly* better than not understanding the thing at all! Furthermore, the former might already be an immense undertaking, as understanding human intelligence and the physical universe turned out to be (even setting aside the "mysteries" of consciousness and metaphysics). Indeed I submit that, for the past four centuries, "start with the shadow" has been a spectacularly fruitful approach to unravelling the mysteries of the universe: one that's succeeded where greedy attempts to go behind the shadow have failed. If one likes, the goal of this essay is to explore what happens when one applies a "start with the shadow" approach to the free-will debate.

Personally, I'd go even further than claiming a vague connection between unpredictability and free will. Just as displaying intelligent behavior (by passing the Turing Test or some other means) might be thought a *necessary condition* for consciousness if not a sufficient one, so I tend to see Knightian unpredictability as a necessary condition for free will. In other words, if a system were completely predictable (even probabilistically) by an outside entity—not merely in principle but in practice—then I find it hard to understand why we'd still want to ascribe "free will" to the system. Why not admit that we now fully understand what makes this system tick?

However, I'm aware that many people sharply reject the idea that unpredictability is a necessary condition for free will. Even if a computer in another room perfectly predicted all of their actions, days in advance, these people would still call their actions "free," so long as "they themselves chose" the actions that the computer also predicted for them. In Section 2.5, I'll explore some of the difficulties that this position leads to when carried to science-fiction conclusions. For now, though, it's not important to dispute the point. I'll happily settle for the weaker claim that unpredictability has *something* to do with free will, just as intelligence has something to do with consciousness. More precisely: in both cases, even when people *think* they're asking purely philosophical questions about the latter concept, much of what they want to know often turns out to hinge on "grubby empirical questions" about the former concept![4] So if the philosophical questions seem too ethereally inaccessible, then we might as well focus for a while on the scientific ones.

1.2 Note on the Title

The term "ghost in the machine" was introduced in 1949 by Gilbert Ryle [73]. His purpose was to ridicule the notion of a "mind-substance": a mysterious entity that exists outside of space and ordinary physical causation; has no size, weight, or other material properties; is knowable by its possessor with absolute certainty (while the minds of others are *not* so knowable); and somehow receives signals from the brain and influences the brain's activity, even though it's nowhere to be found *in* the brain. Meanwhile, a *quantum Turing machine*, defined by Deutsch [30] (see also Bernstein and Vazirani [13]), is a Turing machine able to exploit the principle of quantum superposition. As far as anyone knows today [2], our universe seems to be efficiently simulable by—or even "isomorphic to"—a quantum Turing machine, which would take as input the universe's quantum initial state (say, at the Big Bang), then run the evolution equations forward.

1.3 Level

Most of this essay should be accessible to any educated reader. In a few sections, though, I assume familiarity with basic concepts from quantum mechanics, or (less often) relativity, thermodynamics, Bayesian probability, or theoretical computer science. When I *do* review concepts from those fields, I usually focus only on the ones most relevant to whatever point I'm making. To do otherwise would make the essay even more absurdly long than it already is! Readers seeking an accessible introduction to some of the established theories invoked in this essay might enjoy my recent book *Quantum Computing Since Democritus* [4].[5]

[4] A perfect example of this phenomenon is provided by the countless people who claim that even if a computer program passed the Turing Test, it still wouldn't be conscious—and then, without batting an eye, defend that claim using arguments that presuppose that the program *couldn't* pass the Turing Test after all! ("Sure, the program might solve math problems, but it could never write love poetry," etc. etc.) The temptation to hitch metaphysical claims to empirical ones, without even realizing the chasm one is crossing, seems incredibly strong.

[5] For the general reader, other good background reading for this essay might include *From Eternity to Here* by Sean Carroll [20], *The Beginning of Infinity* by David Deutsch [31], *The Emperor's New Mind* by Roger Penrose

In the main text, I've tried to keep the discussion extremely informal. I've found that, with a contentious subject like free will, mathematical rigor (or the pretense of it) can easily obfuscate more than it clarifies. However, for interested readers, I did put some more technical material into appendices: a suggested formalization of "Knightian freedom" in Appendix 12; some observations about prediction, Kolmogorov complexity, and the universal prior in Appendix 13; and a suggested formalization of the notion of "freebits" in Appendix 3.1.

2 FAQ

In discussing a millennia-old conundrum like free will, a central difficulty is that *almost everyone already knows what he or she thinks*—even if the certainties that one person brings to the discussion are completely at odds with someone else's. One practical consequence is that, no matter how I organize this essay, I'm bound to make a large fraction of readers impatient; some will accuse me of dodging the real issues by dwelling on irrelevancies. So without further ado, I'll now offer a "Frequently Asked Questions" list. In the thirteen questions below, I'll engage determinists, compatibilists, and others who might have strong *a priori* reasons to be leery of my whole project. I'll try to clarify my areas of agreement and disagreement, and hopefully convince the skeptics to read further. Then, after developing my own ideas in Sections 3 and 4, I'll come back and address still further objections in Section 5.

2.1 Narrow Scientism

For thousands of years, the free-will debate has encompassed moral, legal, phenomenological, and even theological questions. You seem to want to sweep all of that away, and focus exclusively on what would some would consider a narrow scientific issue having to do with physical predictability. Isn't that presumptuous?

On the contrary, it seems presumptuous *not* to limit my scope! Since it's far beyond my aims and abilities to address all aspects of the free-will debate, as discussed in Section 1.1 I decided to focus on one issue: the physical and technological questions surrounding how well human and animal brains can ever be predicted, in principle, by external entities that also want to keep those brains alive. I focus on this for several reasons: because it seems underexplored; because I might have something to say about it; and because even if what I say is wrong, the predictability issue has the appealing property that *progress* on it seems possible. Indeed, even if one granted—which I don't—that the predictability issue had nothing to do with the "true" mystery of free will, I'd still care about the former at least as much as I cared about the latter!

However, in the interest of laying all my cards on the table, let me offer some brief remarks on the moral, legal, phenomenological, and theological aspects of free will.

On the moral and legal aspects, my own view is summed up beautifully by the Ambrose Bierce poem:

> There's no free will, says the philosopher
> To hang is most unjust.
> There's no free will, assent the officers

[67], or *Free Will as an Open Scientific Problem* by Mark Balaguer [9]. Obviously, none of these authors necessarily endorse everything I say (or vice versa)! What the books have in common is simply that they explain one or more concepts invoked in this essay in much more detail than I do.

We hang because we must. [14]

For the foreseeable future, I can't see that the legal or practical implications of the free-will debate are nearly as great as many commentators have made them out to be, for the simple reason that (as Bierce points out) any implications would apply "symmetrically," to accused and accuser alike.

But I would go further: I've found many discussions about free will and legal responsibility to be downright *patronizing*. The subtext of such discussions usually seems to be:

> We, the educated, upper-class people having this conversation, *should* accept that the entire concept of "should" is quaint and misguided, when it comes to the uneducated, lower-class sorts of people who commit crimes. Those poor dears' upbringing, brain chemistry, and so forth absolve them of any real responsibility for their crimes: the notion that they had the "free will" to choose otherwise is just naïve. My friends and I are *right* because we accept that enlightened stance, while other educated people are *wrong* because they fail to accept it. For us educated people, of course, the relevance of the categories "right" and "wrong" requires no justification or argument.

Or conversely:

> Whatever the truth, we educated people *should* maintain that all people are responsible for their choices—since otherwise, we'd have no basis to punish the criminals and degenerates in our midst, and civilization would collapse. For *us*, of course, the meaningfulness of the word "should" in the previous sentence is not merely a useful fiction, but is clear as day.

On the phenomenological aspects of free will: if someone claimed to know, from introspection, either that free will exists or that it doesn't exist, then of course I could never refute that person to his or her satisfaction. But precisely because one can't decide between conflicting introspective reports, in this essay I'll be exclusively interested in what can be learned from scientific observation and argument. Appeals to inner experience—including my own and the reader's—will be out of bounds. Likewise, while it might be impossible to avoid grazing the "mystery of consciousness" in a discussion of human predictability, I'll do my best to avoid addressing that mystery head-on.

On the theological aspects of free will: probably the most relevant thing to say is that, even if there existed an omniscient God who knew all of our future choices, that fact wouldn't concern us in this essay, *unless* God's knowledge could somehow be made manifest in the physical world, and used to *predict* our choices. In that case, however, we'd no longer be talking about "theological" aspects of free will, but simply again about scientific aspects.

2.2 Bait-and-Switch

Despite everything you said in Section 1.1, I'm still not convinced that we can learn anything about free will from an analysis of unpredictability. Isn't that a shameless "bait-and-switch"?

Yes, but it's a shameless bait-and-switch with a distinguished history! I claim that, *whenever* it's been possible to make definite progress on ancient philosophical problems, such progress has almost always involved a similar "bait-and-switch." In other words: one replaces an unanswerable philosophical riddle Q by a "merely" scientific or mathematical question Q', which captures *part* of what people have wanted to know when they've asked Q. Then, with luck, one solves Q'.

Of course, even if Q' is solved, centuries later philosophers might still be debating the exact relation between Q and Q'! And further exploration might lead to *other* scientific or mathematical questions—Q'', Q''', and so on—which capture aspects of Q that Q' left untouched. But from my perspective, this process of "breaking off" answerable parts of unanswerable riddles, then trying to answer those parts, is the closest thing to philosophical progress that there is.

Successful examples of this breaking-off process fill intellectual history. The use of calculus to treat infinite series, the link between mental activity and nerve impulses, natural selection, set theory and first-order logic, special relativity, Gödel's theorem, game theory, information theory, computability and complexity theory, the Bell inequality, the theory of common knowledge, Bayesian causal networks—each of these advances addressed questions that could rightly have been called "philosophical" before the advance was made. And after each advance, there was *still* plenty for philosophers to debate about truth and provability and infinity, space and time and causality, probability and information and life and mind. But crucially, it seems to me that the technical advances transformed the philosophical discussion as philosophical discussion *itself* rarely transforms it! And therefore, if such advances don't count as "philosophical progress," then it's not clear that anything should.

Appropriately for this essay, perhaps the *best* precedent for my bait-and-switch is the Turing Test. Turing began his famous 1950 paper "Computing Machinery and Intelligence" [89] with the words:

> I propose to consider the question, "Can machines think?"

But after a few pages of ground-clearing, he wrote:

> The original question, "Can machines think?" I believe to be too meaningless to deserve discussion.

So with legendary abruptness, Turing simply *replaced* the original question by a different one: "Are there imaginable digital computers which would do well in the imitation game"—i.e., which would successfully fool human interrogators in a teletype conversation into *thinking* they were human? Though some writers would later accuse Turing of conflating intelligence with the "mere simulation" of it, Turing was perfectly clear about what he was doing:

> I shall replace the question by another, which is closely related to it and is expressed in relatively unambiguous words ... We cannot altogether abandon the original form of the problem, for opinions will differ as to the appropriateness of the substitution and we must at least listen to what has to be said in this connexion [sic].

The claim is not that the new question, about the imitation game, is *identical* to the original question about machine intelligence. The claim, rather, is that the new question is a worthy candidate for what we *should* have asked or *meant* to have asked, if our goal was to learn something new rather than endlessly debating definitions. In math and science, the process of revising one's original question is often the core of a research project, with the actual answering of the revised question being the relatively easy part!

A good replacement question Q' should satisfy two properties:

(a) Q' should capture some *aspect* of the original question Q—so that an answer to Q' would be *hard to ignore* in any subsequent discussion of Q.

(b) Q' should be precise enough that one can see what it would mean to make *progress* on Q': what experiments one would need to do, what theorems one would need to prove, etc.

The Turing Test, I think, captured people's imaginations precisely because it succeeded so well at (a) and (b). Let me put it this way: if a digital computer were built that aced the imitation game, then *it's hard to see what more science could possibly say* in support of machine intelligence being possible. Conversely, if digital computers were proved unable to win the imitation game, then it's hard to see what more science could say in support of machine intelligence *not* being possible. Either way, though, we're no longer "slashing air," trying to pin down the true meanings of words like "machine" and "think": we've hit the relatively-solid ground of a science and engineering problem. Now if we want to go further we need to *dig* (that is, do research in cognitive science, machine learning, etc). This digging might take centuries of backbreaking work; we have no idea if we'll ever reach the bottom. But at least it's something humans know how to do and have done before. Just as important, diggers (unlike air-slashers) tend to uncover countless treasures besides the ones they were looking for.

By analogy, in this essay I advocate replacing the question of whether humans have free will, by the question of how accurately their choices can be predicted, in principle, by external agents compatible with the laws of physics. And while I don't pretend that the "replacement" question is identical to the original, I do claim the following: if humans turned out to be arbitrarily predictable in the relevant sense, then *it's hard to see what more science could possibly say in support of "free will being a chimera."* Conversely, if a fundamental reason were discovered why the appropriate "prediction game" *couldn't* be won, then it's hard to see what more science could say in support of "free will being real."

Either way, I'll try to sketch the research program that confronts us if we take the question seriously: a program that spans neuroscience, chemistry, physics, and even cosmology. Not surprisingly, much of this program consists of problems that scientists in the relevant fields are already working on, or longstanding puzzles of which they're well aware. But there are also questions—for example, about the "past macroscopic determinants" of the quantum states occurring in nature—which as far as I know *haven't* been asked in the form they take here.

2.3 Compatibilism

Like many scientifically-minded people, I'm a *compatibilist*: someone who believes free will can exist even in a mechanistic universe. For me, "free will is as real as baseball," as the physicist Sean Carroll memorably put it.[6] That is, the human capacity to weigh options and make a decision "exists" in the same sense as Sweden, caramel corn, anger, or other complicated notions that might interest us, but that no one expects to play a role in the fundamental laws of the universe. As for the fundamental laws, I believe them to be completely mechanistic and impersonal: as far as they know or care, a human brain is just one more evanescent pattern of computation, along with sunspots and hurricanes. Do you dispute any of that? What, if anything, can a compatibilist take from your essay?

[6] See blogs.discovermagazine.com/cosmicvariance/2011/07/13/free-will-is-as-real-as-baseball/

I have a lot of sympathy for compatibilism—certainly more than for an incurious mysticism that doesn't even try to reconcile itself with a scientific worldview. So I hope compatibilists will find much of what I have to say "compatible" with their own views!

Let me first clear up a terminological confusion. Compatibilism is often defined as the belief that free will is compatible with *determinism*. But as far as I can see, the question of determinism versus indeterminism has almost nothing to do with what compatibilists actually believe. After all, most compatibilists happily accept quantum mechanics, with its strong indeterminist implications (see Question 2.8), but regard it as having almost no bearing on their position. No doubt some compatibilists find it important to stress that *even if classical physics had been right*, there still would have been no difficulty for free will. But it seems to me that one can be a "compatibilist" even while denying that point, or remaining agnostic about it. In this essay, I'll simply define "compatibilism" to be the belief that free will is compatible with a *broadly mechanistic worldview*—that is, with a universe governed by impersonal mathematical laws of *some* kind. Whether it's important that those laws be probabilistic (or chaotic, or computationally universal, or whatever else), I'll regard as internal disputes within compatibilism.

I can now come to the question: is my perspective compatible with compatibilism? Alas, at the risk of sounding lawyerly, I can't answer without a further distinction! Let's define *strong compatibilism* to mean the belief that the statement "Alice has free will" is compatible with the actual, physical existence of a machine that predicts all of Alice's future choices—a machine whose predictions Alice herself can read and verify after the fact. (Where by "predict," we mean "in roughly the same sense that quantum mechanics predicts the behavior of a radioactive atom": that is, by giving arbitrarily-accurate probabilities, in cases where deterministic prediction is physically impossible.) By contrast, let's define *weak compatibilism* to mean the belief that "Alice has free will" is compatible with Alice living in a mechanistic, law-governed universe—but *not* necessarily with her living in a universe where the prediction machine can be built.

Then my perspective is compatible with weak compatibilism, but incompatible with strong compatibilism. My perspective *embraces* the mechanical nature of the universe's time-evolution laws, and in that sense is proudly "compatibilist." On the other hand, I care whether our choices can *actually* be mechanically predicted—not by hypothetical Laplace demons but by physical machines. I'm troubled if they are, and I take seriously the possibility that they aren't (e.g., because of chaotic amplification of unknowable details of the initial conditions).

2.4 Quantum Flapdoodle

The usual motivation for mentioning quantum mechanics and mind in the same breath has been satirized as "quantum mechanics is mysterious, the mind is also mysterious, ergo they must be related somehow"! Aren't you worried that, merely by writing an essay that *seems* to take such a connection seriously, you'll fan the flames of pseudoscience? That any subtleties and caveats in your position will quickly get lost?

Yes! Even though I can only take responsibility for what I write, not for what various Internet commentators, etc. might mistakenly *think* I wrote, it would be distressing to see this essay twisted to support credulous doctrines that I abhor. So for the record, let me state the following:

(a) I don't think quantum mechanics, or anything else, lets us "bend the universe to our will," except through interacting with our external environments in the ordinary causal ways. Nor do I think that quantum mechanics says "everything is holistically connected to everything else" (whatever that means). Proponents of these ideas usually invoke the phenomenon of *quantum entanglement* between particles, which can persist no matter how far apart the particles are. But contrary to a widespread misunderstanding encouraged by generations of "quantum mystics," it's an elementary fact that entanglement does *not* allow instantaneous communication. More precisely, quantum mechanics is "local" in the following sense: if Alice and Bob share a pair of entangled particles, that nothing that Alice does to her particle only can affect the probability of any outcome of any measurement that Bob performs on his particle only.[7] Because of the famous Bell inequality, it's crucial that we don't interpret the concept of "locality" to mean *more* than that! But quantum mechanics' revision to our concept of locality is so subtle that neither scientists, mystics, nor anyone else anticipated it beforehand.

(b) I don't think quantum mechanics has vindicated Eastern religious teachings, any more than (say) Big Bang cosmology has vindicated the Genesis account of creation. In both cases, while there are interesting parallels, I find it dishonest to seek out only the points of convergence while ignoring the many inconvenient parts that don't fit! Personally, I'd say that the quantum picture of the world—as a complex unit vector $|\psi\rangle$ evolving linearly in a Hilbert space—is not a close match to *any* pre-20^{th}-century conception of reality.

(c) I don't think quantum mechanics has overthrown Enlightenment ideals of science and rationality. Quantum mechanics does overthrow the "naïve realist" vision of particles with unambiguous trajectories through space, and it does raise profound conceptual problems that will concern us later on. On the other hand, the point is still to describe a physical world external to our minds by positing a "state" for that world, giving precise mathematical rules for the evolution of the state, and testing the results against observation. Compared to classical physics, the reliance on mathematics has only increased; while the Enlightenment ideal of describing Nature as we find it to be, rather than as intuition says it "must" be, is emphatically upheld.

(d) I don't think the human brain is a quantum computer in any interesting sense. As I explain in [4], at least three considerations lead me to this opinion. First, it would be nearly miraculous if complicated entangled states—which today, can generally survive for at most a few seconds in near-absolute-zero laboratory conditions—could last for any appreciable time in the hot, wet environment of the brain. (Many researchers have made some version of that argument, but see Tegmark [88] for perhaps the most detailed version.) Second, the sorts of tasks quantum computers are known to be good at (for example, factoring large integers and simulating quantum systems) seem like a terrible fit to the sorts of tasks that *humans* seem be good at, or that could have plausibly had survival value on the African savannah! Third, and most importantly, I don't see anything that the brain being a quantum computer would plausibly help to *explain*. For example, why would a conscious quantum computer be any less mysterious than a conscious *classical* computer? My conclusion is that, *if* quantum effects play any role in the brain, then such effects are almost certainly short-lived and microscopic.[8] At the "macro" level of most interest to neuroscience, the evidence is overwhelming that

[7] Assuming, of course, that we don't *condition* on Alice's knowledge—something that could change Bob's probabilities even in the case of mere classical correlation between the particles.

[8] This is not to say, of course, that the brain's activity might not *amplify* such effects to the macroscopic, classical scale—a possibility that will certainly concern us later on.

the brain's computation and information storage are classical. (See Section 6 for further discussion of these issues in the context of Roger Penrose's views.)

(e) I don't think consciousness is in any sense necessary to bring about the "reduction of the wavefunction" in quantum measurement. And I say that, despite freely confessing to unease with all existing accounts of quantum measurement! My position is that, to whatever extent the reduction of the wavefunction is a real process at all (as opposed to an artifact of observers' limited perspectives, as in the Many-Worlds Interpretation), it must be a process that can occur even in interstellar space, with no conscious observers anywhere around. For otherwise, we're forced to the absurd conclusion that the universe's quantum state evolved linearly via the Schrödinger equation for billions of years, *until* the first observers arose (who: humans? monkeys? aliens?) and looked around them—at which instant the state suddenly and violently collapsed!

If one likes, whatever I *do* say about quantum mechanics and mind in this essay will be said in the teeth of the above points. In other words, I'll regard points (a)-(e) as sufficiently well-established to serve as useful *constraints*, which a new proposal ought to satisfy as a prerequisite to being taken seriously.

2.5 Brain-Uploading: Who Cares?

Suppose it were possible to "upload" a human brain to a computer, and thereafter predict the brain unlimited accuracy. Who cares? Why should anyone even worry that that would create a problem for free will or personal identity?

For me, the problem comes from the observation that it seems impossible to give any operational difference between a perfect predictor of your actions, and a second copy or instantiation of yourself. If there are two entities, both of which respond to every situation exactly as "you" would, then by what right can we declare that only one such entity is the "real" you, and the other is just a predictor, simulation, or model? But having multiple copies of you in the same universe seems to open a Pandora's box of science-fiction paradoxes. Furthermore, these paradoxes aren't "merely metaphysical": they concern *how you should do science* knowing there might be clones of yourself, and which predictions and decisions you should make.

Since this point is important, let me give some examples. Planning a dangerous mountain-climbing trip? Before you go, make a backup of yourself—or two or three—so that if tragedy should strike, you can restore from backup and then continue life as if you'd never left. Want to visit Mars? Don't bother with the perilous months-long journey through space; just use a brain-scanning machine to "fax yourself" there as pure information, whereupon another machine on Mars will construct a new body for you, functionally identical to the original.

Admittedly, some awkward questions arise. For example, after you've been faxed to Mars, what should be done with the "original" copy of you left on Earth? Should it be destroyed with a quick, painless gunshot to the head? Would *you* agree to be "faxed" to Mars, knowing that that's what would be done to the original? Alternatively, if the original were left alive, then what makes you sure you would "wake up" as the copy on Mars? At best, wouldn't you have 50/50 odds of still finding yourself on Earth? Could that problem be "solved" by putting a thousand copies of you on Mars, while leaving only one copy on Earth? Likewise, suppose you return unharmed from your mountain-climbing trip, and decide that the backup copies you made before you left are now an expensive nuisance. If you destroy them, are you guilty of murder? Or is it more like suicide? Or neither?

Here's a "purer" example of such a puzzle, which I've adapted from the philosopher Nick Bostrom [18]. Suppose an experimenter flips a fair coin while you lie anesthetized in a white, windowless hospital room. If the coin lands heads, then she'll create a thousand copies of you, place them in a thousand identical rooms, and wake each one up. If the coin lands tails, then she'll wake you up without creating any copies. You wake up in a white, windowless room just like the one you remember. Knowing the setup of the experiment, at what odds should you be willing to bet that the coin landed heads? Should your odds just be 50/50, since the coin was fair? Or should they be biased 1000:1 in favor of the coin having landed heads—since if it *did* land heads, then there are a thousand of you confronting the same situation, compared to only one if the coin landed tails?

Many people immediately respond that the odds should be 50/50: they consider it a metaphysical absurdity to adjust the odds based on the number of copies of yourself in existence. (Are we to imagine a "warehouse full of souls," with the odds of any particular soul being taken out of the warehouse proportional to the number of suitable bodies for it?) However, those who consider 50/50 the obvious answer should consider a slight variant of the puzzle. Suppose that, if the coin lands tails, then as before the experimenter leaves a single copy of you in a white room. If the coin lands heads, then the experimenter creates a thousand copies of you and places them in a thousand windowless rooms. Now, though, 999 of the rooms are painted blue; only one of the rooms is white like you remember.

You wake up from the anesthesia and find yourself in a white room. *Now* what posterior probability should you assign to the coin having landed heads? *If* you answered 50/50 to the first puzzle, then a simple application of Bayes' rule implies that, in the *second* puzzle, you should consider it overwhelmingly likely that the coin landed tails. For if the coin landed heads, then presumably you had a 99.9% probability of being one of the 999 copies who woke up in a blue room. So the fact that you woke up in a white room furnishes powerful evidence about the coin. Not surprisingly, many people find *this* result just as metaphysically unacceptable as the 1000:1 answer to the first puzzle! Yet as Bostrom points out, it seems mathematically inconsistent to insist on 50/50 as the answer to *both* puzzles.

Probably the most famous "paradox of brain-copying" was invented by William Newcomb, then popularized by Robert Nozick [66] and Martin Gardner [39]. In *Newcomb's paradox*, a superintelligent "Predictor" presents you with two closed boxes, and offers you a choice between opening the first box only or opening both boxes. Either way, you get to keep whatever you find in the box or boxes that you open. The contents of the first box can vary—sometimes it contains $1,000,000, sometimes nothing—but the second box always contains $1,000.

Just from what was already said, it seems that it must be preferable to open both boxes. For whatever you would get by opening the first box only, you can get $1,000 more by opening the second box as well. But here's the catch: using a detailed brain model, the Predictor has already foreseen your choice. *If* it predicted that you would open both boxes, then the Predictor left the first box empty; while if it predicted that you would open the first box only, then the Predictor put $1,000,000 in the first box. Furthermore, the Predictor has played this game hundreds of times before, both with you and with other people, and its predictions have been right every time. Everyone who opened the first box ended up with $1,000,000, while everyone who opened both boxes ended up with only $1,000. Knowing all of this, what do you do?

Some people dismiss the problem as contradictory—arguing that, if the assumed Predictor exists, then you have no free will, so there's no use fretting over how many boxes to open since your choice is already predetermined anyway. Among those willing to play along, opinion has been split for decades between "one-boxers" and "two-boxers." Lately, though, the one-boxers seem to have been gaining the upper hand—and reasonably so in my opinion, since by the assumptions of the thought experiment, the one-boxers *do* always walk away richer!

As I see it, the real problem is to *explain* how one-boxing could possibly be rational, given that, at the time you're contemplating your decision, the million dollars are either in the first box or not. Can a last-minute decision to open both boxes somehow "reach backwards in time," causing the million dollars that "would have been" in the first box to disappear? Do we need to distinguish between your "actual" choices and your "dispositions," and say that, while one-boxing is admittedly irrational, *making yourself into the sort of person* who one-boxes is rational?

While I consider myself a one-boxer, the only justification for one-boxing that makes sense to me goes as follows.[9] In principle, you could base your decision of whether to one-box or two-box on anything you like: for example, on whether the name of some obscure childhood friend had an even or odd number of letters. However, this suggests that the problem of predicting whether you will one-box or two-box is "you-complete."[10] In other words, if the Predictor can solve this problem reliably, then it seems to me that it must possess a simulation of you so detailed as to constitute another *copy* of you (as discussed previously).

But in that case, to whatever extent we want to think about Newcomb's paradox in terms of a freely-willed decision at all, we need to imagine *two* entities separated in space and time—the "flesh-and-blood you," and the simulated version being run by the Predictor—that are nevertheless "tethered together" and share common interests. *If* we think this way, then we can easily explain why one-boxing can be rational, even without backwards-in-time causation. Namely, as you contemplate whether to open one box or two, who's to say that you're not "actually" the simulation? If you are, then of course your decision can affect what the Predictor does in an ordinary, causal way.

For me, the takeaway is this. *If* any of these technologies—brain-uploading, teleportation, the Newcomb predictor, etc.—were actually realized, then all sorts of "woolly metaphysical questions" about personal identity and free will would start to have *practical consequences*. Should you fax yourself to Mars or not? Sitting in the hospital room, should you bet that the coin landed heads or tails? Should you expect to "wake up" as one of your backup copies, or as a simulation being run by the Newcomb Predictor? These questions all seem "empirical," yet one can't answer them without taking an implicit stance on questions that many people would prefer to regard as outside the scope of science.

Thus, the idea that we can "escape all that philosophical crazy-talk" by declaring that the human mind is a computer program running on the hardware of the brain, and that's all there is to it, strikes me as ironically backwards. Yes, we can say that, and we might even be right. But far from bypassing all philosophical perplexities, such a move lands in a *swamp* of them! For now we need to give some account of how a rational agent ought to make decisions and scientific predictions, in situations where it knows it's only one of several exact copies of itself inhabiting the

[9] I came up with this justification around 2002, and set it out in a blog post in 2006: see www.scottaaronson.com/blog/?p=30. Later, I learned that Radford Neal [64] had independently proposed similar ideas.

[10] In theoretical computer science, a problem belonging to a class C is called *C-complete*, if solving the problem would suffice to solve any other problem in C.

same universe.

