HOR:RATIO
Instructions for use

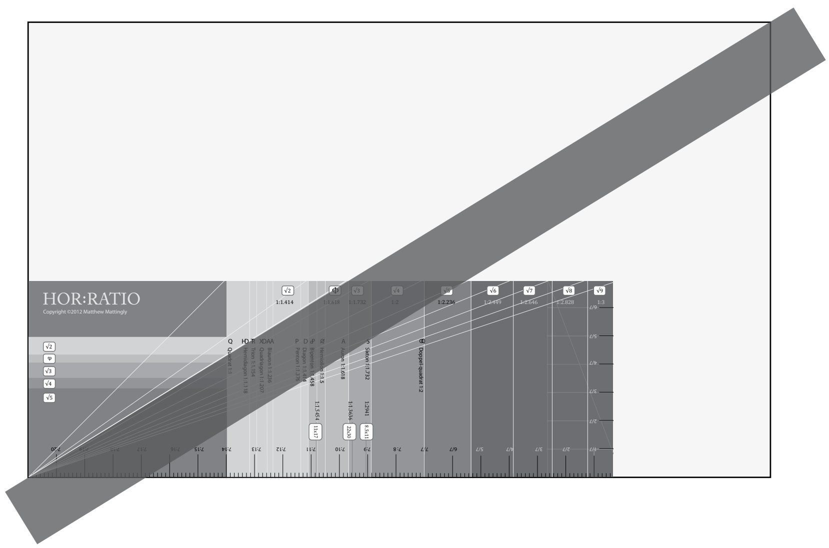
Hor:ratio is a pocket reference and proportional protractor for artists, designers, art historians, or anyone interested in systems of proportion. It enables the creation, scaling and measurement of rectangular ratios, making it easy to experiment with proportional combinations for new works or to test hypotheses about the composition of existing works.

Throughout human history, artists, designers and architects have been guided by various systems of proportion. Sometimes these systems were mathematically rigorous, strictly adhered to and thought to be divinely inspired. In other times they have been employed more loosely as aids to aesthetically pleasing designs. Whether adopted as a tool of creation or analysis, they are fascinating in their own right.

Hor:ratio folds to a 3” square and can be easily carried in a wallet or as a bookmark. Unfolded to a 3”x 9” rectangle it is a convenient size to use with a pocket sketchbook.

Using Hor:ratio
1. Print out on ordinary printer paper
2. Cut out using crop marks as guides. For best results use sharp knife and straightedge.
3. Fold lengthwise, being careful to line up edges.
4. Fold in thirds so the title square is on one side and the armature square on the reverse.

Constructing a Rectangle
Hor:ratio can be used folded into a square or opened out into a rectangle to reveal additional ratios. To construct a rectangle of any size and a given proportion, place the lower left corner of Hor:ratio where you want the lower left corner of your desired rectangle to be. Lay a straight edge along the diagonal of the selected proportional rectangle, and extend it out to the desired length and height.



Alternatively, you may make a small tick mark at the crossing point so you can remove Hor:ratio but still find the diagonal. This is useful if you wish to make a rectangle that is smaller than Hor:ratio.

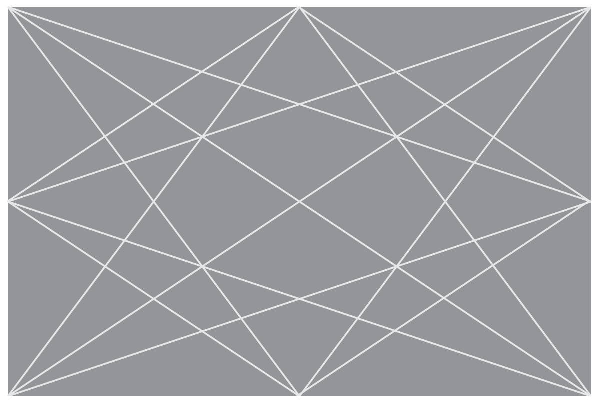
Measuring a Rectangle
To check the proportions of an existing rectangle, align Hor:ratio with the lower left corner and lay a straight edge from the upper right corner of the rectangle of interest. The point where it crosses the top (or right side, if used folded) edge of Hor:ratio will indicate which (if any) of the ratios applies.

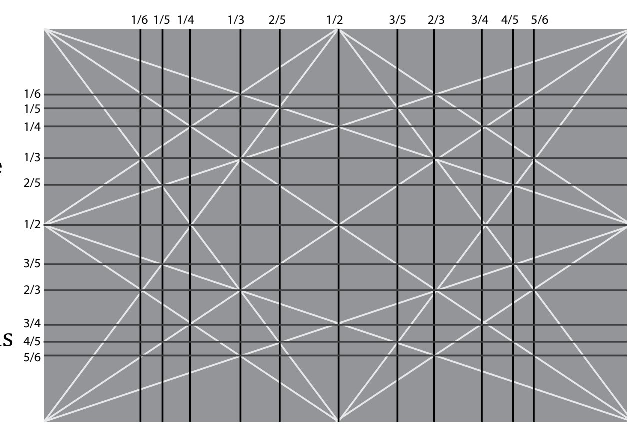
Although using Hor:ratio is simple, it presents a lot of densely packed information, encompassing several different systems and their interrelationships. The rest of this guide addresses each system in turn, explaining its history, theory and practical use in design and compositional analysis.

