children who experience difficulty in learning through language to have another avenue of attack; to allow children with a strong spatial orientation to make better use of it in school learning; to increase the spatial awareness of all children and help them to become aware of the way space can be mathematized (i.e., not just a label, 'circle' but an understanding of circularity); and to provide a basis for the learning of geometry which is an area of mathematical education that appears to be neglected.

To achieve these goals, Bishop (1986, in press) believes that children need to be encouraged to reflect on their particular spatial view of the world and through discussion be helped to focus on features of space that are of significance to mathematics. He also believes that children should be encouraged to represent their spatial understandings through modelling, drawing and language. In this way, spatial strengths can be used to provide a foundation for verbal learning. These insights are of particular importance if more effective mathematics programs are to be provided for Aboriginal children who have a strong spatial orientation and yet need to develop the language skills through which much school learning is mediated.

4. THINKING, TALKING and LEARNING

4.1 Thinking and Learning

It may be appropriate at this time to briefly reflect on some of the literature about cognition and about when and how children learn. The influence of Piaget is still felt in this area and many writers refer to his findings (Liebeck, 1984; Dickson et al, 1984; Lewis, 1979, 1980, 1983a, 1983b). Although the results of his particular studies are not in doubt, many writers appear to question the interpretation of his results and have, for example, demonstrated that, while conservation indicates a certain level of logico-mathematical development, children can use a different kind of logic to solve a range of mathematical problems (Lewis, 1983b; Gelman & Gallistel, 1978; Donaldson, 1978). Unfortunately some of Piaget's findings have led to the suggestion that until children can conserve number only pre-number activities should be introduced. For Aboriginal children who will not be exposed to number experiences in the home it is, as Boulton-Lewis and Halford (1985, p.7) point out, a little like expecting children to learn to swim while keeping them out of the water.

Piagetian tests when carried out in Aboriginal communities have indicated developmental lag. (See Hunting & Whitely, 1983, pp.15-17, for a summary of the findings of major studies.) However, there has been some criticism of testing procedures and certain language problems have been identified (Boulton - Lewis & Halford, 1985. See also work of Hastie 1984, and Hastie & Treagust, 1985 for discussion of results obtained when both context and language were modified.) Nevertheless, the typical explanation for the results of Piagetian-type testing has been to see them as a function of environmental factors or different cognitive strategies. For example, a recent study by Seagrim and Lendon (1980) found that Aboriginal children adopted into European homes performed better than those in a boarding home in Alice Springs and both groups did significantly better than children living at Hermansburg, an isolated Aboriginal community. These findings are in many ways similar to results that are obtained from intelligence tests. In some recent research in this area, Clark & Halford (1983) found that all the cultural and location differences that were being accounted for by cognitive style could be better accounted for by psychometric measures of intelligence and, in addition, these measures tended to be more predictive than measures of cognitive styles.

Recent procedures developed by Halford (1984, 1985) have aimed to account for cognitive capabilities through measuring children's ability to process information.. The levels of development that have been identified (Halford, 1984, 1985) relate to the fact that children's ability to understand is a function of the amount of information that they can integrate in a single mental process. Such levels have curriculum implications for they enable teachers to relate mathematical instruction to children's ability to process information. (Halford & Boulton-Lewis, 1984; Boulton-Lewis & Irons, 1984). As Lewis (1979) points out they may also account for some of the difficulties experienced by young children who have to learn in a second language, and who if translating, are contending with an extra 'bit' of information.

Boulton-Lewis and Halford (1985) believed that assessment of the information processing capacity of Aboriginal children would provide a more definitive indication of underlying cognitive capacity than previous approaches. They found that a group of Aboriginal children at Cherbourg were able to process information as well as European children of the same age. Such a finding is encouraging for all those working with Aboriginal children who felt that this was the case but could not produce evidence to support their intuitions. However, the environmental factor that is revealed in Piagetian and intelligence testing procedures simply cannot be ignored by teachers who must work in these communities. It would seem then, that if Aboriginal children are to learn mathematics successfully in school, classrooms will need to resemble mathematical homes. An environment will thus be created in which the cognitive processes which underlie the technological component of the MT culture can be revealed to children as they are involved in both living and learning mathematics in school.