Many will try to escape the problem, by saying that such an agent, being (by assumption) "just a computer program," simply does whatever its code determines it does given the relevant initial conditions. For example, if a piece of code says to bet heads in a certain game, then all agents running that code will bet heads; if the code says to bet tails, then the agents will bet tails. Either way, an *outside* observer who knew the code could easily calculate the probability that the agents will win or lose their bet. So what's the philosophical problem?

For me, the problem with this response is simply that it gives up on science as *something agents can use to predict their future experiences.* The agents wanted science to tell them, "given such-and-such physical conditions, here's what you *should* expect to see, and why." Instead they're getting the worthless tautology, "if your internal code causes you to expect to see X, then you expect to see X, while if your internal code causes you to expect to see Y, then you expect to see Y." But the same could be said about *anything*, with no scientific understanding needed! To paraphrase Democritus,[11] it seems like the ultimate victory of the mechanistic worldview is also its defeat.

As far as I can see, the only hope for avoiding these difficulties is if—because of chaos, the limits of quantum measurement, or whatever other obstruction—minds *can't* be copied perfectly from one physical substrate to another, as can programs on standard digital computers. So that's a possibility that this essay explores at some length. To clarify, we can't use any philosophical difficulties that would arise if minds were copyable, as evidence for the empirical claim that they're *not* copyable. The universe has never shown any particular tendency to cater to human philosophical prejudices! But I'd say the difficulties provide more than enough reason to *care* about the copyability question.

2.6 Determinism versus Predictability

I'm a determinist: I believe, not only that humans lack free will, but that everything that happens is completely determined by prior causes. So why should an analysis of "mere unpredictability" change my thinking at all? After all, I readily admit that, despite being metaphysically determined, many future events are unpredictable in practice. But for me, the fact that we can't predict something is *our* problem, not Nature's!

There's an observation that doesn't get made often enough in free-will debates, but that seems extremely pertinent here. Namely: *if you stretch the notion of "determination" far enough, then events become "determined" so trivially that the notion itself becomes vacuous.*

For example, a religious person might maintain that all events are predetermined by God's Encyclopedia, which of course only God can read. Another, secular person might maintain that *by definition*, "the present state of the universe" contains all the data needed to determine future events, even if those future events (such as quantum measurement outcomes) aren't *actually* predictable via present-day measurements. In other words: if, in a given conception of physics, the present state does *not* fully determine all the future states, then such a person will simply add "hidden variables" to the present state until it does so.

[11] In Democritus's famous dialogue between the intellect and the senses, the intellect declares: "By convention there is sweetness, by convention bitterness, by convention color, in reality only atoms and the void." To which the senses reply: "Foolish intellect! Do you seek to overthrow us, while it is from us that you take your evidence? Your victory is your defeat."

Now, if our hidden-variable theorist isn't careful, and piles on additional requirements like spatial locality, then she'll quickly find herself contradicting one or more of quantum mechanics' no-hidden-variable theorems (such as the Bell [10], Kochen-Specker [51], or PBR [71] theorems). But the bare assertion that "everything is determined by the current state" is no more disprovable than the belief in God's Encyclopedia.

To me, this immunity from any possible empirical discovery shows just how *feeble* a concept "determinism" really is, unless it's supplemented by further concepts like locality, simplicity, or (best of all) actual predictability. A form of "determinism" that applies not merely to our universe, but to any *logically possible* universe, is not a determinism that has "fangs," or that could credibly threaten any notion of free will worth talking about.

2.7 Predictability in the Eye of the Beholder

A system that's predictable to one observer might be unpredictable to another. Given that predictability is such a relative notion, how could it possibly play the fundamental role you need it to?

This question was already briefly addressed in Section 1.1, but since it arises so frequently, it might be worth answering again. In this essay, I call a physical system S "predictable" if (roughly speaking) there's *any possible technology*, *consistent with the laws of physics*, that would allow an external observer to gain enough information about S, without destroying S, to calculate well-calibrated probabilities (to any desired accuracy) for the outcomes of all possible future measurements on S within some allowed set. Of course, this definition introduces many concepts that require further clarification: for example, what do we mean by "destroying" S? What does it mean for probabilities to be "well-calibrated"? Which measurements on S are "allowed"? What can the external observer be assumed to know about S before encountering it? For that matter, what exactly counts as an "external observer," or a "physical system"? I set out my thoughts about these questions, and even suggest a tentative formal definition of "Knightian freedom" in terms of other concepts, in Appendix 12.

For now, though, the main point is that, whenever I talk about whether a system "can" be predicted, the word "can" has basically the same meaning as when physicists talk about whether information "can" get from point A to point B. Just like in the latter case, we don't care whether the two points are *literally* connected by a phone line, so too in the former case, we don't care whether the requisite prediction machine has actually been built, or could plausibly be built in the next millennium. Instead, we're allowed to imagine *arbitrarily-advanced technologies*, so long as our imaginations are constrained by the laws of physics.[12]

(Observe that, were our imaginations not constrained *even* by physics, we'd have to say that *anything whatsoever* "can" happen, except outright logical contradictions. So in particular, we'd reach the uninteresting conclusion that *any* system can be perfectly predicted—by God, for example, or by magical demons. For more see Question 2.6.)

[12] By "the laws of physics," I mean the currently-accepted laws—*unless* we're explicitly tinkering with those laws to see what happens. Of course, whenever known physics imposes an inconvenient limit (for example, no faster-than-light communication), the convention in most science-fiction writing is simply to stipulate that physicists of the future will discover some way around the limit (such as wormholes, tachyons, "hyperdrive," etc). In this essay I take a different approach, trying to be as conservative as I can about fundamental physics.

2.8 Quantum Mechanics and Hidden Variables

Forget about free will or Knightian uncertainty: I deny even that *probability* plays any fundamental role in physics. For me, like for Einstein, the much-touted "randomness" of quantum mechanics merely shows that we humans haven't yet discovered the underlying deterministic rules. Can you prove that I'm wrong?

With minimal use of Occam's Razor, yes, I can! In 1926, when Einstein wrote his famous aphorism about God and dice, the question of whether quantum events were "truly" random or merely pseudorandom could still be considered metaphysical. After all, common sense suggests we can never say with confidence that *anything* is random: the most we can ever say is that *we* failed to find a pattern in it.

But common sense is flawed here. A large body of work, starting with that of Bell in the 1960s [10], has furnished evidence that quantum measurement outcomes *can't* be governed by any hidden pattern, but must be random in just the way quantum mechanics says they are. Crucially, this evidence doesn't circularly assume that quantum mechanics is the final theory of nature. Instead, it assumes just a few general principles (such as spatial locality and "no cosmic conspiracies"), together with the results of specific experiments that have already been done. Since these points are often misunderstood, it might be worthwhile to spell them out in more detail.

Consider the *Bell inequality*, whose violation by entangled particles (in accord with quantum mechanics) has been experimentally demonstrated more and more firmly since the 1980s [7]. From a modern perspective, Bell simply showed that certain games, played by two cooperating but non-communicating players Alice and Bob, can be won with greater probability if Alice and Bob share entangled particles than if they merely share correlated *classical* information.[13] Bell's theorem is usually presented as ruling out a class of theories called *local hidden-variable theories*. Those theories sought to explain Alice and Bob's measurement results in terms of ordinary statistical correlations between two random variables X and Y, which are somehow associated with Alice's and Bob's particles respectively, and which have the properties that nothing Alice does can affect Y and that nothing Bob does can affect X. (One can imagine the particles flipping a coin at the moment of their creation, whereupon one of them declares, "OK, if anyone asks, I'll be spinning up and you'll be spinning down!")

In popular treatments, Bell's theorem is usually presented as demonstrating the reality of what Einstein called "spooky action at a distance."[14] However, as many people have pointed out over the

[13] The standard example is the *CHSH game* [23]. Here Alice and Bob are given bits x and y respectively, which are independent and uniformly random. Their goal is for Alice to output a bit a, and Bob a bit b, such that $a + b \pmod 2 = xy$. Alice and Bob can agree on a strategy in advance, but can't communicate after receiving x and y. Classically, it's easy to see that the best they can do is always to output $a = b = 0$, in which case they win the game with probability $3/4$. By contrast, if Alice and Bob own one qubit each of the entangled state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, then there exists a strategy by which they can win with probability $\cos^2(\pi/8) \approx 0.85$. That strategy has the following form: Alice measures her qubit in a way that depends on x and outputs the result as a, while Bob measures his qubit in a way that depends on y and outputs the result as b.

[14] The reason why many people (including me) cringe at that sort of talk is the *no-communication theorem*, which explains why, despite Bell's theorem, entanglement *can't* be used to send actual messages faster than light. (Indeed, if it could, then quantum mechanics would flat-out contradict special relativity.)

The situation is this: *if* one wanted to violate the Bell inequality using classical physics, *then* one would need faster-than-light communication. But that doesn't imply that quantum mechanics' violation of the same inequality should *also* be understood in terms of faster-than-light communication! We're really dealing with an intermediate case here—"more than classical locality, but less than classical nonlocality"—which I don't think anyone even recognized as a logical possibility until quantum mechanics forced it on them.

years—see, for example, my 2002 critique [1] of Stephen Wolfram's *A New Kind of Science* [93]—one can also see Bell's theorem in a different way: as using the *assumption* of no instantaneous communication to address the even more basic issue of *determinism*. From this perspective, Bell's theorem says the following:

> *Unless* Alice and Bob's particles communicate faster than light, the results of all possible measurements that Alice and Bob could make on those particles *cannot have been determined prior to measurement*—not even by some bizarre, as-yet-undiscovered uncomputable law—assuming the statistics of all the possible measurements agree with the quantum predictions. Instead, the results *must* be "generated randomly on-the-fly" in response to whichever measurement is made, just as quantum mechanics says they are.

The above observation was popularized in 2006 by John Conway and Simon Kochen, who called it the "Free Will Theorem" [25]. Conway and Kochen put the point as follows: if there's no faster-than-light communication, *and* Alice and Bob have the "free will" to choose how to measure their respective particles, then the particles must have their own "free will" to choose how to respond to the measurements.

Alas, Conway and Kochen's use of the term "free will" has generated confusion. For the record, *what Conway and Kochen mean by "free will" has only the most tenuous connection to what most people (including me, in this essay) mean by it!* Their result might more accurately be called the "freshly-generated randomness theorem."[15] For the indeterminism that's relevant here is "only" probabilistic: indeed, Alice and Bob could be replaced by simple dice-throwing or quantum-state-measuring automata without affecting the theorem at all.[16]

Another recent development has made the conflict between quantum mechanics and determinism particularly vivid. It's now known how to exploit Bell's theorem to generate so-called "Einstein-certified random bits" for use in cryptographic applications [70, 90], starting from a much smaller number of "seed" bits that are known to be random.[17] Here "Einstein-certified" means that, *if* the bits pass certain statistical tests, then they *must* be close to uniformly-random, unless nature resorted to "cosmic conspiracies" between separated physical devices to bias the bits.

Thus, if one wants to restore determinism while preserving the empirical success of quantum mechanics, then one has to posit a conspiracy in which every elementary particle, measuring device, and human brain potentially colludes. Furthermore, this conspiracy needs to be so diabolical as to leave essentially no trace of its existence! For example, in order to explain why we can't exploit the conspiracy to send faster-than-light signals, one has to imagine that the conspiracy prevents our own brains (or the quantum-mechanical random number generators in our computers, etc.) from making the choices that would cause those signals to be sent. To my mind, this is no better than the creationists' God, who planted fossils in the ground to confound the paleontologists.

I should say that at least one prominent physicist, Gerard 't Hooft, actually advocates such a cosmic conspiracy [46] (under the name "superdeterminism"); he speculates that a yet-undiscovered

[15] Or the "Free Whim Theorem," as Conway likes to suggest when people point out the irrelevance of human free will to the theorem.

[16] This point was recently brought out by Fritz, in his paper "Bell's theorem without free will" [37]. Fritz replaces the so-called "free will assumption" of the Bell inequality—that is, the assumption that Alice and Bob get to choose which measurements to perform—by an assumption about the *independence* of separated physical devices.

[17] The protocol of Vazirani and Vidick [90] needs only $O(\log n)$ seed bits to generate n Einstein-certified output bits.

replacement for quantum mechanics will reveal its workings.[18] For me, though, the crux is that once we start positing conspiracies between distant regions of spacetime, or between the particles we measure and our own instruments or brains, determinism becomes consistent with *any* possible scientific discovery, and therefore retreats into vacuousness. As the extreme case, as pointed out in Question 2.6, someone could always declare that everything that happens was "determined" by God's unknowable book listing everything that will ever happen! That sort of determinism can never be falsified, but has zero predictive or explanatory power.

In summary, I think it's fair to say that *physical indeterminism is now a settled fact to roughly the same extent as evolution, heliocentrism, or any other discovery in science.* So *if* that fact is considered relevant to the free-will debate, then all sides might as well just accept it and move on! (Of course, we haven't yet touched the question of whether physical indeterminism *is* relevant to the free-will debate.)

2.9 The Consequence Argument

How does your perspective respond to Peter van Inwagen's Consequence Argument?

Some background for non-philosophers: the Consequence Argument [48] is an attempt to formalize most people's intuition for why free will is incompatible with determinism. The argument consists of the following steps:

(i) If determinism is true, then our choices today are determined by whatever the state of the universe was (say) 100 million years ago, when dinosaurs roamed the earth.

(ii) The state of the universe 100 million years ago is clearly outside our ability to alter.

(iii) Therefore, if determinism is true, then our choices today are outside our ability to alter.

(iv) Therefore, if determinism is true, then we don't have free will.

(A side note: as discussed in Question 2.3, the traditional obsession with "determinism" here seems unfortunate to me. What people really mean to ask, I think, is whether free will is compatible with *any* mechanistic account of the universe, regardless of whether the account happens to be deterministic or probabilistic. On the other hand, one could easily rephrase the Consequence Argument to allow for this, with the state of the universe 100 million years ago now fully determining the *probabilities* of our choices today, if not the choices themselves. And I don't think that substitution would make any essential difference to what follows.)

One can classify beliefs about free will according to how they respond to the Consequence Argument. If you accept the argument as well as its starting premise of determinism (or mechanism), and hence also the conclusion of no free will, then you're a *hard determinist (or mechanist).* If you accept the argument, but reject the conclusion by denying the starting premise of determinism

[18] Some people might argue that Bohmian mechanics [16], the interpretation of quantum mechanics that originally inspired Bell's theorem, is also "superdeterministic." But Bohmian mechanics is empirically equivalent to standard quantum mechanics—from which fact it follows immediately that the "determinism" of Bohm's theory is a formal construct that, whatever else one thinks about it, has no actual consequences for prediction. To put it differently: at least in its standard version, Bohmian mechanics buys its "determinism" via the mathematical device of pushing all the randomness back to the beginning of time. It then accepts the nonlocality that such a tactic inevitably entails because of Bell's theorem.

or mechanism, then you're a *metaphysical libertarian*. If you reject the argument by denying that steps (iii) or (iv) follow from the previous steps, then you're a *compatibilist*.

What about the perspective I explore here? It denies step (ii)—or rather, it denies the usual notion of "the state of the universe 100 million years ago," insisting on a distinction between "macrofacts" and "microfacts" about that state. It agrees that the past *macro*facts—such as whether a dinosaur kicked a particular stone—have an objective reality that is outside our ability to alter. But it denies that we can always speak in straightforward causal terms about the past *micro*facts, such as the quantum state $|\psi\rangle$ of some particular photon impinging on the dinosaur's tail.

As such, my perspective can be seen as an example of a little-known view that Fischer [35] calls *multiple-pasts compatibilism*. As I'd put it, multiple-pasts compatibilism agrees that the past microfacts about the world determine its future, and it also agrees that the past *macro*facts are outside our ability to alter. However, it maintains that there might be many possible settings of the past microfacts—the polarizations of individual photons, etc.—that all coexist in the same "past-ensemble." By definition, such microfacts can't possibly have made it into the history books, and a multiple-pasts compatibilist would deny that they're necessarily "outside our ability to alter." Instead, our choices today might play a role in selecting *one* past from a giant ensemble of macroscopically-identical but microscopically-different pasts.

While I take the simple idea above as a starting point, there are two main ways in which I try to go further than it. First, I insist that whether we *can* make the needed distinction between "microfacts" and "macrofacts" is a *scientific* question—one that can only be addressed through detailed investigation of the quantum decoherence process and other aspects of physics. Second, I change the focus from unanswerable metaphysical conundrums about what "determines" what, toward empirical questions about the actual *predictability* of our choices (or at least the probabilities of those choices) given the past macrofacts. I argue that, by making progress on the predictability question, we can learn something about whether multiple-pasts compatibilism is a *viable* response to the Consequence Argument, even if we can never know for certain whether it's the *right* response.

2.10 Paradoxes of Prediction

You say you're worried about the consequences for rational decision-making, Bayesian inference, and so forth if our choices were all mechanically predictable. Why isn't it reassurance enough that, logically, predictions of an agent's behavior can never be known ahead of time to the agent itself? For if they *were* known ahead of time, then in real life—as opposed to Greek tragedies, or stories like Philip K. Dick's *Minority Report* [32]—the agent could simply defy the prediction by doing something else! Likewise, why isn't enough that, because of the Time Hierarchy Theorem from computational complexity, predicting an agent's choices might require as much computational power as the agent itself expends in making those choices?

The obvious problem with such computational or self-referential arguments is that they can't possibly prevent one agent, say Alice, from predicting the behavior of a *different* agent, say Bob. And in order to do that, Alice doesn't need unlimited computational power: *she only needs a bit more computational power than Bob has.*[19] Furthermore, Bob's free will actually seems *more*

[19] This is analogous to how, in computational complexity theory, there exists a program that that uses (say) $n^{2.0001}$ time steps, and that simulates any n^2-step program provided to it as input. The time hierarchy theorem, which is close to tight, only rules out the simulation of n^2-step programs using significantly *less* than n^2 time steps.

threatened by Alice predicting his actions than by Bob predicting his own actions, supposing the latter were possible! This explains why I won't be concerned in this essay with computational obstacles to prediction, but only with obstacles that arise from Alice's physical inability to gather the requisite information about Bob.

Admittedly, as MacKay [59], Lloyd [57], and others have stressed, if Alice wants to predict Bob's choices, then she needs to be careful not to *tell* Bob her predictions before they come true! And that does indeed make predictions of Bob's actions an unusual sort of "knowledge": knowledge that can be falsified by the very fact of Bob's learning it.

But unlike some authors, I don't make much of these observations. For even if Alice can't tell Bob what he's going to do, it's easy enough for her to demonstrate to him afterwards that *she* knew. For example, Alice could put her prediction into a sealed envelope, and let Bob open the envelope only after the prediction came true. Or she could send Bob a *cryptographic commitment* to her prediction, withholding the decryption key until afterward. If Alice could do these things reliably, then it seems likely that Bob's self-conception would change just as radically as if he knew the predictions in advance.

2.11 Singularitarianism

How could it possibly make a difference to anyone's life whether his or her neural computations were buffeted around by microscopic events subject to Knightian uncertainty? Suppose that only "ordinary" quantum randomness and classical chaos turned out to be involved: how on earth could that matter, outside the narrow confines of free-will debates? Is the variety of free will that apparently interests you—one based on the physical unpredictability of our choices—really a variety "worth wanting," in Daniel Dennett's famous phrase [27]?

As a first remark, if there's *anything* in this debate that all sides can agree on, hopefully they can agree that the truth (whatever it is) doesn't care what we want, consider "worth wanting," or think is necessary or sufficient to make our lives meaningful!

But to lay my cards on the table, my interest in the issues discussed in this essay was sparked, in part, by considering the arguments of the so-called Singularitarians.[20] These are people who look at current technological trends—including advances in neuroscience, nanotechnology, and AI, as well as the dramatic increases in computing power—and foresee a "technological singularity" perhaps 50-200 years in our future (not surprisingly, the projected timescales vary). By this, they mean not a mathematical singularity, but a rapid "phase transition," perhaps analogous to the appearance of the first life on earth, the appearance of humanity, or the invention of agriculture or of writing. In Singularitarians' view, the next such change will happen around the time when humans manage to build artificial intelligences that are smarter than the smartest humans. It stands to reason, the Singularitarians say, that such AIs will realize they can best further their goals (whatever those might be) by building AIs even smarter than themselves—and then the super-AIs will build yet smarter AIs, and so on, presumably until the fundamental physical limits on intelligence (whatever they are) are reached.

Following the lead of science-fiction movies, of course one might wonder about the role of humans in the resulting world. Will the AIs wipe us out, treating us as callously as humans have

[20] Personally, I'd suggest a shortening of the name to "Singulatarian," but have been assured that the longer version is standard.

treated each other and most animal species? Will the AIs keep us around as pets, or as revered (if rather dimwitted) creators and ancestors? Will humans be invited to upload their brains to the post-singularity computer cloud—with each human, perhaps, living for billions of years in his or her own simulated paradise? Or will the humans simply merge their consciousnesses into the AI hive-mind, losing their individual identities but becoming part of something unimaginably greater?

Hoping to find out, many Singularitarians have signed up to have their brains cryogenically frozen upon their (biological) deaths, so that some future intelligence (before or after the singularity) might be able to use the information therein to revive them.[21] One leading Singularitarian, Eliezer Yudkowsky, has written at length about the irrationality of people who *don't* sign up for cryonics: how they value social conformity and "not being perceived as weird" over a non-negligible probability of living for billions of years.[22]

With some notable exceptions, academics in neuroscience and other relevant fields have tended to dismiss Singularitarians as nerds with hyperactive imaginations: people who have no idea how great the difficulties are in modeling the human brain or building a general-purpose AI. Certainly, one could argue that the Singularitarians' *timescales* might be wildly off. And even if one accepts their timescales, one could argue that (almost by definition) the unknowns in their scenario are so great as to negate any *practical* consequences for the humans alive now. For example, suppose we conclude—as many Singularitarians have—that the greatest problem facing humanity today is how to ensure that, when superhuman AIs are finally built, those AIs will be "friendly" to human concerns. The difficulty is: *given our current ignorance about AI, how on earth should we act on that conclusion?* Indeed, how could we have any confidence that whatever steps we *did* take wouldn't backfire, and increase the probability of an *un*friendly AI?

Yet on questions of principle—that is, of what the laws of physics could ultimately allow—I think the uncomfortable truth is that it's the Singularitarians who are the scientific conservatives, while those who reject their vision as fantasy are scientific radicals. For at some level, all the Singularitarians are doing is taking conventional thinking about physics and the brain to its logical conclusion. If the brain is a "meat computer," then given the right technology, why *shouldn't* we be able to copy its program from one physical substrate to another? And why couldn't we then run multiple copies of the program in parallel, leading to all the philosophical perplexities discussed in Section 2.5?

Maybe the conclusion is that we should all become Singularitarians! But given the stakes, it seems worth exploring the possibility that there are scientific reasons why human minds *can't* be casually treated as copyable computer programs: not just practical difficulties, or the sorts of question-begging appeals to human specialness that are child's-play for Singularitarians to demolish. If one likes, the origin of this essay was my own refusal to accept the lazy cop-out position, which answers the question of whether the Singularitarians' ideas are *true* by repeating that their ideas are crazy and weird. If uploading our minds to digital computers is indeed a fantasy, then I demand to know what it is about the physical universe that *makes* it a fantasy.

2.12 The Libet Experiments

Haven't neuroscience experiments already proved that our choices aren't nearly as unpredictable as we imagine? Seconds before a subject is conscious of making a de-

[21]The largest cryonics organization is Alcor, www.alcor.org.
[22]See, for example, lesswrong.com/lw/mb/lonely_dissent/

cision, EEG recordings can already detect the neural buildup to that decision. Given that empirical fact, isn't any attempt to ground freedom in unpredictability doomed from the start?

It's important to understand what experiments have and haven't shown, since the details tend to get lost in popularization. The celebrated experiments by Libet (see [55]) from the 1970s used EEGs to detect a "readiness potential" building up in a subject's brain up to a second and a half before the subject made the "freely-willed decision" to flick her finger. The implications of this finding for free will were avidly discussed—especially since the subject might not have been *conscious* of any intention to flick her finger until (say) half a second before the act. So, did the prior appearance of the readiness potential prove that what we perceive as "conscious intentions" are just window-dressing, which our brains add after the fact?

As Libet acknowledged, an important gap in his experiment was that it had inadequate "control." That is, how often did the readiness potential form, *without* the subject flicking her finger (which might indicate a decision that was "vetoed at the last instant")? Because of this gap, it was unclear to what extent the signal Libet found could actually be used for prediction.

More recent experiments—see especially Soon et al. [82]—have tried to close this gap, by using fMRI scans to predict *which* of two buttons a person would press. Soon et al. [82] report that they were able to do so four or more seconds in advance, with success probability significantly better than chance (around 60%). The question is, how much should we read into these findings?

My own view is that the quantitative aspects are crucial when discussing these experiments. For compare a (hypothetical) ability to predict human decisions a full minute in advance, to an ability to predict the same decisions 0.1 seconds in advance, in terms of the intuitive "threat" to free will. The two cases seem utterly different! A minute seems like clearly enough time for a deliberate choice, while 0.1 seconds seems like clearly *not* enough time; on the latter scale, we are only talking about physiological reflexes. (For intermediate scales like 1 second, intuition—or at least my intuition—is more conflicted.)

Similarly, compare a hypothetical ability to predict human decisions with 99% accuracy, against an ability to predict them with 51% accuracy. I expect that only the former, and not the latter, would strike anyone as threatening or even uncanny. For it's obvious that human decisions are *somewhat* predictable: if they weren't, there would be nothing for advertisers, salespeople, seducers, or demagogues to exploit! Indeed, with zero predictability, we couldn't even talk about *personality* or *character* as having any stability over time. So *better-than-chance* predictability is just too low a bar for clearing it to have any relevance to the free-will debate. One wants to know: *how much* better than chance? Is the accuracy better than what my grandmother, or an experienced cold-reader, could achieve?

Even within the limited domain of button-pressing, years ago I wrote a program that invited the user to press the 'G' or 'H' keys in any sequence—'GGGHGHHHHGHG'—and that tried to predict which key the user would press next. The program used only the crudest pattern-matching—"in the past, was the subsequence GGGH more likely to be followed by G or H?" Yet humans are so poor at generating "random" digits that the program regularly achieved prediction accuracies of around 70%—no fMRI scans needed![23]

[23] Soon et al. [82] (see the Supplementary Material, p. 14-15) argue that, by introducing a long delay between trials and other means, they were able to rule out the possibility that their prediction accuracy was due purely to "carryover" between successive trials. They also found that the probability of a run of N successive presses of the same button decreased exponentially with N, as would be expected if the choices were independently random. However, it would be interesting for future research to compare fMRI-based prediction "head-to-head" against prediction using carefully

In summary, I believe neuroscience might *someday* advance to the point where it completely rewrites the terms of the free-will debate, by showing that the human brain is "physically predictable by outside observers" in the same sense as a digital computer. But it seems nowhere close to that point today. Brain-imaging experiments have succeeded in demonstrating predictability with better-than-chance accuracy, in limited contexts and over short durations. Such experiments are deeply valuable and worth trying to push as far as possible.

On the other hand, the mere *fact* of limited predictability is something that humans already knew, millennia before brain-imaging technologies became available! And to whatever extent the intuition for free-will-as-unpredictability survived that earlier folk-knowledge, it seems like it mostly also survives the recent fMRI experiments, whose predictive successes haven't been obviously more impressive.

2.13 Mind and Morality

Notwithstanding your protests that you won't address the "mystery of consciousness," your entire project seems geared toward the old, disreputable quest to find some sort of dividing line between "real, conscious, biological" humans and digital computer programs, even supposing that the latter could perfectly emulate the former. Many thinkers have sought such a line before, but most scientifically-minded people regard the results as dubious. For Roger Penrose, the dividing line involves neural structures called microtubules harnessing exotic quantum-gravitational effects (see Section 6). For the philosopher John Searle [78], the line involves the brain's unique "biological causal powers": powers whose existence Searle loudly asserts, but which he never actually explains. For you, the line seems to involve hypothesized limits on predictability imposed by the No-Cloning Theorem. But regardless of where the line gets drawn, let's discuss where the rubber meets the road. Suppose that in the far future, there are trillions of emulated brains (or "ems") running on digital computers. In such a world, would you consider it acceptable to "murder" an em (say, by deleting its file), simply because it lacked the "Knightian unpredictability" that biological brains might or might not derive from amplified quantum fluctuations? If so, then isn't that a cruel, arbitrary, "meatist" double standard—one that violates the most basic principles of your supposed hero, Alan Turing?

For me, this *moral* objection to my project is possibly the most pressing objection of all. Will *I* be the one to shoot a humanoid robot pleading for its life, simply because the robot lacks the supposedly-crucial "freebits," or "Knightian unpredictability," or whatever else the magical stuff is supposed to be that separates humans from machines?

