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Predatory Trading

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ABSTRACT

This paper studies *predatory trading*, trading that induces and/or exploits the need of other investors to reduce their positions. We show that if one trader needs to sell, others also sell and subsequently buy back the asset. This leads to price overshooting and a reduced liquidation value for the distressed trader. Hence, the market is illiquid when liquidity is most needed. Further, a trader profits from triggering another trader's crisis, and the crisis can spill over across traders and across markets.

LARGE TRADERS FEAR A FORCED LIQUIDATION, especially if their need to liquidate is known by other traders. For example, hedge funds with (nearing) margin calls may need to liquidate, and this could be known to certain counterparties such as the bank financing the trade. Similarly, traders who use portfolio insurance, stop loss orders, or other risk management strategies can be known to liquidate in response to price drops; a short-seller may need to cover his position if the price increases significantly or if his share is recalled (i.e., a "short squeeze"); certain institutions have an incentive to liquidate bonds that are downgraded or in default; and, intermediaries who take on large derivative positions must hedge them by trading the underlying security. A forced liquidation is often very costly since it is associated with large price impact and low liquidity.

We provide a new framework for studying the strategic interaction among large traders who have market impact. Traders trade continuously and limit their trading intensity to minimize temporary price impact costs. Some of the traders may end up in financial difficulty, and the resulting need to liquidate is known by the other strategic traders.

Our analysis shows that if a distressed large investor is forced to unwind his position (i.e., when he needs liquidity the most), other strategic traders initially trade in the *same* direction. That is, to profit from price swings, other traders

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conduct predatory trading and withdraw liquidity instead of providing it. This predatory activity makes liquidation costly and leads to price overshooting. Moreover, predatory trading can even induce the distressed trader's need to liquidate; hence, predatory trading can enhance the risk of financial crisis. We show that predation is profitable if the market is illiquid and if the distressed trader's position is large relative to the buying capacity of other traders. Further, predation is most fierce if there are few predators.

These findings are in line with anecdotal evidence as summarized in Table I. A well-known example is the alleged trading against Long Term Capital Management's (LTCM's) positions in the fall of 1998. *Business Week* wrote:

> ... if lenders know that a hedge fund needs to sell something quickly, they will sell the same asset—driving the price down even faster. Goldman, Sachs & Co. and other counterparties to LTCM did exactly that in 1998.[1]

Cramer (2002, p. 182) describes hedge funds' predatory intentions in colorful terms:

> When you smell blood in the water, you become a shark .... when you know that one of your number is in trouble ... you try to figure out what he owns and you start shorting those stocks ...

Also, Cai (2002) finds that "locals" on the Chicago Board of Exchange (CBOE) pits exploited knowledge of LTCM's short positions in the treasury bond futures market. Another indication of the fear of predatory trading is evident in the opposition to UBS Warburg's proposal to take over Enron's traders without taking over its trading positions. This proposal was opposed on the grounds that "it would present a 'predatory trading risk' because Enron's traders would effectively know the contents of the trading book."[2] Similarly, many institutional investors are forced by law or their own charter to sell bonds of companies which undergo debt restructuring procedures. Hradsky and Long (1989), for example, documents price overshooting in the bond market after default announcements.

Furthermore, our model shows that an adverse wealth shock to one large trader, coupled with predatory trading, can lead to a price drop that brings other traders into financial difficulty, leading in turn to further predation, and so on. This ripple effect can cause a widespread crisis in the financial sector. Accordingly, the testimony of Alan Greenspan in the U.S. House of Representatives on October 1, 1998 indicates that the Federal Reserve Bank was worried that LTCM's financial difficulties might destabilize the financial system as a whole:

> ... the act of unwinding LTCM's portfolio would not only have a significant distorting impact on market prices but also in the process could produce large losses, or worse, for a number of creditors and counterparties, and for other market participants who were not directly involved with LTCM.[3]

[1] "The Wrong Way to Regulate Hedge Funds," *Business Week*, February 26, 2001, p. 90.

[2] AFX News Limited, AFX—Asia, January 18, 2002.

[3] Testimony of Alan Greenspan, U.S. House of Representatives, October 1, 1998, http://www.federalreserve.gov/boarddocs/testimony/19981001.htm.

Also, the Brady Report (Brady et al. (1988), p. 15) suggests that the 1987 stock market crash was partly due to predatory trading in the spirit of our model:

> . . . This precipitous decline began with several "triggers," which ignited mechanical, price-insensitive selling by a number of institutions following portfolio insurance strategies and a small number of mutual fund groups. The selling by these investors, and the prospect of further selling by them, encouraged a number of aggressive trading-oriented institutions to sell in anticipation of further declines. These aggressive trading-oriented institutions included, in addition to hedge funds, a small number of pension and endowment funds, money management firms and investment banking houses. This selling in turn stimulated further reactive selling by portfolio insurers and mutual funds.

Predation risk affects the optimal risk management strategy for large institutional investors who hold illiquid assets. The optimal risk management strategy should depend on the liquidity of the assets and on the positions and financial standing of other large investors. Indeed, JP Morgan Chase and Deutsche Bank recently developed a "dealer exit stress-test" to assess the risk that a rival is forced to withdraw from the market (Jeffery (2003)). Further, risk managers should consider the risk that fund outflows can lead to predatory trading, resulting in losses that could fuel further outflows, and so on. Hence, the more likely fund outflows are, the more liquid the fund's asset holdings should be. The danger of predatory trading might make it impossible for a fund to raise money in order to temporarily bridge some financial short-falls, since doing so requires that it reveals its financial need. More generally, the possibility of predatory trading is an argument against very strict disclosure policy. In the same spirit, the disclosure guidelines of the IAFE Investor Risk Committee (IRC) (2001) maintain that "large hedge funds need to limit granularity of reporting to protect themselves against predatory trading against the fund's position." Likewise, market makers at the London Stock Exchange prefer to delay the reporting of large transactions since it gives them "a chance to reduce a large exposure, rather than alerting the rest of the market and exposing them to predatory trading tactics from others."[4]

Our model also provides guidance for the valuation of large security positions. We distinguish between three forms of value, with increasing emphasis on the position's liquidity. Specifically, the "paper value" is the current mark-to-market value of a position, the "orderly liquidation value" reflects the revenue one could achieve by secretly liquidating the position, and the "distressed liquidation value" equals the amount which can be raised if one faces predation by other strategic traders, that is, with endogenous market liquidity. We show that under certain conditions, the paper value exceeds the orderly liquidation value, which in turn exceeds the distressed liquidation value. Hence, if a large trader estimates "impact costs" based on normal (orderly) market behavior, then he

[4] *Financial Times*, June 5, 1990, section I, p. 12.

may underestimate his actual cost in case of an acute need to sell because predation makes liquidity time-varying. In particular, predation reduces liquidity when large traders need it the most. Along these lines, Pastor and Stambaugh (2003) and Acharya and Pedersen (2005) find measures of liquidity risk to be priced.

Our work is related to several strands of literature. First, our model provides a natural example of "destabilizing speculation" by showing that although strategic traders stabilize prices most of the time, their predatory behavior can destabilize prices in times of financial crises. Our model thus contributes to an old debate; see Friedman (1953), Hart and Kreps (1986), DeLong et al. (1990), and Abreu and Brunnermeier (2003). Trading based on private information about security fundamentals is studied by Kyle (1985), whereas, in our model, agents trade to profit from their information about the future order flow coming from the distressed traders. Order flow information is also studied by Madrigal (1996), Vayanos (2001), and Cao, Evans, and Lyons (2006), but these papers do not consider the strategic effects of forced liquidation. The notion of predatory trading partially overlaps with that of stock price manipulation, which is investigated by Allen and Gale (1992) among others. One distinctive feature of predatory trading is that the predator derives profit from the price impact of the prey and not from his own price impact. Attari, Mello, and Ruckes (2002) and Pritsker (2003) are close in spirit to our paper. Pritsker (2003) also finds price overshooting in an example with heterogeneous risk-averse traders. Attari et al. (2002) focus, in a two-period model, on a distressed trader's incentive to buy in order to temporarily push up the price when facing a margin constraint, and a competitor's incentive to trade in the opposite direction and to lend to this trader. The systemic risk component of our paper is related to the literature on financial crisis. Bernardo and Welch (2004) provide a simple model of "financial market runs" in which traders join a run out of fear of having to liquidate before the price recovers, and Morris and Shin (2004) study how sales can reinforce sales.

The remainder of the paper is organized as follows. Section I introduces the model. Section II provides a preliminary result which simplifies the analysis. Section III derives the equilibrium and discusses the nature of predatory trading, with both a single and multiple predators. Further, Section III shows how predation can drive an otherwise solvent trader into financial distress and discusses implications for risk management. Section IV studies the valuation of large positions in light of illiquidity caused by predation. Section V considers the buildup of the traders' positions and implications of disclosure requirements. Front-running, circuit breakers, up-tick rule, and contagion are discussed in Section VI. Proofs are relegated to Appendix A. Appendix B provides a generalized model with noisy asset supply.

I. Model

We consider a continuous-time economy with two assets, a riskless bond and a risky asset. For simplicity we normalize the risk-free rate to 0. The risky asset has an aggregate supply of $S > 0$ and a final payoff v at time T, where v is a random variable[5] with an expected value of $\mathrm{E}(v) = \mu$. One can view the risky asset as the payoff associated with an arbitrage strategy consisting of multiple assets. The price of the risky asset at time t is denoted by $p(t)$. The economy has two kinds of agents: large strategic traders (arbitrageurs) and long-term investors. We can think of the strategic traders as hedge funds and proprietary trading desks, and the long-term investors as pension funds and individual investors.

Strategic traders, $i \in \{1, 2, \ldots, I\}$, are risk neutral and seek to maximize their expected profit. Each strategic trader is large, and hence, his trading impacts the equilibrium price. He therefore acts strategically and takes his price impact into account when trading. Each strategic trader i has a given initial endowment, $x^i(0)$, of the risky asset and he can continuously trade the asset by choosing his trading intensity, $a^i(t)$. Hence, at time t his position, $x^i(t)$, in the risky asset is

$$x^i(t) = x^i(0) + \int_0^t a^i(\tau)\, d\tau. \tag{1}$$

We assume that each large strategic trader is restricted to hold

$$x^i(t) \in [-\bar{x}, \bar{x}]. \tag{2}$$

This position limit can be interpreted more broadly as a risk limit or a capital constraint. The specific constraint on asset holdings is not crucial for our results. What is crucial is that strategic traders cannot take unlimited positions, because if they could, they would drive the price to the expected value $p = \mu$, a trivial outcome. To consider the case of limited capital, we assume that $\bar{x}I < S$.

Strategic traders are subject to a risk of financial distress at time t_0. We consider both the case in which an exogenous set of agents is in distress (Section III.A) and the case of endogenous distress (Section III.B). In any case, we denote the set of distressed strategic traders by $\mathcal{I}^d$ and the set of unaffected strategic traders, the "predators," by $\mathcal{I}^p$. Similarly, the number of distressed traders is I^d and the number of predators is I^p. A strategic trader in financial distress must liquidate his position in the risky asset, that is,

$$i \in \mathcal{I}^d \Rightarrow \begin{cases} a^i(t) \leq -\frac{A}{I} & \text{if } x(t) > 0 \quad \text{and} \quad t > t_0 \\ a^i(t) = 0 & \text{if } x(t) = 0 \quad \text{and} \quad t > t_0 \\ a^i(t) \geq \frac{A}{I} & \text{if } x(t) < 0 \quad \text{and} \quad t > t_0 \end{cases} \tag{3}$$

where $A \in \mathbb{R}$ is related to the market structure described below. This statement says that a distressed trader must liquidate his position at least as fast as A/I until he reaches his final position $x^i(T) = 0$. Below we show that this is the

[5] All random variables are defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$.

fastest rate at which an agent can liquidate without risking temporary price impact costs.[6]

The assumption of forced liquidation can be explained by (external or internal) agency problems. Bolton and Scharfstein (1990) show that an optimal financial contract may leave an agent cash constrained even if the agent is subject to predation risk.[7] Also, the need to liquidate can be the result of a company's own risk management policy. We note that our results do not depend qualitatively on the nature of the troubled agents' liquidation strategy, nor do they depend on the assumption that such agents must liquidate their entire position. It suffices that a troubled large trader must reduce his position before time T.

