MILITARY TOPOGRAPHIC DEPARTMENT
OF THE GENERAL STAFF OF ARMED FORCES
OF THE RUSSIAN FEDERATION

"PARAMETRY ZEMLI 1990"
(PZ-90.11)

Reference Document

Moscow – 2014

The “Parametry Zemli 1990”[1] (PZ-90.11) reference document contains geometric and physical numerical geodetic parameters, characteristics of fundamental geodetic constants, reference ellipsoid, geocentric coordinate system and the Earth’s gravity field as well as information about their representation and implementation recommendations.

Given data meet the modern requirements and are intended for geodesists, specialists in orbital calculations, designers and users of navigation systems.

The reference document was developed in Scientific-research center for survey and navigation support of the 27th Central research institute of Ministry of Defense of the Russian Federation.

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[1] In English “The Earth Parameters 1990”

TABLE OF CONTENTS

LIST OF SYMBOLS AND ABBREVIATIONS

AEGF – Abnormal Earth's Gravity Field
GA – Gravity Anomaly
QH – Quasigeoid Height
GLONASS – GLObal NAvigation Satellite System
EGF – Earth's Gravity Field
SV – Space Vehicle
SGC – Space Geodetic Complex
SGN – Space Geodetic Network
PZ-90 – Earth Parameters "Parametry Zemli 1990" (PZ-90)
PZ-90.02 – Earth Parameters "Parametry Zemli 1990" (PZ-90.02)
PZ-90.11 – Earth Parameters "Parametry Zemli 1990" (PZ-90.11)
RAS – Russian Academy of Science
RMS – Root-Mean-Square
SD AF RF – Survey Department of Armed Forces of the Russian Federation
DM – Digital Model
BIH – Bureau International de l'Heure (International Time Bureau)
FK – Fundamental Katalog (Catalogue of Fundamental Stars)
GPS – Global Positioning Systems
IERS – International Earth Rotation and Reference Systems Service
IGS – International GNSS Service
ITRF – International Terrestrial Reference Frame
ICRS – International Celestial Reference System
ICRF – International Celestial Reference Frames
ITRS – International Terrestrial Reference System
WGS – World Geodetic System

INTRODUCTION

With the Decree of the Government of the Russian Federation No.568 dated July 28, 2000 the geocentric coordinate system being part of the “Earth Parameters 1990” (PZ-90) was given national status for navigation and geodetic support of orbital missions. Rapidly growing requirements to navigation accuracy, widespread use of GLONASS/GPS receivers have raised the need to improve accuracy of parameter definition specifying shape and dimensions of the Earth and its gravity field as well as to refine the entire system of geodetic Earth parameters on a regular basis.

The first modernization of the geocentric coordinate system “Parametry Zemli 1990” (PZ-90) was completed in 2002 (PZ-90.02) with extensive scope of SGC GEO-IK measurement information obtained since 1990 that was not processed by commissioning PZ-90 and precise measurements from stations of space geodetic network (SGN) obtained with GLONASS/GPS equipment. It has led to considerable improvement in accuracy of the state geocentric coordinate system relative to PZ-90, accuracy of geodetic binding of GLONASS ground control segment measuring tools and calculation of GLONASS satellite ephemeris.

With the Order of the Government of the Russian Federation No.797-r dated June 20, 2007 a refined version of the state geocentric coordinate system being part of the “Parametry Zemli 1990” (PZ-90.02) was implemented to improve GLONASS performances and geodetic support of orbital missions and navigation.

In 2011 the state geocentric coordinate system “Parametry Zemli 1990” (PZ-90.02) was specified (PZ-90.11) with extensive scope of GLONASS/GPS precise measurements from SGN stations and a number of IGS stations. With the Decree of the Government of the Russian Federation No.1463 dated December 28, 2012 the global terrestrial geocentric reference system “Parametry Zemli 1990” (PZ-90.11) became the national reference system for geodetic support of orbital missions and navigation.

Geometrical and physical numeric geodetic parameters related to the global terrestrial geocentric reference system “Parametry Zemli 1990” (PZ-90.11) were approved by the Order of the Minister of Defense of the Russian Federation No.11 dated January 15, 2014.

1 GENERAL PROVISIONS

The "Parametry Zemli 1990" and its last version PZ-90.11 represent the system of geodetic parameters including fundamental geodetic constants, reference ellipsoid parameters, the Earth's gravity field parameters, geocentric coordinate system and transformation parameters to other reference systems.

Values of geodetic parameters were defined on the basis of co-processed space- and ground-based data. Satellite dynamic method was taken as a methodological basis for measurement processing. Compensation computations were based on general least square method. List and numerical characteristics of parameters of reference ellipsoid and the Earth's gravity field models remain unchanged in PZ-90.11. Only the geocentric coordinate system being part of the "Parametry Zemli 1990" was specified.

By clarifying the geocentric coordinate system the best use was made of data on defining global terrestrial reference systems obtained by national and international scientific organizations that were based on satellite and space object observations. In PZ-90.11 axes orientation, scale and origin ensured alignment to the similar ITRF parameters at the cm level.

For improving accuracy of SGN station relative position and alignment accuracy to ITRF a representative number of GPS and GLONASS observations were used that were collected after commissioning PZ-90.02. A new step in clarifying geocentric position of station network that fix PZ-90.11 was to include in processing a number of measuring and relevant DORIS information (2002, 2008 and 2010) obtained from combined stations of this system and IGS network.

The Addendum to the reference document includes geopotential model parameters, a list of SGN station coordinates in PZ-90.11 and other information including IGS station coordinates in PZ-90.11. These data can be provided in the prescribed manner.

2 BASIC CONSTANTS OF THE SYSTEM OF GEODETIC EARTH PARAMETERS

Set of geodetic parameters included in PZ-90.11 has been defined with regard to practical requirements of geodesy, geophysics, navigation and ballistics concerning figure and dimensions of the Earth and its gravity field. Values of universal physical constants used by geodetic parameter output are shown in Table 1.

Table 1

Constant	Symbol	Unit	Value
Velocity of light in vacuum	c	m/s	299 792 458
Gravitational constant	f	$m^3/(kg \cdot s^2)$	$6.672\,59 \cdot 10^{-11}$

Geodetic reference system is based on the theory of level ellipsoid of revolution that uniquely determines figure of reference ellipsoid and normal gravity field through four independent parameters fM, ω, a and α – fundamental geodetic constants. Their values are shown in Table 2.

Table 2

Parameter	Symbol	Unit	Value
Gravitational constant (mass of Earth's atmosphere included)	fM	m^3/s^2	$398\,600.4418 \cdot 10^{+9}$
Angular velocity of the Earth	ω	rad/s	$7.292\,115 \cdot 10^{-5}$
Semi-major axis of reference ellipsoid	a	m	6 378 136
Flattering factor of the Earth	α	–	1/298.257 84

All other geodetic constants defining physical and geometric Earth characteristics represented in the form of level ellipsoid of revolution (Earth with normal gravity field) are related to these parameters through strict mathematical relations.

Parameters a and α defining the shape of reference ellipsoid remain unchanged related to PZ-90 and PZ-90.02. In some applications value of gravitational constant fM' is used with mass of Earth's atmosphere not included. This value is defined by the equation

$$fM' = fM - fM_a, \tag{2.1}$$

where fM_a – a component of geocentric gravitational constant due to existence of the atmosphere,

$$fM_a = 0.3500 \cdot 10^{+9} \text{ m}^3/\text{s}^2.$$

Table 3 shows geodetic constants specifying geometric and physical characteristics of the Earth in general that are calculated with values of fundamental geodetic constants given in Table 2.

Table 3

Constant	Symbol	Unit	Value
Geometric constants			
Semi-minor axis	b	m	6 356 751.3618
Ellipsoid eccentricity squared	e^2	–	0.006 694 3662
Second ellipsoid eccentricity squared	e'^2	–	0.006 739 4828
Physical constants			
Normal potential on the ellipsoid surface	U_0	m^2/s^2	62 636 861.4
Normal gravity at the ellipsoid equator	γ_a	mGal	978 032.84
Normal gravity at the ellipsoid pole	γ_b	mGal	983 218.80
Coefficients in equation for acceleration of normal gravity	β	–	0.005 3024
	β_1	–	0.000 0058
Correction in acceleration of normal gravity due to attraction of the atmosphere on the sea level	$\delta\gamma_a$	mGal	– 0.87
Second degree zonal coefficient of normal potential	J_2^0	–	$1\,082.625\,75 \cdot 10^{-6}$
Fourth degree zonal coefficient of normal potential	J_4^0	–	$-2.370\,89 \cdot 10^{-6}$
Sixth degree zonal coefficient of normal potential	J_6^0	–	$6.08 \cdot 10^{-9}$
Eighth degree zonal coefficient of normal potential	J_8^0	–	$-1.40 \cdot 10^{-11}$

3 REFERENCE SYSTEMS

Global terrestrial and celestial reference systems and local coordinate systems are used in geodetic applications. Among local coordinate systems are national coordinate systems.

3.1 Global terrestrial reference systems

By definition global terrestrial reference system is a geocentric spatial reference system with the center of mass of the Earth System being as origin (fig. 1). *Z*-axis is directed to the Conventional Reference Pole (Conventional International Origin) that was defined by the International Earth Rotation and Reference System Service (IERS), *X*-axis is directed to the intersection point of the equatorial plane and the Zero Meridian, defined by IERS and Bureau International de l’Heure (BIH), *Y*-axis completes a right-handed system. Global terrestrial reference system rotates along with the Earth.

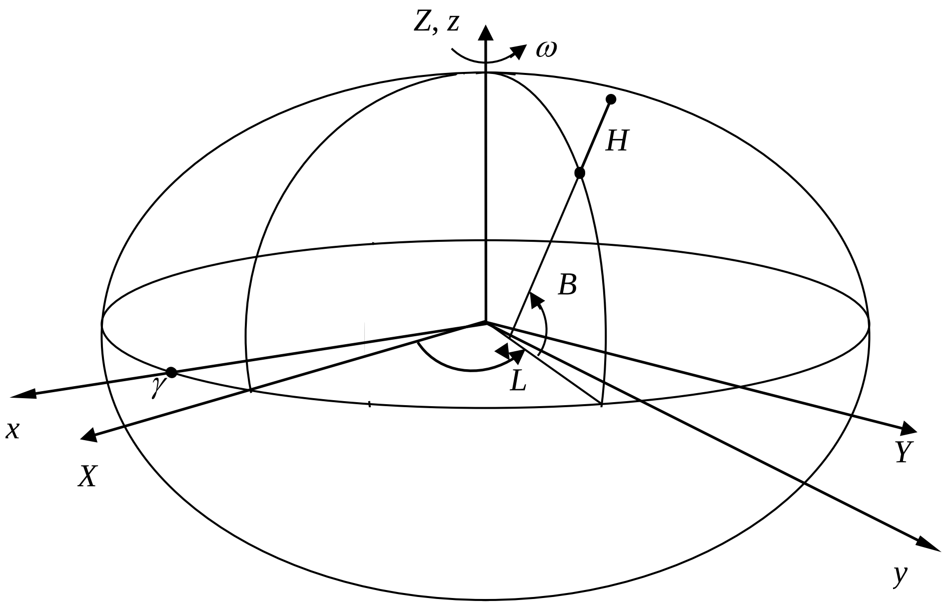

Fig. 1

In geocentric reference system point position is defined by *X*, *Y*, *Z* coordinates. In geodetic applications for the same purpose geodetic coordinates *B*, *L*, *H* are used that are related to reference ellipsoid – ellipsoid of revolution which geometric center coincides with the center of global terrestrial reference system (fig. 1). *Z*-axis serves as rotational axis of ellipsoid.

Geodetic latitude B is determined as the angle between normal line passing through the given point to ellipsoid and equatorial plane; geodetic longitude L – the dihedral angle between the Zero Meridian plane and the plane of the meridian passing through the given point (positive direction of longitudes is the direction from the Zero Meridian to the east, from 0° to 360°); geodetic height H – the segment of normal line to reference ellipsoid from its surface to the given point. Mathematical relationship of geodetic and spatial Cartesian coordinates is defined by the equations given in Appendix 1.

Geocentric reference frame being part of PZ–90.11 is a practical realization of global terrestrial reference system at epoch 2010.0. It is fixed by globally allocated SGN stations, which coordinates and velocities are determined from the processing of satellite measurements. Accuracy of geocentric reference system PZ–90.11 regarding the center of mass of the Earth System is determined by RMS valued 0.05 m and for the direction of the axes – valued 0.001″. RMS of relative position of stations is calculated to be 0.005 – 0.01 m. Scale accuracy of the reference system conforms to the current state of knowledge of the speed of light, the geocentric gravitational constant as well as the accuracy of the satellite laser ranging measurements characterized by RMS 0.001 – 0.005 m.

PZ–90.11 reference system is distributed on a number of IGS stations. Coordinates of SGN and IGS stations on the Russian territory in PZ-90.11 at epoch 2010.0 are presented in Appendix 2.

World Geodetic System WGS 84 (USA) and International Terrestrial Reference System ITRS, which is supported and regularly updated by IERS, are analogues of geocentric reference system PZ–90. International Terrestrial Reference Frame (ITRF) is a practical realization of ITRS.

By determining PZ-90, WGS 84 and ITRS the same theoretical principles were used. However, by their practical realization small divergences between referred systems stand out that can be explained by differences in structure and scope of used measurement information and methodological differences. To evaluate the value of these divergences the (3.2) equation is used.

3.2 Celestial reference system

International Celestial Reference System (ICRS) is the equivalent to the inertial reference system of Newtonian astrometry. The ICRS is realized by the International Celestial Reference Frame (ICRF). A realization of the ICRF consists of a set of precise coordinates of compact extragalactic radio sources. Nowadays for computing ephemerides of stars fundamental lists FK6 and HIPPARCOS that are fixed to ICRS are used where the position of reference stars is specified in mean celestial reference system at standard epoch 2000.0 (JD2000.0). Relationship of global terrestrial and celestial reference systems is defined by the equations given in Appendix 3.

The origin of celestial reference system coincides with the center of global terrestrial reference system (fig. 1), z-axis is directed to the Celestial North Pole, x-axis is located in the equatorial plane and directed to the First Point of Aries γ, *y*-axis completes a right-handed system. Celestial pole and the First Point of Aries do not retain unchanged its position in space as time passes that's why they take standard epoch 2000.0 to fix the inertial reference system.

Celestial reference system is used for orbital and ballistic analysis. For translation to global terrestrial reference system the equation (A.3.1) is used.

3.3 Reference geodetic coordinate systems

Reference surface in reference geodetic coordinate systems as well as in global terrestrial reference system is ellipsoid of revolution. The difference is that origin and orientation of axes in these systems may not coincide.

When translating geodetic coordinates *B*, *L*, *H* in spatial Cartesian coordinates *X*, *Y*, Z the same equations as for global terrestrial geodetic reference system (given in Appendix 1) are used for reference system. Relationship of spatial Cartesian coordinates *X*, *Y*, *Z* in reference and global terrestrial reference systems is defined by the equation (3.2).

Since 1946 reference coordinate system 1942 ("Sistema Koordinat 1942" – SK–42) has been in place in Russia. With the Decree of the Government of the Russian Federation No.568 dated July 28, 2000 since July 1, 2002 reference coordinate system 1995 ("Sistema Koordinat 1995" – SK–95) has been implemented. Reference coordinate systems SK–42 and SK–95 are fixed by stations of the state geodetic network. With the Decree of the Government of the Russian Federation No.1463 dated December 28, 2012 the geodetic reference coordinate system GSK–2011 ("Geodezicheskaya Sistema Koordinat 2011") was defined as the national system for geodetic survey and mapping.

3.3.1 Coordinate system 1942

The Krasovsky ellipsoid with semi-major axis a_{Kr} = 6 378 245 m and flattering factor α_{Kr} = 1/298.3 is taken as reference surface. Center of the Krasovsky ellipsoid coincides with the center of the reference coordinate system. At the epoch of SK–42 generation this was achieved by implementing two conditions: parallelism of Z-axis of reference coordinate system and mean axis of Earth's rotation and parallelism of planes of prime astronomical and geodetic meridians.

