The On-Line Encyclopedia of Integer Sequences

Neil J. A. Sloane
Visiting Scholar, Rutgers University
President, OEIS Foundation
11 South Adelaide Avenue
Highland Park, NJ

THE ON–LINE ENCYCLOPEDIA OF INTEGER SEQUENCES®

founded in 1964 by N. J. A. Sloane

The On-Line Encyclopedia of Integer Sequences® (OEIS®)

Enter a sequence, word, or sequence number:

1,2,3,6,11,23,47,106,235

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For more information about the Encyclopedia, see the Welcome page.

Languages: English Shqip العربية Bangla Български Català 中文 (正體字, 简化字 (1), 简化字 (2))
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Last modified March 16 18:25 EDT 2015. Contains 255620 sequences.

A087097 Lunar primes (cf. A087062).

19, 29, 39, 49, 59, 69, 79, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 109, 209, 219, 309, 319, 329, 409, 419, 429, 439, 509, 519, 529, 539, 549, 609, 619, 629, 639, 649, 659, 709, 719, 729, 739, 749, 759, 769, 809, 819, 829, 839, 849, 859, 869, 879, 901, 902, 903, 904, 905, 906, 907, 908, 909, 912, 913, 914, 915, 916, 917, 918, 919, 923, 924, 925, 926, 927, 928, 929, 934, 935, 936, 937, 938, 939, 945, 946, 947, 948, 949, 956, 957, 958, 959, 967, 968, 969, 978, 979, 989 (list; graph; listen; history; edit; internal format)

OFFSET 1,1

COMMENTS 9 is the multiplicative unit. A number n is a dismal prime if it is not a dismal product (see A087062 for definition) r*s where neither r nor s is 9.
All dismal primes must contain a 9, so this is a subset of A011539.
Also, numbers n such that the dismal sum of the dismal prime divisors of n is n. (From N. J. A. Sloane, Aug 23 2010)

LINKS David Applegate and N. J. A. Sloane, Table of n, a(n) for n = 1..22095 [all primes with at most 6 digits]
D. Applegate, C program for dismal arithmetic and number theory
Index entries for sequences related to dismal arithmetic

EXAMPLE 8 is not prime since 8 = 8*8. 9 is not prime since it is the multiplicative unit. 10 is not prime since 10 = 10*8. Thus 19 is the smallest prime.

CROSSREFS Cf. A087062, A087636, A087638, A087984.
Sequence in context: A047985 A061763 A088474 * A038364 A151360 A109276
Adjacent sequences: A087094 A087095 A087096 * A087098 A087099 A087100

KEYWORD nonn,easy,base

AUTHOR Marc LeBrun (mlb(AT)well.com), Oct 20 2003

Outline of Talk

- About the OEIS
- Sequences from Geometry
- Sequences from Arithmetic
- "Music" and Videos
- The BANFF "Integer Sequences and K12" Conference

OEIS.org

- Fun: 2, 4, 6, 3, 9, 12, 8, 10, 5, 15, ... ?
- Addictive (better than video games)
- Accessible (free, friendly)
- Street creds (4000 citations)
- Interesting, educational
- Essential reference
- Low-hanging fruit
- Need editors

Facts about the OEIS

- Accurate information about 250000 sequences
- Definition, formulas, references, links, programs
- View as list, table, graph, music!
- 200 new entries and updates every day
- 4000 articles and books cite the OEIS
- Often called one of best math sites on the Web
- Since 2010, a moderated Wiki

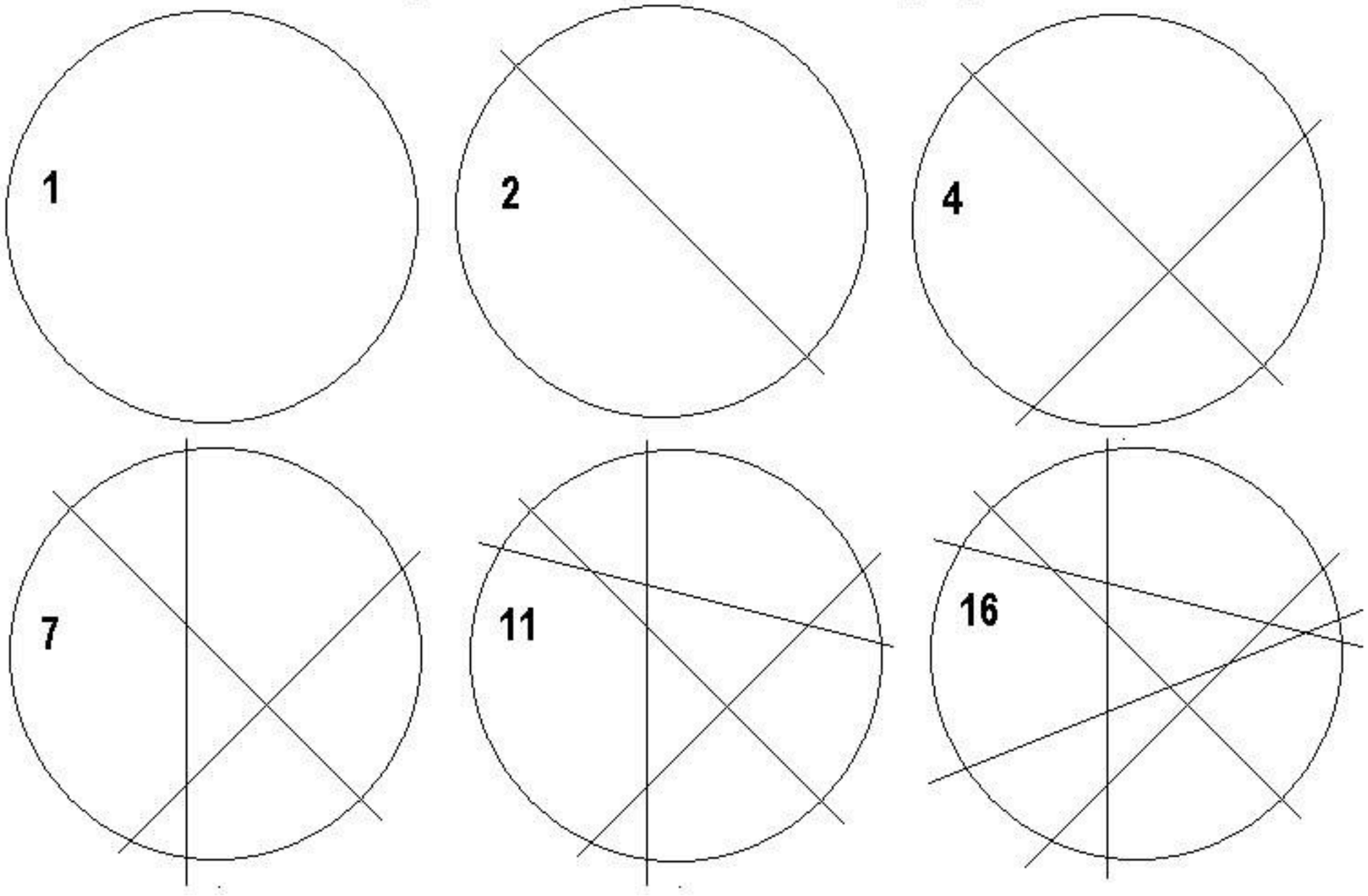
Main Uses for OEIS

- To see if your sequence is new, to find references, formulas, programs
- Catalan or Collatz? (Very easy or very hard?)
- Many collaborations, very international
- Source of fascinating research problems
- Has led many people into mathematics
- Fun, Escape

Sequences from Geometry

- Slicing a pancake
- Kobon triangles
- Lines in plane
- Circles in plane
- Tiling a square with dominoes