In addition to the strategic traders, the market is populated by long-term investors. The long-term traders are price-takers and have, at each point in time, an aggregate demand of

$$Y(p) = \frac{1}{\lambda}(\mu - p), \tag{4}$$

depending on the current price p. This demand schedule by long-term traders is based on two assumptions. First, it is downward sloping since in order to get long-term traders to hold more of the risky asset, they must be compensated in terms of lower prices. This could be because of risk aversion or because of institutional frictions that make the risky asset less attractive for long-term traders. For instance, long-term traders may be reluctant to buy complicated derivatives such as asset-backed securities. (This institutional friction, of course, is what makes it profitable for strategic traders to enter the market.) A downward sloping demand curve also arises in a price pressure model à la Grossman and Miller (1988) since the competitive but risk-averse market-making sector is only willing to absorb the selling pressure at a lower price. Price pressure implies a temporary price decline, and, similarly, in our model the price decline vanishes at time T. Alternatively, if strategic traders have private information about the fundamental value v, then the long-term traders face an adverse selection problem that naturally leads to a downward sloping demand curve (Kyle (1985)). As in Kyle (1985), λ measures the market liquidity of the risky asset.[8]

The second assumption underlying (4) is that long-term traders' demand depends only on the current price p. That is, they do not attempt to profit from price swings. This behavior by the long-term investors is motivated by

[6] We will see later that, in equilibrium, a troubled trader who must liquidate maximizes his profit by liquidating at this speed. Liquidating fast minimizes the costs of front-running by other traders.

[7] Bolton and Scharfstein (1990) consider predation in product markets, not in financial markets.

[8] While the long-term traders have a downward sloping demand curve, we shall see that the strategic traders' actions tend to flatten the curve, except during crisis periods. Empirically, Shleifer (1986), Chan and Lakonishok (1995), Wurgler and Zhuravskaya (2002), and others document downward sloping demand curves, disputing Scholes (1972) who concludes that the demand curve is almost flat.

an assumption that they do not have sufficient information, skills, or time to predict future price changes.

The trading mechanism works in the following way. The market clearing price $p(t)$ solves $Y(p(t)) + X(t) = S$, where X is the aggregate holding of the risky asset by strategic traders,

$$X(t) = \sum_{i=1}^{I} x^i(t). \tag{5}$$

Market clearing and (4) imply that the price is

$$p(t) = \mu - \lambda(S - X(t)). \tag{6}$$

Hence, while in the "long term" at time T, the price is expected to be μ, in the "medium term" the demand curve is downward sloping as described in (6). Further, in a given instant, that is, "in the very short term," the strategic investors do not have immediate access to the entire demand curve (6). As Longstaff (2001) documents, in the real world one cannot trade infinitely fast in illiquid markets. To capture this phenomenon, we assume that strategic traders can as a whole trade at most $A \in \mathbb{R}$ shares per time unit at the current price $p(t)$. Rather than simply assuming that orders beyond A cannot be executed, we assume that traders suffer temporary impact costs if

$$\left| \sum_i a^i(t) \right| > A. \tag{7}$$

Orders are executed with equal priority in the sense that trader i incurs a cost of

$$G(a^i(t), a^{-i}(t)) := \gamma \max\left\{0, a^i - \bar{a}, \underline{a} - a^i\right\}, \tag{8}$$

where $\bar{a} = \bar{a}(a^{-i}(t))$ and $\underline{a} = \underline{a}(a^{-i}(t))$ are, respectively, the unique solutions to

$$\bar{a} + \sum_{j, j \neq i} \min\{a^j, \bar{a}\} = A, \tag{9}$$

$$\underline{a} + \sum_{j, j \neq i} \max\{a^j, \underline{a}\} = A, \tag{10}$$

and where $a^{-i}(t) := (a^1(t), \ldots, a^{i-1}(t), a^{i+1}(t), \ldots, a^I(t))$. In words, $\bar{a}$ ($\underline{a}$) is the highest intensity with which trader i can buy (sell) without incurring the cost associated with a temporary price impact. Further, G is the product of the per-share cost, γ, multiplied by the number of shares exceeding $\bar{a}$ or $\underline{a}$. We assume for simplicity that the temporary price impact is large, that is, $\gamma \geq \lambda I \bar{x}$.

There are several possible interpretations of this market structure. First, we can think of a limit order book with a finite depth as follows: each instant, long-term traders submit A new buy-limit orders and A new sell-limit orders at the current price level, while old limit orders are canceled. This implies that the depth of the limit order book is always a flow of $A\,dt$. Hence, as long as the strategic traders trade at a total speed lower than A, their orders are absorbed by the limit order book, new limit orders flow in, and the price walks up or down the demand curve (6). Orders that exceed A cannot be executed. More generally, one could assume that such excess orders would hit limit orders far away from the current price, and consequently suffer temporary impact costs in line with our model.[9]

Alternatively, one could interpret the model as an over-the-counter market in which it takes time to find counterparties. In order to trade, strategic traders must make time-consuming phone calls to long-term traders. As each strategic trader goes through his "rolodex"—his list of customer phone numbers ordered by reservation value—they walk along the demand curve.[10] If strategic traders share the same customer base, they face an aggregate speed constraint in line with our model. If traders' customer bases are distinct, the speed constraint is trader-specific.[11]

Importantly, our qualitative results do not depend on the specific assumptions of the model; for example, they also arise in a discrete-time setting. The results rely on: (i) strategic traders have limited capital, that is, $\bar{x} << \infty$, (otherwise, the price is always $\mu = \mathrm{E}(v)$); and (ii) markets are illiquid in the sense that large trades move prices ($\lambda > 0$), and traders avoid trading arbitrarily fast ($A < \infty$). The latter assumption is relaxed in Section VI. B in which all long-term traders participate in a batch auction and orders of any size can be executed immediately.

Strategic trader i's objective is to maximize his expected wealth subject to the constraints described above. His wealth is the final value, $x^i(T)v$, of his stock holdings reduced by the cost, $a^i(t)p(t) + G$, of buying the shares, where G is the temporary impact cost. That is, a strategic trader's objective is

$$\max_{a^i(\cdot)\in\mathcal{A}^i} \mathrm{E}\left(x^i(T)v - \int_0^T [a^i(t)p(t) + G(a^i(t), a^{-i}(t))]\,dt\right), \tag{11}$$

where $\mathcal{A}^i$ is the set of feasible trading processes, that is, the $\{\mathcal{F}_t^i\}$-adapted piecewise-continuous processes that satisfy (2) and (3). The filtration $\{\mathcal{F}_t^i\}$ represents trader i's information. We assume that each strategic trader learns, at time t_0, which traders are in distress. We consider both the case in which the size of any distressed trader's position is disclosed at t_0 and the case in

[9] Our interpretation of the limit order book implicitly assumes that new orders arrive close to the current price, even if some trader hits limit orders far away from the current price. If new orders flow in at the last execution price, then hitting orders far away from the current price becomes even more costly as it permanently moves the price.

[10] If the strategic traders must contact the long-term traders in random order, the model needs to be slightly adjusted, but would qualitatively be the same. Duffie, Gârleanu, and Pedersen (2003a, 2003b) provide a search framework for over-the-counter markets.

[11] Longstaff (2001) assumes that an agent must choose a limited trading intensity, that is, $|a^i(t)| \leq$ constant. Making this assumption separately for each trader would not change our results qualitatively.

which it is not. With no disclosure of positions, the filtration $\{\mathcal{F}_t^i\}$ is generated by $\mathcal{I}^d 1_{(t\geq t_0)}$. With disclosure of positions, the filtration is additionally generated by $x^i(t_0)1_{(t\geq t_0)}$ for $i \in \mathcal{I}^d$.

DEFINITION 1: *An equilibrium is a set of feasible processes* $(a^1, \ldots, a^I)$ *such that, for each* i, $a^i \in \mathcal{A}^i$ *solves* (11)*, taking* $a^{-i} = (a^1, \ldots, a^{i-1}, a^{i+1}, \ldots, a^I)$ *as given.*

If investors could learn from the price, then they could essentially infer other traders' actions since there is no noise in our model. Assuming that the strategic traders can perfectly observe the actions of other strategic traders seems unrealistic and complicates the game. Therefore, in Appendix B, we consider a more general economy with supply uncertainty and show that, even though traders observe prices, they cannot infer other traders' actions. For ease of exposition, we analyze a setting with the same equilibrium actions but which abstracts from supply uncertainty; that is, we simply consider a filtration $\{\mathcal{F}_t^i\}$ that does not include the price and the temporary impact costs. This means that a trader's strategy depends on the current time, whether or not he is in distress, and how many other traders are in distress.[12]

II. Preliminary Analysis

In this section, we show how to solve a trader's problem. For this, we rewrite trader i's problem (11) as a constant (which depends on $x(0)$) plus

$$\mathrm{E}\left(\lambda S x^i(T) - \frac{1}{2}\lambda[x^i(T)]^2 - \int_0^T [\lambda a^i(t) X^{-i}(t) + G(a^i(t), a^{-i}(t))]\, dt\right), \quad (12)$$

where we use $\mathrm{E}(v) = \mu$, expression (6) for the price, equation (1), which implies that $\int_0^T a^i(t)\, x^i(t)\, dt = \frac{1}{2}[x^i(t)^2]_0^T$, and where we define

$$X^{-i}(t) := \sum_{j=1,\ldots,I,\, j\neq i} x^j(t). \quad (13)$$

Under our standing assumptions, $p(t) < \mathrm{E}(v)$ at any time, and hence, any optimal trading strategy satisfies $x^i(T) = \bar{x}$ if trader i is not in distress. That is, the trader ends up with the maximum capital in the arbitrage position. Furthermore, it is not optimal to incur the temporary impact cost, that is, each trader optimally keeps his trading intensity within his bounds $\underset{\sim}{a}$ and $\bar{a}$. These considerations imply that the trader's problem can be reduced to minimizing the third term in (12), which is useful in solving a trader's optimization problem and in deriving the equilibrium.

[12] If one assumes that prices and temporary impact costs are observable, then our equilibrium remains a Nash equilibrium since it is optimal to choose a strategy that is a function of time if everyone else does so. This assumption would, however, raise additional technical issues related to differential games (see, e.g., Clemhout and Wan (1994)). Also, such an assumption might lead to multiple equilibria, for instance, because deviations could be detected and followed by a punishment.

LEMMA 1: *If $T > 2\bar{x}I/A$ and $X^{-i} \geq 0$, trader i's problem can be written as*

$$\min_{a^i(\cdot)\in\mathcal{A}^i} \mathrm{E}\left(\int_0^T a^i(t)X^{-i}(t)\,dt\right) \tag{14}$$

$$s.t. \quad x^i(T) = x^i(0) + \int_0^T a^i(t)\,dt = \bar{x} \quad \text{if } i \in \mathcal{I}^p \tag{15}$$

$$a^i(t) \in [\underline{a}(a^{-i}(t)), \bar{a}(a^{-i}(t))]. \tag{16}$$

Note that a distressed trader $i \in \mathcal{I}^d$ must have $x^i(T) = 0$ in order to have a feasible strategy $a^i \in \mathcal{A}^i$ that satisfies (3). The lemma shows that the trader's problem is to minimize $\int a^i(t)X^{-i}(t)\,dt$, that is, to minimize his trading cost, not taking into account his own price impact. This is because the model is set up such that the trader cannot make or lose money based on the way his own trades affect prices. (For example, λ is assumed to be constant.) Rather, the trader makes money by exploiting the way in which the *other* traders affect prices (through X^{-i}). This distinguishes predatory trading from price manipulation.

III. The Predatory Phase ($t \in [t_0, T]$)

We first consider the "predatory phase," that is, the period $[t_0, T]$ in which some strategic traders face financial distress. In Section V, we analyze the full game including the "investment phase" $[0, t_0)$ in which traders decide the size of their initial (arbitrage) positions. We assume that each strategic large trader has the same position, $x(t_0) \in (0, \bar{x}]$, in the risky asset at time t_0. Furthermore, we assume for simplicity that there is "sufficient" time to trade, that is, $t_0 + 2\bar{x}I/A < T$.

We proceed in two stages: in Section A, certain traders are already in distress and we analyze the behavior of the undistressed predators. Section B endogenizes agents' distress and studies how predation and "panic" can lead to a widespread crisis.

A. Exogenous Distress

Here, we take as given the set of distressed traders, $\mathcal{I}^d$, and the common initial holding, $x(t_0)$, of all strategic traders. A distressed trader j sells, in equilibrium, his shares at constant speed $a^j = -A/I$ from t_0 until $t_0 + x(t_0)I/A$, and thereafter $a^j = 0$. This behavior is optimal, as will be clear later. This liquidation strategy is known, in equilibrium, by all the strategic traders.

The predators' strategies are more interesting. We first consider the simplest case in which there is a single predator, and we subsequently consider the case with multiple competing predators.

A.1. Single Predator ($I^p = 1$)

In the case with a single predator, the strategic interaction is simple: the predator, say i, is merely choosing his optimal trading strategy given the known liquidation strategy of the distressed traders. Specifically, the distressed traders' total position, X^{-i}, is decreasing to 0, and it is constant thereafter. Hence, using Lemma 1, we get the following equilibrium.

