Computer Models Define Tide Variability

FEATURE

by Derek Goring

Solving complex forecasting problems

The tide consists of as many as 600 harmonic constituents. Of these components, however, eight are primary tides—four diurnal (once daily) and four semidiurnal (twice daily). Of these eight, just three semidiurnal tides comprise more than 90% of the tidal energy in most places. These three tides are the lunar, solar, and elliptic, the last arising because the moon's orbit is elliptical rather than circular. For most sites, you can forecast the tide to centimeter accuracy in height and a few minutes accuracy in time if you know just the eight primary tide constituents.

We can accurately identify all eight constituents by installing a tide gauge and recording hourly data for 206 days, the time span between the tides of highest range, called perigean spring tides, which occur when a full or new moon coincides with the moon's closest approach to Earth. Once identified for a particular location, hydrographers can use the eight constituents to forecast or hindcast the tide at that site for decades or even centuries. In some places, however, tides along a coast can be quite different just short distances apart. One such place is Cook Strait, between the North and South Islands of New Zealand (Figure 1).

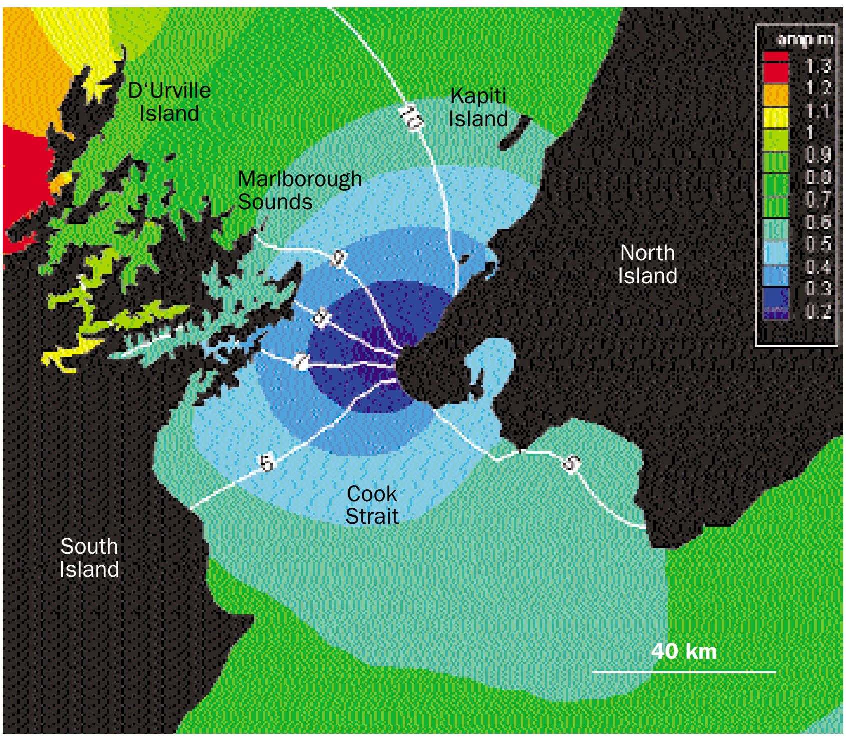

Figure 1. In this cotidal chart of Cook Strait for the primary lunar semidiurnal tide (M_2), colors denote the amplitude and lines denote the time of high tide in hours after midnight on January 1, 1900.

Along the southwestern coast of North Island, the time of high tide varies by 3 h within a few kilometers, although its amplitude is constant and only about 20 cm. However, along the western side of D'Urville Island to the north of South Island, the time of high tide is constant but the amplitude varies by 30 cm. This spatial variability in the time and height of high tide is important to the whole range of marine stakeholders, from fishers to yachtsmen and from marine farmers to hydrographers. Yet deploying tide gauges every few kilometers along the coast is impractical for a country such as New Zealand, which has an 11,000-km coastline but only 3.5 million people.

Applying physics

Using the two most basic laws of physics—conservation of mass and conservation of momentum (Newton's second law)—and a lot of modern technology, we can now solve the problem of defining the spatial variability of tides.

Applying these two laws to fluid flow results in a set of partial differential equations: a continuity equation and the Navier–Stokes equations. This complicated set of equations must be simplified for most practical problems. First, we eliminate turbulence by averaging to give the Reynolds-averaged Navier–Stokes equations. Then we apply a shallow-water assumption—the simplification that the waves under consideration are very long compared to the water depth.

Using this simplification, we can eliminate the vertical dimension by integrating velocities over depth and by assuming that the pressure distribution over depth will be hydrostatic. This results in a two-dimensional, time-varying problem with three equations—a continuity equation and two momentum equations—and three unknowns: water surface displacement (η), and eastward and northward depth-averaged velocities (u and v). Because we are solving for tides, which are harmonic, we can eliminate time by converting all variables into the frequency domain using Fourier transformation and solve for each tide with its unique frequency in turn.