Maximal number of pieces formed when slicing a pancake with n cuts



A124

$a(n)=a(n-1)+n; \quad a(n) = n(n+1)/2 + 1$

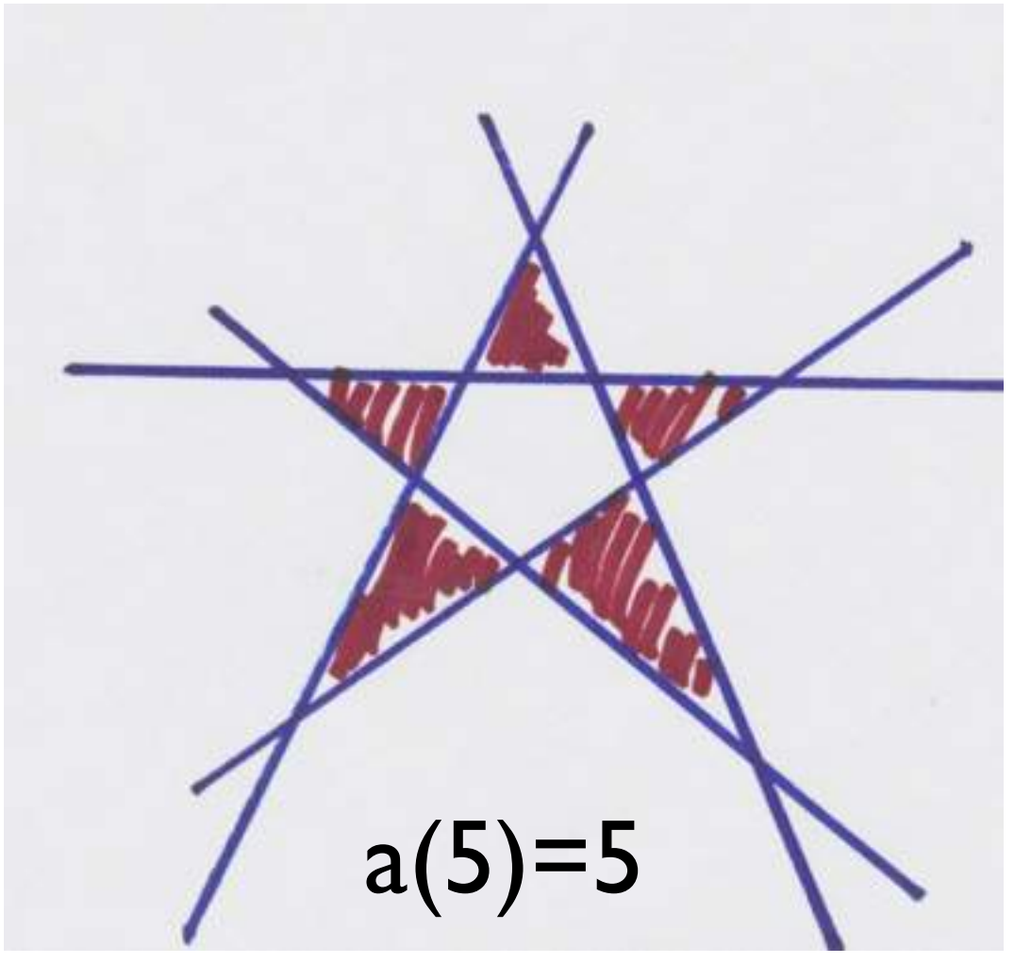
Kobon Triangles

Kobon Fujimura, circa 1983

A6066

Kobon Triangles

How many non-overlapping triangles can you draw with n lines?



1	2	3	4	5	6	7	8	9
0	0	1	2	5	7	11	15	21

A6066

a(10) is 25 or 26

Kobon triangles (cont.)

A6066

a(17)=85

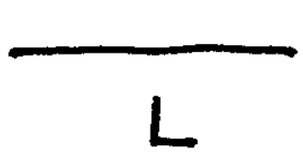
Johannes Bader: 17 lines, 85 triangle, maximum possible

No. of ways to arrange n lines in the plane

1, 2, 4, 9, 47, 791, 37830

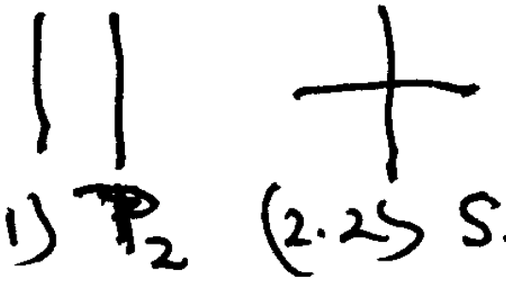
A241600

$a(1) = 1$



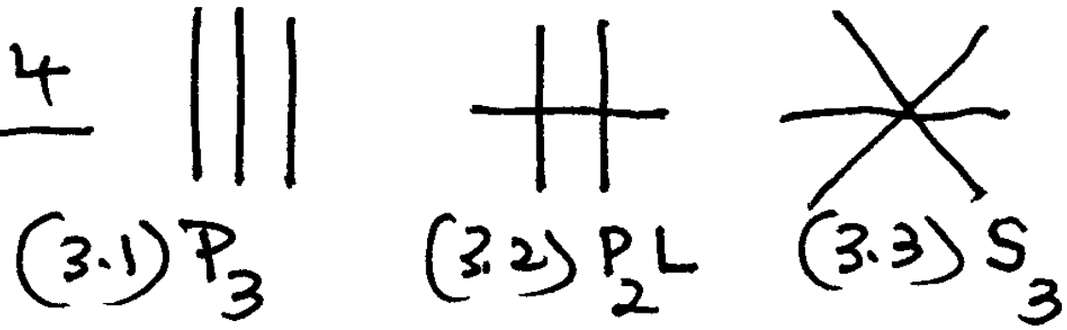
(1.1)

$a(2) = 2$



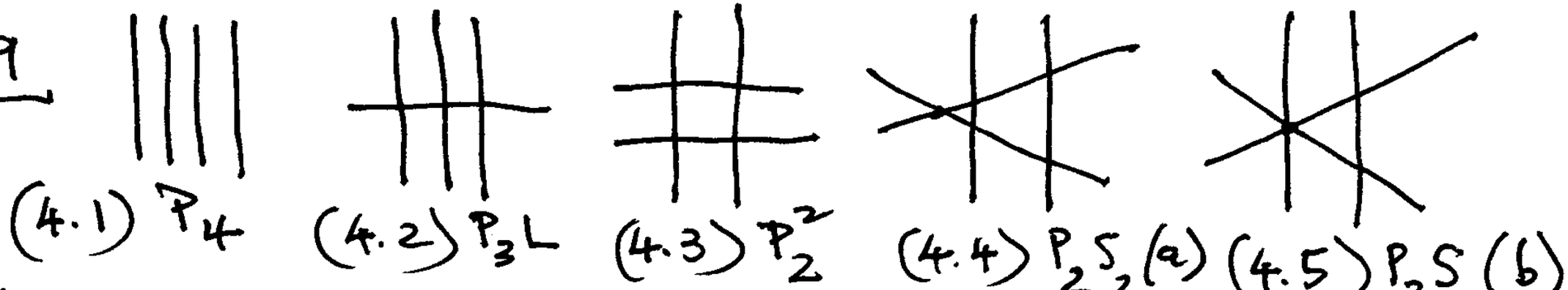
(2.1) P_2 (2.2) S_2

$a(3) = 4$

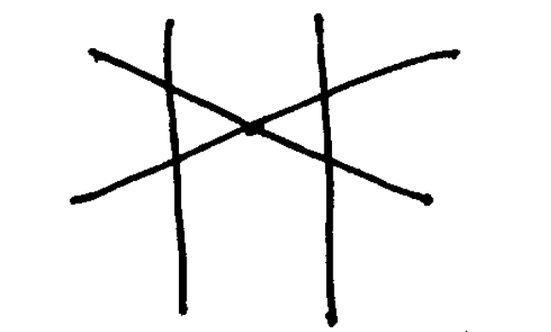


(3.1) P_3 (3.2) P_2L (3.3) S_3 (3.4) G_3

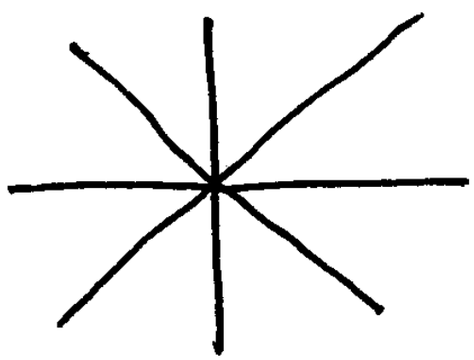
$a(4) = 9$



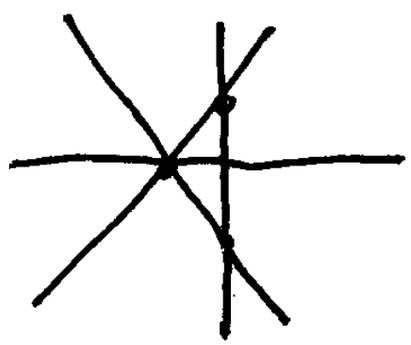
(4.1) P_4 (4.2) P_3L (4.3) P_2^2 (4.4) $P_2S_2(a)$ (4.5) $P_2S_2(b)$



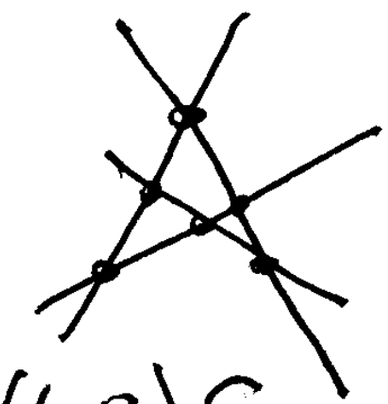
(4.6) $P_2S_2(c)$



(4.7) S_4



(4.8) S_3L



(4.9) G_4

A241600 (cont.)