PROPOSITION 1: *With $I^p = 1$, the following describes an equilibrium:*[13] *each distressed trader sells with constant speed A/I for $\tau = \frac{x(t_0)}{A/I}$ periods. The predator sells as fast as he can without incurring temporary impact costs for τ time periods, and then buys back for $\bar{x}/A$ periods. That is,*

$$a^{i*}(t) = \begin{cases} -A/I & \text{for } t \in [t_0, t_0 + \tau), \\ A & \text{for } t \in \left[t_0 + \tau, t_0 + \tau + \frac{\bar{x}}{A}\right), \\ 0 & \text{for } t \geq t_0 + \tau + \frac{\bar{x}}{A}. \end{cases} \tag{17}$$

The price overshoots; the price dynamics are

$$p^*(t) = \begin{cases} p(t_0) - \lambda A[t - t_0] & \text{for } t \in [t_0, t_0 + \tau), \\ p(t_0) - \lambda I x(t_0) + \lambda A[t - (t_0 + \tau)] & \text{for } t \in \left[t_0 + \tau, t_0 + \tau + \frac{\bar{x}}{A}\right), \\ \mu + \lambda[\bar{x} - S] & \text{for } t \geq t_0 + \tau + \frac{\bar{x}}{A}, \end{cases} \tag{18}$$

where $p(t_0) = \mu + \lambda(Ix(t_0) - S)$.

We see that although the surviving strategic trader wants to end up with all his capital invested in the arbitrage position ($x^i(T) = \bar{x}$), he is selling as long as the liquidating trader is selling. He is selling to profit from the price swings that occur in the wake of the liquidation. The predatory trader would like to "front-run" the distressed trader by selling before him and buying back shares after the distressed trader has pushed down the price further. Since both traders can sell at the same speed, the equilibrium is that they sell simultaneously and the predator buys back in the end. (The case in which predators can sell earlier than distressed traders is considered in Section VI.A.)

The selling by the predatory trader leads to price "overshooting." The price falls not only because the distressed trader is liquidating, but also because the predatory trader is selling as well. After the distressed trader is done selling, the predatory trader starts buying until he is at his capacity $\bar{x}$, and this pushes the price up toward its new equilibrium level.

The predatory trader profits from the distressed trader's liquidation for two reasons. First, the predator can sell his assets for an average price that is higher than the price at which he can buy them back after the distressed trader has left the market. Second, the predator can buy additional units cheaply until

[13] The predator's profit does not depend on how fast he buys back his shares as long as he does not incur temporary impact costs and he ends fully invested. Hence, there are other equilibria in which the predator buys back at a slower rate. These equilibria are, however, qualitatively the same as the one stated in the proposition, and there are no other equilibria than these.

he reaches his capacity. Since the price of the predator's existing position $x(t_0)$ goes down, the predator may appear to be losing money on a mark-to-market basis as the liquidation takes place. In the real world, holding a position that is loosing money on a mark-to-market basis can be problematic and this could further entice the predator's selling.

The predatory behavior by the surviving agent makes liquidation excessively costly for the distressed agent. To see this, suppose a trader estimates the liquidity in "normal times," that is, when no trader is in distress. The liquidity—as defined by the price sensitivity to demand changes—is given by λ in equation (6). When liquidity is needed by the distressed trader, however, the liquidity is lower due to the fact that the market becomes "one-sided" since the predator is selling as well. Specifically, the price moves by $I\lambda$ for each unit the distressed trader is selling.

The distressed trader's excess liquidation cost equals the predator's profit from preying. Note that the predator does not exploit the group of long-term investors. The price overshooting implies that long-term investors are buying and selling shares at the same price. Hence, it does not matter for the group of long-term investors whether the predator preys or not.[14]

Numerical example. We illustrate this predatory behavior with a numerical example. The supply of the risky asset is $S = 40$ and there are $I = 2$ strategic traders, each of whom has a capacity of $\bar{x} = 10$ shares. At time $t_0 = 5$, each trader has a position of $x(t_0) = 8$ and $I^d = 1$ trader becomes distressed while the other trader acts as a predator. At time $T = 7$ the asset is liquidated with expected value $\mu = 140$ (or, equivalently, the market becomes perfectly liquid). Before that time, the price liquidity factor is $\lambda = 1$ and $A = 20$ shares can be traded per time unit without temporary impact costs.

Figure 1 (Panel A) illustrates the holdings of the distressed trader: this trader starts liquidating his position of 8 shares at time $t_0 = 5$ with a trading intensity of $A/2 = 10$ shares per time unit. He is done liquidating at time 5.8. At time 5, the predator knows that this liquidation will take place, and, further, he realizes that the price will drop in response. Hence, he wants to sell high and buy back low. The predator optimally sells all his 8 shares simultaneously with the distressed trader's liquidation, and, thereafter, he buys back $\bar{x} = 10$ shares as shown in Figure 1 (Panel B).

Figure 2 shows the price dynamics. The price is falling from time 5 to time 5.8, when both strategic traders are selling. Since 16 shares are sold and $\lambda = 1$, the price drops 16 points, falling to 100. As the predator rebuilds his position from time 5.8 to time 6.3, the price recovers to 110. Hence, there is a price overshooting of 10 points.

It is intriguing that the predator is selling even when the price is below its long-run level 110. This behavior is optimal because, as long as the distressed

[14] Recall that even though long-term investors could profit from using a predatory strategy themselves, we assume that they do not have sufficient information or skills to do so.

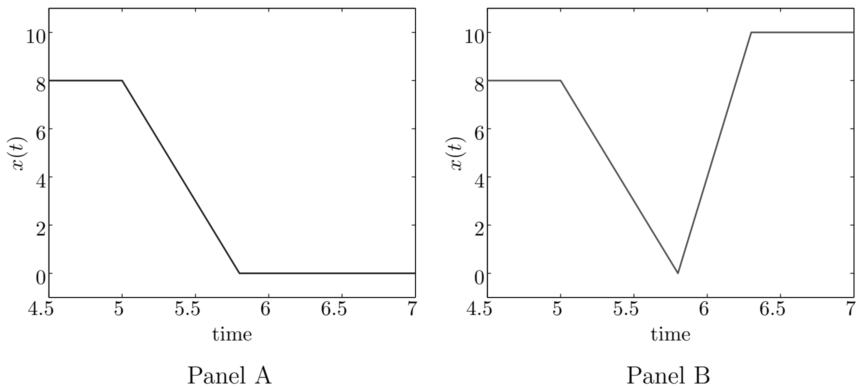

Figure 1. Holdings of distressed trader (Panel A) and of single predator (Panel B). Starting with an initial holding of $x^i(t_0) = 8$, both traders sell at maximum intensity of $A/2 = 10$ from $t_0 = 5$ until $t_0 + \frac{x(t_0)}{A/2} = 5.8$. By then, the distressed trader has completed his liquidation and subsequently the predator buys back shares.

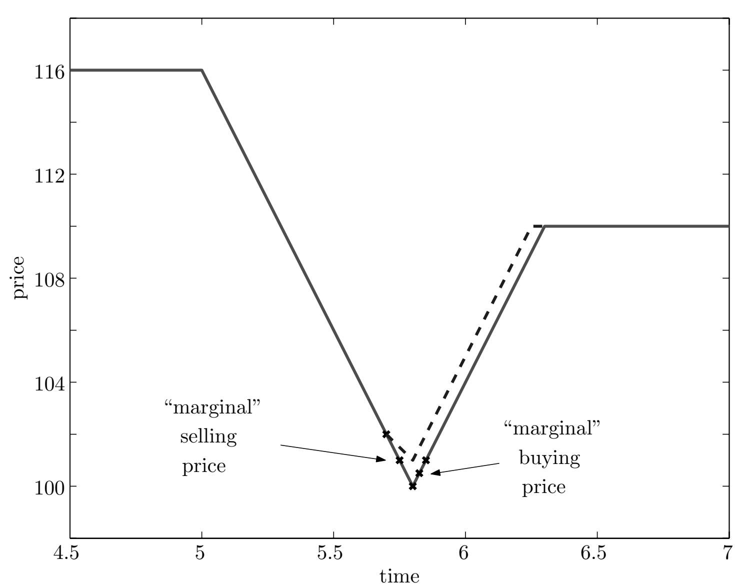

Figure 2. Price dynamics with single predator. The price falls as the distressed trader and the predator sell from time 5 to time 5.8, and rebounds as the predator buys back. The predation leads to price overshooting and a low liquidation value for the distressed trader—the market is illiquid when the distressed trader needs liquidity. The dotted line represents the hypothetical price dynamics if the predator sells one share less, that is, if only the distressed trader sells from time 5.7 to time 5.8. This hypothetical behavior is not optimal since the last "marginal" share can be sold at an average price of 101.00 and then can be bought back cheaper at 100.50.

trader is selling, the price will drop further and the predator can profit from selling additional shares and later repurchasing them. To further explain this point, we consider the predator's profit if he sells one share less. In this case, the predator sells 7 shares from time 5 to time 5.7, waits for the distressed trader to finish selling at time 5.8, and then buys 9 shares from time 5.8 to time 6.25. The price dynamics in this case are illustrated by the dotted line in Figure 2. We see that the 9 shares are bought back at the same prices as the *last* 9 shares were bought in the case in which the predator continues to sell as long as the distressed trader does. Hence, to compare the profit in the two cases, we focus on the price at which the 10th (and last) share is sold and bought back. This share is sold at prices between 102 and 100, that is, at an average price of 101. It is bought back at prices between 100 and 101, that is, at an average price of 100.50. Hence, this "extra" trade is profitable, earning a profit of $101 - 100.50 = 0.50$.

A.2. Multiple Predators ($I^p \geq 2$)

We saw in the previous example how a single predatory trader has an incentive to "front-run" the distressed trader by selling as long as the distressed trader is selling. With multiple surviving traders this incentive remains, but another effect is introduced: these predators want to end up with all their capital in the arbitrage position and they want to buy their shares sooner than the other strategic traders do.

The proposition below shows that, in equilibrium, predators trade off these incentives by selling for a while and then start buying back before the distressed traders have finished their liquidation.

PROPOSITION 2: *In the unique symmetric equilibrium with $I^p \geq 2$ and $x(t_0) \geq \frac{I^p-1}{I-1}\bar{x}$, each distressed trader sells with constant speed A/I for $\frac{x(t_0)}{A/I}$ periods. Each predator sells at trading intensity A/I for $\tau := \frac{x(t_0) - \frac{I^p-1}{I-1}\bar{x}}{A/I}$ periods and buys back shares at a trading intensity of $\frac{AI^d}{I(I^p-1)}$ until $t_0 + \frac{x(t_0)}{A/I}$. That is,*

$$a^{i*}(t) = \begin{cases} -A/I & \textit{for} \quad t \in [t_0, t_0+\tau), \\ \frac{AI^d}{I(I^p-1)} & \textit{for} \quad t \in \left[t_0+\tau, t_0 + \frac{x(t_0)}{A/I}\right), \\ 0 & \textit{for} \quad t \geq t_0 + \frac{x(t_0)}{A/I}. \end{cases} \tag{19}$$

The price overshoots; the price dynamics are

$$p^*(t) = \begin{cases} p(t_0) - \lambda A[t-t_0] & \textit{for} \quad t \in [t_0, t_0+\tau), \\ p(t_0) - \lambda A\tau + \lambda \frac{AI^d}{I(I^p-1)}[t-(t_0+\tau)] & \textit{for} \quad t \in \left[t_0+\tau, t_0+\frac{x(t_0)}{A/I}\right), \\ \mu + \lambda[\bar{x}I^p - S] & \textit{for} \quad t \geq t_0 + \frac{x(t_0)}{A/I}, \end{cases} \tag{20}$$

where $p(t_0) = \mu + \lambda I x(t_0) - \lambda S$.

The proposition shows that price overshooting also occurs in the case of multiple predators if $x(t_0)$ is large relative to $\bar{x}$.[15] This is because the predators strategically sell excessively at first, and start buying relatively late.

It is instructive to consider why it cannot be an equilibrium that there is no price overshooting and predators start buying back already at time $t' < t_0 + \tau$ when the price reaches its long-run level. To see that, suppose all predators start buying back at time t'. Then, if a single predator deviated and postponed buying, the price would continue to fall after t'. Hence, this deviating predator could buy back his position cheaper after other traders have completed their liquidations, and hence, increase his profit.

The equilibrium has the property that, from each predator's perspective, $X^{-i}(t)$ (the total asset holdings of other strategic traders) is declining until $t_0 + \tau$ and is constant thereafter. Since predator i also sells until $t_0 + \tau$, aggregate stock holdings $X(t)$ and the price overshoot.

The price overshooting is lower if there are more predators since more predators behave more competitively:

PROPOSITION 3: *Keep constant the fraction, I^p/I, of predators, the total arbitrage capacity, $I\bar{x}$, and the total initial stock holding, $Ix(t_0)$, and assume that $Ix(t_0) \geq I^p\bar{x}$. Then, the price overshooting*

(i) is strictly positive for all nonzero $I^p < \infty$;
(ii) is decreasing in the number of predators I^p; and,
(iii) approaches zero as I^p approaches infinity.

Numerical example. We consider cases with a total number of traders $I = 3$, 9, and 27. For each case, we assume that a third of the traders are in distress, that is, $I^d/I = 1/3$. As in the previous example, we let $\lambda = 1$, $\mu = 140$, $S = 40$, $t_0 = 5$, $T = 7$, the total trading speed be $A = 20$, the total initial holding be $x(t_0)I = 16$, and the total trader holding capacity be $\bar{x}I = 20$. Figure 3 (Panel A) illustrates the asset holdings of predators and Figure 3 (Panel B) shows the price dynamics.

