The On-Line Encyclopedia of Integer Sequences

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President, OEIS Foundation
11 South Adelaide Avenue
Highland Park, NJ

The new OEIS: oeis.org

- 206,500 sequences (20,000 in first year)
- Owned and maintained by OEIS Foundation (a 501(c)3 public charity)
- Need more editors to help
- Need more eyes searching for sequences
- Need help with "kiosk" for science museums
- Need help with "music"

Facts about the OEIS

- Accurate information about 200,000 sequences
- Definition, formulas, references, links, programs
- View as list, table, graph, sounds!

Euler totient A10 Recaman A5132

- 50 new entries, 50 updates every day

(150 emails per day)

- Traffic: 155 GB/month
- 2000 articles and books cite the OEIS
- Often called one of best math sites on the Web
- Maintained by NJAS for 45 years

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How It All Began

Dec. 1963: Average height of random node in rooted labeled tree on n nodes:

n:	1	2	3	4	5	6	...
a(n):	0	1	8	78	944	13800	...

A000435

Question: $\frac{a(n)}{n^n} \to \ ?\ \ as\ n \to \infty$

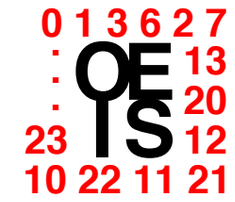
Polya counting theory (and John Riordan) ➔

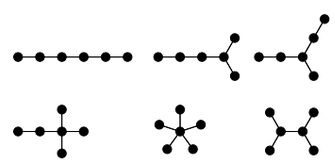
$$a(n) \ = \ (n-1)! \sum_{k=0}^{n-2} \frac{n^k}{k!}$$

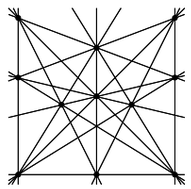
$$\sim \ n^n \sqrt{\frac{2\pi}{n}} \ \text{as}\ n \to \infty$$

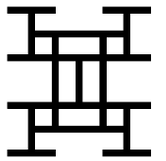
Poster

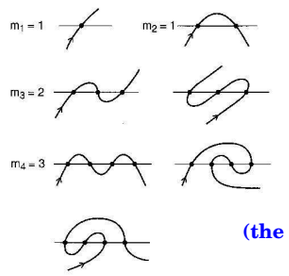
See OEIS Foundation web site: oeisf.org





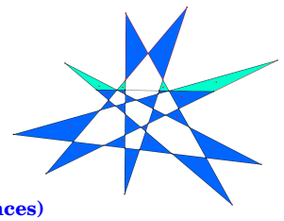




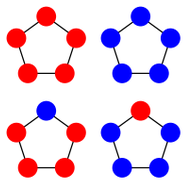


The OEIS Foundation Inc.

Launches the OEIS Wiki!



(the new version of the On–Line Encyclopedia of Integer Sequences)



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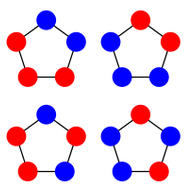

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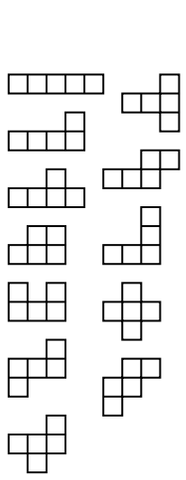

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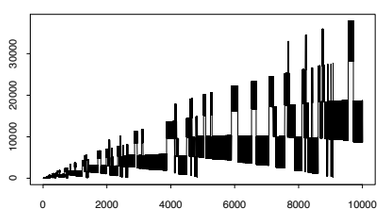

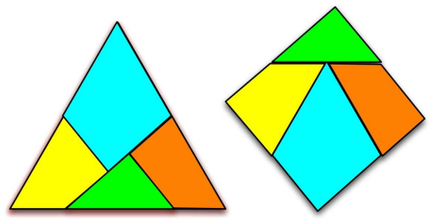

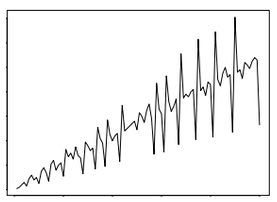

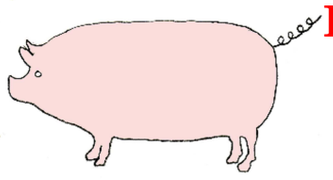

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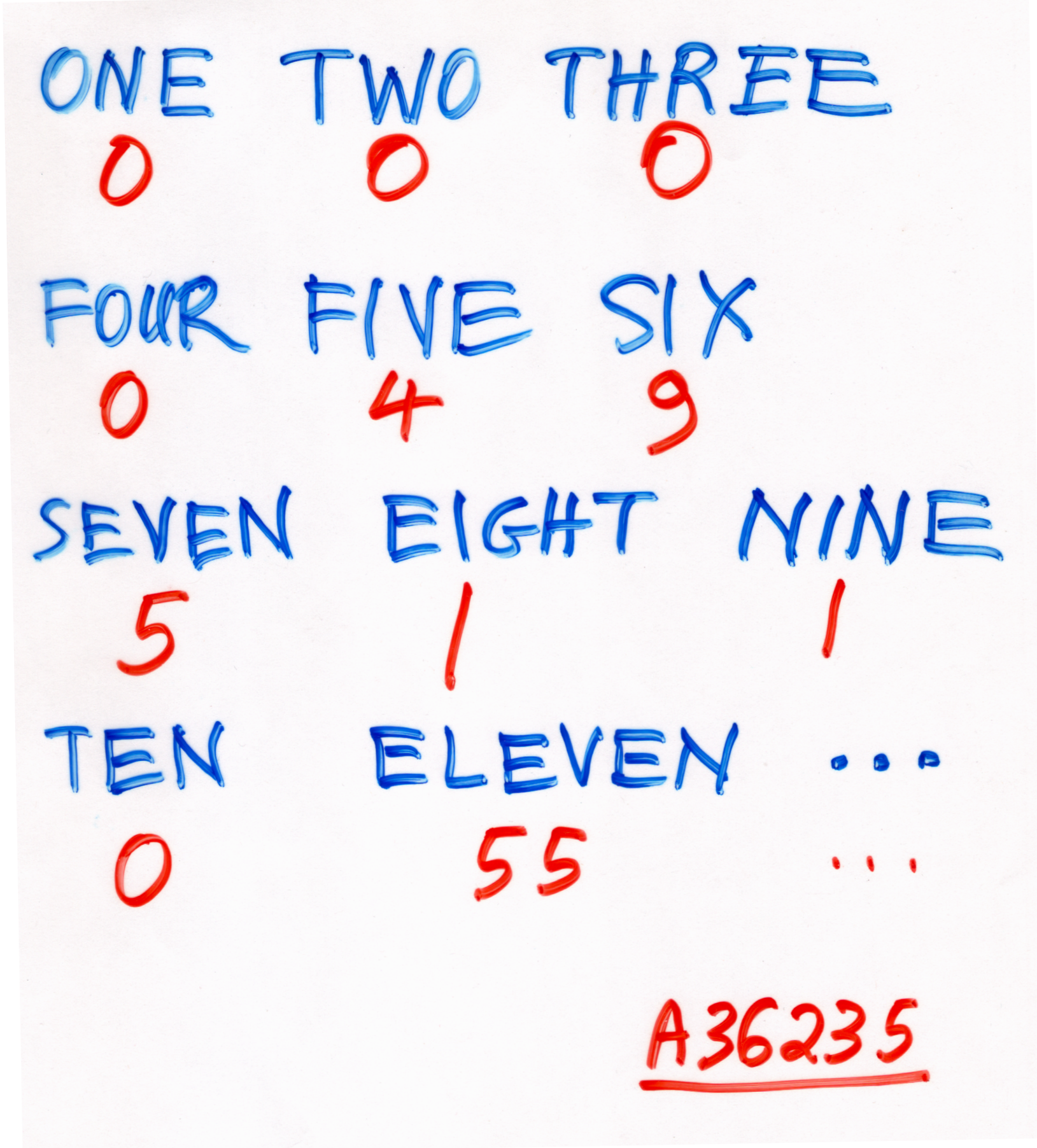
(For a Key to this poster, see http://www.oeisf.org/key.pdf)