These conditions could not be properly monitored at that time because of measurment accurracy and limited capacity of processing (adjustment) that led to rotation of axes of the reference coordinate system that was reliably estimated only in the beginning of the 80s with using satellite data.

3.3.2 Coordinate system 1995

The Krasovsky ellipsoid is taken as reference surface in coordinate system 1995 as well as in SK–42. Axes of SK–95 are determined under condition of being parallel with axes of global terrestrial coordinate system PZ–90.

3.3.3 Geodetic reference system GSK-2011

Geodetic reference system GSK–2011 is practical realization of terrestrial spatial reference system with origin in center of mass of the Earth System. *Z*-axis is directed to the Conventional Reference Pole that was defined by IERS, *X*-axis is directed to the intersection point of the equatorial plane and the Zero Meridian, defined by IERS and BIH, *Y*-axis completes a right-handed system. Origin of the reference system serves as geometric center of reference ellipsoid with following parameters a = 6 378 136.5 m, α = 1/298,256 415 1. Reference frame GSK–2011 is fixed by stations of fundamental astronomic geodetic network (about 50 stations) at epoch 2011.0.

3.4 Height systems

By determining height system it is critical to choose reference surface and origin, the height of which is taken to be zero. The geoid and the quasigeoid are the reference surfaces. Heights above the geoid surface are known as orthometric heights, above the quasigeoid – normal heights. Also in practice geodetic heights can be used that are measured above the ellipsoid surface (global terrestrial or reference).

The geoid is a model of the figure of the Earth formed by level (equipotential) surface of gravity potential that coincides with mean sea level over the oceans at a standstill and extendable under the mainland. It is impossible to determine the geoid position in relation to the chosen ellipsoid only on the basis of ground measurements. Quasigeoid that represents surface close to the geoid can be determined with mathematical certainty based on ground gravity data. Offset between the geoid and quasigeoid comes to 2-5 cm on average in flat country and up to 2 m in high mountain area.

The Baltic normal height system was adopted in the Russian Federation. In this system normal heights are transmitted from the datum level (zero-mark of the Kronstadt tide gauge) to the stations of the altitude (levelling) network by means of geometrical levelling. Nowadays the Baltic height system 1977 is in use. Relationship between geodetic H and normal H^γ heights are defined by the equation:

$$H = H^\gamma + \zeta, \qquad (3.1)$$

where ζ is a quasigeoid height above the given ellipsoid.

Normal heights do not depend on selected reference ellipsoid. Geodetic heights and quasigeoid heights depend on the selected reference ellipsoid. Normal heights are displayed on topographic maps and are listed in catalogues of coordinates of geodetic stations.

3.5 Coordinate transformations

<u>Transformation of spatial Cartesian coordinates</u>

Transforming coordinates in system 1 to coordinates in system 2 is made through the equation

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_2 = (1+m)\begin{pmatrix} 1 & +\omega_Z & -\omega_Y \\ -\omega_Z & 1 & +\omega_X \\ +\omega_Y & -\omega_X & 1 \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_1 + \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix}, \qquad (3.2)$$

where $\Delta X, \Delta Y, \Delta Z$ – linear elements of reference system transformation for transforming system 1 to system 2, m;

$\omega_X, \omega_Y, \omega_Z$ – angular elements of reference system transformation for transforming system 1to system 2, rad;

m – scale element of reference system transformation for transforming system 1 to system 2

By reverse transformation of spatial Cartesian Coordinates transformation elements have the same values but with reverse sign. Values of transformation elements for the most widespread reference systems are shown in Appendix 4.

Transformation of geodetic coordinates

Transforming coordinates in system (1) to coordinates in system 2 is made through the equation:

$$\left.\begin{aligned} B_2 &= B_1 + \Delta B \\ L_2 &= L_1 + \Delta L \\ H_2 &= H_1 + \Delta H \end{aligned}\right\}, \qquad (3.3)$$

where B, L, H – geodetic point latitude, longitude and height;

$\Delta B, \Delta L, \Delta H$ – corrections to geodetic coordinates.

Corrections to geodetic coordinates are defined by the following equations:

$$\left.\begin{aligned} &\Delta B = \frac{\rho''}{(M+H)}[\frac{N}{a}e^2 \sin B \cos B \Delta a + \left(\frac{N^2}{a^2}+1\right) N \sin B \cos B \frac{\Delta e^2}{2} - \\ &-(\Delta X \cos L + \Delta Y \sin L)\sin B + \Delta Z \cos B] - \\ &-\omega_X \sin L\left(1+e^2 \cos 2B\right) + \omega_Y \cos L\left(1+e^2\cos 2B\right) - \rho'' m e^2 \sin B \cos B \\ &\Delta L = \frac{\rho''}{(N+H)\cos B}(-\Delta X \sin L + \Delta Y \cos L) + tgB(1-e^2)(\omega_X \cos L + \\ &+\omega_Y \sin L) - \omega_Z \\ &\Delta H = -\frac{a}{N}\Delta a + N \sin^2 B \frac{\Delta e^2}{2} + (\Delta X \cos L + \Delta Y \sin L)\cos B + \Delta Z \sin B - \\ &-Ne^2 \sin B \cos B(\frac{\omega_X}{\rho''}\sin L - \frac{\omega_Y}{\rho''}\cos L) + \left(\frac{a^2}{N}+H\right)m \end{aligned}\right\}, \qquad (3.4)$$

where $\Delta B, \Delta L, \Delta H$ corrections to geodetic latitude, geodetic longitude, arcsec and to height, m;

B, L, H – geodetic latitude, longitude, rad and height, m;

ΔX, ΔY, ΔZ – linear elements of reference systems transformation for transforming system 1to system 2, m;

ω_X, ω_Y, ω_Z – angular elements of reference systems transformation for transforming system 1 to system 2, arcsec;

m – scale elements of reference systems transformation for transforming system 1to system 2;

M – radius of curvature in the meridian $M = a(1-e^2)(1-e^2\sin^2 B)^{-\frac{3}{2}}$;

N – radius of curvature in the prime vertical $N = a(1-e^2\sin^2 B)^{-\frac{1}{2}}$;

$\rho'' = 206\ 264{,}806\ 25$ arcsec.

In the equations (3.4) values a и e^2 are taken equal

$$a = \frac{a_1 + a_2}{2}, \quad e^2 = \frac{e_1^2 + e_2^2}{2},$$

and Δa и Δe^2 are equal

$$\Delta a = a_2 - a_1, \quad \Delta e^2 = e_2^2 - e_1^2,$$

where a_1, a_2 – semi-major axes of ellipsoids in reference systems 1 and 2 correspondingly;

e_1^2, e_2^2 – squared ellipsoid eccentricities in reference systems 1 and 2 correspondingly.

By reverse transformation from system 2 to system 1 in the equation (3.3) values of geodetic coordinates in system 2 are used and correction sign ΔB, ΔL, ΔH is changed to reverse one.

Equations (3.4) ensure computation of corrections to geodetic coordinates with an error not exceeding 0.3 m (in linear measure) and in order to get the error not more than 0.001 m the second iteration is performed. It means that the values of corrections to geodetic coordinates computed by the equation (3.3) are taken into consideration and the computations are performed over again by the equations (3.4). In the second iteration the following values are used:

$$B = \frac{B_1 + (B_1 + \Delta B)}{2},$$

$$L = \frac{L_1 + (L_1 + \Delta L)}{2},$$

$$H = \frac{H_1 + (H_1 + \Delta H)}{2}.$$

Equations (3.3), (3.4) and accuracy specifications of transformation apply to latitudes ± 89°.

Transforming geodetic coordinates to plane Cartesian coordinates

Along with spatial Cartesian and geodetic coordinates point position can be specified by plane Cartesian coordinates x, y and geodetic heights H. By computing plane Cartesian coordinates in reference coordinate systems SK-42 and SK-95 Gauss–Kruger projection with parameters of the Krasovsky ellipsoid is used.

Plane Cartesian coordinates are computed by the equations:

$$\begin{aligned} \mathrm{x} = {} & 6\,367\,558.4968\,B - \sin 2B\,(16\,002.8900 + 66.9607\sin^2 B + \\ & + 0.3515\sin^4 B - l^2(1\,594\,561.25 + 5\,336.535\sin^2 B + \\ & + 26.790\sin^4 B + 0.149\sin^6 B + l^2(672\,483.4 - 811\,219.9\sin^2 B + \\ & + 5\,420.0\sin^4 B - 10.6\sin^6 B + l^2(278\,194 - 830\,174\sin^2 B + \\ & + 572\,434\sin^4 B - 16\,010\sin^6 B + l^2(109\,500 - 574\,700\sin^2 B + \\ & + 863\,700\sin^4 B - 398\,600\sin^6 B))))), \end{aligned} \tag{3.5}$$

$$\begin{aligned} \mathrm{y} = {} & (5 + 10\,n)\,10^5 + l\cos B\,(6\,378\,245 + 21\,346.1415\sin^2 B + \\ & + 107.1590\sin^4 B + 0.5977\sin^6 B + l^2(1\,070\,204.16 - \\ & - 2\,136\,826.66\sin^2 B + 17.98\sin^4 B - 11.99\sin^6 B + \\ & + l^2(270\,806 - 1\,523\,417\sin^2 B + 1\,327\,645\sin^4 B - \\ & - 21\,701\sin^6 B + l^2(79\,690 - 866\,190\sin^2 B + 1\,730\,360\sin^4 B - \\ & - 945\,460\sin^6 B)))), \end{aligned} \tag{3.6}$$

where x, y – plane Cartesian coordinates (abscissa and ordinate) of the given point in Gauss–Kruger projection, m;

B – geodetic latitude of the given point, rad;

l – distance from the given point to the central meridian zone, computed by the equation $l = \{L - [3 + 6(n - 1)]\}/57{,}295\ 77951$, rad;

L – geodetic longitude of the given point, degree;

n – number of six-degree-zone in Gauss–Kruger projection computed by the equation $n = E[(6 + L)/6]$;

$E[...]$ – whole part of the expression enclosed in square brackets.

Transforming plane Cartesian coordinates in Gauss–Kruger projection to geodetic coordinates on the Krasovsky ellipsoid is computed by the equations:

$$\begin{aligned} & B = B_0 + \Delta B; \\ & L = 6(n - 0.5)\,/\,57.29577951 + l, \end{aligned} \tag{3.7}$$

where B, L – geodetic latitude and longitude of the point of interest, rad;

B_0 – geodetic latitude of the point with abscissa being equal to the abscissa of the point of interest and ordinate being equal to zero, rad;

n – number of six-degree-zone in Gauss–Kruger projection computed by the equation $n = E[\mathrm{y}\cdot 10^{-6}]$;

Values B_0, ΔB and l are computed by the following equations:

$$B_0 = \beta + \sin 2\beta(0.002\ 52588685 - 0.000\ 01491860 \sin^2\beta + +0.000\ 00011904 \sin^4\beta), \quad (3.8)$$

$$\Delta B = -\mathrm{z}_0^2 \sin 2B_0(0.251\ 684631 - 0.003\ 369263 \sin^2 B_0 + 0.000\ 011276 \sin^4 B_0 - -\mathrm{z}_0^2(0.105\ 00614 - 0.045\ 59916 \sin^2 B_0 + 0.002\ 28901 \sin^4 B_0 - -0.000\ 02987 \sin^6 B_0 - \mathrm{z}_0^2(0.042\ 858 - 0.025\ 318 \sin^2 B_0 + 0.014\ 346 \sin^4 B_0 - - 0.001\ 264 \sin^6 B_0 - \mathrm{z}_0^2(0.01672 - 0.00630 \cdot \sin^2 B_0 + 0.01188 \sin^4 B_0 - - 0.00328 \sin^6 B_0)))), \quad (3.9)$$

$$l = \mathrm{z}_0(1 - 0.003\ 3467108 \sin^2 B_0 - 0.000\ 0056002 \sin^4 B_0 - -0.000\ 0000187 \sin^6 B_0 - \mathrm{z}_0^2(0.167\ 78975 + 0.162\ 73586 \sin^2 B_0 - -0.000\ 52490 \sin^4 B_0 - 0.000\ 00846 \sin^6 B_0 - z_0^2(0.042\ 0025 + +0.148\ 7407 \sin^2 B_0 + 0.005\ 9420 \sin^4 B_0 - 0.000\ 0150 \sin^6 B_0 - -\mathrm{z}_0^2(0.01225 + 0.09477 \sin^2 B_0 + 0.03282 \sin^4 B_0 - 0.00034 \sin^6 B_0 - -\mathrm{z}_0^2(0.0038 + 0.0524 \sin^2 B_0 + 0.0482 \sin^4 B_0 + 0.0032 \sin^6 B_0))))), \quad (3.10)$$

where β – auxiliary quantity computed by the equation
$\beta = \mathrm{x}/6\,367\,558.496\,8$;

z_0 – auxiliary quantity computed by the equation
$\mathrm{z}_0 = (\mathrm{y} - (10n + 5)10^5) / (6\ 378\ 245 \cos B_0)$;

x, y – abscissa and ordinate of the given point in Gauss–Kruger projection, m.

Error of coordinate transformation made by the equations (3.5) – (3.10) does not exceed 0.001 m.

Transformation of increase of spatial Cartesian coordinates

Transformation of increase of spatial Cartesian coordinates from system 1 to system 2 is performed by the equation

$$\begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix}_2 = (1+m)\begin{pmatrix} 1 & +\omega_Z & -\omega_Y \\ -\omega_Z & 1 & +\omega_X \\ +\omega_Y & -\omega_X & 1 \end{pmatrix}\begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix}_1 \quad (3.11)$$

By reverse transformation of increase of spatial Cartesian coordinates transformation elements have the same values but with reverse sign.

4 The Earth's gravity field

The Earth's gravity field represented by the gravity potential *W* is divided into two components – normal and abnormal potential

$$W = U + T \tag{4.1}$$

where *U* – normal potential defined as gravity potential of reference ellipsoid;

T – disturbing potential (gravitational potential) representing abnormal Earth's gravity field.

Normal potential is defined as a sum of gravitational potential of reference ellipsoid V_0 (normal gravitational potential) and potential of centrifugal force *Q*

$$U = V_0 + Q. \tag{4.2}$$

Potential *W* can be also defined as a sum of gravitational potential *V* and potential of centrifugal force *Q* where

$$W = V + Q, \tag{4.3}$$

$$V = V_0 + T. \tag{4.4}$$

To allow for influence of the Earth gravity field in orbital and ballistic computations and for solving geodetic and geophysical tasks numerical analytic Earth's gravitational models (EGMs) are used that are representing gravitational potential *V* and digital models of transformants[2] of disturbing potential *T* describing detailed specifics of abnormal Earth's gravity field.

By describing EGF planetary characteristics it is conveniently to use a unified form for both items (V_0 and *T*) of gravitational potential *V* as expansion in terms of spherical harmonics

$$V(\rho,\varphi,\lambda) = \frac{fM}{\rho}\left[1 + \sum_{n=2}^{N}\left(\frac{a}{\rho}\right)^{n}\sum_{m=0}^{n}\left(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}(\sin\varphi)\right], \tag{4.5}$$

where ρ, φ, λ – spherical geocentric point coordinates;

a – semi-major axis of reference ellipsoid;

fM – geocentric gravitational constant (mass of Earth's atmosphere included);

N – maximum degree of the expansion;

$\bar{C}_{nm}, \bar{S}_{nm}$ – fully normalized coefficients of degree *n* and order *m* of gravitational potential expansion in terms of spherical harmonics;

$\bar{P}_{nm}(\sin\varphi)$ – fully normalized Legendre polynomials of degree *n* and order *m* calculated by the equation (4.28).