We see that there is a substantial price overshooting when the number of predators is small, and that the overshooting is decreasing as the number

[15] We assume that all strategic traders' positions at t_0 are the same, that is, $x^i(t_0) = x(t_0)\,\forall i$. The analysis extends to a setting in which strategic traders hold different positions at t_0. In such a setting the equilibrium strategies are described as follows: initially, all predators and distressed sellers sell at full speed $\frac{A}{I}$. Hence, each trader's X^{-i} is declining. When $X^{-i}(t) = X^{-i}(T) = \bar{x}(I^p - 1)$ for the strategic trader with the smallest initial position $x^i(t_0)$, all predators start repurchasing shares at a speed of $\frac{I^d}{I[I^p - 1]}A$. Note that this speed guarantees that each predator's X^{-i} is flat. When the predator with the highest $x(t_0)$ reaches his final holding $\bar{x}$, he stops buying shares and the remaining predators increase their trading intensity to $\frac{I^d}{I[(I^p - 1) - 1]}A$. Similarly, as predators complete their repurchases, the remaining predators adjust their trading speed to $\frac{I^d}{I[I^{\text{remaining}} - 1]}A$. Interestingly, for fixed aggregate holdings of all predators at t_0, the price overshooting increases with the dispersion of the initial holdings. To see this, note that the length of the initial selling spree is determined by the predator with the smallest initial position $x^i(t_0)$, whose $X^{-i}(t_0)$ is the highest.

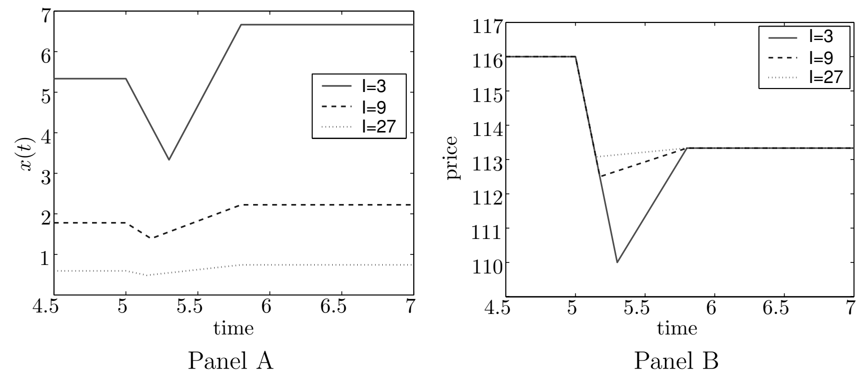

Figure 3. Holdings (Panel A) and price dynamics (Panel B) with multiple predators. The solid line shows each predator's holdings $x^i(t)$ and the price dynamics for the case in which two predators prey on one distressed trader. The dashed line shows holdings and prices when six predators prey on three distressed traders. The dotted lines correspond to the case with 18 predators and 9 distressed traders. As the number of predators increases, the predators start buying back earlier and the price overshooting decreases.

of predators increases. With more predators, the competitive pressure to buy shares early is larger. Hence, the liquidation cost for a distressed trader is decreasing in the number of predators (even holding the total trading capacity fixed).

Collusion. The predators can profit from collusion. In particular, they could increase their revenue from predation by selling until the troubled traders were finished liquidating and only then start rebuilding their positions. Hence, through collusion, the predators could jointly act like a single predator (with the slight modification that multiple predators have more capital). Collusive and noncollusive outcomes are qualitatively different. A collusive outcome is characterized by predators buying shares only after the troubled traders have left the market and by a large price overshooting. In contrast, a noncollusive outcome is characterized by predators buying all the shares they need by the time the troubled traders have finished liquidating and by a relatively smaller price overshooting.

Collusion could potentially occur through an explicit arranged agreement or implicitly without arrangement, called "tacit" collusion. Tacit collusion means that the collusive outcome is the equilibrium in a noncooperative game. In our model, tacit collusion cannot occur. However, if strategic traders could observe (or infer) each others' trading activity, then tacit collusion might arise because predators could "punish" a predator that deviates from the collusive strategy.[16]

[16] If traders could observe each others' trades, then we would have to change our definition of strategies and equilibrium accordingly. A rigorous analysis of such a model is beyond the scope of this paper.

Large amounts of sidelined capacity, $\bar{x} - x(t_0)$. Proposition 2 states that predatory trading and the overshooting occur as long as traders' initial holding is large enough relative to their position limit, that is, $x(t_0) \geq \frac{I^p - 1}{I - 1}\bar{x}$. Proposition 2′ analyzes the complementary case in which $x(t_0) < \frac{I^p - 1}{I - 1}\bar{x}$, that is, the capacity on the sideline is large relative to the selling of the distressed traders. Since the amount of available (sidelined) capacity is large, the competitive pressure among undistressed traders to buy shares overwhelms the incentive to front-run, and therefore there is no predatory trading. Instead, undistressed traders start buying immediately.

PROPOSITION 2′: *In the unique symmetric equilibrium with* $I^p \geq 2$ *and* $x(t_0) < \frac{I^p - 1}{I - 1}\bar{x}$, *each distressed trader sells with constant speed A/I. Each predator buys initially at the high trading intensity of* $\frac{A(I + I^d)}{I^p I}$ *for* $\tau := -\frac{(I-1)x(t_0) - (I^p - 1)\bar{x}}{A(1 - \frac{I + I^d}{I^p I})}$ *periods and goes on buying at the lower trading intensity of* $\frac{AI^d}{I(I^p - 1)}$ *until* $t_0 + \frac{x(t_0)}{A/I}$. *The price is increasing.*

In Section V we study the equilibrium determination of $x(t_0)$ and show that $x(t_0)$ is so large that predatory trading happens with positive probability.

B. Endogenous Distress, Systemic Risk, and Risk Management

So far, we have assumed that certain strategic traders fall into financial distress, without specifying the underlying cause. In this section, we endogenize distress and study how predatory activity can lead to contagious default events. We assume that a trader must liquidate if his wealth drops to a threshold level $\underline{W}$. This is because of margin constraints, risk management, or other considerations in connection with low wealth. Trader *i*'s wealth at *t* consists of his position, $x^i(t)$, of the asset that our analysis focuses on, as well as wealth held in other assets $O^i(t)$. That is, his mark-to-market wealth is $W^i(t) = x^i(t)p(t) + O^i(t)$. The value of the other holdings, $O^i(t)$, is subject to an exogenous shock at time t_0, which can be observed by all traders. At other times, $O^i(t)$ is constant.

Obviously, if the wealth shock ΔO^i at t_0 is so negative that $W^i(t_0) \leq \underline{W}$, the trader is immediately in distress and must liquidate. Smaller negative shocks that result in $W^i(t_0) > \underline{W}$ can, however, also lead to an endogenous distress, since the potential selling behavior of predators and other distressed traders may erode the wealth of trader *i* even further. A trader who knows that he must liquidate in the future finds it optimal to start selling already at time t_0 because he foresees the price decline caused by the selling pressure of other strategic traders. Interestingly, whether an agent anticipates having to liquidate depends on the number of other agents who are expected to be in distress. As in the previous sections, we consider the set $\mathcal{I}^d$ of liquidating traders.

We let $\underline{W}(I^d)$ be the maximum wealth at t_0 such that trader *i* cannot avoid financial distress if I^d traders are *expected* to be in distress. More precisely, for $I^d > 0$, it is the maximum wealth $W^i(t_0)$ such that

$$\max_{a^i \in \mathcal{A}^i} \min_{t \in [t_0, T]} W^i(t, a^i, a^{-i}) \leq \underline{W}, \tag{21}$$

where a^{-i} has $I^d - 1$ strategies of liquidating and $I - I^d$ strategies of preying in a time period of $\tau(I^d)$. Further, for $I^d = 0$, $\underline{W}(0) = \underline{W}$. To understand this definition, suppose trader i expects that $I^d - 1$ other traders will be in distress with resulting selling pressure. Further, he expects that $I - I^d$ other traders will act as predators, preying with a vigor that corresponds to I^d defaults. That is, the predators sell in anticipation of all of the defaults including trader i's own default. If, under these circumstances, trader i will sooner or later be in default no matter what he does, then his wealth is less than $\underline{W}(I^d)$.

With this definition of $\underline{W}(I^d)$, it follows directly that—in an equilibrium[17] in which I^d traders immediately liquidate and $I^p = I - I^d$ traders prey as in Propositions 1, 2, and 2′—every distressed trader $i \in \mathcal{I}^d$ has wealth $W^i(t_0) \leq \underline{W}(I^d)$, and every predator $i \in \mathcal{I}^p$ has wealth $W^i(t_0) > \underline{W}(I^d)$.

Interestingly, the higher the expected number, I^d, of distressed traders, the higher is the "survival hurdle" $\underline{W}(I^d)$.

PROPOSITION 4: *The more traders are expected to be in distress, the harder it is to survive. That is, $\underline{W}(I^d)$ is increasing in I^d.*

This insight follows from two facts: first, even without predatory trading, a higher number of distressed traders leads to more sell-offs and a larger price decline, thereby eroding each trader's wealth. Second, a higher number of distressed traders also makes predation more fierce since there are fewer competing predators and more prey to exploit. This fierce predation lowers the price even further, making survival more difficult.

Proposition 4 is useful in understanding systemic risk. Financial regulators are concerned that the financial difficulty of one or two large traders can drag down many more investors, thereby destabilizing the financial sector. Our framework helps explain why this spillover effect occurs. To see this, consider the economy depicted in Figure 4 (Panel A). Trader A's wealth is in the range of $(\underline{W}(1), \underline{W}(2)]$, trader B's wealth is in $(\underline{W}(2), \underline{W}(3)]$, and trader C's is in $(\underline{W}(3), \underline{W}(4)]$. The three remaining traders (D, E, and F) have enough reserves to fight off any crisis, that is, their wealth is above $\underline{W}(I)$.

With these wealth levels, the unique equilibrium is such that no strategic trader is in distress and all of them immediately start to increase their position from $x(t_0)$ to $\bar{x}$. To see this, note first that it cannot be an equilibrium that one agent defaults. If one agent is expected to default, no one defaults because no one has wealth below $\underline{W}(1)$. Similarly, it is not an equilibrium that two traders default, because only trader A has wealth below $\underline{W}(2)$, and so on.

[17] There may be other kinds of equilibria in which a surviving trader does not prey because of fear of driving himself in distress. For ease of exposition, we do not consider these equilibria. Equilibria of the form that we consider exist under certain conditions on the initial holdings and wealth levels.

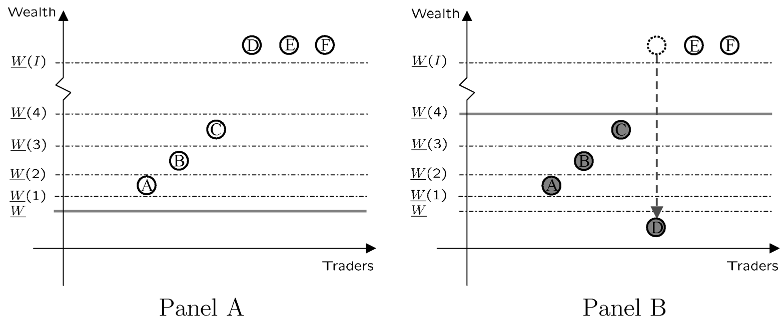

Figure 4. Systemic risk in setting with endogenous distress. This figure shows the wealth levels of traders A, ..., E and several survival hurdles $\underline{W}(I^d)$, that is, the wealth necessary to survive if the market believes that I^d traders will be in distress. In Panel A, traders' wealth levels are high enough that all traders survive in the unique equilibrium. In Panel B, trader D is in distress because of a wealth shock. This leads to predatory trading which can drag traders A, B, and C into distress too.

On the other hand, if trader D faces a wealth shock at t_0 such that $W^D(t_0) < \underline{W}$, he can drag down traders A, B, and C, as shown in Figure 4 (Panel B). If it is expected that four traders will be in distress, then traders A, B, C, and D will liquidate their position since their wealth is below $\underline{W}(4)$. Intuitively, the fact that trader D is forced to liquidate his position encourages predation and the price is depressed. This, in turn, brings three other traders into financial difficulty. This situation captures the notion of systemic risk. The financial difficulty of one trader endangers the financial stability of three other traders.

In the economy of Figure 4 (Panel B), there are also other equilibria in which 1, 2, or 3 traders face distress. For instance, it is an equilibrium that only trader D liquidates, since if everybody expects that only trader D will go under, traders A, B, C, E, and F prey only briefly and buy back after a short while. The predation is less fierce in this equilibrium in the sense that predators start repurchasing shares earlier (i.e., the turning point $t_0 + \tau$ occurs earlier).