Two Domains
Systems of proportion can be classed under two principle domains, sometimes referred to as "static" and "dynamic." The static ratios are based on ends and sides of commensurable length: both sides can be measured or divided evenly by the same number. Dynamic ratios are based on commensurate areas: the larger rectangle may be divided evenly by smaller rectangles of similar or related proportions. The names come from the work of Jay Hambidge, and reflect his bias in favor of the dynamic. However, both systems have been favored at different times; dynamic ratios often feature incommensurate end to side ratios, which have been seen as either a blessing or a curse. It's also possible to make too much of the dichotomy - each domain can generate overlapping or very similar ratios. Hor:ratio's title side is primarily dedicated to the dynamic domain, the reverse side to the static.

The Static Ratios
The static ratios include any pair of commensurable numbers, but certain ratios have historically been considered more harmonious than others, though tastes and aesthetic canons have changed over time. Artists and theoreticians have sought guidance in arithmetic or geometric relationships, analogies with musical harmony, and religious or philosophical doctrines. Hor:ratio consolidates some of these traditional systems while seeking to open up new ways of using simple mathematical and geometric techniques to structure designs.

The Armature of the Rectangle
The left hand square is based on the "Armature of the Rectangle." This simple diagram of diagonals and their intersections is a powerful framework for composition, and can be applied to any rectangle regardless of its proportions. It has been argued that the armature is an inherent attribute of the rectangle; like its borders or the flatness of its surface, this hidden structure is always present and perceived by the viewer whether or not the artist chooses to acknowledge it.

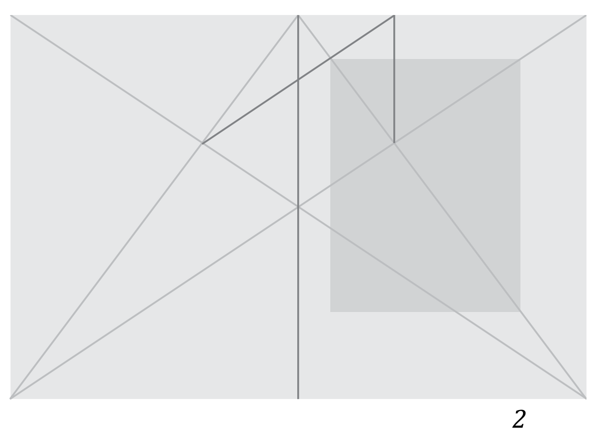




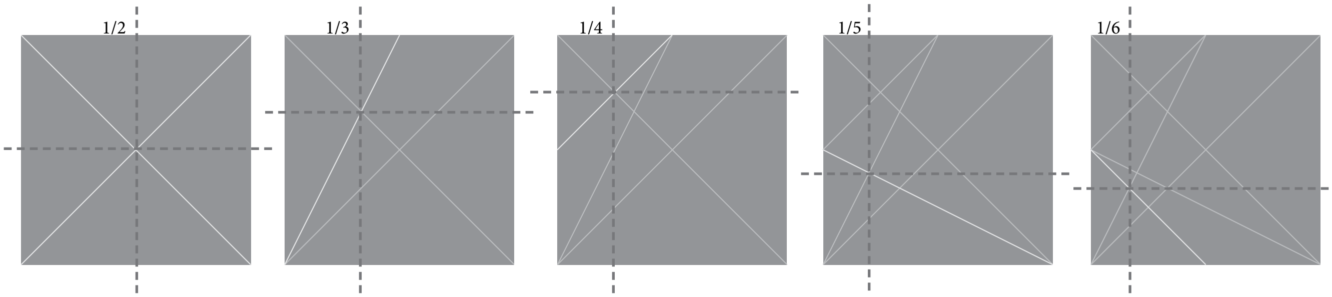
The points established by the intersections of the diagonals serve as landmarks within the rectangle and locate horizontal and vertical divisions of the rectangle's ends and sides into halves, thirds, quarters, fifths, and sixths. The divisions create grids of rectangles proportional to the base rectangle. They may be further subdivided and used in conjunction with other proportioning systems.

If using the armature as a foundation for a composition it is a good idea to erase the diagonals once the divisions are found, as they can exert an undue influence on the design.

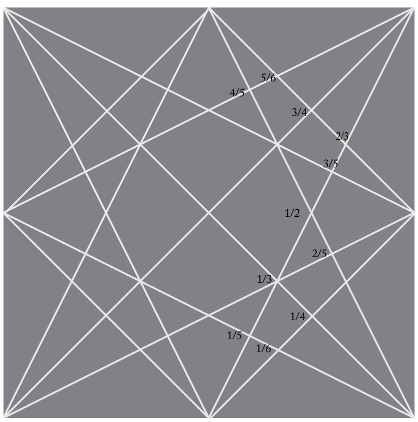
It is not necessary to construct the entire armature if you only want to locate specific divisions. It can be adapted for specific proportioning systems, and has been used in various forms by artists and designers going back at least to the Medieval period. In this variation, known as Villard's Diagram, and used in book design to lay out the margins and type areas on two page spreads, the type area's top left corner is located 1/9 of the way across the page, as defined by the intersection of the armature lines.



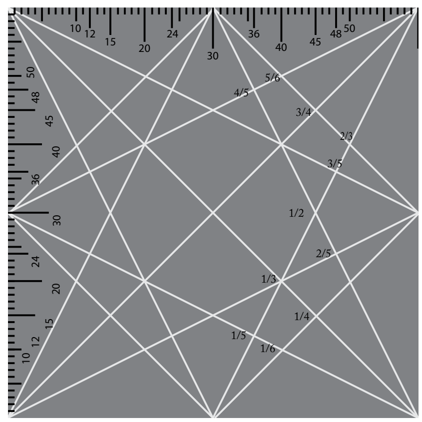
Hor:ratio bases the armature on a square, creating a scale with equal divisions for each dimension.



