Structural Acoustics: A General Form of Reciprocity Principles in Acoustics

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Abstract

A generalized Reciprocity Principle for Acoustics is obtained. By specialization, various principles which appear in the literature are obtained.

1 INTRODUCTION

According to reference [1], Russian measurements on the acoustic response of submarines to internal and external excitation make extensive use of the "Reciprocity Principle." While this can hardly be anything but an application of Green's theorem, the way the theorem is stated seems to be slightly different than that in the usual literature — and no derivation is given. Accordingly, it may be useful to see what forms of a "Reciprocity Principle" one might have.

The standard literature is a little vague. Rayleigh's [2] form relates the pressure at a point B due to a source at A to the pressure at A with the same source at B. The derivation assumes a bounded region with rigid boundaries. It is indicated that the results hold for an infinite region with rigid finite boundaries and a radiation condition at infinity.

Landau and Lifshitz [3][a] discuss only an infinite region with no finite boundaries. The statement is the same as Rayleigh but there is a generalization to inhomogeneous media. Addition [3][b] there is a problem given which relates velocities due to dipole sources. Morse and Ingard [4] give the same result as Rayleigh but for both rigid and compliant boundary conditions. The result implied in [1] seems to relate the pressure at A due to a localized force at B to the velocity at B due to a mass source at A.

Below we give a very general relation which requires only an impedance type boundary condition on finite boundaries. By specializing, we immediately obtain the three kinds of reciprocity principles discussed above. Clearly other specializations will yield additional principles. We leave it to the reader to determine these — and their usefulness.

2 BASIC EQUATIONS

These are:

a) The Euler Equation

$$\rho\left\{\frac{\partial\tilde{v}}{\partial t} + (\tilde{v}\cdot\nabla)\tilde{v}\right\} = -\nabla P$$

b) The Mass Conservation Equation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\tilde{v} = 0$$

c) An Equation of State

$$P = P(\rho).$$

For acoustics we linearize putting

$$\rho = \rho_0 + {\rho'}_1, \quad \tilde{v} = \tilde{v}_0 + \tilde{v}'.$$

Here, as compared to Landau and Lifshitz [3], we choose the homogeneous case ρ_o = constant and $\tilde{v}_0 = 0$. The linearized equations are (after dropping the primes)

$$\begin{aligned} \rho_0\frac{\partial\tilde{v}}{\partial t} &= -\nabla P \\ \frac{\partial\rho}{\partial t} + \rho_0\,\nabla\cdot\tilde{v} &= 0. \end{aligned}$$

Now $\frac{\partial\rho}{\partial t} = \frac{\partial\rho}{\partial P}\,\frac{\partial P}{\partial t} \equiv \frac{1}{c^2}\,\frac{\partial P}{\partial t}$. The resulting equations are

$$\rho_0\frac{\partial\tilde{v}}{\partial t} = -\nabla P$$

$$\frac{1}{c^2}\frac{\partial P}{\partial t} + \rho_0 \nabla \cdot \tilde{v} = 0.$$

Assuming simple harmonic motion, i.e., all quantities $\sim e^{-i\omega t}$ yields.

$$-i\omega\rho_o\tilde{v} = -\nabla P$$

$$\frac{-i\omega}{c^2}P + \rho_0 \nabla \cdot \tilde{v} = 0.$$

Let us introduce forcing terms into both equations. Then

$$-i\omega\rho_0\tilde{v} = -\nabla P + \tilde{F}(\tilde{r} - \tilde{r}_1) \qquad (2-1)$$

$$\frac{-i\omega P}{c^2} + \rho_o\nabla \cdot \tilde{v} = M(\tilde{r} - \tilde{r}_2). \qquad (2-2)$$

Eliminating $\tilde{v}$ we obtain

$$(\nabla^2 + k^2)P = \nabla \cdot \tilde{F} + i\omega M, \qquad (2-3)$$

where $k^2 = \omega^2/c^2$.