In the case of multiple equilibria, interesting coordination issues arise: a widespread crisis can be caused by coordinated selling by predators or by "panic" selling by vulnerable traders. For example, it could be that neither trader E nor trader F alone can cause trader A's distress, but that the joint selling of E and F will push the price sufficiently down to drive A into financial distress.

Alternatively, suppose C expects A and B to be selling along with aggressive trading by predators. Then, C will sell himself, and this panic selling by C will in turn warrant the selling by A and B. Alternatively, if A, B, and C could coordinate on not panicking, then selling is not warranted and the widespread crisis will be avoided.

We note that the multiplicity in our example does not arise when trader E also faces a wealth shock at t_0 such that $W^E(t_0) < \underline{W}(1)$. In this case, at least two traders must liquidate, which drives A into default since A has wealth less than $\underline{W}(2)$. Hence, at least three traders must liquidate, which makes predation yet fiercer and drives B into default. Similarly, this results in C's default, and we see that the "ripple-effect" equilibrium is unique in this case.

The dangers of systemic risk in financial markets provide an argument for intervention by regulatory bodies such as central banks. A bailout of one or two traders or even only a coordination effort can stabilize prices and ensure the survival of numerous other vulnerable traders. However, it also spoils the profit opportunity for the remaining predators who would otherwise benefit from the financial crisis. From an ex ante perspective, the anticipation of crisis-preventive action by the central bank reduces the systemic risk of the financial sector, and hence, traders are more willing to exploit arbitrage opportunities. This reduces initial mispricings, but it could also worsen agency problems not considered here.

In light of our model, the 1987 crash can be viewed as an example of predatory trading enhancing systemic risk. The Brady Report (Brady et al. (1988)) argues that an initial price decline triggered price insensitive selling by institutions that followed portfolio insurance trading strategies. This encouraged aggressive trading-oriented institutions to sell. That is, they preyed on portfolio insurance traders. Less informed long-term traders did not step in to provide liquidity since they underestimated the amount of uninformed trading—portfolio insurance trading and predatory trading—and interpreted it as informed selling. The latter point is emphasized by Grossman (1988) and Gennotte and Leland (1990).

Risk management. The 1987 crash also illustrates the danger of using a rigid risk management strategy that is known to certain other strategic traders. It is preferable to keep the risk management strategy confidential and sufficiently flexible.

The systemic risk in our model implies that risk management should take into consideration other traders' exposures and financial soundness. Indeed, JP Morgan Chase and Deutsche Bank have recently started conducting "dealer exit stress tests" in which a bank estimates "the impact on its own book caused by a rival being forced to withdraw" (Jeffery (2003)).

Further, the less liquid the security (i.e., the higher λ), the larger is the price decline due to predatory trading and the associated wealth deterioration. Formally, this means that $\underline{W}(I^d)$ is increasing in λ. Consequently, a fund with illiquid assets must have a careful risk management strategy.

Also, the risk management strategy should take into account that asset correlations can be different during a liquidity crisis because price movements are caused by distressed selling and predatory trading rather than fundamental news. Suppose, for example, that the risky asset represents a long-short position in securities with identical cash flows. These securities will move together in normal times, but during a liquidity crisis their prices can depart as represented by $p(t)$ declining in our model. Hence, a seemingly perfect hedge based on fundamentals can cause losses during a crisis as the mispricing widens, forcing a trader to liquidate at the least favorable terms. Risk managers should be aware that the past empirical correlation structure might ignore possible predatory trading attacks and separate stress tests are needed to account for predation risk.

A fund's wealth might not only suffer from selling illiquid assets, but also from fund outflows. The risk of fund outflows effectively increases the fund's ultimate survival threshold $\underline{W}$ and makes it even more vulnerable to predatory trading. Hence, open-end funds are more subject to predatory trading than closed-end funds and, consequently, should hold more liquid assets.

Furthermore, traders who hold illiquid assets might be unable to seek outside financing to bridge temporary liquidity needs. This is because the trader may have to reveal his position and trading strategy to possible creditors, such as the trader's brokers, exposing him to predatory trading.

Finally, risk management should take into account the way in which assets are marked-to-market. Suppose, for instance, that a position is financed by collateralized loan by a broker, who can sell the asset if margin requirements are not met. Then, the broker has some discretion in setting the price used to mark the position to market if the market is highly illiquid. Hence, the broker can enhance the trader's problems by marking-to-market aggressively and forcing a fire-sale of the illiquid asset, depressing the price and causing losses for the distressed trader. The broker may have an incentive to do this in order to be able to sell the collateral early.[18] An illustrative example is the case of Granite Partners (Askin Capital Management), who held very illiquid fixed-income securities. Its main brokers—Merrill Lynch, DLJ, and others—gave the fund less than 24 hours to meet a margin call. Merrill Lynch and DLJ then allegedly sold off collateral assets at below market prices at an insider-only auction in which bids were solicited from a restricted number of other brokers excluding retail institutional investors.

Extensions. In our perfect information setting, all traders know how the equilibrium will play out at the instant after t_0. That is, they know the entire future price path as well as the number of predators I^p and victims I^d. In a more complex setting in which traders' wealth shocks are not perfectly observable and the price process is noisy, this need not be the case. A trader might start selling shares not knowing when the price decline stops. He might expect to act as a predator but may actually end up as prey.

Finally, while in our equilibrium all vulnerable traders start liquidating their position from t_0 onwards, one sometimes observes that these traders miss the opportunity to reduce their position early. This exacerbates the predation problem, since a delayed reaction on the part of the distressed traders allows the predators to front-run as discussed in Section VI.A. The phenomenon of delayed reaction by vulnerable traders may be explained in an enriched version of our framework. First, if prices are fluctuating, the trader might "gamble for resurrection" by not selling early, in the hope that a positive price shock will

[18] Futures exchanges can also induce predatory trading by imposing tighter margin constraints.

liberate him from financial distress. Second, if selling activity cannot be kept secret, a desire to appear solvent might prevent a troubled trader from selling early.

IV. Valuation with Endogenous Liquidity

Predatory trading has implications for valuation of large positions. We consider three levels of valuation with increasing emphasis on the position's liquidity:

DEFINITION 2:

(i) The "paper value" of a position x at time t is $V^{\text{paper}}(t, x) = xp(t)$;
(ii) the "orderly liquidation value" is $V^{\text{orderly}}(t, x) = x[p(t) - \frac{1}{2}\lambda x]$; *and,*
(iii) the "distressed liquidation value", $V^{\text{distressed}}(t, x, I^p)$, *is the revenue raised in equilibrium when* I^p *predators are preying.*

The paper value is the simple mark-to-market value of the position. The orderly liquidation value is the revenue raised in a secret liquidation, taking into account the fact that the demand curve is downward sloping. The downward sloping demand curve implies that liquidation makes the price drop by λx, resulting in an average liquidation price of $p(t) - \frac{1}{2}\lambda x$.

The distressed liquidation value takes into account not only the downward sloping demand curve, but also the strategic interaction between traders and, specifically, the costs of predation. We note that $V^{\text{distressed}}$ depends on the characteristics of the market such as the number of predators, the number of troubled traders, and their initial holdings. For instance, the distressed valuation of a position declines if other traders also face financial difficulty.

Clearly, the orderly liquidation value is lower than the paper value. The distressed liquidation value is even lower if the predators have initially large positions.

PROPOSITION 5: *If* $x(t_0) \geq \frac{\sqrt{I^p(I^p-1)}}{I-1}\bar{x}$, *then*

$$V^{\text{paper}}(t_0, x(t_0)) > V^{\text{orderly}}(t_0, x(t_0)) > V^{\text{distressed}}(t_0, x(t_0), I^p).$$

The low distressed liquidation value is a consequence of predation. In particular, predation causes the price to initially drop much faster than what is warranted by the distressed trader's own sales. Hence, the market is endogenously more illiquid when a distressed trader needs liquidity the most.

It is interesting to consider what happens as the number of predators grows, keeping constant their total size. More predators implies that their behavior is more similar to that of a price-taking agent. This more competitive behavior makes predation less fierce, reduces the price overshooting, and increases the distressed liquidation value. As the number of predators grows, the price overshooting disappears (Proposition 3). Importantly, however, even in the limit with infinitely many predators, the distressed liquidation value is strictly lower than the orderly liquidation value. This is because predatory trading makes the price drop faster than without predatory trading, implying that the distressed traders sell most of their shares at the low price.

PROPOSITION 6: *Keep constant the fraction of predators, I^p/I, the total arbitrage capital, $I\bar{x}$, and the total initial holding, $Ix(t_0)$, and suppose that $x(t_0) \geq \bar{x}\sqrt{I^p/I}$. Then, the total distressed liquidation value, $I^d V^{\text{distressed}}$, is increasing in the number of predators, I^p. In the limit as I^p approaches infinity, the total distressed liquidation revenue remains strictly smaller than the total orderly liquidation value,*

$$\lim_{I^p\to\infty} I^d V^{\text{distressed}}(t_0, x(t_0), I^p) < V^{\text{orderly}}\big(t_0, I^d x(t_0)\big).$$

If the predators' initial position $x(t_0)$ is low relative to their capacity $\bar{x}$, then the distressed liquidation value can be greater than the orderly liquidation value. This is because, in this case, the announcement of a distressed liquidation will cause the other traders to compete for the shares and immediately start buying (Proposition 2′). Hence, announcing an intention to sell—called sunshine trading—is profitable if there is enough available capacity on the sideline among relevant investors; otherwise, it will cause predatory trading.

V. The Investment Phase ($t \in [0, t_0]$)

So far, we have taken as given the position, $x(t_0)$, that strategic traders want to acquire prior to t_0. Here, we endogenize $x(t_0)$ and thereby determine the capacity $\bar{x} - x(t_0)$ that traders leave on the sideline to reduce their risk exposure or to be able to exploit cheap buying opportunities that may arise later. We show that the sidelined capacity in equilibrium is so small that predatory trading has to occur with strictly positive probability, and we study how $x(t_0)$ depends on disclosure policies.

For simplicity, we assume that with probability π, a randomly chosen trader is in distress ($I^d = 1$), and with probability $1 - \pi$, no trader is in distress ($I^d = 0$). Note that this implies that the risk of distress is exogenous and independent of the position size. The strategic traders' initial position at time 0—when they learn of the arbitrage opportunity—is assumed to be 0. To separate the investment phase from the predatory phase, we assume that the time, t_0, of possible financial distress is sufficiently late, that is, $t_0 > \frac{\bar{x}}{A/I}$.

Proposition 7 describes the initial trading by large strategic investors.

PROPOSITION 7: *First, all traders buy at the rate A/I until they have accumulated a position of $x(t_0)$. If $I > 2$ and a distressed trader's position is not disclosed, then*

$$x(t_0) = \left(1 - \frac{\pi}{I}\right)\bar{x}. \tag{22}$$

If $I = 2$ or if a distressed trader's position is disclosed, then

$$x(t_0) = \left(1 - \frac{\pi}{I-1}\right)\bar{x}. \tag{23}$$

If a trader is distressed at t_0, then $x(t_0)$ is so large—with or without disclosure—that the remaining strategic traders use the predatory strategies described in Propositions 1 *and* 2*. If no one is in distress at t_0, then all traders buy at the rate A/I until they reach their capacity $\bar{x}$.*

All traders have an initial desire to buy their preferred position $x(t_0)$ without any delay since the acquisitions by other traders increase the price. Importantly, it is this desire of the traders to quickly acquire a large position that later leaves them vulnerable to predation.

The optimal position $x(t_0)$ balances the costs and benefits associated with the three possible outcomes after t_0: (i) no trader faces distress, (ii) another trader faces distress, and (iii) the trader himself faces distress. In case (i) all traders immediately start buying additional shares and the price increases after t_0. In the other two cases, the behavior of the surviving strategic traders depends on the position size $x(t_0)$. For $x(t_0) \geq \frac{I^p-1}{I-1}\bar{x}$, they sell and prey on the distressed trader as described in Propositions 1 and 2, while for $x(t_0) < \frac{I^p-1}{I-1}\bar{x}$, they buy assets and provide liquidity as outlined in Proposition 2′.

Proposition 7 shows that $x(t_0) \geq \frac{I^p-1}{I-1}\bar{x}$, which implies that predatory trading is an inherent part of equilibrium. To see why, suppose to the contrary that $x(t_0)$ is so small that there is always enough available capital to absorb a distressed trader's position as described in Proposition 2′. Then, the price increases after t_0 not only in case (i), but also if a trader is in distress as in (ii) and (iii). Therefore, in all three cases, a trader would profit from having built up a larger position prior to t_0, and this is inconsistent with equilibrium.

In fact, it is a general result, beyond our specific assumptions, that *in any equilibrium, predatory trading occurs with positive probability*. The general argument is that keeping capacity on the sideline has opportunity costs, which must be offset by profits earned during a crisis with capital shortage. Hence, such a "liquidity crisis" must happen with positive probability and predatory trading is profitable during a liquidity crisis.