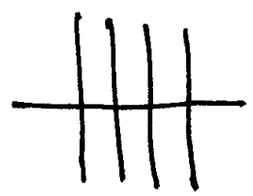
$a(5) = 47$. Summary:

$P_5: 1$, $P_4L: 1$, $P_3P_2: 1$, $P_3S_2: 4$, $P_2^2L: 6$,

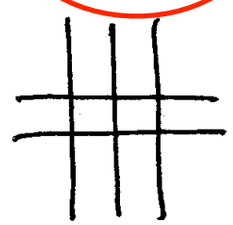
$P_2G_3: 14$, $P_2S_3: 3$, $S_5: 1$, $S_4L: 1$, ~~$S_3^2: 3$, $S_3G_2: 6$, $G_5: 6$.~~

$S_3^2: 3$, $S_3S_2: 6$, $G_5: 6$.

(5.1) P_5

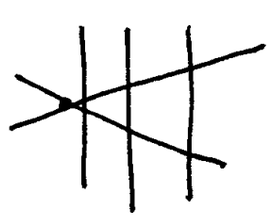


(5.2) P_4L

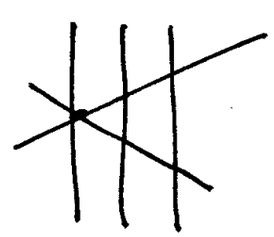


(5.3) P_3P_2

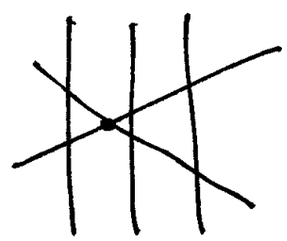
(5.4)–(5.7) P_3S_2:



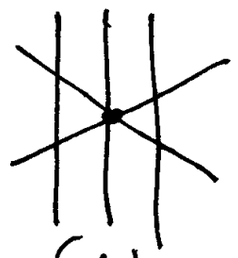
(a)



(b)

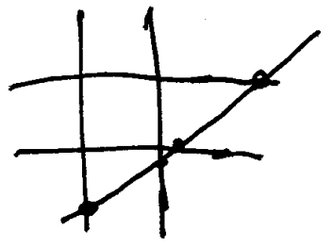


(c)

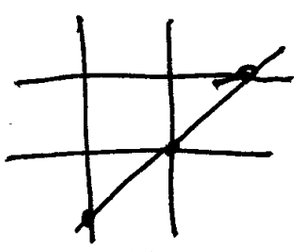


(d)

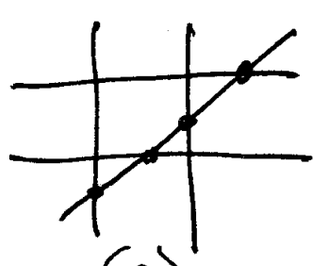
(5.8)–(5.13) P_2^2L:



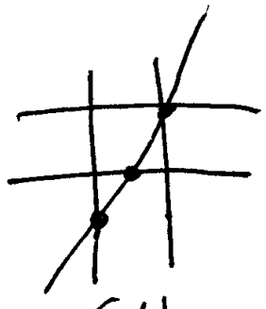
(a)



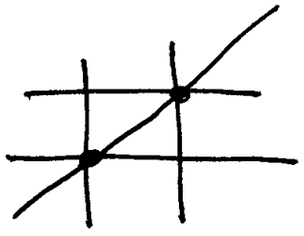
(b)



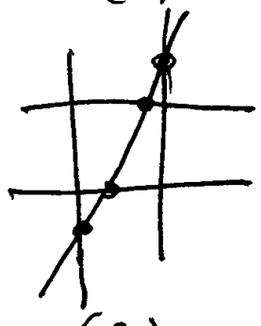
(c)



(d)



(e)



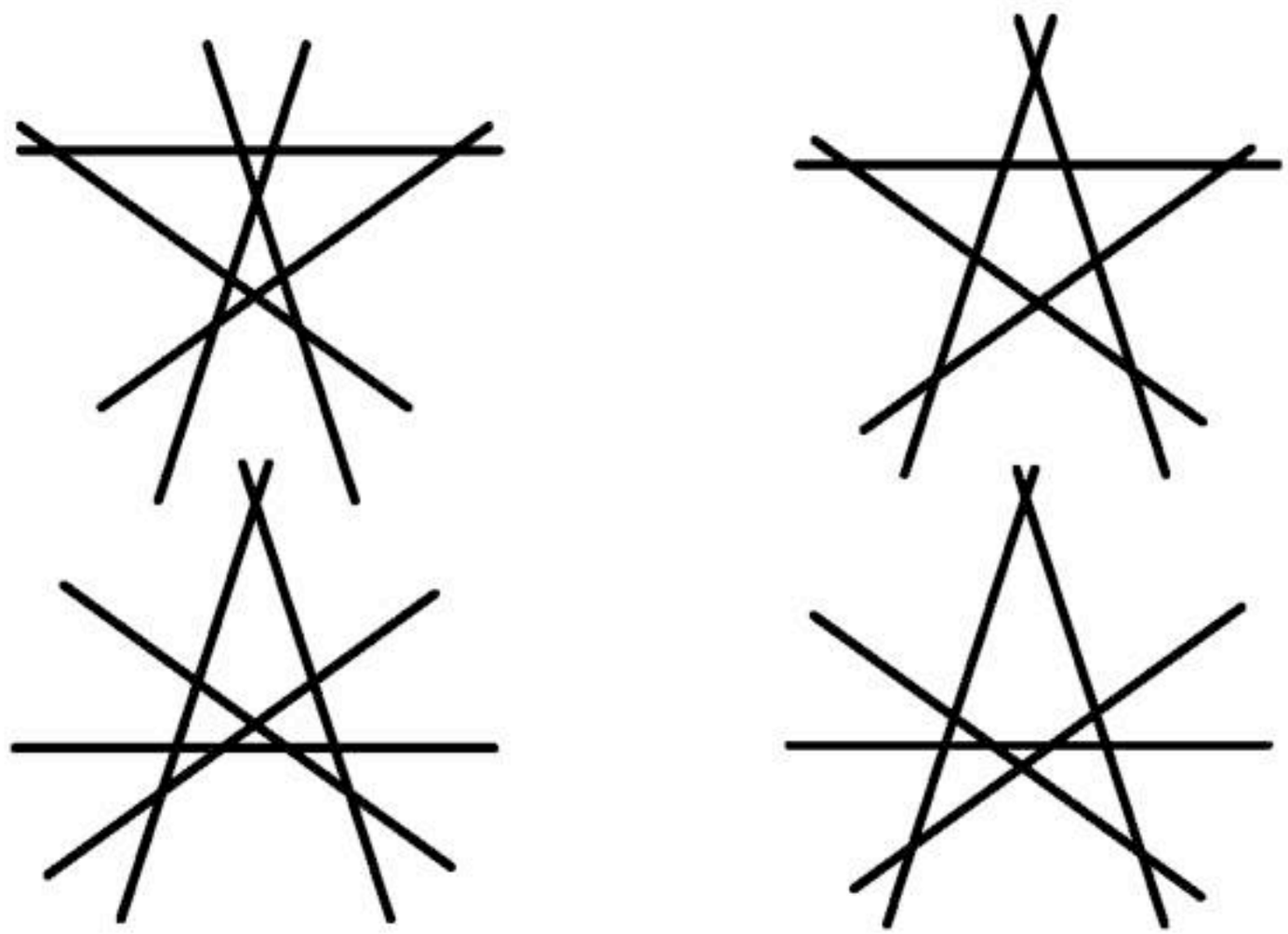
(f)