4.2 Constructing Mathematical Meanings

The book How Children Learn Mathematics (Liebeck, 1984) provides a handy guide for both teachers and parents in mainstream society. The teaching/learning strategy is categorized as E (experience with physical objects), L (spoken language about the experience), P (pictures that represent the experience) and S (written symbols that generalize the experience). This approach is very similar to that recommended for use in the Aboriginal schools in the NT. One problem that is beginning to emerge, however, is that concrete experiences do not necessarily create reality. While using blocks or counters for operations may make the process 'real' they do not always make it 'real life' and the social meaning and thus the purpose of the activity is frequently not made available to children.

Bishop (1984) sees approaches that enable children to be involved in the 'social construction of meaning' as a significant development in our understanding of the process of mathematics education. He believes that teachers need to move away from 'thinking' too much about content, knowledge and topics. Rather they need to think about the kind of experiences that children can be involved in that will enable them to construct mathematical meanings for themselves. Key features of this approach are activities or experiences, communication, which is to do with sharing meanings, and negotiation, which is to do with developing meanings. (Bishop, 1984, p.26) Such approaches see mathematics as a language activity and researchers such as Bourke (1982), Dekker (1985), Romberg (1985) and Hirabayashi (1985) have all begun to study the patterns of discourse through which mathematical meanings are made and shared in classrooms.

Bauersfeld (1980) has also carried out research in this area. After analysing classroom texts in relation to mathematics lessons, he demonstrated that, while teachers and children are using language to interact, they are all behaving according to their own actual subjective realities. Hence, teachers and students are frequently at cross purposes even though they both believe that they understand what the other person is saying. He, like Bishop, also favours approaches that enable students to be involved in social negotiation of mathematical meanings but points out that in mathematics such negotiations need to continue until students become aware of the performance of meaning that gets the teacher's sanction. (Bauersfeld, 1980, p.35). For Aboriginal children involved in learning Western mathematics in school such negotiations need to be lengthy so they can, if necessary, recognize their particular

Aboriginal view of reality and also come to perceive the meaning inherent in the MT culture.

Such approaches, while essential if Aboriginal children are to gain mathematical meanings and not just skill in responding correctly in some narrowly defined mathematical situation, present other problems for classroom teachers. Malcolm (1980) demonstrated that many Aboriginal children do not find it easy to take part in the participant structures through which meanings are made and shared in the classroom. Kearin's (1985), Harris (1985) and Christie and Harris (1985) have also documented the difficulties Aboriginal children experience in classrooms and the many factors that lead to communication breakdown in that context.

The strategies developed by Gray (1983) for enabling Aboriginal children to become more effective language learners in the school context need to be examined in this regard. Through an approach that has come to be known as Concentrated Language Encounters, he has enabled children to encounter language in a context that provides meaning on which language can be mapped. Negotiation is a key feature of the approach and observers are impressed not only with the improved language abilities of the children but with the knowledge that children have gained through meaningful interactions in 'real-life' contexts. If Aboriginal children are to find meaning in their school mathematics programs, Concentrated 'Mathematics' Encounters may provide an approach through which this may be achieved.

4.3 Learning Through Language

Austin and Howson (1979) in response to the increased awareness among mathematics educators about the importance of language in mathematical education reviewed the literature in that area. To provide a framework for a discussion that preceded the bibliography they grouped language into that used by the Learner, that of the teacher and the language of mathematics itself. Many of the issues referred to have also been addressed in 'A Review of Research in Mathematical Education (Bell, Costello & Küchemann, 1983, Ch.11).

The role of language in helping the learner learn has become increasingly clear in recent years. Munro (1982, p.70) points out that children learn more efficiently when the information with which they have to cope is well organized and sequenced so that patterns and trends emerge easily and clearly. Munro claims that verbalization greatly facilitates this process. He found (1982, p.71) that verbalising instances of a concept and talking about commonalities between the instances assisted children in generalization and learning of the concept.