We want to put this in a form to apply Green's Theorem. We consider pressures P_1 (corresponding to an $\tilde{F}_1, M_1$) and P_2 (corresponding to a $\tilde{F}_2, M_2$). Thus, we have

$$(\nabla'^2 + k^2)P_1(\tilde{r}') = \nabla' \cdot \tilde{F}_1(\tilde{r}' - \tilde{r}_1') + i\omega M_1(\tilde{r}' - \tilde{r}_1) \qquad (2\text{-}4)$$

$$(\nabla'^2 + k^2)P_2(\tilde{r}') = \nabla' \cdot \tilde{F}_2(\tilde{r}' - \tilde{r}_2') + i\omega M_2.(\tilde{r}' - \tilde{r}_2) \qquad (2\text{-}5)$$

Multiply Equation (2-4) by $P_2(\tilde{r}')$ and Equation (2-5) by $P_1(\tilde{r}')$. Subtract and integrate over the fluid. The result is:

$$\int d^3\tilde{r}' \left\{P_2(\tilde{r}')\nabla'^2\, P_1(\tilde{r}') - P_1(\tilde{r}')\nabla'^2 P_2(\tilde{r}')\right\}$$

$$
\begin{aligned}
&= \int d^3r' \left\{P_2(\tilde{r}')\nabla' \cdot \tilde{F}_1(\tilde{r}' - \tilde{r}_1') + i\omega P_2(\tilde{r}')M_1(\tilde{r}' - r_1)\right\} \\
&- \int d^3r' \left\{P_1(\tilde{r}')\nabla' \cdot \tilde{F}_2(\tilde{r}' - \tilde{r}_2') + i\omega P_1(\tilde{r}')M_2(\tilde{r}' - r_2)\right\}
\end{aligned}
\tag{2-6}
$$

Using Green's Theorem and assuming impedance boundary conditions (i.e. P and $\frac{\partial P}{\partial n}$ are proportional on the boundary) we see that the left side of Equation (2-7) is zero.

Thus

$$
\begin{aligned}
&\int d^3r' \left\{P_2(\tilde{r}')\nabla' \cdot \tilde{F}_1(\tilde{r}' - \tilde{r}_1') + i\omega P_2(\tilde{r}')M_1(\tilde{r}' - r_1)\right\} \\
&= \int d^3r' \left\{P_1(\tilde{r}')\nabla' \cdot \tilde{F}_2(\tilde{r}' - \tilde{r}_2')| + i\omega P_1(\tilde{r}')M_2(\tilde{r}' - r_2)\right\}.
\end{aligned}
\tag{2 - 7}
$$

This is our general reciprocity principle. Specializing the $\tilde{F}$s and Ms will give the specific principles mentioned in the introduction.

3 SPECIAL CASES

Here we will only consider the situation when the Fs and Ms are constants time δ functions. To keep things straight, it is convenient to introduce a special notation. Thus

P_1 which corresponds to $\tilde{F}_1(\tilde{r} - \tilde{r}_1')$ equal to $\tilde{F}_1\delta(\tilde{r} - \tilde{r}_1')$ ($\tilde{F}_1$ now a constant vector) and $M_1(\tilde{r} - \tilde{r}_1)$ equal to $M_1\delta(\tilde{r} - \tilde{r}_2)$ will be denoted as

$$P(\tilde{F}_1, \tilde{r}_1', M_1, \tilde{r}_1; \tilde{r}).$$

a) Let $\tilde{F}_1 = \tilde{F}_2 = 0$.

$$\begin{aligned} M_1(\tilde{r}' - \tilde{r}_1) &= M\delta(\tilde{r}' - \tilde{r}_1) \\ M_2(\tilde{r}' - r_2) &= M\delta(\tilde{r}' - \tilde{r}_2) \end{aligned}$$

(M a constant).

Then Equation (2-7) reads

$$P(\tilde{F} = 0, M, \tilde{r}_1; \tilde{r}_2) = P(\tilde{F} = 0, M, \tilde{r}_2; \tilde{r}_1). \qquad (3-1)$$

Since both sides are proportional to M, we can take this equal to 1 and Equation (3-1) is the statement that

$$P(\tilde{r}_1; \tilde{r}_2) = P(\tilde{r}_2; \tilde{r}_1). \qquad (3-2)$$

This is the standard Rayleigh form of the reciprocity principle. In words: the pressure at $\tilde{r}_1$ due to a unit source at $\tilde{r}_2$ is the same as the pressure at $\tilde{r}_2$ due to a unit source at $\tilde{r}_1$.

b) Let $M_1 = M_2 = 0$.

$$\begin{aligned} \tilde{F}_1(\tilde{r}' - \tilde{r}'_1) &= \tilde{F}_1\delta(\tilde{r}' - \tilde{r}'_1) \\ \tilde{F}_2(\tilde{r}' - \tilde{r}'_2) &= \tilde{F}_2\delta(\tilde{r}' - \tilde{r}'_2). \end{aligned}$$

