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Mar. 31, 1974

Dear Neil,

Please send me a copy of your "Handbook of Integer Sequences" I saw the review in S.A. April 1974.

Yes, I was heartily surprised at how many sequences have convolution arrays with the unit determinant property. All (1, 1, x, y, z, ...) sequences do x, y, z, ... are arbitrary.} The proof of the theorem (Hoggatt-Kramer) is in a thesis of Judy Kramer available from the Fibonacci Association (Brother Alfred above) for \$10.00.

Enclosed is the paper by Hoggatt & Bergum.

Most sincerely,

Vern

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April 20, 1974

Dear Neil,

I see from your Format you can insert new sequences endlessly with references for each one. I appreciate your Supplement I. I'll be happy to send you several (hundred) more sequences but alas I'll have to wait until I see your general plan ad format. (ad what you already have. I eagerly await your handbook via Academic Press.

In two of our publications edited by Brother Alfred there are several sequences, too! When I get a few days I'll do you some real good!!

For instance: 1, 1, 2, 5, 14, 42, ... CATALAN NUMBERS $\frac{1}{n+1}\binom{2n}{n}$ etc. However 1, 2, 5, 14, 42, ... is the FIRST CONVOLUTION SEQUENCE FOR THE CATALAN NUMBERS.

Reference (~~appendix~~ Supplement your pg 22) Sequence 1174 1, 3, 12, 55, ...

This the second Convolution sequence for 1, 1, 3, 12, 55, ...

your sequence 1174 appears in L. Carlitz's article "Two-line Enumeration" APRIL 1973 FQJ

I'll have much more to say on ~~all~~ this.

Sincerely, Vern

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April 22, 1975

Dear Neil,

you almost threw me when you asked about

1, 2, 5, 12, 28, 64, ... $a_{n+1} = 2a_n + 2^{n-2}$

this occurs in the counting the Compositions of n using all positive integers as possible summands. If one then displays all such Compositions all the same time the number of ones used is given by the sequence above

	ones
1 = 1	1
2 = 2, 1+1	2
3 = 2+1, 1+2, 1+1+1, 3	5
4 = 2+1+1, 1+2+1, 1+1+2, 3+1, 1+3, 1+1+1+1, 4, 2+2	12
5: 2+2+1, 2+1+2, 1+2+2; 2+1+1+1, 1+2+1+1, 1+1+2+1, 1+1+1+2; 3+1+1, 1+3+1, 1+1+3; 1+4, 4+1, 1+1+1+1+1, 5, 3+2, 2+3	28

this is also the Convolution of 1, 1, 2, 4, 8, ... 2^{n-1}, ... with itself. The reference is "Palindromic Compositions" by V. E. HOGGATT, JR and Marjorie Bicknell to appear in

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the Fibonacci Quarterly Dec 1975, I cannot tell you the page numbers in that issue yet!..

Yes, Neil, I am well aware of my promise. Since by far most of my articles are written with Co-Author Marjorie Bicknell, I'll try to get her to "cooperate" and get you some new sequences real soon. I get new sequences almost every day.

N.B. If $F(X) = X^{a_0} + X^{a_1} + \dots + X^{a_k} + \dots$ then the number of compositions of n using summands from $\{a_n\}_{n=0}^{\infty}$ is given by

$$\sum_{n=0}^{\infty} C_n X^n = \frac{1}{1-F(x)}$$

In case $F(X) = X^1 + X^2 + X^3 + \dots + X^n + \dots = \frac{X}{1-X}$ then the generating function is $\frac{1-X}{1-2X} = \{1, 1, 2, 4, 8, \dots\}$

Now the number of occurrences of 1 in all the compositions of n is given by $\frac{X^1}{[1-F(x)]^2} = \frac{X(1-X)^2}{(1-2X)^2}$

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= the Convolution of sequence 1, 1, 2, 4, 8, ... with itself.

Sincerely, Vern

P.S. The occurrences of k (a positive integer) in the Compositions of n is given by $x^k\left(\frac{(1-x)}{1-2x}\right)^2$ which is the same sequence with $(k-1)$ zeros at the front. Thus the sequence is well used in this particular problem.