Many writers refer to the importance of language and concept development and the work of Vygotsky (1962) is frequently quoted in reference to that issue. Austin and Howson (1979, p.167) and Bell et al (1983, p.275) both report Vygotsky's claim that:

> the birth of a new concept is invariably foreshadowed by a more or less strained or extended use of old linguistic material; the concept does not attain to individual and independent life until it has found a distinctive linguistic embodiment.

The assumption is that if children are to develop mathematical concepts they must talk themselves into those understandings as they experience the idea for themselves. Thinking about mathematical issues is also seen to be a language activity that involves either vocal, sub-vocal or inner speech. These understandings further support the claim that if children are to develop mathematical ideas and to learn to think in mathematical ways, mathematics education needs to be regarded as a language matter. Hence, where possible, children need to be educated in a language in which they are able to talk.

In developing mathematical concepts and in learning to talk about mathematical processes and understandings a large mathematical vocabulary is acquired. Although sometimes referred to as a language it is more correctly a register (Halliday, 1974) and like all such registers provides participants in a particular activity with a kind of shorthand with which ideas can be shared. Ultimately much of this mathematical vocabulary will need to be able to be read as it is an integral part of the language of text books. However, it is important that children learn to talk their way into understandings usually beginning with the language of the everyday world and gradually refining it until the appropriate mathematical vocabulary and usage is acquired (Hough, 1981). Wishart (1977) in a study of the order in which children naturally acquired mathematical language, in this case polarized comparitive terms, demonstrated that children used positive terms (e.g., more, big) before they understood the negative member.

Further up the school Faber-Morris (1982) studied problem solving and Lowenthal (1984) and Wall (1982) examined logical reasoning. Although such areas of mathematics involve cognitive abilities they are, nevertheless, developed through and dependent on language and children in many mathematics programs who find difficulty in talking about mathematical ideas experience difficulty in thinking about them also.

Many of the researchers and academics who addressed language matters inevitably focussed on the readability of texts. It seems that many children who experience difficulty in mathematics, fail to solve mathematical problems because they cannot read the questions. (Austin & Howson, 1979; Bell et al, 1983; Faber-Morris, 1982; and others). As text books make use of exposition and instruction as well as examples and exercises, children who are to work effectively from text books need to learn how to interact with such texts. Richardson and Williamson (1982) and Ellerton (1985) who involved children in creating mathematics problems for each other may have a solution to some of the problems in this area. It has been found (Gray, 1983) that one of the most effective ways to assist Aboriginal children to become effective readers of unfamiliar genres (patterns of discourse) is to help them learn how to write them. In that way, children who are to read mathematics text books would need to first become 'writers' of such texts.

Inevitably a move into written language involves a consideration of the written symbols through which mathematical understandings are encoded. Howson (1984, p.569) refers to the work of Bramford who in <u>A Study of Mathematical Education</u> laid down as a principle of teaching:

> No symbol or contraction should be introduced till the pupil himself so deeply feels the need for such that he is either ready himself to suggest some

> contraction, or at least appreciate reasonably fully the advantage of it when it is supplied by the teacher.

Although Howson (1984), Pellerey (1984), Sinclair (1984) and Healey (1980) all in their own way explore issues related to the development and use of langauge and symbolism in gaining a mathematical education, and thus focus on aspects of problems experienced by children either individually or in groups, nevertheless their recommendations are remarkably similar to those made by Bramford seventy years ago.

5. TWO LANGUAGES IN MATHEMATICS EDUCATION

5.1 Bilingual Children in Mainstream Education

While many children who learn mathematics in their mother tongue experience difficulty in acquiring the register associated with mathematics, these difficulties can be exacerbated for children who must learn in their second language if their particular language needs are not addressed. Many of these children experience difficulties which can clearly be related to their inability to comprehend English mathematical terms and the patterns of discourse found in oral interactions and written texts (Newman, 1981). Turner (1980) found that both first and second language speakers had problems with the language of mathematics. However, for second language speakers, problems were aggravated when teachers used terms but did not ensure that language was learned. In Halliday's view (1975), learning language involves 'learning how to mean' and hence learning the language of mathematics involves learning how to make and share mathematical meanings using language appropriate to the context, which is more than recognizing and responding to words in isolation. Unless teachers of mathematics become more aware of this difference it seems that many second language learners will continue to be disadvantaged in school.

