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A110
A587
A11971
A247108

August 2, 1984
DENYS WUILQUIN
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BAGUIO-CITY 0201
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Mr. NEIL A. SLOANE
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Dear Mr. Sloane,

With a calculator HP41C Hewlett Packard, it was not difficult to compute the Sum S and s of these two CONVERGENT Series:

$$S = \frac{1}{1} + \frac{2^n}{2!} + \frac{3^n}{3!} + \text{etc...} \quad \text{and} \quad s = \frac{1}{1} - \frac{2^n}{2!} + \frac{3^n}{3!} - \text{etc. etc...}$$

However, I was very astonished at first sight with the results I obtained; nevertheless, I immediately thought there was a mathematical relation between the coefficients relative to e, successively calculated for S; (1; 2; 5; 15; 52; etc.) and relative to 1/e for s; (1; 0; -1; -1; 2; 9; 9; etc.).

It was NOT EASY for me to discover these relations; but with A LOT OF PERSEVERANCE, I finally succeeded. I wrote a little after to Mr. Douglas R. HOFSTADTER (the artificial intelligence Laboratory, 545 Technology square, CAMBRIDGE, Massachussetts 02139 - Tel. 617253 USA) in order to let him know my work and to get his advice.

So, he advised me to send my results to you.

Beforehand, I did not know the "Bell numbers", and I discovered them only by chance, (as well as the way to calculate them).

Now, I should be very interested to know what is their EXACT PURPOSE. Is it ONLY as a result of this curious above Series, or for another important use in Mathematics or in Physics?

with the hope to receive some information about the matter,
I am,

Sincerely yours.

D. Wuilquin.

P.S. Please, will you excuse my English, which is sometimes uncorrect, because I am FRENCH.

About some strange convergent series -

if S represents $\frac{1}{1} + \frac{2^n}{2!} + \frac{3^n}{3!} + \frac{4^n}{4!} + \frac{5^n}{5!} + \frac{6^n}{6!} +$ etc, etc.

and if s represents $\frac{1}{1} - \frac{2^n}{2!} + \frac{3^n}{3!} - \frac{4^n}{4!} + \frac{5^n}{5!} - \frac{6^n}{6!} +$ etc. { which is the same as the "above series", but with signs + and − alternated

$e = 2.718281828\ldots$ (which is the base of the natural logarithms).

"!" means "factorial"; (for instance $6! = 1\times2\times3\times4\times5\times6$)

when exponent n =		
0	$S =$ A110 $e-1$	$s =$ A587 $\frac{1}{e-1}$
1	$S = e$	$s = \frac{1}{e}$
2	$S = 2e$	$s = 0$
3	$S = 5e$	$s = \frac{-1}{e}$
4	$S = 15e$	$s = -\frac{1}{e}$
5	$S = 52e$	$s = \frac{2}{e}$
6	$S = 203e$	$s = \frac{9}{e}$
7	$S = 877e$	$s = \frac{9}{e}$
8	$S = 4140e$	$s = \frac{-50}{e}$
9	$S = 21147e$	$s = -\frac{267}{e}$
10	$S = 115975e$	$s = \frac{-413}{e}$
11	$S = 678570e$	$s = \frac{2180}{e}$
12	$S = 4213597e$	$s = \frac{17731}{e}$
13	$S = 27644437e$	$s = \frac{50533}{e}$
14	$S = 190899322e$	$s = -\frac{110176}{e}$
15	$S = 1382958545e$	$s = -\frac{1966797}{e}$

etc. etc.

$$S = \frac{1^n}{1} + \frac{2^n}{2!} + \frac{3^n}{3!} + \frac{4^n}{4!} + \frac{5^n}{5!} + \frac{6^n}{6!} + \frac{7^n}{7!} + \text{etc. etc.}$$

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exponent n =	
1	1
2	1, 2
3	2, 3, 5
4	5, 7, 10, 15
5	15, 20, 27, 37, 52
6	52, 67, 87, 114, 151, 203
7	203, 255, 322, 409, 523, 674, 877
8	877, 1080, 1335, 1657, 2066, 2589, 3263, 4140
	4140

The "Bell numbers" (after Eric Temple Bell) 1, 2, 5, 15, 52, 203, etc.

