Introduction to Integrability in AdS/CFT

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Plan

- Lecture 1. Introduction and overview
- Lecture 2. Classical string solutions
- Lecture 3. S-matrix
- Lecture 4. Finite-size effects

Ref: N. Beisert et.al. “Review of AdS/CFT Integrability” arXiv:1012.3982-4005

Lecture 1. Introduction and Overview

AdS / CFT duality

- Type IIB superstrings on $AdS_5 \times S^5$

dual to

$\mathcal{N} = 4 \quad SU(N_c)$ super-Yang-Mills theory

[Maldacena (1997)]

AdS / CFT duality

- Parameter relations:

$$\boxed{g_s = \frac{4\pi\lambda}{N_c} \quad \& \quad \frac{R^2}{\alpha'} = \sqrt{\lambda}}$$

‘t Hooft coupling constant $\lambda = N_c g_{\mathrm{YM}}^2$

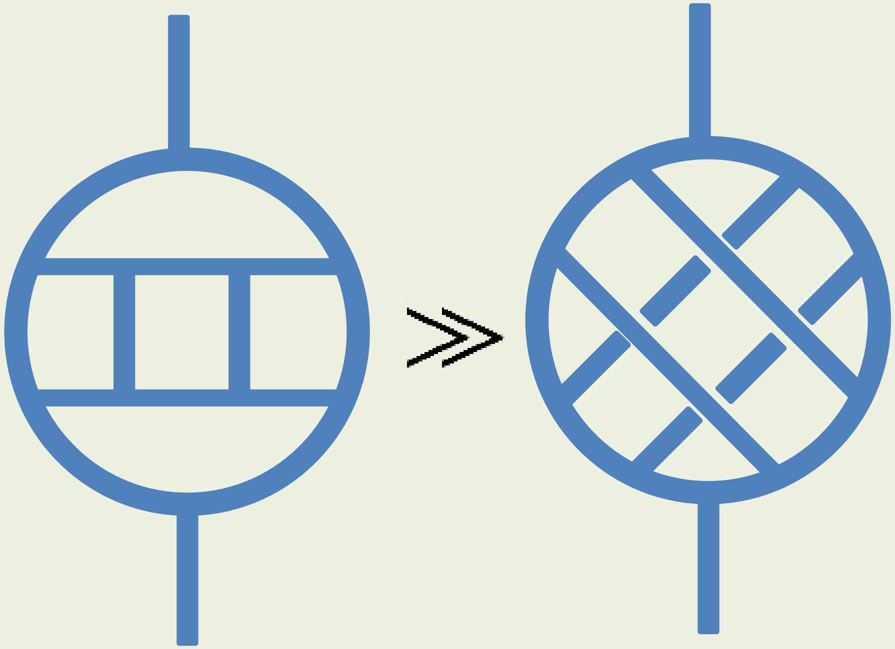
- Free superstring theory corresponds to a planar limit of SYM

$$g_s \to 0 \;\equiv\; N_c \to \infty \text{ with fixed } \lambda$$

- Quantitative check is tricky since it is a strong-weak duality
  - SYM perturbation for $\lambda \ll 1$
  - String perturbation for $\alpha' \ll 1 \quad \Rightarrow \quad \lambda \gg 1$