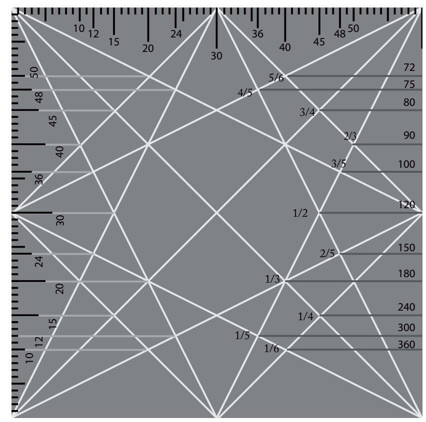
Construction of diagonals to locate half, third, quarter, fifth and sixth divisions.



Hor:ratio's version of the completed armature displays the intersections labeled as fractions of the height.



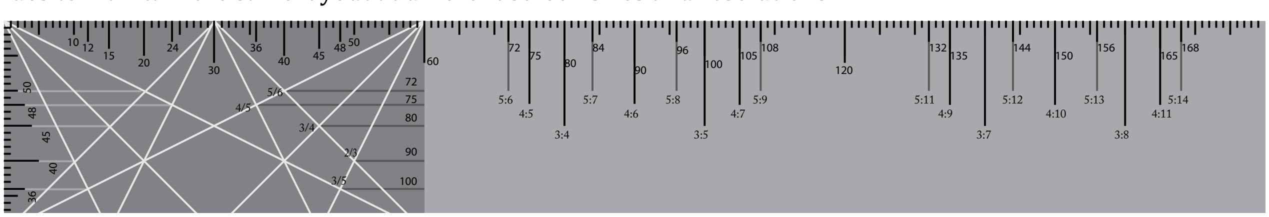
The factors 2,3,4,5, and 6 combined enable division by 60, and in turn by its additional factors 10, 12, 15, 20, and 30. The fractional divisions can now be expressed as multiples of 1/60. The larger factors are indicated by larger markings.



The numbers on the right indicate the length of a rectangle produced by extending a diagonal from the lower left corner through that point. The relationship is the inverse of the fraction times the height, e.g. 5/6 => 6/5 x 60 = 72.

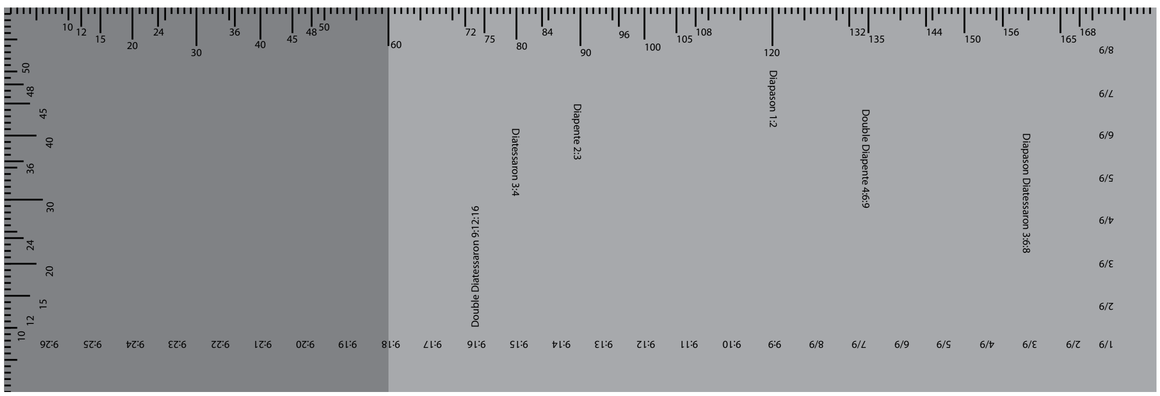
60ths Scale

The top row of marks extend the 60 based scale out to 180. The multiples of factors of 60 get larger marks and are labeled with numerical values and ratios in separate tiers. Thus, 72 has its numerical label in the same tier as the other multiples of 12, and its end to side ratio is given as 5:6 (the end is 60 units high, 60/5 = 12; 12 x 6 =72). Using 60 as a base means that the scale can be divided by many factors. There are book designs based on a square 60 x 60 grid, and web designers often use an even more finely grained system based on 960 (60 x 16) pixels across to allow maximum flexibility with column and gutter widths.   Recently, web designs have moved in the direction of using proportional values to maintain the same layout at different screen sizes and resolutions.