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Dear Neil, (N. J. A. Sloane) Nov. 14, 1976

If you have watched the Fibonacci Quarterly you will have seen: FIVE ARTICLES

Dec 1975 1) The H-Convolution Transform.

2)

APRIL 1976 3) PASCAL, CATALAN and General Sequence Convolution arrays in a matrix

October 1976 4) Sequences of matrix Inverses from Pascal Catalan and Related Convolution arrays.

Dec 1976 5) CATALAN and Related Sequences arising from inverses of Pascal's Triangle Matrices

Feb 1977 6. Numerator Polynomial Coefficient Arrays for Catalan and Related Sequence Convolution Triangles.

February 1976 7. Numerator Polynomial Coefficient array for convolved Fibonacci Sequence.

These articles are chock-full of sequences at some of which are not in your supplemented list. Actually I could send you a new sequence everyday as I am in a creative resurgence right now. I would like to see a section on polynomial recurrence Sequence. It would be messier but a great help in some researches. Every triangular array... (over)

actually carries an infinitude of sequences.
Right now I'm finding Generalized Catalan Numbers
from new Pascal-like Triangles.

CATALAN NUMBERS

Pascal Triangle

$$C_1(X) = \frac{1-\sqrt{1-4X}}{2X}$$

$$\frac{1}{\sqrt{1-4x}} = \{1, 2, 6, 20, 70, \dots\}_2$$

$$C_2(X) = \frac{1}{1-X}\, C_1\left(\frac{X^2}{(1-X)^2}\right) = \frac{1-X-\sqrt{1-2X-3X^2}}{2X^2}$$

1, 3, 7, 19, 51, ... #1070 central terms Trinomial Triangle

$$C_3(X) = \frac{1}{1-X}\, C_1\left(\frac{X}{(1-X)^2}\right) = \frac{1-X-\sqrt{1-6X+X^2}}{2X}$$

from the triangular array

$$\begin{array}{llllll}
1 & & & & & \\
1 & 1 & & & & \\
1 & 3 & 1 & & & \\
1 & 5 & 5 & 1 & & \\
1 & 7 & 13 & 7 & 1 & \\
1 & 9 & 25 & 25 & 9 & 1
\end{array}$$

1, 2, 6, 22, 90, ...

= 1, 2(1, 3, 11, 45, ...)

This is in your table as #1163

~~1, 2~~

Here $\boxed{x}$ $\boxed{y}\ \boxed{z}$ $\boxed{w}$

$$\boxed{W = x+y+z}$$

your #1174 occurs as a generalized Catalan lattice count. See fig 2 Pg 136 April 1976 FQJ.

also #1163 appears in Oct 1976 Page 231

$$\sum_{n=1}^{\infty} b_n X^n = \frac{1+X-\sqrt{1-6X+X^2}}{4X}$$

Some dialogue, please!

Sincerely, Vern.

Recurrence for Column generators of table one is

$$x^2 C^{2n+5}(x) = (1-2x)C^{2n+2}(x) - C^{2n+1}(x) \quad n \geq 0$$

from which the entire table may be created once the coefficients of $C(x)$ are known.

there $C_{n+1} = \sum_{k=0}^{n} C_k C_{n-k} \quad C_0 = 1$

will do it.

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July 6, 1975

Dear Neil,

Thank you for the Copy of Athanasios Papoulis "A new Method of Inversion of the Laplace Transform". I now affirm that those entries in table One are indeed the ~~odd~~ even convolution sequences for the Catalan Sequence.

1 1
1 1
1 1 1 2 1
1 3 3 1
1 1 3 4 2 6 4 1
1 5 10 10 5 1
1 5 6 9 15 5 20

The circled numbers are by Column the Catalan Sequence, $C(x)$, the next left $C^3(x)$, etc
zeroth Convolution 2nd Convolution

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This appear, at least, in a thesis "The Generating functions for the Vertical Columns of the (N+1)-Nomial Triangle by Michel Rondeau written under my guidance in Dec 1973 at San Jose State University. Therein are many relations such as the above.

Sincerely, Vern