The Logic of the Transformation Problem

Alan Freeman

1. Introduction

Since Bohm-Bawerk first criticized Marx's transformation of values into prices of production, almost everyone who has tried to correct or refute Marx's value theory has claimed it is logically flawed. The post-Sraffians are the most emphatic. Steedman writes that the 'central objection' to Marx's approach is that 'even if input prices are transformed, Marx's solution is internally inconsistent.'[1] His argument, which has almost no empirical component, stands or falls on its logical critique. As he himself says, his case 'is the conclusion of an argument in logic; should anyone wish to challenge it, they must do so either by finding a logical flaw in the argument or by rejecting explicitly and coherently one or more of the assumptions on which it is based.'[2]

A footnote adds:

> 'The present type of argument has been examined, in various forms, by many different writers over the last eighty years. The same conclusions have always been reached and no logical flaw has ever been found in such arguments.'[3]

My limited but perhaps ambitious aim is to identify and demarcate this logical flaw.

2. The Argument in Outline

Steedman makes two charges: inconsistency and redundancy. The first allegation dates from von Bortkiewicz. It says that Marx's transformation cannot be applied to a self-reproducing economy without dropping one or other of his famous equalities and his expression for the rate of profit. There is a logical contradiction between hypotheses and results, so the hypotheses must be wrong.

Post-Sraffian writers have developed this idea, for example with claims that labour values lead to negative values, and so on. Nevertheless, what distinguishes writers such as Steedman from all Marx's 'interpreters' and 'correctors' is their use of the second charge: redundancy. They have a distinctive creed, pursued with Jesuitical zeal, and which prescribes that political economy must be reconstructed without labour values.[4] Steedman's argument is succinct. He says that values are not needed to calculate prices and therefore they are not needed at all, because they do not 'determine' prices.

There are four reasons why I shall concentrate on this second charge:

First, the redundancy charge has not been 'studied for eighty years' and is a distinct logical issue from that of inconsistency, deserving separate treatment.

Second, there is no need to repeat Farjoun, Savran and Giussani's refutations of many inconsistency charges. For the same reason I do not propose to adopt the more general joint production framework,[5] the arguments applying *mutatis mutandis*. Third, I wish to re-assess the way in which Marx's equalities have been translated into mathematical terms using simultaneous equation systems, and show that in the sense most important to Marx's analysis, his equalities do hold, even within such systems. But this different interpretation calls for a critical assessment of the post-Sraffian view of causality, the central issue being what 'determines' prices in the real world.

Most important, however, the charge of redundancy is actually the only basis in logic for rejecting labour values. This is not always understood, but becomes clearer if we ask how scientific progress, which constantly encounters contradiction and inconsistency, takes place.

In general two different 'paradigms', or programmes of scientific inquiry, can result from a formal inconsistency. One involves critical revision—reworking existing theory to remove the inconsistency by changing either its hypotheses or the way they are formulated. The other involves critical rejection—transcending the theory as a whole. Within logic as such there is no basis for settling on one or other choice on the grounds of inconsistency. If one assumes $1 + 1 = 4$, one can deduce $1 = 3$, which contradicts an axiom of number theory. Most mathematicians have not rejected number theory, but the hypothesis that $1 + 1 = 4$.[6]

The normal scientific reason for throwing out a theory is that a new one explains the known facts better. Indeed, if inconsistency were sufficient ground to reject an entire theory, the neo-Ricardian school

would be obliged to discard their own theory which contains many inconsistencies, some openly conceded and others brought to light in this volume.

Hence the thrust of this paper. Its argument, in outline, is as follows:

(*i*) The post-Sraffian refutation of labour values cannot be dissociated from a particular formalization (mathematical representation), namely a simultaneous equation system with a uniform profit rate in which input prices are equal to output prices.

(*ii*) This involves 'simplifying assumptions' which turn out to be axioms—indispensable elements of the theory—because without them the neo-Ricardian solutions for prices and profit do not exist. These axioms are incompatible with a real commodity economy and Marx's theory of labour values. Above all they cannot model real causality or real determination, because: (a) They abstract from independent movements in time of economic quantities. Both in reality and in Marx's theory, these movements are the actual causal mechanism through which value magnitudes are transformed into prices. (b) They cannot model capitalist behaviour because they abstract from the real quantities which determine capitalist actions, above all differential profits.

(*iii*) Real causality is therefore replaced by algebraic calculation based on these (false) axioms. The result is a profoundly unscientific theory—in fact idealist—because prices are allegedly determined by metaphysical constructs and not the behaviour of independent private producers.

(*iv*) Further advance demands a different formalization of labour value theory and a critical rejection of simultaneous equation models. The independent variation over time of all economic quantities, particularly differential profit rates, must be given the status Marx himself assigned them, namely that of mechanisms of the law of value.

(*v*) If this is done in accordance with Marx's own suggestions there is every reason to suppose that though new contradictions will certainly emerge, the 'inconsistencies' that arise in the Sraffian formalization will not exist. The alleged inconsistencies in labour value theory turn out to result from the hidden assumptions of this formalization, not from the theory as such.

3. Origins of a Fundamental Error

Sraffa prefaces his work with a statement of intent. He says: 'The investigation is concerned exclusively with such properties of an

economic system as do not depend on changes in the scale of production or in the properties of "factors"... The reason is obvious. The marginal approach requires attention to be focussed on change ... In a system in which, day after day, production continued unchanged in these respects, the marginal product of a factor (or alternatively the marginal cost of a product) would not merely be hard to find—it just would not be there to be found.'[7]

This is more than a restriction of the field of study, for in no real economy does production, day after day, continue unchanged in any respect whatsoever. Sraffa, however, did not claim to present a model of the real workings of a real economy, but concentrated his fire on the internal inconsistencies of the marginalists. He therefore considered it legitimate to abstract from the process of change.

For Steedman the same assumptions take on an enhanced role, since he claims to lay the foundations of a new system of political economy. A founding principle, among those he challenges his critics to refute, is the following: 'The capitalist economies considered are always in a self-reproducing state, whether reproduction be 'simple' or 'expanded' (stationary or growing).'[8]

The term 'self-reproducing' here does not just mean that if the economy is here on Monday, it will also be here on Tuesday. Sraffa and Steedman both repeat a construction which von Bortkiewicz uses when he sets out to solve the alleged 'feedback' failure of Marx's transformation, and which lies at the basis of all such presentations of labour value theory. That is, they say the prices paid for goods at the beginning of a cycle of production are the same as those charged for the same goods at the end of the same cycle. They forcibly equate the results of production to its premises. In short the economy does not merely reproduce itself; it reproduces itself identically. Its past, present and future are locked in a self-sustaining circle.

This is most obvious in relation to prices. Following Steedman, let $\mathbf{p}$ be the price vector, $\mathbf{r}$ the scalar profit rate, $\mathbf{A}$ the matrix of production, $\mathbf{w}$ the real wage, $\mathbf{a}$ the labour employed in each industry, and L the total labour available. The equation

$$\mathbf{p} = (1 + r)(\mathbf{pA} + \mathbf{w}.\mathbf{a}/\mathbf{L}) \qquad (1)$$

is a special case of a more general equation, namely

$$\mathbf{p}^{t+\delta t} = (1 + r)(\mathbf{p}^t\mathbf{A} + \mathbf{w}.\mathbf{a}/\mathbf{L}) \qquad (2)$$

where $\mathbf{p}^t$ are prices at time t. The hidden assumption is that $\mathbf{p}^t = \mathbf{p}^{t+\delta t}$.

Without this we would not have a solvable simultaneous equation system at all but a set of n relations connecting $2n + 2$ variables, relating prices now to prices then.

It is less clear that a similar, but not identical constraint applies to quantities. Neo-Ricardian assumptions require all goods to be consumed; that is, there are no unconsumed stocks, no build-up or decline of use-values in circulation, either of goods or money. In fact the simultaneous equation method, in general, reduces to a treatment of flows, rather than stocks, of commodities.

It might appear that this still leaves room for expansion, provided this is matched either by increased capitalist consumption or by demand arising from investment to meet such consumption. However, matters are not quite so simple if we consider the course of events over time when a new demand arises in the economy. Suppose, say, production increases in the cornflakes sector, either to meet a new demand or in anticipation of it. This creates a demand for inputs of cornflake-making equipment and materials; say, corn and iron. But such a demand cannot be satisfied immediately, because all existing output is allocated to existing consumption, either productive or unproductive.

Within the model as it stands, since these inputs are needed before new production can begin, they cannot be supplied in time to make the extra cornflakes unless the iron and corn manufacturers increase their production in the relevant proportions. Indeed, strictly speaking the extra corn and iron would have to be produced in the previous reproductive cycle to be ready in time, reversing the actual economic sequence and endowing the people concerned with clairvoyance as well as omniscience. Even then, the problem is not solved, since it is unclear where the iron or corn producers can get their own surplus inputs from. Thus the sins of the sons are visited on the fathers, since for all time the economy must already have been preparing itself for the coming cornflake boom.

It may appear that a reduction in production at least is possible. Not so simple; it will lead to temporarily unsold stocks of surplus goods. But unsold goods means a reduction of money profits since it reduces money income. However, profits are already fixed at the same time as the price, and like the price may not vary over the period of reproduction.

These and similar difficulties may be averted only by assuming that production rises everywhere at once in such proportions as perfectly to balance out inputs and outputs. Insofar as changes in the scale of production are even conceivable, they impose a most peculiar

condition, namely that the economy must change all at once or not at all. An unbalanced economy with surplus supply or demand in particular sectors destroys the formal derivation of prices.

Clearly this is at best an abstraction. But it is not a real abstraction. It is an idealization, justified on the basis that more sophisticated analysis can dispense with the simplifications later. An obvious question therefore arises: what happens if these simplifications are dropped? A second question presents itself: what are their logical consequences as they stand?

To answer both questions, we should ask how these simplifications enter the calculation of prices and profits. We have already noted that a solution depends on equating $\mathbf{p}^{t+\delta t}$ to $\mathbf{p}^t$. Can we drop this assumption? No, because without it there are simply too many variables, and no solution exists. Moreover if one did exist, its meaning would be open to question since it would imply that $\mathbf{p}^t$ were determined by events in the future.

But the same argument applies if we try to relax the many other built-in assumptions. In particular, we cannot allow profit rates to become non-uniform, and the matrix $\mathbf{A}$ cannot be made up of less or more columns than rows; that is, there must be exactly as many producers as products.[9] Nor can any of these quantities actually vary while reproduction is going on, for the same reason as prices. Any adjustment to the parameters of the economy must take place in some nether or aetherial region which is not actually part of the space-time continuum occupied by the economy, unless like Joshua we can halt the sun and moon in the sky while the awful business is done.

If any of these assumptions are dropped, instead of an exact determination of $\mathbf{p}$, $\mathbf{w}$ and r we are left with a collection of relations between a large number of variables out of which no definite determination can in general be made, notwithstanding the interesting or insightful relations which can be established between the variables concerned.

There is an instructive way of looking at this, which the non-mathematical reader can omit, moving to the next section, if necessary.

Let us write the equation relating $\mathbf{p}^{t+\delta t}$ to $\mathbf{p}^t$ in a slightly more general form:

$$\mathbf{p}^{t+\delta t} = \mathscr{F}(\mathbf{p}^t, \delta t) \tag{3}$$

where δt is the time interval under consideration, usually the period of

production. Or, bringing all the parameters involved into the expression,

$$\mathbf{p}^{t+\delta t} = \mathscr{F}(\mathbf{p}^t, \mathbf{r}^t, \mathbf{A}^t, \mathbf{w}^t, \mathbf{a}^t, L, \delta t) \tag{4}$$

where now $\mathbf{r}^t$ is a vector of not necessarily equal profit rates.

Two directions of development now suggest themselves. The only fully general mathematical approach would be to derive equations relating $\mathbf{r}^{t+\delta t}$, $\mathbf{A}^{t+\delta t}$, and so on, to the values of all other parameters at time t, and thus derive a differential equation

$$\mathscr{G}(D, \mathbf{p}, \mathbf{r}, \mathbf{A}, \mathbf{w}, \mathbf{a}, L) = 0 \tag{5}$$

where D is the differential operator $\delta/\delta t$. A solution to this equation, together with the appropriate boundary conditions, would in theory define the motion of an economy in time. In my view such an approach, though untried and difficult, is closer to the general method of Marx.