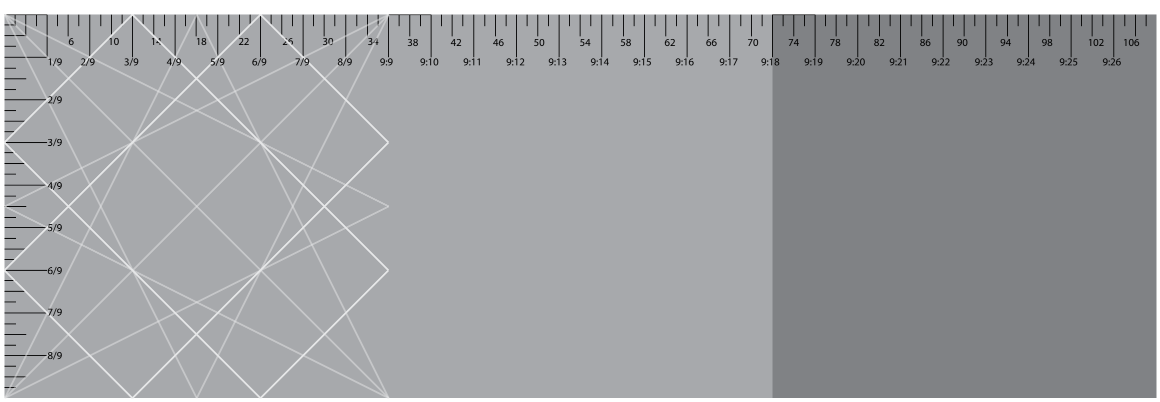
The Musical Consonances
Leon Battista Alberti, writing in the fifteenth century, established a doctrine of porportion based on a combination of ancient Greek and Christian ideals and intimately bound up with theories of musical harmony. The discovery, attributed to Pythagoras, that the division of a vibrating string by simple ratios produced the intervals known to be pleasing to the ear, was seen as evidence of a universal principle of order and divine intent. Alberti used this theory as the basis for a system of porportion to be used in architecture and design. In music, the interval with the ratio 1:2 is one octave, known in Alberti's time as "diapason." 3:4 is the fourth, or diatessaron, and 2:3 is the fifth, or diapente. Alberti set out rules for the application of these proportions and for their combination. The double diapente is a composite consonance whose ratio 4:6:9 bears the 2:3 relationship between its first and second and second and third terms. The double diatessaron has a ratio of 9:12:16, with the relationship 3:4 between each pair of terms. The diapason diatessaron, on the other hand, has the ratio 3:6:8, with the first pair of terms related by 1:2 and the second by 3:4.

Subdividing the ratios in this way establishes system for breaking up space proportionately, especially long narrow shapes like the nave of a church. Although mainly intended for architecture, these principals have been taken up by painters and other artists as well.



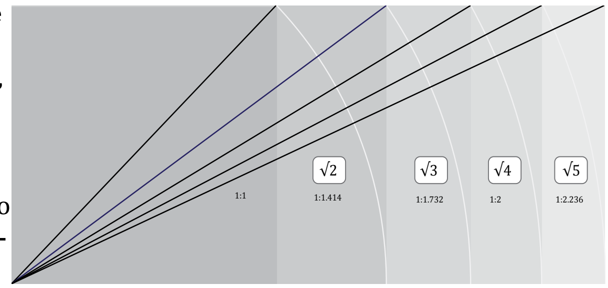
Ninth scale
The armature of the rectangle can be subdivided by any of its simple proportions to construct scales based on these combinations. Here, a variation on the armature is used to divide by 3, then 3 again and finally by 4, to create a scale based on 9 and 36.

Division by 9 was employed by Pieter Bruegel the Elder, (although not as a commensurable scale) and is also significant to the theory of musical consonances.

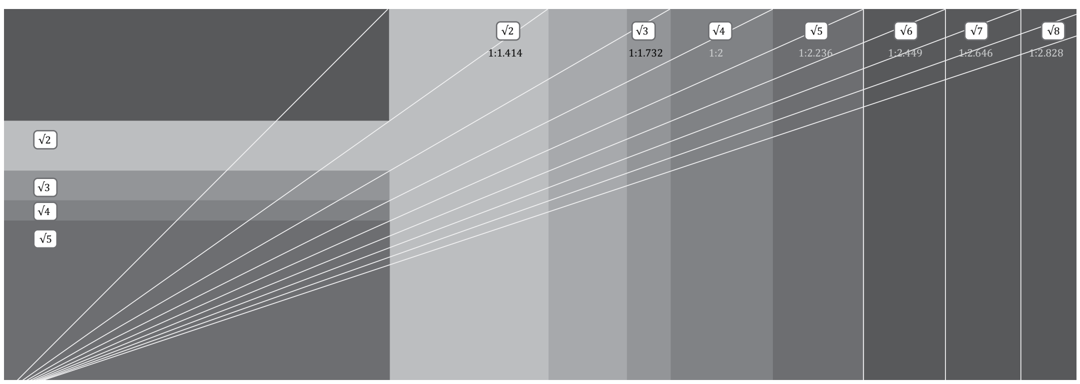


The Dynamic Ratios
The dynamic ratios comprise the "root rectangles," the "golden rectangle," also known by the Greek letter Φ (phi), and the 12 Orthogons of Wolfgang von Wersin. Also included here are the related concepts of reciprocals and complements.

The Root Rectangles
The root rectangles are a series constructed by first rotating the diagonal of a square to become the long side of a new rectangle. If we call the length of each of the sides of the square 1, or unity, the diagonal of the square equals the square root of 2. Rotating this to the horizontal forms a new rectangle with a ratio of 1 to √2, or 1:1.414… The diagonal of this rectangle equals √3; repeating the operation generates a rectangle with a ratio of 1 to √3, or 1:1.732… . The next in the series is an exception to incommensurability of the sides, as the square root of 4 is the whole number 2. This will hold true for all whole number squares such as 9, 16, etc.

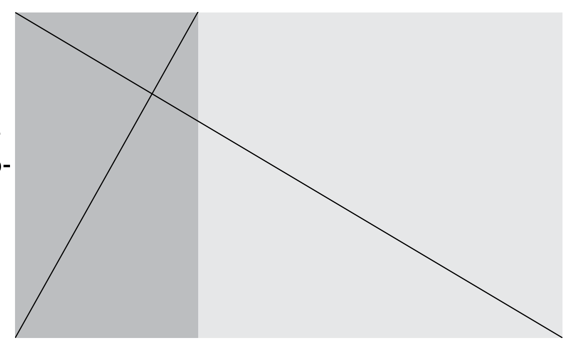


It is possible to go on generating root rectangles indefinitely, and doing so offers new ways of dividing space proportionately. Hor:ratio includes the root rectangles through √9, whose ratio 1:3 equals that of Hor:ratio.