OEIS - The Movie

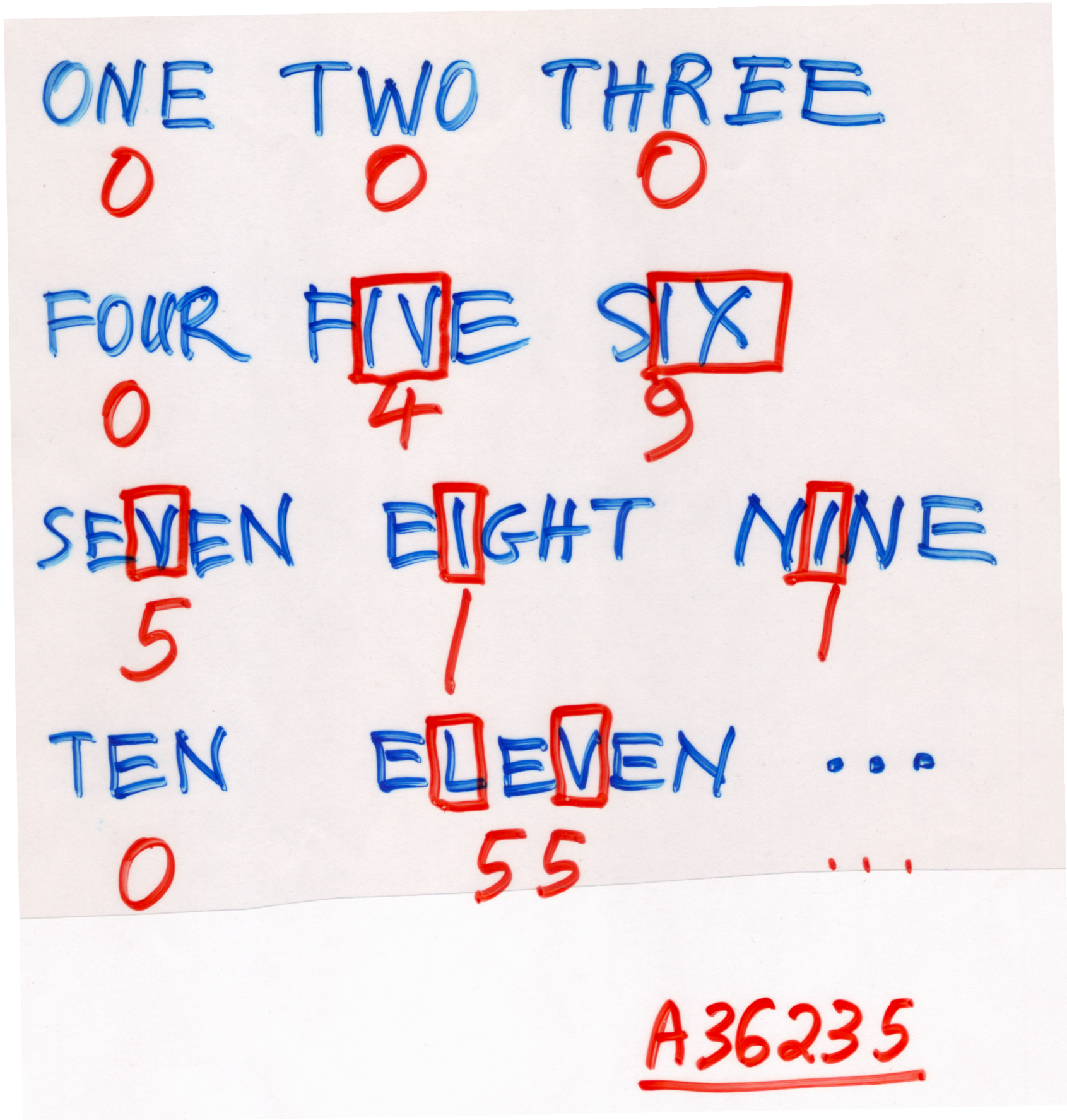
Plots of 1000 sequences from the
On-Line Encyclopedia of Integer Sequences

by T. D. Noe

PUZZLES


ONE TWO THREE
0 0 0
FOUR FIVE SIX
0 4 9
SEVEN EIGHT NINE
5 1 1
TEN ELEVEN ...
0 55 ...
A36235

PUZZLES


ONE TWO THREE
0 0 0
FOUR FIVE SIX
0 4 9
SEVEN EIGHT NINE
5 1 7
TEN ELEVEN ...
0 55 ...
A36235

Two Sequences That Agree For a Long Time

$$\left\lfloor \frac{2n}{\log 2} \right\rfloor$$ = A078608

$$\left\lfloor \frac{2}{2^{1/n} - 1} \right\rfloor$$

Differs for first time at n =

777451915729368

(see A129935)

APPLICATIONS
of OEIS

1988: JOSEPH NORTH OBSERVED THAT IF TRUNCATE GREGORY'S SERIES

$$\pi \approx 4\sum_{k=1}^{5000000} \frac{(-1)^{k+1}}{2k-1} = 4\left(1-\frac{1}{3}+\frac{1}{5}-\cdots\right)$$

THEN GET

3.14159245358979323846464338327950278 41 …
3.14159265358979323846264338327950288 41 …
2 −2 10

2, −2, 10, −122, 2770, … ⇒
1, −1, 5, −61, 1385, … = A364
EULER NUMBERS

J. BORWEIN, P. BORWEIN, K. DILCHER, AMER. MATH. MONTHLY, 1989:

$$\pi - 4\sum_{k=1}^{N/2} \frac{(-1)^{k+1}}{2k-1} \sim 2\sum_{m=0}^{\infty} \frac{E_{2m}}{N^{2m+1}}$$

APPLICATIONS of OEIS

OLD THEOREM

C_n (A108) IS ODD IFF

$n = 1, 3, 7, 15, 31, 63, \ldots$

MOTZKIN NUMBERS (A1006)

$$M_n = \sum \binom{n}{2k} C_k$$

THEOREM (DEUTSCH-SAGAN, JNT, '06)

M_n IS EVEN IFF

EITHER $n \in 4S-2$ or $4S-1$

WHERE $S = \{1, 3, 4, 5, 7, \ldots\}$ (A3159)

= NUMBERS WITH BINARY EXPANSION ENDING IN EVEN NO. OF 0's.

Four Unusual Recurrences

- EKG sequence
- Gijswijt's sequence
- Recamán's sequence
- Van Eck's sequence

EKG Sequence (A64413)

1, 2, 4, 6, 3, 9, 12, 8, 10, 5, 15, ...