However, for some children who are involved in second language education the situation is very different. For example, children in the St. Lambert bilingual program performed at significantly higher levels than controls, on measures of divergent thinking. Examination of the results achieved by individuals within the groups that were studied led to the development of the so called 'threshold' hypothesis (Cummins, 1977, 1981). This hypothesis states that there may be a 'threshold level of linguistic competence which a bilingual child must attain both in order to avoid cognitive defects and to allow the potentially beneficial aspects of becoming bilingual to influence his/her cognitive growth' (Cummins, 1977, p.10). The form of the hypothesis that is most consistent with available data suggests there are two thresholds. Children who know neither language well may experience negative cognitive effects, while those who know both languages extremely well will experience positive cognitive effects. In between these two thresholds neither positive or negative cognitive effects have been noted. Cummins (1981) refers to several studies that have reported findings that are consistent with the general tenets of this threshold hypothesis. Hence, while it has been demonstrated that bilingual education, per se, is not necessarily detrimental and for some can be decidedly advantageous, there are groups of children, whose home language is not being adequately developed and who are not becoming effective speakers of a second language in school, who must be considered to be at risk.

Evidence suggests that such children can easily suffer linguistic, intellectual and academic retardation and may cease to have identity with their cultural group while failing to establish such links with the contact group. (Cummins, 1977; Cummins & Gulutson, 1974). Cummins believes that these conditions can be created when educators endeavour to replace a child's language and culture with that of the dominant group. He describes such an educational program as 'subtractive' while the bilingual education that results in educational advantage he describes as 'additive' (Cummins, 1981, 1985).

Dawe (1983) carried out research to discover if there was any evidence with respect to the ability of bilingual children to reason in mathematics in English as a second language that would support Cummins' hypothesis. He found that mathematical reasoning in the deductive sense is closely related to the ability to use language as a tool for thought, and that the ability of a child to make effective use of the cognitive functions of his first language is a good predictor of his ability to reason deductively in English as a second language. He also found that there was a complex relationship between visuo-spatial and verbal-logical reasoning and that bilingual children often switched from one mode to the other during the reasoning process and this switch was often accompanied by a language switch as well (Dawe, 1983, pp.349-350).

Cathcart (1980) also explored the matter of cognitive flexibility with bilingual and monolingual children. The number of second rationalizations a child could give for conservation was considered to be indicative of this quality, for it demanded that the child look at the phenomena in different ways. The study found in favour of the bilingual children and in Cathcart's view provided further evidence in support of the threshold hypothesis that had been formulated by Cummins (Cathcart 1980, p.8). Both Cathcart and Dawe concluded that first language maintenance for minority language students was an important factor in predicting success in the area of mathematics education, a finding that has implications for mathematical education for remote traditionally oriented Aboriginal children.

5.2 Two Languages in Cross-Cultural Education

The research material that has been reviewed in relation to bilingual education (i.e., children who will be bilingual as a result of the schooling process) has to this point focussed on research evidence to do with children who, at the time of the research, were living in societies within the MT culture. Children who grow up in societies that are not part of the MT culture have other problems. Their first language is often not a good vehicle for encoding the logico-mathematical relationships of the MT culture and their English, which is usually a school language, is frequently not adequate for the task. Because they live their lives within a simple technology, the technological reality encoded in school mathematics is only dimly perceived. In such situations, effective solutions to the difficulties children experience in learning mathematics in school are not easily available. As Aboriginal children, although belonging to a nation which is part of the MT culture, actually live their lives in response to the rhythms and reasons of the Dreamtime, this area of research needs to be explored.