Planar Limit

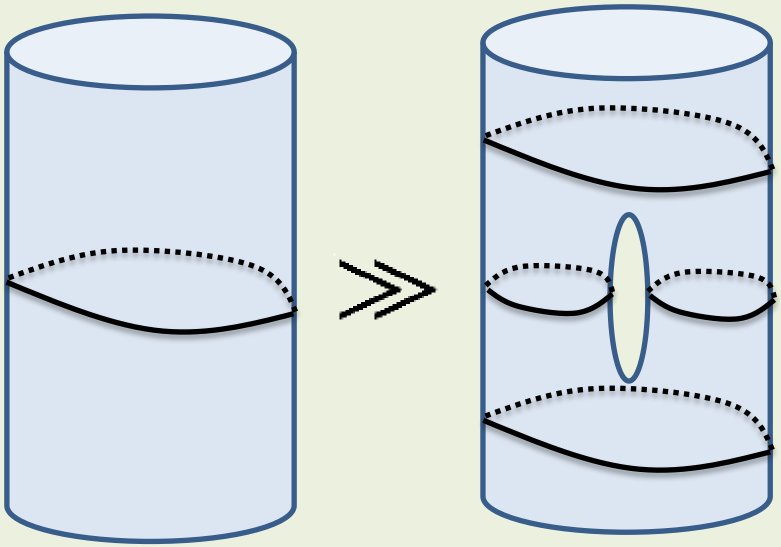
$$N_c \to \infty$$



$$g_{\text{YM}}^{10} N_c^5 = \lambda^5$$

$$g_{\text{YM}}^{10} N_c^3 = \frac{\lambda^5}{N_c^2}$$

$$g_s \to 0$$



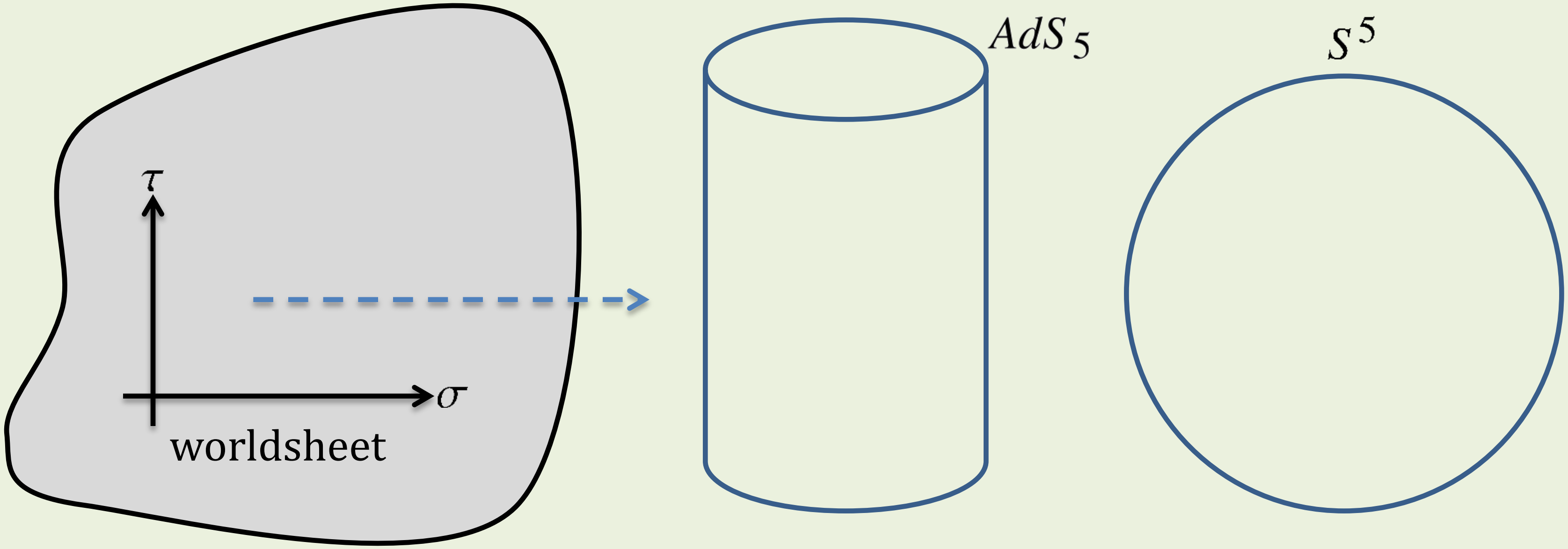
$$O(g_s^2)$$

SYM Operator vs. string configuration

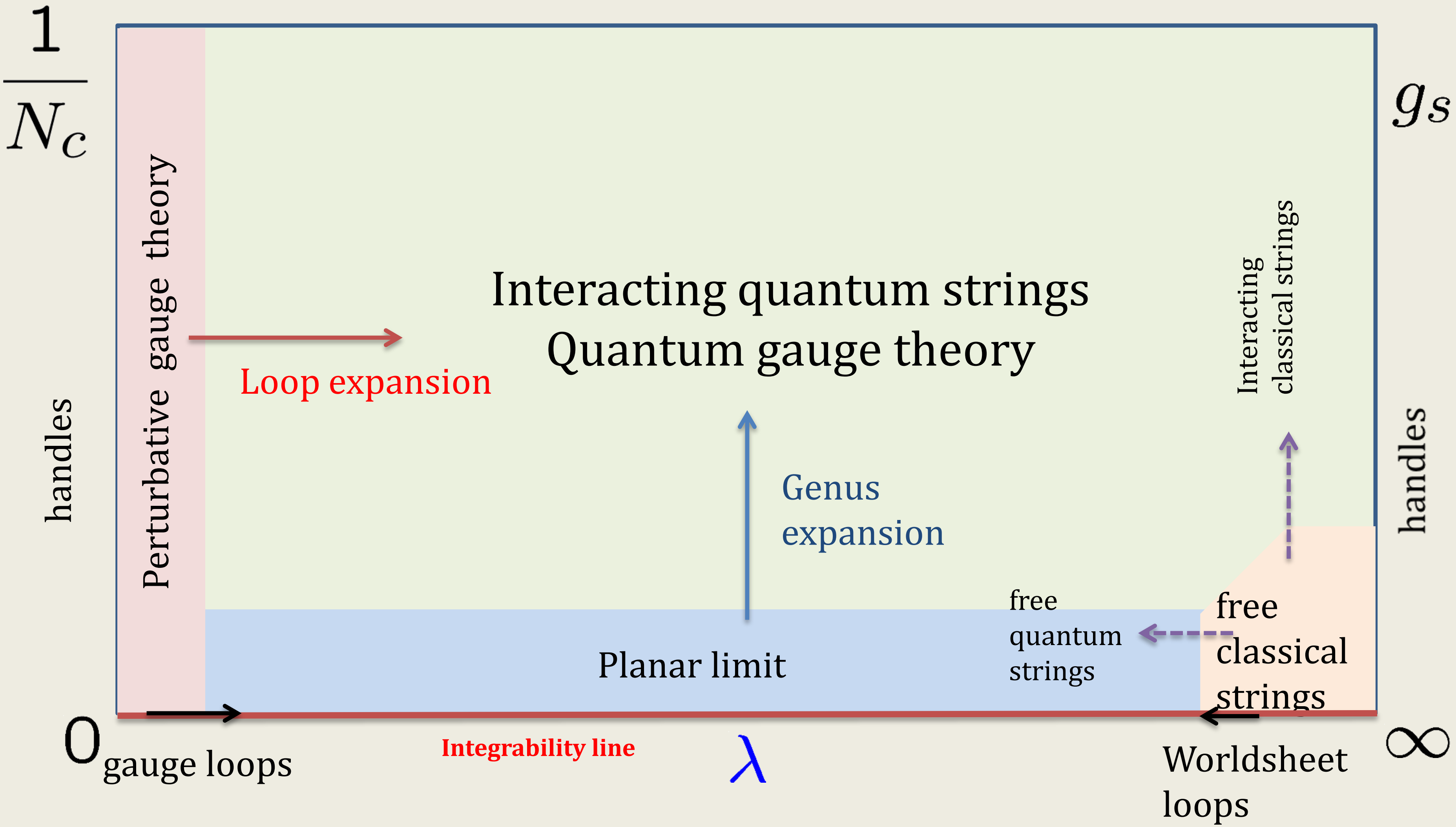
- Composite SYM operator

$$\mathcal{O}(x) = \mathrm{Tr}\left[XYZF_{\mu\nu}\chi^{\alpha}(D_{\mu}Y)\ldots\right]$$