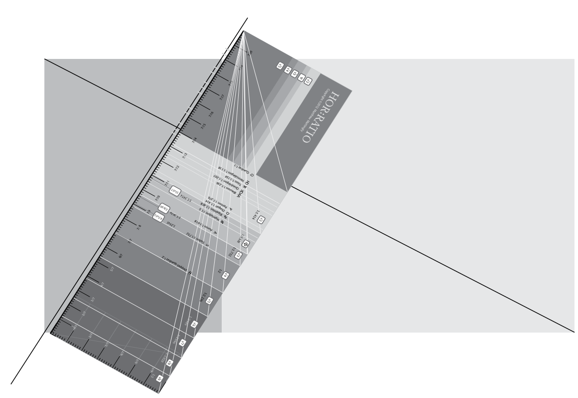


Reciprocals
The reciprocal of a rectangle is similar in shape to the major rectangle, but smaller in size. To obtain the reciprocal, find the line that originates in a corner and intersects the diagonal at a right angle. The point where this line intersects the side defines the end of the reciprocal rectangle.

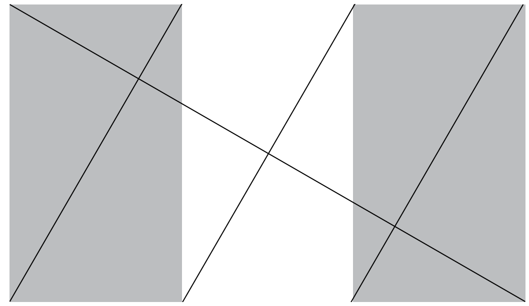
The reciprocal of a number is obtained by division into 1.



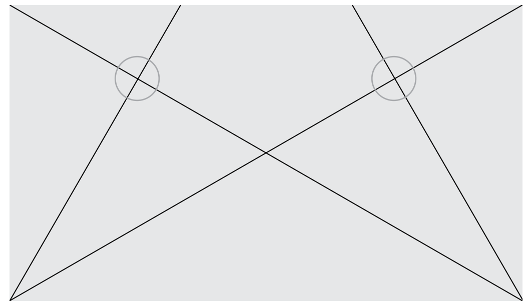
You can use Hor:ratio for a handy square to find the reciprocal of any rectangle.



All root rectangles can be evenly divided by their root number by applying their reciprocals. Here, a √3 rectangle is divided by 3 reciprocal rectangles.

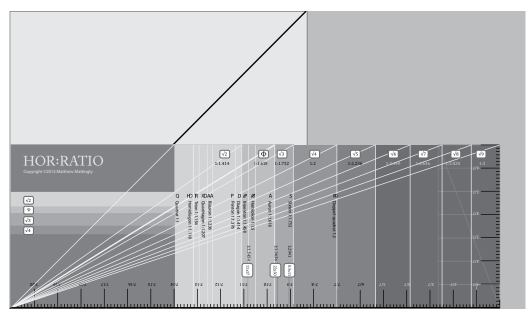


The “eyes of the rectangle” are formed by the intersections of the diagonals of the main rectangle with that of the reciprocal. These can be used as focal points of the design, or to locate further subdivisions or shape relationships.



Complements
The complement of a rectangle is the area left over after the application of a square.

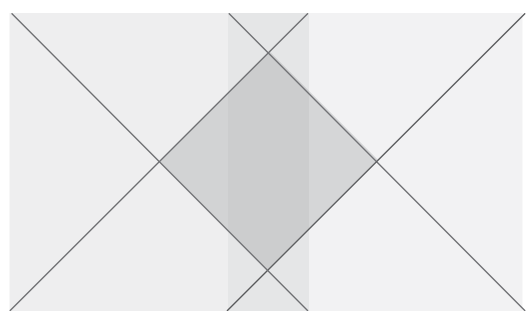
Use the diagonal of Hor:ratio’s square to apply a square to any rectangle



Rebatment
Rebatment is the technique of rotating an end 90˚ to apply a square. Often a square is applied to each end, so that they overlap. The area of overlap establishes off-center vertical subdivisions from which further construction lines can be generated.

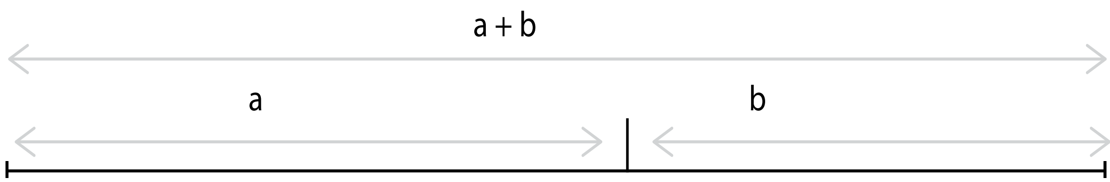
The diagonals of the overlapping squares combine to form the diamond shape in the middle. The corners locate possible horizontal and vertical divisions.

The armatures of the squares can be combined with that of the rectangle as a whole to create compositional structures.