Special care must be taken with the two nonlinear

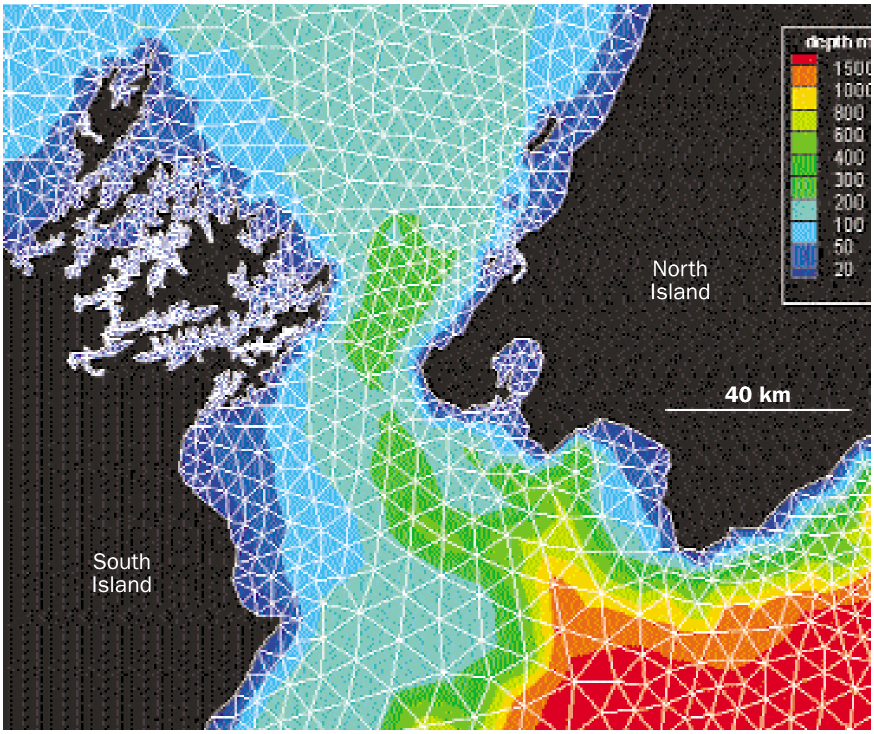

Figure 2. The area of each triangular element of the Cook Strait region is proportional to the average depth under it; long waves take the same time to propagate across all elements.

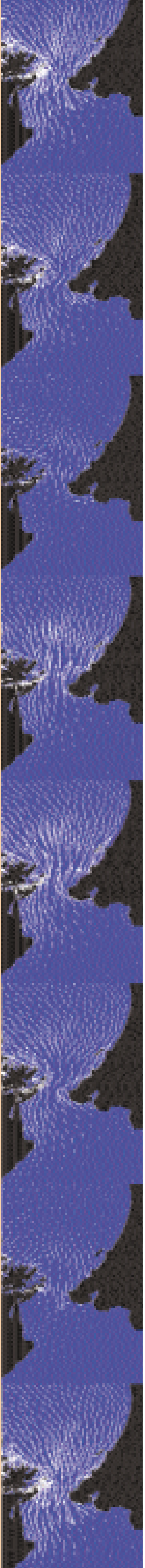

Figure 3. In this animation sequence, velocity vectors are shown for the Cook Strait primary M_2 tide over selected time intervals.

terms (convective accelerations and friction) to properly accommodate the interactions between harmonic components, because such interactions generate harmonics at other frequencies. In addition, for friction, the total velocity for all tides must be used, not just the velocity of the particular tide under consideration. To accommodate these nonlinear effects, we must iterate several times through the tides until the solutions for all tides converge.

Transforming from the time domain to the frequency domain involves substituting each time-series variable with its harmonic equivalent. For example, for the eastward velocity we use $u(t)=Ue^{i(\omega t-\phi)}$, where t is time, U is the amplitude, ϕ is the phase, ω is the angular frequency of the tide under consideration, and $i = \sqrt{-1}$. This makes all the unknown variables complex with the form $u_r + iu_i$, where $u_r = U \cos \phi$ is the real part and $u_i = U \sin \phi$ is the imaginary part.

At one time, we would have had to solve for real and imaginary parts separately, but modern computer languages easily handle complex numbers. The resulting problem consists of three nonlinear, complex partial differential equations in Cartesian coordinates. To accommodate the curvature of the Earth, however, these coordinates must be transformed into spherical polar coordinates.