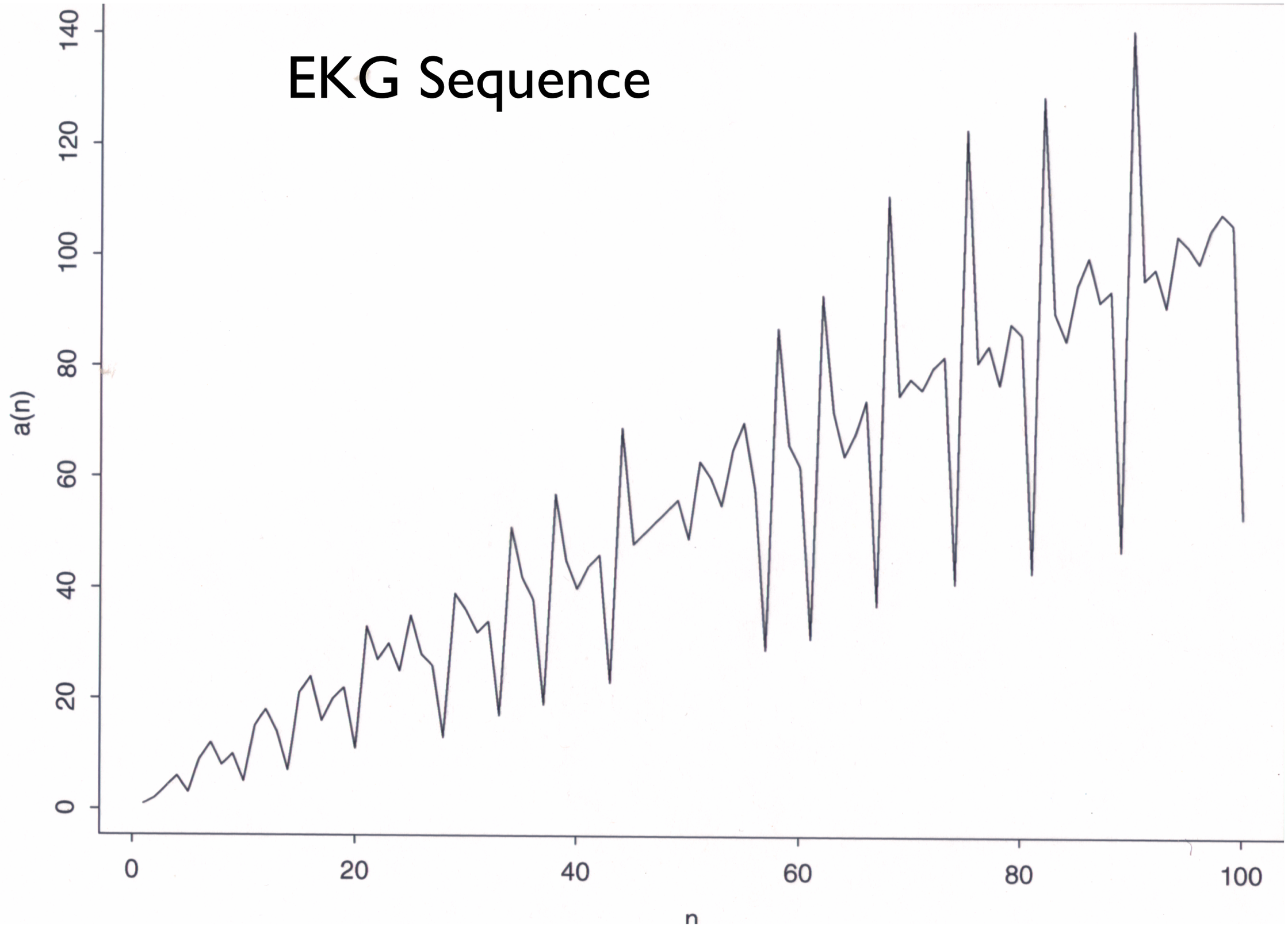
a(1)=1, a(2)=2,

a(n) = min k such that

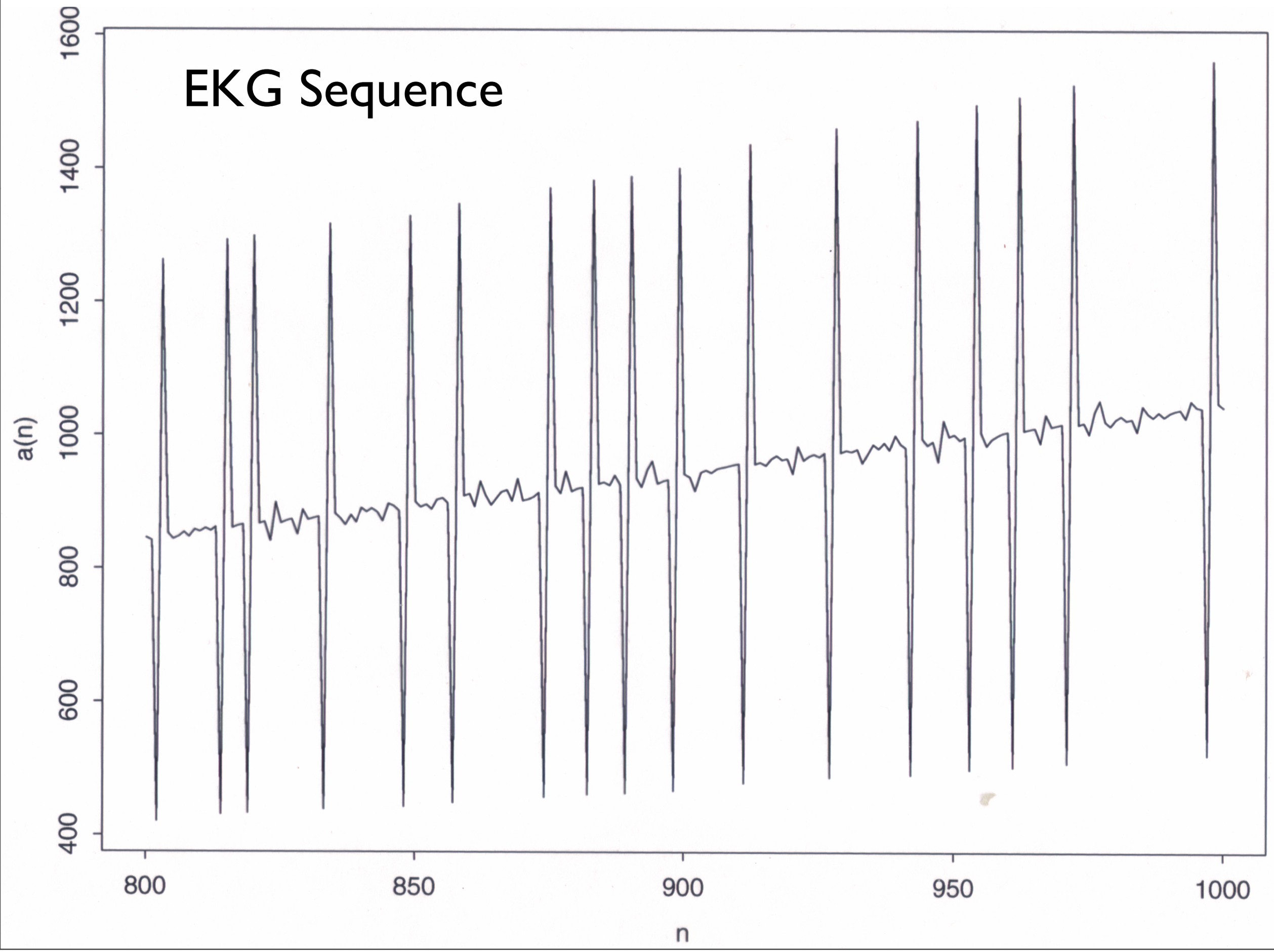
- GCD { a(n-1), k } > 1
- k not already in sequence

- Jonathan Ayres, 2001
- Analyzed by Lagarias, Rains, NJAS, Exper. Math., 2002

EKG Sequence

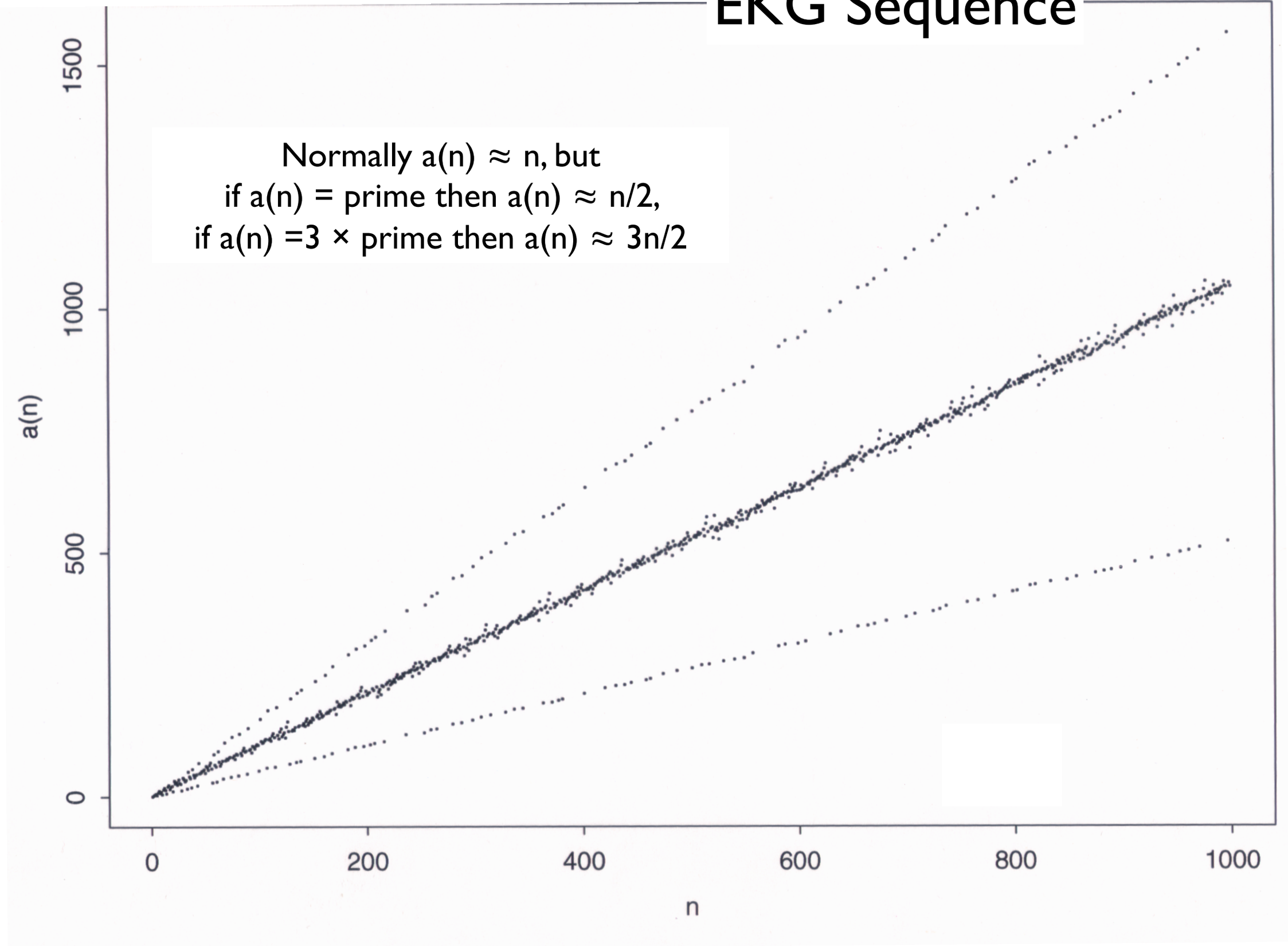


EKG Sequence



EKG Sequence

Normally a(n) ≈ n, but
if a(n) = prime then a(n) ≈ n/2,
if a(n) =3 × prime then a(n) ≈ 3n/2



EKG Sequence

Theorems:

- The sequence is a permutation of the natural numbers
- $c_1\, n \leq a(n) \leq c_2\, n$

Conjectures

- $a(n) \sim n\left(1 + \dfrac{1}{3\log n}\right)$ for the main terms
- $\cdots,\ 2p,\ p,\ 3p, \cdots (\text{primes } p > 2)$

(Proved by Hofman & Pilipczuk, 2008)

EKG Sequence

LEMMA 1 IF ∞ MANY MULTIPLES OF PRIME p APPEAR, THEN ALL MULTIPLES DO.

Pf. kp not in sequence

$\exists n_0$ s.t. $n \geq n_0 \Rightarrow a(n) > kp$

$\therefore a(n) = ip$ $\therefore a(n+1) = kp$, $\ast\ast$

LEMMA 2 IF ALL MULTIPLES OF p APPEAR THEN ALL NUMBERS DO.

Pf. k not in sequence

$a(n) = kip$ $a(n+1) = k$ $\ast\ast$

THEOREM $\{a(n)\}$ IS PERM. OF $\{1, 2, \ldots\}$

Pf. IF ∞ MANY DIFF^{I} PRIMES,

$\therefore \infty$ MANY $2p$'s, USE L1, L2.