In Papua New Guinea where children were receiving schooling in English, Southwell (1977) found that rote learning was synomonous with a mathematics education. Children did not understand what they had learned and so were unable to benefit from the experience or to apply what had been learned to another situation (Southwell, 1977, p.102). Jones (1982) also conducted research in Papua New Guinea. He found that there was a growing mismatch between the mathematical demands of the curriculum and the language capabilities of the students. For example, upper primary school children were expected to use the terms 'more' and 'less' which were introduced in grade 3. However, Jones found that 'more' was not mastered until grade 7 and 'less' was not mastered until grade 10. He concluded that in such circumstances mathematics became a 'meaningless set of rules associated with an equally meaningless set of words and hence little learning of value is likely to take place'. Jones (1982 p.75) and Clarkson (1984) present a resumé of similar findings from other studies undertaken in Papua New Guinea.

5.3 Mathematics in the Mother Tongue

The solution appears to lie in using the first language of the children, for if, as it would seem, becoming a mathematical person involves constructing mathematical meanings and communicating and negotiating about those meanings, it is in the child's first language that such interactions could most easily occur. However, many languages spoken by children in developing countries that are outside the MT culture lack the register - both the vocabulary and the logical connectives - necessary to encode understandings inherent in that culture. As Halliday (1974) points out, this does not necessarily mean that people do not perceive some or many of the classifications inherent in the MT culture, it is just that they do not attend to them. Wilson (1981) in addressing an audience of people involved in cross-cultural education in Papua New Guinea supported this view and claimed that languages are not deficient but different.

All languages have evolved to meet particular needs of their speakers and given time and the need, languages that reflect simple technologies can evolve further and absorb some, at least, of the understandings inherent in the MT culture. Even while such development is taking place Halliday (1974) believes that such languages can be used as a point of departure for helping children learn Western mathematics in school. While these languages may never develop a full register of mathematical terms, concepts can be 'talked around' in the every day language of life. Hence, while there may be no word for 'plus' in a language, children can be involved in and talk about experiences that enable them to 'bring together', 'add together', 'put with' and so on. In doing this it is inevitable that the meaning of some words will change. For example, Christie (1980) notes that in Gupapuyngu, a language spoken in Northeast Arnhem Land, '<u>bulu</u>' the word for 'more' is used to denote 'extra' as in, 'I want more soup', but it does not mean 'relatively greater' as in, 'There is more sugar in this bowl than in that one'. However, it can have its meaning extended to carry that understanding if that is what people want to talk and think about.

Extending meanings of words, borrowing words from the other language, and combining two or three words to create a new term or locution as in 'right-angled triangle', in a planned way, is referred to as language engineering (Morris, 1978). Leeding (1976) in North Australia, Gnerre (1984) from Brazil, Mwombogela (1979) from Tanzania along with a wide range of speakers from third world countries who attended a CASME workshop in Ghana in 1975 can all provide examples of just how this language planning may occur in countries where, frequently, there are political as well as pedagogical reasons for educating children in their mother tongue and thus incentive is provided to extend the local language to carry out the task.

In very small languages of say 1000 speakers or less, there are probably practical rather than theoretical considerations why this development may not extend as far as technically possible and educators need to ensure that children's mathematical developement is not stunted due to a lack of appropriate terms. For example, Clements (1982, p.20) notes that the following could not easily be translated into many of the vernaculars of Papua New Guinea as there would be no word for one-third:

> Here are six buai. If you gave me one-third of them, how many would you give me?

The likely form of the question that would be offered in a vernacular would probably be something like this:

> Here are six buai. Suppose you gave them to Jack, Luke and Mary so that each got the same number of buai. How many would each get?

As Clements points out, although the answers are the same, the former uses the generalized concept of one-third of which makes it more difficult but without such language children will not easily develop generalized concepts. This failure to introduce children to mathematics concepts and the language which encodes their meanings does not only occur in the cross-cultural situation. In Clements' view (1982, p.20) it is a feature of many second language and remedial - and in my experience, Aboriginal - situations and ensures that some children never learn many aspects of mathematics because they are never exposed to them in a way that makes sense, so the concepts remain 'too hard'.