We discretize in space—that is, convert from continuous variables in partial differential equations to their numerical, or discrete, equivalent—by using a finite-element method. In this approach, we divide the geographical domain of interest into a series of triangles called "elements" (Figure 2) and define the tide amplitude and eastward and northward velocities at each node of each triangle. Then, for each element we evaluate the discretized differential equations and assemble them into three global matrix problems in η, u, and v. We solve these problems separately and iteratively, first solving the continuity equation for η using u and v from the previous iteration. Then we solve the momentum equations for u and v using the newly calculated values for η. We repeat this process until the changes in the unknowns become negligible.

Boundary conditions

To drive the model, we need to apply boundary conditions, which are the values of the unknown variables applied at the boundary of the model domain. For these conditions, we use results from the work of researchers who have analyzed data from the U.S.–French oceanographic satellite TOPEX/Poseidon. The satellite measures the water surface displacement of the open ocean well away from the continents. These data have been converted into tidal constituents at 1° spacing around the globe and are available to researchers.

Unfortunately, the satellite gives unreliable results near land, and so we need a model that uses hydrodynamic equations. Thus, for each of the tides in turn, we apply the amplitude and phase (converted to complex numbers) of the water level displacement from TOPEX/Poseidon at the nodes around the open-ocean boundary of the model. The solution gives us the amplitude and phase for the three variables (η, u, and v) at each of the 32,000 nodes in the model.

We repeat this process for 13 tidal constituents (8 major tides and 5 minor tides). Then, to accommodate the nonlinear interactions between constituents and friction, we solve again for the 13 tides using the previous results

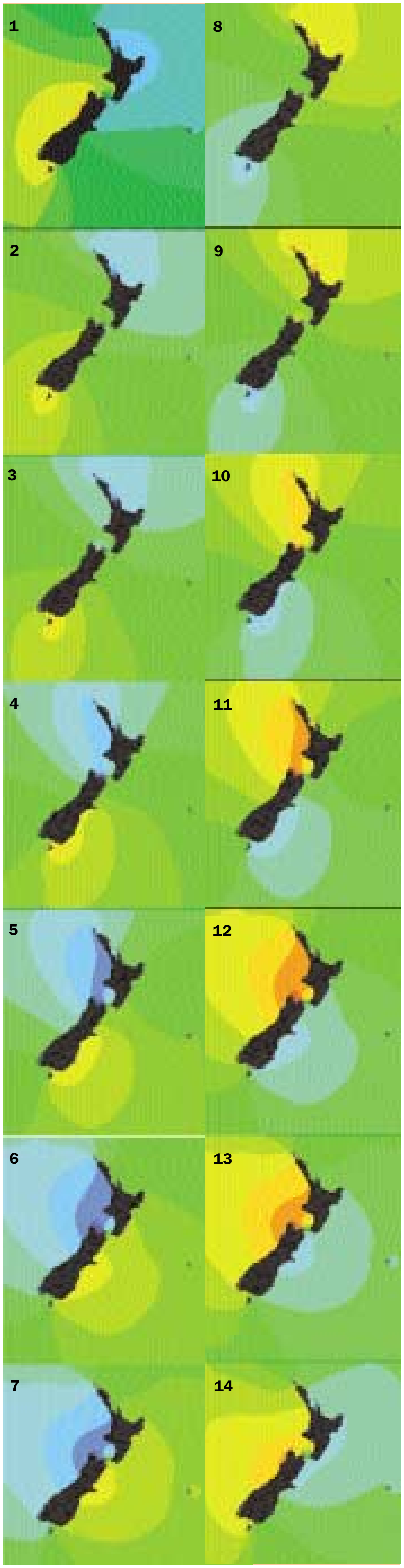

Figure 4. In this animation sequence, the primary M_2 tide amplitude is shown as it rotates counterclockwise around both islands of New Zealand (red is high, blue is low, green is zero elevation).

as starting points. We repeat this iteration until we get convergence for all 3 variables at all 32,000 nodes and for all 13 tides. With modern computers and an efficient matrix solver, this process takes about 12 h on a 600-MHz personal computer.

To check the results, we have installed sea-level recorders at 11 locations around the New Zealand coast (Figure 5a). These highly accurate digital recorders measure sea level every 5 min to millimeter accuracy. The sea-level record contains information not only about tides, but also about storm surges, tsunami, and long-term sea-level changes. The recorders are linked by cellular telephone to our office, and we download the data daily to perform quality assurance checks and archive the data.

Unlike many sea-level recorders worldwide that have been installed to assist in navigation, this network was commissioned for purely scientific purposes. The recorders have been installed on the open coast, well away from ports and harbors. Thus, dredging or silting does not affect them. Many of the recorders are on islands and must be serviced once or twice a year by helicopter. Others are on remote headlands where overland access is difficult, such as Anawhata (Figure 5b), which is also serviced by helicopter.