IF FINITELY MANY DIFF^{I} PRIMES,

ONE APPEARS ∞ OFTEN,

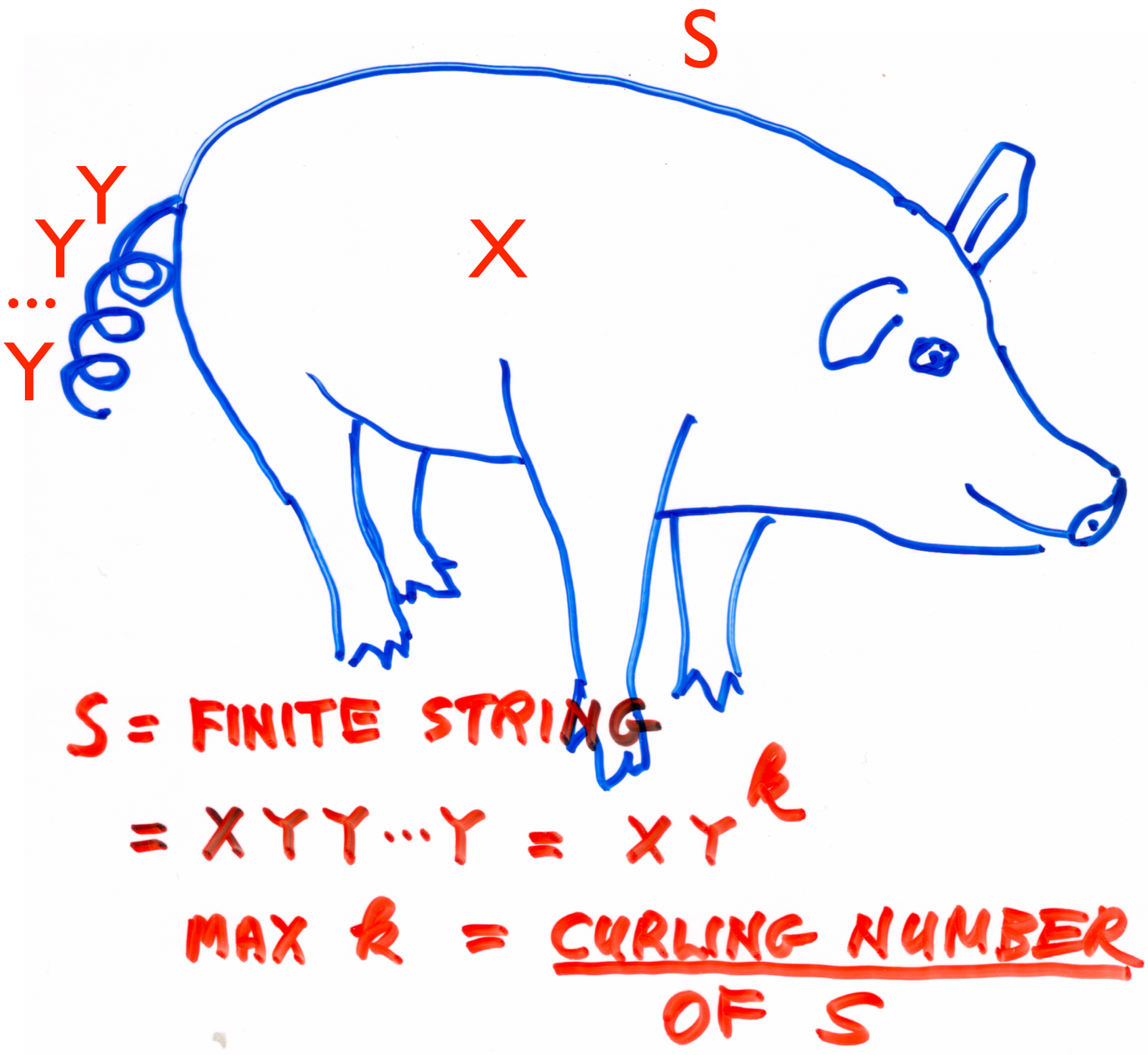
USE L1, L2.

QED

The Curling Number Conjecture

The Curling Number Conjecture

Definition of Curling Number



S = 7 5 2 2 5 2 2 5 2 2, k = 3

CURLING NUMBER CONJECTURE

- START WITH ANY FINITE STRING
- APPEND CURLING NUMBER
- REPEAT
- THEN MUST REACH A 1 !?

E.G.

START : 2 2 2 3 2 2

THEN 2 3 2 2 2 3 3 2 1 ....

BOO!

Gijswijt's Sequence

Dion Gijswijt (Amsterdam), Fokko v. d. Bult,
NJAS, Allan Wilks, John Linderman

Start with 1, always append curling number

1 1 2

1 1 2 2 2 3

1 1 2

1 1 2 2 2 3 2

1 1 2

1 1 2 2 2 3

1 1 2

1 1 2 2 2 3 2 2 2 3 2 2 2 3 3 2

1 1 2

. . . . . .

$a(220) = 4$

(A090822)

Gijswijt, continued

Gijswijt, continued

Is there a 5?

Gijswijt, continued

Is there a 5?

300,000 terms: no 5

Is there a 5?

300,000 terms: no 5

$2 \cdot 10^6$ terms: no 5

Is there a 5?

300,000 terms: no 5

$2 \cdot 10^6$ terms: no 5

10^{120} terms: no 5

Is there a 5?

300,000 terms: no 5

$2 \cdot 10^6$ terms: no 5

10^{120} terms: no 5

NJAS, FvdB: first 5 at about term $10^{10^{23}}$

First n appears at about term

$n-1$

...

5

4

3

2

2

(F.v.d. Bult et al., J. Integer Sequences, 2007)

(A90822)

Proofs could be simplified if Curling Number Conjecture were true

How far can you get with an initial string of n 2's and 3's (before a 1 appears)?

THE UNIQUE RECORD STARTS:

LENGTH 8: 23222323 → 66

LENGTH 22:

23223223223223223223

→ 142

LENGTH 48 → 179

LENGTH 77 → 250

JOINT WORK WITH

BEN CHAFFIN

(INTEL)

THEOREM

LET $\mu(n)$ = MAX LENGTH ATTAINED STARTING WITH n 2's & 3's.

IF S ACHIEVES $\mu(n) > \mu(n-1)+1$ THEN S DOES NOT CONTAIN W^4, $W \neq \phi$.

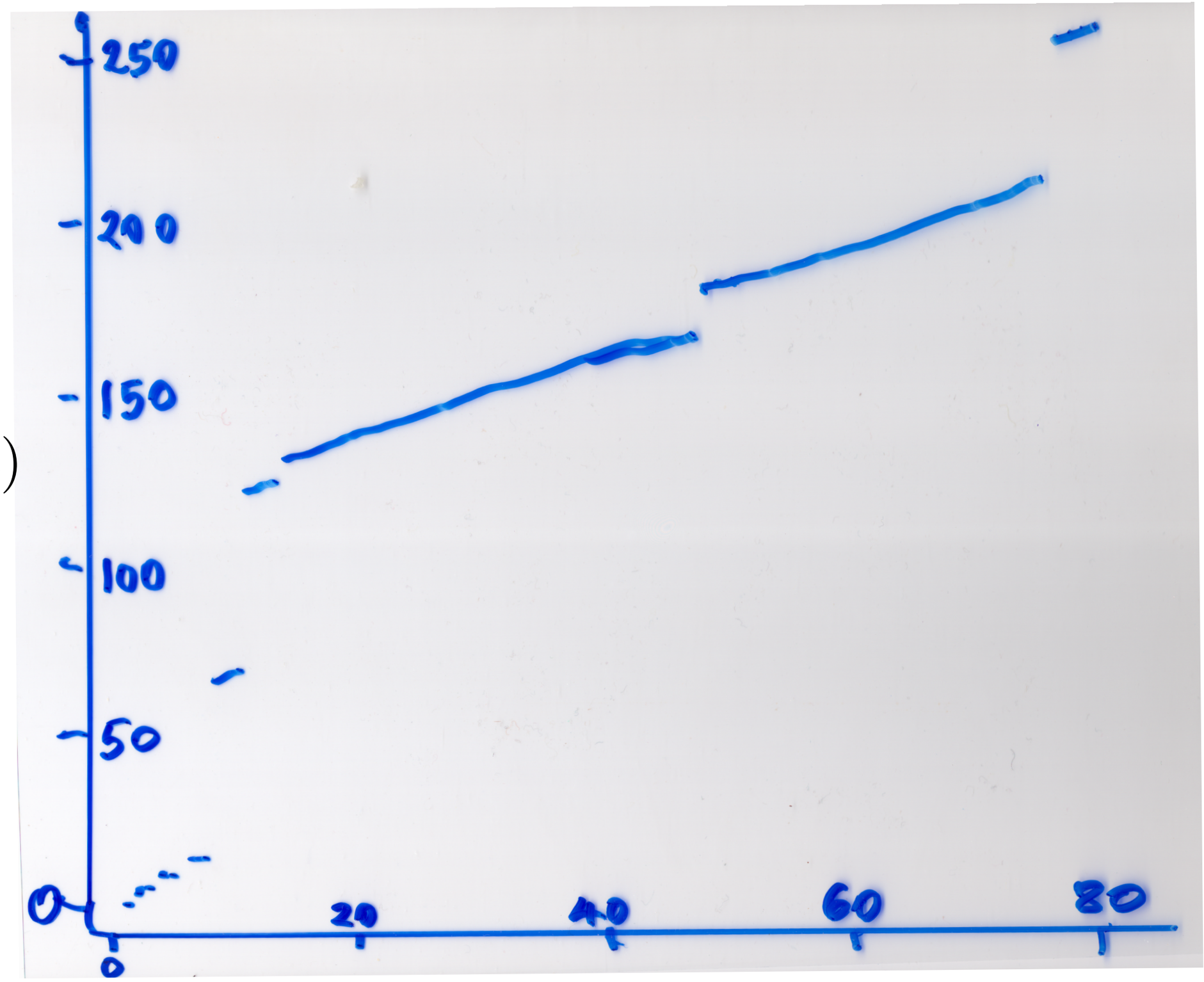
(SO NOT 2222)

Searched n <= 53

CONJECTURE

... S ALSO DOES NOT CONTAIN 33.