A90338

A subset: n lines in general position

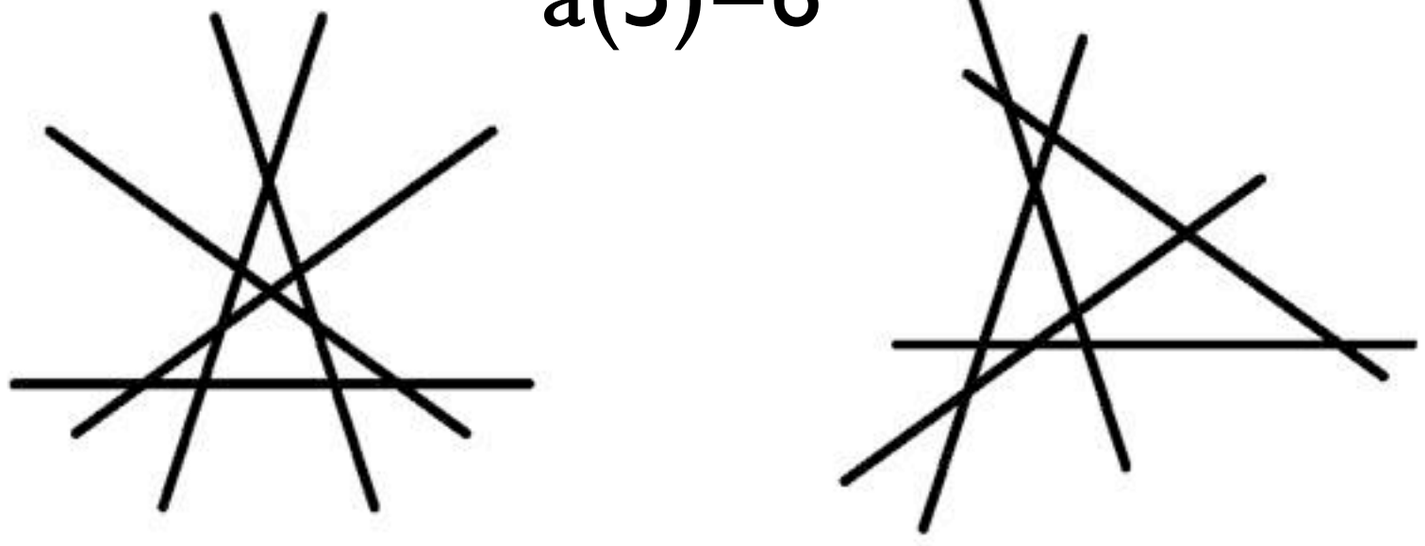
1, 1, 1, 1, 6, 43, 922, 38609

Wild and Reeves, 2004



a(5)=6

5 lines in general position: 6 ways

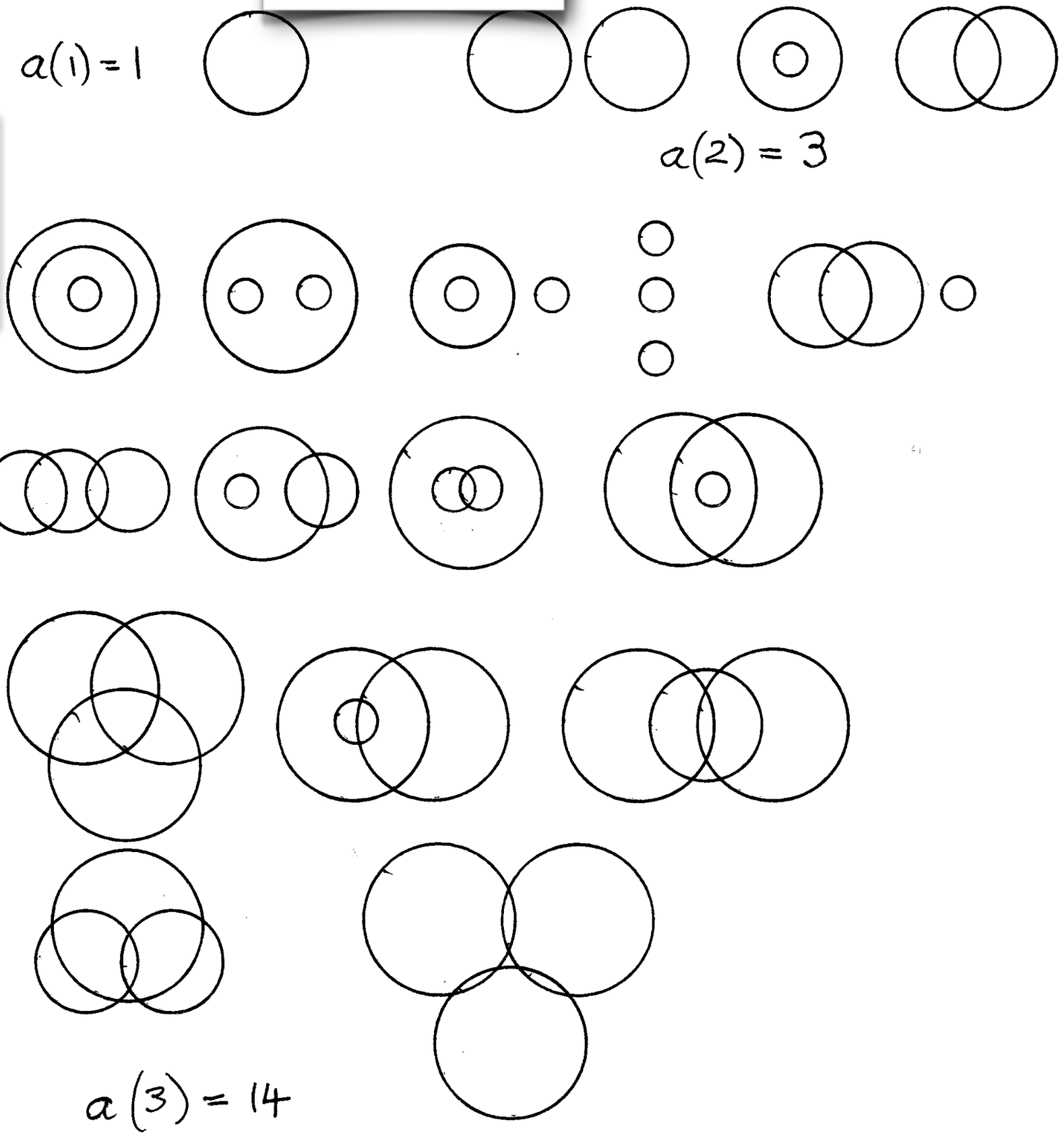


A250001

No. of arrangements of n circles in the plane

1, 3, 14, 168

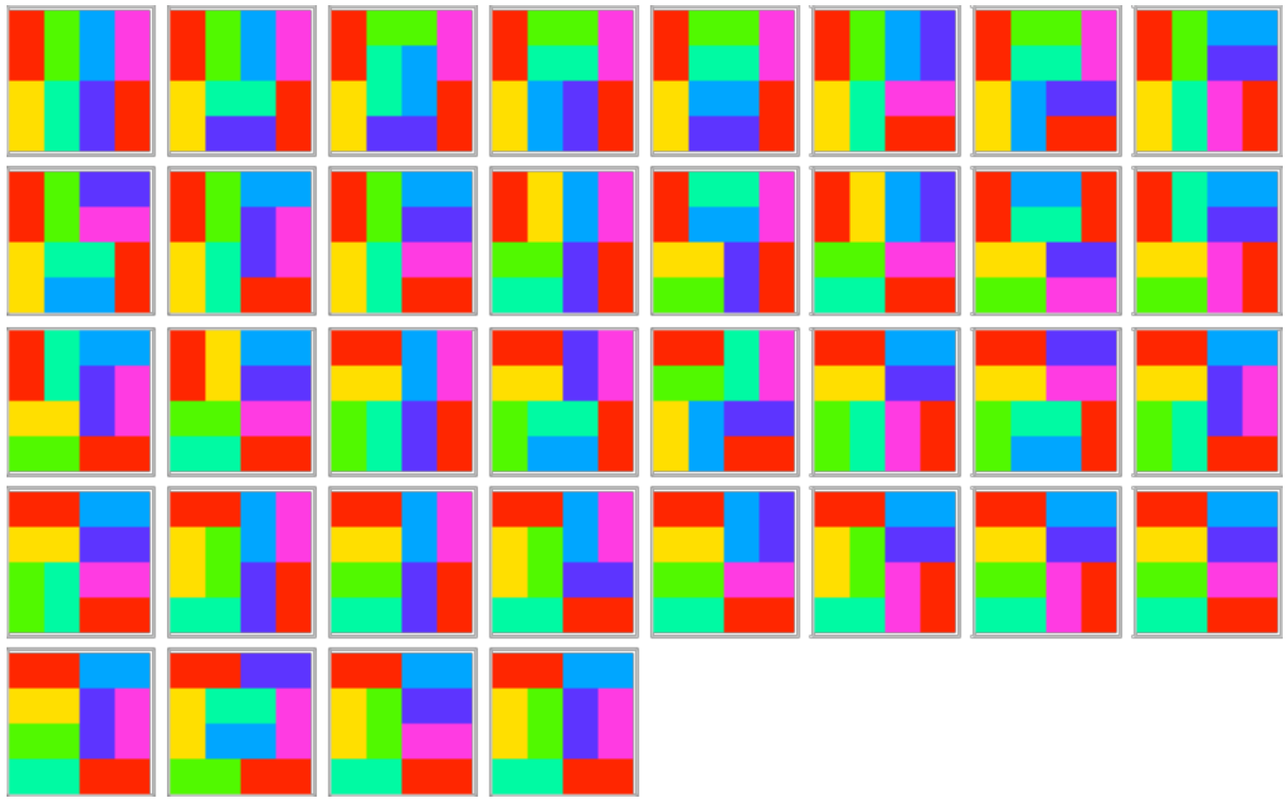
Jonathan Wild



What if allow tangencies?

Tiling a Square with Dominoes

36 ways to tile
a 4X4 square



a(2)=36

1, 2, 36, 6728, 12988816, 258584046368,
5306047752196000, ...