It is instructive to view Sraffa's solution as a second direction of development arising from his desire to abstract from motion. However the method he uses is unnatural. It arrests the moving process neither by recording economic quantities at a particular moment like a photograph, nor by averaging over time, as Marx does. Instead it imposes the boundary condition

$$\frac{\partial}{\partial t}(\mathbf{p}, \mathbf{r}, \mathbf{A}, \mathbf{w}, \mathbf{a}) \equiv 0 \tag{6}$$

for all time and all values of the parameters, corresponding to a particular degenerate case of (3): static equilibrium. It eliminates motion by commanding it to cease.

To do this, the post-Sraffians use one of a class of theorems known as 'fixed-point' theorems. These tell us that under very general conditions, if $\mathscr{F}$ is a function which maps a variable X onto the domain from which X is chosen, then there exist one or more values of X, say X^*, for which

$$X^* = \mathscr{F}(X^*) \tag{7}$$

In this case the domain of X is the space of possible values of p. Moreover, if we impose a particular condition on $\mathbf{w}$, $\mathbf{r}$, $\mathbf{A}$, and $\mathbf{a}$, we can obtain non-zero, positive values of $\mathbf{p}$ which turn out to be

independent of **w** and **r**. The construction also yields a functional relation between **A**, **a**, **w** and **r** if we demand, as we must, that the price vector be non-zero, and be exactly determined, i.e. neither under-determined (too many price solutions) or overdetermined (only zero solutions).

This functional relationship is equivalent to specifying the operator $\mathscr{F}$ as a function of **p** with parameters **r**, **A**, **w**, **a**:

$$\mathscr{F}_{(r,A,w,a)}(\mathbf{p}) = \mathbf{p}\big[(I+r)(A+a.w/L)\big] \tag{8}$$

and requiring it to map **p** strictly onto the set of all prices; that is, it must not add or remove any degrees of freedom from **p**. In more familiar terms, the number of equations must equal the number of variables. One way of satisfying this is to add two conditions:

(i) the profit rate must be scalar and uniform.

(ii) the matrix **A** must be non-singular and hence, in general, square. There must, in other words, be as many producers as products.

These conditions guarantee a unique price vector provided **A** represents an economy producing a physical surplus. The condition for unique prices to exist is that

$$\mathbf{p}\big[(I+r)(A+a.w/L) - I\big] = 0 \tag{9}$$

for some positive p, which implies

$$\det\big[(I+r)(A+a.w/L) - I\big] = 0 \tag{10}$$

or, since **r** is a scalar,

$$\det\big[(1+r)(A+a.w/L) - I\big] = 0 \tag{11}$$

and **p** becomes the dominant characteristic vector of $(A+a.w/L)$, with characteristic root $1/(1+r)$. If wages are paid *post factum* as in Sraffa this becomes

$$\det\big[(1+r)A + a.w/L - I\big] = 0 \tag{12}$$

with a determinate but slightly different relation between **r** and **w**.

These particular solutions suit the post-Sraffians since they yield a relation between the uniform profit rate and the wage which is independent of **p**, so that both **p** and the wage-profit relationship can

be treated as functions of **A** and **a** (the 'technical conditions of production') and independent of each other.

What happens to this solution and its properties if either condition (i) or (ii) above is dropped? This is studied by Albarracin and by Farjoun in this volume. If **r** is not scalar its relation to **w** is no longer independent of **p**, as Albarracin shows, for then relation (10) will give solutions for **p** which depend on the distribution of the elements of **r**. But it is unclear in any case in what sense the system is 'determined', as none of the quantities involved can be exactly calculated.

If **A** is not square or is otherwise singular it ceases to yield unique magnitudes either for **p** or for the wage-profit relation, as Farjoun points out. The maximum profit rate becomes arbitrary and ceases to bear any relation to the 'physical surplus' it is supposed to represent.

These are not mere simplifying assumptions. Without them the solution is not just different or more complex, but *ceases to exist*. The neo-Ricardian construction in general simply stops working. This is not necessarily catastrophic for Sraffa because his restrictions are related to his limited aims. For the post-Sraffians it has far more serious implications, since for them simultaneous equation systems are the foundation of a new system of political economy, to replace labour values. Within their system, these simplifications are in reality structural elements of the theory: axioms. We now turn to the study of their consequences.

4. Price, Supply, Demand and Markets

One of the interesting modern advances in von Bortkiewicz-type equation systems is the discovery, through successive advances by Winternitz, May and Seton,[10] that under the assumption of constant returns to scale, prices do not depend on the scale of production, that is, on the quantity of goods produced in each sector.

It is relatively easy to show from what has already been said that prices in neo-Ricardian systems are generally independent of the quantity of goods produced, and vice versa. This is hardly surprising, since it coincides with Sraffa's general aims. The point is related to the issue of constant returns to scale, which Albarracin in this volume discusses at greater length. Sraffa does not explicitly assume constant returns to scale for parts I and II of his work because no assumption concerning scale appears necessary, though he concedes it to be involved in part III where he discusses the choice of technology.[11]

The assumption is repeated by Steedman when he studies the

'allocation of labour',[12] which in his treatment is equivalent to the scale of production, since under constant returns to scale, production in each sector must everywhere increase in proportion to the employment of labour.

The independence of price from scale of production emerges if we consider Steedman's formulation of the equation system, where he specifies that 'the gross output of each commodity be unity by a suitable choice of units.'[13] This is formally the same as specifying the technological matrix **A** in the normal input–output manner as a matrix of inputs needed to produce one unit of output. Under constant returns to scale, such an equation system clearly does not change with the scale of production, because the elements of the matrix **A** are constants, along with **a**. If production in, say, sector 5 doubles, then the fifth equation is simply multiplied by 2, so that it is in effect the same equation.

Insofar as the scale of production is determined, there is an interesting duality. It would be given by an equation of the form

$$\mathbf{Y} = \mathbf{X} - \left(\mathbf{AX} + \frac{\mathbf{w.a}}{\mathbf{L}} \right)$$

where **Y** is the vector of surplus available for investment or capitalist consumption, and **X** is the vector of the quantity of output in each branch of production. This is in turn independent of the price structure, so that prices are determined independent of quantities and quantities are determined independent of prices.

It might be argued that if **A** varies with changes in **X** (or **a**), that is, if we drop the assumption of constant returns to scale, then the above equations will be interrelated via variations in A or a. Precisely: but under such conditions there is no longer a unique solution for **p**, **w** and **r**, as we have yet another unmanageable system relating, in this case, $n^2 + 2n + 1$ quantities through n equations. Moreover, it becomes absurd to suppose that p will remain constant over time if the scale of production changes over time. The neo-Ricardian construction is not general enough to study such a system.

The independence of price and quantity in the calculation has some unpleasant consequences. We should recall that Steedman says values cannot affect prices, on the grounds that they are not needed to calculate them. But in his system the scale of production need not be known to calculate prices and nor need prices be known to calculate the scale of production. It follows by Steedman's own logic that the price of a good cannot affect how much of it is produced or consumed, nor can the quantity of goods produced affect their prices. This is an

extraordinary conclusion, since in real life these two things have an enormous effect on each other.

Moreover, even discarding Steedman's logic, there is a still more intractable difficulty. If we try to modify the system so that there is a relation between supply, demand and prices, for example by dropping the assumption of constant returns to scale, we find that prices are doubly determined: once by the simultaneous equation model, and once again—differently—by the effects of supply and demand. This is logically impossible, since even in dialectical logic a quantity cannot simultaneously possess two magnitudes.

This adds up to a bald fact: that the interplay of market forces plays no role, and can play no role, in such models. *The market is absent*, in that its mechanisms—the interplay of supply, demand, and movements in prices and profit—are logically incompatible with the post-Sraffian universe.

5. Marx, Markets and Money

By now a vociferous objection will probably have been lodged. Marx himself constantly abstracts from the fluctuations of market prices and frequently explains values and prices of production as 'long term averages' of price movements. Moreover, he explicitly rejects the idea that variations in supply and demand objectively determine the magnitude of prices, the central issue on which labour value theory stands opposed to what has become neo-classical marginal theory.

However, there are two different concepts of determination involved, and two entirely different interpretations of a 'long term average'. In consequence both movements in market prices and the effects of supply and demand do play a role, in Marx's theory, as *mechanisms* of the law of value. Moreover this relates directly to the issues raised by Mandel, Giussani and Salama concerning the role of money and the private character of capitalist production.

The neo-classical interpretation of the average or 'natural price' is that of equilibrium—the level which prices would attain if all variation were to cease. This is a view of price which both neo-classical and neo-Ricardian theory hold in common, and which distinguishes both of them from Marx. Consider, for example, the following passage:

'The value of commodities as determined by labour time is only their average value. This average appears as an external abstraction if it is calculated as the average figure of an epoch, e.g. 1 lb of coffee equals 1 s if the

real average price of coffee is taken over 25 years; but it is very real if it is at the same time recognized as the driving force and the moving principle of the oscillations which commodity prices run through ... The market value is always different, is always below or above this average value of a commodity. Market value equates itself with real value by means of its constant oscillations, never by means of an equation with real value as if the latter were a third party, but rather by means of a constant non-equation of itself ... the two are constantly different and never balance out, or balance out only coincidentally and exceptionally. The price of a commodity constantly stands above or below the value of a commodity, and the value of the commodity itself exists only in this up-and-down movement of commodity prices. Supply and demand constantly determine the prices of commodities; never balance, or only coincidentally; but the cost of production, for its part, determines the oscillations of supply and demand,'[14]

This quite categorical view establishes that Marx by no means denies fluctuations in supply and demand a role in determining the *formation* of values and prices of production; and that his concept of long-term average is precisely what it says: the average of a varying quantity. In no sense is this identical or even comparable to the notion of an equilibrium price. This is scientifically correct, because in all but the simplest of oscillating systems the two magnitudes are numerically different. In mechanics they are different, for example, in any system in which energy of oscillation is transformed into energy of motion, that is, in which net mechanical work is performed. Thus the average behaviour of a surfboard being propelled by a wave is quite different from the behaviour of the same board in a calm sea.

Moreover, where fluctuations in supply and demand are discussed in Chapter 10 of Capital Volume 3, they are not simply noted and passed over, raised in order to be dismissed as so many interpreters imagine. Marx makes it clear that though the magnitude of prices and values are objectively constrained by the law of value, this law includes a mechanism—a qualitative and quantitative process through which commodities come to exchange against money at prices regulated by the labour embodied in them; and that this mechanism can also—as with absolute rent—play a quantitative role where there are natural obstacles that prevent the free oscillation of supply and demand balancing out over time.[15]

This underscores a crucial point about the way the word 'transformation' has been interpreted by Marx's correctors and detractors. The transformation of values into prices is not a calculation through which, given values, one can work out prices, but a process in the real world through which prices come into existence,

quite independent of whether or not the mathematical tools have been developed to calculate the magnitudes involved. In Capital Volume 3 Marx attempts to describe this real process and comes to a definite conclusion on the relation between mechanism and results.

We can fruitfully regard his famous 'two equalities' as a judgement on this relation. While shortages and surpluses can give rise to divergence of market price from value,[16] they cannot create new value in and of themselves. They can play one of two roles. They can either enter the determination of value itself by passing judgement on labour which society has performed, and deciding whether or not it is surplus to requirements; or they can, with the formation of prices of production and the role of rents (not to mention merchant and banking profits) redistribute existing value between capitalists.

This outlook distinguishes him both from marginalists, who only see the mechanism, and the neo-Ricardians, who only see the results.

For the marginalists, the play of demand and supply is in some mysterious way the source of value instead of its regulator. They analyse only fluctuations, and not their objective context. This is like studying wave motion and ignoring the fact that there are definite global quantities associated with a wave: its velocity, amplitude, wavelength and energy, linked by definite objective relations which are more comprehensive than the movement of any particular particle in the wave's path and moreover the key to understanding how the wave connects up with the rest of the world. This is what one must study to see how a board will behave when struck by a wave. But equally one cannot solve the problem by pretending the wave does not exist, as the neo-Ricardian equation systems oblige us to do.

We can illustrate the preceding points with a simple extension to a Sraffian system, also useful in studying the transformation of values into prices, in which I try to make mathematical allowance for the existence of stocks in circulation and their relation to money profits.