Searched n <= 80



$\mu(n)$

n

Recamán's Sequence

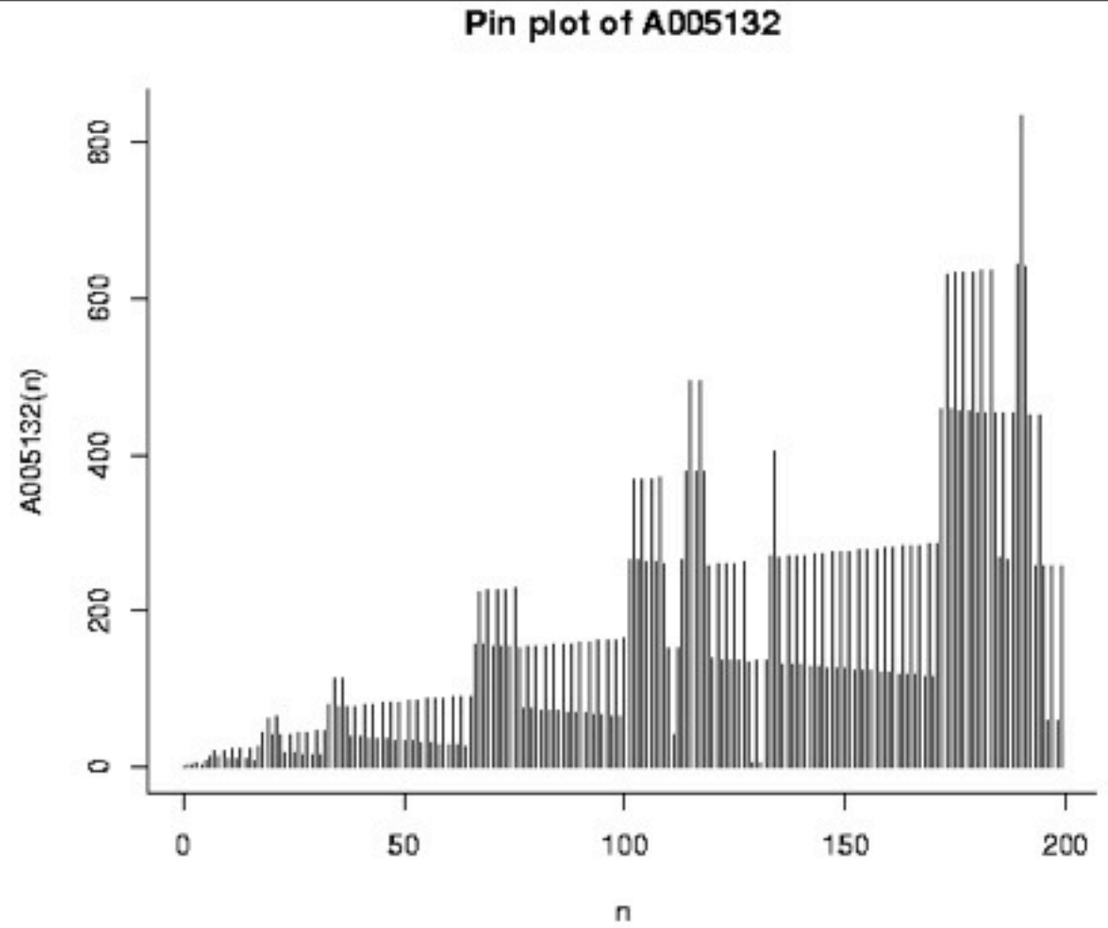
0	1	2	3	4	5	6	7	8	9	...
0	1	3	6	2	7	13	20	12	21	...

$$a_n = a_{n-1} - n$$ (A5132)