Thus, perhaps my most important reason to take the freebit picture seriously is that it *does* suggest a reply to the objection: one that strikes me as both intellectually consistent and moral. I simply need to adopt the following ethical stance: *I'm against any irreversible destruction of knowledge, thoughts, perspectives, adaptations, or ideas, except possibly by their owner.* Such destruction is worse the more valuable the thing destroyed, the longer it took to create, and the harder it is to replace. From this basic revulsion to *irreplaceable loss*, hatred of murder, genocide, the hunting of endangered species to extinction, and even (say) the burning of the Library of Alexandria can all be derived as consequences.

designed machine learning algorithms that see only the sequence of previous button presses.

Now, what about the case of "deleting" an emulated human brain from a computer memory? The same revulsion applies in full force—*if the copy deleted is the last copy in existence.* If, however, there are other extant copies, then the deleted copy can always be "restored from backup," so deleting it seems at worst like property damage. For biological brains, by contrast, whether such backup copies *can* be physically created is of course exactly what's at issue, and the freebit picture conjectures a negative answer.

These considerations suggest that the moral status of ems really *could* be different than that of organic humans, but for straightforward practical reasons that have nothing to do with "meat chauvinism," or with question-begging philosophical appeals to human specialness. The crucial point is that even a program that passed the Turing Test would revert to looking "crude and automaton-like," *if you could read, trace, and copy its code.* And whether the code *could* be read and copied might depend strongly on the machine's physical substrate. Destroying something that's both as complex as a human being *and* one-of-a-kind could be regarded as an especially heinous crime.

I see it as the great advantage of this reply that it makes no direct reference to the first-person experience of *anyone*, neither biological humans nor ems. On this account, we don't *need* to answer probably-unanswerable questions about whether or not ems would be conscious, in order to constructed a workable moral code that applies to them. Deleting the last copy of an em in existence should be prosecuted as murder, *not* because doing so snuffs out some inner light of consciousness (who is anyone else to know?), but rather because it deprives the rest of society of a unique, irreplaceable store of knowledge and experiences, precisely as murdering a human would.

3 Knightian Uncertainty and Physics

Having spent almost half the essay answering *a priori* objections to the investigation I want to undertake, I'm finally ready to start the investigation itself! In this section, I'll set out and defend two propositions, both of which are central to what follows.

The first proposition is that *probabilistic uncertainty (like that of quantum-mechanical measurement outcomes) can't possibly, by itself, provide the sort of "indeterminism" that could be relevant to the free-will debate.* In other words, *if* we see a conflict between free will and the deterministic predictability of human choices, then we should see the same conflict between free will and *probabilistic* predictability, assuming the probabilistic predictions are as accurate as those of quantum mechanics. Conversely, if we hold free will to be compatible with "quantum-like predictability," then we might as well hold free will to be compatible with *deterministic* predictability also. In my perspective, for a form of uncertainty to be relevant to free will, a necessary condition is that it be what the economists call *Knightian uncertainty.* Knightian uncertainty simply refers to uncertainty that we lack a clear, agreed-upon way to quantify—like our uncertainty about existence of extraterrestrial life, as opposed to our uncertainty about the outcome of a coin toss.

The second proposition is that, in current physics, there appears to be only one source of Knightian uncertainty that could possibly be both fundamental and relevant to human choices. That source is *uncertainty about the microscopic, quantum-mechanical details of the universe's initial conditions (or the initial conditions of our local region of the universe).* In classical physics, there's no known fundamental principle that prevents a predictor from learning the relevant initial conditions to whatever precision it likes, without disturbing the system to be predicted. But in quantum mechanics there *is* such a principle, namely the uncertainty principle (or from a more "modern" standpoint, the No-Cloning Theorem). It's crucial to understand that this source of uncertainty is separate from the randomness of quantum measurement *outcomes*: the latter is much more often invoked in free-will speculations, but in my opinion it shouldn't be. If we know a system's quantum state ρ, then quantum mechanics lets us calculate the probability of any outcome of any measurement that might later be made on the system. But if we *don't* know the state ρ, then ρ itself can be thought of as subject to Knightian uncertainty.

In the next two subsections, I'll expand on and justify the above claims.

3.1 Knightian Uncertainty

A well-known argument maintains that the very concept of free will is logically incoherent. The argument goes like this:

> Any event is either determined by earlier events (like the return of Halley's comet), or else not determined by earlier events (like the decay of a radioactive atom). If the event is determined, then clearly it isn't "free." But if the event is *un*determined, it isn't "free" either: it's merely arbitrary, capricious, and random. Therefore no event can be "free."

As I'm far from the first to point out, the above argument has a gap, contained in the vague phrase "arbitrary, capricious, and random." An event can be "arbitrary," in the sense of being undetermined by previous events, without being random in the narrower technical sense of being generated by some known or knowable probabilistic process. The distinction between arbitrary and random is not just word-splitting: it plays a huge practical role in computer science, economics, and other fields. To illustrate, consider the following two events:

> E_1 = Three weeks from today, the least significant digit of the Dow Jones average will be even
>
> E_2 = Humans will make contact with an extraterrestrial civilization within the next 500 years

For both events, we are ignorant about whether they will occur, but we are ignorant in completely different ways. For E_1, we can quantify our ignorance by a probability distribution, in such a way that almost any reasonable person would agree with our quantification. For E_2, we can't.

For another example, consider a computer program, which has a bug that only appears when a call to the random number generator returns the result 3456. That's not necessarily a big deal—since with high probability, the program would need to be run thousands of times before the bug reared its head. Indeed, today many problems are solved using *randomized algorithms* (such as Monte-Carlo simulation), which *do* have a small but nonzero probability of failure.[24] On the other hand, if the program has a bug that only occurs when the user *inputs* 3456, that's a much more serious problem. For how can the programmer know, in advance, whether 3456 is an input (maybe even the *only* input) that the user cares about? So a programmer *must* treat the two types of uncertainty differently: she can't just toss them both into a bin labeled "arbitrary, capricious, and

[24] The probability of failure can be made arbitrarily small by the simple expedient of running the algorithm over and over and taking a majority vote.

random." And indeed, the difference between the two types of uncertainty shows up constantly in theoretical computer science and information theory.[25]

In economics, the "second type" of uncertainty—the type that can't be objectively quantified using probabilities—is called *Knightian uncertainty*, after Frank Knight, who wrote about it extensively in the 1920s [49]. Knightian uncertainty has been invoked to explain phenomena from risk-aversion in behavioral economics to the 2008 financial crisis (and was popularized by Taleb [87] under the name "black swans"). An agent in a state of Knightian uncertainty might describe its beliefs using a convex set of probability distributions, rather than a single distribution.[26] For example, it might say that a homeowner will default on a mortgage with some probability between 0.1 and 0.3, but within that interval, be unwilling to quantify its uncertainty further. The notion of probability intervals leads naturally to various generalizations of probability theory, of which the best-known is the *Dempster-Shafer theory of belief* [79].

What does any of this have to do with the free-will debate? As I said in Section 1.1, from my personal perspective Knightian uncertainty seems like a *precondition* for free will as I understand the latter. In other words, I *agree* with the free-will-is-incoherent camp when it says that a random event can't be considered "free" in any meaningful sense. Several writers, such as Kane [36], Balaguer [9], Satinover [75], and Koch [50], have speculated that the randomness inherent in quantum-mechanical wavefunction collapse, were it relevant to brain function, could provide all the scope for free will that's needed. But I think those writers are mistaken on this point.

For me, the bottom line is simply that it seems like a sorry and pathetic "free will" that's ruled by ironclad, externally-knowable statistical laws, and that retains only a "ceremonial" role, analogous to spinning a roulette wheel or shuffling a deck of cards. Should we say that a radioactive nucleus has "free will," just because (according to quantum mechanics) we can't predict exactly when it will decay, but can only calculate a precise probability distribution over the decay times? That seems perverse—especially since given *many* nuclei, we can predict almost perfectly what *fraction* will have decayed by such-and-such a time. Or imagine an artificially-intelligent robot that used nuclear decays as a source of random numbers. Does anyone seriously maintain that, if we swapped out the actual decaying nuclei for a "mere" pseudorandom computer simulation of such nuclei (leaving all other components unchanged), the robot would suddenly be robbed of its free will? While I'm leery of "arguments from obviousness" in this subject, it really *does* seem to me that if we say the robot has free will in the first case then we should also say so in the second.

And thus, I think that the free-will-is-incoherent camp would be right, *if* all uncertainty were probabilistic. But I consider it far from obvious that all uncertainty *is* (usefully regarded as) probabilistic. Some uncertainty strikes me as Knightian, in the sense that rational people might never even reach agreement about how to assign the probabilities. And while Knightian uncertainty might or might not be relevant to predicting human choices, I definitely (for reasons I'll discuss later) don't think that current knowledge of physics or neuroscience lets us exclude the possibility.

At this point, though, we'd better hear from those who reject the entire concept of Knightian uncertainty. Some thinkers—I'll call them *Bayesian fundamentalists*—hold that Bayesian probability theory provides the only sensible way to represent uncertainty. On that view, "Knightian uncertainty" is just a fancy name for someone's failure to carry a probability analysis far enough.

[25] For example, it shows up in the clear distinctions made between random and adversarial noise, between probabilistic and nondeterministic Turing machines, and between average-case and worst-case analysis of algorithms.

[26] In Appendix 14, I briefly explain why I think convex sets provide the "right" representation of Knightian uncertainty, though this point doesn't much matter for the rest of the essay.

In support of their position, Bayesian fundamentalists often invoke the so-called *Dutch book arguments* (see for example Savage [76]), which say that any rational agent satisfying a few axioms must behave *as if* its beliefs were organized using probability theory. Intuitively, even if you claim not to have any opinion whatsoever about (say) the probability of life being discovered on Mars, I can still "elicit" a probabilistic prediction from you, by observing which bets about the question you will or won't accept.

However, a central assumption on which the Dutch book arguments rely—basically, that a rational agent shouldn't mind taking at least one side of any bet—has struck many commentators as dubious. And if we drop that assumption, then the path is open to Knightian uncertainty (involving, for example, convex *sets* of probability distributions).

Even if we accept the standard derivations of probability theory, the bigger problem is that Bayesian agents can have different "priors." If one strips away the philosophy, Bayes' rule is just an elementary mathematical fact about how one should update one's prior beliefs in light of new evidence. So one can't use Bayesianism to justify a belief in the existence of *objective* probabilities underlying all events, unless one is also prepared to defend the existence of an "objective prior." In economics, the idea that all rational agents can be assumed to start with the same prior is called the *Common Prior Assumption*, or CPA. Assuming the CPA leads to some wildly counterintuitive consequences, most famously *Aumann's agreement theorem* [8]. That theorem says that two rational agents with common priors (but differing information) can never "agree to disagree": as soon as their opinions on any subject become common knowledge, their opinions must be equal.

The CPA has long been controversial; see Morris [63] for a summary of arguments for and against. To my mind, though, the real question is: *what could possibly have led anyone to take the CPA seriously in the first place?*

Setting aside methodological arguments in economics[27] (which don't concern us here), I'm aware of two substantive arguments in favor of the CPA. The first argument is that, if two rational agents (call them Alice and Bob) have different priors, then Alice will realize that *if she had been born Bob*, she would have had Bob's prior, and Bob will realize that if he had been born Alice, he would have had Alice's. But if Alice and Bob are indeed rational, then why should they assign any weight to personal accidents of their birth, which are clearly irrelevant to the objective state of the universe? (See Cowen and Hanson [26] for a detailed discussion of this argument.)

The simplest reply is that, even if Alice and Bob accepted this reasoning, they would *still* generally end up with different priors, unless they furthermore shared the same *reference class*: that is, the set of all agents who they imagine they "could have been" if they weren't themselves. For example, if Alice includes all humans in her reference class, while Bob includes only those humans capable of understanding Bayesian reasoning such as he and Alice are engaging in now, then their beliefs will differ. But requiring agreement on the reference class makes the argument circular—presupposing, as it does, a "God's-eye perspective" transcending the individual agents, the very thing whose existence or relevance was in question. Section 8 will go into more detail about "indexical" puzzles (that is, puzzles concerning the probability of your own existence, the likelihood of having been born at one time rather than another, etc). But I hope this discussion already makes clear how much debatable metaphysics lurks in the assumption that a single Bayesian prior governs (or *should* govern) every probability judgment of every rational being.

[27] E.g., that even if the CPA is wrong, we should assume it because economic theorizing would be too unconstrained without it, or because many interesting theorems need it as a hypothesis.

The second argument for the CPA is more ambitious: it seeks to tell us *what* the true prior is, not merely that it exists. According to this argument, any sufficiently intelligent being ought to use what's called the *universal prior* from algorithmic information theory. This is basically a distribution that assigns a probability proportional to 2^{-n} to every possible universe describable by an n-bit computer program. In Appendix 13, I'll examine this notion further, explain why some people [47, 77] have advanced it as a candidate for the "true" prior, but also explain why, despite its mathematical interest, I don't think it can fulfill that role. (Briefly, a predictor using the universal prior can be thought of as a superintelligent entity that figures out the right probabilities almost as fast as is information-theoretically possible. But that's conceptually very different from an entity that *already knows* the probabilities.)

3.2 Quantum Mechanics and the No-Cloning Theorem

While defending the meaningfulness of Knightian uncertainty, the last section left an obvious question unanswered: where, in a law-governed universe, could we possibly *find* Knightian uncertainty?

Granted, in almost any part of science, it's easy to find systems that are "effectively" subject to Knightian uncertainty, in that we don't yet have models for the systems that capture all the important components and their probabilistic interactions. The earth's climate, a country's economy, a forest ecosystem, the early universe, a high-temperature superconductor, or even a flatworm or a cell are examples. Even if our probabilistic models of many of these systems are improving over time, none of them come anywhere close to (say) the quantum-mechanical model of the hydrogen atom: a model that answers essentially *everything* one could think to ask within its domain, modulo an unavoidable (but precisely-quantifiable) random element.

However, in all of these cases (the earth's climate, the flatworm, etc.), the question arises: what grounds could we ever have to think that Knightian uncertainty was *inherent* to the system, rather than an artifact of our own ignorance? Of course, one could have asked the same question about probabilistic uncertainty, before the discovery of quantum mechanics and its no-hidden-variable theorems (see Section 2.8). But the fact remains that today, we don't have any physical theory that demands Knightian uncertainty in anything like the way that quantum mechanics demands probabilistic uncertainty. And as I said in Section 3.1, I insist that the "merely probabilistic" aspect of quantum mechanics can't do the work that many free-will advocates have wanted it to do for a century.

On the other hand, no matter how much we've learned about the dynamics of the physical world, there remains one enormous source of Knightian uncertainty that's been "hiding in plain sight," and that receives surprisingly little attention in the free-will debate. This is our ignorance of the relevant *initial conditions*. By this I mean both the initial conditions of the entire universe or multiverse (say, at the Big Bang), and the "indexical conditions," which characterize the part of the universe or multiverse in which *we* happen to reside. To make a prediction, of course one needs initial conditions as well as dynamical laws: indeed, outside of idealized toy problems, the initial conditions are typically the "larger" part of the input. Yet leaving aside recent cosmological speculations (and "genericity" assumptions, like those of thermodynamics), the specification of initial conditions is normally not even considered the *task* of physics. So, if there are no laws that fix the initial conditions, or even a distribution over possible initial conditions—if there aren't even especially promising *candidates* for such laws—then why isn't this just what free-will advocates have been looking for?

It will be answered immediately that there's an excellent reason why not. Namely, whatever else we do or don't know about the universe's initial state (e.g., at the Big Bang), clearly nothing about it was determined by any of *our* choices! (This is the assumption made explicit in step (ii) of van Inwagen's "Consequence Argument" from Section 2.9.)

The above answer might strike the reader as conclusive. Yet *if* our interest is in actual, physical predictability—rather than in the metaphysical concept of "determination"—then notice that it's no longer conclusive at all. For we still need to ask: how much can we *learn* about the initial state by making measurements? This, of course, is where quantum mechanics might become relevant.

It's actually easiest for our purposes to forget the famous *uncertainty principle*, and talk instead about the *No-Cloning Theorem*. The latter is simply the statement that there's no physical procedure, consistent with quantum mechanics, that takes as input a system in an arbitrary quantum state $|\psi\rangle$,[28] and outputs two systems *both* in the state $|\psi\rangle$.[29] Intuitively, it's not hard to see why: a measurement of (say) a qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ reveals only a single, probabilistic bit of information about the continuous parameters α and β; the rest of the information vanishes forever. The proof of the No-Cloning Theorem (in its simplest version) is as easy as observing that the "cloning map,"

$$\begin{aligned}(\alpha|0\rangle + \beta|1\rangle)|0\rangle &\longrightarrow (\alpha|0\rangle + \beta|1\rangle)(\alpha|0\rangle + \beta|1\rangle)\\ &= \alpha^2|0\rangle|0\rangle + \alpha\beta|0\rangle|1\rangle + \alpha\beta|1\rangle|0\rangle + \beta^2|1\rangle|1\rangle,\end{aligned}$$

acts nonlinearly on the amplitudes, but in quantum mechanics, unitary evolution must be linear.

Yet despite its mathematical triviality, the No-Cloning Theorem has the deep consequence that quantum states have a certain "privacy": unlike classical states, they can't be copied promiscuously around the universe. One way to gain intuition for the No-Cloning Theorem is to consider some striking cryptographic protocols that rely on it. In *quantum key distribution* [11]—something that's already (to a small extent) been commercialized and deployed—a sender, Alice, transmits a secret key to a receiver, Bob, by encoding the key in a collection of qubits. The crucial point is that if an eavesdropper, Eve, tries to learn the key by measuring the qubits, then the very fact that she measured the qubits will be detectable by Alice and Bob—so Alice and Bob can simply keep trying until the channel is safe. *Quantum money*, proposed decades ago by Wiesner [91] and developed further in recent years [3, 5, 34], would exploit the No-Cloning Theorem even more directly, to create cash that can't be counterfeited according to the laws of physics, but that can nevertheless be verified as genuine.[30] A closely related proposal, *quantum software copy-protection* (see [3, 5]), would exploit the No-Cloning Theorem in a still more dramatic way: to create quantum states $|\psi_f\rangle$ that can be used to evaluate some function f, but that can't feasibly be used to create more states with which f can be evaluated. Research on quantum copy-protection has shown that, at least in a few special cases (and maybe more broadly), it's possible to create a physical object that

(a) interacts with the outside world in an interesting and nontrivial way, yet

[28] Here we assume for simplicity that we're talking about pure states, not mixed states.

[29] The word "arbitrary" is needed because, if we knew how $|\psi\rangle$ was prepared, then of course we could simply run the preparation procedure a second time. The No-Cloning Theorem implies that, if we *don't* already know a preparation procedure for $|\psi\rangle$, then we can't learn one just by measuring $|\psi\rangle$. (And conversely, the inability to learn a preparation procedure implies the No-Cloning Theorem. If we could copy $|\psi\rangle$ perfectly, then we could keep repeating to make as many copies as we wanted, then use quantum state tomography on the copies to learn the amplitudes to arbitrary accuracy.)

[30] Unfortunately, unlike quantum key distribution, quantum money is not yet practical, because of the difficulty of protecting quantum states against decoherence for long periods of time.

(b) effectively hides from the outside world the information needed to predict how the object will behave in future interactions.

When put that way, the *possible* relevance of the No-Cloning Theorem to free-will discussions seems obvious! And indeed, a few bloggers and others[31] have previously speculated about a connection. Interestingly, their motivation for doing so has usually been to *defend compatibilism* (see Section 2.3). In other words, they've invoked the No-Cloning Theorem to explain why, despite the mechanistic nature of physical law, human decisions will nevertheless remain unpredictable *in practice*. In a discussion of this issue, one commenter[32] opined that, while the No-Cloning Theorem *does* put limits on physical predictability, human brains will also remain unpredictable for countless more prosaic reasons that have nothing to do with quantum mechanics. Thus, he said, invoking the No-Cloning Theorem in free-will discussions is "like hiring the world's most high-powered lawyer to get you out of a parking ticket."

Personally, though, I think the world's most high-powered lawyer might ultimately be needed here! For the example of the Singularitarians (see Section 2.11) shows why, in these discussions, it doesn't suffice to offer "merely practical" reasons why copying a brain state is hard. Every practical problem can easily be countered by a speculative technological answer—one that assumes a future filled with brain-scanning nanorobots and the like. If we want a proposed obstacle to copying to survive unlimited technological imagination, then the obstacle had better be grounded in the laws of physics.

So as I see it, the real question is: once we disallow quantum mechanics, does there remain any *classical* source of "fundamental unclonability" in physical law? Some might suggest, for example, the impossibility of keeping a system perfectly isolated from its environment during the cloning process, or the impossibility of measuring continuous particle positions to infinite precision. But either of these would require very nontrivial arguments (and if one wanted to invoke continuity, one would also have to address the apparent breakdown of continuity at the Planck scale). There are also formal analogues of the No-Cloning Theorem in various classical settings, but none of them seem able to do the work required here.[33] As far as current physics can say, *if* copying a bounded physical system is fundamentally impossible, then the reason for the impossibility seems ultimately traceable to quantum mechanics.

Let me end this section by discussing *quantum teleportation*, since nothing more suggestively illustrates the "philosophical work" that the No-Cloning Theorem can potentially do. Recall, from Section 2.5, the "paradox" raised by teleportation machines. Namely, after a perfect copy of you has been reconstituted on Mars from the information in radio signals, what should be done with the "original" copy of you back on Earth? Should it be euthanized?

Like other such paradoxes, this one need not trouble us if (because of the No-Cloning Theorem, or whatever other reason) we drop the assumption that such copying is possible—at least with

[31] See, for example, www.daylightatheism.org/2006/04/on-free-will-iii.html

[32] Sadly, I no longer have the reference.

[33] For example, No-Cloning holds for classical probability distributions: there's no procedure that takes an input bit b that equals 1 with unknown probability p, and produces two output bits that are both 1 with probability p independently. But this observation lacks the import of the quantum No-Cloning Theorem, because regardless of what one wanted to do with the bit b, one might as well have *measured* it immediately, thereby "collapsing" it to a deterministic bit—which *can* of course be copied.

Also, seeking to clarify foundational issues in quantum mechanics, Spekkens [83] constructed an "epistemic toy theory" that's purely classical, but where an analogue of the No-Cloning Theorem holds. However, the toy theory involves "magic boxes" that we have no reason to think can be physically realized.

great enough fidelity for the copy on Mars to be a "second instantiation of you." However, what makes the situation more interesting is that there *is* a famous protocol, discovered by Bennett et al. [12], for "teleporting" an arbitrary quantum state $|\psi\rangle$ by sending classical information only. (This protocol also requires quantum entanglement, in the form of *Bell pairs* $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$, shared between the sender and receiver in advance, and one Bell pair gets "used up" for every qubit teleported.)

Now, a crucial feature of the teleportation protocol is that, in order to determine which classical bits to send, the sender needs to perform a measurement on her quantum state $|\psi\rangle$ (together with her half of the Bell pair) that *destroys* $|\psi\rangle$. In other words, in quantum teleportation, the destruction of the original copy is not an extra decision that one needs to make; rather, it happens as an inevitable byproduct of the protocol itself! Indeed this must be so, since otherwise quantum teleportation could be used to violate the No-Cloning Theorem.

3.3 The Freebit Picture

At this point one might interject: theoretical arguments about the No-Cloning Theorem are well and good, but even if accepted, they still don't provide any concrete picture of how Knightian uncertainty could be relevant to human decision-making.

So let me sketch a possible picture (the only one I can think of, consistent with current physics), which I call the "freebit picture." At the Big Bang, the universe had some particular quantum state $|\Psi\rangle$. If known, $|\Psi\rangle$ would of course determine the universe's future history, modulo the probabilities arising from quantum measurement. However, *because* $|\Psi\rangle$ is the state of the whole universe (including us), we might refuse to take a "God's-eye view" of $|\Psi\rangle$, and insist on thinking about $|\Psi\rangle$ differently than about an ordinary state that we prepare for ourselves in the lab. In particular, we might regard at least some (not all) of the qubits that constitute $|\Psi\rangle$ as what I'll call *freebits*. A freebit is simply a qubit for which the most complete physical description possible involves Knightian uncertainty. While the details aren't so important, I give a brief mathematical account of freebits in Appendix 14. For now, suffice it to say that a *freestate* is a convex set of quantum mixed states, and a freebit is a 2-level quantum system in a freestate.

Thus, by the *freebit picture*, I mean the picture of the world according to which

(i) due to Knightian uncertainty about the universe's initial quantum state $|\Psi\rangle$, at least some of the qubits found in nature are regarded as freebits, and

(ii) the presence of these freebits makes predicting certain future events—possibly including some human decisions—physically impossible, even probabilistically and even with arbitrarily-advanced future technology.

Section 3.4 will say more about the "biological" aspects of the freebit picture: that is, the actual chains of causation that could in principle connect freebits to (say) a neuron firing or not firing. In the rest of this section, I'll discuss some physical and conceptual questions about freebits themselves.

Firstly, why is it important that freebits be qubits, rather than *classical* bits subject to Knightian uncertainty? The answer is that only for qubits can we appeal to the No-Cloning Theorem. Even if the value of a classical bit b can't determined by measurements on the entire rest of the universe, a superintelligent predictor could always learn b by non-invasively measuring b *itself*. But the same is not true for a qubit.

Secondly, isn't Knightian uncertainty in the eye of the beholder? That is, why couldn't one observer regard a given qubit as a "freebit," while a different observer, with more information, described the same qubit by ordinary quantum mixed state ρ? The answer is that our criterion for what counts a freebit is extremely stringent. Given a 2-level quantum system S, *if a superintelligent demon could reliably learn the reduced density matrix ρ of S, via arbitrary measurements on anything in the universe (including S itself), then S is not a freebit.* Thus, to qualify as a freebit, S must be a "freely moving part" of the universe's quantum state $|\Psi\rangle$: it must not be possible (even in principle) to trace S's causal history back to any physical process that generated S according to a known probabilistic ensemble. Instead, our Knightian uncertainty about S must (so to speak) go "all the way back," and be traceable to uncertainty about the initial state of the universe.

To illustrate the point: suppose we detect a beam of photons with varying polarizations. For the most part, the polarizations look uniformly random (i.e., like qubits in the maximally mixed state). But there is a slight bias toward the vertical axis, and the bias is slowly changing over time, in a way not fully accounted for by our model of the photons. So far, we can't rule out the possibility that freebits might be involved. However, suppose we later learn that the photons are coming from a laser in another room, and that the polarization bias is due to drift in the laser that can be characterized and mostly corrected. Then the scope for freebits is correspondingly reduced.

Someone might interject: "but *why* was there drift in the laser? Couldn't freebits have been responsible for the drift itself?" The difficulty is that, *even if so*, we still couldn't use those freebits to argue for Knightian uncertainty in the laser's *output*. For between the output photons, and whatever freebits might have caused the laser to be configured as it was, there stands a classical, macroscopic intermediary: the laser itself. If a demon had wanted to predict the polarization drift in the output photons, the demon could simply have traced the photons back to the laser, then *non-invasively measured the laser's classical degrees of freedom*—cutting off the causal chain there and ignoring any further antecedents. In general, given some quantum measurement outcome Q that we're trying to predict, if there exists a classical observable C that could have been non-invasively measured long before Q, and that if measured, would have let the demon probabilistically predict Q to arbitrary accuracy (in the same sense that radioactive decay is probabilistically predictable), then I'll call C a *past macroscopic determinant (PMD)* for Q.

In the freebit picture, we're exclusively interested in the quantum states—if there are any!—that *can't* be grounded in PMDs, but can only be traced all the way back to the early universe, with no macroscopic intermediary along the way to "screen off" the early universe's causal effects. The reason is simple: because such states, if they exist, are the only ones that our superintelligent demon, able to measure all the macroscopic observables in the universe, would *still* have Knightian uncertainty about. In other words, such states are the only possible freebits.

Of course this immediately raises a question:

> **(*) In the actual universe, *are* there any quantum states that can't be grounded in PMDs?**

A central contention of this essay is that pure thought doesn't suffice to answer question (*): here we've reached the limits of where conceptual analysis can take us. There are possible universes consistent with the rules of quantum mechanics where the requisite states exist, and other such universes where they don't exist, and deciding which kind of universe *we* inhabit seems to require scientific knowledge that we don't have.

Some people, while agreeing that logic and quantum mechanics don't suffice to settle question (*), would nevertheless say we can settle it using simple facts about astronomy. At least near the surface of the earth, they'd ask, what quantum states could there possibly be that *didn't* originate in PMDs? Most of the photons impinging on the earth come from the sun, whose physics is exceedingly well-understood. Of the subatomic particles that could conceivably "tip the scales" of (say) a human neural event, causing it to turn out one way rather than another, others might have causal pathways that lead back to other astronomical bodies (such as supernovae), the earth's core, etc. But it seems hard to imagine how any of the latter possibilities *wouldn't* serve as PMDs: that is, how they wouldn't effectively "screen off" any Knightian uncertainty from the early universe.