<u>Normal gravitational potential</u> V_0 is described by the expansion with only zonal harmonics $\bar{C}^0_{n0}$ of even degrees:

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[2] The transformants of Earth gravitational field are astronomical and geodetic latitudes, longitudes, and azimuths of points on the Earth's surface, normal heights, gravity accelerations in arbitrary points on the local area of the surface, as well as their deviations from corresponding characteristics of the normal field in the form of deviations of the plumb line, height anomalies, gravity acceleration anomalies and other values.

$$V_0\,(\rho,\varphi)=\frac{fM}{\rho}\left[1+\sum_n\left(\frac{a}{\rho}\right)^n\overline{C}^0_{n0}\,\overline{P}_n(\sin\varphi)\right]\quad for\; n=2,4,6,8. \qquad (4.6)$$

where $\overline{P}_n(\sin\varphi)$ – fully normalized Legendre polynomials of degree n and order m = 0.

Disturbing potential T is represented by a complete set of harmonics starting with n = 2

$$T(\rho,\varphi,\lambda)=\frac{fM}{\rho}\sum_{n=2}^{N}\left(\frac{a}{\rho}\right)^n\sum_{m=0}^{n}(\Delta\overline{C}_{nm}\cos m\lambda+\overline{S}_{nm}\sin m\lambda)\overline{P}_{nm}(\sin\varphi), \qquad (4.7)$$

where $\Delta\overline{C}_{nm}=\overline{C}_{nm}$; with the exception of coefficients of zonal harmonics of even degrees n = 2, 4, 6 and 8 used for describing potential V_0. In that case $\Delta\overline{C}_{n0}=\overline{C}_{n0}-\overline{C}^0_{n0}$.

In a general way, coefficients $\overline{C}_{nm},\overline{S}_{nm}$ of planetary EGM are refined by satellite data along with fundamental geodetic constants and parameters that specify geocentric reference system. Listing and parameters of EGMs in the last version of PZ–90.11 were not refined and comply with PZ–90.02. Along with parameters of EGM in the form of coefficients $\overline{C}_{nm},\overline{S}_{nm}$ this edition comprises planetary EGM as point masses gravitational potential as well as samples of digital models of transformants of disturbing potential describing regional and local characteristics of abnormal Earth's gravity field. EGMs are defined as being "tide free".

4.1 Normal Earth's gravity field

Normal Earth's gravity field is represented by normal gravity potential U. The potential of normal gravity field U can be used independently as approximate representation of the Earth's figure and gravity field (Normal Earth). It also can be used as auxiliary quantity as a reference frame when solving boundary problems of potential theory. The U potential is described by the equation (4.2) with normal gravitational potential V_0 as in (4.6) and potential of centrifugal force is described by the equation

$$Q(\rho,\varphi)=\frac{\omega^2\rho^2}{2}\cos^2\varphi. \qquad (4.8)$$

In practical applications along with representing normal gravitational potential V_0 as in the notation (4.6) another notation is used

$$V_0\,(\rho,\varphi)=\frac{fM}{\rho}\left[1-\sum_{n=2}\left(\frac{a}{\rho}\right)^n J^0_n P_n(\sin\varphi)\right]\quad for\; n=2,4,6,8. \qquad (4.9)$$

Coefficients J^0_n and $\overline{C}^0_{n0}$ differ in sign and normalizing factor

$$J^0_n=-\sqrt{2n+1}\;\overline{C}^0_{n0},$$

$$J_2^0 = -\sqrt{5}\ \bar{C}_{20}^0, \qquad J_4^0 = -3\ \bar{C}_{40}^0,$$
$$J_6^0 = -\sqrt{13}\ \bar{C}_{60}^0, \qquad J_8^0 = -\sqrt{17}\ \bar{C}_{80}^0.$$

Harmonic coefficients J_n^0 are computed by values of fundamental geodetic constants *fM*, ω, *a* and α:

$$J_2^0 = \frac{e^2}{3} - \frac{2}{45}\frac{\bar{m}\, e^3}{q_0};$$
$$e^2 = 2\alpha - \alpha^2;$$
$$\bar{m} = \frac{\omega^2 a^3}{fM};$$
$$q_0 = \frac{1}{2}\left[\left(1 + \frac{3}{e'^2}\right)\text{arctg}\, e' - \frac{3}{e'}\right];$$
$$e'^2 = \frac{e^2}{1-e^2} = \frac{a^2 - b^2}{b^2};$$
$$b = \frac{a}{\sqrt{1+e'^2}},$$

and J_4^0, J_6^0, J_8^0 – by the equation:

$$J_n^0 = (-1)^{\frac{n}{2}+1}\frac{3e^{n-2}}{2(n+1)(n+3)}\left[\frac{5n}{2}J_2^0 - (n-2)e^2\right].$$

Values of parameters J_n^0, e^2 and e'^2 are shown in Table 3.

On the surface of reference ellipsoid normal potential is a constant value

$$U_0 = \frac{fM}{a}\left[1 - \sum_n J_n^0 P_n(0)\right] + \frac{\omega^2 a^2}{2} = \text{const},\ n = 2, 4, 6, 8, \tag{4.10}$$

where $P_n(0)$ – Legendre polynomials of degree *n* and order *m* = 0 for $\varphi = 0$. Value U_0 is shown in Table 3.

Normal gravity (mGal) on the ellipsoid surface (*H* = 0) is calculated by the equation:

$$\gamma(B, H_{=0}) = \frac{a\gamma_a \cos^2 B + b\gamma_b \sin^2 B}{\sqrt{a^2\cos^2 B + b^2 \sin^2 B}} \tag{4.11}$$

$$\gamma_a = \frac{fM}{a^2}\left[1 - \sum_n (2n+1) J_n^0 P_n(0)\right] - \omega^2 a,\ \text{for}\ n = 2,\ 4,\ 6,\ 8, \tag{4.12}$$

$$\gamma_b = \frac{fM}{b^2}\left[1 - \sum_n (2n+1)\left(\frac{a}{b}\right)^n J_n^0\right], \text{for } n = 2,\ 4,\ 6,\ 8. \qquad (4.13)$$

Instead of (4.12) the following equation can be used:

$$\gamma_a = \frac{fM}{a^2}\left[1 + \sum_n (2n+1)\overline{C}_{n0}^{0}\,\overline{P}_n(0)\right] - \omega^2 a,\ \ n = 2, 4, 6, 8$$

In simplified notation the equation (4.11) for normal gravity on the ellipsoid surface takes the form:

$$\gamma(B, H_{=0}) \cong \gamma_a(1 + \beta\sin^2 B - \beta_1 \sin^2 2B), \qquad (4.14)$$

where

$$\beta = \frac{\gamma_b - \gamma_a}{\gamma_a},\quad \beta_1 = \frac{\alpha\beta}{4} + \frac{\alpha^2}{8}.$$

Values of coefficients β and β_1 for PZ–90.11 ellipsoid are shown in Table 3.

Relationship between normal gravity and height is given by:

$$\gamma(B,H) \cong \gamma(B, H_{=0})(1 + f_1 H + f_2 H\sin^2 B + f_3 H^2), \qquad (4.15)$$

where $f_1 = -\frac{2}{a}(1 + \alpha + m),\qquad f_2 = \frac{4}{a}\alpha,$

$f_3 = \frac{3}{a^2},\qquad m = \frac{\omega^2 a}{\gamma_a}.$

For approximate computations the change of normal gravity with height is calculated by the equation:

$$\gamma(B,H) - \gamma(B, H_{=0}) \cong -0{,}3086H. \qquad (4.16)$$

In equations (4.15) and (4.16) γ is determined in mGal and height H – in meters.

Normal gravity γ_a, γ_b complies with fundamental Earth's constants. Fundamental constant *fM* takes into account mass of Earth's atmosphere. It works for processing satellite and other space objects observations. By solving boundary value problems of potential theory one should take into account an effect of atmospheric layers locating below the observation point. That is why it is necessary to introduce a correction in the value of normal gravity calculated by the equation (4.15) for the effect of atmospheric layers located above the observation point. Then instead of (4.15) it's necessary to use:

$$\gamma(B,H) \cong \gamma(B, H_{=0})(1 + f_1 H + f_2 H\sin^2 B + f_3 H^2) + \delta\gamma_a. \qquad (4.17)$$

With accuracy sufficient for practical applications the value of correction is computed by the equation

$$\delta\gamma_a = -0{,}87\exp^{-0{,}116 H^{1{,}047}} \qquad (4.18)$$

On ellipsoid surface the correction is $\delta\gamma_a$ = – 0,87 mGal. With the height it changes accordance to exponential law and is almost equal to zero at a height of 35 km.

Equations for computing values of vector components of normal gravity are based on equations (4.8), (4.9) and have the next form:

<u>in spherical geocentric coordinate system</u>

$$\left.\begin{aligned}\gamma_\rho &= -\frac{fM}{\rho^2} - \frac{fM}{\rho^2}(n+1)\sum_n \left(\frac{a}{\rho}\right)^n \overline{C}_{n0}^{0}\,\overline{P}_n(\sin\varphi) + \rho\omega^2 \cos\varphi,\\ \gamma_\varphi &= \frac{fM}{\rho^2}\left[\sum_n \left(\frac{a}{\rho}\right)^n \overline{C}_{n0}^{0}\,\overline{P}_n^{'}(\sin\varphi)\right] - \frac{\omega^2\rho^2}{2}\sin 2\varphi,\\ \gamma_\lambda &= 0\end{aligned}\right\}\ for\ n = 2,4,6,8; \quad (4.19)$$

<u>in geocentric Cartesian coordinate system</u>

$$\begin{bmatrix}\gamma_X\\ \gamma_Y\\ \gamma_Z\end{bmatrix} = \begin{bmatrix}\frac{X}{\rho} & -\frac{XZ}{\rho r} & -\frac{Y}{r}\\ \frac{Y}{\rho} & -\frac{YZ}{\rho r} & \frac{X}{r}\\ \frac{Z}{\rho} & \frac{r}{\rho} & 0\end{bmatrix}\begin{bmatrix}\gamma_\rho\\ \gamma_\varphi\\ \gamma_\lambda\end{bmatrix}, \quad (4.20)$$

where $\overline{P}_n^{'}(\sin\varphi)$ – spherical latitude derivative of fully normalized Legendre polynomial computed by the equation (4.29) for $m = 0$;

$$r = \sqrt{X^2 + Y^2}; \quad (4.21)$$

$$\rho = \sqrt{X^2 + Y^2 + Z^2}; \quad (4.22)$$

$$\begin{cases}X = \rho\cos\varphi\cos\lambda\\ Y = \rho\cos\varphi\sin\lambda\\ Z = \rho\sin\varphi.\end{cases} \quad (4.23)$$

4.2 Abnormal Earth's gravity field

The abnormal EGF component in PZ–90.11 is defined by three planetary models in the form of fully normalized serial expansion coefficients of gravity potential in terms of spherical harmonics to the 70th degree (PZ-2002/70s and PZ-2002/70) and to the 360th degree (PZ-2002/360). Appendixes 5 and 6 comprise their main characteristics. The Earth's gravity field model PZ–2002/70s was obtained by using dynamic method of space geodesy while jointly defining parameters of a priori EGM and geocentric coordinates of the stations from which tracking measurements were taken. This model is recommended for orbital and trajectory calculations. Models of abnormal Earth's gravity field PZ–2002/70 and PZ–2002/360 are obtained as a result of joint processing of PZ–2002/70s model and global catalogue of mean gravity anomalies in trapezoids 30' x 30'.

In a number of practical applications it's convenient to use models in the form of point masses gravitational potential. Appendix 7 illustrates planetary model of abnormal EGF as gravitational potential of 60 point masses (Tochechnaya massa–60, TM-60[3]) obtained as a result of approximation of disturbing potential corresponding to PZ–2002/70s model to 36th degree.

Numerical analytic models of abnormal Earth's gravity field (PZ-2002/70s, PZ-2002/70, PZ-2002/360 and TM-60) make possible to calculate any transformants of disturbing potential on the surface of the Earth and in outer space. Abnormal Earth's gravity field on the surface of the Earth limited by bounded area is more detailed illustrated by digital models of its transformants: gravity anomaly, quasigeoid heights and deviations of the plumb line. Digital models correlate with parameters of normal Earth gravity field.

Representation of Earth's gravity field by gravitational point masses potential can be used not only for describing planetary characteristics but also for modelling a fine structure of abnormal Earth's gravity field on the surface of the Earth and in outer space. For this purpose, local models of abnormal Earth's gravity field are generated that represent abnormal Earth's gravity field as a whole with additional specification in local area. Scope and detail of representing abnormal Earth's gravity field for such models are determined individually taking into account requirements to necessary calculation accuracy of transformants of disturbing potential. Appendix 8 illustrates an example of local model of abnormal Earth's gravity field in the form of gravitational point masses potential generated on the basis of planetary model TM-60.

[3] In English "Point masses–60"

4.2.1 Abnormal Earth's gravity field representation by series of spherical harmonics

Disturbing potential T for points on the surface of the Earth and in outer space is computed by the equation (4.7)

$$T(\rho,\varphi,\lambda)=\frac{fM}{\rho}\sum_{n=2}^{N}\left(\frac{a}{\rho}\right)^{n}\sum_{m=0}^{n}(\Delta\overline{C}_{nm}\cos m\lambda+\overline{S}_{nm}\sin m\lambda)\overline{P}_{nm}(\sin\varphi)\,.$$

Equations for computing transformants of disturbing potential take the form:

– for combined gravity anomalies

$$\Delta g(\rho,\varphi,\lambda)=\frac{fM}{\rho^{2}}\sum_{n=2}^{N}(n-1)\left(\frac{a}{\rho}\right)^{n}\sum_{m=0}^{n}(\Delta\overline{C}_{nm}\cos m\lambda+\overline{S}_{nm}\sin m\lambda)\overline{P}_{nm}(\sin\varphi)\,; \quad (4.24)$$

– for quasigeoid heights

$$\zeta(\rho,\varphi,\lambda)=\frac{fM}{\rho\gamma}\sum_{n=2}^{N}\left(\frac{a}{\rho}\right)^{n}\sum_{m=0}^{n}\left(\Delta\overline{C}_{nm}\cos m\lambda+\overline{S}_{nm}\sin m\lambda\right)\overline{P}_{nm}(\sin\varphi); \quad (4.25)$$

– for components of deviation of the plumb line

$$\xi(\rho,\varphi,\lambda)=-\rho''\frac{fM}{\gamma\rho^{2}}\sum_{n=2}^{N}\left(\frac{a}{\rho}\right)^{n}\sum_{m=0}^{N}\left(\Delta\overline{C}_{nm}\cos m\lambda+\overline{S}_{nm}\sin m\lambda\right)\overline{P}'_{nm}(\sin\varphi),$$

$$\eta(\rho,\varphi,\lambda)=\rho''\frac{fM}{\gamma\rho^{2}\cos\varphi}\sum_{n=2}^{N}\left(\frac{a}{\rho}\right)^{n}\sum_{m=0}^{n}m\left(\Delta\overline{C}_{nm}\sin m\lambda-\overline{S}_{nm}\cos m\lambda\right)\overline{P}_{nm}(\sin\varphi), \quad (4.26)$$

где $\rho''=206\ 264,\ 806\ 25$ угл. с.