(A4003)

$$a(n) = \prod_{j=1}^{n} \prod_{k=1}^{n} \left(4\cos^2 \frac{j\pi}{2n+1} + 4\cos^2 \frac{k\pi}{2n+1} \right)$$

(Kastelyn, 1961)

Two days ago:

Laura Florescu, Daniela Morar, David Perkinson, Nicholas Salter, Tianyuan Xu, Sandpiles and Dominos, 2015

1, 2, 36, 6728, 12988816, 258584046368, 5306047752196000**/5**, ... !!

(A256043)

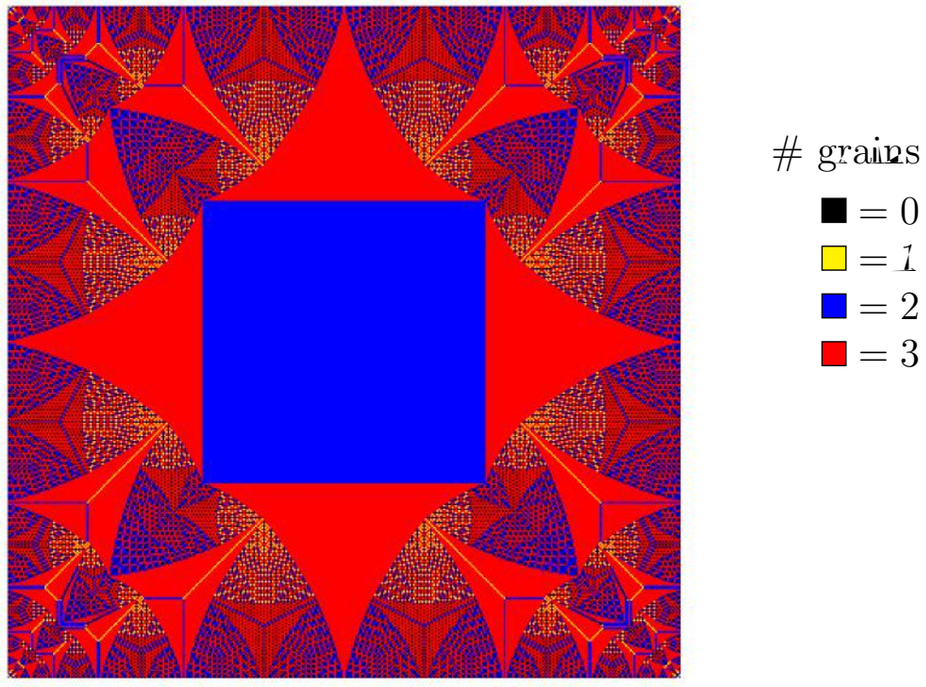


Figure 1: Identity element for the sandpile group of the 400×400 sandpile grid graph.

Two Sequences That Agree For a Long Time

$$\left\lfloor \frac{2n}{\log 2} \right\rfloor$$ = A078608

$$\left\lfloor \frac{2}{2^{1/n} - 1} \right\rfloor$$

Differs for first time at n =

777451915729368

(see A129935)

Sequences from Arithmetic

- Lunar primes
- The EKG sequence
- Curling number conjecture
- Gijswijt's sequence

Lunar Arithmetic

David Applegate, Marc LeBrun and NJAS (J.I.S. 2011)

(For Martin Gardner)

Arithmetic on the moon!

- $i + j = \mathrm{MAX}\{i, j\}$
- $i \times j = \mathrm{MIN}\{i, j\}$
- No carries!

```
  17
+ 23
-----
  27
```

```
  19
× 24
------
  14
 12
------
 124
```

Thm.: Lunar arithmetic is commutative, associative, distributive

Lunar squares

```
0, 1, 2, 3, 4, 5, 6, 7, 8, 9,
100, 111, 112, 113, 114, 115,
116, 117, 118, 119, 200,   ...
```

(A87019)

```
  19
×19
-----
  19
 110
-----
 119
```

Lunar primes?

What is a prime?

Ans. Only factorization is $p = 1 \times p$

But what is "1"?

Ans. The multiplicative identity:
"1" $\times$ n = n for all n

But $1 \times 3 = 1$, so "1" $\neq$ 1

So "1" = 9, since $9 \times n = n$ for all n.

If $u \times v = 9$ then $u = v = 9$, so 9 is the only unit.

So p is prime if its only factorization is $p = 9 \times p$

Lunar primes (cont.)

p is prime if only factorization is p = 9 × p

7? No, 7 = 7 × 7

13? No, 13 × 4

So must have 9 as a digit.

9? No, 9 is the unit

Lunar primes: (A87097)

```
19, 29, 39, 49, 59, 69, 79, 89, 90,
91, 92, 93, 94, 95, 96, 97, 98, 99,
109, 209, ...
```

(119 = 19 × 19 is not a prime)

There are infinitely many primes

109 is prime!

Proof:

```
    ab          1 9
× cd            c 9
                ---
                1 9
              1 c 0
              -----
              1 ? 9
```

Can't get 109

Similarly, 10...009 is prime!

The EKG Sequence

Jonathan Ayres, 2001

EKG Sequence (A64413)

1, 2, 4, 6, 3, 9, 12, 8, 10, 5, 15, ...