To show that the above argument is inconclusive, one need only mention the cosmic microwave background (CMB) radiation. CMB photons pervade the universe, famously accounting for a few percent of the static in old-fashioned TV sets. Furthermore, many CMB photons are believed to reach the earth having maintained quantum coherence ever since being emitted at the so-called *time of last scattering*, roughly $380,000$ years after the Big Bang. Finally, unlike (say) neutrinos or dark matter, CMB photons readily interact with matter. In short, we're continually bathed with at least one type of radiation that seems to satisfy most of the freebit picture's requirements!

On the other hand, no sooner is the CMB suggested for this role than we encounter two serious objections. The first is that the time of last scattering, when the CMB photons were emitted, is separated from the Big Bang itself by $380,000$ years. So if we wanted to postulate CMB photons as freebit carriers, then we'd also need a story about why the hot early universe should *not* be considered a PMD, and about how a qubit might have "made it intact" from the Big Bang—or at any rate, from as far back as current physical theory can take us—to the time of last scattering. The second objection asks us to imagine someone *shielded* from the CMB: for example, someone in a deep underground mine. Would such a person be "bereft of Knightian freedom," at least while he or she remained in the mine?

Because of these objections, I find that, while the CMB might be one *piece* of a causal chain conveying a qubit to us from the early universe (without getting screened off by a PMD), it can't possibly provide the full story. It seems to me that convincingly answering question (*) would require something like a census of the possible causal chains from the early universe to ourselves that are allowed by particle physics and cosmology. I don't know whether the requisite census is beyond present-day science, but it's certainly beyond *me*! Note that, if it could be shown that *all* qubits today can be traced back to PMDs, and that the answer to (*) is negative, then the freebit picture would be falsified.

3.4 Amplification and the Brain

We haven't yet touched on an obvious question: *once freebits have made their way into (say) a brain, by whatever means, how could they then tip the scales of a decision?* Luckily, it's not hard to suggest plausible answers to this question, without having to assume anything particularly exotic about either physics or neurobiology. Instead, one can appeal to the well-known phenomenon of chaos (i.e., sensitive dependence on initial conditions) in dynamical systems.

The idea that chaos in brain activity might somehow underlie free will is an old one. However, that idea has traditionally been rejected, on the sensible ground that a classical chaotic system is still perfectly deterministic! Our inability to measure the initial conditions to unlimited precision, and our consequent inability to predict very far into the future, seem at best like practical limitations.

Thus, a revised idea has held that the role of chaos for free will might be to take quantum fluctuations—which, as we know, are *not* deterministic (see Section 2.8)—and amplify those fluctuations to macroscopic scale. However, this revised idea has also been rejected, on the (again sensible) ground that, even if true, it would only make the brain a *probabilistic* dynamical system, which still seems "mechanistic" in any meaningful sense.

The freebit picture makes a further revision: namely, it postulates that chaotic dynamics in the brain can have the effect of amplifying *freebits* (i.e., microscopic Knightian uncertainty) to macroscopic scale. If nothing else, this overcomes the elementary objections above. Yes, the resulting picture might still be wrong, but not for some simple *a priori* reason—and to me, that represents progress!

It's long been recognized that neural processes relevant to cognition can be sensitive to microscopic fluctuations. An important example is the opening and closing of a neuron's sodium-ion channels, which normally determines whether and for how long a neuron fires. This process is modeled probabilistically (in particular, as a Markov chain) by the standard *Hodgkin-Huxley equations* [44] of neuroscience. Of course, one then has to ask: is the apparent randomness in the behavior of the sodium-ion channels ultimately traceable back to quantum mechanics (and if so, by what causal routes)? Or does the "randomness" merely reflect our ignorance of relevant classical details?

Balaguer [9] put the above question to various neuroscientists, and was told that either that the answer is unknown or that it's outside neuroscience's scope. For example, he quotes Sebastian Seung as saying: "The question of whether [synaptic transmission and spike firing] are 'truly random' processes in the brain isn't really a neuroscience question. It's more of a physics question, having to do with statistical mechanics and quantum mechanics." He also quotes Christof Koch as saying: "At this point, we do not know to what extent the random, i.e. stochastic, neuronal processes we observe are due to quantum fluctuations (à la Heisenberg) that are magnified by chemical and biological mechanisms or to what extent they just depend on classical physics (i.e. thermodynamics) and statistical fluctuations in the underlying molecules."

In his paper "A scientific perspective on human choice" [81], the neuroscientist Haim Sompolinsky offers a detailed review of what's known about the brain's sensitivity to microscopic fluctuations. Though skeptical about any role for such fluctuations in cognition, he writes:

> In sum, given the present state of our understanding of brain processes and given the standard interpretation of quantum mechanics, we cannot rule out the possibility that the brain is truly an indeterministic system; that because of quantum indeterminism, there are certain circumstances where choices produced by brain processes are not fully determined by the antecedent brain process and the forces acting on it. If this is so, then the first prerequisite [i.e., indeterminism] of "metaphysical free will" ... may be consistent with the scientific understanding of the brain.[34]

To make the issue concrete: suppose that, with godlike knowledge of the quantum state $|\Psi\rangle$ of the entire universe, you wanted to change a particular decision made by a particular human being—and do so *without* changing anything else, except insofar as the other changes flowed from

[34] However, Sompolinsky then goes on to reject "metaphysical free will" as incompatible with a scientific worldview: if, he says, there were laws relevant to brain function beyond the known laws of physics and chemistry, then those laws would themselves be incorporated into science, leaving us back where we started. I would agree if it weren't for the logical possibility of "Knightian laws," which explained to us why we couldn't even predict the probability distributions for certain events.

the changed decision itself. Then the question that interests us is: *what sorts of changes to $|\Psi\rangle$ would or wouldn't suffice to achieve your aim?* For example, would it suffice to change the energy of a single photon impinging on the subject's brain? Such a photon might get absorbed by an electron, thereby slightly altering the trajectories of a few molecules near one sodium-ion channel in one neuron, thereby initiating a chain of events that ultimately causes the ion channel to open, which causes the neuron to fire, which causes other neurons to fire, etc. If that sort of causal chain is plausible—which, of course, is an empirical question—then at least as far as neuroscience is concerned, the freebit picture would seem to have the raw material that it needs.

Some people might shrug at this, and regard our story of "the photon that broke the camel's back" as so self-evidently plausible that the question isn't even scientifically interesting! So it's important to understand that there are two details of the story that matter enormously, if we want the freebit picture to be viable. The first detail concerns the *amount of time* needed for a microscopic change in a quantum state to produce a macroscopic change in brain activity. Are we talking seconds? hours? days?[35] The second detail concerns *localization*. We'd like our change to the state of a single photon to be "surgical" in its effects: it should change a person's neural firing patterns enough to alter that person's actions, but any other macroscopic effects of the changed photon state should be mediated through the change to the brain state. The reason for this requirement is simply that, if it fails, then a superintelligent predictor might "non-invasively" learn about the photon by measuring its *other* macroscopic effects, and ignoring its effects on the brain state.

To summarize our questions:

> *What are the causal pathways by which a microscopic fluctuation can get chaotically amplified, in human or other animal brains? What are the characteristic timescales for those pathways? What "side effects" do the pathways produce, separate from their effects on cognition?*[36]

In Section 9, I'll use these questions—and the freebit picture's dependence on their answers—to argue that the picture makes falsifiable predictions. For now, I'll simply say that these questions strike me as wide open to investigation, and not only in principle. That is, I can easily imagine that in (say) fifty years, neuroscience, molecular biology, and physics will be able to say more about these questions than they can today. And crucially, the questions strike me as scientifically interesting regardless of one's philosophical predilections. Indeed, one could reject the freebit picture completely, and still see progress on these questions as a "rising tide that lifts all boats" in the scientific understanding of free will. The freebit picture seems unusual only in *forcing* us to address these questions.

3.5 Against Homunculi

A final clarification is in order about the freebit picture. One might worry that freebits are playing the role of a homunculus: the "secret core" of a person; a smaller person inside who directs the brain like a manager, puppeteer, or pilot; Ryle's ghost in the machine. But in philosophy and

[35] Or one could ask: if we model the brain as a chaotic dynamical system, what's the Lyapunov exponent?

[36] Another obvious question is whether brains *differ* in any interesting way from other complicated dynamical systems like lava lamps or the Earth's atmosphere, in terms of their response to microscopic fluctuations. This question will be taken up in Sections 5.2 and 5.3.

cognitive science, the notion of a homunculus has been rightly laughed off the stage. Like the theory that a clock can only work if there's a smaller clock inside, the homunculus theory blithely offers a black box where a *mechanism* is needed, and it leads to an obvious infinite regress: who's in charge of the homunculus?

Furthermore, if this were really the claim at issue, one would want to know: why do humans (and other animals) *have* such complicated brains in the first place? Why shouldn't our skulls be empty, save for tiny "freebit antennae" for picking up signals from the Big Bang?

But whatever other problems the freebit picture has, I think it's innocent of the charge of homunculism. On the freebit picture—as, I'd argue, on *any* sane understanding of the world—the physical activity of the brain retains its starring role in cognition. To whatever extent your "true self" has *any* definite location in spacetime, that location is in your brain. To whatever extent your behavior is predictable, that predictability ultimately derives from the predictability of your brain. And to whatever extent your choices have an author, *you* are their author, not some homunculus secretly calling the shots.

But if this is so, one might ask, then *what role could possibly be left for freebits*, besides the banal role of an unwanted noise source, randomly jostling neural firing patterns this way and that? Perhaps the freebit picture's central counterintuitive claim is that freebits *can* play a more robust role than that of glorified random-number generator, without usurping the brain's causal supremacy. Or more generally: *an organized, complex system can include a source of "pure Knightian unpredictability," which foils probabilistic forecasts made by outside observers, yet need not play any role in explaining the system's organization or complexity.* While I confess that this claim is strange, I fail to see any *logical* difficulty with it, nor do I see any way to escape it if the freebit picture is accepted.

To summarize, on the freebit picture, freebits are simply part of the explanation for how a brain can reach decisions that are not probabilistically predictable by outside observers, and that are therefore "free" in the sense that interests us. As such, on this picture freebits are just one ingredient among many in the physical substrate of the mind. I'd no more consider them the "true essence" of a person than the myelin coating that speeds transmission between neurons.

4 Freedom from the Inside Out

The discussion in Section 3.5 might remind us about the importance of stepping back. Setting aside any other concerns, *isn't it anti-scientific insanity to imagine that our choices today could correlate nontrivially with the universe's microstate at the Big Bang? Why shouldn't this idea just be thrown out immediately?*

In this section, I'll discuss an unusual perspective on time, causation, and boundary conditions: one that, *if* adopted, makes the idea of such a correlation seem not particularly crazy at all. The interesting point is that, despite its strangeness, this perspective seems perfectly compatible with everything we know about the laws of physics. The perspective is not new; it was previously suggested by Carl Hoefer [45] and independently by Cristi Stoica [85, 86]. (As Hoefer discusses, centuries before either of them, Kant appears to have made related observations in his *Critique of Practical Reason*, while trying to reconcile moral responsibility with free will! However, Kant's way of putting things strikes me as obscure and open to other interpretations.) Adopting Hoefer's terminology, I'll call the perspective "Freedom from the Inside Out," or FIO for short.

The FIO perspective starts from the familiar fact that the known equations of physics are *time-reversible*: any valid solution to the equations is still a valid solution if we replace t by $-t$.[37] Hence, there seems to be no particular reason to imagine time as "flowing" from past to future. Instead, we might as well adopt what philosophers call the "Block Universe" picture, where the whole 4-dimensional spacetime manifold is laid out at once as a frozen block.[38] Time is simply one more coordinate parameterizing that manifold, along with the spatial coordinates x, y, z. The equations *do* treat the t coordinate differently from the other three, but not in any way that seems to justify the intuitive notion of t "flowing" in a particular direction, any more than the x, y, z coordinates "flow" in a particular direction. Of course, with the discovery of special relativity, we learned that the choice of t coordinate is no more unique than the choice of x, y, z coordinates; indeed, an event in a faraway galaxy that you judge as years in the future, might well be judged as years in the past by someone walking past you on the street. To many philosophers, this seems to make the argument for a Block-Universe picture even stronger than it had been in Newtonian physics.[39]

The Block-Universe picture is sometimes described as "leaving no room for free will"—but that misses the much more important point, that the picture also leaves no room for *causation*! If we adopt this mentality consistently, then *it's every bit as meaningless to say that "Jack and Jill went up the hill because of the prior microstate of the universe," as it is to say that "Jack and Jill went up the hill because they wanted to."* Indeed, we might as well say that Jack and Jill went up the hill because of the *future* microstate of the universe! Or rather, the concept of "because" plays no role in this picture: there are the differential equations of physics, and the actual history of spacetime is one particular solution to those equations, and that's that.

Now, the idea of "Freedom from the Inside Out" is simply to embrace the Block-Universe picture of physics, then turn it on its head. We say: if this frozen spacetime block has no "intrinsic" causal arrows anyway, then *why not annotate it with causal arrows ourselves*, in whichever ways seem most compelling and consistent to us? And at least *a priori*, who's to say that some of the causal arrows we draw can't point "backwards" in time—for example, from human mental states and decisions to "earlier" microstates consistent with them? Thus, we might decide that yesterday your brain absorbed photons in certain quantum states *because* today you were going to eat tuna casserole, and running the differential equations backward from the tuna casserole produced the photons. In strict Block-Universe terms, this seems *absolutely* no worse than saying that you ate tuna casserole today because of the state of the universe yesterday.

I'll let Hoefer explain further:

> The idea of freedom from the inside out is this: we are perfectly justified in viewing our own actions *not* as determined by the past, *nor* as determined by the future, but rather as simply determined (to the extent that this word sensibly applies) *by ourselves, by our own wills*. In other words, they need not be viewed as *caused* or *explained* by the physical states of other, vast regions of the block universe. Instead, we can view our own actions, *qua* physical events, as primary explainers, determining—in a very partial

[37] One also needs to interchange left with right and particles with antiparticles, but that doesn't affect the substance of the argument.

[38] Of course, if the physical laws were probabilistic, then we'd have a probability distribution over possible blocks. This doesn't change anything in the ensuing discussion.

[39] As Einstein himself famously wrote to Michele Besso's family in 1955: "Now Besso has departed from this strange world a little ahead of me. That means nothing. People like us, who believe in physics, know that the distinction between past, present and future is only a stubbornly persistent illusion."

> way—physical events outside ourselves to the past and future of our actions, in the block. We adopt the perspective that the determination or explanation that matters is from the *inside* (of the block universe, where we live) *outward*, rather than from the *outside* (e.g. the state of things on a time slice 1 billion years ago) *in*. And we adopt the perspective of downward causation, thinking of our choices and intentions as primary explainers of our physical actions, rather than letting microstates of the world usurp this role. We are free to adopt these perspectives because, quite simply, physics—including [a] postulated, perfected deterministic physics—is perfectly compatible with them. [45, p. 207-208, emphases in original]

Some readers will immediately object as follows:

> Yes, "causality" in the Block Universe might indeed be a subtle, emergent concept. But doesn't the FIO picture take that observation to a ludicrous extreme, by turning causality into a free-for-all? For example, why couldn't an FIO believer declare that the dinosaurs went extinct 65 million years ago *because* if they hadn't, today I might not have decided to go bowling?

The reply to this objection is interesting. To explain it, we first need to ask: if the notion of "causality" appears nowhere in fundamental physics, then why does it *look* like past events constantly cause future ones, and never the reverse? Since the late 19^{th} century, physicists have had a profound answer to that question, or at least a reduction of the question to a different question.

The answer goes something like this: causality is an "emergent phenomenon," associated with the Second Law of Thermodynamics. Though no one really knows why, the universe was in a vastly "improbable," low-entropy state at the Big Bang—which means, for well-understood statistical reasons, that the further in time we move *away* from the Big Bang, the greater the entropy we'll find. Now, the creation of reliable memories and records is essentially always associated with an increase in entropy (some would argue by definition). And in order for us, as observers, to speak sensibly about "A causing B," we must be able to create records of A happening *and then* B happening. But by the above, this will essentially never be possible unless A is closer in time than B to the Big Bang.

We're now ready to see how the FIO picture evades the unacceptable conclusion of a causal free-for-all. It does so by *agreeing* with the usual account of causality based on entropy increase, in all situations where the usual account is relevant. While Hoefer [45] and Stoica [85, 86] are not explicit about this point, I would say that on the FIO picture, it can only make sense to draw causal arrows "backwards in time," in those rare situations where entropy is *not* increasing with time.

What are those situations? To a first approximation, they're the situations where physical systems are allowed to evolve reversibly, free from contact with their external environments. In practice, such perfectly-isolated systems will almost always be microscopic, and the reversible equation relevant to them will just be the Schrödinger equation. One example of such a system would be a photon of cosmic background radiation, which was emitted in the early universe and has been propagating undisturbed ever since. But these are precisely the sorts of systems that I'd consider candidates for "freebits"! As far as I can see, if we want the FIO picture not to lead to absurdity, then we can entertain the possibility of backwards-in-time causal arrows *only* for these systems. For this is where the "normal" way of drawing causal arrows breaks down, and we have nothing else to guide us.

4.1 The Harmonization Problem

There's another potential problem with the FIO perspective—Hoefer [45] calls it the "Harmonization Problem"—that is so glaring that it needs to be dealt with immediately. The problem is this: once we let certain causal arrows point backward in time, say from events today to the microstate at the Big Bang, we've set up what a computer scientist would recognize as a giant *constraint satisfaction problem*. Finding a "solution" to the differential equations of physics is no longer just a matter of starting from the initial conditions and evolving them forward in time. Instead, we can now also have constraints involving *later* times—for example, that a person makes a particular choice—from which we're supposed to propagate backward. But if this is so, then *why should a globally-consistent solution even exist, in general?* In particular, what happens if there are *cycles* in the network of causal arrows? In such a case, we could easily face the classic grandfather paradox of time travel to the past: for example, if an event A causes another event B in its future, but B also causes $\text{not}(A)$ in its past—which in turn causes $\text{not}(B)$, which in turn causes A. Furthermore, even if globally-consistent solutions *do* exist, one is tempted to ask what "algorithm" Nature uses to build a solution up. Should we imagine that Nature starts at the initial conditions and propagates them forward, but then "backtracks" if a contradiction is found, adjusting the initial conditions slightly in an attempt to avoid the contradiction? What if it gets into an infinite loop this way?

With hindsight, we can see the discussion from Section 2.5 about brain-uploading, teleportation, Newcomb's Paradox, and so on as highlighting the same concerns about the consistency of spacetime—indeed, as using speculative future technologies to make those concerns more vivid. Suppose, for example, that your brain has been scanned, and a complete physical description of it (sufficient to predict all your future actions) has been stored on a computer on Pluto. And suppose you then make a decision. Then from an FIO perspective, the question is: can we take your decision to explain, not only the state of your brain at the moment of your choice, and not only various microstates in your past lightcone[40] (insofar as they need to be compatible with your choice), but *also what's stored in the computer memory billions of kilometers away*? Should we say that you and the computer now make your decisions in synchrony, whatever violence that might seem to do to the locality of spacetime? Or should we say that the very act of copying your brain state removed your freedom where previously you had it, or proved that you were never free in the first place?

Fortunately, it turns out that in the freebit picture, *none of these problems arise*. The freebit picture might be rejected for other reasons, but it can't be charged with logical inconsistency or with leading to closed timelike curves. The basic observation is simply that we have to distinguish between what I'll call *macrofacts* and *microfacts*. A macrofact is a fact about a "decohered, classical, macroscopic" property of a physical system S at a particular time. More precisely, a macrofact is a fact F that could, in principle, be learned by an external measuring device without disturbing the system S in any significant way. Note that, for F to count as a macrofact, it's not necessary that anyone has ever known F or will ever know F: only that F *could have been*

[40]Given a spacetime point x, the past lightcone of x is the set of points from which x can receive a signal, and the future lightcone of x is the set of points to which it can send one.

known, if a suitable measuring device had been around at the right place and the right time. So for example, there is a macrofact about whether or not a Stegosaurus kicked a particular rock 150 million years ago, even if no human will ever see the rock, and even if the relevant information can no longer be reconstructed, even in principle, from the quantum state of the entire solar system. There are also macrofacts constantly being created in the interiors of stars and in interstellar space.

By contrast, a microfact is a fact about an undecohered quantum state ρ: a fact that couldn't have been learned, even in principle, without the potential for altering ρ (if the measurement were performed in the "wrong" basis). For example, the polarization of some particular photon of the cosmic microwave background radiation is a microfact. A microfact might or might not concern a freebit, but the quantum state of a freebit is always a microfact.

Within the freebit picture, the solution to the "Harmonization Problem" is now simply to impose the following two rules.

(1) Backwards-in-time causal arrows can point only to microfacts, never to macrofacts.

(2) No microfact can do "double duty": if it is caused by a fact to its future, then it is not itself the cause of anything, nor is it caused by anything else.

Together, these rules readily imply that no cycles can arise, and more generally, that the "causality graph" never produces a contradiction. For whenever they hold, the causality graph with be a *directed acyclic graph* (a dag), with all arrows pointing forward in time, except for some "dangling" arrows pointing backward in time that never lead anywhere else.

Rule (1) is basically imposed by fiat: it just says that, for all the events we actually observe, we must seek their causes only to their past, never to their future. This rule might someday be subject to change (for example, if closed timelike curves were discovered), but for now, it seems like a pretty indispensable part of a scientific worldview. By contrast, rule (2) can be justified by appealing to the No-Cloning Theorem. If a microfact f is directly caused by a macrofact F to its future, then at some point a *measurement* must have occurred (or more generally, some decoherence event that we can *call* a "measurement"), in order to amplify f to macroscopic scale and correlate it with F. In the freebit picture, we think of the correlation with F as completely fixing f, which explains why f can't also be caused by anything other than F. But why can't f, itself, cause some macrofact F' (which, by rule (1), would need to be to f's future)? Here there are two cases: $F' = F$ or $F' \neq F$. If $F' = F$, then we've simply created a "harmless" 2-element cycle, where f and F cause each other. It's purely by convention that we disallow such cycles, and declare that F causes f rather than the reverse. On the other hand, if $F' \neq F$, then we have two independent measurements of f to f's future, violating the No-Cloning Theorem.[41] Note that this argument wouldn't have worked if f had been a macrofact, since macroscopic information *can* be measured many times independently.

Subtle questions arise when we ask about the possibility of microfacts causing other microfacts. Rules (1) and (2) allow that sort of causation, as long as it takes place forward in time—or, if backward in time, that it consists of a single "dangling" arrow only. If we wanted, without causing any harm we could allow long chains (and even dags) of microfacts causing other microfacts backward in time, possibly originating at some macrofact to their future. We would need to be careful, though, that none of those microfacts ever caused any facts to their future, since that would

[41] For the same reason, we also have the rule that a microfact cannot cause two macrofacts to its future via disjoint causal pathways. The only reason this rule wasn't mentioned earlier is that it plays no role in eliminating cycles.

create the possibility of cycles. A simpler option is just to declare the entire concept of causality irrelevant to the microworld. On that view, whenever a microfact f "causes" another microfact f', unitarity makes it just as legitimate to say that f' causes f, or that neither causes the other. Because of the reversibility of microscopic laws, the temporal order of f and f' is irrelevant: if $U|\psi\rangle = |\varphi\rangle$, then $U^\dagger|\varphi\rangle = |\psi\rangle$. This view would regard causality as inextricably bound up with the thermodynamic arrow of time, and therefore with *ir*reversible processes, and therefore with macrofacts.

4.2 Microfacts and Macrofacts

An obvious objection to the distinction between microfacts and macrofacts is that it's poorly-defined. The position of a rock might be "obviously" a macrofact, and the polarization of a photon "obviously" a microfact, but there is a continuous transition between the two. Exactly how decohered and classical does a fact have to be, before it counts as a "macrofact" for our purposes? This, of course, is reminiscent of the traditional objection to Bohr's Copenhagen Interpretation of quantum mechanics, and in particular, to its unexplained yet crucial distinction between the "microworld" of quantum mechanics and the "macroworld" of classical observations.

Here, my response is basically to admit ignorance. The freebit picture is not particularly sensitive to the precise *way* we distinguish between microfacts and macrofacts. But if the picture is to make sense, it does require that there *exist* a consistent way to make this distinction. (Or at least, it requires that there exist a consistent way to *quantify* the macro-ness of a fact f. The degree of macro-ness of f might then correspond to the "effort of will" needed to affect f retrocausally, with the effort becoming essentially infinite for ordinary macroscopic facts!)

One obvious way to enforce a macro/micro distinction would be via a *dynamical collapse theory*, such as those of Ghirardi, Rimini, and Weber [41] or Penrose [68]. In these theories, all quantum states periodically undergo "spontaneous collapse" to some classical basis, at a rate that grows with their mass or some other parameter, and in such a way that the probability of spontaneous collapse is close to 0 for all quantum systems that have so far been studied, but close to 1 for ordinary "classical" systems. Unfortunately, the known dynamical-collapse theories tend to suffer from technical problems, such as small violations of conservation of energy, and of course there is as yet no experimental evidence for them. More fundamentally, I personally cannot believe that Nature would solve the problem of the "transition between microfacts and macrofacts" in such a seemingly ad hoc way, a way that does so much violence to the clean rules of linear quantum mechanics.

As I'll discuss further in Section 5.5, my own hope is that a principled distinction between microfacts and macrofacts could ultimately emerge from cosmology. In particular, I'm partial to the idea that, in a deSitter cosmology like our own, *a "macrofact" is simply any fact of which the news is already propagating outward at the speed of light*, so that the information can never, even in principle, be gathered together again in order to "uncause" the fact. A microfact would then be any fact for which this propagation *hasn't* yet happened. The advantage of this distinction is that it doesn't involve the slightest change to the principles of quantum mechanics. The disadvantage is that the distinction is "teleological": the line between microfacts and macrofacts is defined by what *could* happen arbitrarily far in the future. In particular, this distinction implies that if, hypothetically, we could surround the solar system by a perfect reflecting boundary, then within the solar system, there would no longer be *any* macrofacts! It also implies that there can be no macrofacts in an *anti*-deSitter (adS) cosmology, which *does* have such a reflecting boundary.

Finally, it suggests that there can probably be few if any macrofacts inside the event horizon of a black hole. For even if the information in the black hole interior *eventually* emerges, in coded form, in the Hawking radiation, the Hawking evaporation process is so slow ($\sim 10^{67}$ years for a solar-mass black hole) that there would seem to be plenty of time for an observer outside the hole to gather most of the radiation, and thereby prevent the information from spreading further.[42]

Because I can't see a better alternative—and also, because I rather like the idea of cosmology playing a role in the foundations of quantum mechanics!—my current inclination is to bite the bullet, and accept these and other implications of the macro/micro distinction I've suggested. But there's enormous scope here for better ideas (or, of course, new developments in physics) to change my thinking.

5 Further Objections

In this section, I'll present five objections to the freebit picture, together with my responses. Some of these objections are obvious, and are generic to *any* analysis of "freedom" that puts significant stock in the actual unpredictability of human choices. Others are less obvious, and are more specific to the freebit picture.

5.1 The Advertiser Objection

The first objection is simply that human beings are depressingly predictable in practice: if they weren't, then they wouldn't be so easily manipulable! So, does surveying the sorry history of humankind—in which most people, most of the time, did exactly the boring, narrowly self-interested things one might have expected them to do—furnish any evidence at all for the *existence* of freebits?

Response. I already addressed a related objection in Section 2.12, but this one seems so important that it's worth returning to it from a different angle.

It's obvious that humans are at least partly predictable—and sometimes *extremely* predictable, depending on what one is trying to predict. For example, it's vanishingly unlikely that tomorrow, the CEO of General Motors will show up to work naked, or that Noam Chomsky will announce his support for American militarism. On the other hand, that doesn't mean we know how to program a computer to *simulate* these people, anticipating every major or minor decision they make throughout their lives! It seems crucial to maintain the distinction between the partial predictability that even the most vocal free-will advocate would concede, and the "physics-like" predictability of a comet. To illustrate, imagine a machine that correctly predicted *most* decisions of *most* people: what they'll order for dinner, which movies they'll select, which socks they'll take out of the drawer, and so on. (In a few domains, this goal is already being approximated by recommendation systems, such as those of Amazon and Netflix.) But imagine that the machine was regularly blindsided by the most *interesting, consequential, life-altering* decisions. In that case, I suspect most people's intuitions about their own "freedom" would be shaken slightly, but would basically remain intact. By analogy, for most computer programs that arise in practice, it's easy to decide whether they halt, but that hardly decreases the importance of the *general* unsolvability of the halting problem. Perhaps, as Kane [36] speculates, we truly exercise freedom only for a relatively small number of "self-forming actions" (SFAs)—that is, actions that help to *define* who

[42] To be more precise here, one would presumably need to know the detailed *mapping* between the qubits of Hawking radiation and the degrees of freedom inside the hole, which in turn would require a quantum theory of gravity.

we are—and the rest of the time are essentially "running on autopilot." Perhaps these SFAs are common in childhood and early adulthood, but become rare later in life, as we get set in our ways and correspondingly more predictable.