5.4 Language, Culture and Meaning

There are other difficulties in the cross-cultural situation, however. Bishop (1983) described how a Teachers College student in Papua New Guinea found the area of a piece of paper in the classroom by multiplying length and width and the area of his garden in the village by adding length and width. The student correctly saw them as different processes, each belonging to a particular culture. The student when presented with two rectangles drawn on paper was unable to select the better (bigger) garden because, 'It depends on many things. I cannot say. The soil, the shade'. As Bishop (1983, p.196) concluded, the question was inappropriate. Harris (S., 1980) experienced similar difficulties when posing problems about the Aboriginal kinship system. His query, 'What would happen if so and so married so and so?' was simply met with a blank look and the response that it could not happen. Thus, while it appears that there are many situations in developing countries that could be used by the educator to develop understandings related to the MT culture this may not always be the case, and efforts by teachers from other countries or cultures to embed mathematical understandings in real-life contexts may be, in reality, most inappropriate and ones to which children cannot respond 'intelligently'.

Even when people appear to be engaged in the same activity the cognitive processing involved may be different. Watson (1984) described the cognitive processes that were used by children when they measured in English and wón- ed (i.e., measured) in Yoruba, a Western African Language. In the study, children were asked to comment, either in English or in Yoruba when water, coca-cola or peanuts were poured from one container to another container of a different shape. The children's comments were classified to discover whether they regarded the quality (attribute) of what they were constructing as permanent and whether they gave that quality a temporal or spatial dimension. She found that mostly when children worked in Yoruba they were concerned with the temporal dimension and when they worked in English they were concerned with the spatial dimension. That is, although the task was the same, different languages resulted in different cognitive processes. She concluded that bilingual/bicultural children had to become conscious of the different ways they thought about reality and that they needed to develop competence in the conceptual system inherent in their first language and culture, while bridges were built to the conceptual system of the MT culture (Watson, 1984, p.10).

5.5 Moving into the MT Culture

Horton (1971) described similarities and differences between African traditional thought and Western science. Although, he found many similarities, he also documented vast differences. He saw the key difference to be a very simple one:

> It is that in traditional cultures there is no developed awareness of the established body of theoretical tenets; whereas in scientifically oriented cultures such an awareness is highly developed. It is this difference that we refer to when we say that traditional cultures are "closed" and scientifically oriented cultures are "open".
>
> (Horton, 1971, p.230)

In the view of Fasheh (1982, p.3) it is this difference between open and closed systems that indicates how mathematics should be taught in developing countries. Such communities must avoid approaches that result in rote learning and a continuation of a closed system. Rather, mathematics needs to be taught in ways that will enable children to become open to new thoughts and new ideas. Such approaches need, in Fasheh's view, to encourage children to doubt, enquire, discover, see alternatives and most important of all construct new perspectives and convictions. Through such programs of mathematics education, students should be able to discover new 'facts' about themselves and their own society and culture and so be able to make better judgements and decisions when required.

Such a view of mathematics is not one that sees transmission of knowledge to be paramount. Instead it is the 'reciprocal-interaction' (Cummins, 1985, p.8) between teachers and students as they make and share meanings, either in the students' mother tongue or in a world language, and so construct for themselves the meanings inherent in the MT culture, that is important. As has been indicated, the task is not easy in countries that are striving for nationhood and a place in the MT culture. In traditional Aboriginal society, where the way forward is often perceived in terms of a move back to traditional values, the situation is even more complex.

6. AN ABORIGINAL VIEW OF REALITY

Aboriginal children grow up in a society in which the system that controls the economic realities of life are based on relationships between people rather than relationships between quantities of money, time, goods and other services as it is in the MT culture. Bain in Christie (1985, p.9) has described it as 'interactional' rather than 'transactional'. Thus, Aboriginal children are much better at talking to establish personal relationships with their teachers than they are at talking to transact knowledge inside the classroom. The environment in which people live is also grounded in such interactional relationships which extend back to the Dreamtime and relate Aboriginal people to the land and to the dominant features of the land. Hence, questions like, 'How much land?' are immaterial. Instead, people focus on the relationship between a particular group of people who are 'owned' by the land.

6.1 A Concern for Quality

In such a society the emphasis is not on the quantity of the relationship but on the quality. Rudder (1983) examined the classificatory systems, the evaluative systems and cognitive structures of the Yolnu people of Northeast Arnhem Land. He used the term 'qualitative thinking' to describe the way Yolnu people reflect on their