– for components of abnormal gravity vector

$$\delta g_{\rho}=-\frac{fM}{\rho^{2}}\sum_{n=2}^{N}(n+1)\left(\frac{a}{\rho}\right)^{n}\sum_{m=0}^{n}\left(\Delta\overline{C}_{nm}\cos m\lambda+\overline{S}_{nm}\sin m\lambda\right)\overline{P}_{nm}(\sin\varphi),$$

$$\delta g_{\varphi}=\frac{fM}{\rho^{2}}\sum_{n=2}^{N}\left(\frac{a}{\rho}\right)^{n}\sum_{m=0}^{n}\left(\Delta\overline{C}_{nm}\cos m\lambda+\overline{S}_{nm}\sin m\lambda\right)\overline{P}'_{nm}(\sin\varphi), \quad (4.27)$$

$$\delta g_{\lambda}=\frac{fM}{\rho^{2}\cos\varphi}\sum_{n=2}^{N}\left(\frac{a}{\rho}\right)^{n}\sum_{m=0}^{n}m\left(-\Delta\overline{C}_{nm}\sin m\lambda+\overline{S}_{nm}\cos m\lambda\right)\overline{P}_{nm}(\sin\varphi).$$

Fully normalized Legendre polinomials $\overline{P}_{nm}(\sin\varphi)$ and their latitude derivatives $\overline{P}_{nm}^{'}(\sin\varphi)$ are computed by recurrent formulas:

$$\overline{P}_{nm}(\sin\varphi)=\begin{cases}\overline{P}_{n-1,m-1}(\sin\varphi)\cos\varphi\sqrt{\dfrac{2n+1}{2n}\dfrac{1}{\delta_{m-1}}} & \text{при } n=m\neq 0,\\ \overline{P}_{n-1,m}(\sin\varphi)\sin\varphi\sqrt{\dfrac{4n^2-1}{n^2-m^2}}-\overline{P}_{n-2,m}(\sin\varphi)\times & \\ \times\sqrt{\dfrac{\left[(n-1)^2-m^2\right](2n+1)}{(n^2-m^2)(2n-3)}} & \text{при } n>m,\\ 0 & \text{при } n<m,\\ 1 & \text{при } n=m=0,\end{cases} \tag{4.28}$$

$$\overline{P}_{nm}^{'}(\sin\varphi)=-mtg\varphi\overline{P}_{nm}(\sin\varphi)+\sqrt{\delta_m(n-m)(n+m+1)}\overline{P}_{n,m+1}(\sin\varphi), \tag{4.29}$$

In equations (4.28), (4.29) symbols δ_{m-1} and δ_m are presented in general

$$\delta_k=\begin{cases}\dfrac{1}{2} & \text{при } k=0\\ 1 & \text{при } k\neq 0\end{cases}.$$

4.2.2 Abnormal Earth's gravity field representation by point masses system

Disturbing potential T approximates by gravitational potential of point masses system being specified by value of mass m and its position in the Erath figure. Value of disturbing potential on the surface of the Earth and in any point of outer space is computed by the equation:

$$T=fM\sum_{i=1}^{N}\frac{\varepsilon_i}{r_i}, \tag{4.30}$$

where N – number of point masses;

ε_i – value of point mass expressed in mass of the Earth unit M ($\varepsilon_i=\dfrac{m_i}{M}$);

r_i – distance between the i-th point mass and the given point.

Combined gravity anomaly Δg, quasigeoid height ζ above reference ellipsoid and components of deviation of the plumb line ξ, η on the Earth's surface are calculated by equations (Earth is approximated by sphere)

$$\Delta g(\rho,\varphi,\lambda) \cong fM\sum_{i=1}^{N}\varepsilon_i \frac{R^2-\rho_i^2-3r_i^2}{2Rr_i^3},$$
$$\zeta(\rho,\varphi,\lambda) = \frac{fM}{\gamma}\sum_{i=1}^{N}\frac{\varepsilon_i}{r_i},$$
$$\xi(\rho,\varphi,\lambda) = -\rho'' \frac{fM}{\gamma}\sum_{i=1}^{N}\varepsilon_i \frac{\rho_i}{r_i^3}\left[\sin\varphi_i\cos\varphi - \sin\varphi_i\cos\varphi\cos(\lambda_i-\lambda)\right],$$
$$\eta(\rho,\varphi,\lambda) = -\rho'' \frac{fM}{\gamma}\sum_{i=1}^{N}\varepsilon_i \frac{\rho_i}{r_i^3}\cos\varphi_i\sin(\lambda_i-\lambda), \tag{4.31}$$

where ρ, φ, λ – spherical geocentric coordinates of the given point;
$\rho_i, \varphi_i, \lambda_i$ – spherical geocentric coordinates of the *i*-th point mass;
R – mean radius of the Earth (R = 6 371 000 m)
r_i – distance from *i*-th point mass to the given point with coordinates ρ, φ, λ

$$r_i = \sqrt{\rho^2+\rho_i^2-2\rho\,\rho_i\cos\psi}, \quad \cos\psi = \sin\varphi_i\sin\varphi + \cos\varphi_i\cos\varphi\cos(\lambda_i-\lambda). \tag{4.32}$$

Components of abnormal gravity vector by axes of geocentric Cartesian coordinate system are computed by the equations:

$$\delta g_X = -fM\sum_{i=1}^{N}\varepsilon_i\frac{X-X_i}{r_i^3},$$
$$\delta g_Y = -fM\sum_{i=1}^{N}\varepsilon_i\frac{Y-Y_i}{r_i^3}, \tag{4.33}$$
$$\delta g_Z = -fM\sum_{i=1}^{N}\varepsilon_i\frac{Z-Z_i}{r_i^3}.$$

For translation spherical coordinates to geocentric Cartesian equations (4.23) are used. Value r_i is computed by the equation:

$$r_i = \sqrt{(X-X_i)^2+(Y-Y_i)^2+(Z-Z_i)^2}\,. \tag{4.34}$$

Recalculation of point masses parameters into coefficients of series of spherical harmonics is made by the equations:

$$\bar{C}_{nm} = \frac{1}{2n+1}\sum_{i=1}^{N}\varepsilon_i\left(\frac{\rho_i}{a}\right)^n\cos m\lambda_i\bar{P}_{nm}(\sin\varphi_i),$$
$$\bar{S}_{nm} = \frac{1}{2n+1}\sum_{i=1}^{N}\varepsilon_i\left(\frac{\rho_i}{a}\right)^n\sin m\lambda_i\bar{P}_{nm}(\sin\varphi_i). \tag{4.35}$$

4.3 Digital models (DM) of transformants of abnormal Earth's gravity field

Digital models of abnormal Earth gravity field are created in the form of ordered complex of individual values of transformants of disturbing potential (characteristics of abnormal Earth's gravity field) computed in points of analytical grid generated by meridians and parallels. Values of characteristics of abnormal Earth's gravity field in passing points can be obtained by using interpolation algorithms. Bilinear interpolation algorithm is sufficient for majority of practical applications of DM of abnormal Earth's gravity field.

Gravimetric maps with information about gravity anomalies and quasigeoid heights obtained by processing satellite altimetry data and GLONASS/GPS measurements on bench marks serve as initial information for creating digital models and their graphical analogues. SD AF RF generates digital models (and their graphical analogues) of gravity anomalies (Appendix 9), quasigeoid heights (Appendix 10) and components of deviation of the plumb line (Appendix 11) on the base on this information. Values of gravity anomalies in these models correspond to adopted in the Russian Federation gravimetric system of 1971, quasigeoid heights and deviations of the plumb line – to Earth Parameters system (PZ-90.11). The following algorithms perform translation of values of transformants of disturbing potential to another system of parameters.

Gravity anomalies. Gravity anomalies in gravimetric system of 1971 correspond to normal Helmert formula with a correction (–14 mGal). The correction for difference between coefficients in equation for normal gravity on the ellipsoid PZ–90.11 (4.14) must be taken into account when translating gravity anomaly in gravimetric system of 1971 to normal gravity field PZ–90.11:

$$\Delta g_{\text{PZ-90.11}} = \Delta g_{1971} - 15.9 - 0.40\sin^2\varphi - 1.17\sin^2 2\varphi. \tag{4.36}$$

Quasigeoid heights. By translating quasigeoid heights in global terrestrial reference system 1 to reference coordinate system 2 transformation equations for geodetic heights H are used

$$\zeta_2 = \zeta_1 + \Delta\zeta,$$

$$\Delta\zeta = (H_2 - H^\gamma) - (H_1 - H^\gamma) = -\Delta H, \tag{4.37}$$

$$\begin{aligned}\Delta H = &-\frac{a}{N}\Delta a + N\sin^2 B\frac{\Delta e^2}{2} + (\Delta X\cos L + \Delta Y\sin L)\cos B + \Delta Z\sin B - \\ &-Ne^2\sin B\cos B\left(\frac{\omega_X}{\rho''}\sin L - \frac{\omega_Y}{\rho''}\cos L\right) + \left(\frac{a^2}{N} + H\right)m.\end{aligned} \tag{4.38}$$

Deviations of the plumb line. By recalculation deviations of the plumb line from global terrestrial reference system 1 to reference coordinate system 2 the following equations are used:

$$\xi_2 = \xi_1 + \Delta\xi, \quad \eta_2 = \eta_1 + \Delta\eta, \tag{4.39}$$

$$\Delta\xi = \frac{\rho''}{M+H}\left\{\frac{1}{2}\left[\frac{Ne^2}{a}\Delta a + N\left(\frac{N^2}{a^2}+1\right)\frac{\Delta e^2}{4}\right]\sin 2B - (\Delta X\cos L + \Delta Y\sin L)\sin B + \Delta Z\cos B\right\} -$$

$$-(\omega_X \sin L - \omega_Y \cos L)e^2 \cos 2B,$$

(4.40)

$$\Delta\eta = \frac{\rho''}{N+H}(-\Delta X\sin L + \Delta Y\cos L) + e^2(\omega_X \cos L + \omega_Y \sin L)\sin B, \tag{4.41}$$

$$a = \frac{a_1 + a_2}{2}, \quad e^2 = \frac{e_1^2 + e_2^2}{2}, \qquad \Delta a = a_2 - a_1, \quad \Delta e^2 = e_2^2 - e_1^2 .$$

In equations (4.37) – (4.41) the same symbols are taken as in the equations (3.4).

Values of gravity anomalies and deviations of the plumb line contained in digital models of them are computed for the points of the Earth’s physical surface. Digital models of gravity anomaly are used as usual along with digital terrain models where normal heights are determined in the same mesh points as in DM of gravity anomalies.

Along with digital models of gravity anomalies and quasigeoid heights for detailed representation of abnormal Earth’s gravity field in large regions catalogues (lists) of mean values of gravity anomalies and quasigeoid heights for standard trapezoids (5' x 7.5', 10' x 15', 15' x 15', 20' x 30', 30' x 30' и 1° x 1°) can be generated. For values of abnormal Earth’s gravity field characteristics contained in these lists the same transformations as for digital models are used.

Appendix 1 Transformation of spatial Cartesian Coordinates into geodetic coordinates

Constraint equations between these coordinates are the basis for transformations

$$\begin{cases} X = (N+H)\cos B\cos L \\ Y = (N+H)\cos B\sin L \\ Z = [(1-e^2)N+H]\sin B \end{cases}, \qquad \text{(A.1.1)}$$

where X, Y, Z – spatial Cartesian coordinates;
B, L, H – geodetic coordinates;
N – radius of curvature in the prime vertical;
e – ellipsoid eccentricity.

Radius of curvature in the prime vertical and squared ellipsoid eccentricity are computed by the equations

$$N = \frac{a}{\sqrt{1-e^2\sin^2 B}}, \qquad \text{(A.1.2)}$$

$$e^2 = 2\alpha - \alpha^2, \qquad \text{(A.1.3)}$$

where a – semi-major axis;
α – flattering factor.

It is necessary to perform iterations by calculating geodetic latitude and geodetic height for transforming spatial Cartesian coordinates to geodetic coordinates. For this purpose the following algorithm is used:

1) value of auxiliary quantity r is computed by the equation

$$r = \sqrt{X^2 + Y^2}; \qquad \text{(A.1.4)}$$

2) value r is analyzed:

a) if $r = 0$, then

$$B = \frac{\pi Z}{2|Z|}, \qquad \text{(A.1.5)}$$

$$L = 0, \qquad \text{(A.1.6)}$$

$$H = Z\sin B - a\sqrt{1-e^2\sin^2 B}; \qquad \text{(A.1.7)}$$

б) if $r > 0$, then

$$L_a = \left|\arcsin\left(\frac{Y}{r}\right)\right|, \qquad \text{(A.1.8)}$$

at the same time

$$\left.\begin{array}{l} \text{if } Y < 0,\ X > 0, \text{ then } L = 2\pi - L_a \\ \text{if } Y < 0,\ X < 0, \text{ then } L = \pi + L_a \\ \text{if } Y > 0,\ X < 0, \text{ then } L = \pi - L_a \\ \text{if } Y > 0,\ X > 0, \text{ then } L = L_a \end{array}\right\}; \tag{A.1.9}$$

3) value Z is analyzed:

a) if $Z = 0$, then

$$B = 0, \quad H = r - a;$$

б) in all other cases computations are made as follows:

– compute auxiliary quantities ρ, c, p by the equations:

$$\rho = \sqrt{X^2 + Y^2 + Z^2}\,, \tag{A.1.10}$$

$$c = \arcsin\left(\frac{Z}{\rho}\right), \tag{A.1.11}$$

$$p = \frac{e^2 a}{2\rho}, \tag{A.1.12}$$

– iterative process is implemented:

$$s_1 = 0, \tag{A.1.13}$$

$$b = c + s_1, \tag{A.1.14}$$

$$s_2 = \arcsin\left(\frac{p \sin(2b)}{\sqrt{1 - e^2 \sin^2 b}}\right), \tag{A.1.15}$$

$$d = \left| s_2 - s_1 \right|. \tag{A.1.16}$$

If value d defined by the equation (A.1.16) is less than set tolerance value, then

$$B = b, \tag{A.1.17}$$

$$H = r \cos B + Z \sin B - a\sqrt{1 - e^2 \sin^2 B}\,. \tag{A.1.18}$$

If value d is equal or more than set tolerance value, then

$$s_1 = s_2 \tag{A.1.19}$$

and the computations are repeated starting with formula (A.1.14).

By transforming coordinates as tolerance value for extinction of iterative process value d = 0,0001″ is taken. In this case computational error of geodetic latitude does not exceed 0,003 m.

Appendix 2 SGN, IGS, DORIS station locations on the territory of Russia

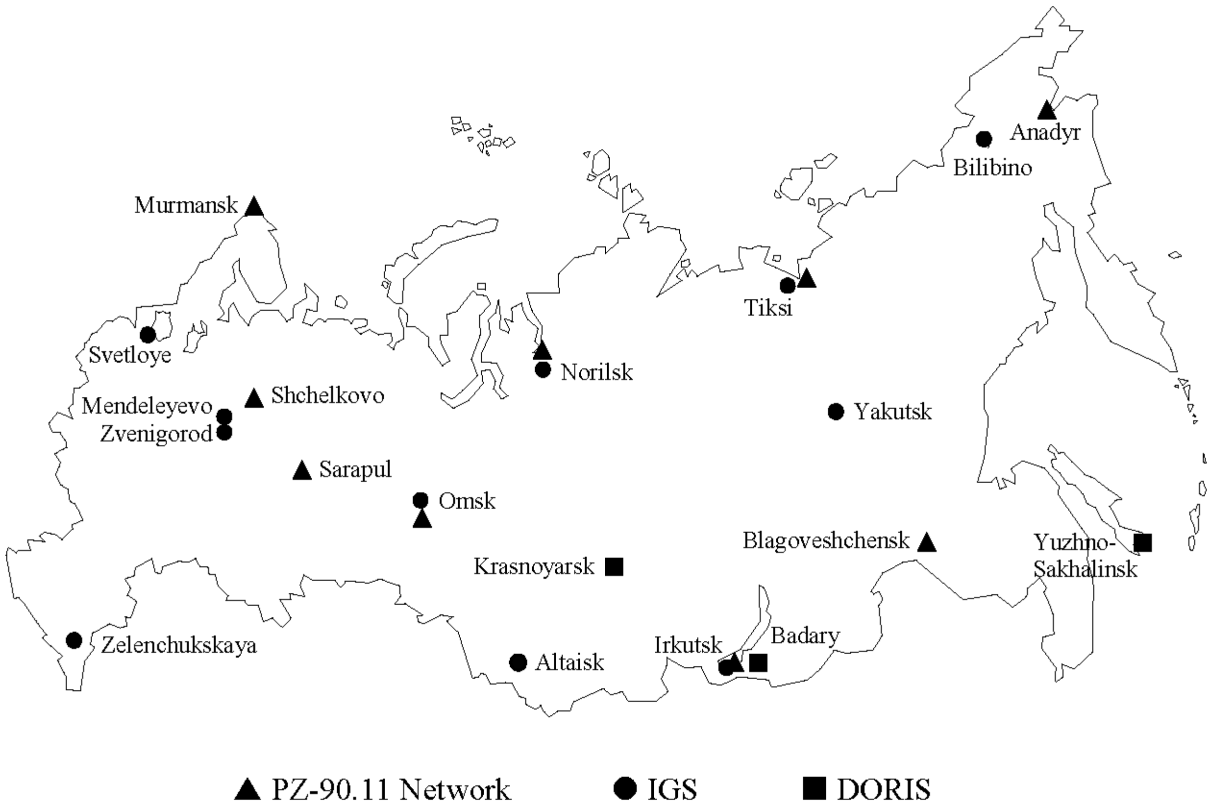


Fig. A2.1 – Plan of SGN(▲), IGS(●) и DORIS(■) station location on the territory of Russia

Information about stations (coordinates and rates) defined in PZ-90.11 at epoch 2010.0 is presented in Addendum to the 3rd edition.