We begin from the first surplus-producing economy cited by Sraffa on p7 of his book. This is as follows:

Figure 1

280 qr. wheat + 12 t. iron	→	575 qr. wheat
120 qr. wheat + 8 t. iron	→	20 t. iron

I choose such a simple system, Sraffa's most basic surplus-producing economy, because my aim is to show what happens to the

most basic category of the Sraffian system—the maximum profit rate—during disequilibrium. I choose, without loss of generality, and for simplicity of illustration, a model in which labour exists only as a co-participant in the Sraffian 'surplus', so that 'profits' here actually represents a surplus to be shared between workers and capitalists, as explained by Guillén Romero in his piece. However, the points made apply equally well in the more developed versions of this system, as the reader can easily verify.

In the above system the rate of profit is 25% and the price of a quarter of wheat is equal to one-fifteenth of a ton of iron.

We now suppose a disturbance to this economy, resulting from a decision by wheat-producing capitalists to increase their supply of wheat by 20%. This decision is taken on an individual basis and without consultation or prior arrangement with the iron-producing capitalists. It is therefore only possible if there are already stocks of wheat and iron from which investment goods may be purchased. We assume that the capitalists possess such stocks, the size of which will in general be related to the time of circulation.

In order to present the analysis in its clearest possible way we assume that they possess these stocks initially in such proportions that the rate of profit remains uniform. The rate of profit will be lower since the capitalists must advance working capital to cover the costs of these stocks.

The absolute quantities of stocks of goods being processed, and tied up in circulation, are laid out below with the prices in brackets, measured in units of iron.

Figure 2

	Production wheat	iron	Stocks	Advanced capital
wheat	280 (18.67)	12 (12)	287.5 (19.17)	(49.83)
iron	120 (8)	8 (8)	10 (10)	(26)
total	400 (26.67)	20 (20)	(29.17)	(75.83)

If trading and production continue as before, the reduced uniform

profit rate is 15.38%. Now consider the effects of the investment. Assume this happens at the same time that productively consumed goods are replaced after a productive cycle. Our table will now read

Figure 3

	Productive Capital	Stocks	Advanced capital
wheat	336 (22.4)	14.4 (14.4)	231.5 (15.43)	(52.23)
iron	120 (8)	8 (8)	7.6 (7.6)	(23.6)
Total	456 (30.4)	22.4 (22.4)	(23.03)	(75.56)

We cannot yet calculate profit on the new investment because nothing has been produced or sold. Assume a complete cycle of reproduction takes place, at the end of which all productively consumed goods are simply replaced without further investment. It is still not possible to determine sales, because we have not said how the 20% increase in wheat production will be absorbed by consumption. Nor can we; and this already reveals one of the problems. Nevertheless, let us make an assumption as close as possible to general neo-Ricardian principles, which is to assume that consumption (by both capitalists and workers combined) increases in proportion to the increase in wheat production, that is, also by 20%. We can now calculate sales as the sum of productive consumption and other consumption (replacement of used up inputs plus the wage plus capitalist consumption), as follows:

Figure 4

	Output	Sales	Costs
wheat	690 (46)	456 +210 = 666 (30.4)+ (14) = (44.4)	(36.8)
iron	20 (20)	22.4 + 0 = 22.4 (22.4)+ 0 = (22.4)	(16)
total	(66)	(52.8)+ (14) = (66.8)	(52.8)

Profits can now be calculated in each sector along with a sectoral profit rate. Wheat sellers realize 7.6 in money profits on an advanced capital of 52.23; a profit rate of 14.5%. Iron sellers realize 6.4 in money profits on an advanced capital of 23.6; a profit rate of 27%. The average profit rate in the economy is 18.4%; the theoretical equilibrium maximum profit rate is exactly what it was before, namely 15.38%. These quantities are nowhere near each other.

The origin of the difference in profit rates is twofold; first, because of the increased demand for their output resulting from investment, iron producers have realized some of the capital previously tied up in stocks, whereas wheat producers have overproduced. Second, since iron stocks have diminished and wheat stocks have increased, the iron producers' profit rate is calculated on less advanced capital. This is not at all unrealistic and such effects figure in all capitalist balance sheets as a matter of course. In 1982, for example, British manufacturing industry recorded a book value of £36,567 m in stocks and work in progress of which £11,107 m were in finished goods.

It may be argued that our assumption about consumption has 'cooked the books' and that a different assumption will equalize profit rates. Yes: profit rates would be equalized at a consumption level of 307 qrs of wheat, representing a 75% increase. Which figure is the most arbitrary? Moreover whatever assumption is made, the iron-makers' profit will be 27%, nearly double the theoretical equilibrium.

The analysis above is in no sense intended to be a real analysis of a real economy nor even a correct approach to such. It is chosen to illustrate our basic point about simultaneous equation systems, which is that the standard solution simply ceases to exist in any meaningful sense once the equilibrium of the economy is disturbed, even as in this case by a relatively small amount. For example, above we assumed that goods continued to sell at the same price following the neo-Ricardian assumption. But there is in fact no a priori way to decide whether sales would actually take place at the indicated prices, whether the iron-makers would be able to put their prices up to reach an even higher profit rate, or whatever.

However, this is only half the story and the worst is yet to come. In principle, there is an escape route for the simultaneous equation method. Following a process analogous to Shaikh's iterative solution to the derivation of labour values,[17] we could 'follow through' the disturbance created by the new investment decision by assuming that in the next period there will be increased investment in iron production to cash in on the higher profits. Given stable technology,

prices and quantities will converge to a new equilibrium in which prices are determined as before and the scale of production is determined by the (exogenous) demand for the physical surplus, i.e. by some form of combination of capitalist greed and the class struggle.

In essence, this is the argument that tends to be put forward by all who use simultaneous equation systems to represent real economies. They choose to ignore the process of attaining a new equilibrium on the basis that, provided it can be shown that such a convergence could theoretically take place, economists should study not the process but the end result.[18]

But this convergence is absolutely not guaranteed if technology changes and continues to change while the adjustment process is going on, above all if the changes in the deployment of technology are a product of the adjustment process itself and take place over a comparable span of time.

In the normal course of events—taking the above as an example—investment will be in more productive technology, so that for the same or comparable deployment of capital (in price terms), physically more goods will be produced. However, while investment in new technology is going on, the old technology is still in use. Investors in new processes can realize exceptional profits precisely because they can produce their goods more cheaply without having to pass on the cost reduction to their purchasers, as long as the market price is determined by costs of production in more backward sectors. If we assume in the above model that investment in wheat production started because a new wheat production process was discovered, and 50% more could be produced for the same investment, then of course the new wheat production process would yield still higher profits than the iron makers.

At this point the neo-Ricardian system ceases to offer any guidance whatsoever. If we stick to fixed prices, the iterative process simply does not converge. If we drop the assumption of fixed prices, there is no basis either for saying what the new prices will be or what the 'physical surplus' will be, or what profits will be, at least until the new production technique has completely displaced the old. But this cannot happen rapidly, if for no other reason that that only 25% of economic production is available for capitalist consumption, workers' consumption and investment all combined. Even assuming 10% of the entire resources of the economy go into investment, and half of this into investment in the new wheat production process, it would still take nearly ten reproductive cycles to replace one process by another. What happens in the ten intervening years? What

happens if yet another technological advance comes along in five years time?

The distinctive weakness of simultaneous equation systems and particularly their post-Sraffian interpretation can be summarized quite conscisely. In Sraffa's desire to abstract from all marginal effects and all process of change, a system has been created in which the economy has no means of reaching its ideal state. It has no economic mechanisms: only economic results. It is therefore incapable of studying the economic mechanism most characteristic of industrial capitalism, the central feature of the 'production of commodities by means of commodities', namely the pursuit of differential profit originating in differential rent derived from advances in labour productivity occasioned by technical advance. This is how values are actually transformed into prices of production under advanced, industrial capitalism. We now turn our attention to this process.

6. Price, Value, and Technological Change

It could be argued that so far we have only unearthed a secondary mechanism connecting market price movements to some form of 'natural' long-term average price, and that the variations concerned are all extremely short term and will cancel out over a period of production, so that for all practical intents and purposes constant prices are a reasonable approximation. It could be argued that since Marx himself abstracts for the most part from the market mechanism, the neo-Ricardian construction is simply Marx's under a cleaner and tidier guise.

This objection cannot be sustained if it can be established that in addition to short-term fluctuations in market prices, there are also medium and long-term movements in average prices interacting with supply and demand to shape the behaviour of a capitalist economy. Are there price movements with the same sort of time scale as variations in either A or a? If so, the neo-Ricardian model collapses into vacuousness, since all quantities are changing with comparable periodicity, the system never settles down, and no simple mutual determination emerges at all. And indeed, both in reality and according to Marx, prices of production move as a function of technical change itself, that is, over the medium and long term.

Steedman's treatment of technological change makes a strange assumption, which has been less searchingly analysed than it ought. Capitalist choice is in effect treated as if all producers at once switched

between two or more alternative technologies with two profit rates and two sets of prices. Yet no indication is given of the process of change itself. A series of bizarre consequences follow, not the least of which is that capitalists would be obliged almost instantaneously to liquidate their entire stock of fixed capital almost at will in order to embark on a new technique of production, without regard either to the time it takes to supply the new fixed capital required—that is, the rate of investment—or the effect on profits of suddenly liquidating old factories, tools and stocks which have not yet realized their value.

Furthermore, Steedman and others make equally strange assumptions about what influences capitalists when they choose a new technique. Investors decide, it appears, not by looking at the profits they will get now, while they are ahead of their competitors, but on the basis of the profit they will later get when their competitors catch up, using the very technique introduced to steal a march on these same competitors. Stranger still, entrepreneurs do not look at their own individual profit rate, but at the average profit rate in the sector as a whole and, indeed, in the economy as a whole.

But this does not at all approximate to the real process of technological change, and certainly not to anything Marx ever discussed. Why does a capitalist invest in a new technique? Why, for example, was car production automated? Not at all because of the average profit which Henry Ford expected the car industry to be making in fifteen years' time, but because by stealing a march on all his competitors, he could for an extended period sell cars for the same price as them but much less than they cost him to make, at a far higher profit rate than the prevailing average and higher than the average eventually attained. It is not the average profit in a sector which influences capitalists, but the prospect of making an exceptional profit while price is still determined by the backward producers in the market, because new technology has not yet augmented supply enough to make the price fall.[19]

Such exceptional profits can exist for some period of time because capital destined for investment is not in infinite supply but is also quantitatively restricted and cannot meet all available investment opportunities at once. Supply of every commodity is therefore restricted below the maximum possible using the newest technology. No single technology is ever, therefore, totally generalized.

Therefore, the normal condition of a capitalist economy is not at all that a single technology rules, but that a variety of technologies coexist along with a distribution of profit rates within, as well as between, sectors. The neo-Ricardian discussion of technical change

introduces equilibrium considerations in the least appropriate place to do so. As fast as old technology is replaced by new, still newer technology is invented. The basis of production is, in Marx's words, 'continuously revolutionized'.

However, can simultaneous equation systems be modified to deal with this process? Not at all. The hidden axioms of the neo-Ricardian system, which we identified earlier, rule it out. First, there must be a uniform profit rate. Differential profits cannot act as a motor of change; they cannot even exist. Second, and even more devastating, the matrix of technical conditions of production is not allowed to introduce more than one functioning producer for each product.

Sraffa at least acknowledges this problem,[20] but falls back on a peculiar construction. If two producers coexist using different methods, then one is assumed to be producing a different commodity from another so as to get an extra equation. This second commodity must be 'non-basic', that is, must not enter the production of any other commodity.

It is very obliging of the producer concerned to show such respect for Mr Sraffa, but the idea is to say the least arbitrary. When you, I, or Mr Sraffa buy a pound of copper, we get a pound of a salmon-coloured malleable conductive substance and we neither know nor can find out whether it is basic or non-basic copper, whether it came from a backyard scrap firm or a third world copper-mine. This is the whole point about what a commodity is under capitalism; it acquires exchange-value because exchange abstracts from all the concrete labours which went to make it, so it becomes indistinguishable from all other commodities of the same type no matter where they came from.