- String configuration in a target space



Parameter space



Integrability

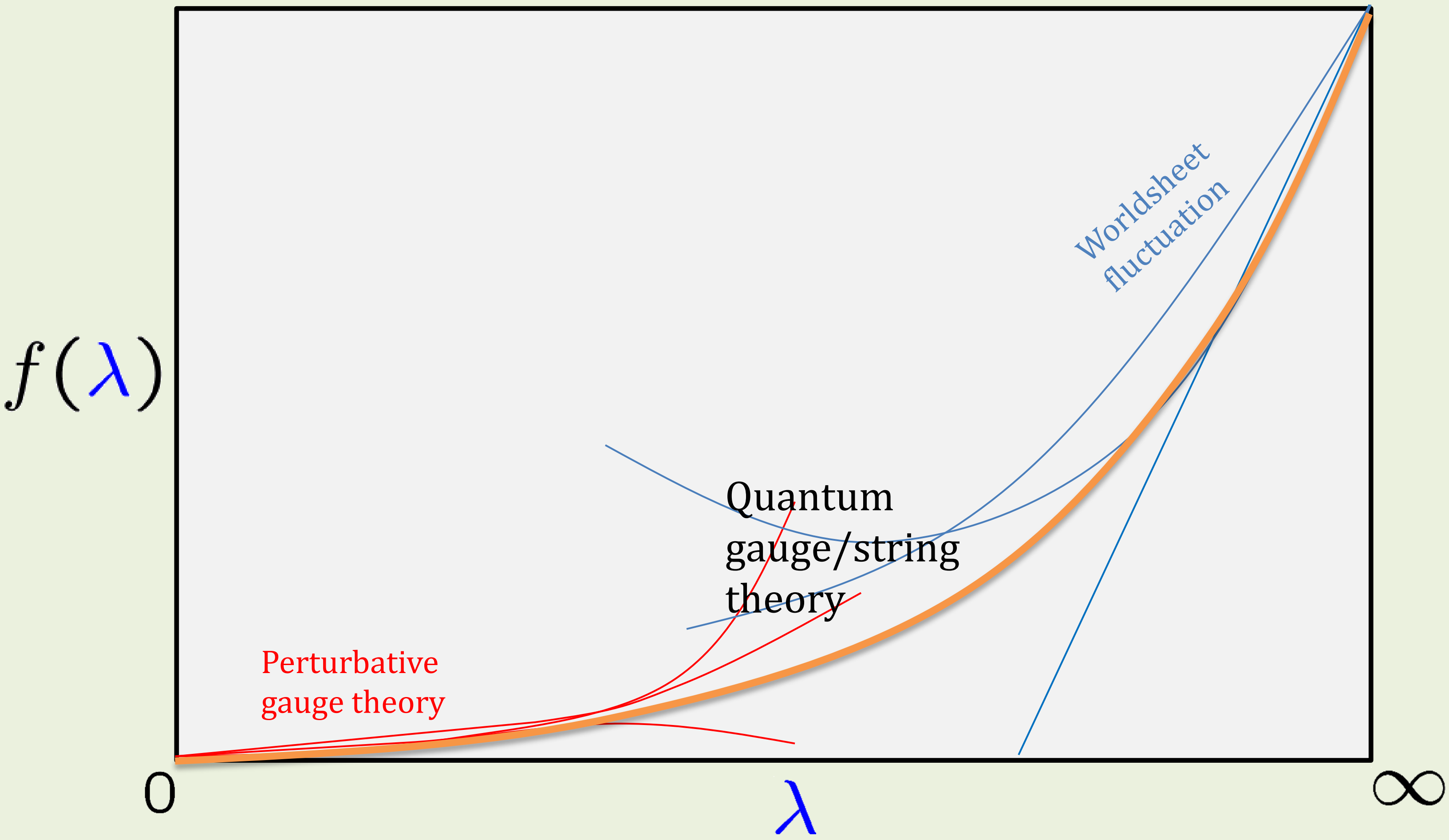
- Appears in the planar limit

- Perturbative integrability
  - Certain integrable models appear in perturbative computations
  - Classical string solutions from some classical integrable systems

- Nonperturbative integrability
  - Exact results for any value of λ

- Only a few physical quantities are exactly computable so far
  - Anomalous dimensions
  - Worldsheet S-matrix

Nonperturbative



Perturbative integrability in N=4 SYM

N=4 Super Yang-Mills theory

- $\mathcal{N}=4\ \ SU(N_c)$ SYM

$$\boxed{S=\frac{\mathrm{Tr}}{g_{\mathrm{YM}}^2}\int d^4x\left\{-\frac{1}{4}F_{\mu\nu}^2+(D_\mu\Phi^a)^2+\left[\Phi^a,\Phi^b\right]^2+\bar{\chi}\not{D}\chi-i\bar{\chi}\Gamma_a[\Phi^a,\chi]\right\}}$$

