

ON THE COEFFICIENTS IN THE EXPANSION OF e^{e^x}

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R. E. BEARD (*J.I.A.* LXXVI, 152) has derived the expansion of e^{e^x} and e^{-e^x} and has shown that the coefficients which he obtains can be expressed in terms of Stirling's numbers of the second kind ('reduced differences of zero').

The following demonstration derives the expansion of e^{e^x} , or $(e^{e^x})^{\lambda}$, directly in this form from first principles, thus embracing e^{e^x} and e^{-e^x} as special cases.

By expanding $(e^x - 1)^n$ by the binomial theorem the coefficient of x^r in $(e^x - 1)^n$ is seen at once to be

$$\begin{aligned} & \left\{ n^r - \binom{n}{1} (n-1)^r + \binom{n}{2} (n-2)^r - \dots \right\} / r! \\ & = (E-1)^n \circ^r / r! \\ & = \Delta^n \circ^r / r!. \end{aligned}$$

Hence the coefficient of x^r in $\sum_{n=1}^{\infty} \frac{\lambda^n}{n!} (e^x - 1)^n$ is $\frac{1}{r!} \sum_{n=1}^{\infty} \frac{\lambda^n \Delta^n \circ^r}{n!}$, where, since $\Delta^n \circ^r = 0$ for $n > r$, the upper limit ∞ may be replaced by r .

But $\sum_{n=1}^{\infty} \frac{\lambda^n}{n!} (e^x - 1)^n = \exp\{\lambda(e^x - 1)\} - 1$, so that the coefficient of x^r in $e^{\lambda(e^x - 1)}$ is

$$\frac{1}{r!} \sum_{n=1}^r \frac{\lambda^n \Delta^n \circ^r}{n!},$$

i.e. the coefficient of x^r in $e^{\lambda e^x}$ is $\frac{e^{\lambda}}{r!} \sum_{n=1}^r \lambda^n \frac{\Delta^n \circ^r}{n!}$.

We have thus obtained the expansion of $e^{\lambda e^x}$ for all λ . Setting $\lambda = 1$ and $\lambda = -1$ respectively we have the expansions of e^{e^x} and e^{-e^x} in terms of reduced differences of zero, as given by Beard (*loc. cit.* p. 154).

HET ACTUARIEEL GENOOTSCHAP

At its annual general meeting held on 16 October 1951 Het Actuarieel Genootschap (the Actuarial Society of the Netherlands) adopted new bye-laws which brought to a fitting conclusion the steps taken by its Council during the last ten years to enhance the importance and dignity of the actuarial profession in the Netherlands. It is thought that British actuaries will be interested in the following brief description of the steps taken and the effect of the bye-laws, based on information supplied by the Institute's Corresponding Member, Jhr. G. M. M. Alting von Geusau.

The Council first reorganized the system of actuarial tuition, and two special chairs in actuarial science were established at the Municipal University of Amsterdam. A course at the University followed by success in its actuarial examinations is therefore one of the ways in which a student can become an actuary. For those who cannot afford a university education, or who do not wish for one, Het Actuarieel Genootschap established an examination syllabus, the examinations consisting of a preliminary and three actuarial examinations—A, B and C.

The system of tuition and examination having been improved, the time was considered ripe to modernize the bye-laws.

The new bye-laws introduced five classes of members.

(a) **WERKENDE LEDEN.** This class can be compared with that of Fellow of the Institute. To be admitted as a *werkend lid* the candidate must have passed all the examinations, either the appropriate ones of the University or those held by the Genootschap, and have been *aspirant lid* for at least two years. In addition, a candidate who wishes to be admitted as a *werkend lid* must submit to the *Reglement van Orde* (dealing with professional conduct) and to the *Reglement van Tuchtrechtspraak* (dealing with questions of discipline). He is entitled to describe himself as *Actuaris A.G.*

(b) **ASPIRANT LEDEN.** This is similar in some respects to Associate of the Institute. To be admitted as an *aspirant lid* the candidate must have made such progress in his studies of actuarial science that it may be expected that he will pass all his future actuarial examinations.

(c) **BELANGSTELLELENDE LEDEN.** For admission to this class the only qualification necessary is that the candidate should be proved to have a great interest in actuarial science or practice.

(d) **LEDEN VAN VERDIENSTE.** The honour *lid van verdienste* is conferred on a member

- (1) who has received five times a prize for the best essay on a subject selected by the Genootschap, or
- (2) who has rendered distinguished service to actuarial science or to the Genootschap.

(e) **BUITENGEWONE LEDEN.** These are extraordinary members, not possessing Dutch nationality, who are elected in special recognition of distinguished service to the Genootschap. As with Corresponding Members of the Institute, *buitengewone leden* are expected to render assistance in promoting the objects of the Genootschap.