a(1)=1, a(2)=2,

a(n) = min k such that

- GCD { a(n-1), k } > 1
- k not already in sequence

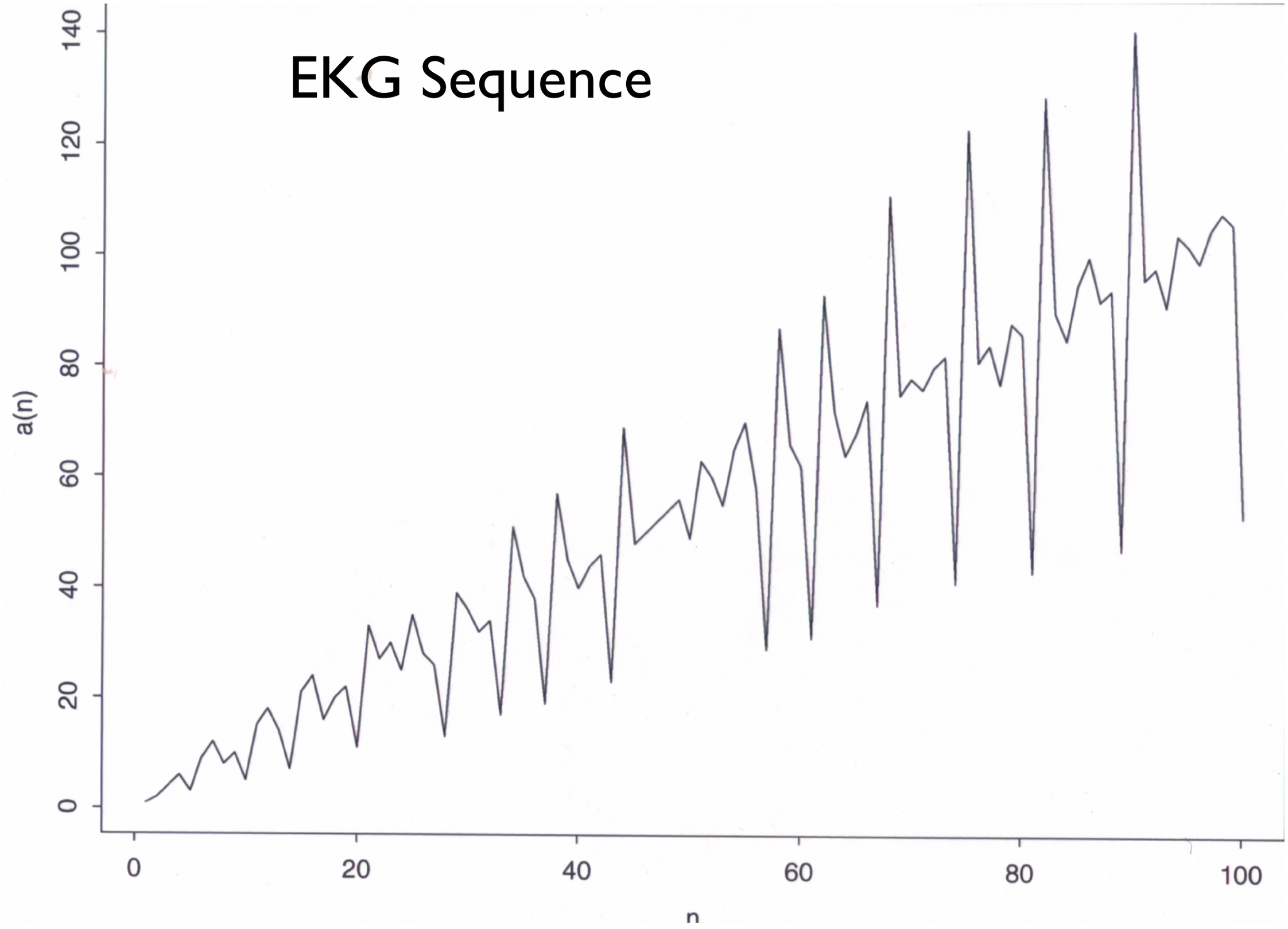
- Jonathan Ayres, 2001

- Analyzed by Lagarias, Rains, NJAS, Exper. Math., 2002

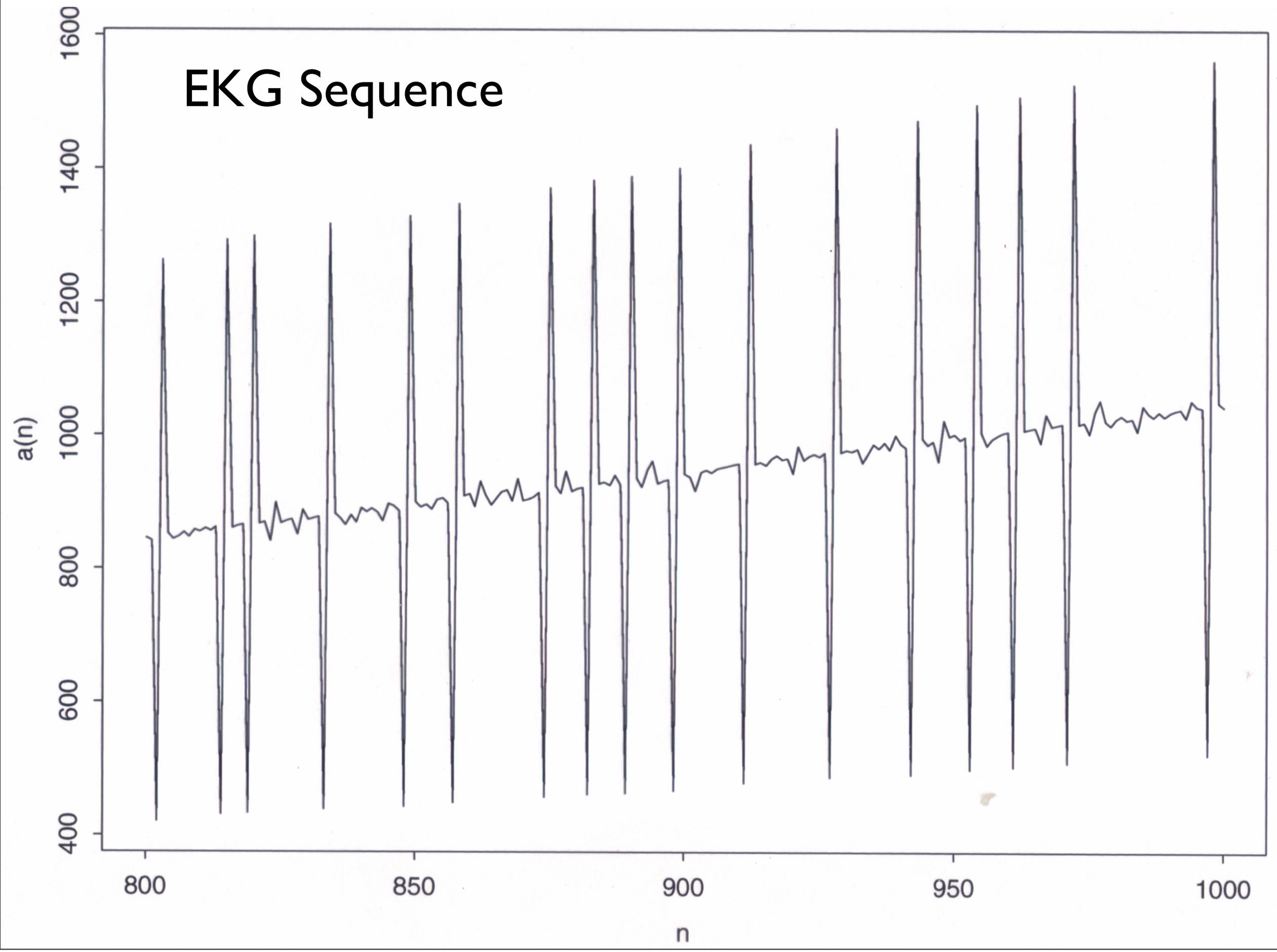
- Gordon Hamilton, Videos related to this sequence:

https://www.youtube.com/watch?v=yd2jr30K2R4&feature=youtu.be http://m.youtube.com/watch?v=Y2KhEW9CS0A

EKG Sequence

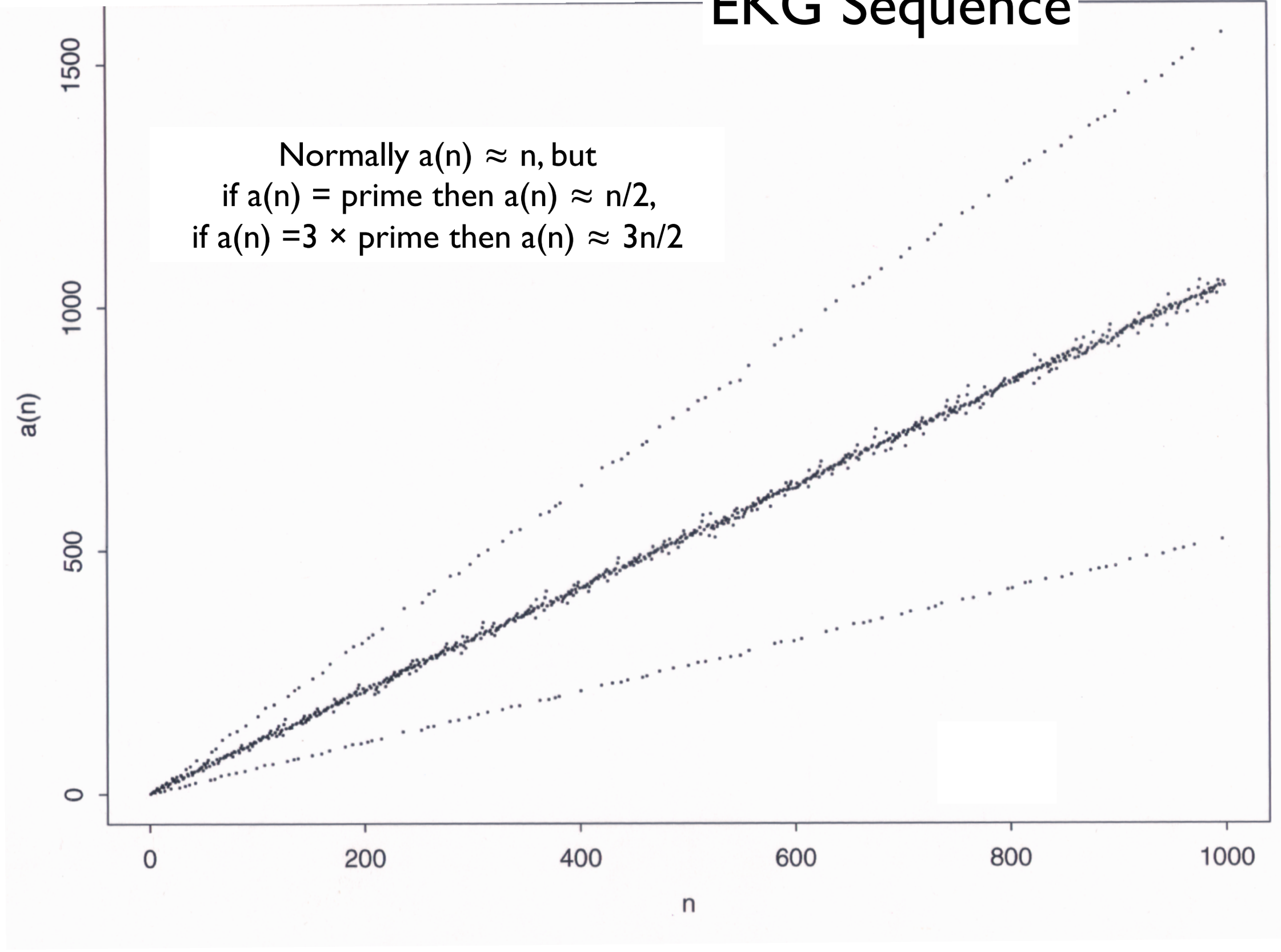


EKG Sequence



EKG Sequence

Normally a(n) ≈ n, but
if a(n) = prime then a(n) ≈ n/2,
if a(n) =3 × prime then a(n) ≈ 3n/2



EKG Sequence

Theorems:

- The sequence is a permutation of the natural numbers
- $c_1\, n \leq a(n) \leq c_2\, n$

Conjectures

- $a(n) \sim n\left(1 + \frac{1}{3\log n}\right)$ for the main terms
- $\cdots, 2p, p, 3p, \cdots (\text{primes } p > 2)$

(Proved by Hofman & Pilipczuk, 2008)

EKG Sequence

LEMMA 1 IF ∞ MANY MULTIPLES OF PRIME p APPEAR, THEN ALL MULTIPLES DO.