If this were not so, if one paid a different price for copper depending on who made it and independent of its chemical or physical properties, and put it to different use depending on who one bought it from, then one would cease to have 'production by means of commodities' and would have production through a series of planned bilateral or multilateral arrangements. Price paid would cease to represent real transfers of money and would become instead a mere book-keeping arrangement, as it is within a large enterprise whose departments supply each other and charge each other 'shadow prices' fixed by decree and not by the market.

There is yet a third point. Sraffa's construction also serves to derive an 'independent standard of value'—the standard commodity—to use as a yardstick in comparing physical quantities of different commodities. Sraffa rightly criticizes neo-classical capital theory

because it cannot establish any independent measure of the 'quantity of capital', whence its derivation of global quantities such as the marginal productivity of capital is next to meaningless.

But Sraffa's construction by no means escapes the problem. It is hard enough to use the standard commodity, as Sraffa acknowledges, to compare physical quantities of fixed capital in different systems, that is, systems employing different technologies. But if the technology of a single capitalist economy undergoes constant change, the standard commodity itself undergoes constant change even within that system, and no invariable measure of the neo-Ricardians' beloved 'physical quantities' exists.

This is precisely the importance of labour values. If we try to use 'physical quantities' to compare the results of production using different techniques, we find ourselves unable to do so because changes in technique invalidate Sraffa's construction of a standard commodity. If we try to use prices, we find the standard of measure varies over time in an unpredictable manner and in response to factors extraneous to production as such. Labour values behave differently. They do vary over time, but in a manner which we can keep strict account of, and which is rooted in production itself, because reflective of one of the most fundamental relations between human and machine—namely, the productivity of labour.

The value added to a commodity during production, critics often forget nowadays, is not a metaphysical quantity defined by a set of equations, but in the last instance a real quantity measurable with a stopwatch, even though it owes its existence to exchange. Every capitalist company keeps the most detailed record of its labour statistics. Even in the depths of capitalist crisis we can visit any functioning factory and make a plausible provisional estimate of the value it is adding to its product. Using backward extrapolation as Shaikh proposes in this volume we can make fair estimates, not just of value added but of the total value of any stock of use-values. To the extent that we are inaccurate, the problem is one of measurement and lack of data, not one of theoretical principle. We do not need to assume that the whole system can reproduce itself for this calculation; only that private exchange takes place on a sufficient scale to abstract from the concrete labours involved and thus establish exchange value, hence that the commodities involved should be capable of being sold for money. Values *exist* and are empirically measurable, redundant or not.

This is not substantively altered because we might include a correction in later accounts if we find that some of the labour was

wasted because products went unsold. The initial statistics serve as the basis of valid calculations which can later be adjusted, just as any good capitalist bookkeeper will calculate provisional sales and profits without full knowledge of bad debts, returns, or the value of stock in hand, and will carry the difference between estimate and final figure over to the next accounting period as accrued costs or benefits. Nor is the principle altered if a commodity is devalued through technical change, and value thereby destroyed. On the contrary, this brings to light an important difference between value and the neo-Ricardian concept of dated labour. Dated labour measures the labour which has actually been expended on a commodity. If productivity does not change, this is theoretically the same as value.

However, suppose a car is made using presses made twenty years ago, when the presses required 100,000 hours of labour to construct. If the same presses or their equivalent are now made using 50,000 hours, then even the old presses will now pass progressively less value to the cars as the new presses come into use, eventually being found socially surplus to requirements as a result of technical progress. Iterative calculations with input-output matrices yield *values*, not dated labours, which could only be calculated (with difficulty) from a succession of input-output tables of different dates. Finally, the calculation of values is not invalidated if certain labours must be valued higher or lower than others either because of skills, or for other reasons, provided the difference is quantifiable.

Measurements of labour time are thus the best objective basis for studying technical change precisely because they are not derived from a future reproductive process which may well fail to work, but from the private circumstances of each individual producer as they arise from previous phases of reproduction.[21] What is Marx's presentation of technical change?

Its crucial component is identified by Savran in his piece, and touched on also by Salama: that is, the role of 'individual values'. Their existence, which cannot be comprehended by Steedman's derivation of value magnitude, is not just a convenient means of escaping criticism; it is the mechanism of superprofit.

Consider a single branch of production in which there are two capitals. One turns over values each year according to the following:

Figure 5

Constant	Variable	Surplus-value
4000	1000	1000

producing, let us assume, 6000 units of use-value. The second turns over the same values but produces 7000 units of use-value because it has a higher productivity of labour. We have assumed an identical value-composition of capital only to illustrate our point and a more general treatment is perfectly simple.

According to Marx three circumstances can arise. If supply exceeds demand, social value will be determined by the most productive capital. If supply falls behind demand, value will be determined by the least productive producer. We will treat in this example the third and most general case of a balance between supply and demand; in this case the value of the 13000 produced commodities will be equal to the total labour time added or transferred in their production, namely 12000.[22] The average value of a unit of use-value is $12000/13000 = 12/13$. At this point it is convenient to define the inverse of this as the *specific productivity* in the sector concerned: 13/12. (This is not the same as labour productivity, since it will vary with changes in the value of constant capital, though the two quantities are clearly and easily related.)

What is the individual value of the use-values produced by each capital? Simply the quantity of labour added or transferred divided by the quantity of use-values produced: for capital 1, this will be 1, for the second 6/7. Specific productivity of each capital is 1 and 7/6 respectively.

The differences in productivity will have an effect on profits. Suppose initially that goods exchange at their social value, that is, at 12/13 per unit of use-value. Suppose for convenience that 1 unit of exchange value is priced at £1, that is, £1 represents one hour of abstract labour.

Capital 1 will realize $6000 \times £1 \times 12/13 = £5,538.46$, Capital 2 will realize $7000 \times £1 \times 12/13 = £6461.53$. The 6000 hours of labour added or transferred by capital 2 have yielded a differential rent of £461.53, or a *specific differential rent* of 7.6 pence per hour, 6.6 pence

per unit of use-value sold. Capital 1 has suffered a negative differential rent of the same amount, equivalent to 7.1 pence per unit sold.

There is no essential difference if we move from values to prices of production. Let us assume that constant capital is divided into 400 in fixed, and 3600 in circulating capital in each case, again for simplicity, and that fixed capital turns over in ten years whereas circulating capital turns over four times a year. Assume variable capital turns over once a week.

In order to begin production, the two capitalists will require stocks of productive capital with the following values:

Figure 6

Fixed constant	Circulating constant	Variable
4000	900	20

In the case of variable capital, money sufficient to buy 20 units of value is advanced but the 'stock' possessed by the capitalist takes the form of hired labour-power, or the right to use the labourers' time—in our case, 40 units of such time. Strictly speaking, the 20 units of variable capital are advanced as money by the capitalist and maintained as commodities by the labourers in the shape of the week's purchases of food, clothing, and so on.

Let us assume that inputs were all bought at a specific price of £1 per unit of value. This assumption is for simplicity only and the essential results are no different if input prices differ from values. Assume the ruling rate of profit is 20%. Capital advanced is then £4920 for capital 1 and for capital 2, so that total capital advanced in the sector is £9840. The calculation can be followed through with the same essential results if input prices are higher or lower.

On the output of the sector, if the sectoral average profit is assumed equal to the global average of 20%, a mark-up on costs yields a price of £11808 for 13000 use-values, or 90.8 pence per unit. Individual sales will realize £5449 for capital 1, and £6358 for capital 2. The producers will calculate their individual annual profit rates by subtracting the money they spend over the preceding year, namely £5000 each, from their sales. This yields the following table, dividing by capital advanced to get profit rate:

Figure 7

	Mass of Profit	Rate of Profit
Capital 1	£449	9.1%
Capital 2	£1358	27.6%

This considerable difference results almost entirely from the productivity difference of 16.6%. In the next reproductive cycle things will change depending on a number of circumstances outside this analysis but not outside value analysis in general. If an individual profit rate of 28% is attractive enough for investment capital—that is, if there are not even higher individual rates of profit to be had elsewhere—new capital will flow into process 2, either because capital 2 invests its (much greater) profits in expanding production or because other capitals will get in on the act. The average composition of capital in the sector will fall at a definite rate related to the rate of investment; the specific productivity of the sector will rise and the social value of its product will in general fall, as will its price of production. Beyond a certain point, capital 1 will cease to yield any profit at all and will go bankrupt; in any case it will decline because its rate of profit is lower than that of capital 2, so that its owners will tend to disinvest, contributing to cheapening the output.

Finally, the output of this sector will, of course, gradually decline in value in a clearly measurable and definable way. As this happens, capitalists who use it as input will be affected, because their stocks of this commodity will be revalued; that is, value will be destroyed through technological obsolescence. If we want to keep track of all these processes, it then turns out that it is no longer sufficient, as in neo-Ricardian models, just to keep a record of capital turned over in a given period; one must keep a record of the stock of capital kept in each of the forms of capital identified by Marx: commodity capital about to enter production (C); productive capital (P); commodity capital seeking realization (C' and hence c); and not least, to study investment behaviour and price phenomena in their full complexity, some hypotheses and analysis must be made about the behaviour of hoards of the money-form of capital, M and M'.

All these quantities are in principle empirically measurable or deducible from empirically measurable quantities. They give us a measure of capital independent of price movements, though not of course fixed in time, and also traceable to empirically measurable

quantities. Most important, however, they permit us to study precisely what neo-Ricardian systems do not, namely the movement of capital consequent on variations in individual profit rates.

The neo-Ricardians may object that the analysis does not allow us to calculate prices. Precisely so, but neither does Sraffa's analysis. In general, the whole idea of calculating prices, as we will discuss in the final section, is vain because prices, like values, are *data*. They are empirically given, the result of a complex process with a visible end result. The problem, if one wants to make useful predictions, is not to make bets on the end result, which can be more quickly ascertained from the nearest grocer, but to find out about the process which produced it. But, as we have established, price-value deviations are not the result of the aggregate masses of value in various parts of the economy, but result from the changes in these masses, from the process of capital movement. Of course, if one abstracts from this movement, one will be unable to find any connection between value and price, because one has abstracted from the process that produces prices in the first place. If one stops a clock, one will be unable to tell the time; this does not stop time passing.[23]

Two questions then remain. First, what are the factors which determine differences in profits, as opposed to their average values? Second, what is the relevance of Marx's two equalities, and his rate of profit formula, to the above analysis?

The first question yields an important answer. Despite deviations of average prices of production from average values, there is every reason to suppose that the deviation of individual values from average values is far greater, and that the movement of capital is ultimately determined by these differences. Value magnitudes, though disguised in the price form, can and do exert a decisive influence on the very factor from which the neo-Ricardians abstract—economic change.

The point can be studied both theoretically and empirically. A model, which space does not permit us to exhibit in full, can be constructed in which each sector comprises b_i capitals ($i = 1, \ldots, n$) with outputs X_i^k ($k = 1, \ldots, b_i$), requiring use-values to be advanced in the form of productive constant capital in quantities U_{ij}^k ($j = 1, \ldots, n$) and with turnover T_{ij}^k (so that the quantity of a use value turned over in unit time will be $T_{ij}^k U_{ij}^k$),[24] and variable capital sufficient to maintain a workforce of L_i^k workers. Following the method just used we can define specific productivities τ_i^k and market shares $Z_i^k = X_i^k / X_i$ where $X_i = \sum_k X_i^k$.

We can derive a formula for differential rent per unit of use-value

$$\rho_i^k = \frac{Z_i^k}{\tau_i^k} - \frac{1}{\tau_i} \tag{13}$$

A profit and price analysis can be defined using price-value multipliers λ_i, which it is convenient to write in the form $(1 + \mu_i)$.[25] Prices are of course the same for each of the b_i capitals producing commodity i. Profits and prices are related through the formula

$$(1 + r_i^k(t))(K_i^k(t)) = (1 + \mu_i(t + \delta t))X_i^k / \tau_i^k$$
$$= (1 + \mu_i)V_i^k(1 - \tau_i \rho_i^k) \tag{14}$$

where K_i^k is the price of advanced capital and can be calculated from prices and the quantities U_{ij}^k, τ_j, μ_j as they stand at the beginning of production, and V_i^k is the value of X_i^k.