The Golden Ratio (ϕ)

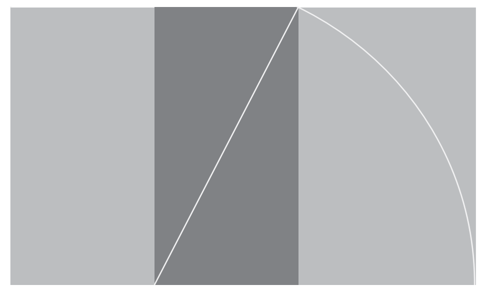
The golden ratio, sometimes represented by the Greek letter phi (ϕ), has a special place in art, architecture and design. It appears in the works of ancient Egypt and Greece, and has been periodically rediscovered and put to use ever since. It has a set of unique mathematical properties as well as intriguing relationships to natural forms, and has been the subject of much speculation and analysis by theorists and historians. Its salient feature is the fact that if a line is bisected by ϕ, the ratio of the larger part to the smaller is the same as that of the whole to the larger part. Expressed as a numerical ratio, it is 1: 1.618033988749895... but is usually rounded to 1:1.618.



a+b is to a as a is to b

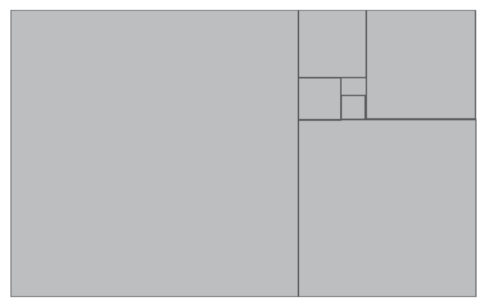
The Golden Rectangle

The golden rectangle is one whose end:side ratio is 1:1.618… . It is constructed by finding the diagonal from the midpoint of the side of a square to the top corner, and rotating it to the horizontal to form a new rectangle. This shape, including the original square, is a golden rectangle.



The Rectangle of the Whirling Squares

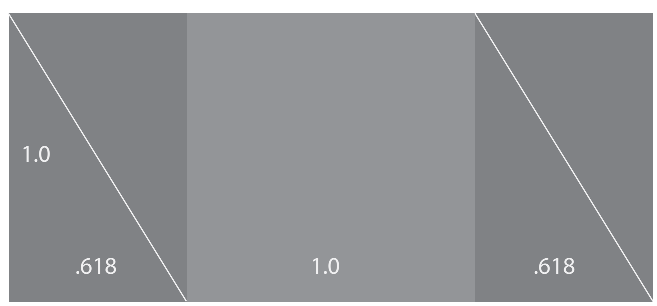
The area added to the original square is also a golden rectangle, rotated 90˚. It can in turn be divided into another square and a golden rectangle, in ever smaller proportions. Because the squares take the form of a spiral, the golden rectangle has also been called the “rectangle of the whirling squares.”



Relation to Root 5 Rectangle

A root 5 rectangle can be broken up into a square and 2 golden rectangles.
The formula for ϕ is:
$1 + \sqrt{5}/2 = 1.618$
ϕ has the property $1/1.618 = 1.618 - 1 = .618$
For a $\sqrt{5}$ rectangle whose end = 1,
its side = $\sqrt{5} = 2.23606797749979$
Minus the square: $2.236 - 1 = 1.236...$
Divided by 2: $2.236/2 = .618$

.618 is the end of a Golden Rectangle with side = 1.

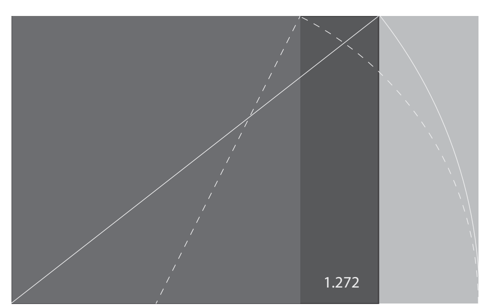


Root Phi Rectangle

The root phi (√ ϕ) rectangle may be constructed by first making a golden rectangle, then rotatating the long side up until it intersects the top. The resulting rectangle’s proportions are 1: √ ϕ or 1:1.272.

This format is popular for portraits and other artwork.

Divided diagonally, the root phi rectangle forms two Kepler triangles with geometrically progressive sides in the ratio 1: √ϕ: ϕ or 1:1.272:1.618.