Further, Proposition 7 determines $x(t_0)$ exactly and shows how it depends on the granularity of disclosure. If agents anticipate that their position will be disclosed when in distress, then they choose smaller initial positions, that is, $(1 - \frac{\pi}{I-1})\bar{x} < (1 - \frac{\pi}{I})\bar{x}$. This is because disclosure makes it more costly to liquidate a larger position because predators will prey more fiercely (i.e., start buying at a later time τ).

The link between disclosure and predation risk is relevant more generally, that is, even if disclosure is not tied directly to the distress event. Enforcing strict disclosure rules concerning a fund's security positions or risk management strategy can increase the fund's exposure to predation risk. This helps explain the secrecy of large hedge funds and why they deal with multiple brokers and banks to reduce the amount of sensitive information known by each counterparty. Consistently, IAFE Invertor Risk Committee (IRC) (2001) emphasizes that disclosure increases predation risk for hedge funds and favors less stringent disclosure rules for large funds. The risk of predation is reduced if the disclosure pertains only to portfolio characteristics and not to specific positions, or if the disclosure is delayed in time.

Also, our analysis suggests that any disclosed information should be dispersed as broadly as possible in order to minimize the implications of predatory trading since, with more strategic traders, predation is less fierce. Also, a public disclosure could be helpful if it could attract liquidity from long-term traders by creating attention and convincing them that a selling pressure was due to distress, not due to adverse information about the security. While this is outside our model, attracting long-term traders might flatten their demand curve (i.e., lower λ).

VI. Further Implications of Predatory Trading

A. Front-Running

So far, we have considered equilibria in which the distressed traders sell at the same time as the predators. Anecdotal evidence suggests that, in some cases, the predators are selling *before* the distressed trader. That is, the predators are truly front-running. There are various potential reasons for the delayed selling by the distressed traders. They might hope that they will face a positive wealth shock that will allow them to overcome the financial difficulty and avoid liquidation costs. Alternatively, the distressed trader may not be aware that the predator—for instance, the trader's own investment bank—is preying on him. Finally, the predators could simply have an ability to trade faster. In any case, front-running makes predation even more profitable.

The equilibrium with a single predator is simple: first, the predator sells as much as possible. Then, he waits for the distressed trader to depress the price by liquidating his position, and finally the predator repurchases his position. Clearly, the price overshoots, and the predation makes liquidation costly.

The equilibrium with many predators can also easily be analyzed within our framework. Suppose that at time t_0 it is clear that I^d traders are in financial distress and that these traders start selling at time t_1, where $t_1 > t_0 + \frac{I^d I^p}{A(I^p - 1)}\bar{x}$.

The predatory trading plays out as follows: first, the predators front-run by selling. This leads to a large price drop. When the distressed traders start selling, the predators start buying back, and the price recovers to its new equilibrium level.

PROPOSITION 8: *In the unique symmetric equilibrium with $I^p \geq 2$ and $x(t_0) \geq \frac{I^p-1}{I-1}\bar{x}$, each distressed trader sells with constant speed A/I for $\frac{x(t_0)}{A/I}$ periods starting at time t_1. Each predator starts selling from t_0 onwards at trading intensity A/I^p for $\tau := \frac{(I-1)x(t_0)-\bar{x}(I^p-1)}{A(I^p-1)/I^p}$ periods and buys back shares at a trading intensity of $\frac{A}{I}\frac{I^d}{I^p-1}$ from t_1 onwards. That is,*

$$a^{i*}(t) = \begin{cases} -A/I^p & \text{for} \quad t \in [t_0, t_0+\tau), \\ 0 & \text{for} \quad t \in [t_0+\tau, t_1), \\ \frac{AI^d}{I(I^p-1)} & \text{for} \quad t \in \left[t_1, t_1+\frac{x(t_0)}{A/I}\right), \\ 0 & \text{for} \quad t \geq t_1+\frac{x(t_0)}{A/I}. \end{cases} \tag{24}$$

The price overshoots; the price dynamics are

$$p^*(t) = \begin{cases} p(t_0) - \lambda A[t-t_0] & \text{for} \quad t \in [t_0, t_0+\tau), \\ p(t_0) - \lambda A\tau & \text{for} \quad t \in [t_0+\tau, t_1), \\ p(t_0) - \lambda A\tau + \lambda\frac{A}{I}\frac{I^d}{I^p-1}[t-t_1] & \text{for} \quad t \in \left[t_1, t_1+\frac{x(t_0)}{A/I}\right), \\ \mu + \lambda[\bar{x}I^p - S] & \text{for} \quad t \geq t_1+\frac{x(t_0)}{A/I}, \end{cases} \tag{25}$$

where $p(t_0) = \mu + \lambda Ix(t_0) - \lambda S$. The ability to front-run by predators implies larger liquidation costs for distressed traders and greater price overshooting.

Changes in the composition of main stock indices force index funds to rebalance their portfolios to minimize their tracking errors. While prior to 1989 changes in the composition of the S&P occurred without prior notice, from 1989 onwards they were announced 1 week in advance. The price dynamics during these intermediate weeks suggest that index tracking funds rebalance their portfolio around the inclusion/deletion date, while strategic traders front-run by trading immediately after the announcement. In particular, Lynch and Mendenhall (1997) document a sharp price rise (drop) on the announcement day, a continued rise (decline) until the actual inclusion (deletion), and a partial price reversal on the days following the inclusion (deletion). Hence, consistent with our model's predictions, there appears to be front-running and price overshooting. If index tracking funds start rebalancing their portfolios at the time of the index inclusion, then our model replicates exactly the documented stylized price pattern. However, high observed trading volume on the day prior to the inclusion (deletion) indicates that many of the index funds trade already prior to this day. If so, our model would predict that the price reversal occurs the day before inclusion (deletion) unless there is a monopolist strategic trader or traders collude. We note that the observed persistence of price overshooting might partially be due to price pressure in the spirit of Grossman and Miller (1988), although simple price pressure would not explain the price adjustment and large trading volume around the announcement (which, in our model, are caused by front-running).

B. Batch Auction Markets, Trading Halts, and Circuit Breakers

In this subsection, we study how certain market practices can alleviate the problem of predatory trading. We consider a setting in which trading is halted, all of the long-term traders are contacted, and all traders—strategic and long-term—can participate in a batch auction. Hence, while shares are traded continuously outside the batch auction, blocks can be sold in the auction. We assume that long-term traders provide a continuum of limit orders, distressed traders submit market orders for their entire holdings, and predators submit market orders to maximize profit. After all orders are collected, they are executed at a single price in the auction, and, thereafter, sequential trading resumes as described previously in the paper. Real-world trading halts and circuit breakers work essentially in this way.

Proposition 9 describes the equilibrium behavior of the predators and the price dynamics for this setting.

PROPOSITION 9: *With $x(t_0) \geq \frac{I^p-1}{I-1}\bar{x}$, each predator submits a buy order of size $\frac{I^p-1}{I^p}[\bar{x}-x(t_0)]$ at the batch auction at t_0. Thereafter, each predator buys at a trading intensity of A/I^p for $[\bar{x}-x(t_0)]/A$ periods. The price dynamics are*

$$p^*(t) = \begin{cases} \mu - \lambda S + \lambda(I^p-1)\bar{x} + \lambda x(t_0) & \text{at the batch auction at } t_0 \\ \mu - \lambda S + \lambda(I^p-1)\bar{x} + \lambda x(t_0) + \lambda A[t-t_0] & \text{for } t \in \left[t_0, t_0 + \frac{\bar{x}-x(t_0)}{A}\right) \\ \mu - \lambda S + \lambda I^p \bar{x} & \text{for } t \geq t_0 + \frac{\bar{x}-x(t_0)}{A}. \end{cases} \tag{26}$$

The price overshooting is smaller compared to the setting without batch auction.

In contrast to the sequential market structure, predators do not sell shares. This is because the batch auction prevents predators from walking down the demand curve. However, predators are still reluctant to provide liquidity as long as competitive forces are weak. To see this, note that a single predator does not participate in the batch auction at all, while in the case of multiple predators each individual predator's order size is limited to $\frac{I^p-1}{I^p}[\bar{x}-x(t_0)]$. This explains why some price overshooting remains. After the batch auction, the surviving predators build up their final position $x(T)=\bar{x}$ as fast as possible in continuous trading. Hence, the price gradually increases until it reaches the same long-run level $p(T) = \mu - \lambda S + \lambda I^p \bar{x}$. In summary, the price overshooting is substantially lower compared to the sequential trading mechanism and the new long-run equilibrium price is reached more quickly.

C. Bear Raids and the Up-tick Rule

A bear raid is a special form of predatory trading, which was not uncommon prior to 1933 according to Eiteman, Dice, and Eiteman (1966).[19] A ring of traders identifies and sells short a stock that other investors hold long on their margin accounts. This depresses the stock's price and triggers margin calls for the long investors, who are then forced to sell their shares, further deflating the price. Based on his (allegedly first-hand) knowledge of these practices, Joe Kennedy, the first head of the Securities and Exchange Commission (SEC), introduced the so-called up-tick rule to prevent bear raids. This rule bans short-sales during a falling market. In the context of our model, this means that strategic traders with small initial positions, $x(t_0)$, cannot undertake predatory trading. This reduces the price overshooting and increases the distressed traders' liquidation revenue.

D. Contagion

Predatory trading suggests a novel mechanism for financial contagion. Suppose that the strategic traders have large positions in several markets. Further, suppose that a large strategic trader incurs a loss in one market, bringing this trader into financial trouble. Then, this large trader must downsize his operations and reduce all his positions. Kyle and Xiong (2001) model this direct contagion result due to a wealth effect. Our model shows that predatory trading by other traders amplifies contagion and the price impact in all affected markets.

This amplification is not driven by a correlation in economic fundamentals or by information spillovers, but rather by the composition of the holdings of large traders who must significantly reduce their positions. This insight has the following empirical implication: a shock to one security, which is held by large vulnerable traders, may be contagious to other securities that are also held by the vulnerable traders.

VII. Conclusion

This paper provides a new framework for studying the phenomenon of predatory trading. Predatory trading is important in connection with large security trades in illiquid markets. We show that predatory trading leads to price overshooting and amplifies a large trader's liquidation cost and default risk. Hence, the risk management strategy of large traders should account for "predation risk." Predatory trading enhances systemic risk, since a financial shock to one trader may spill over and trigger a crisis for the whole financial sector. Consequently, our analysis provides an argument in favor of coordinated actions by regulators or bailouts. Our analysis has further implications for the regulation of securities trading and disclosure rules of large traders, and it explains certain advantages of trading halts, batch auctions, and of the up-tick rule.

[19] The origin of the term "bear" goes back to the 18th century, where it described a trader who sold the bear's skin before he had caught it.

Table I
Examples of Risks Associated with Predatory Trading

Issue	Source	Quotation
Enron, UBS Warburg	AFX News Limited, AFX—Asia, January 18, 2002.	UBS Warburg's proposal to take over Enron's traders without taking over the trading book was opposed on the ground that "it would present a 'predatory trading risk,' as Enron traders effectively know the contents of the trading book."
Disclosure	IAFE Investor Risk Committee (IRC), July 27, 2001, p. 6	For large portfolios, granular disclosure is far from costless and can be ruinous. Large funds need to limit granularity of reporting sufficiently to protect against predatory trading.
Predation, LTCM	*Business Week*, February 26, 2001, p. 90	. . . if lenders know that a hedge fund needs to sell something quickly, they will sell the same asset—driving the price down even faster. Goldman, Sachs & Co. and other counterparties to LTCM did exactly that in 1998. (Goldman admits it was a seller but says it acted honorably and had no confidential information.)
LTCM	*New York Times Magazine*, January 24, 1999, p. 24	Meriwether quoting another LTCM principal: "the hurricane is not more or less likely to hit because more hurricane insurance has been written. In financial markets this is not true . . . because the people who know you have sold the insurance can make it happen."
Systemic risk	Testimony of Alan Greenspan, U.S. House of Representatives, October 1, 1998	It was the judgment of officials at the Federal Reserve Bank of New York, who were monitoring the situation on an ongoing basis, that the act of unwinding LTCM's portfolio in a forced liquidation would not only have a significant distorting impact on market prices but also in the process could produce large losses, or worse, for a number of creditors and counterparties, and for other market participants who were not directly involved with LTCM.
Hedge funds as predators	Cramer (2002), p. 182, 200	"When you smell blood in the water, you become a shark. . . . when you know that one of your number is in trouble . . . you try to figure out what he owns and you start shorting those stocks . . . "; "even though brokers are never supposed to 'give up' their clients' names, there is something about a dying client that sent these brokers to go to the untapped pay phone . . . and tell their buddies . . . "
Credit General	Harvard Business School case 9-296-011 by Andre F. Perold	In June 1995, Credit General bought a yard (billion pounds) of sterling from a client, an unusually large amount. Then, "Credit General immediately moved to execute the trade as planned, attempting to sell the yard sterling for marks. However, the sterling market suddenly seemed to have 'evaporated.' The price of sterling fell rapidly, as did liquidity."