- R-symmetry : $\mathcal{N}$=4 SUSY so(6) ≅ su(4)
- Scalar fields : $\Phi^a,\ a=1,\ldots,6$ $\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline\end{array}$
- Gauginos : $\chi,\ \bar{\chi}$ fundamental in su(4) □
- All in adjoint rep. in $SU(N_c)$

	R-charge
A_μ	$\mathbf{1}$
χ_α^A $\bar{\chi}_{\bar{\alpha}}^{\bar{A}}$	$\mathbf{4}\oplus\bar{\mathbf{4}}$
Φ^a	$\mathbf{6}$

4d conformal field theory

- One-loop β-function

$$\beta \equiv \mu \frac{\partial g_{\mathrm{YM}}}{\partial \mu} = -\frac{g_{\mathrm{YM}}^3}{16\pi^2}\left(\frac{11}{3}N_c - \frac{1}{6}\overset{6N_c}{\sum_i} C_i - \frac{1}{3}\overset{8N_c}{\sum_j} \tilde{C}_j\right) \quad = 0$$

- β=0 at all orders of perturbation
  - Three loops in superspace formulation
  - All loops in light-cone gauge
- No scale dependence

N=4 superconformal algebra

- Lorentz generators : $L_{\mu\nu}$
- Translations : P_μ
- Conformal boosts : K_μ
- Dilatation : D
- Supercharges : $Q_{a\alpha}, \quad \overline{Q}^{a}_{\dot{\alpha}}$
- Superconformal boosts : $S^{a}_{\alpha}, \quad \overline{S}_{\dot{\alpha}a}$
- R-symmetry : su(4)

su(2,2) ≅ so(2,4)

psu(2,2|4)

32 super charges

$$\left(\begin{array}{c|c|c} L & Q & P \\ \hline S & R & \overline{Q} \\ \hline K & \overline{S} & \overline{L} \end{array}\right)$$

- psu(2,2|4) commutation relations

$$\boxed{\begin{array}{l} \left[D, P_\mu\right] = -iP_\mu, \qquad \left[D, L_{\mu\nu}\right] = 0, \qquad \left[D, K_\mu\right] = iK_\mu \\ \left[D, Q_{\alpha a}\right] = -\frac{i}{2} Q_{\alpha a}, \quad \left[D, \overline{Q}{}^a_{\dot\alpha}\right] = -\frac{i}{2}\overline{Q}{}^a_{\dot\alpha}, \quad \left[D, S^{\,a}_{\alpha}\right] = \frac{i}{2} S^{\,a}_{\alpha}, \quad \left[D, \overline{S}_{\dot\alpha a}\right] = \frac{i}{2}\overline{S}_{\dot\alpha a} \end{array}}$$

$$\left[L_{\mu\nu}, P_\lambda\right] = -i(\eta_{\mu\lambda}P_\nu - \eta_{\lambda\nu}P_\mu), \qquad \left[L_{\mu\nu}, K_\lambda\right] = -i(\eta_{\mu\lambda}K_\nu - \eta_{\lambda\nu}K_\mu)$$

$$\left[P_\mu, K_\nu\right] = 2i(L_{\mu\nu} - \eta_{\mu\nu}D)$$

$$\left\{Q_{\alpha a}, \overline{Q}{}^b_{\dot\alpha}\right\} = \gamma^\mu_{\alpha\dot\alpha}\delta^b_a P_\mu, \quad \left\{Q_{\alpha a}, Q_{\alpha b}\right\} = \left\{\overline{Q}{}^a_{\dot\alpha}, \overline{Q}{}^b_{\dot\alpha}\right\} = 0$$

$$\left[P_\mu, Q_{\alpha a}\right] = \left[P_\mu, \overline{Q}{}^a_{\dot\alpha}\right] = 0, \quad \left[L^{\mu\nu}, Q_{\alpha a}\right] = i\gamma^{\mu\nu}_{\alpha\beta}\epsilon^{\beta\gamma}Q_{\gamma a}, \quad \left[L^{\mu\nu}, \overline{Q}{}^a_{\dot\alpha}\right] = i\gamma^{\mu\nu}_{\dot\alpha\dot\beta}\epsilon^{\dot\beta\dot\gamma}\overline{Q}{}^a_{\dot\gamma}$$

$$\left[K^\mu, Q_{\alpha a}\right] = \gamma^\mu_{\alpha\dot\alpha}\epsilon^{\dot\alpha\dot\beta}\overline{S}_{\dot\beta a}, \quad \left[K^\mu, \overline{Q}{}^a_{\dot\alpha}\right] = \gamma^\mu_{\alpha\dot\alpha}\epsilon^{\alpha\beta}S^{\,a}_{\beta}$$

$$\left\{Q_{\alpha a}, \overline{Q}{}^b_{\dot\alpha}\right\} = \gamma^\mu_{\alpha\dot\alpha}\delta^b_a P_\mu, \quad \left\{S^{\,a}_{\alpha}, \overline{S}_{\dot\alpha b}\right\} = \gamma^\mu_{\alpha\dot\alpha}\delta^a_b K_\mu, \quad \left\{S^{\,a}_{\alpha}, S^{\,a}_{\alpha}\right\} = \left\{\overline{S}_{\dot\alpha a}, \overline{S}_{\dot\alpha b}\right\} = 0$$

$$\left[K_\mu, S^{\,a}_{\alpha}\right] = \left[K_\mu, \overline{S}_{\dot\alpha a}\right] = 0$$