Having said this, I concede that the intuition in favor of humans' predictability is a powerful one. Indeed, even supposing humans *did* have the capacity for Knightian freedom, one could argue that that capacity can't be particularly important, if almost all humans choose to "run on autopilot" almost all of the time!

However, against the undeniable fact of humans so often being manipulable like lab rats, there's a *second* undeniable fact, which should be placed on the other side of the intuitive ledger. This second fact is the conspicuous failure of investors, pundits, intelligence analysts, and so on *actually to predict*, with any reliability, what individuals or even entire populations will do. Again and again the best forecasters are blindsided (though it must be admitted that *after the fact*, forecasters typically excel at explaining the inevitability of whatever people decided to do!).

5.2 The Weather Objection

What's special about the human brain? If we want to describe it as having "Knightian freedom," then why not countless *other* complicated dynamical systems, such as the Earth's weather, or a lava lamp?

Response. For systems like the weather or a lava lamp, I think a plausible answer can actually be given. These systems are indeed unpredictable, but the unpredictability seems much more probabilistic than Knightian in character. To put it another way, the famous "butterfly effect" seems likely to be an artifact of *deterministic* models of those systems; one expects it to get washed out as soon as we model the systems' microscopic components probabilistically. To illustrate, imagine that the positions and momenta of all the molecules in the Earth's atmosphere had been measured to roughly the maximum precision allowed by quantum mechanics; and that, on the basis of those measurements, a supercomputer had predicted a 23% probability of a thunderstorm in Shanghai at a specific date next year. Now suppose we changed the initial conditions by adding a single butterfly. In classical, deterministic physics, that could certainly change whether the storm happens, due to chaotic effects—but that isn't the relevant question. The question is: how much does adding the butterfly change the *probability* of the storm? The answer seems likely to be "hardly at all." After all, the original 23% was already obtained by averaging over a huge number of possible histories. So unless the butterfly somehow changes the *general features* of the statistical ensemble, its effects should be washed out by the unmeasured randomness in the millions of *other* butterflies (and whatever else is in the atmosphere).

Yet with brains, the situation seems plausibly different. For brains seem "balanced on a knife-edge" between order and chaos: were they as orderly as a pendulum, they couldn't support interesting behavior; were they as chaotic as the weather, they couldn't support rationality. More concretely, a brain is composed of neurons, each of which (in the crudest idealization) has a firing rate dependent on whether or not the sum of signals from incoming neurons exceeds some threshold. As such, one expects there to be many molecular-level changes one could make to a brain's state that don't affect the overall firing pattern at all, but a few changes—for example, those that push a critical neuron "just over the edge" to firing or not firing—that affect the overall firing pattern drastically. So for a brain—unlike for the weather—a single freebit *could* plausibly influence the probability of some macroscopic outcome, even if we model all of the system's constituents quantum-mechanically.

A closely-related difference between brains and the weather is that, while both are presumably chaotic systems able to amplify tiny effects, only in the case of brains are the amplifications likely to have "irreversible" consequences. Even if a butterfly flapping its wings can cause a thunderstorm halfway around the world, a butterfly almost certainly won't change the average *frequency* of thunderstorms—at least, not without changing something other than the weather as an intermediary. To change the frequency of thunderstorms, one needs to change the trajectory of the earth's *climate*, something less associated with butterflies than with macroscopic "forcings" (for example, increasing the level of carbon dioxide in the atmosphere, or hitting the earth with an asteroid). With brains, by contrast, it seems perfectly plausible that a single neuron's firing or not firing could affect the rest of a person's life (for example, if it caused the person to make a *very* bad decision).

5.3 The Gerbil Objection

Even if it's accepted that brains are very different from lava lamps or the weather considered purely as dynamical systems, one could attempt a different *reductio ad absurdum* of the freebit picture, by constructing a physical system that *behaves* almost exactly like a brain, yet whose Knightian uncertainty is "decoupled" from its intelligence in what seems like an absurd way. Thus, consider the following thought experiment: on one side of a room, we have a digital computer, whose internal operations are completely deterministic. The computer is running an AI program that easily passes the Turing test: many humans, let's say, have maintained long Internet correspondences with the AI on diverse subjects, and not one ever suspected its consciousness or humanity. On the other side of the room, we have a gerbil in a box. The gerbil scurries in its box, in a way that we can imagine to be subject to at least some Knightian uncertainty. Meanwhile, an array of motion sensors regularly captures information about the gerbil's movements and transmits it across the room to the computer, which uses the information as a source of random bits for the AI. Being extremely sophisticated, of course the AI doesn't make *all* its decisions randomly. But if it can't decide between two roughly-equal alternatives, then it sometimes uses the gerbil movements to break a tie, much as an indecisive human might flip a coin in a similar situation.

The problem should now be obvious. By assumption, we have a system that *acts* with human-level intelligence (i.e., it passes the Turing test), and that's *also* subject to Knightian uncertainty, arising from amplified quantum fluctuations in a mammalian nervous system. So if we believe that humans have a "capacity for freedom" because of those two qualities, then we seem *obligated* to believe that the AI/gerbil hybrid has that capacity as well. Yet if we simply disconnect the computer from the gerbil box, then the AI loses its "capacity for freedom"! For then the AI's responses, though they might still *seem* intelligent, could be "unmasked as mechanistic" by anyone who possessed a copy of the AI's program. Indeed, even if we replaced the gerbil box by an "ordinary" quantum-mechanical random number generator, the AI's responses would still be *probabilistically* predictable; they would no longer involve Knightian uncertainty.

Thus, a believer in the freebit picture seems forced to an insane conclusion: that the gerbil, though presumably oblivious to its role, is like a magic amulet that gives the AI a "capacity for freedom" it wouldn't have had otherwise. Indeed, the gerbil seems uncomfortably close to the soul-giving potions of countless children's stories (stories that always end with the main character realizing that she *already had a soul*, even without the potion!). Yet, if we reject this sort of thinking in the gerbil-box scenario, then why shouldn't we also reject it for the human brain? With the brain, it's true, the "Knightian-indeterminism-providing gerbil" has been moved physically closer to the locus of thought: now it scurries around in the synaptic junctions and neurons, rather than in a box on the other side of the room. But why should proximity make a difference? *Wherever* we put the gerbil, it just scurries around aimlessly! Maybe the scurrying is probabilistic, maybe it's "Knightian," but either way, it clearly plays no more *explanatory* role in intelligent decision-making than the writing on the Golem's forehead.

In summary, it seems the only way to rescue the freebit picture is to paper over an immense causal gap—between the brain's "Knightian noise" and its cognitive information processing—with superstition and magic.

Response. Of all the arguments directed specifically against the freebit picture, this one strikes me as the most serious, which is why I tried to present it in a way that would show its intuitive force.

On reflection, however, there *is* at least one potentially-important difference between the AI/gerbil system and the brain. In the AI/gerbil system, the "intelligence" and "Knightian noise" components were *cleanly separable* from one another. That is, the computer *could* easily be disconnected from the gerbil box, and reattached to a *different* gerbil box, or an ordinary random-number generator, or nothing at all. And after this was done, there's a clear sense in which the AI would still be running "the exact same program": only the "indeterminism-generating peripheral" would have been swapped out. For this reason, it seems best to think of the gerbil as simply *yet another part of the AI's external environment*—along (for example) with all the questions sent to the AI over the Internet, which could *also* be used as a source of Knightian indeterminism.

With the brain, by contrast, it's not nearly so obvious that the "Knightian indeterminism source" *can* be physically swapped out for a different one, without destroying or radically altering the brain's *cognitive* functions as well. Yes, we can imagine futuristic nanorobots swarming through a brain, recording all the "macroscopically measurable information" about the connections between neurons and the strengths of synapses, then building a new brain that was "macroscopically identical" to the first, differing only in its patterns of microscopic noise. But how would we know whether the robots had recorded *enough* information about the original brain? What if, in addition to synaptic strengths, there was also cognitively-relevant information at a smaller scale? Then we'd need more advanced nanorobots, able to distinguish even smaller features. Ultimately, we could imagine robots able to record *all* "classical" or even "quasi-classical" features. But by definition, the robots would then be measuring features that were *somewhat* quantum-mechanical, and therefore inevitably changing those features.

Of course, this is hardly a conclusive argument, since maybe there's a gap of many orders of magnitude between (a) the smallest possible scale of cognitive relevance, and (b) the scale where the No-Cloning Theorem becomes relevant. In that case, at least in principle, the nanorobots really *could* complete their scan of the brain's "cognitive layer" without risking the slightest damage to it; only the (easily-replaced?) "Knightian indeterminism layer" would be altered by the nanorobots' presence. Whether this is possible is an empirical question for neuroscience.

However, the discussion of brain-scanning raises a broader point: that, against the gerbil-box scenario, we need to weigh some other, older thought experiments where many people's intuitions go the other way. Suppose the nanorobots *do* eventually complete their scan of all the "macroscopic, cognitively-relevant" information in *your* brain, and suppose they then transfer the information to a digital computer, which proceeds to run a macroscopic-scale simulation of your brain. Would that simulation *be* you? If your "original" brain were destroyed in this process, or simply anesthetized, would you expect to wake up as the digital version? (Arguably, this is not even a philosophical question, just a straightforward empirical question asking you to predict a future observation!)

Now, suppose you believe there's some conceivable digital doppelgänger that would *not* "be you," and you also believe that the difference between you and it resides somewhere in the physical world. Then since (by assumption) the doppelgänger is functionally indistinguishable from you, it would seem to follow that the difference between you and it *must* reside in "functionally-irrelevant" degrees of freedom, such as microscopic ones. Either that, or else the boundary between the "functional" and "non-functional" degrees of freedom is not even sharp enough for the doppelgängers to be created in the first place.

My conclusion is that *either* you can be uploaded, copied, simulated, backed up, and so forth, leading to all the puzzles of personal identity discussed in Section 2.5, or else you *can't* bear the same sort of "uninteresting" relationship to the "non-functional" degrees of freedom in your brain that the AI bore to the gerbil box.

To be clear, nothing I've said even hints at any *sufficient condition* on a physical system, in order for the system to have attributes such as free will or consciousness. That is, even if human brains were subject to Knightian noise at the microscopic scale, and even if the sources of such noise could *not* be cleanly separated from the cognitive functions, human beings might still fail to be "truly" conscious or free—whatever those things mean!—for other reasons. At most, I'm investigating plausible *necessary* conditions for Knightian freedom as defined in this essay (and hence, in my personal perspective, for "free will" also).

5.4 The Initial-State Objection

The fourth objection holds that the notion of "freebits" from the early universe nontrivially influencing present-day events is not merely strange, but inconsistent with known physics. More concretely, Stenger [84] has claimed that it *follows* from known physics that the initial state at the Big Bang was essentially random, and can't have encoded any "interesting" information.[43] His argument is basically that the temperature at the Big Bang was enormous; and in quantum field theory (neglecting gravity), such extreme temperatures seem manifestly incompatible with any nontrivial structure.

Response. Regardless of whether Stenger's *conclusion* holds, today there are strong indications, from cosmology and quantum gravity, that something has to be wrong with the above *argument* for a thermal initial state.

First, by this argument, the universe's entropy should have been maximal at the Big Bang, but the Second Law tells us that the entropy was minimal! Stenger [84] notices the obvious problem, and tries to solve it by arguing that the entropy at the Big Bang really *was* maximal, given the tiny size of the observable universe at that time. On that view, the reason why entropy can increase as we move away from the Big Bang is simply that the universe is expanding, and with it the dimension of the state space. But others, such as Carroll [20] and Penrose [69], have pointed out severe problems with that answer. For one thing, if the dimension of the state space can increase, then we give up on *reversibility*, a central feature of quantum mechanics. For another, this answer has the unpalatable implication that the entropy should turn around and decrease in a hypothetical Big Crunch. The alternative, which Carroll and Penrose favor, is to hold that *despite* the enormous temperature at the Big Bang, the universe's state then was every bit as "special" and low-entropy as the Second Law demands, but its specialness must have resided in *gravitational*

[43] Here Stenger's concern was not free will or human predictability, but rather ruling out the possibility (discussed by some theologians) that God could have arranged the Big Bang with foreknowledge about life on Earth.

degrees of freedom that we don't yet fully understand.

The second indication that the "thermal Big Bang" picture is incomplete is that *quantum field theory has misled us in a similar way before.* In 1975, Hawking [42] famously used quantum field theory in curved spacetime to calculate the temperature of black holes, and to prove the existence of the Hawking radiation by which black holes eventually lose their mass and disappear. However, Hawking's calculation seemed to imply that the radiation was thermal—so that in particular, it couldn't encode any nontrivial information about objects that fell into the black hole. This led to the *black-hole information-loss paradox*, since quantum-mechanical reversibility forbids the quantum states of the infalling objects simply to "disappear" in this way. Today, largely because of the *AdS/CFT correspondence* (see [15]), there's a near-consensus among experts that the quantum states of infalling objects *don't* disappear as Hawking's calculation suggested. Instead, at least from the perspective of someone outside the black hole, the infalling information should "stick around" on or near the event horizon, in not-quite-understood quantum-gravitational degrees of freedom, before finally emerging in garbled form in the Hawking radiation. And if quantum field theory says otherwise, that's because quantum field theory is only a limiting case of a quantum theory of gravity.

The AdS/CFT correspondence is just one realization of the *holographic principle*, which has emerged over the last two decades as a central feature of quantum theories of gravity. It's now known that many physical theories have both a D-dimensional "bulk" formulation and a $(D-1)$-dimensional "boundary" formulation. In general, these two formulations *look* completely different: states that are smooth and regular in one formulation might look random and garbled in the other; questions that are trivial to answer in one formulation might seem hopeless in the other.[44] Nevertheless, there exists an isomorphism between states and observables, by which the boundary formulation "holographically encodes" everything that happens in the bulk formulation. As a classic example, if Alice jumps into a black hole, then she might perceive herself falling smoothly toward the singularity. Meanwhile Bob, far from the black hole, might describe exactly the same physical situation using a "dual description" in which Alice never makes it past the event horizon, and instead her quantum information gets "pancaked" across the horizon in a horrendously complicated way. In other words, not only is not absurd to suppose that a "disorganized mess of entropy" on the boundary of a region could be "isomorphic" to a richly-structured state in the region's interior, but there are now detailed examples where that's exactly what happens.[45]

The bottom line is that, when discussing extreme situations like the Big Bang, it's *not okay* to ignore quantum-gravitational degrees of freedom simply because we don't yet know how to model them. And including those degrees of freedom seems to lead straight back to the unsurprising conclusion that *no one knows* what sorts of correlations might have been present in the universe's initial microstate.

[44] As the simplest example, the boundary formulation makes it obvious that the total entropy in a region should be upper-bounded by its *surface area*, rather than its volume. In the bulk formulation, that property is strange and unexpected.

[45] Admittedly, the known examples involve isomorphisms between two theories with different numbers of spatial dimensions but both with a time dimension. There don't seem to be any nontrivial examples where the boundary theory lives on an initial spacelike or null hypersurface of the bulk theory. (One could, of course, produce a trivial example, by simply defining the "boundary theory" to consist of the initial conditions of the bulk theory, with no time evolution! By "nontrivial," I mean something more interesting than that.)

5.5 The Wigner's-Friend Objection

A final objection comes from the Many-Worlds Interpretation of quantum mechanics—or more precisely, from taking seriously the possibility that a conscious observer could be measured in a coherent superposition of states.

Let's start with the following thought experiment, called *Wigner's friend* (after Eugene Wigner, who wrote about it in 1962 [92]). An intelligent agent A is placed in a coherent superposition of two different mental states, like so:

$$|\Phi\rangle = \frac{|1\rangle |A_1\rangle + |2\rangle |A_2\rangle}{\sqrt{2}}.$$

We imagine that these two states correspond to two different questions that the agent could be asked: for example, $|1\rangle |A_1\rangle$ represents A being asked its favorite color, while $|2\rangle |A_2\rangle$ denotes A being asked its favorite ice cream flavor. Then, crucially, a second agent B comes along and measures $|\Phi\rangle$ in the basis

$$\left\{ \frac{|1\rangle |A_1\rangle + |2\rangle |A_2\rangle}{\sqrt{2}}, \frac{|1\rangle |A_1\rangle - |2\rangle |A_2\rangle}{\sqrt{2}} \right\}, \tag{1}$$

in order to verify that A really *was* in a superposition of two mental states, not just in one state or the other.

Now, there are many puzzling questions one can ask about this scenario: most obviously, "what is it like" for A to be manipulated in this way? If A perceives itself in a definite state—either A_1 *or* A_2, but not both—then will B's later measurement of A in the basis (1) appear to A as a violation of the laws of physics?

However, let's pass over this well-trodden ground, and ask a different question more specific to the freebit picture. According to that picture, A's "free decision" of how to answer whichever question it was asked should be correlated with one or more freebits w. But if we write out the combined state of the superposed A and the freebits,

$$\frac{|1\rangle |A_1\rangle + |2\rangle |A_2\rangle}{\sqrt{2}} \otimes |w\rangle ,$$

then a problem becomes apparent. Namely, the same freebits w need to do "double duty," correlating with A's decision in both the $|1\rangle$ branch and the $|2\rangle$ branch! In other words, even supposing microscopic details of the environment could somehow "explain" what happens in one branch, how could the *same* details explain both branches? As A_1 contemplated its favorite color, would it find itself oddly constrained by A_2's simultaneous contemplations of its favorite ice cream flavor, or vice versa?

One might think we could solve this problem by stipulating that w is split into two collections of freebits, $|w\rangle = |w_1\rangle \otimes |w_2\rangle$, with $|w_1\rangle$ corresponding to A_1's response and $|w_2\rangle$ corresponding to A_2's. But this solution quickly runs into an exponential explosion: if we considered a state like

$$\frac{1}{2^{500}} \sum_{x \in \{0,1\}^{1000}} |x\rangle |A_x\rangle ,$$

we would find we needed 2^{1000} freebits to allow each A_x to make a yes/no decision independently of all the other A_x's.

Another "obvious" way out would be if the freebits were entangled with A:

$$\left|\Phi'\right\rangle = \frac{|1\rangle |A_1\rangle |w_1\rangle + |2\rangle |A_2\rangle |w_2\rangle}{\sqrt{2}}.$$

The problem is that there seems to be no way to *produce* such entanglement, without violating quantum mechanics. If w is supposed to represent microscopic details of A's environment, ultimately traceable to the early universe, then it would be extremely mysterious to find A and w entangled. Indeed, in a Wigner's-friend experiment, such entanglement would show up as a *fundamental decoherence source*: something which was *not* traceable to any "leakage" of quantum information from A, yet which somehow prevented B from observing quantum interference between the $|1\rangle |A_1\rangle$ and $|2\rangle |A_2\rangle$ branches, when B measured in the basis (1). If, on the other hand, B *did* ultimately trace the entanglement between A and w to a "leakage" of information from A, then w would have been revealed to have never contained freebits at all! For in that case, w would "merely" be the result of unitary evolution coupling A to its environment—so it could presumably be predicted by B, who could even verify its hypothesis by measuring the state $|\Phi'\rangle$ in the basis

$$\left\{ \frac{|1\rangle |A_1\rangle |w_1\rangle + |2\rangle |A_2\rangle |w_2\rangle}{\sqrt{2}}, \frac{|1\rangle |A_1\rangle |w_1\rangle - |2\rangle |A_2\rangle |w_2\rangle}{\sqrt{2}} \right\}.$$

Response. As in Section 4.2—where asking about the definition of "microfacts" and "macrofacts" led to a closely-related issue—my response to this important objection is to *bite the bullet*. That is, I accept the existence of a deep incompatibility between the freebit picture and the physical feasibility of the Wigner's-friend experiment. To state it differently: *if* the freebit picture is correct, *and* the Wigner's-friend experiment can be carried out, then I think we're forced to conclude that—at least for the duration of the experiment—*the subject no longer has the "capacity for Knightian freedom,"* and is now a "mechanistic," externally-characterized physical system similar to a large quantum computer.

I realize that the position above sounds crazy, but it becomes less so once one starts thinking about what would *actually* be involved in performing a Wigner's-friend experiment on a human subject. Because of the immense couplings between a biological brain and its external environment (see Tegmark [88] for example), the experiment is likely impossible with any entity we would currently recognize as "human." Instead, as a first step (!), one would presumably need to solve the problem of *brain-uploading*: that is, transferring a human brain into a digital substrate. Only then could one even *attempt* the second part, of transferring the now-digitized brain onto a quantum computer, and preparing and measuring it in a superposition of mental states. I submit that, while the resulting entity *might* be "freely-willed," "conscious," etc., it certainly wouldn't be uncontroversially so, nor can we form any intuition by reasoning by analogy with ourselves (even if we were inclined to try).

Notice in particular that, if the agent A could be manipulated in superposition, then as a direct byproduct of those manipulations, A would presumably undergo *the same mental processes over and over, forwards in time as well as backwards in time*. For example, the "obvious" way for B to measure A in the basis (1), would simply be for B to "uncompute" whatever unitary transformation U had placed A in the superposed state $|\Phi\rangle$ in the first place. Presumably, the process would then be repeated many times, as B accumulated more statistical evidence for the quantum interference pattern. So, during the uncomputation steps, would A "experience time running backwards"? Would the inverse map $U^\dagger$ feel different than U? Or would all the applications of U and $U^\dagger$

be "experienced simultaneously," being functionally indistinguishable from one another? I hope I'm not alone in feeling a sense of vertigo about these questions! To me, it's at least a plausible speculation that A doesn't experience *anything*, and that the reasons why it doesn't are related to B's very ability to manipulate A in these ways.

More broadly, the view I've taken here on superposed agents strikes me as almost a *consequence* of the view I took earlier, on agents whose mental states can be perfectly measured, copied, simulated, and predicted by other agents. For there's a close connection between being able to *measure* the exact state $|S\rangle$ of a physical system, and being able to detect quantum interference in a superposition of the form

$$|\psi\rangle = \frac{|S_1\rangle + |S_2\rangle}{\sqrt{2}},$$

consisting of two "slight variants" of $|S\rangle$. If we know $|S\rangle$, then among other things, we can load a copy of $|S\rangle$ onto a quantum computer, and thereby prepare and measure a superposition like $|\psi\rangle$—provided, of course, that one counts the quantum computer's *encodings* as $|S_1\rangle$ and $|S_2\rangle$ as "just as good as the real thing." Conversely, the ability to detect interference between $|S_1\rangle$ and $|S_2\rangle$ presupposes that we know, and can control, *all* the degrees of freedom that make them different: quantum mechanics tells us that, if any degrees of freedom are left unaccounted for, then we will simply see a probabilistic mixture of $|S_1\rangle$ and $|S_2\rangle$, not a superposition.

But if this is so, one might ask, then *what makes humans any different?* According to the most literal reading of quantum mechanics' unitary evolution rule—which some call the Many-Worlds Interpretation—don't we *all* exist in superpositions of enormous numbers of branches, and isn't our inability to measure the interference between those branches merely a "practical" problem, caused by rapid decoherence? Here I reiterate the speculation put forward in Section 4.2: that the decoherence of a state $|\psi\rangle$ should be considered "fundamental" and "irreversible," precisely when $|\psi\rangle$ becomes entangled with degrees of freedom that are receding toward our deSitter horizon at the speed of light, and that can no longer be collected together even in principle. That sort of decoherence could be avoided, at least in principle, by a fault-tolerant quantum computer, as in the Wigner's-friend thought experiment above. But it plausibly *can't* be avoided by any entity that we would currently recognize as "human."

One could also ask: *if* the freebit picture were accepted, what would be the implications for the foundations of quantum mechanics? For example, would the Many-Worlds Interpretation then have to be rejected? Interestingly, I think the answer is no. Since I haven't suggested any change to the formal rules of quantum mechanics, *any* interpretations that accept those rules—including Many-Worlds, Bohmian mechanics, and various Bayesian/subjectivist interpretations—would in some sense "remain on the table" (to whatever extent they were on the table before!). As far as we're concerned in this essay, if one wants to believe that different branches of the wavefunction, in which one's life followed a different course than what one observes, "really exist," that's fine; if one wants to deny the reality of those branches, that's fine as well. Indeed, if the freebit picture were correct, then *Nature would have conspired so that we had no hope, even in principle, of distinguishing the various interpretations by experiment.*

Admittedly, one might think it's obvious that the interpretations can't be distinguished by experiment, with or without the freebit picture. Isn't that why they're *called* "interpretations"? But as Deutsch [29] already pointed out in 1985, scenarios like Wigner's friend seriously challenge the idea that the interpretations are empirically equivalent. For example, if one could perform a quantum interference experiment on *one's own mental state*, then couldn't one directly experience what it was like for the different components of the wavefunction describing that state to evolve unitarily?[46] And wouldn't that, essentially, vindicate a Many-Worlds-like perspective, while ruling out the subjectivist views that refuse to countenance the reality of "parallel copies of oneself"? By denying that the subject of a Wigner's-friend experiment has the "capacity for Knightian freedom," the freebit picture suggests that maybe there's nothing that it's like to *be* such a subject—and hence, that debates about quantum interpretation can freely continue forever, with not even the in-principle possibility of an empirical resolution.

6 Comparison to Penrose's Views

Probably the most original thinker to have speculated about physics and mind in the last half-century has been Roger Penrose. In his books, including *The Emperor's New Mind* [67] and *Shadows of the Mind* [68],[47] Penrose has advanced three related ideas, all of them extremely controversial:

(1) That arguments related to Gödel's Incompleteness Theorem imply that the physical action of the human brain cannot be algorithmic (i.e., that it must violate the Church-Turing Thesis).

(2) That there must be an "objective" physical process that collapses quantum states and produces the definite world we experience, and that the best place to look for such a process is in the interface of quantum mechanics, general relativity, and cosmology (the "specialness" of the universe's initial state providing the only known source of time-asymmetry in physics, not counting quantum measurement).

(3) That objective collapse events, possibly taking place in the cytoskeletons of the brain's neurons (and subsequently amplified by "conventional" brain activity), provide the best candidate for the source of noncomputability demanded by (1).

An obvious question is how Penrose's views relate to the ones discussed here. Some people might see the freebit picture as "Penrose lite." For it embraces Penrose's core belief in a relationship between the mysteries of mind and those of modern physics, and even follows Penrose in focusing on certain *aspects* of physics, such as the "specialness" of the initial state. On the other hand, the account here rejects almost all of Penrose's further speculations: for example, about noncomputable dynamics in quantum gravity and the special role of the cytoskeleton in exploiting those dynamics.[48] Let me now elaborate on seven differences between my account and Penrose's.

[46] A crucial caveat is that, after the interference experiment was over, one would retain no reliable memories or publishable records about "what it was like"! For the very fact of such an experiment implies that one's memories are being created and destroyed at will. Without the destruction of memories, we can't get interference. After the experiment is finished, one might have *something* in one's memory, but what it is could have been probabilistically predicted even before the experiment began, and can in no way depend on "what it was like" in the middle. Still, *at least while the experiment was underway*, maybe one would know which interpretation of quantum mechanics was correct!

[47] A later book, *The Road to Reality* [69], says little directly about mind, but is my favorite. I think it makes Penrose's strongest case for a gap in our understanding of quantum mechanics, thermodynamics, and cosmology that radical new ideas will be needed to fill.

[48] Along another axis, though, some people might see the freebit picture as *more* radical, in that it suggests the impossibility of *any* non-tautological explanation for certain events and decisions, even an explanation invoking oracles for Turing-uncomputable problems.

(1) **I make no attempt to "explain consciousness."** Indeed, that very goal seems misguided to me, at least if "consciousness" is meant in the phenomenal sense rather than the neuroscientists' more restricted senses.[49] For as countless philosophers have pointed out over the years (see McGinn [62] for example), *all* scientific explanations seem equally compatible with a "zombie world," which fulfills the right causal relations but where no one "really" experiences anything. More concretely, even if Penrose were right that the human brain had "super-Turing" computational powers—and I see no reason to think he is—I've never understood how that would help with what Chalmers [22] calls the "hard problem" of consciousness. For example, could a Turing machine equipped with an oracle for the halting problem perceive the "redness of red" any better than a Turing machine without such an oracle?

Given how much Penrose says about consciousness, I find it strange that he says almost nothing about the related mystery of *free will*. My central claim, in this essay, is that there exists an "empirical counterpart" of free will (what I call "Knightian freedom"), whose investigation really *does* lead naturally to questions about physics and cosmology, in a way that I don't know to happen for any of the usual empirical counterparts of consciousness.

(2) **I make no appeal to the Platonic perceptual abilities of human mathematicians.** Penrose's arguments rely on human mathematicians' supposed power to "see" the consistency of (for example) the Peano axioms of arithmetic (rather than simply *assuming* or *asserting* that consistency, as a computer program engaging in formal reasoning might do). As far as I can tell, to whatever extent this "power" exists at all, it's just a particularly abstruse type of *qualia* or *subjective experience*, as empirically inaccessible as any other type. In other words, instead of talking about the consistency of Peano arithmetic, I believe Penrose might as well have fallen back on the standard arguments about how a robot could never "really" enjoy fresh strawberries, but at most *claim* to enjoy them.