(continued)

Table I—*Continued*

Issue	Source	Quotation
Askin/Granite Hedge Fund collapse 1994	Friedman, Kaplan, Seiler & Adelman LLP http://www.fklaw.com/news-28.html	[During] the period around which the rumors as to the Funds' difficulties were circulating, DLJ quickly repriced the securities resulting in significant margin deficits. . . . The court also cited evidence that Merrill may have improperly diverted profits to itself.
Market making	*Financial Times* (London), June 5, 1990, section I, p. 12.	U.K. market makers wanted to keep the right to delay reporting of large transactions since this would give them "a chance to reduce a large exposure, rather than alerting the rest of the market and exposing them to predatory trading tactics from others."
Crash 1987	Brady Report, p. 15	This precipitous decline began with several "triggers," which ignited mechanical, price-insensitive selling by a number of institutions following portfolio insurance strategies and a small number of mutual fund groups. The selling by these investors, and the prospect of further selling by them, encouraged a number of aggressive trading-oriented institutions to sell in anticipation of further declines. These aggressive trading-oriented institutions included, in addition to hedge funds, a small number of pension and endowment funds, money management firms and investment banking houses. This selling in turn stimulated further reactive selling by portfolio insurers and mutual funds.
Risk management	Jeffery (2003)	JP Morgan Chase and Deutsche Bank use dealer exit stress tests in which a bank can "estimate the impact on its own book caused by a rival being forced to withdraw." "Almost all risk officers and traders agree that dealer exits are perhaps their greatest risk in highly concentrated markets."
Corners and short-squeezes	Jarrow (1992) p. 311	Famous market manipulations, corners, and short-squeezes form an important part of American securities industry folklore. Colorful episodes include the collapse of a gold corner on Black Friday, September 24, 1869, corners on the Northern Pacific Railroad (1901), Stultz Motor Car Company (1920), and the Radio Corporation of America (1928). More recent alleged corners include the soy bean market (1977 and 1989), silver market (1979–1980), tin market (1981–1982 and 1984–1985), and the Treasury bond market (1986). All of these episodes were characterized by extraordinary price increases followed by dramatic collapses . . .

Appendix A: Proofs

Proof of Lemma 1: First, we rewrite the objective function (11) as

$$\mathrm{E}\biggl(\lambda S x^i(T) - \frac{1}{2}\lambda[x^i(T)]^2 + x^i(0)(\mu - \lambda S) + \frac{1}{2}\lambda x^i(0)^2 - \int_0^T [\lambda a^i(t)X^{-i}(t) + G(a^i(t), a^{-i}(t))]\,dt\biggr), \tag{A1}$$

where the terms with $x^i(0)$ do not depend on the trading strategy. Consider a strategy, $a(\cdot)$, with $x^i(T) < \bar{x}$. Since $\bar{a} \geq A/I$ and $T > 2\bar{x}I/A$, there must be an interval $[t', t'']$ such that $a(t) < \bar{a}(t) - \epsilon$ for $t \in [t', t'']$. Consider another strategy $\hat{a}$ which is the same as a except that $\hat{a}(t) = a(t) + \epsilon$ for $t \in [t', t'']$, implying that $\hat{x}^i(T) = x^i(T) + \epsilon(t'' - t')$. Then, for small enough ϵ, the objective function changes by

$$\begin{aligned}\lambda &\left(\epsilon(t'' - t')(S - x^i(T)) - \frac{1}{2}\epsilon^2(t'' - t')^2 - \int_{t'}^{t''} \epsilon X^{-i}(t)\,dt\right) \\ &\geq \lambda\left(\epsilon(t'' - t')(S - x^i(T) - (I - 1)\bar{x}) - \frac{1}{2}\epsilon^2(t'' - t')^2\right) > 0,\end{aligned} \tag{A2}$$

where we use $X^{-i} \leq (I - 1)\bar{x}$. This shows that a is not optimal, and hence, any optimal strategy must have $x^i(T) = \bar{x}$.

Next, consider a strategy with $a(t) \geq \bar{a}(t) + \epsilon$ for $t \in [t', t' + \tau']$. (The case with $a(t) < \underline{a}(t)$ is similar.) Then, the profit can be increased by using another strategy $\hat{a}$ which is the same as a except that $\hat{a}(t) = a(t) - \epsilon$ for $t \in [t', t' + \tau']$ and $\hat{a}(t) = a(t) + \epsilon$ on some other interval $[t'', t'' + \tau']$, where $a(t) \leq \bar{a}(t) - \epsilon$. The change in objective function is

$$\begin{aligned}&\gamma\epsilon\tau' - \lambda\left(\int_{t'}^{t'+\tau'} \epsilon X^{-i}(t)\,dt - \int_{t''}^{t''+\tau'} \epsilon X^{-i}(t)\,dt\right) \\ &\quad\geq \epsilon\tau'\left(\gamma - \lambda(I - 1)\bar{x}\right) > 0,\end{aligned} \tag{A3}$$

using $0 \leq X^{-i} \leq (I - 1)\bar{x}$. This implies that a is not optimal. Q.E.D.

Proof of Proposition 1: The distressed trader's strategy is optimal since any faster liquidation leads to temporary price impact costs.

The surviving trader, i, wants to minimize $\int_{t_0}^T a^i(t)X^{-i}(t)\,dt$ subject to the constraints $x^i(T) = \bar{x}$ and $|a^i(t)| \leq A/I$. Here, $X^{-i}(t)$ is the position of the trader in financial trouble, so

$$X^{-i}(t) = \begin{cases} x(t_0) - tA/I & \text{for} \quad t \in [t_0, t_0 + x(t_0)I/A], \\ 0 & \text{for} \quad t > t_0 + x(t_0)I/A. \end{cases} \tag{A4}$$

Since $X^{-i}(t)$ is decreasing, $\int_{t_0}^T a^i(t)X^{-i}(t)\,dt$ is minimized by choosing the control variable as stated in the proposition. Q.E.D.

Proof of Propositions 2 and 2′: Suppose that $x(t_0) \geq \frac{I^p-1}{I-1}\bar{x}$. A distressed trader's strategy is optimal, given the other traders' actions, since: (i) until time $t_0+\tau$, the price is falling and the distressed trader is selling as fast as he can without incurring temporary impact costs; and, (ii) after time $t_0+\tau$, the price is rising and the distressed trader is selling at the minimal required speed.

To see the optimality of a predator's strategy, suppose, without loss of generality, that trader i is not the trader in financial distress and that all other traders are using the proposed equilibrium strategies. Then, the total position, $X^{-i}(t)$, of the other traders is

$$X^{-i}(t) = \begin{cases} (I-1)\left(x(t_0) - \frac{A}{I}t\right) & \text{for } \; t \in [t_0, t_0+\tau], \\ (I^p-1)\bar{x} & \text{for } \; t > t_0+\tau. \end{cases} \tag{A5}$$

Trader i wants to minimize $\int_{t_0}^{T} a^i(t) X^{-i}(t)\,dt$ subject to the constraints $x^i(T) = \bar{x}$ and $a^i(t) \in [\underline{a}, \bar{a}]$. Since $X^{-i}(t)$ is first decreasing and then constant, $\int_{t_0}^{T} a^i(t) X^{-i}(t)\,dt$ is minimized by choosing $a^i = \underline{a}$ as long as $X^{-i}(t)$ is decreasing and by choosing a positive a^i thereafter. Hence, the a^i that is described in the proposition is optimal. We note that trader i's objective function does not depend on the speed with which he buys back after time τ. There is a single speed, however, which is consistent with the equilibrium.

To prove the uniqueness of this equilibrium, we first note that, in any symmetric equilibrium, $X^{-i}(t)$ must be (weakly) monotonic. To see this, suppose to the contrary that there exists t', t'', and t''' such that $t' < t'' < t'''$ and both $X^{-i}(t') < X^{-i}(t'')$ and $X^{-i}(t'') > X^{-i}(t''')$. Since $X^{-i}(t') < X^{-i}(t'')$ and since the distressed traders are selling, the predators must be buying while $X^{-i}(t)$ is arbitrarily close to $X^{-i}(t'')$ (on a set of nonzero measure). Further, since $X^{-i}(t'') > X^{-i}(t''')$, $a^i(t) < \bar{a}$ for t arbitrarily close to t'''. Hence, trader i can decrease his trading cost (14) by buying less around $X^{-i}(t'')$ and more around $X^{-i}(t''')$, while incurring no temporary impact costs and keeping $x^i(T)$ unchanged. Similar arguments show that there cannot exist $t' < t'' < t'''$ such that $X^{-i}(t') > X^{-i}(t'')$ and $X^{-i}(t'') < X^{-i}(t''')$.

Next, for $x(t_0) \geq \frac{I^p-1}{I-1}\bar{x}$, monotonicity of X^{-i} implies that X^{-i} is nonincreasing since $X^{-i}(0) \geq (I^p-1)\bar{x} = X^{-i}(T)$. It follows directly from Lemma 1 that as long as $X^{-i}(t) > (I^p-1)\bar{x}$ and t is not too large, an optimal strategy satisfies $a^i = \underline{a}$, that is, $a^i = -A/I$. Hence, $\dot{X}^{-i}(t) = -A\frac{I-1}{I}$ until $X^{-i}(t) = (I^p-1)\bar{x}$.

The proof of Proposition 2′ is analogous. In this case, each predator's $X^{-i}(t)$ is increasing until it reaches its final level $(I^p-1)\bar{x}$ at $t_0+\tau$ and is flat thereon, in equilibrium. That is, from $t_0+\tau$ onwards, the sell orders of I^d distressed traders are exactly offset by the buy orders of I^p-1 predators. The price dynamics are

$$p^*(t) = \begin{cases} p(t_0) + \lambda A(t-t_0) & \text{for } \; t \in [t_0, t_0+\tau), \\ p(t_0) + \lambda A\tau + \lambda A\frac{I^d}{I}\left(\frac{I^p}{I(I^p-1)} - 1\right)(t-t_0-\tau) & \text{for } \; t \in \left[t_0+\tau, t_0+\frac{x(t_0)}{A/I}\right), \\ \mu + \lambda(S - I^p\bar{x}) & \text{for } \; t \geq t_0 + \frac{x(t_0)}{A/I}, \end{cases} \tag{A6}$$

where $p(t_0) = \mu - \lambda(S - Ix(t_0))$. Q.E.D.

Proof of Proposition 3: The size of the overshooting, that is, the difference between the lowest price (which is achieved at time $t_0 + \tau$) and the new equilibrium price is $\bar{x} I^d/(I-1)$. This difference decreases toward 0 as the number of agents increases, that is, as $I \to \infty$,

$$\frac{\bar{x} I^d}{I-1} = \frac{(\bar{x} I)(I^d/I)}{I-1} \searrow 0 \tag{A7}$$

since $\bar{x} I$ and I^d/I are constant. Q.E.D.

Proof of Proposition 4: To show that $\underline{W}(I^d)$ is increasing in I^d, we show that the paper wealth, $W^i(t, a^i, a^{-i})$, at any time t is decreasing in I^d. The paper wealth is increasing in the holdings, X^{-i}, of the other traders. With higher I^d, more agents are liquidating their entire holdings, reducing X^{-i}. Further, a higher I^d implies that the remaining predators reverse from selling to buying at a later time τ (defined in Proposition 2), which also reduces X^{-i}. Q.E.D.

Proof of Proposition 5: Clearly, $V^{\text{paper}} > V^{\text{orderly}}$. If there is only one predator, then it follows immediately from Proposition 1 that $V^{\text{orderly}} > V^{\text{distressed}}$. If there are multiple predators, the distressed liquidation value is computed using Proposition 2. After tedious calculations, the result is

$$V^{\text{distressed}} = x(t_0)p(t_0) - \frac{1}{2}\lambda\left(Ix(t_0)^2 - \frac{I^p(I^p-1)}{I-1}\bar{x}^2\right). \tag{A8}$$

It follows that $V^{\text{orderly}} > V^{\text{distressed}}$ under the condition stated in the proposition. Q.E.D.

Proof of Proposition 6: We first note that $x(t_0) \geq \bar{x}\sqrt{I^p/I}$ implies that, for all I, $x(t_0) \geq \frac{I^p-1}{I-1}\bar{x}$. Hence, Proposition 2 applies and we can use (A8) to compute the total distressed liquidation value:

$$I^d V^{\text{distressed}} = I^d x(t_0)p(t_0) - \frac{1}{2}\lambda\left(I^d I x(t_0)^2 - \frac{(I^p-1)}{I-1} I^d I^p \bar{x}^2\right). \tag{A9}$$

Since $Ix(t_0)$, $I^p x(t_0)$, and $I^d x(t_0)$ are assumed independent of I, all the terms in (A9) are independent of I, except the term involving $(I^p - 1)/(I - 1)$. This term is increasing in the number of agents, yielding the first result in the proposition. In the limit as the number of agents increases, the total distressed liquidation value is

$$I^d x(t_0)p(t_0) - \frac{1}{2}\lambda\left(I^d I x(t_0)^2 - \frac{I^p}{I} I^d I^p \bar{x}^2\right). \tag{A10}$$

This value is greater than the orderly liquidation value, $I^d x(t_0)(p(t_0) - \frac{1}{2}\lambda I^d x(t_0))$, under the condition $Ix(t_0) \geq \sqrt{I^p I}\bar{x}$. Q.E.D.