$$\boxed{\begin{array}{l} \left\{Q_{\alpha a}, S^{\,b}_{\beta}\right\} = -i\epsilon_{\alpha\beta}\sigma^{IJ\,b}{}_{a}R_{IJ} + \gamma^{\mu\nu}_{\alpha\beta}\delta^b_a L_{\mu\nu} - \frac{1}{2}\epsilon_{\alpha\beta}\delta^b_a D \\ \left\{\overline{Q}{}^a_{\dot\alpha}, \overline{S}_{\dot\beta b}\right\} = i\epsilon_{\dot\alpha\dot\beta}\sigma^{IJ\,a}{}_{b}R_{IJ} + \gamma^{\mu\nu}_{\dot\alpha\dot\beta}\delta^a_b L_{\mu\nu} - \frac{1}{2}\epsilon_{\dot\alpha\dot\beta}\delta^a_b D \end{array}}$$

- Conformal symmetry → No mass spectrum

- Conformal dimension spectrum for a local operator $[D, \mathcal{O}(0)] = -i\,\Delta\,\mathcal{O}(0)$

- Lowered by K : $\mathcal{O}'(0) \equiv \left[K_\mu, \mathcal{O}(0)\right] \to \left[D, \mathcal{O}'(0)\right] = -i\,(\Delta - 1)\,\mathcal{O}'(0)$

$$\left[D, K_\mu\right] = +i\,K_\mu$$

- Primary operator : $\left[K_\mu, \widetilde{\mathcal{O}}(0)\right] = 0$

- Descendent operators : (ex) $\left[P_\mu, \widetilde{\mathcal{O}}\right] = -i\partial_\mu \widetilde{\mathcal{O}}$ $\quad \left[D, P_\mu\right] = -i\,P_\mu$

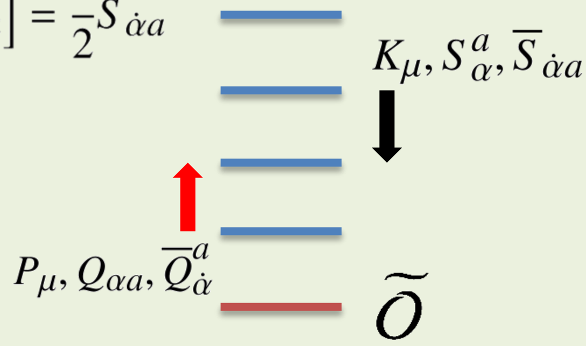
$$\left[D, \partial_\mu \tilde{\mathcal{O}}\right] = -i\,(\Delta + 1)\,\partial_\mu \tilde{\mathcal{O}}$$

- Superconformal raising and lowering ops.

$$[D, Q_{\alpha a}] = -\frac{i}{2} Q_{\alpha a}, \quad \left[D, \overline{Q}{}^a_{\dot\alpha}\right] = -\frac{i}{2}\overline{Q}{}^a_{\dot\alpha}, \quad [D, S^{\,a}_{\alpha}] = \frac{i}{2} S^{\,a}_{\alpha}, \quad \left[D, \overline{S}_{\dot\alpha a}\right] = \frac{i}{2}\overline{S}_{\dot\alpha a}$$

- Superconformal primary :

$$\left[S^{\,a}_{\alpha}, \widetilde{\mathcal{O}}(0)\right] = \left[\overline{S}_{\dot\alpha a}, \widetilde{\mathcal{O}}(0)\right] = 0$$



- Cartan subalgebra $$[D,R] = \left[L_{\mu\nu},R\right] = \left[D,L_{\mu\nu}\right] = 0$$

- Irreducible rep. are given by eigenvalues of these operators

$$\Big(\overbrace{\quad\Delta\quad}^{D},\overbrace{S_1,S_2}^{L_{\mu\nu}}\,|\,\overbrace{J_1,J_2,J_3}^{R}\Big)$$

- Scalar fields

$$Z \equiv \Phi_1 + i\Phi_2, \quad Y \equiv \Phi_3 + i\Phi_4, \quad X \equiv \Phi_5 + i\Phi_6$$
$$\overline{Z} \equiv \Phi_1 - i\Phi_2, \quad \overline{Y} \equiv \Phi_3 - i\Phi_4, \quad \overline{X} \equiv \Phi_5 - i\Phi_6$$
$$(1,0,0|\pm 1,0,0),\ (1,0,0|0,\pm1,0),\ (1,0,0|0,0,\pm1)$$

- Gauginos and gauge fields

$$\overset{\chi_\alpha^A}{\left(\frac{3}{2},\pm\frac{1}{2},0|\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2}\right)},\ \overset{F_+}{(2,m,0|0,0,0)},\ \overset{\mathcal{D}}{\left(1,\pm\frac{1}{2},\pm\frac{1}{2}|0,0,0\right)}$$

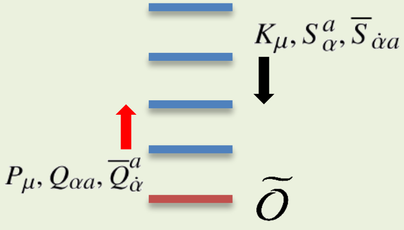
- General gauge invariant composite operators

$$\tilde{O}(x) = \mathrm{Tr}\left[O_1(x)O_2(x)\ldots O_L(x)\right]$$

- ½ -BPS operator $\mathrm{Tr}\left[Z^L\right] \quad \rightarrow \quad (L,0,0|L,0,0)$