In general r_i^k is of course different from the surplus-value added in capital K. However, it then becomes relevant to find out the relative magnitudes of the different components of the deviation of r_i^k from this surplus. In static models, attention has always focussed on the price–value deviation, and not on the deviation of individual value from average value. But in the above equation for individual profit there are two terms. One represents the price–value deviation, and one the variation in individual values. If the second turns out to be in general greater than the first, then the movement of capital will be dominated by value quantities even though in the aggregate quantities of production resulting from these movements, values are disguised as prices.

But this is in fact the case. A substantial amount of data exists, particularly the material collected by the US Bureau of Labour Statistics in the 1950s, the material from the European Productivity Association in the 1960s, and more recent studies, among others by Salter, in which inter-firm differences in productivity have been studied.[26] It turns out that differences in labour productivity in quite settled industries regularly amount to some 100–200%, vastly in excess of the deviations of price from value. With the introduction of a complete new technology such as the production line, or electrical power, differences in labour productivity can be quite phenomenal and out of all proportion to price-value deviations.

Indeed the US Bureau of Labour Statistics, which persists in the best interests of the capitalist class in collecting detailed figures on

labour productivity despite dogmatic attacks in the capitalist finan-
cial press, has seen fit to justify this in terms which all participants in
the value debate should frame in gold and install on their walls:

> 'The indexes (of labour productivity) do not measure the specific contri-
> bution of labour or of capital or of any other factor of production. Changes
> in the ratio between output and man-hours of work show the joint effect of
> a number of separate though interrelated influences such as technological
> improvements, the rate of operations, the relative contributions of
> production of plants at different levels of efficiency, the flow of materials
> and components, as well as the skill and effect of workforce, the efficiency of
> management and the status of labour relations.'[27]

7. Revisiting the Two Equalities

We now return to our starting point and to von Bortkiewicz's demand
for 'feedback'. I hope by now I have convinced the reader that there is
an insuperable logical error in his approach, which carries over into
the simultaneous equation method in general; and that it is illegi-
timate to equate the results of production to its premises, because this
imposes a forced abstraction from economic motion, and hence from
all the central characteristics of commodity production.

This does not mean that the results of production have no relation
to its premises. An economy emerges from its past and perforce gives
birth to its future. However, value theory must clear away the fog of
eighty years of confusion heaped on confusion and permit the past to
produce the future instead of the other way round. The discussion has
to be dragged from the eternal present and put back in the green
world of real history.

In my view, therefore, the question to be addressed is slightly
different from von Bortkiewicz's, and arises naturally from the
discussion: Given that the actions of private commodity producers
are socialized through exchange, how do the social results of
exchange in turn impose themselves on private individuals?

To see why the issue needs to be posed this way, let us look at the
theoretical movement involved in neo-Ricardianism. At first sight,
production is private and exchange is social, in that producers take
independent decisions, and only through exchange do they discover
they are part of a social organism, when the market passes judgement
on their actions. The marginalists leave the matter there, believing
without proof that the market can instantly reconcile all private
fantasies.

Nevertheless, scientific study reveals that the apparent privacy of
production has limits. The social results of exchange enter production
as soon as circulation broadens to include the means of production:
when they become commodities. Producers must then pay apparently
given prices, apparently given wages, content themselves with an
apparently given average profit rate, and in general cannot exercise
private control over their inputs. Therefore, says von Bortkiewicz, we
must take the results of circulation as an immediate premise of
production.

But this leads to the converse error. Reason displaces animal spirits
as the guardian angel of a system which is neither wholly animal nor
wholly rational. But not all that is rational is real. Though social
constraints are imposed by previous history, capitalists still cannot
and do not plan, because they still do not know, and cannot know,
what will happen when they take their plans to market, which is
anonymous and unconscious. Commodity production remains quin-
tessentially private even in the epoch of monopoly, cartel, and state
intervention. Fluctuations in supply and demand, and capital
movements, even within definite constraints, still prevent the next
price round matching up to capitalist expectations, and their best-laid
plans go wrong.

However, these deviations from private plans are not arbitrary.
They are arrayed on a definite lawlike framework. Capitalists cannot
set fantastic prices or seek ludicrous profits, or they perish. There are
limits on what they can do, and these limits are social. When venture
capital pursues superprofit, only to find output prices collapse so that
superprofit evaporates, it confronts the social effects of its private
behaviour. Moreover these are not the social effects of exchange in
general, but specific results of the circulation of aggregate capital: of
what happens when social *aggregate* demand meets social *aggregate*
supply in the market place. The neo-Ricardians assume a priori that
these match. They do not; but the deviations between them are the
key to economic motion.

These effects, studied and codified, constitute the formal closure of
the mathematical systems I have exhibited, and make them decidable,
i.e. make it possible to produce definite results from them, either in the
form of a class of differential equation systems, or a class of computer
simulations. But they also correspond to the way Marx himself
approaches reproduction.

In Volume 2 of *Capital* Marx asks: how does circulation, which is
regulated by exchange-value, lead to the distribution of commodities
to producers for whom they serve as use-value?[28] How can an

individual capital be sure of retrieving the factories, tools, raw materials, labourers it needs to resume production, when it does not itself produce them? Marx approaches the issue by looking at the totality of produced commodities and asking how they find their way from initial sellers to final buyers. He gives a precise solution in volume 2, where goods exchange at their value, but appears not to give one in volume 3, where they exchange at prices of production.

Is this an omission? Commentators have often failed to ask the obvious question: what *constitutes* a precise solution? The problem is that *at this level of concreteness, there is no single general solution to be derived solely from the conditions of production*, because the solution depends on the economy being studied including its conditions of circulation, distribution, class structure, and so on. Even with a widely shared technology, social reproduction takes completely different forms, for example, in Britain and in Germany—not least because of the different relations between the banks and industry.

Does this mean Marxism should cease to seek such precise solutions? Does it mean Marx 'forgets' the problem? In my view, not at all. For the social effects one must study in order to see how capitalist plans are reconciled with market reality are no more or less than the *competitive struggle between capitals for a share of the annually-produced surplus-value*, which is the subject matter of the whole of volume 3.[29]

This restores the proper and legitimate subject matter of both politics and economics, namely political economy; it connects up economics and politics and studies the class struggle in all its richness. Marx's concern, which I think is the only correct one, is to explain what lies behind the class struggle—not just between workers and capitalists, but between capitalists and capitalists—by showing how battles over rent, rates of interest, relative profits, battles to raise or lower prices, tariff and tax battles, and even wars, all repose on a common substratum: the battle for the redistribution of the spoils of exploitation, in its value form.[30] What I hope I have shown with the above argument is that this concern is not a narrow political concern which can be hived off from economics, as Steedman tends to do, but is on the contrary the only formally correct way to close the mathematical models we have been discussing; different structures and relations of class forces defining different ground rules for capital and price movements.

And this is what defines the scientific function of the famous 'two equalities'; not, as von Bortkiewicz and his successors would have it, as a device for calculating prices which are already known anyway, but as an analytic instrument for going *behind* these prices and finding out how they distribute the results of production to the capitalists.

What determines that any given capitalist cannot raise her or his rate of profit to 100%, 200% or 300%? What determines that if one individual profit rate goes up, others must go down? What determines that bankers, to take a topical example, cannot extract arbitrary debt repayments from Mexico or Argentina? Ultimately the fact that there is a finite and definite quantity of new exchange value produced each year, that a finite and definite proportion of it goes to the capitalists as a whole, and that try as they might they can do no more than redistribute this amongst each other. Thus supply and demand do not cause profits and prices to vary arbitrarily but within definite limits which can be mathematically prescribed.

This basic fact emerges even if one works directly from prices, even paper money prices. If total profits are £75bn and if the banks take £15bn and the merchants £25bn, then industry will take £35bn and no book-juggling can alter it. If, moreover, commercial capital has advanced £100bn and industry £200bn, then the gross average profit rate in commerce will be 25% and in industry 17.5%, again no matter how the books are juggled—even if the issue of fictitious capital disguises the fact for a period, only to vanish with the onset of crisis. And if industry forces commerce to cut its margins and thence its profits, it cannot thereby make more than £60bn, a profit rate of 30%, by any means at all.

To express this algebraically, if the mass of realized profits is P in price terms, being $P_1, \ldots, P_k$ for each of k capitals; and if these capitals, again in price terms, add up to $K_1, \ldots, K_k$ with $\sum_i K_i = K$, the total advanced social capital; then there is a definite relation between profit rates and share of profit, namely

$$\sum_i r_i K_i = P = \sum_i P_i \qquad (15)$$

But prices are not enough to express what is going on. Suppose there is an inflationary issue of paper money which doubles paper money prices. None of the profit ratios will change, nor will the ratios P_i/P, except insofar as those capitals K_i containing a high proportion of money, as opposed to other commodities, will be reduced relative to the others; or except insofar as workers fail to recoup the loss of

purchasing power. Something real lies behind these ratios; some social substance is being divided up. What is it? Marx is clear: it is value. In order to express this division as a distribution of value, price of production is analysed as a transformed form of value and profit as a transformed form of surplus-value. Expressed in the simplest possible way, the sum of prices equals the sum of values, and the sum of profits equals the sum of surplus-values.[31]

Before we turn, finally, to assess these two assertions mathematically, we ought to ask whether there is an alternative way of discussing distribution. From the outset we note that neo-Ricardian systems in general are badly suited to the job, because in them profit rates are permanently and everywhere equal, so there can be no competitive struggle. There are, however, deeper methodological objections.

The Sraffian school in general has made a lot of representing distribution between workers and capitalists as a battle over surplus product, rather than surplus-value. However, this becomes very dubious once we allow for any variation in the physical make-up of the national product. If workers buy videos and stop going to the cinema, who is to say whether the real wage in physical terms has risen or fallen? Indeed, if workers buy videos and capitalists visit the theatre, who is to say which has appropriated the biggest share of the social product? Once constructions such as the 'standard commodity' fall by the wayside, the whole project of measuring distribution of physical terms gets very arbitrary, as emerges in the problem known in economics as the 'index number problem'.[31]

Now, things improve if we use price measures, in that prices at least make unlike goods commensurable, but awkward problems remain. In 1961 British workers made £16,396 m; in 1981 £146,310 m. Are they nine times better off? Clearly not, because the money now buys less. But how much less? The orthodox solution is to compare the physical bundles of goods which could be bought with the wage in the two different years. But this puts us right back where we started, with the index number problem.

The only half-sane, and intuitively reasonable approach is to express the price of a share of the national product as a proportion of national income, as a proportion of the total price of commodities thrown into circulation. But then the issue is posed with a vengeance: what real substance does this total price represent? Clearly the total price of the commodities produced in 1982, with three million unemployed, does not represent the same thing as in 1962, when under half a million were unemployed.

The only *genuinely* invariable 'standard of measure' for assessing

the share of social product which anyone or any capital appropriates is its value, for the simple reason that , abstracting from relativistic time-dilatation, an hour in 1982 had just as many minutes in it as an hour in 1962.

Only one single, accountable source of variation in labour values exists; its productivity, which even the Bureau of Labour Statistics acknowledges as the finest synthetic measure of the diverse effects of the many 'factors of production'.

More precisely, because different concrete labours are reduced in exchange to homogeneous abstract labour and because labour-power is the only commodity which appears as a direct input in every other commodity, it and only it can serve as a universal standard of measure; moreover to the extent that money can be used as a standard of measure, it is precisely and only because the money-commodity itself directly represents a determinate quantity of social labour.

However, when we approach the problem in this way, that is, when we understand that value must serve as a measure of what is appropriated in circulation, as well as what emerges from production, a question immediately arises. The total process of circulation includes not just the exchange of commodities for sale against money, but the subsequent purchase of commodities for use, with that same money. The movement of circulation is not just C–M, but C–M–C, or to be more precise still,

Figure 8

$$C' \begin{Bmatrix} C \\ -M' \\ c \end{Bmatrix} \begin{cases} M - C \!\! \diagup^{L} _{\diagdown mp} \\ m - c \end{cases}$$
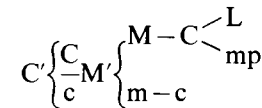
Von Bortkiewicz's presentation of Marx's 'equalities' is a very strange one, because it arrests the circulation process midway. It compares an aggregate of commodities in the form C' or c with a second aggregate in the form M' or m. Marx poses it rather differently:

'It is clear enough that the average profit can be nothing other than the total mass of surplus-value, distributed between the masses of capital in each sphere of production in proportion to their size. It is the sum total of the realized unpaid labour, dead and living, *in the total mass of commodities and money that accrues to the capitalists.*'[33]

At the very least, this is a different and more sophisticated way of putting it. For the neo-Ricardians, obsessed with the problem of numerical calculation, the issue is to compare capitals or sums of capitals entirely in their M form, with the same capitals in their C form. For Marx, the problem is to establish what share of produced value is appropriated by the different classes and sub-classes in society. The neo-Ricardians' mathematical formalizations do not permit us to answer the question Marx was asking.