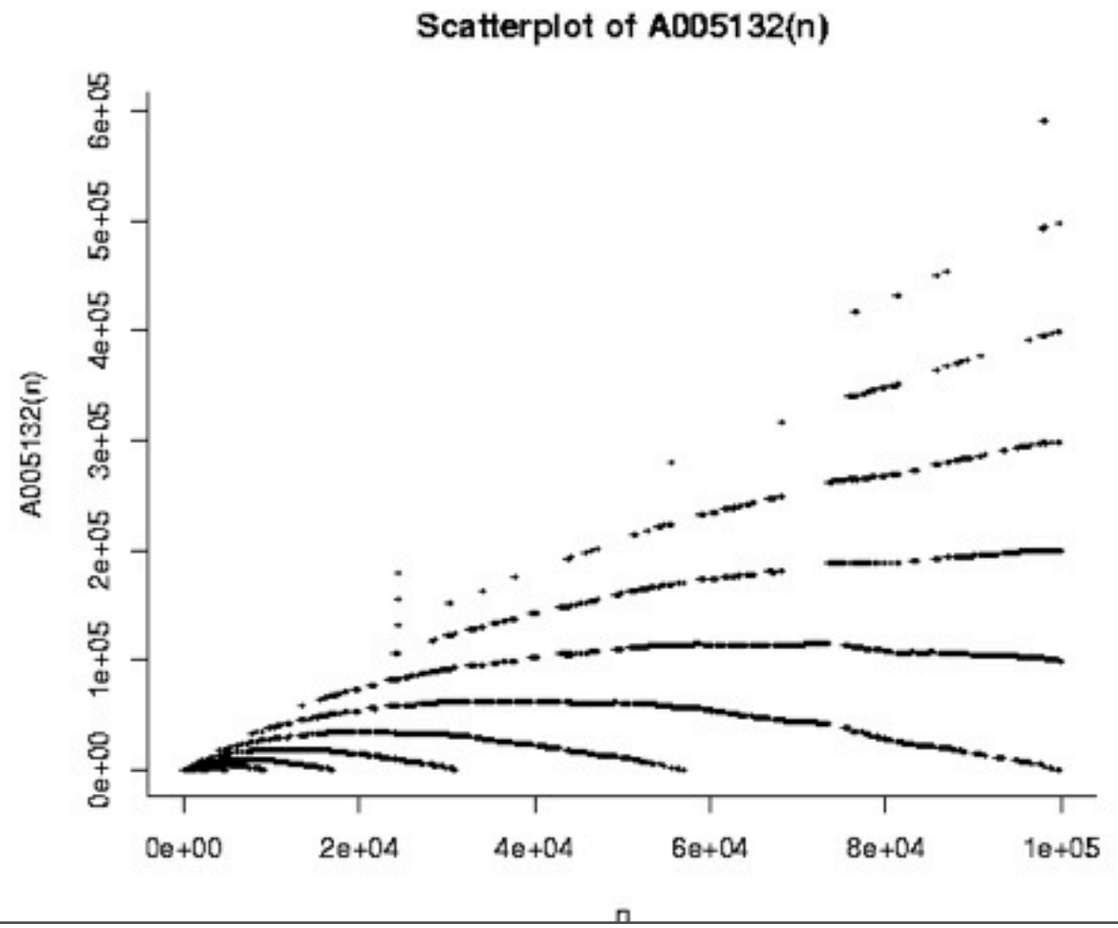
if positive and new, otherwise

$$a_n = a_{n-1} + n$$

- from Bernardo Recamán Santos (Colombia), circa 1992



Recamán, continued



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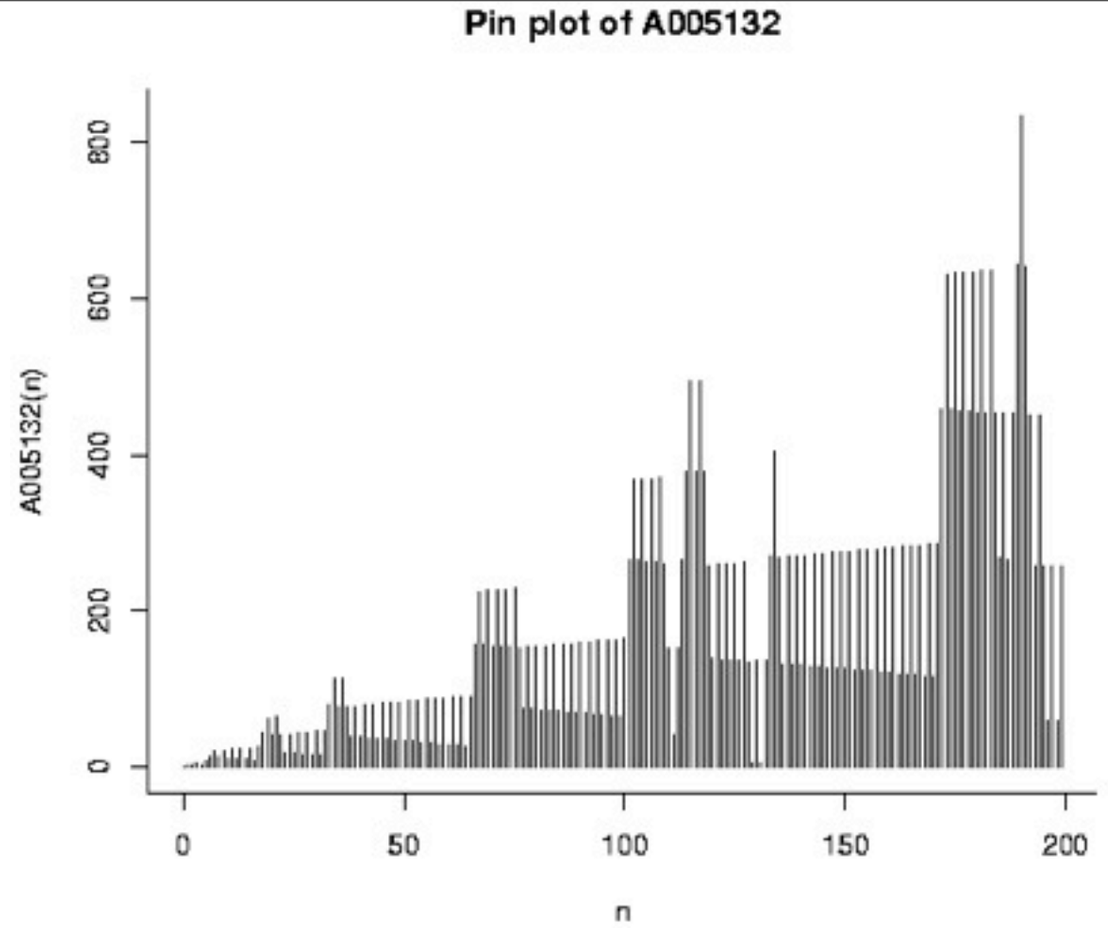
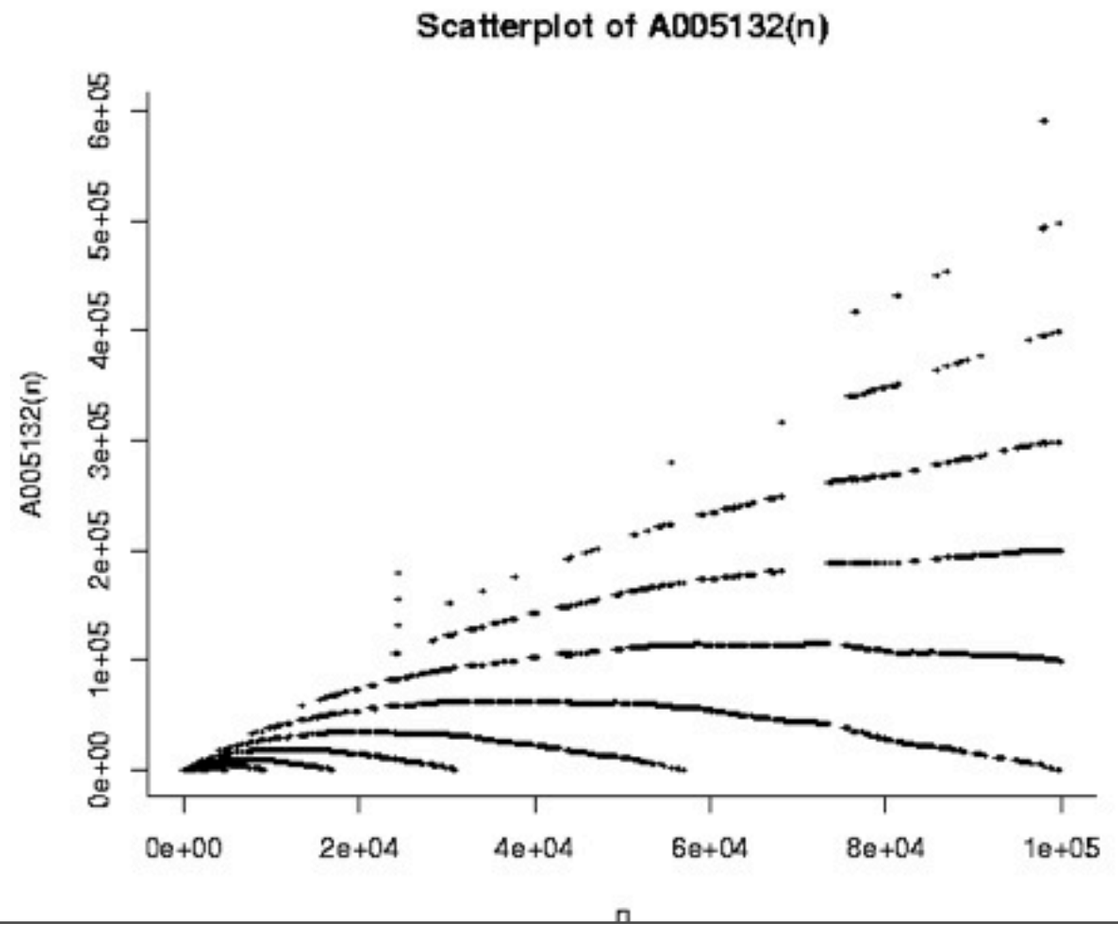


Recamán, continued



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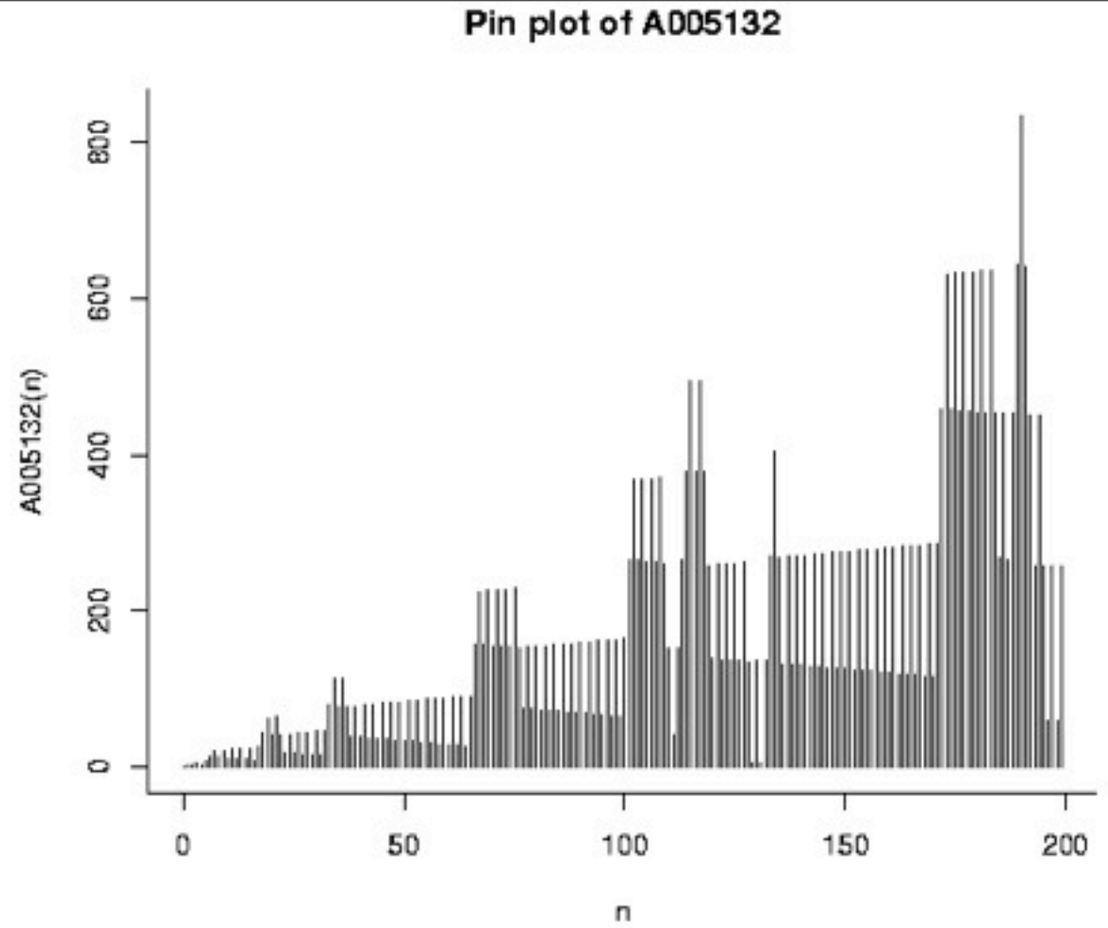
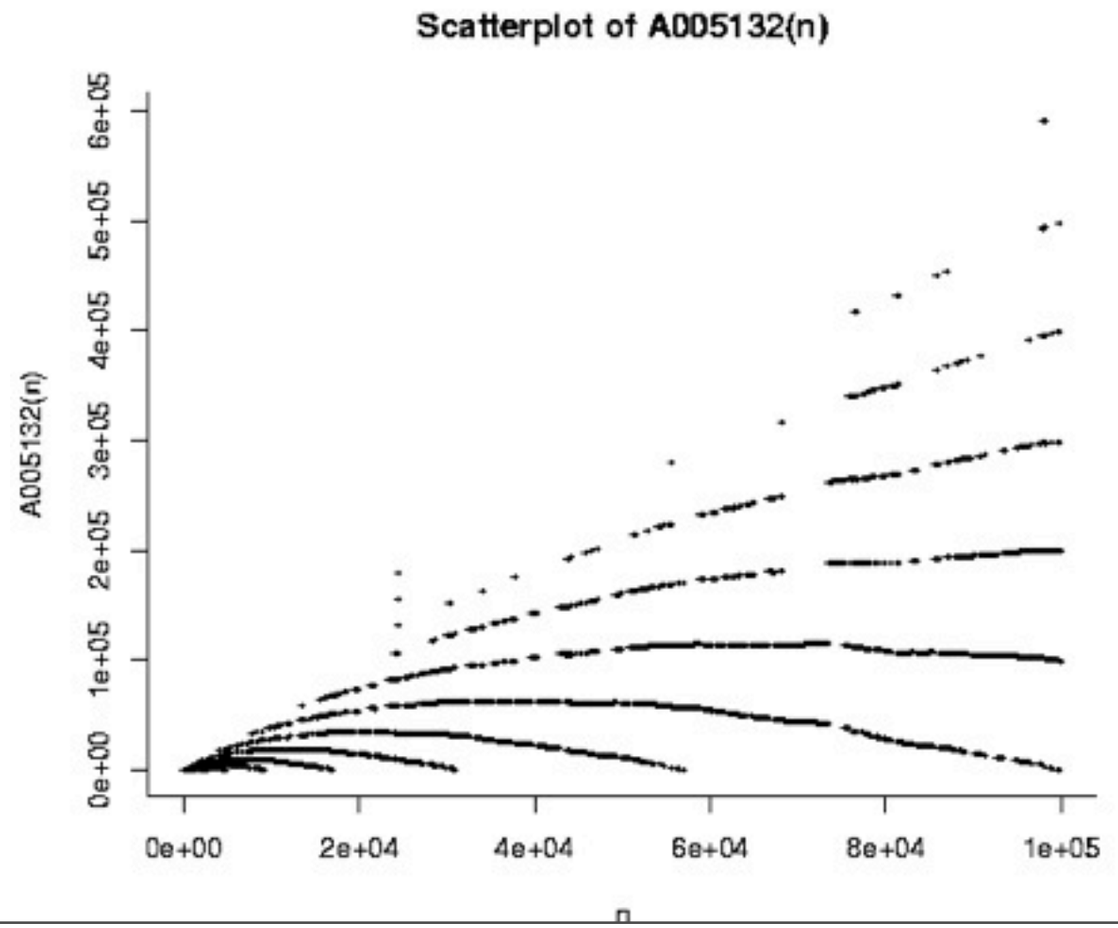