Chiral primary or BPS operator

- Impose further condition $\left[Q_{a\alpha}, \widetilde{O}(0)\right] = 0, \qquad \text{for some } \alpha, a$
  - Jacobi identity $\left[\left\{Q_{a\alpha}, S^{b}_{\beta}\right\}, \widetilde{O}(0)\right] = \left[-i\varepsilon_{\alpha\beta}\left(\sigma^{IJ}\right)^{b}_{a} R_{IJ} - \varepsilon_{\alpha\beta}\delta^{b}_{a} D + \sigma^{\mu\nu}_{\alpha\beta}\delta^{b}_{a} L_{\mu\nu}, \widetilde{O}(0)\right] = 0$
  - For the Lorentz scalar operator : $\left[L_{\mu\nu}, \widetilde{O}(0)\right] = 0$



$$\left(\sigma^{IJ}\right)^{b}_{a}\left[R_{IJ}, \widetilde{O}(0)\right] = \Delta\ \delta^{b}_{a}\ \widetilde{O}(0) \qquad \sigma^{12} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

  - Satisfied if R-charge = conformal dimension $\Delta = J_1$

$$\mathrm{Tr}\left[Z^{L}\right] \quad \rightarrow \quad (L, 0, 0|L, 0, 0)$$

  - This commutes with half SUSY charges and conformal dimension is protected and gets no quantum corrections

Anomalous Dimension

- Conformal dimensions of composite operators :

$$\langle \mathcal{O}_n(x)\mathcal{O}_m(0)\rangle = \frac{\delta_{mn}}{|x|^{2\Delta_n}}$$

- Anomalous dimension is defined by $\Delta = \Delta_0 + \gamma$

- can be calculated by
  - Direct perturbation theory
  - Renormalization group under dilatation

- Operator mixing by RG dilatation

Perturbative computation

- (ex) Konishi operator $\mathcal{O}(x) = \mathrm{Tr}\left(\sum_a \Phi_a(x)^2\right)$ $\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{1}{|x-y|^{2\Delta}}$
- Tree-level

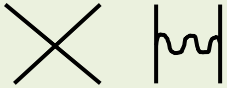
$$\left\langle : \Phi_a(x)^A_B \Phi_a(x)^B_A :: \Phi_b(y)^C_D \Phi_b(y)^D_C : \right\rangle_0 = \left(\frac{g^2_{\mathrm{YM}}}{8\pi^2}\right)^2 \frac{N_c^2 \cdot 6 \cdot 2}{|x-y|^4}$$

$$\frac{g^2_{\mathrm{YM}}}{8\pi^2}\frac{\delta^A_C\delta^D_B\delta_{ab}}{|x-y|^2}$$

$$\int \frac{d^4z}{|z-x|^4|z-y|^4} \approx \frac{2i}{|x-y|^4}\int_{\Lambda^{-1}}^{|x-y|}\frac{d\xi d\Omega_3}{\xi} = \frac{2\pi^2 i}{|x-y|^4}\ln(\Lambda^2|x-y|^2)$$

- One-loop

$$\left\langle : \Phi_a(x)^A_B \Phi_a(x)^B_A :: \Phi_b(y)^C_D \Phi_b(y)^D_C : \left(\frac{g^2_{\mathrm{YM}}}{4}\int d^4z \mathrm{Tr}(\Phi_c\Phi_c\Phi_d\Phi_d)(z)\right)\right\rangle_0 + \dots$$



$$\langle \mathcal{O}_R(x)\mathcal{O}_R(y)\rangle = \left(\frac{\lambda}{8\pi^2}\right)^2 \frac{12}{|x-y|^4}\left[1 - \frac{3\lambda}{4\pi^2}\ln(|x-y|^2)\right] \sim \frac{1}{|x-y|^{2(2+\underbrace{3\lambda/4\pi^2}_{\gamma})}}$$

RG method

- Under dilatation $x \to \alpha x$

$$\mathcal{O}(x) = \alpha^{-\Delta}\mathcal{O}(\alpha x)$$

- One-loop corrections

- Operator mixing (ex) su(2) sector

$$\left\{\mathrm{Tr}\,[ZZZZXX]\,,\mathrm{Tr}\,[ZZZXZX]\,,\mathrm{Tr}\,[ZZXZZX]\,,\mathrm{Tr}\,[ZXZZZX]\right\}$$

$$\mathcal{O}_a = \mathcal{Z}_a^b(\Lambda)\mathcal{O}_b$$

- Dilatation matrix

$$\Gamma = \frac{d\mathcal{Z}}{d\ln\Lambda}\cdot\mathcal{Z}^{-1}$$

Mapping to integrable spin chain

- Finding the eigenvalues of the dilatation matrix is very difficult problem but fortunately …
- Mapping the matrix to a Hamiltonian of integrable spin chain has been discovered [(ex) so(6), su(2) spin chains]
- (ex) su(2) sector $\left\{\mathrm{Tr}\left[Z^L\right], \mathrm{Tr}\left[Z^{L-1}X\right], \mathrm{Tr}\left[Z^{L-n-1}XZ^{n-1}X\right], \ldots, \mathrm{Tr}\left[X^L\right]\right\}$
- One-loop dilatation → Heisenberg spin chain model
  - Map: $|\Uparrow\rangle \equiv |Z\rangle, \quad |\Downarrow\rangle \equiv |X\rangle$
  - Vacuum state: Ferromagnetic vac. ←→ BPS $|\Uparrow \ldots \Uparrow\rangle \equiv \mathrm{Tr}\left[Z^L\right]$
  - Excited states:

$$|\Uparrow\Downarrow\Uparrow\Downarrow\Uparrow\Uparrow\ldots\rangle + \ldots \equiv \mathrm{Tr}\left[ZXZXZZ + \ldots\right] + \ldots$$