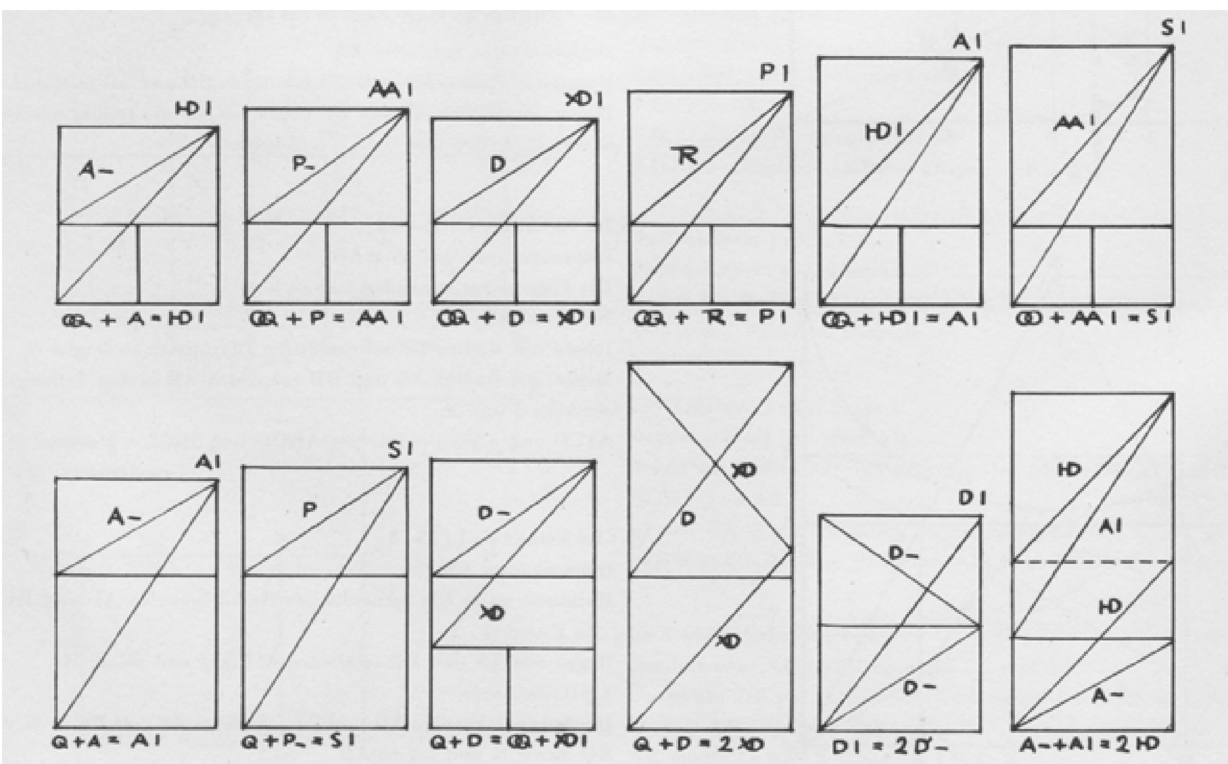


The 12 Orthogons of Wolfgang von Wersin

Wolfgang von Wersin, a 20th century designer, published in 1956 *Das Buch vom Rechteck Gesetz und Gestik des Raumlichen die Othogone-scheibe. Die Orthogone-scheibe (The Book of Rectangles, Spatial Law and Gestures of The Orthogons Described. The Orthogons Described),* which introduced his system of proportional rectangles which he called the "orthogons." The system is based on the "mother square" and claimed inspiration from earlier systems based on the square, such as Durer's *Of the Just Shaping of Letters*. The set overlaps with the root rectangles and the Golden rectangle ("Auron"), but omits the root 5 rectangle, being confined to the set of rectangles that fit between a square and a double square.

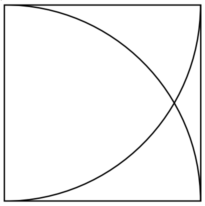
The orthogons appear on Hor:ratio in the space between the square and the root 4 rectangle, identified by Wersin's terms and symbols and their ratios.

The orthogons can be recombined to create other orthogons:



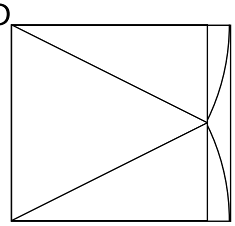
Drawing from *Das Buch vom Rechteck*

Q



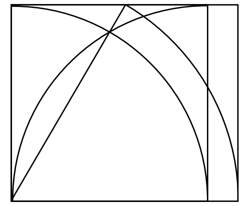
Quadrat 1:1

HD



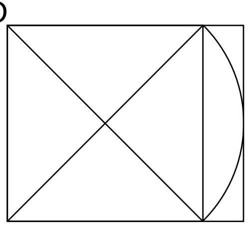
Hemidiagon 1:1.118

TR



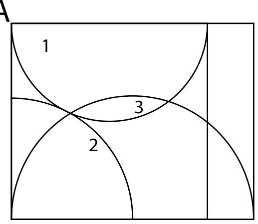
Trion 1:1.154

XD



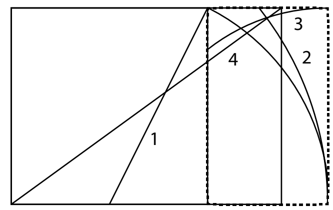
Quadriagon 1:1.207

AA



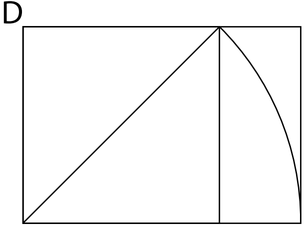
Biauron 1:1.236

P



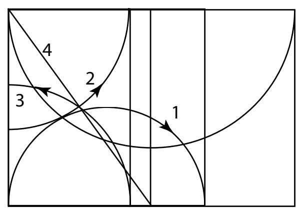
Penton 1:1.376

D



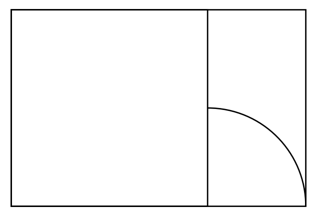
Diagon 1:1.414

dP



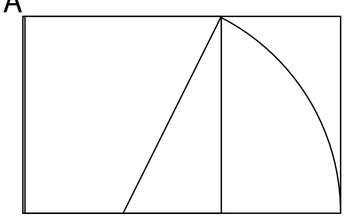
Bipenton 1:1.458

Ø



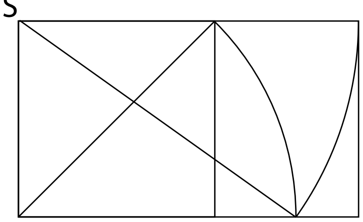
Hemiolion 1:1.5

A



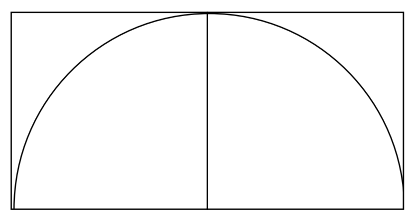
Auron 1:1.618

S



Sixton 1:1.732

QQ



Doppel-quadrat 1:2

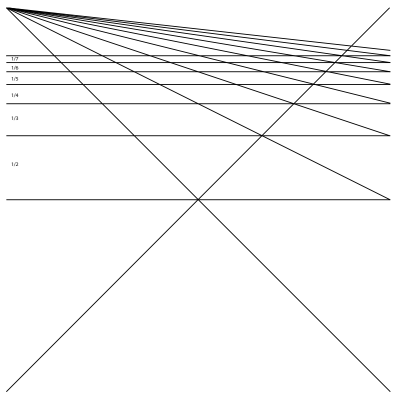
Diagrams based on *Das Buch vom Rechteck*