Each "colored entry" is the sum of the two numbers (of the other color) situated to its left, as shown above.

$$s = \frac{1^n}{1} - \frac{2^n}{2!} + \frac{3^n}{3!} - \frac{4^n}{4!} + \frac{5^n}{5!} - \frac{6^n}{6!} + \frac{7^n}{7!} - \text{etc. etc.}$$

naught

Exponent n =		
	1	(0)
2	0, −1	(−1)
3	−1, −1, 0	(−1)
4	−1, 0, +1, +1	(1 + 1 = 2)
5	+2, +3, +3, +2, +1	(3 + 3 + 2 + 1 = 9)
6	+9, +7, +4, +1, −1, −2	(7 + 4 + 1 − 1 − 2 = 9)
7	+9, 0, −7, −11, −12, −11, −9	etc. etc.
8	−50, −59, −59, −52, −41, −29, −18, −9	
	−267	

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The "Anti-Bell numbers"? 1; 0; −1; −1; +2; +9; +9; −50; −267; etc.

Each "colored entry" corresponds to the difference of the two numbers (of the other colors, the one above, MINUS the one below) situated to its left, as shown above.

~~checkings:~~ Each "Black number" which is situated to the left of this above sketch (at the beginning of every horizontal line) corresponds to the sum of the ALGEBRAICAL numbers situated on the preceding horizontal line, just AFTER the red line: "/" (for instance 0 − 7 − 11 − 12 − 11 − 9 = −50);

CHECKING: all these "Black numbers" (situated to the left of every horizontal line) are also ending (to the extreme right) the horizontal line which is just following, but with the positive or negative sign INVERTED.

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August 6, 1984
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Dear Mr. Sloane,

On August 2, I sent you a letter relative to the "BELL Numbers", which I discovered per chance, after a long research.

As I told you, there is another interesting series of numbers too, (which may be considered belonging to the same mathematical family): 1; 0; -1; -1; +2; +9; +9; -50; -267; -413; +2180; +17731; +50533; etc.

At first, I found a different way to calculate them; but it is also possible to find them by using EXACTLY YOUR PROCESS, that is to say, BY ADDING (instead of SUBTRACTING from each other) the 2 numbers situated at the left of the new result to be found, just as shown below: the ONLY DIFFERENCE is that instead of beginning any new line with EXACTLY the same number which terminates the previous line (as when calculating your "ordinary Bell Numbers" 1; 2; 5; 15; 52; 203 etc.), you begin the following line with this last number always previously multiplied by -1, (that is to say with the sign + or - INVERTED)

with Exponent $n =$

$$s = \frac{1^n}{1} - \frac{2^n}{2!} + \frac{3^n}{3!} - \frac{4^n}{4!} + \frac{5^n}{5!} - \frac{6^n}{7!} + \frac{7^n}{7!} - \text{etc. etc.}$$

$(e = 2.718281828\ldots)$

with $n = 1$; $s = 1/e$	1									← Results	1
with $n = 2$; $s = 0$	-1	0								← "	2
with $n = 3$; $s = -1/e$	0	-1	-1							← "	3
with $n = 4$; $s = -1/e$	1	1	0	-1						← "	4
with $n = 5$; $s = 2/e$	1	2	3	3	2					← "	5
with $n = 6$; $s = 9/e$	-2	-1	1	4	7	9				← "	6
etc. etc.	-9	-11	-12	-11	-7	0	9			← "	7
	-9	-18	-29	-41	-52	-59	-59	-50		← "	8
	50	+41	+23	-6	-47	-99	-158	-217	-267	←	9
	267	etc. etc									

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Each "colored entry" is the SUM of the two numbers (of the other color) situated to its left, as shown above.

✡ Note = my first (or SUBTRACTING) way of calculation had however a USEFUL advantage when adding together ALGEBRAICAL numbers: an EASY CHECKING, which is not possible here with your method.

not exact.

Hoping you received my first invoice, I am
sincerely yours.
D. Wuilquin

✡ Last additional clause concerning "the checking".

It is not exact to tell that there is no checking possible with your own method, when ADDING together ALGEBRAICAL numbers:

When you add together ALGEBRAICALLY all the numbers relative to any horizontal line, (EXCEPT THE LAST NUMBER), the total corresponds always to the LAST NUMBER ENDING THE FOLLOWING LINE.

examples of checking:

$1+2+3+3 = \to 9$

$-2-1+1+4+7 = \to 9$

$-9-11-12-11-7+0$
$= -50$

etc, etc.

CHECKING:

naught 1
-1 — $\boxed{0}$
0 --- -1 --- → -1
+1 --- +1 --- 0 --- → $\boxed{-1}$
+1 --- +2 --- +3 --- +3 --- → 2
-2 --- -1 --- +1 --- +4 --- +7 --- → $\boxed{9}$
-9 --- -11 --- -12 --- -11 --- -7 --- 0 --- → 9
-9 --- -18 --- -29 --- -41 --- -52 --- -59 --- -59 --- → $\boxed{-50}$
→ -267

-50

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It is easy to notice that this above sketch is exactly the symmetrical one of the sketch I sent you a few days ago in order to solve the same problem.