  - Dilatation $\Gamma = \frac{\lambda}{8\pi^2}\sum_{l=1}^{L}\left[1 - \vec{\sigma}_l \cdot \vec{\sigma}_{l+1}\right]$

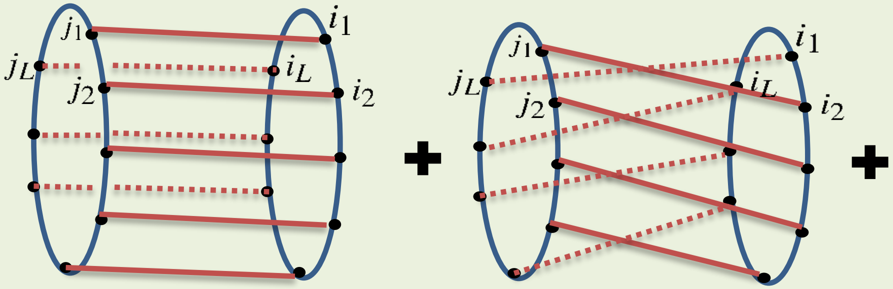
(ex) SO(6) sector

- Scalar fields $\{Z,\ Y,\ X,\ \bar{Z},\ \bar{Y},\ \bar{X}\}$
- Composite operators

$$\left\{\mathrm{Tr}\left[XYZ\bar{X}YZX\bar{Z}\cdots\right],\ldots\right\} = \mathrm{Tr}\left[\Phi_{i_1}\cdots\Phi_{i_L}\right] \equiv \mathcal{O}_{i_1\cdots i_L}(x)$$

- Two-point function $\left\langle \bar{\mathcal{O}}^{j_1\cdots j_L}(x)\,\mathcal{O}_{i_1\cdots i_L}(y)\right\rangle$

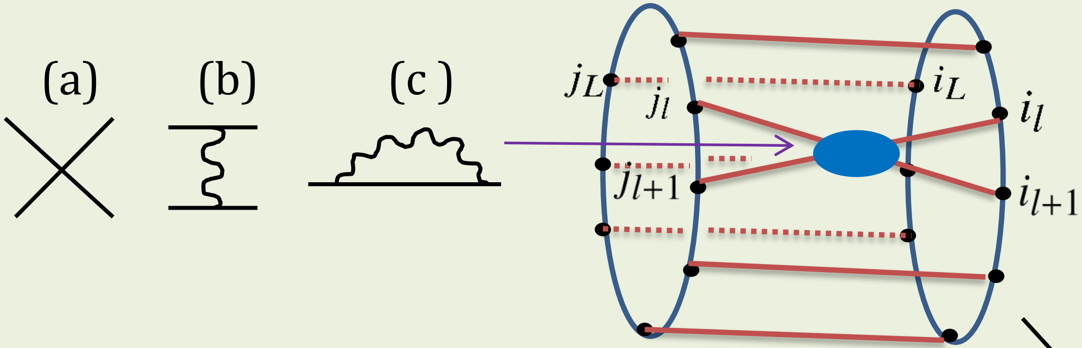
- Tree level

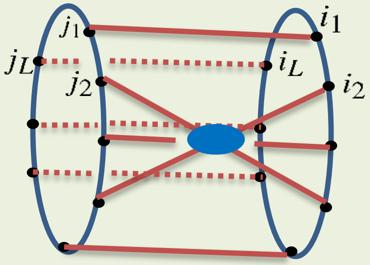


$$\left(\frac{\lambda}{8\pi^2}\right)^L \frac{1}{|x-y|^{2L}}\left[\delta^{j_1}_{i_1}\cdots\delta^{j_L}_{i_L}+\delta^{j_1}_{i_2}\cdots\delta^{j_L}_{i_1}+\ldots\right]$$

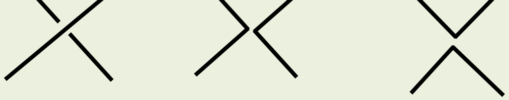
cyclic permutations

- One-loop level : nearest neighbor only





- Wave-function renormalization



$$
\begin{aligned}
Z^{(a)} &= 1 - \frac{\lambda}{16\pi^2} \ln \Lambda \cdot \left(2\delta_{i_l}^{j_{l+1}} \delta_{i_{l+1}}^{j_l} - \delta_{i_l}^{j_l} \delta_{i_{l+1}}^{j_{l+1}} - \delta_{i_l, i_{l+1}} \delta^{j_l j_{l+1}} \right) \\
Z^{(b)} &= 1 - \frac{\lambda}{16\pi^2} \ln \Lambda \cdot \delta_{i_l}^{j_l} \delta_{i_{l+1}}^{j_{l+1}} \\
Z^{(c)} &= 1 + \frac{\lambda}{8\pi^2} \ln \Lambda \cdot \delta_{i_l}^{j_l} \delta_{i_{l+1}}^{j_{l+1}}
\end{aligned}
$$

$$
Z = 1 + \frac{\lambda}{16\pi^2} \ln \Lambda \cdot \left(\delta_{i_l, i_{l+1}} \delta^{j_l j_{l+1}} - 2\delta_{i_l}^{j_{l+1}} \delta_{i_{l+1}}^{j_l} + 2\delta_{i_l}^{j_l} \delta_{i_{l+1}}^{j_{l+1}} \right)
$$