I am not at all sure what will be the eventual mathematical outcome of the debate around the question as posed by von Bortkiewicz. It is a great deal more complex than most commentators have realized, as the contributions from Shaikh, Mandel and Giussani establish not least because money itself is a commodity and a component part of values in circulation. But important though this discussion is, it does not seem to me that the vindication of value theory depends on its outcome. As is explained in the introduction, the substantive issue is whether or not new value can be created in circulation, and whether or not value can be transferred from workers to capitalists in circulation. If the answer to both questions is no, the decisive component of Marx's value analysis survives intact, and in particular it vindicates the project of analysing price formation as the outcome of a competitive struggle between capitals for a share of surplus-value.

But the answer to these questions *is* no, even in von Bortkiewicz's framework, and almost (but not quite) trivially so. Total value appropriated must be equal to total value thrown into circulation, because exchange simply redistributes the same physical products to new owners. And under simple reproduction it is relatively easy to show that the total value appropriated by the capitalists is equal to the total surplus-value thrown into circulation, deviations from this rule being possible if the actual number of workers is expanding or contracting, (more generally, if the absolute mass of variable capital is changing at a different rate from the absolute mass of value in circulation), or if value is carried over from one cycle to the next.

This can be seen in the following example, derived from von Bortkiewicz's example, which displays the total circulation of commodities in the form which Marx considered the most general, namely the circuit beginning with C'.[34]

Let us assume that gross transfers of value take place, in a three-sector economy divided into von Bortkiewicz's sectors I, IIa and IIb (luxury goods), as follows:

Figure 9

	C	V	S		Output
I	300	120	80	→	500
IIa	80	96	64	→	240
IIb	120	24	16	→	160
Total	500	240	160		

The vector of surplus-value is

$$s = \begin{bmatrix} 80 \\ 64 \\ 16 \end{bmatrix}$$

We can also define a vector e, following Seton, of surplus commodities (commodities destined for capitalist consumption); it is

$$e = \begin{bmatrix} 0 \\ 0 \\ 160 \end{bmatrix}$$

Prices of production can be assigned so that these values circulate if exchanged in proportion to these prices in many different ways. We choose one such, which corresponds to an equal profit rate of 1.125, the case studied by the neo-Ricardians. This yields the prices system, with some small errors due to rounding:

Figure 10

	C	V	Profit	Output price
I	309	103	91	514
IIa	82	82	37	205
IIb	123	21	32	100
Total	514	205	180	819

What happens to the produce of sector IIb? Clearly it is purchased

by the capitalists in proportion to their profits. But it is reasonable to ask what are the values of the goods they receive. These are given by the vector

$$\begin{bmatrix} 91 \\ 37 \\ 32 \end{bmatrix}$$

which can be compared with the vector of surplus-values to show that in circulation the capitalists have gained or lost surplus-value according to the vector

$$\begin{bmatrix} +11 \\ -27 \\ +16 \end{bmatrix}$$

This vector would differ, depending on relative profit rates, if prices of production were different, showing that the surplus-value is indeed distributed differently between capitals as a function of profit rates and, consequently, as a function of a competitive struggle. Furthermore the differences are real and not just symbolic. If the luxury sector produces only Jaguar cars then the IIa capitalists have lost 27 cars, and if they push their profits up they will get them back.

We can now display a schema showing how value is transformed for each capitalist at each stage in its circuit.

We emphasize in this diagram, by putting prices and values beside each other for every form of each capital, that commodities possess a value beside their price, even after circulation. That is, if a capitalist uses money valued 120 hours to purchase commodities valued at 100 hours, then these commodities transfer 100 hours of value into production, not 120. It seems to me this is the only reasonable way to express what goes on in production, in which living and dead labour confront each other. Living labour, no matter what the price of production of variable capital, discharges its function as *labour-power*, as work measured in time. If a labourer works eight hours, these eight hours do not expand or contract with the price of food; and they cannot be properly added to the value coming from constant capital unless this too is expressed in terms of the value it acquires through production, unmodified by circulation.

It may be argued that this introduces redundancy. Not so; price, value, and use-value are all necessary to a complete analysis. But price

Figure 11
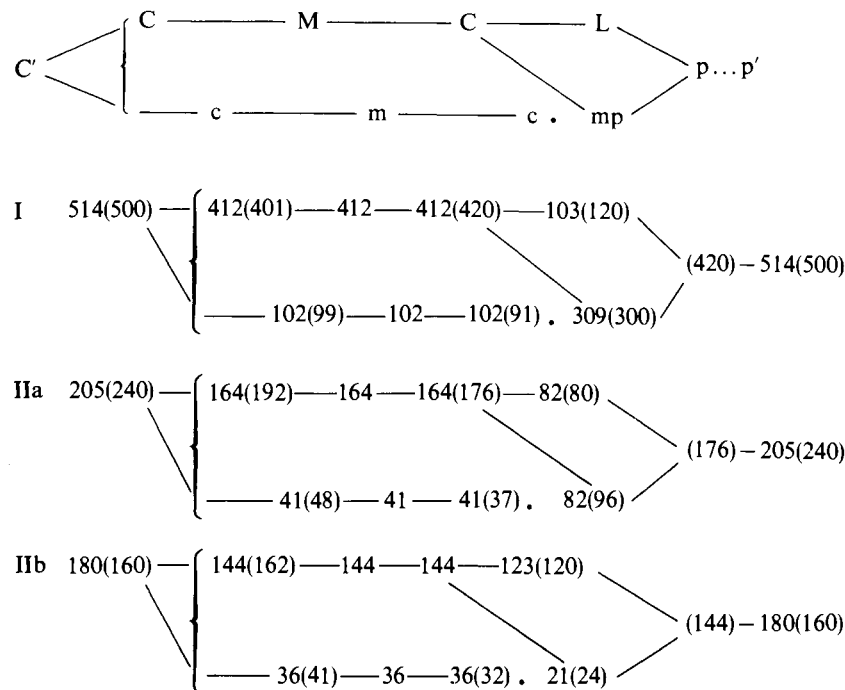
Figures in brackets are values

is the transformed form of value, which therefore comes to the fore in circulation (for example, when the capitalists calculate their profit rates, or the size of their advanced capital) rather than in production. To put it as we did earlier; the social product is ultimately appropriated in the form of value, not in the form of price. In this form, Marx's two equalities hold, and make perfect sense.

We are now in a position to assess both Steedman's logical case, and the direction of development which future formalizations of labour value theory might most fruitfully take.

8. Real and Metaphysical Determination

Steedman's main redundancy argument against labour values is that prices may be determined without reference to them. However, what

does he mean by 'determination'? In his summary statement he writes the following:

'the physical quantities of commodities and of labour specifying the methods of production, together with the physical quantities of commodities specifying the given real wage rate, suffice to determine the rate of profit (and the associated prices of production); ... the labour-time required (directly and indirectly) to produce any commodity—and thus the value of any commodity—is determined by the physical data relating to the methods of production; it follows that value magnitudes are, at best, redundant in the determination of the rate of profit (and prices of production).'[35]

However, on p.47 of his book, next to his oft-discussed diagram showing the relations of determination between prices, use-values and values, the accompanying text reads:

'Starting from the physical conditions of production and the real wage, one can derive values and surplus-value, showing how the values of commodities other than labour-power depend only on the (technically and socially determined) physical conditions of production, while the value of labour-power and surplus-value depend, in addition, on the real wages of the workers ... one can also derive from the physical picture of the economy a coherent theory of profits and prices. In doing so, however, one finds that, in general, profits and prices cannot be derived from the ordinary value schema, that $S/(C + V)$ is not the rate of profit and that total profit is not equal to total surplus-value.'

The word 'determine' does not appear here; its place is taken by the word 'derive'. The two concepts are, for Steedman, identical. There is only one other reference in the text to a concept of determination which might differ from the above; this is on p.30, where he asks which of the two profit rates (according to his definition of prices, or his definition of values (will 'affect the capitalists' decisions and actions'. Elsewhere the concept of determination is unequivocal, repeated many times, and always in contexts which make it clear that when Steedman says a quantity is determined, he means it can be calculated, and vice versa.

In short, causality and calculation are for Steedman one and the same thing. This notion of causality has to be rejected on no less than four distinct grounds.

First, there is an inherent logical problem in such a view of determination, well known in econometrics. Suppose a set of

quantities x, y, z, and so on are interrelated by a set of equations. How do we know whether x and y determine the value of z, or whether z and y determine the magnitude of x, or whether z and x determine the magnitude of y? In general there is no intrinsic basis for deciding. Thus, suppose in a Sraffian system that profits, prices and physical conditions in all but one sector of production were given exogenously. It would then be possible to calculate the necessary physical composition of the final sector of production. Can one infer that the technology of iron production is 'determined' by prices, the remaining technology, and the profit rate? In formal logical terms, the argument is identical. One requires an external, i.e. an economic argument, to explain why technology must be treated as predetermined and prices as endogenous. But no such argument is provided. It is simply 'written in' to the equations.

The *second* point is that it is not true that simply because a variable does not enter a calculation, particularly a summary or final calculation—what econometrics terms a 'reduced form'—it must be excluded from all causal mechanisms. This is easily established with an example from mechanics. Using Newton's three laws, one can write an equation for the motion of a pendulum in which the mass of the pendulum turns out to be irrelevant, because it moves with a periodicity related only to its length and the acceleration due to gravity. This does not mean the concept of mass is an irrelevancy to determining the motion of a pendulum, as you will discover if you try to build a weightless pendulum.

An even more apposite example is that of electromagnetic radiation. In the nineteenth century, Maxwell wrote down a set of differential equations explaining the relation between varying electrical and magnetic fields. In free space, the solution to these equations turns out to define the motion of light. This discovery was one of the most exciting of the nineteenth century, the foundation of all modern telecommunications and a great deal of modern physics notwithstanding subsequent advances in quantum mechanics. Nevertheless Maxwell's equations still play little or no role in the science of optics, for the simple reason that the path of a beam of light can be calculated on the basis of a number of general equations most of which in fact apply equally both to waves and particles, and involve no mention of electrical or magnetic fields. It would be absolutely absurd, however, on this basis to claim that electromagnetic phenomena are redundant in the study of light, since they illustrate all its deepest properties.[36]

However, a *third*, more telling point is this: there is no branch of

science whatsoever in which any serious investigator uses a concept of causality independent of *time*. Of course, it goes almost without saying the Marx's concept of 'laws' constitutes a concept of 'laws of motion'. The study of motion and change is the essence of dialectics. But one need not be a Marxist to reject the idea that two simultaneous events can 'cause' each other. We deduce that a bullet causes death because a person is alive before being shot and dead afterwards; that a jet causes a plane to fly because the plane takes off after the jet has been started and not before. This is no less true for static equilibria where forces such as gravity, tension, pressure and so on are said to be the cause of the equilibrium. When a roof rests on a wall, we say the wall causes the roof to stay up because when the wall is removed, the roof falls. If the roof failed to fall we would not say the wall supported it. Behind all equilibrium is movement, and even equilibrium relations cannot be revealed without disequilibrium analysis. The most general study of equilibrium in mechanics, namely Lagrange's method, operates precisely by studying the effect of small perturbations on the energy of a system.

It follows that even if it were permissible to study economics by analysing the behaviour of static equilibrium models, which it is not, we could only make inference about causality by studying the effects of a disturbance to the equilibrium, and that it is entirely wrong to try and infer causality from static relations between moving objects.