In both cases, the reply seems obvious: *how do you know* that the robot doesn't really enjoy the strawberries that it claims to enjoy, or see the consistency of arithmetic that it claims to see? And how do you know other people *do* enjoy or see those things? In any case, none of the arguments in this essay turn on these sorts of considerations. If any important difference is to be claimed between a digital computer and a human brain, then I insist that the difference correspond to something empirical: for example, that computer memories can be reliably measured and copied without disturbing them, while brain states quite possibly can't. The difference must *not* rely, even implicitly, on a question-begging appeal to the author's or reader's own subjective experience.

(3) **I make no appeal to Gödel's Incompleteness Theorem.** Let me summarize Penrose's (and earlier, Lucas's [58]) "Gödelian argument for human specialness." Consider any finitely

[49] Just like "free will," the word "consciousness" has been the victim of ferocious verbal overloading, having been claimed for everything from "that which disappears under anesthesia," to "that which a subject can give verbal reports about," to "the brain's executive control system"! Worse, "consciousness" has the property that, even if one specifies *exactly* what one means by it, readers are nevertheless certain to judge anything one says against their own preferred meanings. For this reason, just as I ultimately decided to talk about "freedom" (or "Knightian freedom") rather than "free will" in this essay, so I'd much rather use less fraught terms for executive control, verbal reportability, and so on, and restrict the word "consciousness" to mean "the otherwise-undefinable thing that people have tried to get at for centuries with the word 'consciousness,' supposing that thing exists."

describable machine M for deciding the truth or falsehood of mathematical statements, which never errs but which might sometimes output "I don't know." Then by using the code of M, it's possible to construct a mathematical statement S_M—one example is a formal encoding of "M will never affirm this statement"—that we humans, "looking in from the outside," can clearly see is true, yet that M itself can't affirm without getting trapped in a contradiction.

The difficulties with this argument have been explained at length elsewhere [4, 28, 72]; some standard replies to it are given in a footnote.[50]

Here, I'll simply say that I think the Penrose-Lucas argument establishes *some* valid conclusion, but the conclusion is much weaker than what Penrose wants, and it can also be established much more straightforwardly, without Gödelian considerations. The valid conclusion is that, *if you know the code of an AI,* then regardless of how intelligent the AI seems to be, you can "unmask" it as an automaton, blindly following instructions. To do so, however, you don't need to trap the AI in a self-referential paradox: it's enough to verify that the AI's responses are precisely the ones predicted (or probabilistically predicted) by the code that you possess! Both with the Penrose-Lucas argument and with this simpler argument, it seems to me that the real issue is not whether the AI follows a program, but rather, whether it follows a program that's *knowable by other physical agents.* That's why this essay focusses from the outset on the latter issue.

(4) **I don't suggest any barrier to a suitably-programmed digital computer passing the Turing Test.** Of course, if the freebit picture were correct, then there *would* be a barrier to duplicating the mental state and predicting the responses of a *specific* human. Even here, though, it's possible that, through non-invasive measurements, one could learn enough to create a digital "mockup" of a given person that would fool that person's closest friends and relatives, possibly for decades. (For this purpose, it might not matter if the mockup's responses eventually diverged badly from the original person's!) And such a mockup would certainly succeed at the "weaker" task of passing the Turing Test—i.e., of fooling interrogators into thinking it was human, at least until its code was revealed. If these sorts of mockups *couldn't* be built, then it would have to be for reasons well beyond anything explored in this essay.

(5) **I don't imagine anything particularly exotic about the biology of the brain.** In *The Emperor's New Mind* [67], Penrose speculates that the brain might act as what today we would call an adiabatic quantum computer: a device that generates highly-entangled quantum states, and might be able to solve certain optimization problems faster than any classical algorithm. (In this case, presumably, the entanglement would be *between neurons.*) In *Shadows* [68], Penrose goes further, presenting a proposal of himself and Stuart Hameroff that ascribes a central role to *microtubules*, a component of the neuronal cytoskeleton. In this

[50] Why is it permissible to assume that M never errs, if no *human* mathematician (or even, arguably, the entire mathematical community) has ever achieved that standard of infallibility?

Even if M never *did* affirm S_M, or never erred more generally, how could we ever *know* that? Indeed, much like with consciousness itself, even if one person had the mysterious Platonic ability to "see" M's soundness, how could that person ever convince a skeptical third party?

Finally, if we believed that the human brain was itself finitely describable, then why couldn't we construct a similar mathematical statement (e.g., "Penrose will never affirm this statement"), which *Penrose* couldn't affirm without contradicting himself, even though a different human, or indeed an AI program, could easily affirm it?

proposal, the microtubules would basically be "antennae" sensitive to yet-to-be-discovered quantum-gravity effects. Since Penrose *also* conjectures that a quantum theory of gravity would include Turing-uncomputable processes (see point 7 below), the microtubules would therefore let human beings surpass the capabilities of Turing machines.

Unfortunately, subsequent research hasn't been kind to these ideas. Calculations of decoherence rates leave almost no room for the possibility of quantum coherence being maintained in the hot, wet environment of the brain for anything like the timescales relevant to cognition, or for long-range entanglement between neurons (see Tegmark [88] for example). As for microtubules, they are common structural elements in cells—not only in neurons—and no clear evidence has emerged that they are particularly sensitive to the quantum nature of spacetime. And this is setting aside the question of the evolutionary pathways by which the "quantum-gravitational antennae" could have arisen.

The freebit perspective requires none of this: at least from a biological perspective, its picture of the human brain is simply that of conventional neuroscience. Namely, the human brain is a massively-parallel, highly-connected *classical* computing organ, whose design was shaped by millions of years of natural selection. Neurons perform a role vaguely analogous to that of *gates* in an electronic circuit (though neurons are far more complicated in detail), while synaptic strengths serve as a readable and writable memory. If we restrict to issues of principle, then perhaps the most salient *difference* between the brain and today's electronic computers is that the brain is a "digital/analog hybrid." This means, for example, that we have no practical way to measure the brain's exact "computational state" at a given time, copy the state, or restore the brain to a previous state; and it is not even obvious whether these things can be done in principle. It also means that the brain's detailed activity might be sensitive to microscopic fluctuations (for example, in the sodium-ion channels) that get chaotically amplified; this amplification might even occur over timescales relevant to human decision-making (say, 30 seconds). Of course, if those fluctuations were quantum-mechanical in origin—and at a small enough scale, they *would* be—then they couldn't be measured even in principle without altering them.

From the standpoint of neuroscience, the last parts of the preceding paragraph are certainly not established, but neither does there seem to be any good evidence against them. I regard them as plausible guesses, and hope future work will confirm or falsify them. To the view above, the freebit picture adds only a single further speculation—a speculation that, moreover, I think *does not even encroach on neuroscience's "turf."* This is simply that, if we consider the quantum states ρ relevant to the microscopic fluctuations, then those states are subject to at least some Knightian uncertainty (i.e., they are "freestates" as defined in Appendix 14); and furthermore, at least some of the Knightian uncertainty could ultimately be traced, if we wished, back to our ignorance of the detailed microstate of the early universe. This might or might not be true, but it seems to me that it's not a question for neuroscience at all, but for physics and cosmology (see Section 9). What the freebit picture "needs" from neuroscience, then, is extremely modest—certainly compared to what Penrose's picture needs!

(6) **I don't propose an "objective reduction" process that would modify quantum mechanics.** Penrose speculates that, when the components of a quantum state achieve a large enough energy difference that they induce "appreciably different" configurations of the spacetime metric (roughly, configurations that differ by one Planck length or more), new

quantum-gravitational laws beyond of unitary quantum mechanics should come into effect, and cause an "objective reduction" of the quantum state. This hypothetical process would underlie what we perceive as a measurement or collapse. Penrose has given arguments that his reduction process, if it existed, would have escaped detection by current quantum interference experiments, but *could* conceivably be detected or ruled out by experiments in the foreseeable future [61]. Penrose's is far from the only "objective reduction" model on the table: for example, there's a well-known earlier model due to Ghirardi, Rimini, and Weber (GRW) [41], but that one was purely "phenomenological," rather than being tied to gravity or some other known part of physics.

If an objective reduction process were ever discovered, then it would provide a ready-made distinction between microfacts and macrofacts (see Section 4.2), of exactly the sort the freebit picture needs. Despite this, I'm profoundly skeptical that any of the existing objective reduction models are close to the truth. The reasons for my skepticism are, first, that the models seem too ugly and ad hoc (GRW's more so than Penrose's); and second, that the AdS/CFT correspondence now provides evidence that quantum mechanics can emerge unscathed even from the combination with gravity. That's why, in Sections 4.2 and 5.5, I speculated that the distinction between microfacts and macrofacts might ultimately be defined in terms of deSitter space cosmology, with a macrofact being any fact "already irreversibly headed toward the deSitter horizon."

(7) **I don't propose that quantum gravity leads to Turing-uncomputable processes.** One of Penrose's most striking claims is that the laws of physics should involve *uncomputability*: that is, transitions between physical states that cannot in principle be simulated by a Turing machine, even given unlimited time. Penrose arrives at this conclusion via his Gödel argument (see point 3); he then faces the formidable challenge of where to *locate* the necessary uncomputability in anything plausibly related to physics. Note that this is *separate* from the challenge (discussed in point 5) of how to make the human brain sensitive to the uncomputable phenomena, supposing they exist! In *Shadows* [68], Penrose seems to admit that this is a weak link in his argument. As evidence for uncomputability, the best he can offer is a theorem of Markov that the 4-manifold homeomorphism problem is undecidable (indeed, equivalent to the halting problem) [60], and a speculation of Geroch and Hartle [40] that maybe that fact has something to do with quantum gravity, since some attempted formulations of quantum gravity involve sums over 4-manifolds.

Personally, I see no theoretical or empirical reason to think that the laws of physics should let us solve Turing-uncomputable problems—either with our brains, or with any other physical system. Indeed, I would go further: in [2], I summarized the evidence that the laws of physics seem to "conspire" to prevent us from solving NP-complete problems (like the Traveling Salesman Problem) in polynomial time. But the NP-complete problems, being solvable in "merely" exponential time, are child's play compared to Turing-uncomputable problems like the halting problem! For this reason, I regard it as a serious drawback of Penrose's proposals that they demand uncomputability in the dynamical laws, and as an advantage of the freebit picture that it suggests nothing of the kind. Admittedly, the freebit picture does require that there be no complete, computationally-simple description of the *initial conditions*.[51] But it

[51] More precisely, the initial state, when encoded in some natural way as a binary string, must have non-negligibly large "sophistication": see Appendix 13.

seems to me that nothing in established physics should have led us to expect that such a description would exist *anyway*![52] The freebit picture is silent on whether detailed properties of the initial state can be actually be *used* to solve otherwise-intractable computational problems, such as NP-complete problems, in a reliable way. But the picture certainly gives no reason to *think* this is possible, and I see no evidence for its possibility from any other source.

7 "Application" to Boltzmann Brains

In this section, I'll explain how the freebit perspective, if adopted, seems to resolve the notorious "Boltzmann brain problem" of cosmology. No doubt some people will feel that the cure is even worse than the disease! But even if one thinks that, the mere fact of a connection between freebits and Boltzmann brains seems worth spelling out.

First, what is the Boltzmann brain problem? Suppose that—as now seems all but certain [74]—our universe will *not* undergo a Big Crunch, but will simply continue to expand forever, its entropy increasing according to the Second Law. Then eventually, after the last black holes have decayed into Hawking radiation, the universe will reach the state known as *thermal equilibrium*: basically an entropy-maximizing soup of low-energy photons, flying around in an otherwise cold and empty vacuum. The difficulty is that, even in thermal equilibrium, there's still a tiny but nonzero probability that *any given (finite) configuration* will arise randomly: for example, via a chance conglomeration of photons, which could give rise to other particles via virtual processes. In general, we expect a configuration with total entropy S to arise at a particular time and place with probability of order $\sim 1/\exp(S)$. But eternity being a long time, even such exponentially-unlikely fluctuations should not only occur, but occur *infinitely often*, for the same reason why all of Shakespeare's plays presumably appear, in coded form, infinitely often in the decimal expansion of π.[53]

So in particular, we would eventually expect (say) beings physically identical to you, who'd survive just long enough to have whatever mental experiences you're now having, then disappear back into the void. These hypothetical observers are known in the trade as *Boltzmann brains* (see [33]), after Ludwig Boltzmann, who speculated about related matters in the late 19^{th} century. So, how do you know that *you* aren't a Boltzmann brain?

But the problem is worse. Since in an eternal universe, you would have infinitely many Boltzmann-brain doppelgängers, any observer with your memories and experiences seems infinitely *more* likely to be a Boltzmann brain, than to have arisen via the "normal" processes of Darwinian evolution and so on starting from a Big Bang! Silly as it sounds, this has been a major problem plaguing recent cosmological proposals, since they keep wanting to assign enormous probability measure to Boltzmann brains (see [20]).

But now suppose you believed the freebit picture, and also believed that possessing Knightian freedom is a necessary condition for counting as an observer. Then I claim that the Boltzmann brain problem would immediately go away. The reason is that, in the freebit picture, Knightian

[52] By contrast, when it comes to dynamical behavior, we have centuries of experience discovering laws that can indeed be simulated on a computer, given the initial conditions as input, and no experience discovering laws that can't be so simulated.

[53] This would follow from the conjecture, as yet unproved, that π is a "base-10 normal number": that is, that just like for a random sequence, every possible sequence of k consecutive digits appears in π's decimal expansion with asymptotic frequency $1/10^k$.

freedom *implies* a certain sort of correlation with the universe's initial state at the Big Bang—so that lack of complete knowledge of the initial state corresponds to lack of complete predictability (even probabilistic predictability) of one's actions by an outside observer. But a Boltzmann brain wouldn't have that sort of correlation with the initial state. By the time thermal equilibrium is reached, the universe will (by definition) have "forgotten" all details of its initial state, and any freebits will have long ago been "used up." In other words, there's no way to make a Boltzmann brain think one thought rather than another by toggling freebits. So, on this account, Boltzmann brains wouldn't be "free," even during their brief moments of existence. This, perhaps, invites the further speculation that there's nothing that it's like to be a Boltzmann brain.

7.1 What Happens When We Run Out of Freebits?

The above discussion of Boltzmann brains leads to a more general observation about the freebit picture. Suppose that

(1) the freebit picture holds,

(2) the observable universe has a finite extent (as it does, assuming a positive cosmological constant), and

(3) the holographic principle (see Section 5.4) holds.

Then the number of freebits accessible to any one observer must be finite—simply because the number of bits of *any* kind is then upper-bounded by the observable universe's finite holographic entropy. (For details, see Bousso [19] or Lloyd [56], both of whom estimate the observable universe's information capacity as roughly $\sim 10^{122}$ bits.)

But the nature of freebits is that they get permanently "used up" whenever they are amplified to macroscopic scale. So under the stated assumptions, we conclude that only a finite number of "free decisions" can possibly be made, before the observable universe runs out of freebits! In my view, this should not be too alarming. After all, even *without* the notion of freebits, the Second Law of Thermodynamics (combined with the holographic principle and the positive cosmological constant) already told us that the observable universe can witness at most $\sim 10^{122}$ "interesting events," of any kind, before it settles into thermal equilibrium. For more on the theme of freebits as a finite resource, see Appendix 13.

8 Indexicality and Freebits

The Boltzmann brain problem is just one example of what philosophers call an *indexical* puzzle: a puzzle involving the "first-person facts" of who, what, where, and when *you* are, which seems to persist even after all the "third-person facts" about the physical world have been specified. Indexical puzzles, and the lack of consensus on how to handle them, underlie many notorious debates in science and philosophy—including the debates surrounding the anthropic principle and "fine-tuning" of cosmological parameters, the multiverse and string theory landscape, the Doomsday argument, and the Fermi paradox of where all the extraterrestrial civilizations are. I won't even try to summarize the vast literature on these problems (see Bostrom [18] for an engaging introduction). Still, it might be helpful to go through a few examples, just to illustrate what we mean by indexical puzzles.

- When doing Bayesian statistics, it's common to use a *reference class*: roughly speaking, the set of observers from which you consider yourself to have been "sampled." For an uncontroversial example, suppose you want to estimate the probability that you have some genetic disease, in order to decide (say) whether it's worth getting tested. In reality, you either have the disease or you don't. Yet it seems perfectly unobjectionable to estimate the *probability* that you have the disease, by imagining that you were "chosen randomly" from among all people with the same race, sex, and so forth, then looking up the relevant statistics. However, things quickly become puzzling when we ask how *large* a reference class can be invoked. Can you consider yourself to have been "sampled uniformly at random" from the set of all humans who ever lived or ever will live? If so, then why not also include early hominids, chimpanzees, dolphins, extraterrestrials, or sentient artificial intelligences? Many would simply reply that you're *not* a dolphin or any of those other things, so there's no point worrying about the hypothetical possibility of having been one. The problem is that you're *also* not some other person of the same race and sex—you're *you*—but for medical and actuarial purposes, we clearly *do* reason as if you "could have been" someone different. So why can your reference class include those other people but not dolphins?

- Suppose you're an astronomer who's trying to use Bayesian statistics to estimate the probability of one cosmological model versus another one, conditioned on the latest data about the cosmic background radiation and so forth. Of course, as discussed in Section 3.1, any such calculation requires a specification of *prior probabilities*. The question is: should your prior include the assumption that, all else being equal, we're twice as likely to find ourselves in a universe that's twice as large (and thus presumably has twice as many civilizations, in expectation)? If so, then how do we escape the absurd-seeming conclusion that we're *certain* to live in an infinite universe, if such a universe is possible at all—since we expect there to be infinitely many more observers in an infinite universe than in a finite one? Surely we can't deduce the size of the universe without leaving our armchairs, like a medieval scholastic? The trouble is that, as Bostrom [18] points out, *not* adjusting the prior probabilities for the expected number of observers leads to its own paradoxes. As a fanciful example, suppose we're trying to decide between two theories, which on physical grounds are equally likely. Theory A predicts the universe will contain a single civilization of two-legged creatures, while theory B predicts the universe will contain a single civilization of two-legged creatures, *as well as* a trillion equally-advanced civilizations of nine-legged creatures. Observing ourselves to be two-legged, can we conclude that theory A is overwhelmingly more likely—since if theory B were correct, then we would almost certainly have been pondering the question on nine legs? A straightforward application of Bayes' rule seems to imply that the answer should be yes—*unless* we perform the adjustment for the number of civilizations that led to the first paradox!

- Pursuing thoughts like the above quickly leads to the notorious *Doomsday argument*. According to that argument, the likelihood that human civilization will kill itself off in the near future is much larger than one would naïvely think—where "naïvely" means "before taking indexical considerations into account." The logic is simple: suppose human civilization will continue for billions of years longer, colonizing the galaxy and so forth. In that case, our own position near the very beginning of human history would seem absurdly improbable—the more so when one takes into account that such a long-lived, spacefaring civilization would probably have a much larger population than exists today (just as *we* have a much larger population than existed hundreds of years ago). If we're Bayesians, and willing to speak at all about ourselves as drawn from an ensemble (as most people are, in the medical example), that presumably means we should revise downward the probability that the "spacefaring" scenario is correct, and revise upward the probability of scenarios that give us a more "average" position in human history. But because of exponential population growth, one expects the latter to be heavily weighted toward scenarios where civilization kills itself off in the very near future. Many commentators have tried to dismiss this argument as flat-out erroneous (see Leslie [52] or Bostrom [18] for common objections to the argument and responses to them).[54] However, while the "modern, Bayesian" version of the Doomsday argument might indeed be wrong, it's not wrong because of some trivial statistical oversight. Rather, the argument might be wrong because it embodies an interestingly false way of thinking about indexical uncertainty.

Perplexing though they might be, what do any of these indexical puzzles have to do with *our* subject, free will? After all, presumably no one thinks that we have the "free will" to choose where and when we're born! Yet, while I've never seen this connection spelled out before, it seems to me that indexical puzzles like those above *do* have some bearing on the free will debate. For the indexical puzzles make it apparent that, even if we assume the laws of physics are completely mechanistic, there remain large aspects of our experience that those laws fail to determine, even probabilistically. Nothing in the laws picks out one particular chunk of suitably organized matter from the immensity of time and space, and says, "here, *this* chunk is you; its experiences are your experiences." Nor does anything in the laws give us even a probability distribution over the possible such chunks. Despite its obvious importance even for empirical questions, our uncertainty about who we are and who we "could have been" (i.e., our reference class) bears all the marks of Knightian uncertainty. Yet once we've admitted indexical uncertainty into our worldview, it becomes less clear why we should reject the sort of uncertainty that the freebit picture needs! If whether you find yourself born in 8^{th}-century China, 21^{st}-century California, or Planet Zorg is a variable subject to Knightian uncertainty, then why not what you'll have for dessert tonight?

More concretely, suppose that there are numerous planets nearly identical to Earth, down to the same books being written, people with the same names and same DNA being born, etc. If the universe is spatially infinite—which cosmologists consider a serious possibility[55]—then there's no need to imagine this scenario: for simple probabilistic reasons, it's almost certainly true! Even

[54] Many people point out that cavemen could have made exactly the same argument, and would have been wrong. This is true but irrelevant: the whole point of the Doomsday argument is that *most* people who make it will be right!

Another common way to escape the argument's conclusion is to postulate the existence of large numbers of extraterrestrial civilizations, which are there regardless of what humans do or don't do. If the extraterrestrials are included in our reference class, they can then "swamp" the effect of the number of future humans in the Bayesian calculation.

[55] Astronomers can only see as far as light has reached since the Big Bang. If a positive spatial curvature was ever detected on cosmological scales, it would strongly suggest that the universe "wraps around"—much like hypothetical ancients might have deduced that the earth was round by measuring the curvature of a small patch. So far, though, except for local perturbations, the universe appears perfectly flat to within the limits of measurement, suggesting that it is either infinite or else extends far beyond our cosmological horizon. On the other hand, it is logically possible that the universe could be *topologically* closed (and hence finite), despite having zero spatial curvature. Also, assuming a positive cosmological constant, sufficiently far parts of the universe would be forever out of causal contact with us—leading to philosophical debate about whether those parts should figure into scientific explanations, or even be considered to "exist."

if the universe is spatially finite, the probability of such a "twin Earth" would approach 1 as the number of stars, galaxies, and so on went to infinity. Naturally, we'd expect any two of these "twin Earths" to be separated by exponentially large distances—so that, because of the dark energy pushing the galaxies apart, we would *not* expect a given "Earth-twin" ever to be in communication with any of its counterparts.

Assume for simplicity that there are at most two of these Earth-twins, call them A and B. (That assumption will have no effect on our conclusions.) Let's suppose that, because of (say) a chaotically amplified quantum fluctuation, these two Earths are about to diverge significantly in their histories for the first time. Let's further suppose that *you*—or rather, beings on A and B respectively who look, talk, and act like you—are the proximate cause of this divergence. On Earth A, the quantum fluctuation triggers a chain of neural firing events in "your" brain that ultimately causes "you" to take a job in a new city. On Earth B, a different quantum fluctuation triggers a chain of neural firings that causes "you" to stay where you are.

We now ask: from the perspective of a superintelligence that knows everything above, what's the total probability p that "you" take the new job? Is it simply $\frac{1}{2}$, since the two actual histories should be weighted equally? What if Earth B had a greater probability than Earth A of having formed in the first place—or did under one cosmological theory but not another? And why do we need to average over the two Earths at all? Maybe "you$_A$" is the "real" you, and taking the new job is a defining property of who you are, much as Shakespeare "wouldn't be Shakespeare" had he not written his plays. So maybe you$_B$ isn't even part of your reference class: it's just a faraway doppelgänger you'll never meet, who looks and acts like you (at least up to a certain point in your life) but *isn't* you. So maybe $p = 1$. Then again, maybe you$_B$ is the "real" you and $p = 0$. Ultimately, not even a superintelligence could calculate p without knowing something about *what it means to be "you,"* a topic about which the laws of physics are understandably silent.

Now, someone who accepted the freebit picture would say that the superintelligence's inability to calculate p is no accident. For whatever quantum fluctuation separated Earth A from Earth B could perfectly well have been a freebit. In that case, before you made the decision, the right representation of your physical state would have been a Knightian combination of you$_A$ and you$_B$. (See Appendix 14 for details of how these Knightian combinations fit in with the ordinary density matrix formalism of quantum mechanics.) After you make the decision, the ambiguity is resolved in favor of you$_A$ or you$_B$. Of course, you$_A$ might then turn out to be a Knightian combination of two further entities, you$_{A_1}$ and you$_{A_2}$, and so on.

For me, the appeal of this view is that it "cancels two philosophical mysteries against each other": free will and indexical uncertainty. As I said in Section 1.1, free will seems to me to *require* some source of Knightian uncertainty in the physical world. Meanwhile, indexical uncertainty is a type of Knightian uncertainty that's been considered troublesome and unwanted—though attempts to replace it with probabilistic uncertainty have led to no end of apparent paradoxes, from the Doomsday argument to Bostrom's observer-counting paradoxes to the Boltzmann brain problem. So it seems natural to try to fit the free-will peg into the indexical-uncertainty hole.

9 Is The Freebit Picture Falsifiable?

An obvious question about the freebit picture is whether it leads to any new, falsifiable predictions. At one level, I was careful not to "commit" myself to such predictions in this essay! My goal was to clarify some conceptual issues about the physical predictability of the brain. Whenever I ran up against an unanswered scientific question—for example, about the role of amplified quantum events in brain activity—I freely confessed my ignorance.

On the other hand, it's natural to ask: *are there empirical conditions that the universe has to satisfy, in order for the freebit perspective to have even a* ***chance*** *of being related to "free will" as most people understand the concept?*

I submit that the answer to the above question is yes. To start with the most straightforward predictions: first, it's necessary that psychology will never become physics. If human beings could be predicted as accurately as comets, then the freebit picture would be falsified. For in such a world, it would have *turned out* that, whatever it is we called "free will" (or even "the illusion of free will") in ordinary life, that property was not associated with any fundamental unpredictability of our choices.

It's also necessary that a quantum-gravitational description of the early universe will *not* reveal it to have a simply-describable pure or mixed state. Or at least, it's necessary that indexical uncertainty—that is, uncertainty about our own location in the universe or multiverse—will forever prevent us from reducing arbitrary questions about our own future to well-posed mathematical problems. (Such a math problem might ask, for example, for the probability distribution $\mathcal{D}$ that results when the known evolution equations of physics are applied to the known initial state at the Big Bang, marginalized to the vicinity of the earth, and conditioned on some relevant subset of branches of the quantum-mechanical wavefunction in which we happen to find ourselves.)

However, the above "predictions" have an unsatisfying, "god-of-the-gaps" character: they're simply predictions that certain scientific problems will never be completely solved! Can't we do better, and give *positive* predictions? Perhaps surprisingly, I think we can.

The first prediction was already discussed in Section 3.4. In order for the freebit picture to work, *it's necessary that quantum uncertainty—for example, in the opening and closing of sodium-ion channels—can not only get chaotically amplified by brain activity, but can do so "surgically" and on "reasonable" timescales.* In other words, the elapsed time between (a) the amplification of a quantum event and (b) the neural firings influenced by that amplification, must not be so long that the idea of a connection between the two retreats into absurdity. (Ten seconds would presumably be fine, whereas a year would presumably not be.) Closely related to that requirement, the quantum event must not affect countless *other* classical features of the world, separately from its effects on the brain activity. For if it did, then a prediction machine could in principle measure those other classical features to forecast the brain activity, with no need for potentially-invasive measurements on either the original quantum state or the brain itself.

It's tempting to compare the empirical situation for the freebit picture to that for supersymmetry in physics. Both of these frameworks are very hard to falsify—since no matter what energy scale has been explored, or how far into the future neural events have been successfully predicted, a diehard proponent could always hold out for the superparticles or freebits making their effects known at the *next* higher energy, or the next longer timescale. Yet despite this property, supersymmetry and freebits are both "falsifiable in degrees." In other words, if the superparticles can be chased up to a sufficiently high energy, then *even if present, they would no longer do most of the work they were originally invented to do.* The same is true of freebits, if the time between amplification and decision is long enough.