For translation to another (given) epoch, it is necessary to allow for rates $\dot{X}, \dot{Y}, \dot{Z}$ by the equation

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{t_2} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{t_1} + (t_2 - t_1) \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix}_{t_1}, \qquad (A.2.1)$$

where t_1, t_2 – initial and given epochs.

Practical example of using the equation (A.2.1) is given in Appendix 4.

Appendix 3 Constraint equation between global terrestrial and celestial reference systems

Constraint equation between global terrestrial and celestial (inertial) reference systems has the next form:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \boldsymbol{P}_t\, \overline{\boldsymbol{S}}_t\, \boldsymbol{N}_t\, \boldsymbol{Pr}_{t-T_0} \boldsymbol{B}_{T_0} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{T_0}, \qquad \text{(A.3.1)}$$

where	$\boldsymbol{B}_{T_0}$	– shifting matrix of mean pole JD2000.0 relatively ICRS taking into account terms second order of smallness;
	$\boldsymbol{Pr}_{t-T_0}$	– transfer matrix for translating celestial system at standard epoch T_0 (JD2000.0) to celestial system at epoch t (mean celestial reference system) or precession matrix for time interval $t-T_0$;
	$\boldsymbol{N}_t$	– transfer matrix for translating mean to true celestial reference system at epoch t or nutation matrix;
	$\overline{\boldsymbol{S}}_t$	– transfer matrix for translating true celestial reference system to instantaneous terrestrial reference system or diurnal rotation matrix;
	$\boldsymbol{P}_t$	– transfer matrix for translating instantaneous terrestrial reference system to terrestrial reference system or pole movement matrix;

Transformation matrixes $\boldsymbol{B}, \boldsymbol{P}, \overline{\boldsymbol{S}}, \boldsymbol{N}$ and $\boldsymbol{Pr}$ have the following structures.

$$\boldsymbol{B} = \begin{pmatrix} +0.999\,999\,999\,999\,9942 & -0.000\,000\,070\,782\,7974 & +0.000\,000\,080\,562\,17151 \\ +0.000\,000\,070\,782\,7948 & +0.999\,999\,999\,999\,9969 & +0.000\,000\,033\,060\,4145 \\ -0.000\,000\,080\,562\,1738 & -0.000\,000\,033\,060\,4088 & +0.999\,999\,999\,999\,9962 \end{pmatrix}. \qquad \text{(A.3.2)}$$

Shifting matrix of mean pole JD2000.0 relatively ICRS covers the classic concept of reference system related to equinox.

$$\boldsymbol{P} = \begin{pmatrix} 1 & 0 & x_p \\ 0 & 1 & -y_p \\ -x_p & y_p & 1 \end{pmatrix}, \qquad \text{(A.3.3)}$$

where x_p, y_p – pole coordinates (in radian measure) published in “Universal time and pole coordinates” bulletin.

$$\overline{\boldsymbol{S}} = \begin{pmatrix} \cos S & \sin S & 0 \\ -\sin S & \cos S & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{A.3.4}$$

Where S – apparent sidereal time of Greenwich meridian computed by the equation given in RAS “Almanac”.

$$\boldsymbol{N} = \begin{pmatrix} \mathrm{N}_{11} & \mathrm{N}_{12} & \mathrm{N}_{13} \\ \mathrm{N}_{21} & \mathrm{N}_{22} & \mathrm{N}_{23} \\ \mathrm{N}_{31} & \mathrm{N}_{32} & \mathrm{N}_{33} \end{pmatrix}, \tag{A.3.5}$$

where N_{11} = cosB,

N_{12} = sinBcosC,

N_{13} = sinBsinC,

N_{21} = – sinBcosA,

N_{22} = cosAcosBcosC – sinAsinC,

N_{23} = cosAcosBsinC + sinAcosC,

N_{31} = sinAsinB,

N_{32} = – sinAcosBcosC – cosAsinC,

N_{33} = – sinAcosBsinC + cosAcosC;

A, B, C – reduction values,

$\mathrm{A} = -\varepsilon_0 - \Delta\varepsilon - d\varepsilon,$

$\mathrm{B} = -\Delta\psi - d\psi,$

$\mathrm{C} = \varepsilon_0;$

ε_0 – mean obliquity of ecliptic;

$\Delta\varepsilon, d\varepsilon$ – short and long-periodic terms of nutation in obliquity;

$\Delta\psi, d\psi$ – short and long-periodic terms of nutation in longitude.

$$\boldsymbol{Pr} = \begin{pmatrix} \mathrm{Pr}_{11} & \mathrm{Pr}_{12} & \mathrm{Pr}_{13} \\ \mathrm{Pr}_{21} & \mathrm{Pr}_{22} & \mathrm{Pr}_{23} \\ \mathrm{Pr}_{31} & \mathrm{Pr}_{32} & \mathrm{Pr}_{33} \end{pmatrix}, \tag{A.3.6}$$

where $\mathrm{Pr}_{11} = \cos\varsigma_0 \cos Z \cos\Theta - \sin\varsigma_0 \sin Z$,

$\mathrm{Pr}_{12} = -\sin\varsigma_0 \cos Z \cos\Theta - \cos\varsigma_0 \sin Z$,

$\mathrm{Pr}_{13} = -\cos Z \sin\Theta$,

$\mathrm{Pr}_{21} = \cos\varsigma_0 \sin Z \cos\Theta + \sin\varsigma_0 \cos Z$,

$\mathrm{Pr}_{22} = -\sin\varsigma_0 \sin Z \cos\Theta + \cos\varsigma_0 \cos Z$,

$\mathrm{Pr}_{23} = -\sin Z \sin\Theta$,

$\mathrm{Pr}_{31} = \cos\varsigma_0 \sin\Theta$,

$\mathrm{Pr}_{32} = -\sin\varsigma_0 \sin\Theta$,

$\mathrm{Pr}_{33} = \cos\Theta$.

In accordance with resolution B1 (Prague, 2006) the following equations are used for precession parameters ξ, z, Θ

ξ = (2.650 545 + (2306.083 227 + (0.298 8499 + (0.018 01828 – (0.000 005 791 + 0.000 000 3173 × T) × T)× T) × T) × T) / 206 264.806 247 097

z = (– 2.650 545 + (2306.077 181 + (1.092 7348 + (0.018 2683 – (0.000 028 596 + 0.000 000 2904 × T) × T) × T) × T) × T) / 206 264.806 247 097

Θ = (2004.191 903 – (0.429 4934 + (0.041 82264 + (0.000 007 089 +
+ 0.000 000 1274 × T) × T) × T) × T) × T / 206 264.806 247 097,

where T = (JD(t) – 2451 545.0) / 36525 in terrestrial time scale realized by ground time and frequency standards.

Appendix 4 Relationship between geodetic reference systems

Transformation of spatial Cartesian coordinates from system 1 to system 2 is made by the equation

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_2 = (1+m)\begin{pmatrix} 1 & +\omega_Z & -\omega_Y \\ -\omega_Z & 1 & +\omega_X \\ +\omega_Y & -\omega_X & 1 \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_1 + \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} \qquad \text{(A.4.1)}$$

where	ΔX, ΔY, ΔZ –	linear transformation elements for translating reference system 1 to system 2, m;
	$\omega_X, \omega_Y, \omega_Z$ –	angular transformation elements for translating reference system 1 to system 2, rad;
	m –	scale transformation element for translating reference system 1 to system 2.

By reverse transformation of spatial Cartesian coordinates transformation elements have the same values but with reverse sign.

Values of transformation elements of reference systems are obtained by differences of coordinates of the same points in both systems. In Table A4.1 values of transformation elements and their RMS errors (written in universal way «±») are given for all reference systems indicated in Section 3.

Transformation element errors SK–42 – PZ–90 belong to global model of transformation elements. Transformation elements for coordinate systems SK–95 – PZ–90 are set prescriptively (fixed) that's why values of RMS errors for them are not given.

Transformation elements for reference systems PZ–90.02 – ITRF2000 are given at epoch 2002.0 and coincide with transformation elements for PZ–90.02 – WGS 84 (G1150), where G1150 – number of GPS-week.

Table A4.1 – Reference systems transformation elements and their RMS errors

#	From system 1	To system 2	Δ*X*, m	Δ*Y*, m	Δ*Z*, m	ω_X, 10^{-3} угл.с	ω_Y, 10^{-3} угл.с	ω_Z, 10^{-3} угл.с	*m*, 10^{-6}	Epoche
1	SK-42	PZ-90	+25 ± 2	–141 ± 2	–80 ± 3	0 ± 100	–350 ±100	–660 ±100	0 ±0.250	–
2	SK-95	PZ-90	+25.90	–130.94	–81.76	0	0	0	0	–
3	PZ-90	PZ-90.02	–1.07 ±0.1	–0.03 ±0.1	+0.02 ±0.1	0	0	–130 ±10	–0.220 ±0.020	2002.0
4	WGS 84 (G1150)	PZ-90.02	+0.36 ±0.1	–0.08 ± 0.1	–0.18 ±0.1	0	0	0	0	2002.0
5	PZ-90.02	PZ-90.11	–0.373 ±0.027	+0.186 ±0.056	+0.202 ±0.033	–2.30 ±2.11	+3.54 ±0.87	–4.21 ±0.82	–0.008 ±0.004	2010.0
6	GSK-2011	PZ-90.11	0.000 ±0.008	+0.014 ±0.018	–0.008 ±0.011	–0.562 ±0.698	–0.019 ±0.259	+0.053 ±0.227	–0.0006 ±0.0010	2011.0
7	PZ-90.11	ITRF2008	–0.003 ±0.002	–0.001 ±0.002	0.000 ±0.002	+0.019 ±0.072	–0.042 ±0.073	+0.002 ±0.090	–0.000 ±0.0003	2010.0

Table A4.2 shows transformation elements for transforming given systems to PZ–90.11. Therewith transformation elements for a number of systems are obtained by combining data (algebraic addition) from Table A4.1.

Table A4.2 – transformation elements by transfer to system PZ-90.11 (system 2)

#	From system 1	ΔX, m	ΔY, m	ΔZ, мм	ω_X, 10^{-3} arcsec	ω_Y, 10^{-3} arcsec	ω_Z, 10^{-3} arcsec	*m*, 10^{-6}
1	SK–42	+23.557	–140.844	–79.778	–2.30	–346.46	–794.21	–0.228
2	SK–95	+24.457	–130.784	–81.538	–2.30	+3.54	–134.21	–0.228
3	PZ–90	–1.443	+0.156	+0.222	–2.30	+3.54	–134.21	–0.228
4	WGS 84 (G1150)	–0.013	+0.106	+0.022	–2.30	+3.54	–4.21	–0.008
5	PZ–90.02	–0.373	+0.186	+0.202	–2.30	+3.54	–4.21	–0.008
6	ITRF2008	+0.003	+0.001	0.000	–0.019	+0.042	–0.002	0.000

Reference systems PZ–90.02, PZ–90.11, WGS 84(G1150), GSK-2011 and ITRF2008 excel at improved accuracy. That’s why before applying formula (A.4.1) it’s necessary to recalculate point coordinates in both systems 1 and 2 by the equation (A.2.1) at epoch of transformation elements derivation for these reference systems using rates of point coordinates.

As an example, numeric results of transformation of MDVJ coordinates (IGS network station) presented in ITRF2008 at epoch 2005.0 to PZ-90.11 at any epoch, eg. 2013.9, are given.

MDVJ coordinates in ITRF2008 at epoch 2005.0 and rates have the following values:

$X = 2845\ 456.0813$ m; $V_x = -\ 0.0212$ m/year;
$Y = 216\ 0954.2453$ m; $V_Y = +\ 0.0124$ m/ year;
$Z = 5265\ 993.2296$ m; $V_Z = +\ 0.0072$ m/ year.

Three steps of the calculation:

1. Recalculation by the equation (A.2.1) of MDVJ coordinates in ITRF2008 from epoch 2005.0 to epoch 2010.0, because transformation elements (ITRF2008 – PZ–90.11, Table A4.2) are obtained at epoch 2010.0:

$X = 2845\ 456.0813 + (-\ 0.0212)\times(2010.0 - 2005.0) = 2845\ 455.9753;$
$Y = 216\ 0954.2453 + (+\ 0.0124)\times(2010.0 - 2005.0) = 216\ 0954.3073;$
$Z = 5265\ 993.2296 + (+\ 0.0072)\times(2010.0 - 2005.0) = 5265\ 993.2656.$

2. Recalculation by the equation (A.4.1) of point coordinates from ITRF2008 to PZ–90.11 at epoch 2010.0 (epoch of transformation elements derivation) using transformation elements from Table A4.2:

$X = 2845\ 455.9772;$
$Y = 216\ 0954.3078;$
$Z = 5265\ 993.2664.$

3. Recalculation by the equation (A.2.1) MDVJ coordinates in PZ-90.11 at epoch 2013.9:

$X = 2845\ 455.9772 + (-\ 0.0212)\times(2013.9-2010.0) = 2845\ 455.8945;$
$Y = 216\ 0954.3078 + (+\ 0.0124)\times(2013.9-2010.0) = 216\ 0954.3562;$
$Z = 5265\ 993.2652 + (+\ 0.0072)\times(2013.9-2010.0) = 5265\ 993.2945.$

Equations for coordinate transformations from above listed systems in Table A4.2 to PZ–90.11 (after transformation at the epoch of transformation elements derivation) are given below. As an example an equation for reverse coordinate transformation from PZ–90.11 to PZ–90.02 is also given.