There is, finally, a *fourth* and slightly distinct point which perhaps affords the deepest insight of all. Steedman's reference to the 'rate of profit which affects the capitalists' actions' contains the germ of a more correct approach to causality, if we take it to be the basis of a real study of the role of capitalist consciousness in economic movement. The difficulty with marginalism is that it seeks an explanation of consumer behaviour solely in subjective consciousness, in the secret desires of the buyers. Both Marxism and neo-Ricardianism reject this. Nevertheless, consciousness does play a definite role in economic analysis, because when one has outlined the objective laws governing its movement, one must also show how these manifest themselves in the consciousness of agents. There is, one must agree, no point in producing a completely coherent theory of price and value determination that cannot show how capitalist behaviour (and workers' behaviour) actually implements this determination.[37]

This might appear to be the post-Sraffians' strongest point; in reality it is their weakest. What does actually affect capitalist behaviour? To be sure, it is affected by price phenomena and they are not necessarily conscious of the value relations behind prices. But

their behaviour is not governed by the hypothetical equilibrium profit rate predicted by the post-Sraffian models, for the simple reason that this theoretical ideal is never attained. The actual quantities affecting capitalist behaviour—individual profit rates—are not visible in a neo-Ricardian system. So what does determine capitalist behaviour for the post-Sraffians?

In section 3 we observed that, strictly speaking, a Sraffian system cannot meet new demand except through a balanced and simultaneous increase in all sectors to ensure there is no excess product.

How could such an increase take place? What form of consciousness must be assumed so that capitalists in widely different parts of the economy can co-ordinate their actions to bring about a harmonious result? Only *conscious co-ordinated planning* could achieve it. Only if each capitalist knew what every other capitalist were doing, where to obtain each part of their inputs and where to dispose of each part of their outputs, could they ensure that there was no disturbance of prices caused by fluctuations in supply or demand.

In short, the post-Sraffian concept of causality excludes the central feature of capitalism, which all contributors to this book stress—that production is private and producers are not conscious of each other's actions or the social results of their own actions. This concept of causality cannot model the consciousness of agents in a commodity economy.[38]

But this is not all. Where are the planners? There are none, so that the system takes on a profoundly idealist character. The planning agent is the equation system itself, which has incarnated itself in the real world as a causal agent. Descending like cabbalistic lightning from mathematical heaven to vulgar earth, it demands that the inner thoughts of every capitalist and every worker become miniature reproductions of its mystical inner self.

There is a striking duality between such systems and the general equilibrium systems devised by Walras in the 1930s using marginal methods. Their weakness, on which even sympathetic interpreters agree, are twofold. They have no market mechanism, and they behave unpredictably if trading goes on at disequilibrium prices. General equilibrium theory creates a *deus ex machina* in the shape of the Walrasian auctioneer, a benign but mythical figure who consults all agents concerning their inmost desires, and then announces optimum equilibrium positions, which agents then adopt and everyone lives happily ever after.

Sraffian systems encounter the same problem from the opposite

side. A simultaneous equation system is neither more nor less than a Calvinist Walrasian auctioneer, austerely indifferent to agent's desires, who assigns them to their predestined role in the great eternal equilibrium on the basis of their allotted portion of technology, condemning them to live out their days forever producing and consuming the same thing at the same price.

This brings us to a final point concerning the fundamental difference in goals between Marx's inquiry and Steedman's, and its consequences for the study of transformation and social reproduction.

What is the purpose of economic inquiry into capitalism? Not, fundamentally, to take its existence for granted and explore its ideal forms, but to take its existence as fact and study its historical limits. Not to study why it can theoretically survive, but how it is actually breaking up. Not to study its ideal equilibria but its real crises. This is so, not just for moral but for scientific reasons. We can readily agree that any scientist who assumes that a theory is immutable and not subject to change and evolution is a fool and a bad scientist; but even more so someone who assumes the same thing of her or his object of study!

What, therefore, is the purpose of studying social reproduction? Marx's reproduction schemas in Volume 2 are not dedicated to the same aim as the Sraffians'. He does not begin by assuming that the economy reproduces itself, in order to find out how goods exchange. He begins by assuming that goods exchange, in order to find out how the economy reproduces itself. The purpose is to study no less than Adam Smith's 'hidden hand' — how it can be that private decisions by independent producers can lead to a coherent social effect which was not consciously planned by any of them.

Many inconsistences and contradictions arise from this study, because generally speaking capitalism does not reproduce itself. The problem of research is fundamentally an empirical one, to determine which of these contradictions is a pure theoretical fiction, a misrepresentation of the real world, and which is empirically true. Theory must be revised to follow reality, not vice versa as with Steedman. In this research, values, prices and profits are not deductions but *data: given* measurable quantities. Reproduction is not given: it is deduced. The problems, I repeat, is to show how exchange causes reproduction—not how reproduction causes exchange. Marx's own statements in Volume 2 make this clear. Thus

'The continuous supply of labour-power on the part of the working class in department I, the transformation of one part of department I's commodity capital back into the money form of variable capital, the replacement of a part of department II's commodity capital by natural elements of constant capital II_c — these necessary preconditions all mutually require one another, but they are mediated by a very complicated process which involves three processes of circulation that proceed independently, even if they are intertwined with one another. The very complexity of the process provides many occasions for it to take an abnormal course.'[39]

This is very remote from the 'feedback' assumption in the form which von Bortkiewicz demands. Marx merely sets out to show that it is *possible* for an economy to reproduce the use-values used in production even though the producers do not know how this is done. In the normal course of events, this will not happen perfectly, or not at all. Hence the very careful basis on which he explains how he uses his 'abstraction' of simple reproduction:

'Simple reproduction on the same scale seems to be an abstraction, both in the sense that the absence of any accumulation or reproduction on an expanded scale is an assumption foreign to the capitalist basis, and in the sense that the conditions in which production takes place do not remain absolutely the same in different years (which is what is assumed here). But since, when accumulation takes place, simple reproduction still remains a part of this, and is a real factor in accumulation, this can also be considered by itself.'[40]

It is a far cry from saying that simple reproduction is the actual state of any economy, even an abstract one. To say that simple reproduction 'is a part of' a real economy means that a real economy is to be treated as simple reproduction plus additional elements, that is, plus some use-values which are not properly circulated, plus some use-values which are not realized, plus some use-values which are used in accumulation, plus sectors of the economy where used-up means of production are not replaced because they are obsolete — and so on.

The distinction in logical method is so emphatic that we can illustrate it as follows: suppose it were finally and conclusively proved that simple reproduction could not take place if the sum of values were not equal to the sum of prices and the sum of profits to the sum of surplus-values. *One would then have to conclude, as a Marxist, that the economy could not properly reproduce itself for this reason*, and begin to treat the transformation of value into price as a real factor in

capitalist crises. Only if this prediction failed to find empirical confirmation could one finally reject value theory as unfounded.

One and only one test, a test which is remarkably and singularly absent from post-Sraffian writings can be the final arbiter of theory: the test of practice. As Albert Einstein, whose authority on such matters can hardly be questioned, remarked: 'The sceptic will say "It may well be true that this system of equations is reasonable from a logical standpoint. But this does not prove it corresponds to nature." You are right, dear sceptic. Experience alone can decide on truth.'[41]

3. Ibid., p. 49.

4. 'It has been shown that the proximate determinants of the rate of profit, the rate of accumulation, the prices of production, the social allocation of labour, etc., are the physical conditions of production, the real wage and the capitalist drive to accumulate. The next step is then to investigate the social, economic, political, technical, etc., determinants of those proximate determinants... Such study can no doubt draw on much of Marx's work, as one source amongst the many which will be needed. *But it will involve no reference to Marx's value magnitudes*, which are mere derivates of the things to be explained.' Ibid., p. 207, emphasis in original. This is a complete scientific project involving the reconstruction of political economy, and though it shares the mathematical system of Sraffa it is not the same project. See Sweezy, 'Marxian Value Theory and Crises', *The Value Controversy*, London, 1981, p. 21, who quotes the following remark of Joan Robinson: 'Piero has always stuck close to pure unadulterated Marx and regards my amendments with suspicion.' Robinson, 'The Labour Theory of Value', *Monthly Review*, December 1977, p. 56n.

5. Though value theory is not refuted but confirmed by a proper study of joint production, as Farjoun shows in this volume, joint production nevertheless plays a special role in Sraffian theory which is irrelevant to value theory, and very unsatisfactory in general. Its real purpose is the basis of a strange treatment of fixed capital in which capitalists 'produce' their unused fixed capital and resell it to themselves annually. This is very doubtful since it is at best an accounting transaction and certainly not a real sale; for example the capitalists do not have the option of buying someone else's unused capital instead of their own. Is the entire universe in a permanent frenzy of exchange by virtue of merely existing? This aside, 'genuine' joint production where outputs emerge in fixed proportions is extremely rare, and is confused by the Sraffians with multiple production in which the same factory can produce many different things but there is a choice of what to make. Even in the case of coking and oil fractionation, to which the Sraffians refer in support of their case, study shows that in practice refineries and coking plants can and do vary the proportions of outputs by using different admixtures of varying grades of crude oil or coal, so that in reality virtually all industries can independently control the quantity of every product sold. This is not to say that multiple production is not deserving of study; but it is not what the Sraffians are talking about.

6. A less trivial example is the famous correspondence between Bertrand Russell and the German logician Frege. Frege devised the first complete formalisation of mathematical set theory. Russell sent him a note expressing a very deep paradox in Frege's concise notation, a paradox which turns out to be the archetype of an entire class of logical paradoxes including the famous 'liar' paradox. Frege had unwittingly produced a system in which this could be very concisely expressed, so that when he read Russell's one-line note, he is said to have exclaimed that his life's work was ruined. Yet his system remains the foundation of most modern mathematics and, had it been junked because of this paradox, mathematics as we know it would not exist. This is not to say there are not inherent limitations in the Frege–Russell approach, which derived from the 'corrections' introduced by Russell. But these limitations derive from the attempt to reduce all mathematics to logic, rather than the paradox itself. See Andrzej Mostowski, *Thirty Years of Foundational Studies*, Oxford, 1966.

7. Piero Sraffa, *Production of Commodities by Means of Commodities*, Cambridge, 1960, preface.

8. Steedman, p. 18.

9. It is, of course, possible to exhibit systems in which, for example, a vector of profit rates replaces the single uniform rate, as Steedman does on p. 180. The point is,

Chapter Ten

1. Ian Steedman, *Marx After Sraffa*, London 1977, p. 29.
2. Ibid., p. 49.

however, that when this is done there are no longer the same number of equations as variables and neither prices nor profits can be uniquely determined. See note 20 on Morishima's treatment.

10. K. May, 'Value and Price of Production: A Note on Winternitz's Solution', *Economic Journal*, December 1951, Francis Seton, 'The Transformation Problem', *Review of Economic Studies*, vol. 24, 1957, pp. 149–160.

11. Sraffa, preface.

12. Steedman, p. 184, where he says 'If there are constant returns to scale and if wages are paid in advance...' and adds in his footnote 'Both assumptions will be made throughout this chapter, the former being essential to the argument, the latter merely a convenience.'

13. Steedman, p. 50.

14. Karl Marx, *Grundrisse*, Harmondsworth, 1977, pp. 137–39. See Rosdolsky, *The Making of Marx's Capital*, ch. 4, London, where there is an excellent discussion of this and related passages.

15. Space does not permit a full discussion of the relation between this mechanism and the role of money, which Mandel deals with at greater length. It should be clear with a little thought, however, that the mechanism is possible precisely because goods exchange against a third commodity, money, and not against each other. In a barter or semi-barter society, supply and demand variations would call forth *independent* movements in the ratios of exchange of each commodity against each other commodity so that a price vector in the normal sense would cease to exist.

16. Market price also, of course, diverges from price of production. Value, price of production and market price are three distinct entities, both conceptually and actually.

17. 'Marx's Theory of Value and the "Transformation Problem"', in Jesse Schwartz, ed., *The Subtle Anatomy of Capitalism*, Santa Monica, California, 1977.

18. See, for example, Steedman, p. 128, where he discusses the falling rate of profit under the assumption that all capitalists will adopt the technology which gives optimum profits for society as a whole. He continues 'some writers have been tempted to confuse this straightforward argument by asserting (correctly) that decentralized, individual decisions need not always lead, in aggregate, to the achievement of the commonly pursued objective... This 'argument' is just silly. For unless the previously adopted technique is no longer available, it is being asserted that, after the change, capitalists are no longer maximising the rate of profit attainable with the given wage! Even if a new invention should lead many capitalists mistakenly to adopt it, as soon as it is found to be less profitable than the previously used technique, all capitalists will revert to the latter. With a given real wage, the rate of profit can be lowered only by technical *regress*, never by technical progress.'