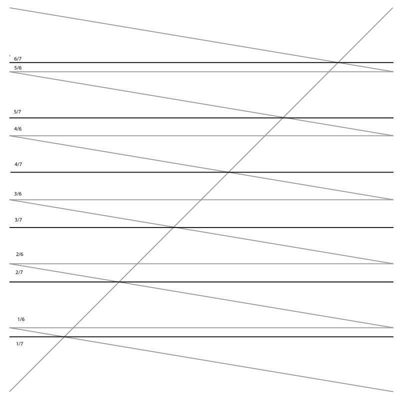
Orthogon Construction

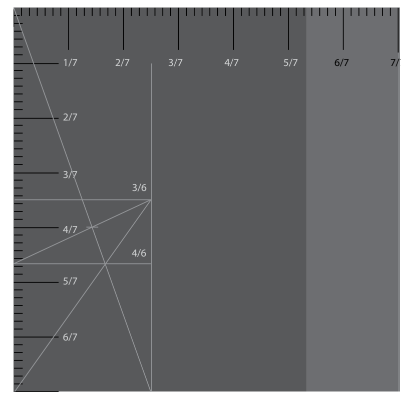
The orthogons are all constructed by starting with a square and using a compass and straightedge to extend or subdivide it. These diagrams assume that the mid-points of the sides have been established. Most are simple operations; a few, like the Biauron, Penton and Bipenton, are more complex. For these the order of operations is indicated.

Wersin's book illustrates many application in design and architecture. His examples tend to be rather rectilinear, but the system of finely tuned ratios could be applied to any artistic design. They offer formats closer to the square, and suggest ways of dividing space that complement and extend the set of root rectangles. The golden rectangle still holds a special place, but now it can be seen to be a member of a family of related proportional shapes.

Seventh Scale
This scale of ratios based on seven is based on commensurable numbers, but can't be generated directly from the armature of the rectangle. However, division by seven, and in theory, any number can be accomplished by means of the Villard diagram, which can in turn be used to extend the armature:







This simplified Villard diagram shows how the rectangle can be divided into successive fractions using diagonals. First, find the center point and draw a line dividing it in half. Draw a diagonal from one end of that line to the opposite corner. The intersection of this line with the main diagonal divides the rectangle by 3, as we already know from the armature. Draw a line through this point and another diagonal to the opposite corner to divide by 4. Repeating the operation yields the 5th, 6th, 7th, and so on to infinity, limited only by the practical problem of drawing infinitesimally fine lines.

However, we don't have to plod from one step to the next. Knowing the secrets of the armature, we can leapfrog to the nearest factor of sixty and proceed stepwise from there. Here, as indicated by the gray lines, we have used the armature to establish the divisions by 6 and drawn the diagonal of each 1/6th segment.The intersections of these diagonals with the main diagonal locate each adjacent 7th division (black lines.) The same could be done with 10ths and 11ths, 12ths and 13ths, etc.

Hor:ratio provides a scale of 7ths and 49ths.

The Villard diagram to find 4/7ths between 3/6ths (1/2) and 4/6ths(2/3rds) is shown.

7 column grids are commonly used in typography and graphic design, and they present some interesting compositional possibilities for artists. Combining odd fractions to build composite grids of, say, 7ths and 13ths might yield some interesting dissonances.

Conclusion
Like perspective, color theory and anatomy, proportional systems can be powerful artistic tools. Like any tool, they can be ineffective or destructive if misapplied or misused. Theories, rules and schematic systems must always be in service to artistic intent. The proportional layout of the rectangle is only one component in a good composition; factors such as tonal contrast, orientation and the shapes themselves are all factors that can potentially outweigh size and position in compositional importance. The real value of learning about these concepts is not that they offer formulas for success, but that they offer new avenues for creative thought. A refined sense of proportion and knowledge of the properties and interrelationships of these families of rectangular ratios will strengthen your compositional, observational and ultimately, your expressive, abilities.

Options and layers
If you open the Hor:ratio.pdf file in the free Adobe Reader program, you can open the Layers view to expose and enable/disable the main layers. By default, Hor:ratio shows all layers except for

Fibonacci: illustrates the connection between the Fibonacci series and the Golden Ratio. The ratio of Fibonacci numbers 5 and 8 roughly approximates ϕ (8/5 = 1.6, phi = 1.618). The approximation becomes closer as we progress in the series, becoming very close by about the 15th step:

1 1 2 3 5 8 13 21 34 55 89 144 233 377 610
610/377 = 1.618037135278515...
ϕ = 1.618033988749895...

and becoming ever closer but never quite equal after that. Marker width is proportional to the difference: ϕ - 8/5 .
All higher Fibonacci ratios will narrow the width of the marker but never erase it .

Inches: Dimensions in inches (3” x 9”, 1/2,1/4,1/8)

Traditional Formats: Defined by manufacturing requirements or customs, not mathematical ratios. 8.5 x 11

You can show these or hide any of the layers and print custom versions of Hor:ratio.

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