We analyze the data from these 11 sites (and from another 9 sites run by other organizations) to generate the amplitude and phase of the water-level displacement for the 13 tides used, and we compare the results with those from the model. When we first made this comparison, we got a poor fit for some of the constituents. The reason was that we did not take proper account of the ocean-loading tide, which is the deflection of Earth's crust resulting from the change in the weight of the water above it caused by the tide. In some places around New Zealand, this crustal deflection is ±4 cm over a 12-h period and in others, it is almost zero.

The model also overestimates the amplitude of the tide at some places because we have not defined the sea-floor topography or the shoreline accurately enough. This is particularly true in harbors and estuaries where the tide is locally amplified but the model tends to overestimate this amplification if the grid is not fine enough. In most places on the open coast, the model gets the tides right within ±10 cm in amplitude and ±10° in phase (equivalent to ±20 min for semidiurnal tides). This level of accuracy is adequate for almost all applications of tide forecasting.

The accuracy of the model's tidal currents is much harder to verify. Measuring currents is a difficult task that requires lowering valuable equipment over the side of a ship and coming back several months later to recover it, at which time you hope you can find where you put it and that nobody has taken it. Furthermore, each current meter measures the current at only one position in the depth, but our model gives the average current over the depth. Nevertheless, some records exist, and our model provides results that are reasonable considering the simplifications we have made to the governing equations to make the problem tractable.

The results of the model are displayed as cotidal charts (Figure 1) or they are synthesized into time slices and animated (Figures 3 and 4).

Applications

We are using the model results in a wide range of applications. One is to assist the police in recovering bodies or other objects that have been thrown into the sea. Using the velocities from the model, we can estimate the journey that an object will undergo over a tidal cycle (its excursion from one point and back). We plan to extend this capability to tracking the propagation of oil spills. Another application of the model is siting oil rigs in the open ocean, where an accurate knowledge of the tides greatly assists installation.

We also provide tidal corrections to hydrographers surveying the bathymetry of the open ocean and coastal seas. Marine farming of mussels is a booming industry in New Zealand, and defining the tidal regime for new farms is

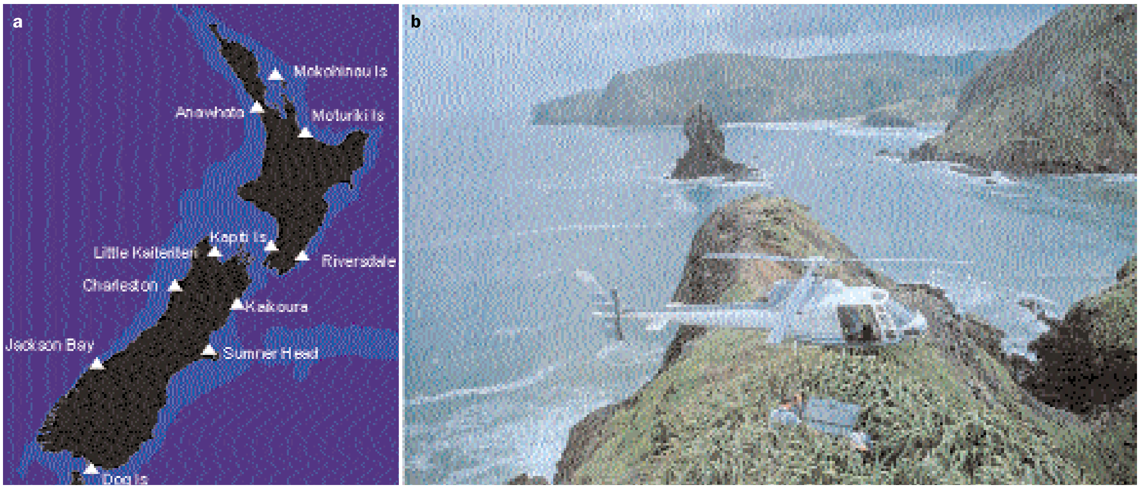

Figure 5. A network of 11 sea-level recorders proivide data used to verify the tide model (a); many are on islands and remote head-lands and must be serviced by heli-copter (b).

important for their design and securing government permits. We also provide a tide-forecasting service for any location in the seas around New Zealand; our scientist colleagues use this service to plan fieldwork and holidays.

Finally, we receive about one inquiry a week from prospective brides planning a wedding on a remote beach and trying to decide on which day and at what time to have it.

Further reading

Cartwright, D. E. *Tides*: *A Scientific History*; Cambridge University Press: Cambridge, England, 1999; 292 pp.

Pugh, D. T. *Tides, Surges and Mean Sea-Level*; John Wiley & Sons: Chichester, England, 1987; 472 pp.

Walters, R. A. A model for tides and currents in the English Channel and North Sea. *Advances in Water Resources* **1987**, *10*, 138–148.

Visit www.amtec.com/contours/issue7/pom.html for animated tidal-model results. Ω

BIOGRAPHY

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