- Dilatation matrix

$$
\Gamma = \frac{\lambda}{8\pi^2} \sum_{l=1}^{L} \left(1 - \mathbf{P}_{l,l+1} + \frac{1}{2} \mathbf{K}_{l,l+1} \right)
$$

Minahan, Zarembo (2003)

Heisenberg model

- 1D spin chain, XXX model

$$H = \sum_{l=1}^{L} \left(1 - \vec{\sigma}_l \cdot \vec{\sigma}_{l+1} \right)$$

$$\sigma_j^a = \mathbf{1} \otimes \cdots \otimes \mathbf{1} \otimes \sigma^a \otimes \mathbf{1} \otimes \cdots \otimes \mathbf{1} : \quad 2^L \times 2^L \text{ Matrix}$$

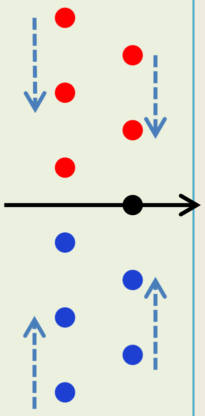
$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Exactly solved by H. Bethe
- Bethe ansatz equations for real Bethe roots
- Lectures by Rafael Nepomechie

$$\left(\frac{u_j + i/2}{u_j - i/2} \right)^L = \prod_{\substack{k=1 \\ k \neq j}}^{M} \frac{u_j - u_k + i}{u_j - u_k - i} \qquad \gamma = \frac{\lambda}{2\pi^2} \sum_{j=1}^{M} \frac{1}{u_j^2 + \frac{1}{4}}$$

Bethe Strings

- Bethe roots so far were real but complex roots can exist
- L→∞ limit:
  - Introduce a complex pair of root with imaginary parts $u_j = u^R \pm i\alpha$
  - For positive imaginary root: LHS of BAE $\left(\frac{u_j + i/2 + i\alpha}{u_j - i/2 + i\alpha}\right)^L \to \infty$
  - RHS of BAE : there should be another Bethe root which makes a denominator vanish : $u_k = u^R + i(\alpha - 1)$
  - Repeat the process until the imaginary part is still positive
  - For negative imaginary root: LHS of BAE $\left(\frac{u_j + i/2 - i\alpha}{u_j - i/2 - i\alpha}\right)^L \to 0$
  - RHS should vanish by adding $u_l = u^R + i(-\alpha + 1)$
  - For finite # of roots, α should be an integer or a half-integer

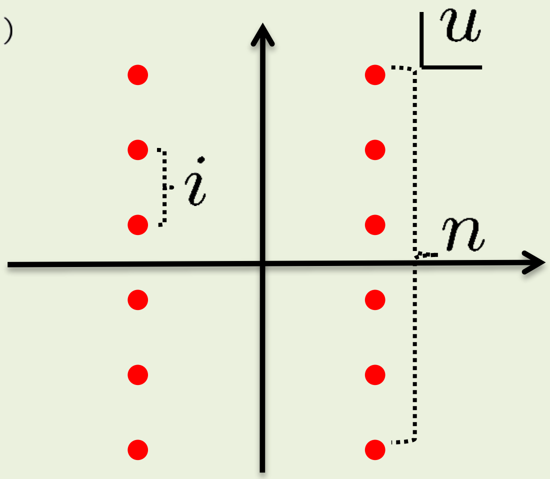


- Bethe string

$$u_j^{(n)} = u^R + \frac{n + 1 - 2j}{2} i, \quad j = 1, \ldots, n$$

- Low lying states are given by “long strings” rather than real roots

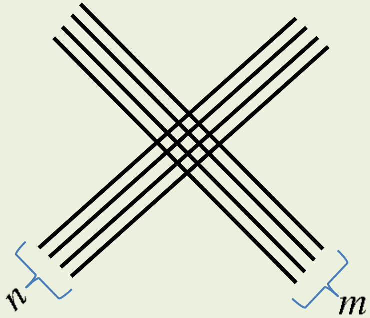
$$E^{(n)} = \sum_{j=1}^{n} \frac{1}{\left(u_j^{(n)}\right)^2 + \frac{1}{4}} = \frac{n}{(u^R)^2 + \frac{n^2}{4}} \le \frac{n}{(u^R)^2 + \frac{1}{4}} = nE^{(1)}$$



- BAE for the strings can be obtained by multiplying elementary BAE for each component of a string
  - Elementary BAE

$$e_1(u_j)^L = \prod_{k\neq j}^{M} e_2(u_j - u_k) \qquad e_{n}(u) \equiv \frac{u + in/2}{u - in/2}$$

  - BAE for strings



$$\prod_{j=1}^{n} \frac{u^R + i(n+1-2j)/2 + i/2}{u^R + i(n+1-2j)/2 - i/2} = \frac{u^R + in/2}{u^R - in/2} = e_n(u^R)$$

$$e_{n_J}(u_J^R)^L = \prod_{K=1}^{M} E_{n_J,n_K}(u_J^R - u_K^R) \qquad E_{n,m} = e_{|n-m|} e^2_{|n-m|+2} \cdots e^2_{n+m-2} e_{n+m}$$

- For finite L : strings are deformed

Summary

- AdS/CFT duality: string theory and SYM theory
- Today: Focused on perturbative SYM side
  - Planar limit: Integrability in computing anomalous dimensions
  - Operator mixing is solved by integrable spin chains in one-loop
  - New states ("Bethe strings") appear
- Tomorrow: String theory side as a strong coupling limit
  - Classical string states
  - Strong coupling limit of all-loop conjectures