1. Transforming coordinates in SK–42 to PZ–90.11

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{PZ-90.11}} = (1 - 0.228\cdot10^{-6}) \begin{pmatrix} 1 & -3.85044\cdot10^{-6} & +1.67968\cdot10^{-6} \\ +3.85044\cdot10^{-6} & 1 & -0.01115\cdot10^{-6} \\ -1.67968\cdot10^{-6} & +0.01115\cdot10^{-6} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{SK-42}} + \begin{pmatrix} +23.557 \\ -140.844 \\ -79.778 \end{pmatrix}.$$

2. Transforming coordinates in SK–95 to PZ–90.11

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{PZ-90.11}} = (1 - 0.228 \cdot 10^{-6}) \begin{pmatrix} 1 & -0.65067 \cdot 10^{-6} & -0.01716 \cdot 10^{-6} \\ +0.65067 \cdot 10^{-6} & 1 & -0.01115 \cdot 10^{-6} \\ +0.01716 \cdot 10^{-6} & +0.01115 \cdot 10^{-6} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{SK-95}} + \begin{pmatrix} +24.457 \\ -130.784 \\ -81.538 \end{pmatrix}.$$

3. Transforming coordinates in PZ–90.02 to PZ–90.11

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{PZ-90.11}} = (1 - 0.228 \cdot 10^{-6}) \begin{pmatrix} 1 & +0.65067 \cdot 10^{-6} & -0.01716 \cdot 10^{-6} \\ -0.65067 \cdot 10^{-6} & 1 & -0.01115 \cdot 10^{-6} \\ +0.01716 \cdot 10^{-6} & +0.01115 \cdot 10^{-6} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{PZ-90}} + \begin{pmatrix} -1.443 \\ +0.156 \\ +0.222 \end{pmatrix}.$$

4. Transforming coordinates in WGS 84 (G1150) to PZ–90.11

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{PZ-90.11}} = (1 - 0.008 \cdot 10^{-6}) \begin{pmatrix} 1 & -0.02041 \cdot 10^{-6} & -0.01716 \cdot 10^{-6} \\ +0.02041 \cdot 10^{-6} & 1 & -0.01115 \cdot 10^{-6} \\ +0.01716 \cdot 10^{-6} & +0.01115 \cdot 10^{-6} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\substack{\text{WGS 84} \\ \text{(G1150)}}} + \begin{pmatrix} -0.013 \\ +0.106 \\ +0.022 \end{pmatrix}.$$

5. Transforming coordinates in PZ–90.02 to PZ–90.11

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{PZ-90.11}} = (1 - 0.008 \cdot 10^{-6}) \begin{pmatrix} 1 & -0.02041 \cdot 10^{-6} & -0.01716 \cdot 10^{-6} \\ +0.02041 \cdot 10^{-6} & 1 & -0.01115 \cdot 10^{-6} \\ +0.01716 \cdot 10^{-6} & +0.01115 \cdot 10^{-6} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{PZ-90.02}} + \begin{pmatrix} -0.373 \\ +0.186 \\ +0.202 \end{pmatrix}.$$

6. Transforming coordinates in ITRF2008 to PZ–90.11

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{PZ-90.11}} = \begin{pmatrix} 1 & -0.00001 \cdot 10^{-6} & -0.00020 \cdot 10^{-6} \\ +0.00001 \cdot 10^{-6} & 1 & -0.00009 \cdot 10^{-6} \\ +0.00020 \cdot 10^{-6} & +0.00009 \cdot 10^{-6} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{ITRF2008}} + \begin{pmatrix} +0.003 \\ +0.001 \\ -0.000 \end{pmatrix}.$$

7. As an example, equation for reverse coordinate transformation from PZ–90.11 to PZ–90.02.

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{PZ-90.02}} = (1 + 0.008 \cdot 10^{-6}) \begin{pmatrix} 1 & +0.02041 \cdot 10^{-6} & +0.01716 \cdot 10^{-6} \\ -0.02041 \cdot 10^{-6} & 1 & +0.01115 \cdot 10^{-6} \\ -0.01716 \cdot 10^{-6} & -0.01115 \cdot 10^{-6} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{PZ-90.11}} + \begin{pmatrix} +0.373 \\ -0.186 \\ -0.202 \end{pmatrix}.$$

Appendix 5 Characteristics of abnormal Earth's gravity field model in the form of series of spherical harmonics to the 70th degree (PZ-2002/70s)

Initial data: satellite observation results
Initial parameters:

semi-major axis *a*	6 378 136.0 m
coefficient $\overline{C}_{20}$	– 484.165 3863·10 $^{-6}$
velocity of the Earth ω	0.729 2115·10 $^{-4}$ rad/s
value *fM*	398 600.4418 km^3/s^2

In Table A5.1 characteristics of abnormal Earth's gravity field model are given in the form of degree variances and their errors for expansion coefficients $\Delta\overline{C}_{nm}, \overline{S}_{nm}$ of disturbing potential in terms of spherical harmonics, gravity anomalies and quasigeoid heights.

Table A5.1

Degree *n*	Characteristic					
	c_n	m_{c_n}	Δg_n, mGal	$m_{\Delta g_n}$, mGal	ζ_n, m	m_{ζ_n}, m
1	2	3	4	5	6	7
2	0.281 265E-05	0.100 826E-09	2.75	0.0001	17.93	0.0006
3	0.296 999E-05	0.292 918E-09	5.82	0.0006	18.94	0.001
4	0.151 337E-05	0.411 659E-09	4.44	0.001	9.65	0.002
5	0.116 896E-05	0.598 152E-09	4.58	0.002	7.45	0.003
6	0.905 458E-06	0.738 650E-09	4.43	0.004	5.77	0.004
7	0.753 147E-06	0.970 655E-09	4.42	0.006	4.80	0.006
8	0.487 644E-06	0.114 753E-08	3.34	0.008	3.11	0.007
9	0.426 644E-06	0.140 826E-08	3.34	0.01	2.72	0.008
10	0.355 045E-06	0.161 139E-08	3.13	0.01	2.26	0.01
11	0.262 531E-06	0.189 590E-08	2.57	0.01	1.67	0.01
12	0.151 398E-06	0.206 915E-08	1.63	0.02	0.96	0.01
13	0.239 996E-06	0.235 061E-08	2.82	0.02	1.53	0.01
14	0.146 748E-06	0.263 123E-08	1.86	0.03	0.93	0.01
15	0.140 885E-06	0.296 830E-08	1.93	0.04	0.89	0.01
16	0.137 230E-06	0.333 901E-08	2.01	0.04	0.87	0.02
17	0.113 641E-06	0.366 910E-08	1.78	0.05	0.72	0.02
18	0.117 559E-06	0.405 278E-08	1.95	0.06	0.74	0.02
19	0.100 039E-06	0.440 281E-08	1.76	0.07	0.63	0.02
20	0.934 784E-07	0.478 448E-08	1.74	0.08	0.59	0.03
21	0.977 608E-07	0.515 717E-08	1.91	0.10	0.62	0.03
22	0.904 674E-07	0.554 179E-08	1.86	0.11	0.57	0.03
23	0.806 259E-07	0.592 484E-08	1.73	0.12	0.51	0.03
24	0.676 026E-07	0.629 790E-08	1.52	0.14	0.43	0.04
25	0.765 550E-07	0.667 096E-08	1.80	0.15	0.48	0.04
26	0.624 120E-07	0.706 333E-08	1.52	0.17	0.39	0.04
27	0.536 845E-07	0.753 045E-08	1.36	0.19	0.34	0.04
28	0.650 545E-07	0.795 718E-08	1.72	0.21	0.41	0.05
29	0.594 330E-07	0.840 042E-08	1.63	0.23	0.37	0.05

30	0.604 324E-07	0.884 043E-08	1.71	0.25	0.38	0.05
1	2	3	4	5	6	7
31	0.568 533E-07	0.929 183E-08	1.67	0.27	0.36	0.05
32	0.548 741E-07	0.974 403E-08	1.66	0.29	0.34	0.06
33	0.546 498E-07	0.102 052E-07	1.71	0.31	0.34	0.06
34	0.608 397E-07	0.106 712E-07	1.96	0.34	0.38	0.06
35	0.584 932E-07	0.111 444E-07	1.94	0.37	0.37	0.07
36	0.505 784E-07	0.116 234E-07	1.73	0.39	0.32	0.07
37	0.518 073E-07	0.121 087E-07	1.82	0.42	0.33	0.07
38	0.453 287E-07	0.125 973E-07	1.64	0.45	0.28	0.08
39	0.483 193E-07	0.130 983E-07	1.79	0.48	0.30	0.08
40	0.400 794E-07	0.136 051E-07	1.53	0.51	0.25	0.08
41	0.438 596E-07	0.141 172E-07	1.71	0.55	0.27	0.09
42	0.433 150E-07	0.146 346E-07	1.74	0.58	0.27	0.09
43	0.404 610E-07	0.151 588E-07	1.66	0.62	0.25	0.09
44	0.396 641E-07	0.156 888E-07	1.67	0.66	0.25	0.10
45	0.396 143E-07	0.162 248E-07	1.70	0.69	0.25	0.10
46	0.427 009E-07	0.167 666E-07	1.88	0.73	0.27	0.10
47	0.417 365E-07	0.173 144E-07	1.88	0.78	0.26	0.11
48	0.373 048E-07	0.178 680E-07	1.71	0.82	0.23	0.11
49	0.324 528E-07	0.184 274E-07	1.52	0.86	0.20	0.11
50	0.377 305E-07	0.189 924E-07	1.81	0.91	0.24	0.12
51	0.358 821E-07	0.195 631E-07	1.75	0.95	0.22	0.12
52	0.338 484E-07	0.201 394E-07	1.69	1.00	0.21	0.12
53	0.378 241E-07	0.207 214E-07	1.92	1.05	0.24	0.13
54	0.343 982E-07	0.213 087E-07	1.78	1.10	0.21	0.13
55	0.319 094E-07	0.219 016E-07	1.68	1.15	0.20	0.13
56	0.345 203E-07	0.224 998E-07	1.86	1.21	0.22	0.14
57	0.345 580E-07	0.231 033E-07	1.89	1.26	0.22	0.14
58	0.301 858E-07	0.237 122E-07	1.68	1.32	0.19	0.15
59	0.334 572E-07	0.243 263E-07	1.90	1.38	0.21	0.15
60	0.302 668E-07	0.249 456E-07	1.74	1.44	0.19	0.15
61	0.293 159E-07	0.255 701E-07	1.72	1.50	0.18	0.16
62	0.285 973E-07	0.261 997E-07	1.70	1.56	0.18	0.16
63	0.276 423E-07	0.268 332E-07	1.67	1.63	0.17	0.17
64	0.269 906E-07	0.274 735E-07	1.66	1.69	0.17	0.17
65	0.240 269E-07	0.281 126E-07	1.50	1.76	0.15	0.17
66	0.272 278E-07	0.287 647E-07	1.73	1.83	0.17	0.18
67	0.267 866E-07	0.293 906E-07	1.73	1.90	0.17	0.18
68	0.251 156E-07	0.300 568E-07	1.64	1.97	0.16	0.19
69	0.283 859E-07	0.306 393E-07	1.89	2.04	0.18	0.19
70	0.229 427E-07	0.313 086E-07	1.55	2.11	0.14	0.20
∑(2 - 70)	**0.476 359E-05**	**0.133 327E-06**	**18.59**	**7.39**	**30.20**	**0.85**

$$c_n = \left(\sum_{m=0}^{n} (\overline{\Delta C}_{nm}^2 + \overline{S}_{nm}^2)\right)^{1/2}, \qquad m_{c_n} = \left(\sum_{m=0}^{n} (m_{\overline{C}_{nm}}^2 + m_{\overline{S}_{nm}}^2)\right)^{1/2},$$

$$\Delta g_n = \left(\frac{fM}{a^2}\right)(n-1)c_n, \qquad m_{\Delta g_n} = \left(\frac{fM}{a^2}\right)(n-1)m_{c_n},$$

$$\zeta_n = Rc_n, \qquad m_{\zeta_n} = Rm_{c_n},$$

where R – mean Earth radius (R = 6 371 000 м).

In the last table line total values of characteristics of abnormal Earth's gravity field satellite model and their errors are in bold.

Appendix 6 Characteristics of abnormal Earth's gravity field model in the form of series of spherical harmonics to the 360th degree (PZ-2002/360)

Initial data:

abnormal Earth's gravity field satellite model PZ-2002/70s (Appendix 5)

catalogue of mean gravity anomalies with resolution 30' x 30'

Characteristics of initial gravity anomalies catalogue, mGal:: 259 200 values of gravity anomalies

minimum	– 300.3
maximum	366.7
mean	– 0.58
rms	28.68
average value of RMS error	6.80

In table A6.1 values of degree variances of values of gravity anomalies, quasigeoid heights and their errors are given calculated by the model.

Table A.6.1

Degree n	Characteristic			
	Δg_n, mGal	$m_{\Delta g_n}$, mGal	ζ_n, m	m_{ζ_n}, m
1	2	3	4	5
2	2.75	0.0001	17.93	0.0006
3	5.82	0.0006	18.94	0.002
4	4.44	0.001	9.65	0.003
5	4.58	0.002	7.45	0.004
6	4.43	0.003	5.77	0.005
7	4.42	0.005	4.80	0.006
8	3.34	0.008	3.11	0.007
9	3.34	0.01	2.72	0.008
10	3.13	0.01	2.26	0.010
11	2.56	0.01	1.66	0.012
12	1.65	0.02	0.98	0.013
13	2.83	0.02	1.53	0.014
14	1.92	0.03	0.96	0.016
15	1.92	0.03	0.89	0.018
16	2.02	0.04	0.87	0.020
17	1.85	0.05	0.75	0.022
18	2.00	0.06	0.76	0.024
19	1.77	0.07	0.64	0.025
20	1.82	0.08	0.62	0.027
21	2.02	0.08	0.65	0.029
22	1.94	0.09	0.60	0.030
23	1.78	0.10	0.52	0.032
24	1.60	0.11	0.45	0.033
25	1.84	0.12	0.50	0.034
26	1.62	0.13	0.42	0.035
27	1.32	0.14	0.33	0.036
28	1.81	0.15	0.43	0.037
29	1.64	0.16	0.38	0.038
30	1.72	0.17	0.38	0.038
31	1.71	0.18	0.37	0.039
32	1.67	0.18	0.35	0.039
33	1.80	0.19	0.36	0.040
34	2.04	0.20	0.40	0.040
35	2.01	0.21	0.38	0.040
36	1.79	0.21	0.33	0.040
37	1.92	0.22	0.34	0.040
38	1.73	0.23	0.30	0.040
39	1.84	0.23	0.31	0.040
40	1.68	0.24	0.28	0.040
41	1.83	0.25	0.29	0.040
42	1.88	0.25	0.29	0.040
43	1.77	0.26	0.27	0.040
44	1.77	0.26	0.26	0.040
45	1.87	0.27	0.27	0.040
46	1.99	0.27	0.28	0.039
47	1.96	0.28	0.27	0.039
48	1.80	0.28	0.24	0.039
49	1.65	0.28	0.22	0.039
50	1.88	0.29	0.25	0.039
51	1.73	0.29	0.22	0.038
52	1.79	0.30	0.22	0.038
53	2.02	0.30	0.25	0.038
54	1.92	0.31	0.23	0.038
55	1.78	0.31	0.21	0.037
56	1.99	0.31	0.23	0.037
57	1.96	0.32	0.22	0.037
58	1.73	0.32	0.19	0.037
59	1.92	0.32	0.21	0.036
60	1.80	0.33	0.19	0.036
61	1.78	0.33	0.19	0.036
62	1.81	0.33	0.19	0.036
63	1.82	0.34	0.19	0.035
64	1.69	0.34	0.17	0.035
1	2	3	4	5
65	1.66	0.34	0.16	0.035
66	1.79	0.35	0.17	0.035
67	1.82	0.35	0.17	0.034
68	1.81	0.35	0.17	0.034
69	1.84	0.35	0.17	0.034
70	1.59	0.36	0.15	0.034
∑(2 - 70)	**18.83**	**1.89**	**30.38**	**0.273**
71	1.61	0.37	0.15	0.034
72	1.86	0.37	0.17	0.034
73	1.68	0.37	0.15	0.034
74	1.90	0.37	0.17	0.033
75	1.71	0.38	0.15	0.033
76	1.65	0.38	0.14	0.033
77	1.70	0.38	0.14	0.033
78	1.69	0.38	0.14	0.032
79	1.73	0.39	0.14	0.032
80	1.68	0.39	0.13	0.032
81	1.87	0.39	0.15	0.032
82	1.99	0.39	0.16	0.032
83	1.92	0.40	0.15	0.031
84	1.74	0.40	0.13	0.031
85	1.69	0.40	0.13	0.031
86	1.84	0.40	0.14	0.031
87	1.74	0.41	0.13	0.031
88	1.71	0.41	0.12	0.030
89	1.68	0.41	0.12	0.030
90	1.57	0.41	0.11	0.030
91	1.77	0.42	0.12	0.030
92	1.70	0.42	0.12	0.030
93	1.73	0.42	0.12	0.030
94	1.78	0.42	0.12	0.029
95	1.73	0.42	0.12	0.029
96	1.56	0.43	0.10	0.029
97	1.70	0.43	0.11	0.029
98	1.80	0.43	0.12	0.029
99	1.63	0.43	0.10	0.029