Pf. kp not in sequence

$\exists n_0$ s.t. $n \geq n_0 \Rightarrow a(n) > kp$

$\therefore a(n) = ip$ $\therefore a(n+1) = kp$, $\ast\ast$

LEMMA 2 IF ALL MULTIPLES OF p APPEAR THEN ALL NUMBERS DO.

Pf. k not in sequence

$a(n) = kip$ $a(n+1) = k$ $\ast\ast$

THEOREM $\{a(n)\}$ IS PERM. OF $\{1, 2, \ldots\}$

Pf. IF ∞ MANY DIFF^{I} PRIMES,

$\therefore \infty$ MANY $2p$'s, USE L1, L2.

IF FINITELY MANY DIFF^{I} PRIMES,

ONE APPEARS ∞ OFTEN,

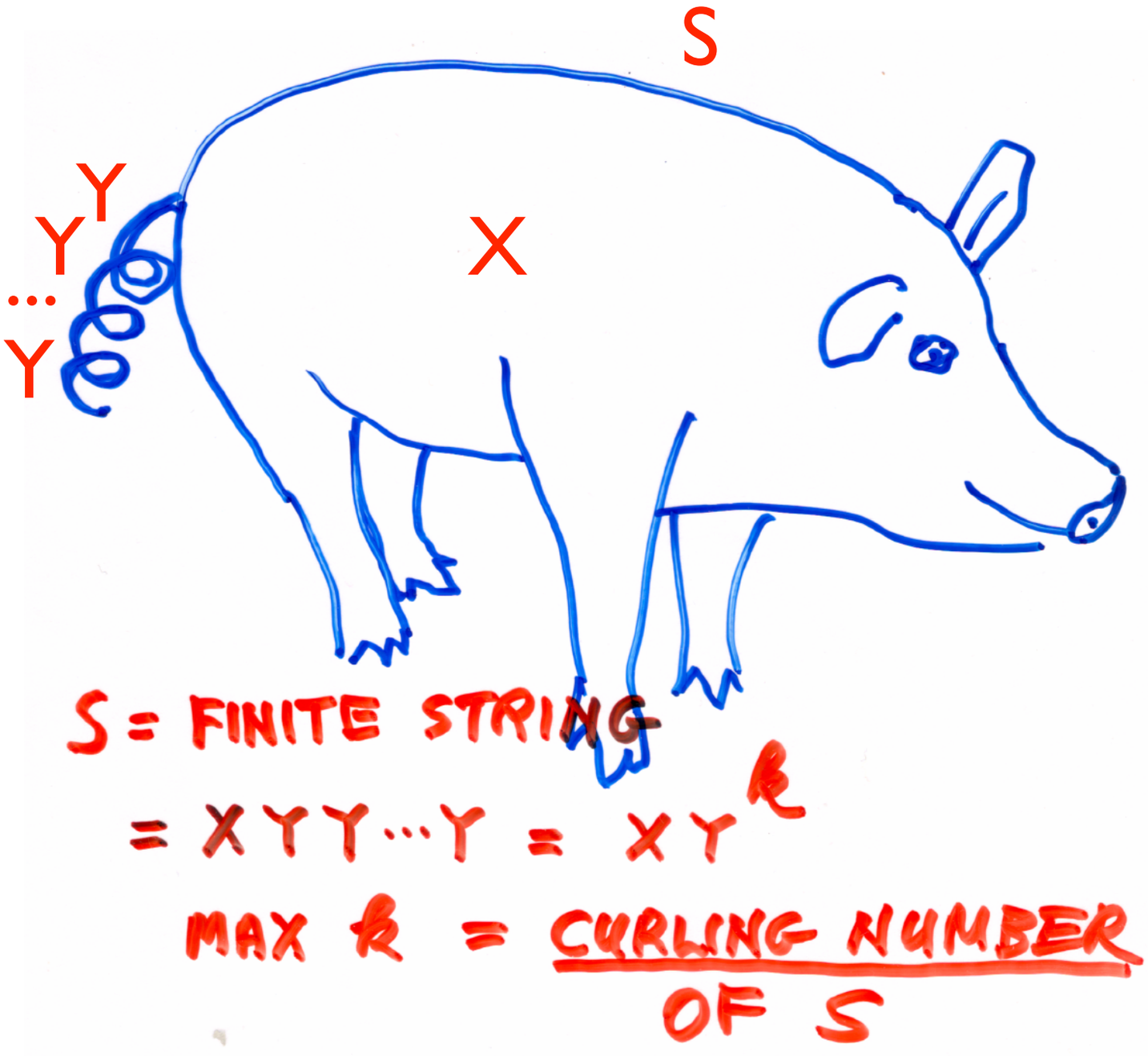
USE L1, L2.

QED

The Curling Number Conjecture

The Curling Number Conjecture

Definition of Curling Number



S = 7 5 2 2 5 2 2 5 2 2, k = 3

CURLING NUMBER CONJECTURE

- START WITH ANY FINITE STRING
- APPEND CURLING NUMBER
- REPEAT
- THEN MUST REACH A 1 !?

E.G.

START : 2 2 2 3 2 2

THEN 2 3 2 2 2 3 3 2 1 ....

BOO!

Gijswijt's Sequence

Dion Gijswijt, 2004

A90822

Gijswijt's Sequence

Fokko v. d. Bult, Dion Gijswijt, John Linderman,
N. J. A. Sloane, Allan Wilks (J. Integer Seqs., 2007)

Start with 1, always append curling number

1 1 2

1 1 2 2 2 3

1 1 2

1 1 2 2 2 3 2

1 1 2

1 1 2 2 2 3

1 1 2

1 1 2 2 2 3 2 2 2 3 2 2 2 3 3 2

1 1 2

. . . . . .

$a(220) = 4$

(A090822)

Gijswijt, continued

Gijswijt, continued

Is there a 5?

Gijswijt, continued

Is there a 5?

300,000 terms: no 5

Is there a 5?

300,000 terms: no 5

$2 \cdot 10^6$ terms: no 5

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10^{120} terms: no 5

Is there a 5?

300,000 terms: no 5

$2 \cdot 10^6$ terms: no 5

10^{120} terms: no 5

NJAS, FvdB: first 5 at about term $10^{10^{23}}$

First n appears at about term

$$n-1$$

...

5

4

3

2

2

(F.v.d. Bult et al., J. Integer Sequences, 2007)

(A90822)

Proofs could be simplified if Curling Number Conjecture were true

How far can you get with an initial string of n 2's and 3's (before a 1 appears)?

THE UNIQUE RECORD STARTS:

LENGTH 8: 23222323 → 66

LENGTH 22:

23223223223223223223

→ 142

LENGTH 48 → 179

LENGTH 77 → 250

JOINT WORK WITH

BEN CHAFFIN

(INTEL)

Conjecture

LET $\mu(n)$ = MAX LENGTH ATTAINED STARTING WITH n 2's & 3's.

IF S ACHIEVES $\mu(n) > \mu(n-1)+1$ THEN S DOES NOT CONTAIN w^4, $w \neq \phi$.