19. 'It has been said that competition equalises profit rates between the different spheres of production to produce an average rate of profit, and that this is precisely the way in which the values of products from these various spheres are transformed into prices of production. This happens, moreover, by the continual transfer of capital from one sphere to another, where profit stands above the average for the time being... This movement of capitals is always brought about in the first place by the state of market prices, which raise profits above the general average level in one place, and reduce it below the average in another.' *Capital*, Volume 3, p. 310. Even more explicitly, 'Market value, (and everything that was said about this applies with the necessary limitations also to price of production) involves a surplus profit for those producing under the best conditions in any particular sphere of production. Excluding all cases of crisis and overproduction, *this holds good for all market prices*, no matter how they might diverge from market values or market prices of production. The concept of market price means

that the same price is paid for all commodities of the same kind, even if these are produced under very different individual conditions and may therefore have very different cost prices.' *Capital*, volume 3, p. 301 (our emphasis). See E. Mandel, *Late Capitalism*, London, 1973, where the role of technological rents as a mechanism of the law of value was first systematically developed.

20. Sraffa, pp. 82–83. An alternative approach is outlined in the final chapter of Morishima, *Marx's Economics*, Cambridge, 1973 — building on the work of J. von Neumann, Steedman uses some of this work in chapter 13 on the determination of labour allocation. The approach uses linear programming methods to determine which of a large number of possible processes will be used under the assumption that society as a whole will try to maximise its rate of profit. However, this does not escape my central point which is that many different techniques of production *simultaneously* in use; furthermore it introduces weird assumptions such as that discussed by Farjoun in this volume, namely that excess products are sold at zero prices. Finally, with the introduction of the completely arbitrary idea that a society of private producers strives consciously to maximise its average rate of profit, all prospect of modelling real economic mechanisms under commodity production are thrown to the winds.

21. Among many reasons for labour-time as the foundation of value is one which receives less allowance than it ought, labour-power is virtually the only commodity whose use-value has strictly speaking, a two-dimensional magnitude, one component being the *time* dimension, the other the number of labourers. If we study speed of any economic change — for example, the rate of new investment in new processes — labour time is the only way we can relate the speed of this process to the rate of creation of new value. Neither 'price' nor 'physical quantities' can do this properly, because neither has a time dimension. See section 8 of this piece and also Mandel's comment in footnote 24.

22. There is a considerable discussion on Marx's analysis of the relation between supply, demand and the formation of market (social) values from individual values, for which see Rosdolsky, pp. 89–93. In the calculations which follow, I treat only Marx's 'intermediate' case in which market value is the average of individual values. The alternative cases can be analysed into the model using the same essential method, but two complications present themselves. First, some assumption must be made (on the basis of empiricial observation) as to what relation must exist between supply and demand (i.e. some assumption on the level and rate of change of unrealised commodities seeking realisation) to determine which case applies. Second, if market value is not the average of individual values, then some labour must be counted as more or less productive than the average and the total labour time of society no longer adds up to the total new value created; as with skilled labour, the reduction of concrete to abstract labour then involves quantitative as well as qualitative changes. An alternative, which I prefer, is to treat market value as always being the average of individual values, and deal with Marx's other two cases (which he himself regards as exceptional) as forming prices of production which include a component of rent as well as average profit.

23. Nor does it make the clock accurate, even though as the logical Charles Dodgson remarked, it will be right twice a day.

24. There is a specific reason for separating out the effects of turnover in this way. The model keeps track of the quantity of goods and values tied up in production, and distinguishes this clearly and from the outset from the quantity turned over. This 'minor' distinction is rarely made although it can be done even in the framework of a simultaneous equation model. One consequence is to systematically obscure the discussion on profit rate. Marx calculates profit as the capitalists do, on capital advanced and not on capital turned over. On this basis his derivation of rising organic

composition is a lot clearer. If one takes the total labour of society as approximately constant (i.e. abstracts from population growth) and assumes that a proportion of it is each year invested and not immediately turned over, then the stock of dead value tied up in production must, ceteris paribus, increase faster than living value. Of course, advances in productivity will then reduce the values of these stocks but the mechanism will be distinctly different from that assumed in most discussions on the question, because these stocks of fixed capital are not devalued gradually through cheaper replacements, but suddenly and abruptly in the crisis phase of the business cycle, when society discovers their new values through a general surplus of unrealised value seeking realisation. Because value tied up in fixed capital does not seek realisation directly (i.e. the products of these factories), the cheapening of the elements of constant capital is by no means as simple as it appears in models where constant capital is assumed to circulate completely in each production period. See Marx's Letter to Engels, April 30, 1868, in *Letters on 'Capital'*, K. Marx, F. Engels, London, 1983.

25. We write our price-value multipliers in this way in order to emphasise Marx's concept that the transformation process involves *transfers of value* from one sector of the capitalist class to another. The quantities represent the proportion of value transferred in or out of a given capital, per unit of value sold. Marx's proposition that total value equals total price then boils down to the equation $\Sigma\mu_i = 0$.

26. See for example, the series of studies reviewed and often presented in the *Bulletin of the European Productivity Association*, Paris, 1956–61 and the series published by the Bureau of Labour Statistics under the general rubric 'Case Study Data of Productivity and Factory Performance' in the 1950s. There is a considerable bibliography in Zoltan Roman, *Productivity and Economic Growth*, Budapest, 1982. The surge of interest in inter-firm productivity comparisons in the fifties and sixties had died away, partly because firms are unwilling to divulge what is clearly sensitive commercial information in more competitive times. See, however, W.E.G. Salter, *Productivity and Technical Change*, Cambridge, 1969. Note that the quantity 'specific productivity' we have defined above is not identical in magnitude to labour productivity since it includes a contribution from constant capital, i.e. from dead labour.

27. US Bureau of Labour Statistics, *Relationship Between Productivity Measurements*, undated. The BLS is simply replying to the charge that, by considering only the productivity of labour it is considering only one 'factor of production'. It is virtually compelled to admit that in comparative studies, labour presents itself as the real basis of all so-called factors of production: Marx's entire point in a nutshell, confirmation of his view that in controversies amongst bourgeois economists, the statisticians turn out to be right against the theoreticians ninety-nine times out of a hundred.

28. 'As long as we were dealing with capital's value production and the value of its product individually, the natural form of the commodity product was a matter of complete indifference for the analysis, whether it was machines or corn or mirrors... Insofar as the reproduction of capital came into consideration, it was sufficient to assume that the opportunity arose within the circulation sphere for the part of the product that represented capital value to be transformed back into its elements of production, and therefore into its shape as productive capital, just as we could assume that worker and capitalist found on the market the commodities on which they spent their wages and surplus-value. But this purely formal manner of presentation is no longer sufficient once we consider the total social capital and the value of its product. The transformation of one portion of the product's value back into capital, the entry of another part into the individual consumption of the capitalist and working classes, forms a movement within the value of the product in which the total capital has

resulted; and this movement is not only a replacement of values, but a replacement of materials, and is therefore conditioned not just by the mutual relations of the value components of the social product but equally by their use-values, by their material shape.' *Capital*, volume 2, Harmondsworth, 1978, p. 470.

29. 'The "social need" which governs the principle of demand is basically conditioned by the relationship of the different classes and their respective economic positions; in the first place, therefore, particularly by the proportion between the total surplus-value and wages, and secondly, by the proportion between the various parts into which surplus-value is itself divided (profit, interest, ground-rent, taxes, etc.)' *Capital*, volume 3, p. 282. 'Demand and supply, on further analysis, imply the existence of various different classes and segments of classes which distribute the total social revenue amongst themselves and consume it as such, thus making up a demand created out of revenue'. p. 296.

30. Most clearly in his summary statement: 'that [Sraffa's] critique is in no way destructive of the project of providing a materialist account of the capitalist mode of production; nor is it in the least inconsistent with the attempt to build a fully articulated social, political and economic account of particular capitalist social formations. More specifically, many aspects of Marx's political economy, because they are independent of his reasoning in terms of value magnitudes, are unaffected by the Sraffa-based critique.' Steedman, p. 206. The point is that value magnitudes supply the hinge, the buckle, the pivotal point which relate class analysis to economic analysis, and cannot be prised loose from the frame without destroying the edifice.

31. See in particular Marx's letter to Engels of April 30, 1868 (quoted above) in which the whole argument is summarised very clearly and concisely.

32. 'If we find that the cost of base-year purchases at current prices is greater than the value of current purchases at current prices; while the cost of current purchases at base-year prices is also greater than the cost of base-year purchases at base-year prices; then we are unable to say whether the standard of living has increased or not, since the current quantities were not available in the base year, and the base year quantities are not available now. This is the index number problem.' G. Bannock, R. E. Baxter and R. Rees, *The Penguin Dictionary of Economics*, Harmondsworth, 1977, p. 211.

33. Capital, volume 3, p. 274, my emphasis.

34. 'But precisely because the circuit C'...C' presupposes in its description the existence of another industrial form C(= L + mp) (and mp comprises other capitals of various kinds, e.g. in our case machines, coal, oil, etc.) it itself demands to be considered not only as the *general* form of the circuit, i.e. as a social form in which every individual industrial capital can be considered (except in the case of its first investment) hence not only as a form of motion common to all individual industrial capitals, but at the same time as the form of motion of the sum of individual capitals, i.e. of the total social capital of the capitalist class, a movement in which the movement of any individual industrial capital simply appears as a partial one, intertwined with the others and conditioned by them... C...C' is the basis of Quesnay's *Tableau Economique*, and it shows great discernment on his part that he selected this form in opposition to M...M' (the form fixed on and isolated by the Mercantile System), and not P...P'. *Capital*, volume 2, p. 179.

35. Steedman, p. 202.

36. Indeed Kuhn, *The Structure of Scientific Revolutions*, Chicago, 1962, remarks that optics is one of the few sciences whose basic techniques have remained virtually static and untouched by the continuous revolutions in its foundations. From time to time post-Sraffians attempt to place themselves in a Kuhnian framework, arguing that they are participating in a 'scientific revolution' following the discovery of

inconsistencies in Marx. It is hard for anyone with a background in the natural sciences to stifle a sense of outrage at this idea: Kuhn's entire work is devoted to explaining how science progresses through inconsistencies between theory and *empirically observed fact*, to which virtually no-one on the post-Sraffian side of the debate makes the slightest reference. It is difficult to see what conceivable relation can exist between observed reality and Steedman's closing remark which I quoted in footnote 17, since every single capitalist observer now concedes that the rate of profit world has been systematically declining *in fact* since the late 1960s, far faster than can be accounted for by changes in the real wage.

37. Hodgson, *Capitalism, Value and Exploitation — A Radical Theory*, Oxford, 1981, p. 95–97, acknowledges that the post-Sraffian system does not have an adequate causal theory, but falls back on the argument that neither has Marxism. We have just seen that Marxism does have a causal theiry: the argument is therefore in the post-Sraffian court.

38. This is yet another reason why Steedman is entirely wrong to consign Marx's concept of fetishism to a *separate* department of political economy from value theory, or even relegate it to philosophy, outside of political economy altogether. The concept of fetishism is integral to Marx's value theory because it expresses the precise fact that the form in which economic laws come into the consciousness of economic agents is not transparent; they are not immediately conscious of the laws which nevertheless govern their actions. This does not mean they are like headless chickens with no consciousness at all, or a consciousness imparted to them by the tabloid newspapers. Crucial determinants of their consciousness are also the expression of laws; the *same* laws. As Marx progresses from volume 1 to volume 3, from abstract to concrete, he also demonstrates the way in which capitalists perceive economic categories, while still showing that, even on the basis of their false perceptions, they can be the agents of the law of value *because their consciousness is also a function of the law of value*. If the disciples of von Neumann had troubled themselves to study his contributions to cybernetics, they would have found out that formal theory is perfectly content with systems, such as cellular automata, in which individual components of the system possess 'consciousness' distinct from the aggregate effects of their interaction, and nevertheless governed by the same law as those same aggregate effects.

39. *Capital*, volume 2, p. 571.

40. *Capital*, volume 2, p. 472.

41. Albert Einstein, *Uber die Allgemeine Gravitationslehre*, in *Ideas and Opinions*, New York, 1960, p. 355.