Equation (2-7) becomes

$$\tilde{F}_1 \cdot \nabla'_1 P(\tilde{F}_2, \tilde{r}'_2; \tilde{r}'_1) = \tilde{F}_2 \cdot \nabla'_2 P(\tilde{F}_1 \tilde{r}'_1; \tilde{r}'_2). \qquad (3-3)$$

Since the gradients are proportional to the velocities we can rewrite Equation (3-3) as

$$\tilde{F}_1 \cdot \tilde{v}(\tilde{F}_2, \tilde{r}'_2; \tilde{r}'_1) = \tilde{F}_2 \cdot \tilde{v}(\tilde{F}_1, r'_1; \tilde{r}'_2) \qquad (3-4)$$

Now both sides are proportional to the magnitudes of both $\tilde{F}_1$ and $\tilde{F}_2$. We can then choose both to be of magnitude 1. We choose $\tilde{F}_1$ and $\tilde{F}_2$ separately to be 1 of 3 orthogonal unit vectors $\tilde{e}_1, \tilde{e}_2, \tilde{e}_3$. This yields the relations

$$\tilde{v}_i(e_i, \tilde{r}'_2; r'_1) = \tilde{v}_i(\tilde{e}_i, \tilde{r}'_1; \tilde{r}'_2) \qquad (3-5)$$

$$(i = 1, 2, 3).$$

Thus the velocity is the ith direction at $\tilde{r}'_1$ due to a force in the ith direction at $\tilde{r}'_2$ is the same as the velocity in the ith direction at r'_2 due to a force in the ith direction at $\tilde{r}'_1$.

If we choose $\tilde{F}_1$ and $\tilde{F}_2$ orthogonal, we get the 6 relations

$$\tilde{v}_i(\tilde{e}_j, \tilde{r}'_2; \tilde{r}'_1) = \tilde{v}_j(\tilde{e}_i, \tilde{r}_1{}'; \tilde{r}'_2) \qquad (3-6)$$

$$i \neq j.$$

Thus the velocity in the ith direction at $\tilde{r}_1'$ due to a force in the jth direction at $\tilde{r}_2'$ is equal to the velocity in the jth direction at $\tilde{r}_2'$ due to a force in the ith direction at $\tilde{r}_1'$.

c) Let $\tilde{F}_1 = 0,\ M_1 = M\delta(\tilde{r}' - \tilde{r}_1)$,

$$\tilde{F}_2 = \tilde{F}_2\delta(\tilde{r}' - \tilde{r}_2'),\ M_2 = 0.$$

Then $P_1(\tilde{r}') = P(\tilde{F}_1 = 0, M, \tilde{r}_2; \cdot\tilde{r}')$

$$P_2 = P(\tilde{F}_2, 0, \tilde{r}_2'; r').$$

Equation (2-7) becomes (on replacing pressure gradients by velocities)

$$i\omega MP(\tilde{F}_2, 0, r_2'; \tilde{r}_1) = -\frac{\tilde{F}_2}{i\omega\rho_o} \cdot \tilde{v}(\tilde{F}_1 = 0, M, \tilde{r}_1; \tilde{r}_2').$$

Since both sides are $\sim$ M and $(\tilde{F}_2)$, we can take $M = 1$, and $\tilde{F}_2 = e_i$ where i = 1, 2, 3 yields 3 orthogonal unit vectors.

Here we have the three relations

$$\omega^2\rho_0 P(\tilde{e}_i, 0, r_2'; \tilde{r}_1) = v_i(\tilde{F}_1 = 0, 1, \tilde{r}_1; \tilde{r}_2') \qquad i = 1, 2, 3 \qquad (3-7)$$

In other words this says: The pressure at $\tilde{r}_1$ due to a force at $\tilde{r}_2'$ in the $\tilde{e}_i$ direction is proportional to the velocity in the ith direction due to a source at $\tilde{r}_1$ at $\tilde{r}_2'$.

We hazard the guess that this is the reciprocity principle referred to in reference [1].

References

[1] Donskoy D., 1991 "Soviet R&D in Low-Frequency Underwater Acoustics," published by Delphic Associates Inc., 7700 Leesburg Pike, No. 250, Falls Church, Va 22043 .

[2] Rayleigh, J. W. S., 1945, "The Theory of Sound", Vol II, p. 145, Dover Publications, NY.

[3] Landau, L. D. and E. M. Lifshitz, 1959, "Fluid Mechanics," Pergamon Press.

a) No. 74

b) Problem on p.291

[4] Morse, P. M. and K. U. Ingrad,(1986), "Theoretical Acoustics", Princeton University Press, Princeton, NJ, p.134, 312, 320, 342, 405 (1986).

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