Recamán, continued



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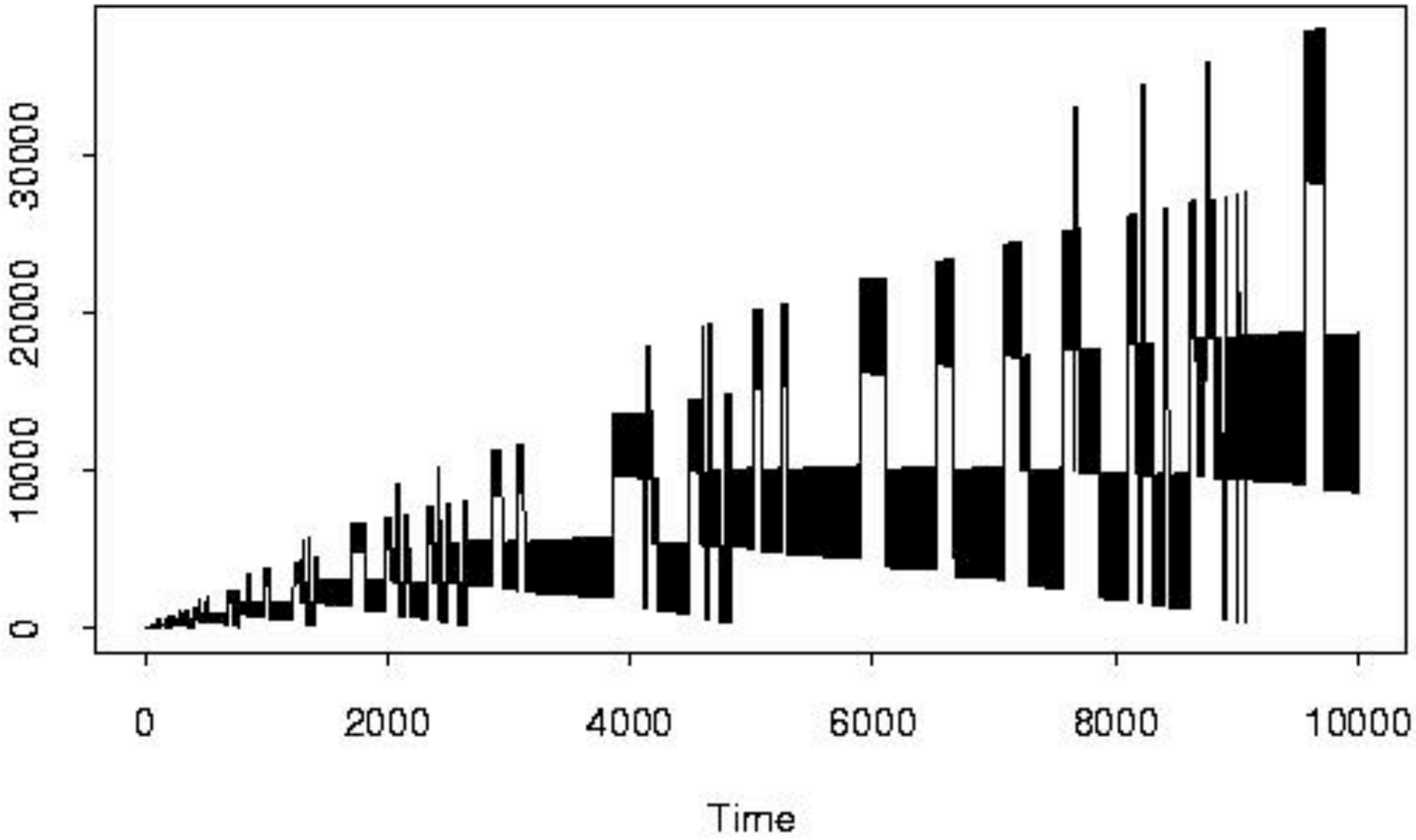


Recamán, continued

When n appears for the first time:

1	2	3	4	5	6	7	8	9	10
1	4	2	131	129	3	5	16	14	12
11	12	13	14	15	16	17	18	19	20
10	8	6	31	29	27	25	23	99734	7

Red: records

Numbers that take a record number of steps to appear:

1	1
2	4
4	131
19	99,734
61	181,653
879	328,002
1355	325,374,625,245
2406	394,178,473,633,984
852655	$> 4.28 \times 10^{73}$

(Benjamin Chaffin, Intel, March 2010)

(A64228) (A64227)

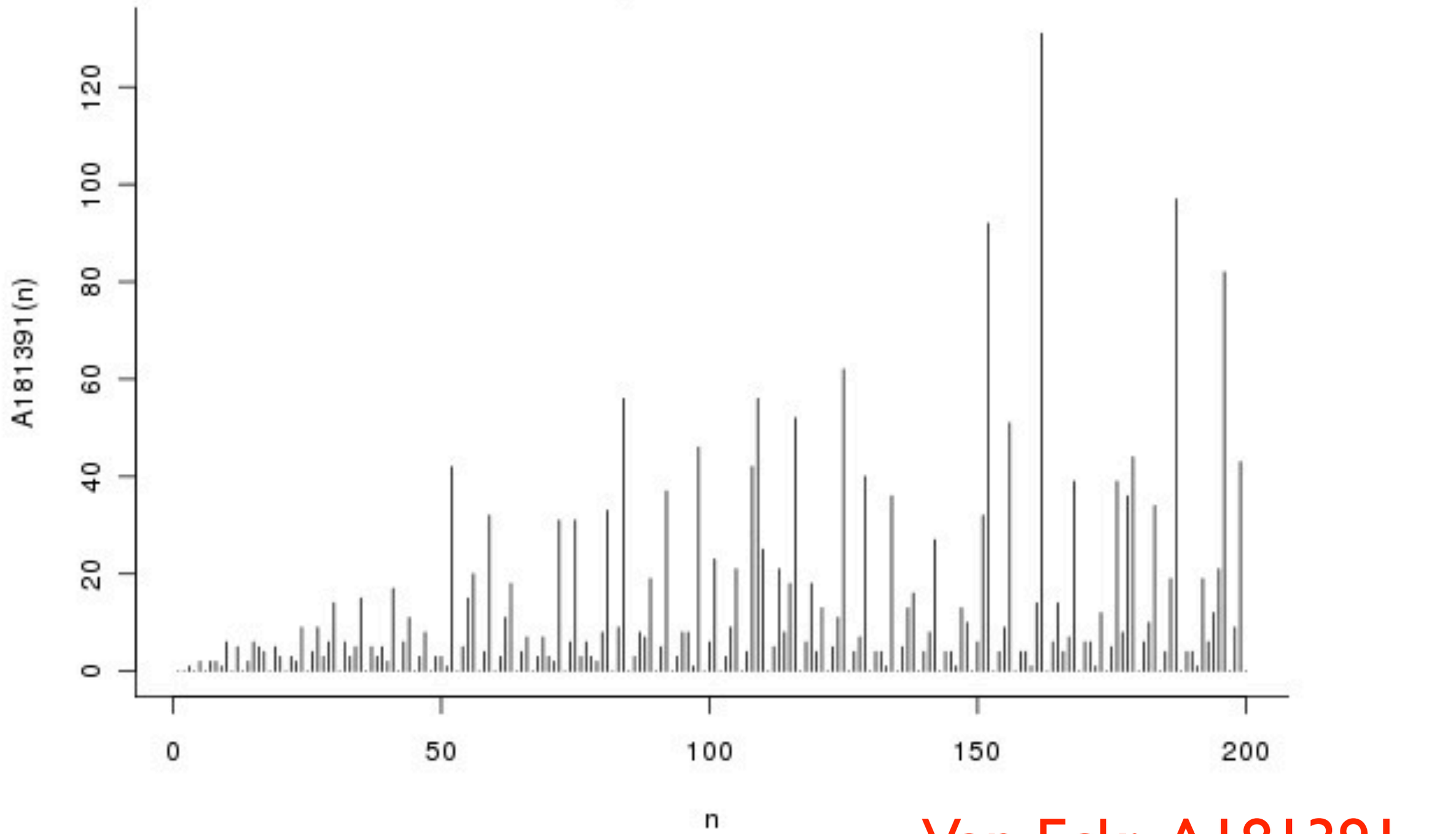
Jan Ritsema van Eck's Sequence

0, 0, 1, 0, 2, 0, 2, 2, 1, 6, 0, 5,
0, 2, 6, 5, 4, 0, 5, 3, 0, 3, 2, 9,
0, 4, 9, 3, 6, 14, 0, 6, 3, 5, 15, 0,
5, 3, 5, 2, 17, 0, 6, 11, 0, 3, 8, 0, ...