Moreover, there's also a *second* empirical prediction of the freebit picture, one that doesn't involve the notion of a "reasonable timescale." Recall, from Section 3.3, the concept of a past macroscopic determinant (PMD): a set of "classical" facts (for example, the configuration of a

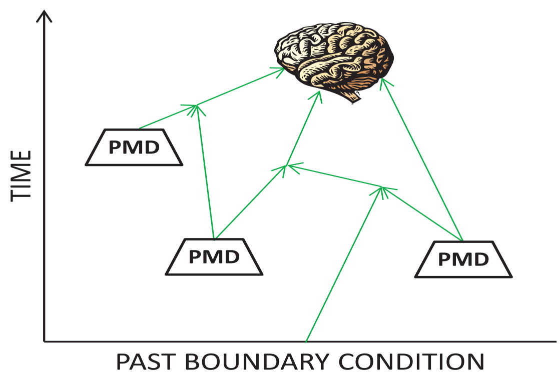


Figure 1: Tracing the causal antecedents of a human decision backwards in time, stopping *either* at past macroscopic determinants (PMDs) or at the initial boundary condition

laser) to the causal past of a quantum state ρ that, if known, completely determine ρ. Now consider an omniscient demon, who wants to influence your decision-making process by changing the quantum state of a single photon impinging on your brain. Imagine that there are indeed photons that would serve the demon for this purpose. However, now imagine that *all such photons can be "grounded" in PMDs*. That is, imagine that the photons' quantum states *cannot* be altered, maintaining a spacetime history consistent with the laws of physics, without *also* altering classical degrees of freedom in the photons' causal past. In that case, the freebit picture would once again fail. For *if* a prediction machine had simply had the foresight to measure the PMDs, then (by assumption) it could also calculate the quantum states ρ, and therefore the probability of your reaching one decision rather than another. Indeed, not only could the machine probabilistically predict your actions; it could even provide a complete quantum-mechanical account of where the probabilities came from. Given such a machine, your choices would remain "unpredictable" only in the sense that a radioactive atom is unpredictable, a sense that doesn't interest us (see Section 3). The conclusion is that, for the freebit picture to work, it's necessary that some of the relevant quantum states *can't* be grounded in PMDs, but only traced back to the early universe (see Figure 1).

10 Conclusions

At one level, all I did in this essay was to invoke what David Deutsch, in his book *The Beginning of Infinity* [31], called the "momentous dichotomy":

> *Either a given technology is possible, or else there must be some reason (say, of physics or logic) why it isn't possible.*

Granted, the above statement is a near-tautology. But as Deutsch points out, the implications of applying the tautology consistently can be enormous. One illustrative application, *slightly* less contentious than free will, involves my own field of quantum computing. The idea there is to apply the principles of quantum mechanics to build a new type of computer, one that could solve certain problems (such as factoring integers) exponentially faster than we know how to solve them with any existing computer. Quantum computing research is being avidly pursued around the world, but there are also a few vocal skeptics of such research. Many (though not all) of the skeptics appear to subscribe to the following three positions:

(a) A useful quantum computer is an almost self-evident absurdity: noise, decoherence, and so forth must conspire to prevent such a machine from working, just as the laws of physics always conspire to prevent perpetual-motion machines from working.

(b) No addition or revision is needed to quantum mechanics: the physical framework that underlies quantum computing, and that describes the state of an isolated physical system as a vector in an exponentially-large Hilbert space (leading to an apparent exponential *slowdown* when simulating quantum mechanics with a conventional computer).

(c) Reconciling (a) and (b) is not even a particularly interesting problem. At any rate, the burden is on the quantum computing *proponents* to sort these matters out! Skeptics can be satisfied that *something* must prevent quantum computers from working, and leave it at that.

What Deutsch's dichotomy suggests is that such blasé incuriosity, in the face of a glaring conflict between ideas, is *itself* the absurd position. Such a position only pretends to be the "conservative" one: secretly it is radical, in that it rejects the whole idea of science advancing by identifying apparent conflicts and then trying to resolve them.

In this essay, I applied Deutsch's momentous dichotomy to a different question:

> Could there exist a machine, consistent with the laws of physics, that "non-invasively cloned" all the information in a particular human brain that was relevant to behavior—so that the human could emerge from the machine unharmed, but would thereafter be fully probabilistically predictable given his or her future sense-inputs, in much the same sense that a radioactive atom is probabilistically predictable?

My central thesis is simply that *there is no "safe, conservative answer" to this question.* Of course, one can debate what exactly the question means, and how we would know whether the supposed cloning machine had succeeded. (See Appendix 12 for my best attempt at formalizing the requirements.) But I contend that philosophical analysis can only take us so far. The question also has an "empirical core" that could turn out one way or another, depending on details of the brain's physical organization that are not yet known. In particular, does the brain possess what one could call a *clean digital abstraction layer*: that is, a set of macroscopic degrees of freedom that

(1) encode everything relevant to memory and cognition,

(2) can be accurately modeled as performing a classical digital computation, and

(3) "notice" the microscopic, quantum-mechanical degrees of freedom at most as pure random-number sources, generating noise according to prescribed probability distributions?

Or is such a clean separation between the macroscopic and microscopic levels unavailable—so that any attempt to clone a brain would either miss much of the cognitively-relevant information, or else violate the No-Cloning Theorem?

In my opinion, *neither* answer to the question should make us wholly comfortable: if it does, then we haven't sufficiently thought through the implications! Suppose, on the one hand, that the brain-cloning device is possible. Then we immediately confront countless "paradoxes of personal identity" like those discussed in Section 2.5. Would you feel comfortable being (painlessly) killed, provided that a perfect clone of your brain's digital abstraction layer remained? Would it matter if the cloned data was moved to a new biological body, or was only "simulated" electronically? If the latter, what would count as an acceptable simulation? Would it matter if the simulation was run backwards in time, or in heavily-encrypted form, or in only one branch of a quantum computation? Would you literally expect to "wake up" as your clone? What if two clones were created: would you expect to wake up as each one with 50% probability? When applying Bayesian reasoning, should you, all other things equal, judge yourself twice as likely to wake up in a possible world with twice as many clones of yourself? The point is that, in a world with the cloning device, these would no longer be metaphysical conundrums, but in some sense, just *straightforward empirical questions* about what you should expect to observe! Yet even people who agreed on every possible "third-person" fact about the physical world and its contents, might answer such questions completely differently.

To be clear: it seems legitimate to me, given current knowledge, to conjecture that there is no principled obstruction to perfect brain-cloning, and indeed that this essay was misguided even to speculate about such an obstruction. However, *if* one thinks that by taking the "pro-cloning" route, one can sidestep the need for any "weird metaphysical commitments" of one's own, then I'd say one is mistaken.

So suppose, on the other hand, that the perfect brain-cloning device is *not* possible. Here, exactly like in the case of quantum computing, no truly inquisitive person will ever be satisfied by a bare *assertion* of the device's impossibility, or by a listing of practical difficulties. Instead, such a person will demand to know: what *principle* explains why perfect brain-cloning can't succeed, not even a million years from now? How do we reconcile its impossibility with everything we know about the mechanistic nature of the brain? Indeed, how should we think about the laws of physics *in general*, so that the impossibility of perfect brain-cloning would no longer seem surprising or inexplicable?

As soon as we try to answer these questions, I've argued that we're driven, more-or-less inevitably, to the view that the brain's detailed evolution would have to be buffeted around by chaotically-amplified "Knightian surprises," which I called freebits. Before their amplification, these freebits would need to live in quantum-mechanical degrees of freedom, since otherwise a cloning machine could (in principle) non-invasively copy them. Furthermore, our ignorance about the freebits would ultimately need to be traceable back to ignorance about the microstate of the early universe—again because otherwise, cloning would become possible in principle, through vigilant monitoring of the freebits' sources.

Admittedly, all this sounds like a tall order! But, strange though it sounds, I don't see that any of it is ruled out by current scientific understanding—though conceivably it *could* be ruled out in the future. In any case, setting aside one's personal beliefs, it seems worthwhile to understand that *this* is the picture one seems forced to, if one starts from the hypothesis that brain-cloning (with all its metaphysical difficulties) should be fundamentally impossible, then tries to make that hypothesis compatible with our knowledge of the laws of physics.

10.1 Reason and Mysticism

At this point, I imagine some readers might press me: but what do I *really* think? Do I actually take seriously the notion of quantum-mechanical "freebits" from the early universe playing a role in human decisions? The easiest response is that, in laying out my understanding of the various alternatives—yes, brain states might be perfectly clonable, but if we want to avoid the philosophical weirdness that such cloning would entail, then we're led in such-and-such an interesting direction, etc.—I *already said* what I really think. In pondering these riddles, I don't have any sort of special intuition, for which the actual arguments and counterarguments that I can articulate serve as window-dressing. The arguments exhaust my intuition.

I'll observe, however, that even uncontroversial facts can be made to sound incredible when some of their consequences are spelled out; indeed, the spelling out of such consequences has always been a mainstay of popular science writing. To give some common examples: everything you can see in the night sky was once compressed into a space smaller than an atom. The entire story of your life, including the details of your death, is almost certainly encoded (in fact, infinitely many times) somewhere in the decimal expansion of π. If Alois Hitler and Klara Pölzl had moved an inch differently while having intercourse, World War II would probably have been prevented. When you lift a heavy bag of groceries, what you're really feeling is the coupling of gravitons to a stress-energy tensor generated mostly by the rapid movements of gluons. In each of these cases, the same point *could* be made more prosaically, and many would prefer that it was. But when we state things vividly, at least we can't be accused of trying to *hide* the full implications of our abstract claims, for fear of being laughed at were those implications understood.

Thus, suppose I'd merely argued, in this essay, that it's possible that humans will never become as predictable (even probabilistically predictable) as digital computers, because of chaotic amplification of unknowable microscopic events, our ignorance of which can be traced as far backward in time as one wishes. In that case, some people would likely agree and others would likely disagree, with many averring (as I do) that the question remains open. Hardly anyone, I think, would consider the speculation an absurdity or a gross affront to reason. But if exactly the same idea is phrased in terms of a "quantum pixie-dust" left over from the Big Bang, which gets into our brains and gives us the capacity for free will—well then, *of course* it sounds crazy! Yet the second phrasing is nothing more than a dramatic rendering of the worldview that the first phrasing implies.

Perhaps some readers will accuse me of mysticism. To this, I can only reply that the view I flirt with in this essay seems like "mysticism" of an unusually tame sort: one that embraces the mechanistic and universal nature of the laws of physics, as they've played out for 13.7 billion years; that can accept even the "collapse of the wavefunction" as an effect brought about by ordinary unitary evolution; that's consumed by doubts; that welcomes corrections and improvement; that declines to plumb the cosmos for self-help tips or moral strictures about our sex lives; and that sees science—not introspection, not ancient texts, not "science" redefined to mean something different, but *just science in the ordinary sense*—as our best hope for making progress on the ancient questions of existence.

To any "mystical" readers, who want human beings to be as free as possible from the mechanistic chains of cause and effect, I say: *this picture represents the absolute maximum that I can see how to offer you, if I confine myself to speculations that I can imagine making contact with our current scientific understanding of the world.* Perhaps it's less than you want; on the other hand, it does seem like more than the usual compatibilist account offers! To any "rationalist" readers, who cheer when consciousness, free will, or similarly woolly notions get steamrolled by the advance of science, I say: you can feel vindicated, if you like, that despite searching (almost literally) to the ends of the universe, I wasn't able to offer the "mystics" anything more than I was! And even what I *do* offer might be ruled out by future discoveries.

Indeed, the freebit picture's falsifiability is perhaps the single most important point about it. Consider the following questions: On what timescales can microscopic fluctuations in biological systems get amplified, and change the probabilities of macroscopic outcomes? What other side effects do those fluctuations have? Is the brain interestingly different in its noise sensitivity than the weather, or other complicated dynamical systems? Can the brain's microscopic fluctuations be fully understood probabilistically, or are they subject to Knightian uncertainty? That is, can the fluctuations all be grounded in past macroscopic determinants, or are some ultimately cosmological in origin? Can we have a complete theory of cosmological initial conditions? Few things would make me happier than if progress on these questions led to the discovery that the freebit picture was wrong. For then at least we would have learned something.

11 Acknowledgments

I thank Yakir Aharonov, David Albert, Julian Barbour, Silas Barta, Wolfgang Beirl, Alex Byrne, Sean Carroll, David Chalmers, Alessandro Chiesa, Andy Drucker, Owain Evans, Andrew Hodges, Sabine Hossenfelder, Guy Kindler, Seth Lloyd, John Preskill, Huw Price, Haim Sompolinsky, Cristi Stoica, Jaan Tallinn, David Wallace, and others I've no doubt forgotten for helpful discussions about the subject of this essay; and especially Dana Moshkovitz Aaronson, David Aaronson, Steve Aaronson, Cris Moore, Jacopo Tagliabue, and Ronald de Wolf for their comments on earlier drafts. Above all, I thank S. Barry Cooper and Andrew Hodges for "commissioning" this essay, and for their near-infinite patience humoring my delays with it. It goes without saying that none of the people mentioned necessarily endorse anything I say here (indeed, some of them definitely don't!).

References

[1] S. Aaronson. Book review on A New Kind of Science. *Quantum Information and Computation*, 2(5):410–423, 2002. quant-ph/0206089.

[2] S. Aaronson. NP-complete problems and physical reality. *SIGACT News*, March 2005. quant-ph/0502072.

[3] S. Aaronson. Quantum copy-protection and quantum money. In *Proc. IEEE Conference on Computational Complexity*, pages 229–242, 2009.

[4] S. Aaronson. *Quantum Computing Since Democritus.* Cambridge University Press, 2013.

[5] S. Aaronson and P. Christiano. Quantum money from hidden subspaces. In *Proc. ACM STOC*, pages 41–60, 2012. arXiv:1203.4740.

[6] L. Antunes and L. Fortnow. Sophistication revisited. *Theory of Computing Systems*, 45(1):150–161, 2009.

[7] A. Aspect, P. Grangier, and G. Roger. Experimental realization of Einstein-Podolsky-Rosen-Bohm gedankenexperiment: a new violation of Bell's inequalities. *Phys. Rev. Lett.*, 49:91–94, 1982.

[8] R. J. Aumann. Agreeing to disagree. *Annals of Statistics*, 4(6):1236–1239, 1976.

[9] M. Balaguer. *Free Will as an Open Scientific Problem*. Bradford Books, 2009.

[10] J. S. Bell. *Speakable and Unspeakable in Quantum Mechanics*. Cambridge, 1987.

[11] C. H. Bennett and G. Brassard. Quantum cryptography: public key distribution and coin tossing. In *Proceedings of IEEE International Conference on Computers Systems and Signal Processing*, pages 175–179, 1984.

[12] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. Wootters. Teleporting an unknown quantum state by dual classical and EPR channels. *Phys. Rev. Lett.*, 70:1895–1898, 1993.

[13] E. Bernstein and U. Vazirani. Quantum complexity theory. *SIAM J. Comput.*, 26(5):1411–1473, 1997. First appeared in ACM STOC 1993.

[14] A. Bierce. *The Collected Works of Ambrose Bierce: 1909-1912*. Cornell University Library, 2009.

[15] J. de Boer. Introduction to the AdS/CFT correspondence. University of Amsterdam Institute for Theoretical Physics (ITFA) Technical Report 03-02, 2003.

[16] D. Bohm. A suggested interpretation of the quantum theory in terms of "hidden" variables. *Phys. Rev.*, 85:166–193, 1952.

[17] N. Bohr. *Atomic Physics and Human Knowledge*. Dover, 2010. First published 1961.

[18] N. Bostrom. *Anthropic Bias: Observation Selection Effects in Science and Philosophy*. Routledge, 2002.

[19] R. Bousso. Positive vacuum energy and the N-bound. *J. High Energy Phys.*, 0011(038), 2000. hep-th/0010252.

[20] S. Carroll. *From Eternity to Here*. Dutton, 2010.

[21] G. J. Chaitin. A theory of program size formally identical to information theory. *J. ACM*, (22):329–340, 1975.

[22] D. J. Chalmers. *The Conscious Mind: In Search of a Fundamental Theory*. Oxford, 1996.

[23] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt. Proposed experiment to test local hidden-variable theories. *Phys. Rev. Lett.*, 23:880–884, 1969.

[24] A. H. Compton. Science and Man's freedom. *The Atlantic*, 200(4):71–74, October 1957.

[25] J. H. Conway and S. Kochen. The strong free will theorem. *Notices of the AMS*, 56(2):226–232, 2009. arXiv:0807.3286.

[26] T. Cowen and R. Hanson. Are disagreements honest? *Journal of Economic Methodology*, 2012. To appear. At hanson.gmu.edu/deceive.pdf.

[27] D. C. Dennett. *Elbow Room: The Varieties of Free Will Worth Wanting*. MIT Press, 1984.

[28] D. C. Dennett. *Darwin's Dangerous Idea: Evolution and the Meanings of Life.* Simon & Schuster, 1995.

[29] D. Deutsch. Quantum theory as a universal physical theory. *International Journal of Theoretical Physics*, 24:1–41, 1985.

[30] D. Deutsch. Quantum theory, the Church-Turing principle and the universal quantum computer. *Proc. Roy. Soc. London*, A400:97–117, 1985.

[31] D. Deutsch. *The Beginning of Infinity: Explanations that Transform the World.* Allen Lane, 2011.

[32] P. K. Dick. *The Collected Stories of Philip K. Dick Volume 4: The Minority Report.* Citadel Twilight, 1998.

[33] L. Dyson, M. Kleban, and L. Susskind. Disturbing implications of a cosmological constant. *Journal of High Energy Physics*, (0210:011), 2002. hep-th/0208013.

[34] E. Farhi, D. Gosset, A. Hassidim, A. Lutomirski, and P. Shor. Quantum money from knots. In *Proc. Innovations in Theoretical Computer Science (ITCS)*, pages 276–289, 2012. arXiv:1004.5127.

[35] J. M. Fischer. *The Metaphysics of Free Will: An Essay on Control.* Wiley-Blackwell, 1995.

[36] J. M. Fischer, R. Kane, D. Pereboom, and M. Vargas. *Four Views on Free Will.* Wiley-Blackwell, 2007.

[37] T. Fritz. Bell's theorem without free will. arXiv:1206.5115, 2012.

[38] P. Gács, J. Tromp, and P. M. B. Vitányi. Algorithmic statistics. *IEEE Trans. Information Theory*, 47(6):2443–2463, 2001.

[39] M. Gardner. Mathematical Games. *Scientific American*, page 102, March 1974. Reprinted with addendum in The Colossal Book of Mathematics.

[40] R. Geroch and J. B. Hartle. Computability and physical theories. *Foundations of Physics*, 16(6):533–550, 1986.

[41] G. C. Ghirardi, A. Rimini, and T. Weber. Unified dynamics for microscopic and macroscopic systems. *Phys. Rev. D*, 34:470–491, 1986.

[42] S. W. Hawking. Black hole explosions? 248(5443):30, 1974.

[43] A. Hodges. *Alan Turing: The Enigma.* Princeton University Press, 2012. Centenary edition. First published 1983.

[44] A. Hodgkin and A. Huxley. A quantitative description of membrane current and its application to conduction and excitation in nerve. *J. Physiol.*, 117:500–544, 1952.

[45] C. Hoefer. Freedom from the inside out. In C. Callender, editor, *Time, Reality, and Experience*, pages 201–222. Cambridge University Press, 2002.

[46] G. 't Hooft. The free-will postulate in quantum mechanics. quant-ph/0701097, 2007.

[47] M. Hutter. On universal prediction and Bayesian confirmation. *Theoretical Computer Science*, 384(1):33–48, 2007. arXiv:0709.1516.

[48] P. van Inwagen. *An Essay on Free Will*. Oxford University Press, 1983.

[49] F. H. Knight. *Risk, Uncertainty, and Profit*. Nabu Press, 2010. First published 1921.

[50] C. Koch. *Consciousness: Confessions of a Romantic Reductionist*. MIT Press, 2012.

[51] S. Kochen and S. Specker. The problem of hidden variables in quantum mechanics. *J. of Math. and Mechanics*, 17:59–87, 1967.

[52] J. Leslie. *The End of the World: The Science and Ethics of Human Extinction*. Routledge, 1998.

[53] S. Levy. *Artificial Life: A Report from the Frontier Where Computers Meet Biology*. Vintage, 1992.

[54] M. Li and P. M. B. Vitányi. *An Introduction to Kolmogorov Complexity and Its Applications (3rd ed.)*. Springer, 2008. First edition published in 1993.

[55] B. W. Libet. Do we have free will? *Journal of Consciousness Studies*, (6):47–57, 1999.

[56] S. Lloyd. Computational capacity of the universe. *Phys. Rev. Lett.*, 88, 2002. quant-ph/0110141.

[57] S. Lloyd. A Turing test for free will. In *This volume*. 2012.

[58] J. R. Lucas. Minds, machines, and Gödel. *Philosophy*, 36:112–127, 1961.

[59] D. M. MacKay. On the logical indeterminacy of a free choice. *Mind*, LXIX(273):31–40, 1960.

[60] A. Markov. Unsolvability of the problem of homeomorphy. In *Proceedings of the International Congress of Mathematicians, Edinburgh*, pages 300–306, 1958.

[61] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester. Towards quantum superpositions of a mirror. *Phys. Rev. Lett.*, 91(130401), 2003. quant-ph/0210001.

[62] C. McGinn. *The Mysterious Flame: Conscious Minds In A Material World*. Basic Books, 2000.

[63] S. Morris. The Common Prior Assumption in economic theory. *Economics and Philosophy*, 11:227–253, 1995.

[64] R. M. Neal. Puzzles of anthropic reasoning resolved using full non-indexical conditioning. Technical Report 0607, Dept. of Statistics, University of Toronto, 2006. www.cs.toronto.edu/~radford/anth.abstract.html.

[65] J. von Neumann. *Mathematical Foundations of Quantum Mechanics*. Princeton University Press, 1996. Translated from German.

[66] R. Nozick. Newcomb's problem and two principles of choice. In Nicholas Rescher, editor, *Essays in Honor of Carl G. Hempel*, pages 114–115. Synthese Library, 1969.

[67] R. Penrose. *The Emperor's New Mind*. Oxford, 1989.

[68] R. Penrose. *Shadows of the Mind: A Search for the Missing Science of Consciousness*. Oxford, 1996.

[69] R. Penrose. *The Road to Reality: A Complete Guide to the Laws of the Universe*. Vintage, 2007.

[70] S. Pironio, A. Acín, S. Massar, A. Boyer de la Giroday, D. N. Matsukevich, P. Maunz, S. Olmschenk, D. Hayes, L. Luo, T. A. Manning, and C. Monroe. Random numbers certified by Bell's theorem. 464:1021–1024, 2010. arXiv:0911.3427.

[71] M. F. Pusey, J. Barrett, and T. Rudolph. On the reality of the quantum state. *Nature Physics*, 8:475–478, 2012. arXiv:1111.3328.

[72] S. Russell and P. Norvig. *Artificial Intelligence: A Modern Approach*. Prentice Hall, 2009. Third edition. First published 1995.

[73] G. Ryle. *The Concept of Mind*. Kessinger Publishing, 2008. First published 1949.

[74] S. Perlmutter and 31 others (Supernova Cosmology Project). Measurements of Ω and Λ from 42 high-redshift supernovae. *Astrophysical Journal*, 517(2):565–586, 1999. astro-ph/9812133.

[75] J. Satinover. *The Quantum Brain*. Wiley, 2001.

[76] L. J. Savage. *The Foundations of Statistics*. Wiley, 1954.

[77] J. Schmidhuber. A computer scientist's view of life, the universe, and everything. In *Foundations of Computer Science: Potential - Theory - Cognition*, pages 201–208. Springer, 1997.

[78] J. Searle. *The Rediscovery of the Mind*. MIT Press, 1992.

[79] G. Shafer. *A Mathematical Theory of Evidence*. Princeton University Press, 1976.

[80] S. M. Shieber, editor. *The Turing Test: Verbal Behavior as the Hallmark of Intelligence*. Bradford Books, 2004.

[81] H. Sompolinsky. A scientific perspective on human choice. In D. Shatz and Y. Berger, editors, *Judaism, Science, and Moral Responsibility*. Rowman and Littlefield, 2005.

[82] C. S. Soon, M. Brass, H.-J. Heinze, and J.-D. Haynes. Unconscious determinants of free decisions in the human brain. *Nature Neuroscience*, 11:543–545, 2008.

[83] R. W. Spekkens. In defense of the epistemic view of quantum states: a toy theory. *Physical Review A*, 75(032110), 2007. quant-ph/0401052.

[84] V. J. Stenger. *Quantum Gods: Creation, Chaos, and the Search for Cosmic Consciousness*. Prometheus Books, 2009.

[85] C. Stoica. Flowing with a frozen river. Runner-up in FQXi essay contest on The Nature of Time. fqxi.org/data/essay-contest-files/Stoica_flowzen_time_2.pdf, 2008.

[86] C. Stoica. Modern physics, determinism and free-will. *Noema*, XI, 2012. www.noema.crifst.ro/doc/2012_5_01.pdf.

[87] N. N. Taleb. *The Black Swan: The Impact of the Highly Improbable.* Random House, 2010. Second edition. First published 2007.

[88] M. Tegmark. The importance of quantum decoherence in brain processes. *Phys. Rev. E*, 61:4194–4206, 1999.

[89] A. M. Turing. Computing machinery and intelligence. *Mind*, 59:433–460, 1950.

[90] U. Vazirani and T. Vidick. Certifiable quantum dice - or, true random number generation secure against quantum adversaries. In *Proc. ACM STOC*, pages 61–76, 2012. arXiv:1111.6054.

[91] S. Wiesner. Conjugate coding. *SIGACT News*, 15(1):78–88, 1983. Original manuscript written circa 1970.

[92] E. P. Wigner. Remarks on the mind-body problem. In I. J. Good, editor, *The Scientist Speculates.* Heinemann, London, 1962.

[93] S. Wolfram. *A New Kind of Science.* Wolfram Media, 2002.

12 Appendix: Defining "Freedom"

In this appendix, I'll use the notion of Knightian uncertainty (see Section 3) to offer a possible mathematical formalization of "freedom" for use in free-will discussions. Two caveats are immediately in order. The first is that my formalization only tries to capture what I've called "Knightian freedom"—a strong sort of in-principle physical unpredictability—and not "metaphysical free will." For as discussed in Section 1.1, I don't see how *any* definition grounded in the physical universe could possibly capture the latter, to either the believers' or the deniers' satisfaction. Also, as we'll see, formalizing "Knightian freedom" is *already* a formidable challenge!

The second caveat is that, by necessity, my definition will be in terms of more "basic" concepts, which I'll need to assume as unanalyzed primitives. Foremost among these is the concept of a *physical system*: something that occupies space; exchanges information (and matter, energy, etc.) with other physical systems; and crucially, retains an identity through time even as its internal state changes. Examples of physical systems are black holes, the earth's atmosphere, human bodies, and digital computers. Without some concept like this, it seems to me that we can never specify *whose* freedom we're talking about, or even which physical events we're trying to predict.

Yet as philosophers know well, the concept of "physical system" already has plenty of traps for the unwary. As one illustration, should we say that a human body remains "the same physical system" after its death? If so, then an extremely reliable method to predict a human subject immediately suggests itself: namely, first shoot the subject; then predict that the subject will continue to lay on the ground doing nothing!

Now, it might be objected that this "prediction method" shouldn't count, since it *changes the subject's state* (to put it mildly), rather than just passively gathering information about the subject.

The trouble with that response is that putting the subject in an fMRI machine, interviewing her, or even just having her sign a consent form or walking past her on the street also change her state! If we don't allow *any* interventions that change the subject's state from what it "would have been otherwise," then prediction—at least with the science-fiction accuracy we're imagining—seems hopeless, but in an uninteresting way. So which interventions are allowed and which aren't?

I see no alternative but to take the *set of allowed interventions* as another unanalyzed primitive. When formulating the prediction task (and hence, in this essay, when defining Knightian freedom), we simply declare that certain interventions—such as interviewing the subject, putting her in an fMRI machine, or perhaps even having nanorobots scan her brain state—are allowed; while other interventions, such as killing her, are not allowed.

An important boundary case, much discussed by philosophers, is an intervention that would destroy each neuron of the subject's brain one by one, replacing the neurons by microchips claimed to be functionally equivalent. Is *that* allowed? Note that such an operation could certainly make it easier to predict the subject—since from that point forward, the predictor would only have to worry about simulating the microchips, not the messy biological details of the original brain. Here I'll just observe that, if we like, we can disallow such drastic interventions, without thereby taking any position on the conundrum of what such "siliconization" would do to the subject's conscious experience. Instead we can simply say that, while the subject might indeed be perfectly predictable after the operation, that fact *doesn't settle the question at hand*, which was about the subject's predictability *before* the operation. For a large part of what we wanted to know was to what extent the messy biological details *do* matter, and we can't answer that question by defining it out of consideration.

But one might object: if the "messy biological details" need to be left in place when trying to predict a brain, what *doesn't* need to be left in place? After all, brains are not isolated physical systems: they constantly receive inputs from the sense organs, from hormones in the bloodstream, etc. So when modeling a brain, do we also need to model the entire environment in which that brain is immersed—or at least, all aspects of the environment that might conceivably affect behavior? If so, then prediction seems hopeless, but again, not for any "interesting" reasons: merely for the boring reason that we can't possibly measure *all* relevant aspects of the subject's environment, being embedded in the environment ourselves.