100	1.67	0.44	0.11	0.028
101	1.44	0.44	0.09	0.028
102	1.71	0.44	0.11	0.028
103	1.89	0.44	0.12	0.028
104	1.67	0.44	0.10	0.028
105	1.54	0.45	0.09	0.028
106	1.67	0.45	0.10	0.028
107	1.65	0.45	0.10	0.027
108	1.72	0.45	0.10	0.027
109	1.65	0.45	0.09	0.027
110	1.80	0.46	0.10	0.027
111	1.70	0.46	0.10	0.027
112	1.58	0.46	0.09	0.027
113	1.63	0.46	0.09	0.027
114	1.73	0.47	0.10	0.027
115	1.74	0.47	0.09	0.027
116	1.62	0.47	0.09	0.026
117	1.63	0.47	0.09	0.026
118	1.66	0.47	0.09	0.026
119	1.66	0.48	0.09	0.026
120	1.66	0.48	0.09	0.026
121	1.59	0.48	0.08	0.026
122	1.48	0.48	0.07	0.026
123	1.65	0.49	0.08	0.026
124	1.57	0.49	0.08	0.026
125	1.66	0.49	0.08	0.025
126	1.66	0.49	0.08	0.025
127	1.72	0.50	0.08	0.025
128	1.47	0.49	0.07	0.025
129	1.53	0.50	0.07	0.025
130	1.49	0.50	0.07	0.025
131	1.49	0.50	0.07	0.025
132	1.46	0.50	0.07	0.025
133	1.56	0.51	0.07	0.025
134	1.51	0.51	0.07	0.025
135	1.50	0.51	0.07	0.025
136	1.53	0.51	0.07	0.024
137	1.50	0.51	0.07	0.024
138	1.60	0.52	0.07	0.024
139	1.43	0.52	0.06	0.024
140	1.54	0.52	0.07	0.024
141	1.48	0.52	0.06	0.024
142	1.34	0.52	0.06	0.024
143	1.43	0.53	0.06	0.024
144	1.49	0.53	0.06	0.024
145	1.42	0.53	0.06	0.024
146	1.41	0.53	0.06	0.024
147	1.42	0.53	0.06	0.024
148	1.41	0.54	0.06	0.023
149	1.41	0.54	0.06	0.023
150	1.37	0.54	0.06	0.023
151	1.52	0.55	0.06	0.023

152	1.36	0.54	0.05	0.023
153	1.41	0.55	0.06	0.023
154	1.27	0.55	0.05	0.023
155	1.45	0.55	0.06	0.023
156	1.39	0.55	0.05	0.023
157	1.37	0.56	0.05	0.023
158	1.29	0.56	0.05	0.023
159	1.40	0.56	0.05	0.023
160	1.28	0.56	0.05	0.023
161	1.33	0.56	0.05	0.023
162	1.38	0.57	0.05	0.023
163	1.41	0.57	0.05	0.023
1	2	3	4	5
164	1.38	0.57	0.05	0.023
165	1.43	0.57	0.05	0.023
166	1.37	0.58	0.05	0.022
167	1.41	0.58	0.05	0.022
168	1.44	0.58	0.05	0.022
169	1.32	0.58	0.05	0.022
170	1.35	0.58	0.05	0.022
171	1.37	0.59	0.05	0.022
172	1.25	0.59	0.04	0.022
173	1.30	0.59	0.04	0.022
174	1.41	0.60	0.05	0.022
175	1.29	0.59	0.04	0.022
176	1.20	0.59	0.04	0.022
177	1.28	0.60	0.04	0.022
178	1.22	0.60	0.04	0.022
179	1.34	0.60	0.04	0.022
180	1.29	0.60	0.04	0.022
181	1.20	0.60	0.04	0.022
182	1.26	0.61	0.04	0.022
183	1.20	0.61	0.04	0.021
184	1.25	0.61	0.04	0.021
185	1.18	0.61	0.04	0.021
186	1.20	0.61	0.04	0.021
187	1.22	0.61	0.04	0.021
188	1.22	0.62	0.04	0.021
189	1.14	0.61	0.03	0.021
190	1.13	0.62	0.03	0.021
191	1.17	0.62	0.04	0.021
192	1.22	0.62	0.04	0.021
193	1.18	0.62	0.04	0.021
194	1.10	0.62	0.03	0.021
195	1.14	0.62	0.03	0.021
196	1.11	0.62	0.03	0.021
197	1.11	0.63	0.03	0.020
198	1.15	0.63	0.03	0.020
199	1.11	0.63	0.03	0.020
200	1.09	0.63	0.03	0.020
201	1.12	0.63	0.03	0.020

202	1.08	0.63	0.03	0.020
203	1.06	0.63	0.03	0.020
204	1.09	0.64	0.03	0.020
205	1.01	0.63	0.03	0.020
206	1.07	0.64	0.03	0.020
207	1.07	0.64	0.03	0.020
208	1.06	0.64	0.03	0.020
209	1.05	0.64	0.03	0.020
210	1.15	0.65	0.03	0.020
211	1.09	0.65	0.03	0.020
212	1.06	0.65	0.03	0.020
213	1.02	0.65	0.03	0.019
214	1.10	0.65	0.03	0.020
215	1.02	0.65	0.03	0.019
216	1.00	0.65	0.03	0.019
217	1.03	0.65	0.03	0.019
218	1.01	0.65	0.03	0.019
219	1.09	0.66	0.03	0.019
220	1.01	0.66	0.03	0.019
221	0.99	0.66	0.02	0.019
222	0.99	0.66	0.02	0.019
223	0.98	0.66	0.02	0.019
224	0.97	0.66	0.02	0.019
225	0.97	0.66	0.02	0.019
226	0.98	0.66	0.02	0.019
227	0.91	0.66	0.02	0.019
228	0.92	0.66	0.02	0.019
229	0.98	0.67	0.02	0.019
230	0.94	0.67	0.02	0.019
231	0.98	0.67	0.02	0.019
232	1.01	0.67	0.02	0.019
233	0.96	0.67	0.02	0.018
234	0.95	0.67	0.02	0.018
235	0.93	0.67	0.02	0.018
236	0.92	0.67	0.02	0.018
237	0.92	0.67	0.02	0.018
238	0.91	0.67	0.02	0.018
239	0.90	0.67	0.02	0.018
240	0.90	0.68	0.02	0.018
241	0.90	0.68	0.02	0.018
242	0.89	0.68	0.02	0.018
243	0.87	0.68	0.02	0.018
244	0.83	0.68	0.02	0.018
245	0.88	0.68	0.02	0.018
246	0.87	0.68	0.02	0.018
247	0.84	0.68	0.02	0.018
248	0.80	0.68	0.02	0.018
249	0.82	0.68	0.02	0.017
250	0.88	0.69	0.02	0.018
251	0.83	0.68	0.02	0.017
252	0.87	0.69	0.02	0.017
253	0.86	0.69	0.02	0.017

254	0.80	0.68	0.02	0.017
255	0.81	0.69	0.02	0.017
256	0.79	0.69	0.02	0.017
257	0.79	0.69	0.02	0.017
258	0.76	0.69	0.01	0.017
259	0.79	0.69	0.01	0.017
260	0.78	0.69	0.01	0.017
261	0.79	0.69	0.01	0.017
262	0.74	0.69	0.01	0.017
263	0.78	0.69	0.01	0.017
264	0.73	0.69	0.01	0.017
1	2	3	4	5
265	0.73	0.69	0.01	0.017
266	0.77	0.69	0.01	0.017
267	0.77	0.70	0.01	0.017
268	0.74	0.69	0.01	0.017
269	0.74	0.70	0.01	0.017
270	0.71	0.69	0.01	0.016
271	0.77	0.70	0.01	0.017
272	0.75	0.70	0.01	0.016
273	0.69	0.69	0.01	0.016
274	0.75	0.70	0.01	0.016
275	0.70	0.70	0.01	0.016
276	0.73	0.70	0.01	0.016
277	0.69	0.70	0.01	0.016
278	0.75	0.71	0.01	0.016
279	0.73	0.70	0.01	0.016
280	0.71	0.70	0.01	0.016
281	0.74	0.71	0.01	0.016
282	0.72	0.71	0.01	0.016
283	0.69	0.70	0.01	0.016
284	0.67	0.70	0.01	0.016
285	0.66	0.70	0.01	0.016
286	0.70	0.71	0.01	0.016
287	0.65	0.70	0.01	0.016
288	0.68	0.71	0.01	0.016
289	0.65	0.71	0.01	0.016
290	0.68	0.71	0.01	0.016
291	0.67	0.71	0.01	0.016
292	0.68	0.71	0.01	0.016
293	0.68	0.71	0.01	0.015
294	0.68	0.71	0.01	0.015
295	0.71	0.72	0.01	0.015
296	0.67	0.71	0.01	0.015
297	0.65	0.71	0.01	0.015
298	0.63	0.71	0.01	0.015
299	0.63	0.71	0.01	0.015
300	0.64	0.71	0.01	0.015
301	0.65	0.71	0.01	0.015
302	0.61	0.71	0.01	0.015
303	0.65	0.72	0.01	0.015

304	0.63	0.71	0.01	0.015
305	0.63	0.72	0.01	0.015
306	0.63	0.72	0.01	0.015
307	0.61	0.71	0.01	0.015
308	0.63	0.72	0.01	0.015
309	0.67	0.72	0.01	0.015
310	0.61	0.72	0.01	0.015
311	0.61	0.72	0.01	0.015
312	0.60	0.72	0.01	0.015
313	0.60	0.72	0.01	0.015
314	0.57	0.71	0.01	0.014
315	0.60	0.72	0.01	0.015
316	0.57	0.72	0.01	0.014
317	0.62	0.72	0.01	0.014
318	0.65	0.73	0.01	0.015
319	0.60	0.72	0.01	0.014
320	0.63	0.73	0.01	0.014
321	0.62	0.73	0.01	0.014
322	0.62	0.73	0.01	0.014
323	0.61	0.73	0.01	0.014
324	0.58	0.72	0.01	0.014
325	0.57	0.72	0.01	0.014
326	0.59	0.72	0.01	0.014
327	0.56	0.72	0.01	0.014
328	0.59	0.73	0.01	0.014
329	0.56	0.72	0.01	0.014
330	0.54	0.72	0.01	0.014
331	0.57	0.72	0.01	0.014
332	0.54	0.72	0.01	0.014
333	0.56	0.72	0.01	0.014
334	0.55	0.72	0.01	0.014
335	0.56	0.72	0.01	0.014
336	0.57	0.73	0.01	0.014
337	0.55	0.72	0.01	0.014
338	0.56	0.73	0.01	0.014
339	0.54	0.72	0.01	0.014
340	0.56	0.73	0.01	0.014
341	0.57	0.73	0.01	0.014
342	0.54	0.73	0.01	0.013
343	0.55	0.73	0.01	0.013
344	0.54	0.73	0.01	0.013
345	0.53	0.73	0.01	0.013
346	0.51	0.72	0.009	0.013
347	0.55	0.73	0.01	0.013
348	0.52	0.73	0.008	0.013
349	0.51	0.72	0.008	0.013
350	0.51	0.72	0.009	0.013
351	0.54	0.73	0.01	0.013
352	0.53	0.73	0.009	0.013
353	0.53	0.73	0.008	0.013
354	0.51	0.73	0.008	0.013
355	0.52	0.73	0.007	0.013
356	0.51	0.73	0.006	0.013
357	0.52	0.73	0.008	0.013
358	0.53	0.73	0.005	0.013
359	0.51	0.73	0.009	0.013
360	0.52	0.73	0.007	0.013
∑(2 - 360)	**27.37**	**10.72**	**30.40**	**0.45**

In the Table total values of degree variances of values of gravity anomalies, quasigeoid heights and their errors are given calculated by the model:

- for degrees *n* from 2 to 70 corresponding to combined abnormal Earth’s gravity field model to 70^{th} degree (PZ-2002/70);
- for degrees *n* from 2 to 360 corresponding to high degree abnormal Earth gravity field model to 360^{th} degree (PZ-2002/360).

Appendix 7 Abnormal Earth's gravity field model TM-60

Table A7.1

#	Mass ($\varepsilon \cdot 10^{10}$)	Geocentric Cartesian point masses coordinates		
		X, km	*Y*, km	*Z*, km
1	2	3	4	5
1	–1 917 861.343	–1 597.53455	3 389.08854	–1 206.07844
2	–7 649.811	5 243.88105	2 173.09105	31.67769
3	–23 204 717.367	–694.74764	–2 543.95209	3 010.82934
4	–8 601 525.203	597.26083	4 124.29275	3 032.72188
5	–39 262 968.108	1 472.01984	2 496.13550	2 305.68074
6	–37 656 928.613	–1 395.72710	522.57926	1 219.80230
7	55 417.233	–4 729.43278	241.06220	–1 386.75430
8	–1 122 388.574	1 613.74542	–1 389.46971	–4 742.37198
9	11 553 321.883	–737.33302	4 223.69658	–89.84823
10	–3 212 166.768	–3 498.93844	2 123.53095	–2 780.28445
11	–127 570.128	2 443.27977	–2 075.43783	–3 115.11405
12	537 718.408	1 399.38281	–2 714.24422	–1 999.26660
13	159 431.526	–1 807.95598	4 038.16007	–1 984.43463
14	–9 621 707.493	500.99426	2 417.51767	2 081.23547
15	14 954 047.019	564.56372	4 169.62120	3 107.28151
16	–1 454 978.047	–3 606.22095	2 202.63232	2 555.27655
17	12 173.463	3 494.94521	–2 225.41218	3 504.56634
18	6 235 318.028	2 819.10268	1 215.19399	3 031.84264
19	–4 022 993.036	830.78873	1 187.36734	2 665.51499
20	–13 233 537.348	–2 407.50792	–987.75326	1 168.97751
21	–837 955.460	2 745.40696	3 312.38455	561.61201
22	66 944.487	474.28039	1 205.35801	–4 546.03756
23	9 533 469.996	–1 675.28817	–797.14875	–3 048.67319
24	13 126.101	1 951.39223	–5 027.13641	–1 609.06073
25	133 490 487.677	–1 131.17750	–625.93077	614.81653
26	11 202.018	4 552.11518	1 485.42650	–1 997.43144
27	–8 306 318.912	32.85983	1 869.79667	–2 877.64543
28	1 563 860.673	–3 588.59466	2 192.00472	2 532.32304
29	–4 110 249.075	–1 688.88208	–2 509.59158	997.39780
30	–708 432.828	142.71007	–2 365.17339	3 559.90481
31	115 758.347	–2 583.62219	3 876.78111	–1 530.49081
32	47 117.768	–4 410.11412	–1 491.17613	1 951.16572
33	3 060 709.563	–3 512.99151	2 136.54197	–2 789.00902
34	125 694.630	2 887.19498	–1 520.35002	–2 929.00588
35	–19 909 835.138	–1 221.99104	–1 430.21334	2 886.63494
36	–25 600 026.158	–795.89525	4 094.06397	–148.43103
37	8 998 492.144	–859.93100	–2 675.58249	3 049.08749
38	6 460 340.213	134.47321	–3 777.76920	–737.29613
39	22 635 646.931	2 030.13210	2 786.99103	1 094.12609
40	14 963 823.854	–894.96828	3 946.71389	–241.43049
41	104 200 607.824	1 424.84214	2 420.76271	1 954.22532
42	–29 025.051	3 640.95418	2 890.38428	–1 299.38754
43	–25 923 827.511	–1 666.65137	–1 236.38163	–177.16683

1	2	3	4	5
44	4 710.121	–2 926.62377	1 305.39588	4 683.40996
45	–72 296 468.140	1 704.30580	2 570.01705	1 437.94150
46	11 488 188.566	–1 329.71266	–1 480.67585	3 122.43463
47	30 871.376	2 993.13075	–3 221.87772	2 310.62018
48	–6 821 341.595	539.11732	4 204.18554	3 168.98432
49	4 053 215.384	520.10547	–3 574.66707	–271.75382
50	2 364 380.549	–1 696.20051	1 611.26783	2 166.50206
51	–5 107 799.845	–2 806.42940	90.03200	–437.20618
52	1 152 914.935	1 613.29970	–1 398.25374	–4 728.63714
53	14 079 228.613	1 331.61419	–1 428.22398	234.32978
54	24 148.390	2 240.63930	2 260.34015	–3 797.81205
55	14 981 240.134	–523.57471	–2 437.64320	3 040.87357
56	–7 746 811.230	2 751.33446	1 231.49383	2 975.90618
57	–52 402 113.809	490.27870	–1 222.02040	299.60165
58	–9 702 053.744	214.47365	–3 717.91494	–621.72796
59	6 998 066.913	43.26692	1 953.11104	–2 943.24995
60	–11 022 424.436	–1 649.80829	–755.81754	–2 992.38699

Appendix 8 Local model of abnormal Earth's gravity field in the form of gravitational potential of point masses

(pattern)

Local model of abnormal Earth's gravity field represents abnormal Earth's gravity field as a whole with additional specification in local area. Every point mass is described by four parameters – *X*, *Y*, *Z* coordinates specifying its spatial position and characteristic of mass ε.