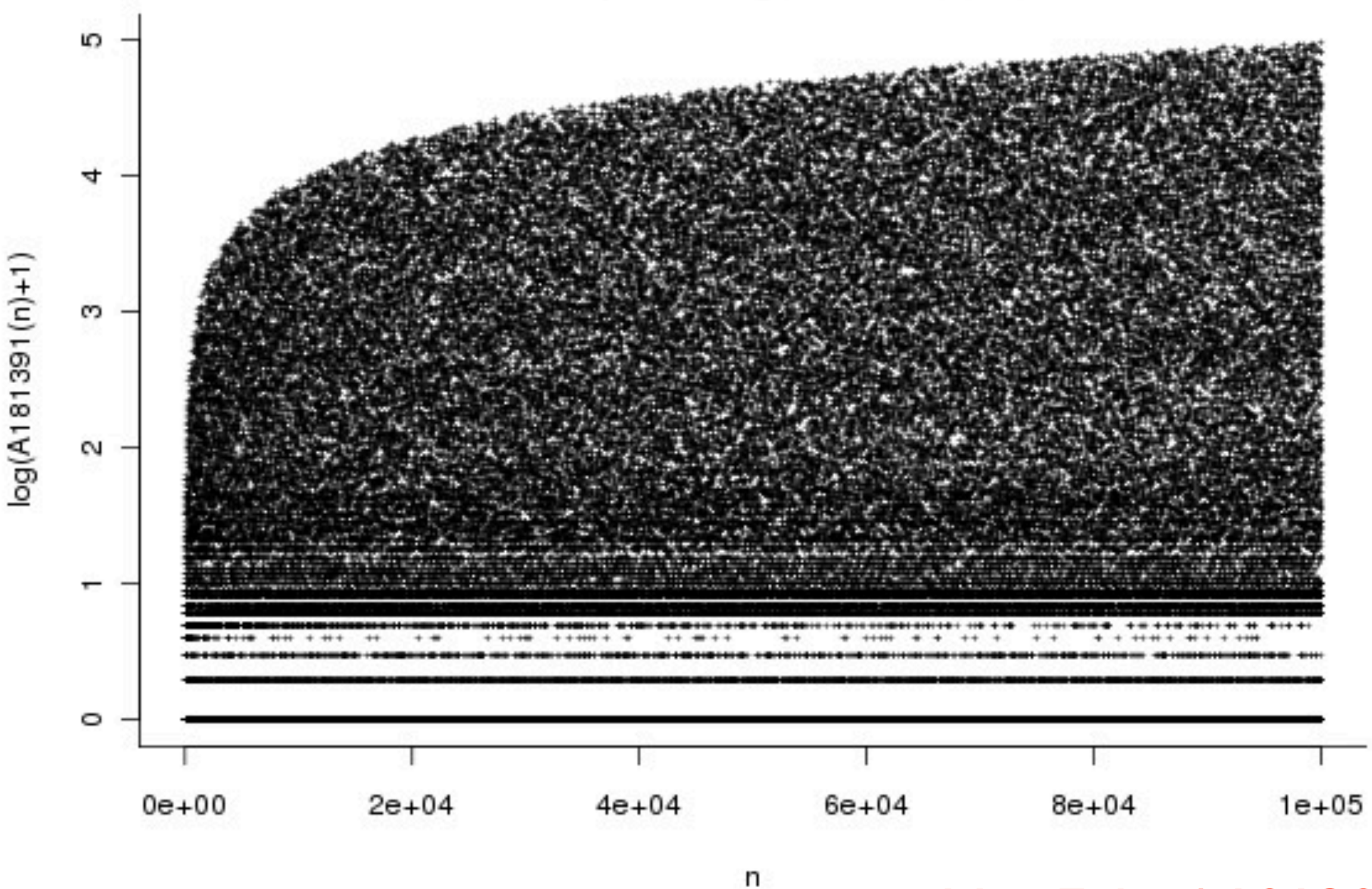
a(n): how far back did we last see a(n-1)?
or 0 if a(n-1) never appeared before.

Van Eck: A181391

A181391 as a graph:



Van Eck: A181391



Van Eck: A181391

Thm. (Van Eck) There are infinitely many zeros.

Proof: (i) If not, no new terms, so bounded.

Let M = max term.

Any block of length M determines the sequence.

Only M^M blocks of length M.

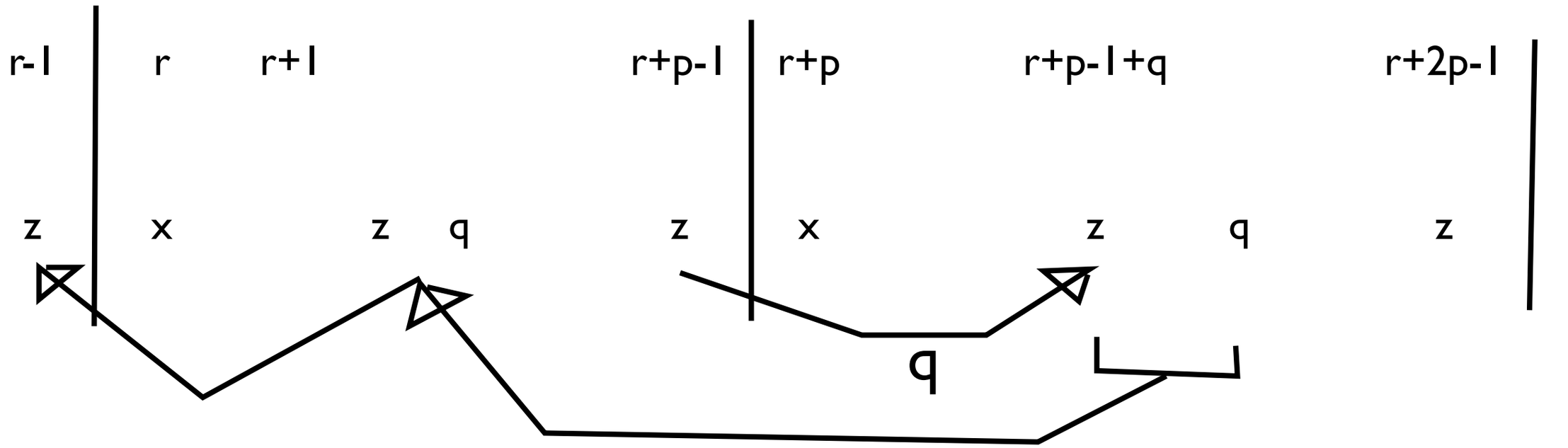
So a block repeats.

So sequence becomes periodic.

Period contains no 0's.

Van Eck: A181391

Proof (ii). Suppose period has length p and starts at term r.



Therefore period really began at term r − 1.

.....

Therefore period began at start of sequence.
But first term was 0, contradiction.

Van Eck: A181391

It seems that:

lim sup a(n) / n = 1

Gaps between 0's roughly log_10 n

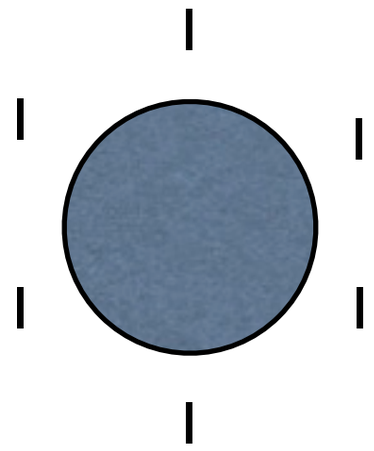
Every number eventually appears

Proofs are lacking!

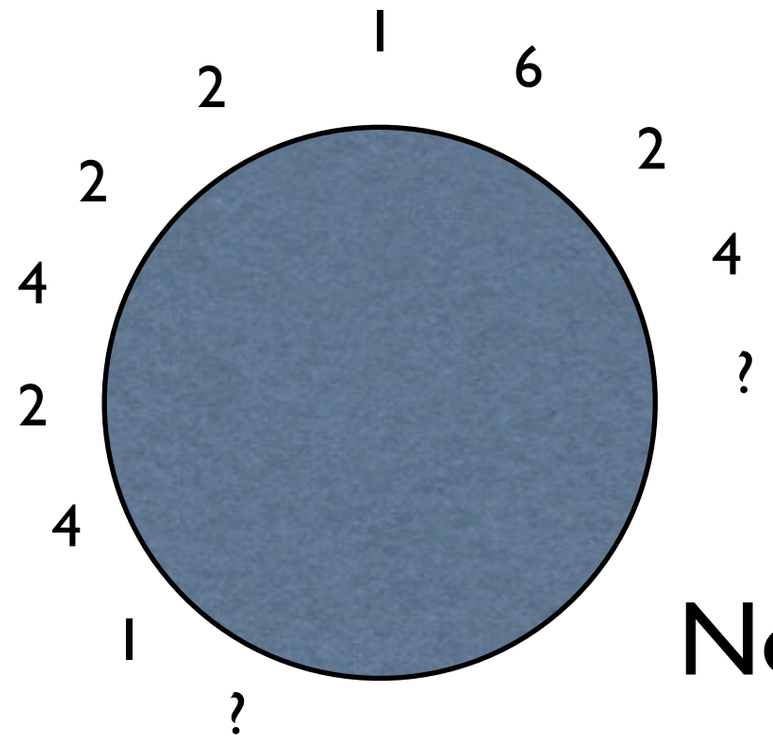
Van Eck: A181391

Van Eck: A181391

Conjecture:
There is no
nontrivial cycle



Trivial cycle



Nontrivial cycle ?

(David Applegate: Only trivial cycles of length up through 14)

The new OEIS: oeis.org

- 206,500 sequences (20,000 in first year)
- Owned and maintained by OEIS Foundation (a 501(c)3 public charity)
- Need more editors to help
- Need more eyes searching for sequences
- Need help with "kiosk" for science museums
- Need help with "music"