Fortunately, I think there's a way around this difficulty, at the cost of one more unanalyzed primitive. Given a physical system S, denote by $I(S)$ the set of *screenable inputs* to S—by which I mean, the inputs to S that we judge that any would-be predictor of S should also be provided, in order to ensure a "fair contest." For example, if S is a human brain, then $I(S)$ would probably include (finite-precision digital encodings of) the signals entering the brain through the optic, auditory, and other sensory systems, the levels of various hormones in the blood, and other measurable variables at the interface between the brain and its external environment. On the other hand, $I(S)$ probably *wouldn't* include, for example, the exact quantum state of every photon impinging on the brain. For arguably, we have no idea how to screen off all those microscopic "inputs," short of siliconization or some equally drastic intervention.

Next, call a system S *input-monitorable* if there exists an allowed intervention to S, the result of which is that, after the intervention, all signals in $I(S)$ get "carbon-copied" to the predictor's computer at the same time as they enter S. For example, using some future technology, a brain might be input-monitored by installing microchips that scan all the electrical impulses in the optic and auditory nerves, the chemical concentrations in the blood-brain barrier, etc., and that faithfully transmit that information to a predictor in the next room via wireless link. Crucially, and in contrast to siliconization, input-monitoring doesn't strike me as raising any profound issues of consciousness or selfhood. That is, it seems fairly clear that an input-monitored human would still be "the same human," just hooked up to some funny devices! Input-monitoring also differs from siliconization in that it seems much closer to practical realization.

The definition of freedom that I'll suggest will only make sense for input-monitorable physical systems. If S is not input-monitorable, then I'll simply hold that the problem of "predicting S's behavior" isn't well-enough defined: S is so intertwined with its environment that one can't say where predicting S ends and predicting its environment begins. One consequence is that, in this framework, we can't even *pose the question* of whether humans have Knightian freedom, unless we agree (at least provisionally) that humans are input-monitorable. Fortunately, as already suggested, I don't see any major scientific obstacles to supposing that humans *are* input-monitorable, and I even think input-monitoring could plausibly be achieved in 50 or 100 years.

Admittedly, in discussing whether humans are input-monitorable, a lot depends on our choice of screenable inputs $I(S)$. If $I(S)$ is small, then input-monitoring S might be easy, but predicting S after the monitoring is in place might be hard or impossible, simply because of the predictor's ignorance about crucial features of S's environment. By contrast, if $I(S)$ is large, then input-monitoring S might be hard or impossible, but supposing S were input-monitored, predicting S might be easy. Since our main interest is the inherent difficulty of prediction, our preference should always be for the largest $I(S)$ possible.

So, suppose S is input-monitorable, and suppose we've arranged things so that all the screenable inputs to S—what S sees, what S hears, etc.—are transmitted in real-time to the predictor. We then face the question: what aspects of S are we trying to predict, and what does it mean to predict those aspects?

Our next primitive concept will be that of S's *observable behaviors.* For the earth's atmosphere, observable behaviors might include snow and thunderstorms; for a human brain, they might include the signals sent out by the motor cortex, or even just a high-level description of which words will be spoken and which decisions taken. Fortunately, it seems to me that, for any sufficiently complex system, the prediction problem is *not* terribly sensitive to which observable behaviors we focus on, provided those behaviors belong to a large "universality class." By analogy, in computability theory, it doesn't matter whether we ask whether a given computer program will ever halt, or whether the program will ever return to its initial state, or whether it will ever print "YES" to the console, or some other question about the program's future behavior. For these problems are all *reducible* to each other: if we had a reliable method to predict whether a program would halt, then we could also predict whether the program would print "YES" to the console, by modifying the program so that it prints "YES" if and only if it halts. In the same way, if we had a reliable method to predict a subject's hand movements in arbitrary situations, I claim that we could also predict the subject's speech. For "arbitrary situations" include those where we direct the subject to translate everything she says into sign language! And thus, assuming we've built a "hand-prediction algorithm" that works in those situations, we must also have built (or had the ability to build) a speech-prediction algorithm as well.

So suppose we fix some set B of observable behaviors of S. What should count as *predicting* the behaviors? From the outset, we should admit that S's behavior might be inherently probabilistic—as, for example, if it depended on amplified quantum events taking place inside S. So we should be satisfied if we can predict S in "merely" the same sense that physicists can predict a radioactive atom: namely, by giving a probability distribution over S's possible future behaviors.

Here difficulties arise, which are well-known in the fields of finance and weather forecasting. How exactly do we test predictions that take the form of probability distributions, if the predictions apply to events that might not be repeatable? Also, what's to prevent someone from declaring success on the basis of absurdly conservative "predictions": for example, "50/50" for every yes/no question? Briefly, I'd say that *if* the predictor's forecasts take the form of probability distributions, then to whatever extent those forecasts are unimpressive ("50/50"), *the burden is on the predictor* to convince skeptics that the forecasts nevertheless encoded everything that *could* be predicted about S via allowed interventions. That is, the predictor needs to rule out the hypothesis that the probabilities merely reflected ignorance about unmeasured but measurable variables. In my view, this would ultimately require the predictor to give a causal account of S's behavior, which showed explicitly how the observed outcome depended on quantum events—the only sort of events that we know to be probabilistic on physical grounds (see Section 2.8).

But it's not enough to let the predictions be probabilistic; we need to scale back our ambitions still further. For even with a system S as simple as a radioactive atom, there's no hope of calculating the *exact* probability that (say) S will decay within a certain time interval—if only because of the error bars in the physical constants that enter into that probability. But intuitively, this lack of precision doesn't make the atom any less "mechanistic." Instead, it seems to me that we should call a probabilistic system S "mechanistic" *if—and only if—the differences between our predicted probabilities for S's behavior and the "true" probabilities can be made as small as desired by repeated experiments.*

Yet we are still not done. For what does the predictor P *already know* about S, before P's data-gathering process even starts? If P were initialized with a "magical copy" of S, then of course predicting S would be trivial.[56] On the other hand, it also seems unreasonable not to tell P *anything* about S: for example, if P could accurately predict S, but only if given the hint that S is a human being, that would still be rather impressive, and intuitively incompatible with S's "freedom." So, as our final primitive concept, we assume a *reference class* C of possible physical systems; P is then told only that $S \in C$ and needs to succeed under that assumption. For example, C might be "the class of all members of the species *Homo sapiens*," or even the class of all systems macroscopically identifiable as some *particular* Homo sapien.[57]

This, finally, leads to my attempted definition of freedom. Before offering it, let me stress that *nothing in the essay depends much on the details of the definition—and indeed, I'm more than willing to tinker with those details.* So then what's the point of *giving* a definition? One reason is to convince skeptics that the concept of "Knightian freedom" *can* be made precise, once one has a suitable framework with which to discuss these issues. A second reason is to illustrate just how much little-examined complexity lurks in the commonsense notion of a physical system's being "predictable"—and to show how non-obvious the questions of freedom and predictability actually become, once we start to unravel that complexity.

> Let S be any input-monitorable physical system drawn from the reference class C. Suppose that, as the result of allowed interventions, S is input-monitored by another

[56] I thank Ronald de Wolf for this observation.

[57] We could also formulate a stronger notion of a "universal predictor," which has to work for *any* physical system S (or equivalently, whose reference class C is the set of all physical systems). My own guess is that, *if* there exists a predictor for "sufficiently complex" systems like human brains, then there also exists a universal predictor. But I won't attempt to argue for that here.

physical system $P = P(C)$ (the "predictor"), starting at some time[58] 0. Given times $0 \leq t < u \leq \infty$, let $I_{t,u}$ encode all the information in the screenable inputs that S receives between times t and u, with $I_{t,\infty}$ denoting the information received from time t onwards. Likewise, let $B_{t,u}$ encode all the information in S's observable behaviors between times t and u. (While this is not essential, we can assume that $I_{t,u}$ and $B_{t,u}$ both consist of finite sequences of bits whenever $u < \infty$.)

Let $\mathcal{D}(B_{t,u}|I_{t,u})$ be the "true" probability distribution[59] over $B_{t,u}$ conditional on the inputs $I_{t,u}$, where "true" means the distribution that would be predicted by a godlike intelligence who knew the exact physical state of S and its external environment at time t. We assume that $\mathcal{D}(B_{t,u}|I_{t,u})$ satisfies the "causal property": that $\mathcal{D}(B_{t,v}|I_{t,u}) = \mathcal{D}(B_{t,v}|I_{t,v})$ depends only on $I_{t,v}$ for all $v < u$.

Suppose that, from time 0 to t, the predictor P has been monitoring the screenable inputs $I_{0,t}$ and observable behaviors $B_{0,t}$, and more generally, interacting with S however it wants via allowed interventions (for example, submitting questions to S by manipulating $I_{0,t}$, and observing the responses in $B_{0,t}$). Then, at time t, we ask P to output a description of a function f, which maps the future inputs $I_{t,\infty}$ to a distribution $\mathcal{E}(B_{t,\infty}|I_{t,\infty})$ satisfying the causal property. Here $\mathcal{E}(B_{t,\infty}|I_{t,\infty})$ represents P's best estimate for the distribution $\mathcal{D}(B_{t,\infty}|I_{t,\infty})$. Note that the description of f might be difficult to "unpack" computationally—for example, it might consist of a complicated algorithm that outputs a description of $\mathcal{E}(B_{t,u}|I_{t,u})$ given as input $u \in (t,\infty)$ and $I_{t,u}$. All we require is that the description be *information-theoretically* complete, in the sense that one *could* extract $\mathcal{E}(B_{t,u}|I_{t,u})$ from it given enough computation time.

Given $\varepsilon, \delta > 0$, we call P a (t, ε, δ)*-predictor* for the reference class C if the following holds. For all $S \in C$, with probability at least $1 - \delta$ over any "random" inputs in $I_{t,u}$ (controlled neither by S nor by P), we have

$$\|\mathcal{E}(B_{t,\infty}|I_{t,\infty}) - \mathcal{D}(B_{t,\infty}|I_{t,\infty})\| < \varepsilon$$

for the actual future inputs $I_{t,\infty}$ (not necessarily for every *possible* $I_{t,\infty}$). Here $\|\cdot\|$ denotes the variation distance.[60]

We call C *mechanistic* if for all $\varepsilon, \delta > 0$, there exists a $t = t_{\varepsilon,\delta}$ and a $P = P_{\varepsilon,\delta}$ such that P is a (t, ε, δ)-predictor for C. We call C *free* if C is not mechanistic.

Two important sanity checks are the following:

(a) According to the above definition, classes C of physical systems like thermostats, digital computers, and radioactive nuclei are indeed mechanistic (given reasonable sets of screenable inputs, allowed interventions, and observable behaviors). For example, suppose that C is the set of all possible configurations of a particular digital computer; the allowed interventions include reading the entire contents of the disk drives and memory and "eavesdropping" on all the input ports (all of which is known to be technologically doable without destroying the computer); and the observable behaviors include everything sent to the output ports. In

[58] Here we don't presuppose that time is absolute or continuous. Indeed, all we need is that S passes through a discrete series of "instants," which can be ordered by increasing values of t.

[59] Or probability measure over infinite sequences, in the case $u = \infty$.

[60] Variation distance is a standard measure of distance between two probability distributions, and is defined by $\|\{p_x\} - \{q_x\}\| := \frac{1}{2}\sum_x |p_x - q_x|$.

that case, even with no further interaction, the predictor can clearly emulate the computer arbitrarily far into the future. Indeed, even if the computer $S \in C$ has an internal quantum-mechanical random number generator, the probability distribution $\mathcal{D}$ over *possible* future behaviors can still be approximately extremely well.

(b) On the other hand, at least mathematically, one can construct classes of systems C that are free. Indeed, this is trivial to arrange, by simply restricting the screenable inputs so that S's future behavior is determined by some input stream to which P does not have access.

Like many involved definitions in theoretical computer science, cryptography, economics, and other areas, my definition of freedom is "merely" an attempt to approximate an informal concept that one had prior to formalization. And indeed, there are many changes one could contemplate to the definition. To give just a few examples, instead of requiring that $\mathcal{E}\left(B_{t,\infty}|I_{t,\infty}\right)$ approximate $\mathcal{D}\left(B_{t,\infty}|I_{t,\infty}\right)$ only for the actual future inputs $I_{t,\infty}$, one could demand that it do so for all *possible* $I_{t,\infty}$. Or one could assume a distribution over the future $I_{t,\infty}$, and require success on *most* of them. Or one could require success only for most $S \in C$, again assuming a distribution over S's. Or one could switch around the quantifiers—e.g., requiring a single predictor P that achieves greater and greater prediction accuracy $\varepsilon > 0$ the longer it continues. Or one could drop the requirement that P forecast all of $B_{t,\infty}$, requiring only that it forecast $B_{t,u}$ for some large but finite u. It would be extremely interesting to develop the mathematical theory of these different sorts of prediction—something I reluctantly leave to future work. Wouldn't it be priceless if, after millennia of debate, the resolution of the question "are humans free?" turned out to be "yes if you define 'free' with the ε, δ quantifiers inside, but no if you put the quantifiers outside?"

A central limitation of the definition, as it stands, is that it's qualitative rather than quantitative, closer in spirit to computability theory than complexity theory. More concretely, the definition of "mechanistic" only requires that there *exist* a finite time t after which the predictor succeeds; it puts no limit on the amount of time. But this raises a problem: what if the predictor could succeed in learning to emulate a human subject, but only after observing the subject's behavior for (say) 10^{100} years? Does making the subject immortal, in order to give the predictor enough time, belong to the set of allowed interventions? Likewise, suppose that, after observing the subject's behavior for 20 years, the predictor becomes able to predict the subject's future behavior probabilistically, but *only for the next* 20 *years*, not indefinitely? The definition doesn't consider this sort of "time-limited" prediction, even though intuitively, it seems almost as hard to reconcile with free will as the unlimited kind. On the other hand, the actual numbers matter: a predictor that needed 20 years of data-gathering, in order to learn enough to predict the subject's behavior for the 5 seconds immediately afterward, would seem intuitively compatible with freedom. In any case, in this essay I mostly ignore quantitative timing issues (except for brief discussions in Sections 2.12 and 9), and imagine for simplicity that we have a predictor that after some finite time learns to predict the subject's responses arbitrarily far into the future.

13 Appendix: Prediction and Kolmogorov Complexity

As mentioned in Section 3.1, some readers will take issue with the entire concept of Knightian uncertainty—that is, with uncertainty that can't even be properly quantified using probabilities. Among those readers, some might be content to assert that there exists a "true, objective" prior probability distribution $\mathcal{D}$ over all events in the physical world—and while we might not know any prescription to *calculate* $\mathcal{D}$ that different agents can agree on, we can be sure that agents are irrational to whatever extent their own priors deviate from $\mathcal{D}$. However, more sophisticated readers might try to *derive* the existence of a roughly-universal prior, using ideas from *algorithmic information theory* (see Li and Vitányi [54] for an excellent introduction). In this appendix, I'd like to sketch how the latter argument would go and offer a response to it.

Consider an infinite sequence of bits $b_1, b_2, b_3, \ldots$, which might be generated randomly, or by some hidden computable pattern, or by some process with elements of both. (For example, maybe the bits are uniformly random, except that every hundredth bit is the "majority vote" of the previous 99 bits.) We can imagine, if we like, that these bits represent a sequence of yes-or-no decisions made by a human being. For each $n \geq 1$, a superintelligent predictor is given $b_1, \ldots, b_{n-1}$, and asked to predict b_n. Then the idea of algorithmic statistics is to give a *single* rule, which can be proved to predict b_n "almost as well" as any other computable rule, in the limit $n \to \infty$.

Here's how it works. Choose any Turing-universal programming language L, which satisfies the technical condition of being *prefix-free*: that is, adding characters to the end of a valid program never yields another valid program. Let P be a program written in L, which runs for an infinite time and has access to an unlimited supply of random bits, and which generates an infinite sequence $B = (b_1, b_2, \ldots)$ according to some probability distribution $\mathcal{D}_P$. Let $|P|$ be the number of bits in P. Then for its initial guess as to the behavior of B, our superintelligent predictor will use the so-called *universal prior* $\mathcal{U}$, in which each distribution $\mathcal{D}_P$ appears with probability $2^{-|P|}/C$, for some normalizing constant $C = \sum_P 2^{-|P|} \leq 1$. (The reason for the prefix-free condition was to ensure that the sum $\sum_P 2^{-|P|}$ converges.) Then, as the bits $b_1, b_2, \ldots$ start appearing, the predictor repeatedly updates $\mathcal{U}$ using Bayes' rule, so that its estimate for $\Pr[b_n = 1]$ is always

$$\frac{\Pr_{\mathcal{U}}[b_1 \ldots b_{n-1}1]}{\Pr_{\mathcal{U}}[b_1 \ldots b_{n-1}]}.$$

Now suppose that the "true" distribution over B is $\mathcal{D}_Q$, for some particular program Q. Then I claim that, in the limit $n \to \infty$, a predictor that starts with $\mathcal{U}$ as its prior will do just as well as if it had started with $\mathcal{D}_Q$. The proof is simple: by definition, $\mathcal{U}$ places a "constant fraction" of its probability mass on $\mathcal{D}_Q$ from the beginning (where the "constant," $2^{-|Q|}/C$, admittedly depends on $|Q|$). So for all n and $b_1 \ldots b_n$,

$$\frac{\Pr_{\mathcal{U}}[b_1 \ldots b_n]}{\Pr_{\mathcal{D}_Q}[b_1 \ldots b_n]} = \frac{\Pr_{\mathcal{U}}[b_1]\Pr_{\mathcal{U}}[b_2|b_1] \cdots \Pr_{\mathcal{U}}[b_n|b_1 \ldots b_{n-1}]}{\Pr_{\mathcal{D}_Q}[b_1]\Pr_{\mathcal{D}_Q}[b_2|b_1] \cdots \Pr_{\mathcal{D}_Q}[b_n|b_1 \ldots b_{n-1}]} \geq 2^{-|Q|}.$$

Hence

$$\prod_{n=1}^{\infty} \frac{\Pr_{\mathcal{U}}[b_n|b_1 \ldots b_{n-1}]}{\Pr_{\mathcal{D}_Q}[b_n|b_1 \ldots b_{n-1}]} \geq 2^{-|Q|}$$

as well. But for all $\varepsilon > 0$, this means that $\mathcal{U}$ can assign a probability to the correct value of b_n less than $1 - \varepsilon$ times the probability assigned by $\mathcal{D}_Q$, only for $O(|Q|/\varepsilon)$ values of n or fewer.

Thus, an "algorithmic Bayesian" might argue that there are only two possibilities: either a physical system is predictable by the universal prior $\mathcal{U}$, or else—to whatever extent it isn't—in any meaningful sense the system behaves randomly. There's no third possibility that we could identify with Knightian uncertainty or freebits.

One response to this argument—perhaps the response Penrose would prefer—would be that we can easily defeat the so-called "universal predictor" $\mathcal{U}$, using a sequence of bits $b_1, b_2, \ldots$ that's deterministic but noncomputable. One way to construct such a sequence is to "diagonalize against $\mathcal{U}$," defining

$$b_n := \begin{cases} 0 & \text{if } \Pr_{\mathcal{U}}[b_1 \dots b_{n-1}1] > \Pr_{\mathcal{U}}[b_1 \dots b_{n-1}0], \\ 1 & \text{otherwise} \end{cases}$$

for all $n \geq 1$. Alternatively, we could let b_n be the n^{th} binary digit of Chaitin's constant Ω [21] (basically, the probability that a randomly generated computer program halts).[61] In either case, $\mathcal{U}$ will falsely judge the b_n's to be random even in the limit $n \to \infty$. Note that an even more powerful predictor $\mathcal{U}'$, equipped with a suitable oracle, could predict either of these sequences perfectly. But then we could construct new sequences $b_1', b_2', \dots$ that were unpredictable even by $\mathcal{U}'$, and so on.

The response above is closely related to a notion called *sophistication* from algorithmic information theory (see [6, 38, 54]). Given a binary string x, recall that the *Kolmogorov complexity* $K(x)$ is the length of the shortest program, in some Turing-universal programming language, whose output (given a blank input) is x. To illustrate, the Kolmogorov complexity of the first n bits of $\pi \approx 11.00100100\ldots_2$ is small ($\log_2 n + O(\log\log n)$), since one only has to provide n (which takes $\log_2 n$ bits), together with a program for computing π to a given accuracy (which takes some small, fixed number of bits, independent of n). By contrast, if x is an n-bit string chosen uniformly at random, then $K(x) \approx n$ with overwhelming probability, simply by a counting argument. Now, based on those two examples, it's tempting to conjecture that "every string is either highly-patterned or random": that is, either

(i) $K(x)$ is small, or else

(ii) $K(x)$ is large, but only because of "boring, random, patternless entropy" in x.

Yet the above conjecture, when suitably formalized, turns out to be false. Given a set of strings $S \subseteq \{0,1\}^n$, let $K(S)$ be the length of the shortest program that lists the elements of S. Then given an n-bit string x and a small parameter c, one can define the *c-sophistication* of x, or $\mathrm{Soph}_c(x)$, to be the minimum of $K(S)$, over all sets $S \subseteq \{0,1\}^n$ such that $x \in S$ and

$$K(S) + \log_2 |S| \leq K(x) + c.$$

Intuitively, the sophistication of x is telling us, in a near-minimal program for x, how many bits of the program need to be "interesting code" rather than "algorithmically random data." Certainly $\mathrm{Soph}_c(x)$ is well-defined and at most $K(x)$, since we can always just take S to be the singleton set $\{x\}$. Because of this, highly-patterned strings are unsophisticated. On the other hand,

[61] For completeness, let me prove that the universal predictor $\mathcal{U}$ fails to predict the digits of $\Omega = 0.b_1b_2b_3\ldots$. Recall that Ω is *algorithmically random*, in the sense that for all n, the shortest program to generate $b_1 \dots b_n$ has length $n - O(1)$. Now, suppose by contradiction that $\Pr_{\mathcal{U}}[b_1 \dots b_n] \geq L/2^n$, where $L \gg n$. Let A_n be a program that dovetails over all programs Q, in order to generate better and better lower bounds on $\Pr_{\mathcal{U}}[x]$ for all n-bit strings x (converging to the correct probabilities in the infinite limit). Then we can specify $b_1 \dots b_n$ by saying: "when A_n is run, $b_1 \dots b_n$ is the j^{th} string $x \in \{0,1\}^n$ such that A_n's lower bound on $\Pr_{\mathcal{U}}[x]$ exceeds $L/2^n$." Since there are at most $2^n/L$ such strings x, this description requires at most $n - \log_2 L + \log_2 n + O(1)$ bits. Furthermore, it clearly gives us a procedure to generate $b_1 \dots b_n$. But if $L \gg n$, then this contradicts the fact that $b_1 \dots b_n$ has description length $n - O(1)$. Therefore

$$\Pr_{\mathcal{U}}[b_1 \dots b_n] = \prod_{i=1}^{n} \Pr_{\mathcal{U}}[b_i | b_1 \dots b_{i-1}] = O\left(\frac{n}{2^n}\right),$$

and $\mathcal{U}$ hardly does better than chance.

random strings are *also* unsophisticated, since for them, we can take S to be the entire set $\{0,1\}^n$. Nevertheless, it's possible to prove [6, 38] that there exist highly sophisticated strings: indeed, strings x such that $\mathrm{Soph}_c(x) \geq n - O(\log n)$. These strings could thus be said to inhabit a third category between "patterned" and "random." Not surprisingly, the construction of sophisticated strings makes essential use of uncomputable processes.

However, for reasons explained in Section 6, I'm exceedingly reluctant to postulate uncomputable powers in the laws of physics (such as an ability to generate the digits of Ω). Instead, I would say that, if there's scope for freedom, then it lies in the fact that even when a sequence of bits $b_1, b_2, \ldots$ is computable, the universal predictor is only guaranteed to work in the limit $n \to \infty$. Intuitively, once the predictor has figured out the program Q generating the b_n's, it can *then* predict future b_n's as well as such prediction is possible. However, the number of serious mistakes that the predictor makes before converging on the correct Q could in general be as large as Q's bit-length. Worse yet, there's no finite time after which the predictor can *know* that it's converged on the correct Q. Rather, in principle the predictor can always be surprised by a bit b_n that diverges from the predictions of whatever hypothesis Q it favored in the past, whereupon it needs to find a new hypothesis Q', and so on.

Some readers might object that, in the real world, it's reasonable to assume an upper bound on the number of bits needed to describe a given physical process (for example, a human brain). In that case, the predictor would indeed have an absolute upper bound on $|Q|$, and hence on the number of times it would need to revise its hypothesis substantially.

I agree that such bounds on $|Q|$ almost certainly exist—indeed, they must exist, if we accept the holographic principle from quantum gravity (see Section 5.4). For me, the issue is simply that the relevant bounds seem too large to be of any practical interest. Suppose, for example, that we believed 10^{14} bits—or roughly one bit per synapse—sufficed to encode everything of interest about a particular human brain. While that strikes me as an underestimate, it still works out to roughly $40,000$ bits per second, assuming an 80-year lifespan. In other words, it seems that a person of normal longevity would have more than enough bits to keep the universal predictor $\mathcal{U}$ on its toes!

The above estimate leads to amusing thought: *if* one lived forever, then perhaps one's "store of freedom" would eventually get depleted, much like an n-bit computer program can surprise $\mathcal{U}$ at most $O(n)$ times. (Arguably, this depletion happens to some extent over our actual lifetimes, as we age and become increasingly predictable and set in our ways.) From this perspective, freedom could be considered merely a "finite-n effect"—but this would be one case where the value of n matters!

14 Appendix: Knightian Quantum States

In Section 3.2, I introduced the somewhat whimsically-named *freestate*: a representation of knowledge that combines probabilistic, quantum-mechanical, and Knightian uncertainty, thereby generalizing density matrices, which combine probabilistic and quantum uncertainty. (The "freebits" referred to throughout the essay are then just 2-level freestates.) While there might be other ways to formalize the concept of freestates, in this appendix I'll give a particular formalization that I prefer.

A good starting point is to combine probabilistic and Knightian uncertainty, leaving aside quantum mechanics. For simplicity, consider a bit $b \in \{0,1\}$. In the probabilistic case, we can specify our knowledge of b with a single real number, $p = \Pr[b=1] \in [0,1]$. In the Knightian case, however, we might have a set of *possible* probabilities: for example,

$$p \in \{0.1\} \cup [0.2, 0.3] \cup (0, 4.0.5)\,. \tag{*}$$

This seems rather complicated! Fortunately, we can make several simplifications. Firstly, since we don't care about infinite precision, we might as well take all the probability intervals to be closed. More importantly, I believe we should assume *convexity*: that is, if $p < q$ are both possible probabilities for some event E, then so is every intermediate probability $r \in [p, q]$. My argument is simply that Knightian uncertainty includes probabilistic uncertainty as a special case: if, for example, we have no idea whether the bit b was generated by process P or process Q, then for all we know, b might *also* have been generated by choosing between P and Q with some arbitrary probabilities.

Under the two rules above, the disjunction (*) can be replaced by $p \in [0.1, 0.5]$. More generally, it's easy to see that our states will always be nonempty, convex regions of the probability simplex: that is, nonempty sets S of probability distributions that satisfy $\alpha\mathcal{D}_1 + (1-\alpha)\,\mathcal{D}_2 \in S$ for all $\mathcal{D}_1, \mathcal{D}_2 \in S$ and all $\alpha \in [0,1]$. Such a set S can be used to calculate upper and lower bounds on the probability $\Pr[E]$ for any event E. Furthermore, there's no redundancy in this description: if $S_1 \neq S_2$, then it's easy to see that there exists an event E for which S_1 allows a value of $\Pr[E]$ not allowed by S_2 or vice versa.

One might worry about the "converse" case: probabilistic uncertainty over different states of Knightian uncertainty. However, I believe this case can be "expanded out" into Knightian uncertainty about probabilistic uncertainty, like so:

$$\frac{(A \text{ OR } B) + (C \text{ OR } D)}{2} = \left(\frac{A+C}{2}\right) \text{ OR } \left(\frac{A+D}{2}\right) \text{ OR } \left(\frac{B+C}{2}\right) \text{ OR } \left(\frac{B+D}{2}\right).$$

By induction, any hierarchy of probabilistic uncertainty about Knightian uncertainty about probabilistic uncertainty about... etc. can likewise be "collapsed," by such a procedure, into simply a convex set of probability distributions.

The quantum case, I think, follows exactly the same lines, except that now, instead of a convex set of probability distributions, we need to talk about a convex set of density matrices. Formally, an *n-dimensional freestate* is a nonempty set S of $n \times n$ density matrices such that $\alpha\rho + (1-\alpha)\,\sigma \in S$ for all $\rho, \sigma \in S$ and all $\alpha \in [0,1]$. Once again, there is no redundancy involved in specifying our knowledge about a quantum system in this way. The argument is simply the following: for all nonempty convex sets $S_1 \neq S_2$, there either exists a state $\rho \in S_1 \setminus S_2$ or a state $\rho \in S_2 \setminus S_1$. Suppose the former without loss of generality. Then by the convexity of S_2, it is easy to find a pure state $|\psi\rangle$ such that $\langle\psi|\rho|\psi\rangle \notin \{\langle\psi|\sigma|\psi\rangle : \sigma \in S_2\}$.