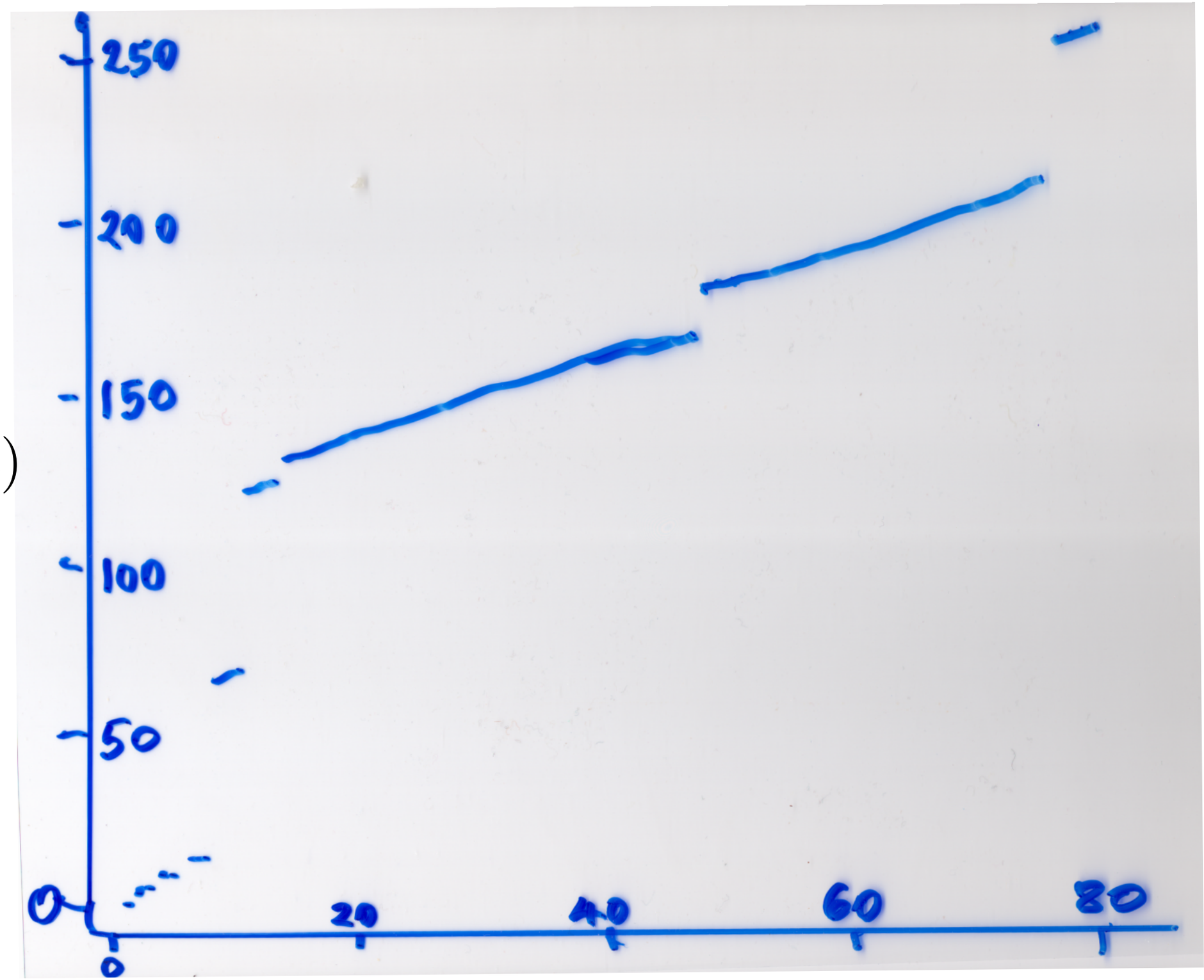
(SO NOT 2222)

Searched n <= 53

Conjecture

... S ALSO DOES NOT CONTAIN 33.

Searched n <= 80



$\mu(n)$

n

“Music” and Videos

Reminder: New keywords “hear” and “look”

Pascal's triangle

A7318

Hofstadter Q sequence
A5185

a(1) = a(2) = 1; a(n) = a(n-a(n-1)) + a(n-a(n-2)) for n > 2.

wt(n) and 4^wt(n) together

(A120 and A102376, Taiko drum and xylophone)

Martin Paech's arrangement of A242353

Recaman's sequence A5132

(Midi "instrument" FX-7)

Samuel Vriezen, Toccata III (2001)

Faure, Prelude, Op. 103, #3

(in G Minor)

Videos about sequences

Charles McKeague, Fibonacci numbers

Dale Gerdemann, Fibonacci tree

Christobal Vila, Nature by numbers

Robert Walker, Golden Rhythmicon

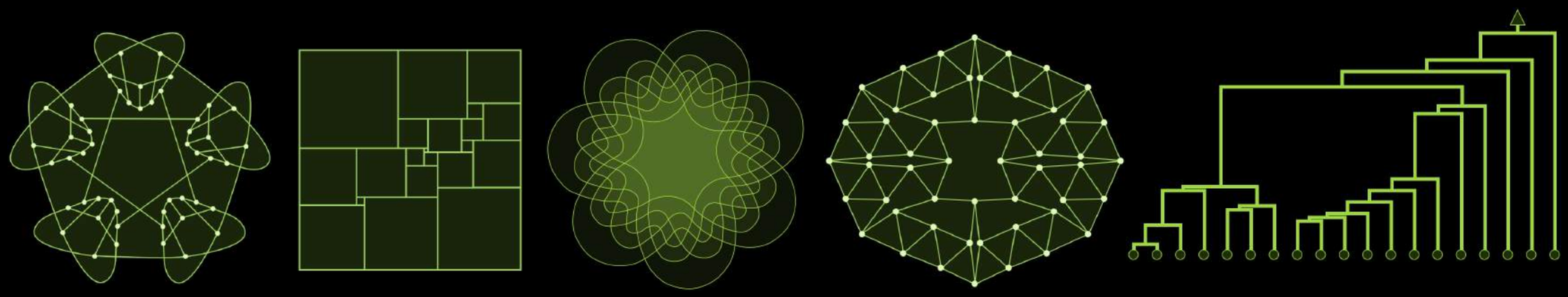
Gordon Hamilton, Wrecker ball sequence
(Recaman's sequence)

There are nearly 200 videos, movies, animations in the OEIS - we need more!

The BANFF Integer Sequences K-12 Conference

Conference organized by Gordon Hamilton and me

Feb. 27 - March 1, 2015



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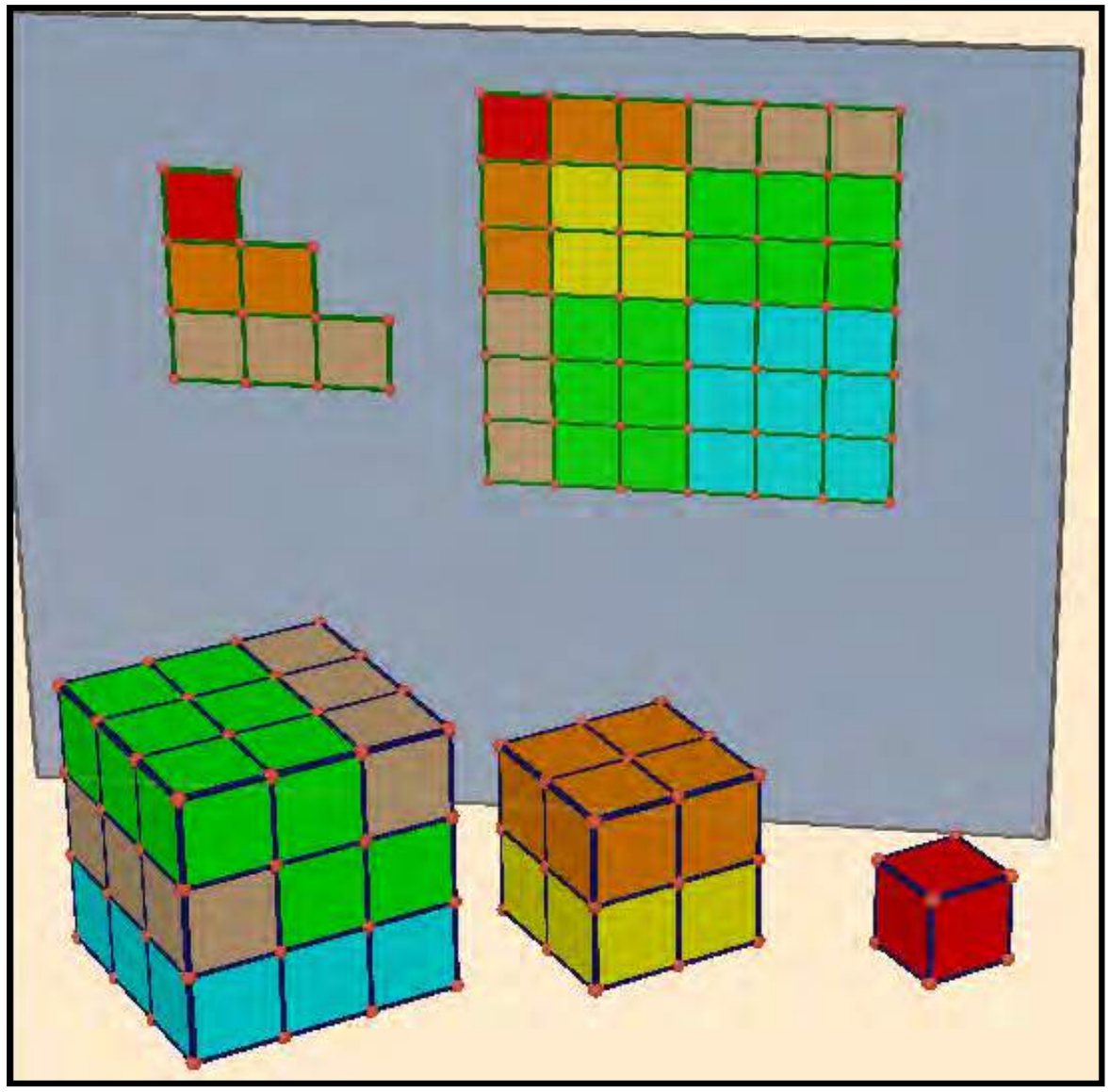
Selected Integer Sequences

Henri Picciotto
www.MathEducationPage.org
henri@MathEducationPage.org



The OEIS

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https://oeis.org

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