Proof of Proposition 7: We give a sketch of the proof. To see the optimality of trader i's strategy, we first note that for any value of $x^i(t_0)$, it is optimal to use the equilibrium strategy after time t_0. The argument for this follows from the proofs of Propositions 1 and 2. Further, prior to t_0, it is optimal to acquire shares at a rate of $\bar{a}$ until the trader has reached his pre-t_0 target. This follows from the incentive to acquire the position before other traders drive up the price.

The equilibrium level of $x(t_0)$ is derived in the remainder of the proof. We consider trader i's expected profit in connection with buying $x(t_0) + \Delta$ shares, given that other traders buy $x(t_0)$ shares. More precisely, we use Lemma 1 and consider how the marginal Δ shares affect the "trading cost" $\int a^i(t)X^{-i}(t)\,dt$. First, buying Δ infinitesimal extra shares prior to time t_0 costs $\Delta(I-1)x(t_0)$ since the shares are optimally bought when the other traders have finished buying and $X^{-i} = (I-1)x(t_0)$.

The benefit, after t_0, of having bought the Δ shares depends on whether: (i) no trader is in distress; (ii) another trader is in distress; or, (iii) the trader himself is in distress, as illustrated below:

(i) If no trader is in distress, then having the extra Δ shares saves a purchase at the per-share cost of $X^{-i} = (I-1)\bar{x}$. This is because the marginal shares are bought in the end when the other $I-1$ traders each have acquired a position of $\bar{x}$.

(ii) If another trader is in financial distress, then having the extra Δ shares saves a purchase at the per-share cost of $X^{-i} = (I-2)\bar{x}$. This is the total position of the other $I-2$ predators when the defaulting trader has liquidated his entire position.

(iii) (a) Suppose $I > 2$ and the position of the distressed trader is not disclosed at time t_0. Then, if the trader himself is in financial distress, the extra Δ shares can be sold when $X^{-i} = (I-1)\bar{x}$. This is the position of the predators when one has just finished liquidating. At that time, the predators have preyed and repurchased their position. (b) Suppose $I = 2$ or that the position of the distressed trader is disclosed at time t_0. Then, if the trader himself is in financial distress, the extra Δ shares can be sold when $X^{-i} = (I-2)\bar{x}$. To see this, note that the extra shares imply that the predators prey longer (τ larger) because they know that one must liquidate a larger position. Hence, the marginal shares are effectively sold at the worst time, when X^{-i} is at its lowest point.

We can now derive the equilibrium $x(t_0)$ by imposing the requirement that the marginal cost of buying the extra shares equals the marginal benefit. In the case in which $I > 2$ and the position of the distressed trader is not disclosed at time t_0, we have

$$(I-1)x(t_0) = (1-\pi)(I-1)\bar{x} + \pi\frac{I-1}{I}(I-2)\bar{x} + \pi\frac{1}{I}(I-1)\bar{x}, \qquad \text{(A11)}$$

implying that

$$x(t_0) = \left(1 - \frac{\pi}{I}\right)\bar{x}. \qquad \text{(A12)}$$

In the case in which $I = 2$ or the position of the distressed trader is disclosed at time t_0, we have

$$(I-1)x(t_0) = (1-\pi)(I-1)\bar{x} + \pi\frac{I-1}{I}(I-2)\bar{x} + \pi\frac{1}{I}(I-2)\bar{x}, \quad \text{(A13)}$$

implying that

$$x(t_0) = \left(1 - \frac{\pi}{I-1}\right)\bar{x}. \quad \text{(A14)}$$

The global optimality of buying $x(t_0)$ shares is seen as follows. First, buying fewer shares than $x(t_0)$ is not optimal since the infra-marginal shares are bought cheaper (in terms of X^{-i}) than $(I-1)x(t_0)$ and their expected benefits are at least $(I-1)x(t_0)$. Second, buying more shares than $x(t_0)$ costs $(I-1)x(t_0)$ per share, and the expected benefit of these additional shares is at most $(I-1)x(t_0)$.

Finally, we have to show that in both cases, $x_0 > \frac{I^p-1}{I-1}\bar{x}$, which implies that after t_0 predatory trading occurs as described in Proposition 1 or 2. For the case in which the position is disclosed, this follows from $x(t_0) = (1 - \frac{\pi}{I-1})\bar{x} \geq (1 - \frac{1}{I-1})\bar{x} = \frac{I^p-1}{I-1}\bar{x}$. This is also sufficient for the other case since then $x(t_0)$ is larger. Q.E.D.

Proof of Proposition 8: The proof of this proposition follows directly from the insight that from each predator's viewpoint, X^{-i} declines with a constant slope $\frac{I^p-1}{I^p}A$ and stays flat as soon as it reaches its final level $X^{-i}(T) = (I^p - 1)\bar{x}$. That front-running implies more costly liquidation and greater price overshooting follows from: (i) each predator has a smaller position at all times; and, (ii) the lowest total holding of all agents (at time t_1) is strictly lower. Q.E.D.

Proof of Proposition 9: The execution price for trader i's order of u^i shares in the batch auction is $p^a(u^i) = \mu - \lambda S + \lambda(I^p - 1)(x(t_0) + u) + \lambda(x(t_0) + u^i)$ if all defaulting traders submit sell orders of size $x(t_0)$ and all other predators submit individual buy orders of u, where u is to be determined. Since X^{-i} is increasing after the auction, trader i optimally buys the remaining $[\bar{x} - x(t_0) - u^i]$ shares at the highest trading intensity A/I^p. If $u^i \geq u$, these buy orders are executed at an average price of $p^a + \frac{1}{2}\lambda I^p[\bar{x} - x(t_0) - u^i]$. Deriving the total cost and taking the first-order condition w.r.t. u^i (evaluated at $u^i = u$) yields an optimal auction order of $u^i = \frac{I^p-1}{I^p}[\bar{x} - x(t_0)]$. If $u^i < u$, then predator i must buy more shares after the auction than the other predators. The first u shares are bought at an average price of $p^a + \frac{1}{2}\lambda I^p[\bar{x} - x(t_0) - u]$ and the last $u - u^i$ ones are bought at an average price of $p^a + \lambda I^p[\bar{x} - x(t_0) - u] + \frac{1}{2}\lambda(u - u^i)$. Taking the first-order condition w.r.t. u^i evaluated at $u^i = u$, we find the same optimality condition as above: $u^i = u = \frac{I^p-1}{I^p}[\bar{x} - x(t_0)]$.

The equilibrium auction price is $p^a = \mu - \lambda S + \lambda(I^p - 1)\bar{x} + \lambda x(t_0)$. The price overshooting is $\lambda(\bar{x} - x(t_0))$, which is smaller than the price overshooting without batch auction, $\lambda \frac{I^d}{I-1}\bar{x}$ for $I^p \geq 2$, and $\lambda\bar{x}$ for $I^p = 1$ (Propositions 1 and 2, respectively). Q.E.D.

Appendix B: Noisy Asset Supply

To motivate our focus on equilibria in which strategies only depend on time (and potential distress), we consider in this section an economy with observable prices and temporary impact costs and add the realistic assumption that there is noise in the supply of assets. In a pure-strategy equilibrium, unexpected price changes are exclusively attributed to the supply noise, and, therefore, the observability of prices does not alter the model's properties. To demonstrate this, we explicitly introduce supply uncertainty in a way that preserves the benchmark model's qualitative features. Then, we show that our equilibrium is also unaffected by the observability of temporary impact costs. We only sketch the analysis, ignoring certain technical difficulties associated with continuous-time differential games.

We assume that the supply, S_t, is a Brownian motion with volatility σ, that is,

$$dS_t = \sigma\, dW_t, \tag{B1}$$

where W is a standard Brownian motion. Agent i maximizes his expected wealth:

$$\max_{a(\cdot)\in\mathcal{A}} \mathrm{E}\left(-\int_0^T a^i(t)p(t)\,dt + x^i(T)v \right), \tag{B2}$$

where $\mathcal{A}$ is the set of $\{\mathcal{F}_t\}$-adapted processes[20] and $\{\mathcal{F}_t\}$ is generated by the price process $\{p_t\}$, the distress indicator $I^p 1_{(t\geq t_0)}$, and $G(a^i(t), a^{-i}(t))$. The price is defined as before by $p(t) = \mu - \lambda(S_t - X(t))$, where $S_0 > \bar{x}I$. We use the definition $\bar{p}(t) = \mu - \lambda(S_0 - X(t))$. With this definition, the agent's objective function can be written as

$$\begin{aligned} &\mathrm{E}\left(x^i(T)v - \int_0^T [a^i(t)p(t) + G(a^i(t), a^{-i}(t))]\,dt \right) \\ &\quad = \mathrm{E}\left(x^i(T)v - \int_0^T [a^i(t)\bar{p}(t) + G(a^i(t), a^{-i}(t))]\,dt \right) + \mathrm{E}\int_0^T a^i(t)[\bar{p}(t) - p(t)]\,dt. \end{aligned} \tag{B3}$$

The first term on the right-hand side is the same as the objective function with a constant supply of S_0. Hence, this term is maximized by the equilibrium strategy if all other agents use the equilibrium strategy. The second term is, as

[20] We ignore that one may need to restrict the set of feasible strategies, for example, to functions of time and current state variables, to have well-defined outcomes for continuous-time differential games.

we show next, 0 under an additional assumption. For any $\{\mathcal{F}_t\}$-adapted process, a, it holds that

$$\begin{aligned}
&\mathrm{E}\int_0^T a^i(t)[\bar{p}(t) - p(t)]\,dt \\
&\quad = \lambda \mathrm{E}\int_0^T a^i(t)[S_0 - S_t]\,dt \\
&\quad = \lambda \mathrm{E}\int_0^T a^i(t)[S_T - S_t]\,dt - \lambda \mathrm{E}\left(\int_0^T a^i(t)\,dt[S_T - S_0]\right) \\
&\quad = \lambda \mathrm{E}\int_0^T a^i(t)E_t[S_T - S_t]\,dt - \lambda \mathrm{E}\left(\int_0^T a^i(t)\,dt[S_T - S_0]\right) \\
&\quad = -\lambda \mathrm{E}\left(\int_0^T a^i(t)\,dt[S_T - S_0]\right).
\end{aligned} \tag{B4}$$

If we assume that the agent must choose a strategy with $\bar{x} = x_T = x_0 + \int_0^T a^i(t)\,dt$, then the last term is 0 since $\int_0^T a^i(t)\,dt = \bar{x} - x_0$ is constant and $\mathrm{E}(S_T - S_0) = 0$. This assumption means that the agent must end up fully invested in the asset. We note that the agent would optimally choose $x_T = \bar{x}$ as long as the supply is not too small. We further note that the supply is close to S_0 with large probability if σ is small.

Even if we do not impose the additional assumption that $x_T = \bar{x}$, we can show that the equilibrium in the model without supply uncertainty is an ε-equilibrium in the model with noisy supply. (See Radner (1980) for a discussion of ε-equilibria.) This property of the strategies follows from the fact that the latter term can be bounded as follows:

$$\begin{aligned}
\left|\mathrm{E}\left(\int_0^T a^i(t)\,dt[S_T - S_0]\right)\right| &\leq \mathrm{E}\left(\int_0^T |a^i(t)|\,dt|S_T - S_0|\right) \\
&\leq \mathrm{E}\left(\int_0^T A\,dt|S_T - S_0|\right) \\
&\leq AT\,\mathrm{E}|S_T - S_0| \\
&= AT\sigma \mathrm{E}|W_T - W_0| \\
&\to 0 \quad \text{as} \quad \sigma \to 0.
\end{aligned} \tag{B5}$$

Hence, agent *i*'s maximal gain from deviating from the strategy of the non-noisy game approaches 0 as the supply uncertainty vanishes ($\sigma \to 0$).

Finally, if traders observe their own temporary impact costs, they could in principle learn about other traders' actions. In our equilibrium, however, no trader can alter any other trader's temporary impact cost. To see this, first note that when traders are selling, each trader sells at an intensity of A/I, and no one incurs temporary impact costs. If one trader would deviate and sell faster, then he would incur a temporary impact cost, but no one else would incur this cost. (This follows from the definition of the impact cost function $G(a^i, a^{-i})$.) The same is true when traders are buying back. This means that no trader has an incentive to deviate: a deviation would not be detected and, therefore, the other traders would continue to trade the same way as otherwise, which would make the deviation unprofitable.

In conclusion, this section shows that our equilibrium can be seen as a perfect equilibrium in a setting in which prices and one's own temporary impacts costs are observable. This is because agents cannot learn from prices when supply is noisy, and because they cannot learn from temporary impact costs in our equilibrium. (They could only learn from temporary impact costs if two or more agents were to deviate simultaneously, but when contemplating to deviate, each agent assigns zero probability that other agents deviate at the same time.) Q.E.D.

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