Local model of abnormal Earth's gravity field consists of planetary, regional and some local subsystems. As planetary subsystem model TM-60 is taken (Appendix 7) obtained by approximation disturbing potential represented by satellite model
PZ–2002/70s to n ~ 14. Regional and local subsystems refine Earth's gravity field representation in bounded areas. Scope and main characteristics of local model of abnormal Earth's gravity field comprising 260 point masses are given in Tables A8.1 and A8.2, structure and parameters of this model – in Table A8.3.

Table A8.1 – Scope of local model of abnormal Earth's gravity field

Latitude, degree		Longitude east, grad	
North	South	West	East
80	68	68	98

Table A8.2 – Main characteristics of local model of abnormal Earth's gravity field

Name	Number of point masses	Initial data resolution
Planetary subsystem	60	12° x 12°
Regional subsystem	25	5° x 5°
Local subsystem 1	49	60' x 90'
Local subsystem 2	50	30' x 60'
Local subsystem 3	76	15' x 30'

Table A8.3 – Parameters of local model of abnormal Earth's gravity field

№ of point mass	Mass ($\varepsilon \cdot 10^{10}$)	*X*, km	*Y*, km	*Z*, km
1	2	3	4	5
1	-1 917 861.343	-1 597.53455	3 389.08854	-1 206.07844
2	-7 649.811	5 243.88105	2 173.09105	31.67769
3	-23 204 717.367	-694.74764	-2 543.95209	3 010.82934
…	…	…	…	…
61	–7 403.655	1 042.131	2 631.301	4 664.285
62	1 958.729	858.888	–877.295	5 701.957
63	–3 978.285	1 224.054	–361.399	5 531.942

1	2	3	4	5
…	…	…	…	…
86	–161.253	2 660.226	1 021.165	5 554.288
87	131.050	1 038.208	1 510.602	5 954.431
88	576.454	611.501	611.501	6 110.897
…	…	…	…	…
135	39.202	1 779.268	1 688.463	5 810.007
136	–60.782	1 516.884	1 776.043	5 814.581
137	–76.032	1 901.081	1 075.580	5 880.511
…	…	…	…	…
185	–21.849	2 057.484	1 578.763	5 751.765
186	13.226	1 460.070	1 080.395	6 042.025
187	–19.403	1 619.154	934.819	6 027.000
…	…	…	…	…
260	–8.296	1 575.183	1 275.558	5 973.493

Appendix 9 Graphical analogue of gravity anomaly digital model

(pattern)

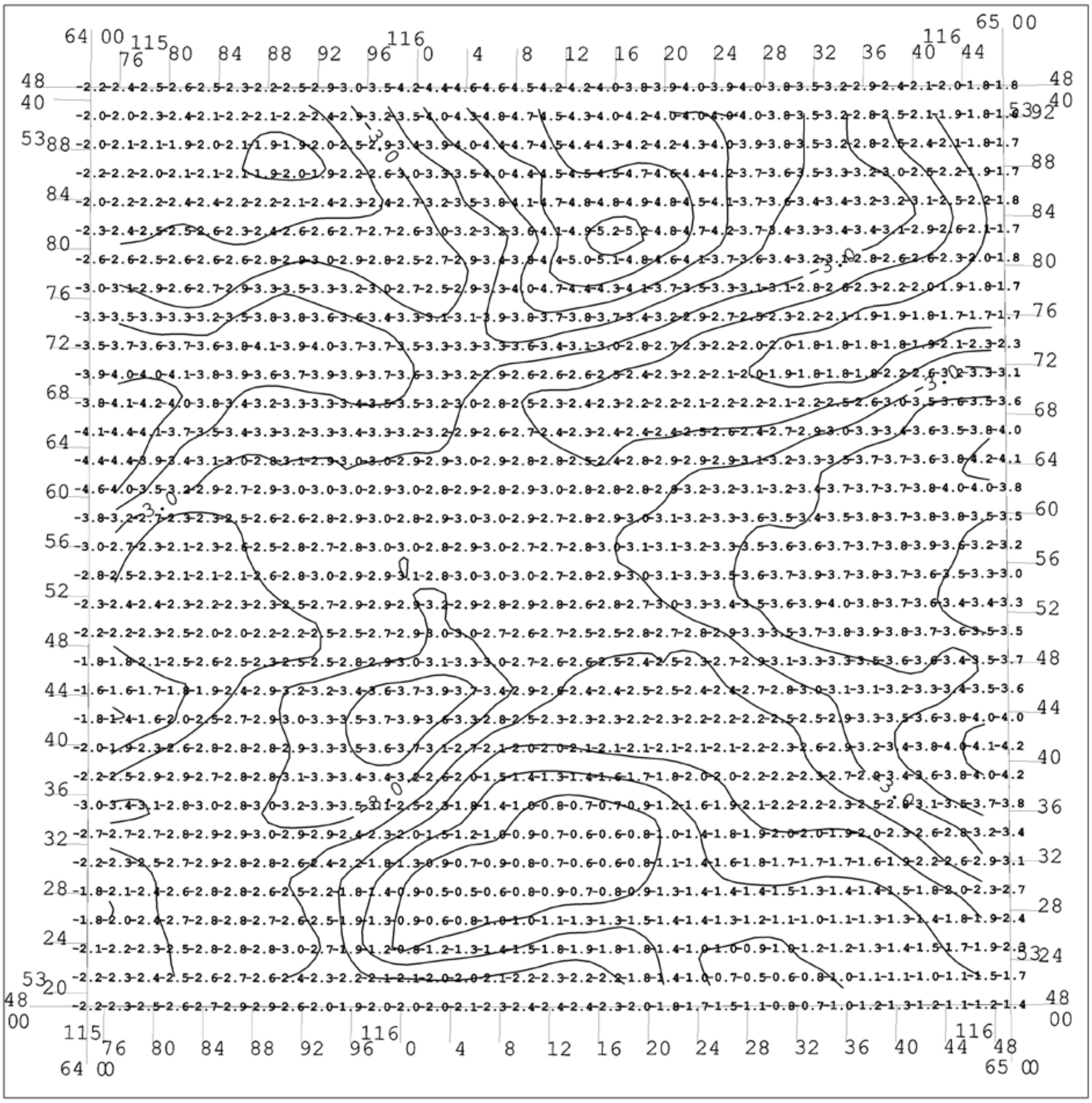
GRAVITY ANOMALY MAP

Bouguer correction ($\sigma = 2.3 g/sm^3$)

M-41-III

Normal Helmert formula 1901 – 1909

Gravimetric system of 1971



Charted in 2002

RMS error of Gravity anomaly determination ±1 mGal

Gravity Contour are drawn through 0.5mGal

Note

The step of named value:

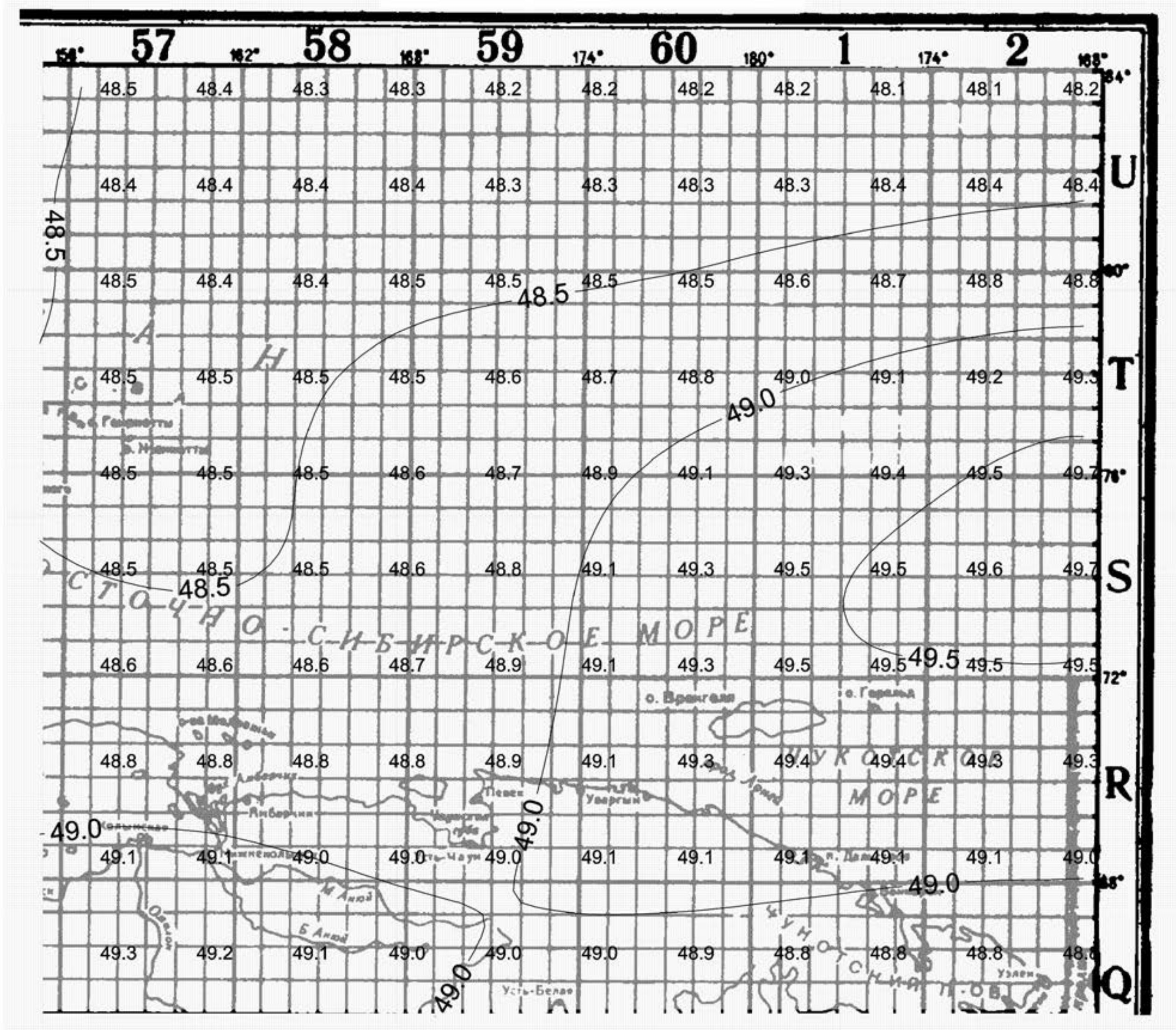
Latitude – 1.25′

Longtitude – 1.875′

Appendix 10 Graphical analogue of quasigeoid height digital model

(pattern)

QUASIGEOID HEIGHT MAP
Above Earth ellipsoid
(Parametry Zemli 1990)



Charted in 1995

RMS error of quasigeoid height determination ±1 m
Continuous contour lines are drawn through 0.5m

Appendix 11 Graphical analogue of digital model of deviations of the plumb line

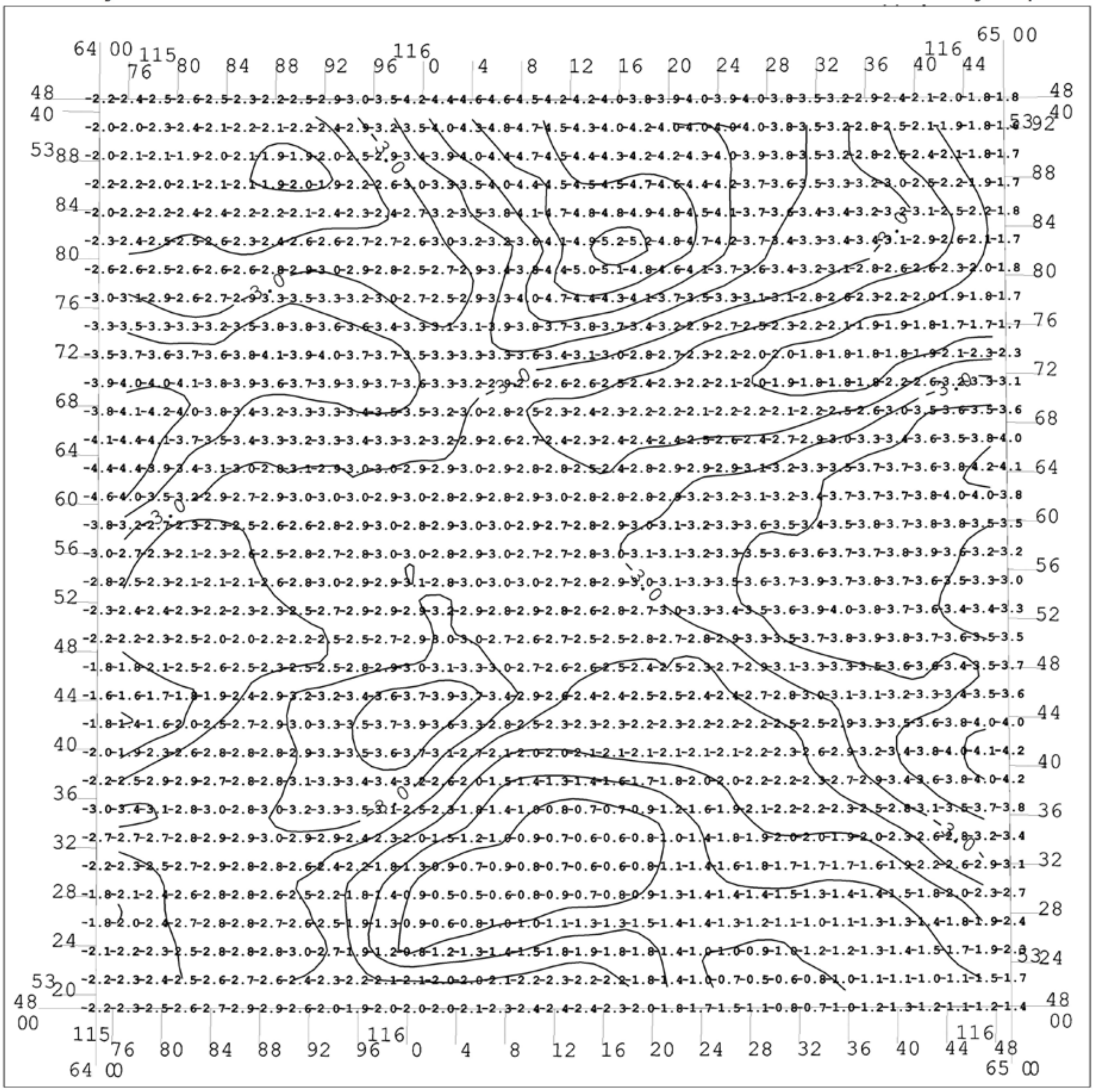
(pattern)

DEVIATIONS OF THE PLUMB LINE MAP
In meridian (ξ)
M-41-III

Coordinate system 1942

Parametry Zemli 1990

Krasovsky ellipsoid



Charted in 2002

RMS error of determination of
component ξ = ±0.5″
correction ΔA = ±0.5″
Continuous lines of ξ equal values are drawn through 0.5″
Note
The step of named ξ value:
latitude – 1.25′
longtitude – 1.875′