Astronomical Society of Victoria

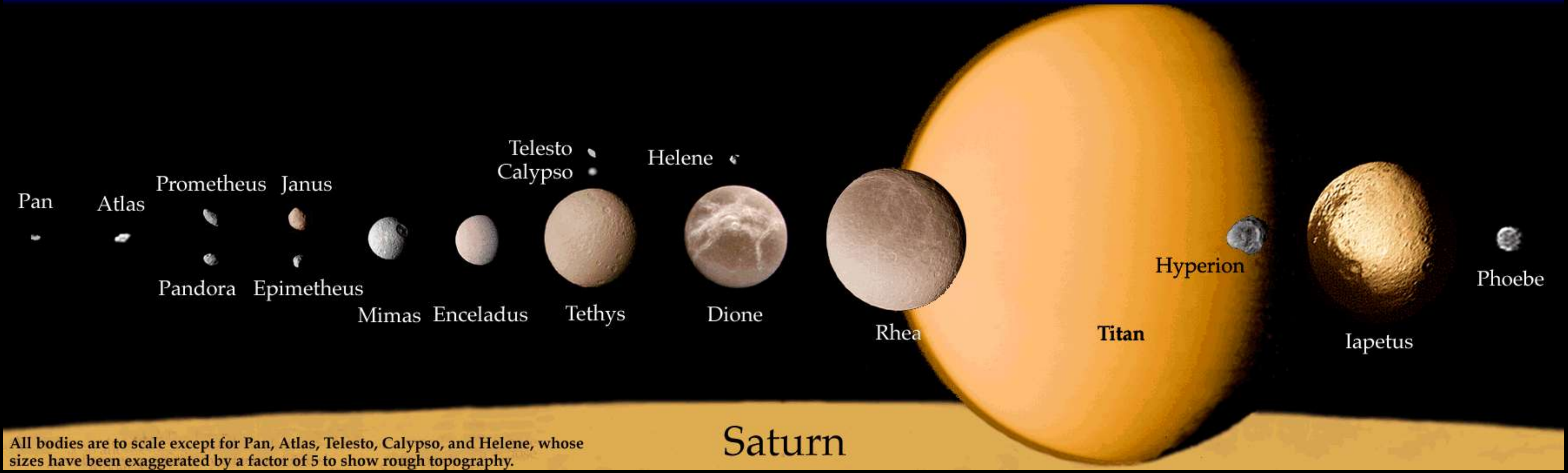
New insights into our Solar System's formation, as gleaned from the MESSENGER, Dawn and Cassini-Huygens spacecraft missions

Andrew Prentice

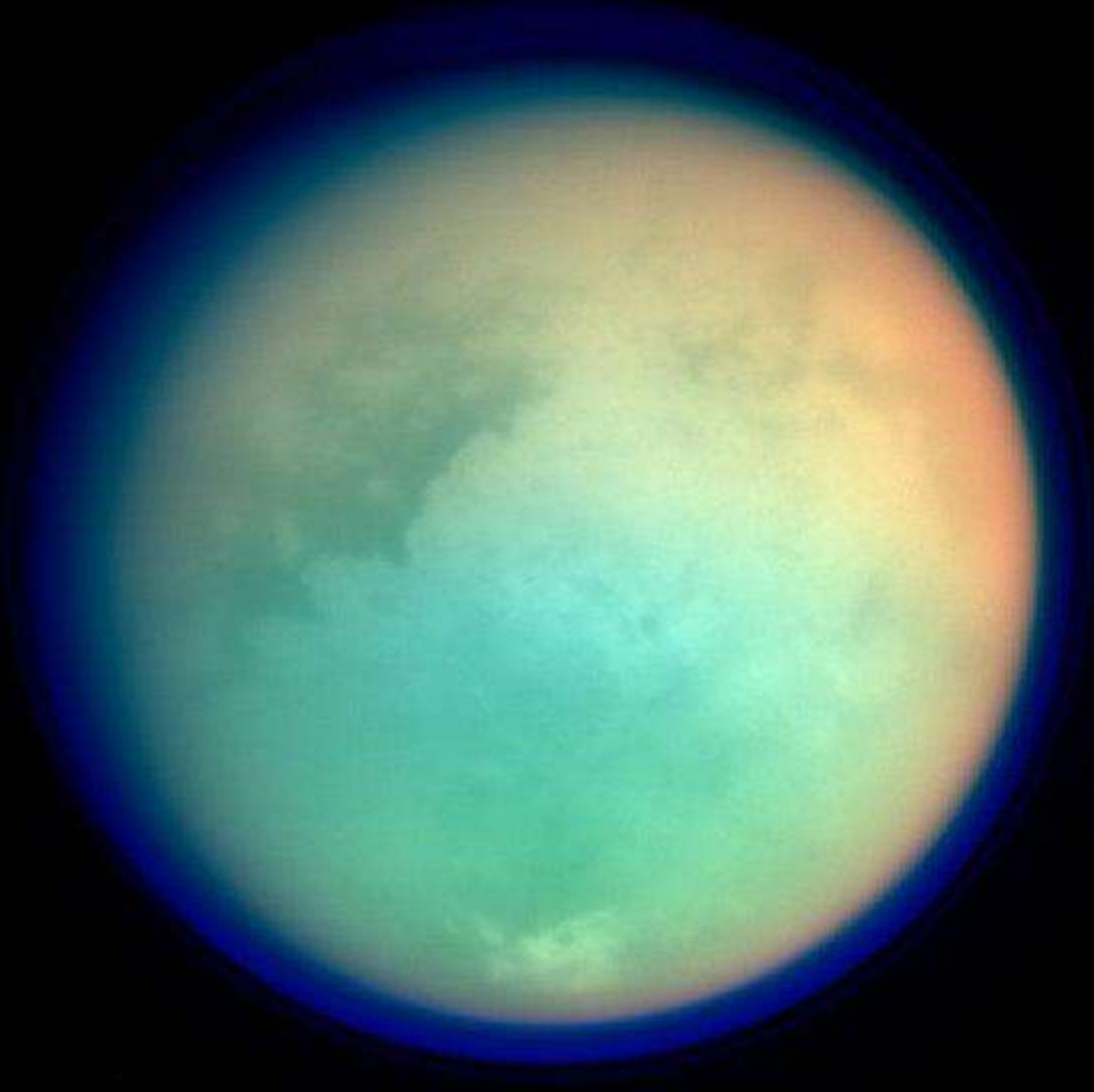
School of Mathematical Sciences, Monash University

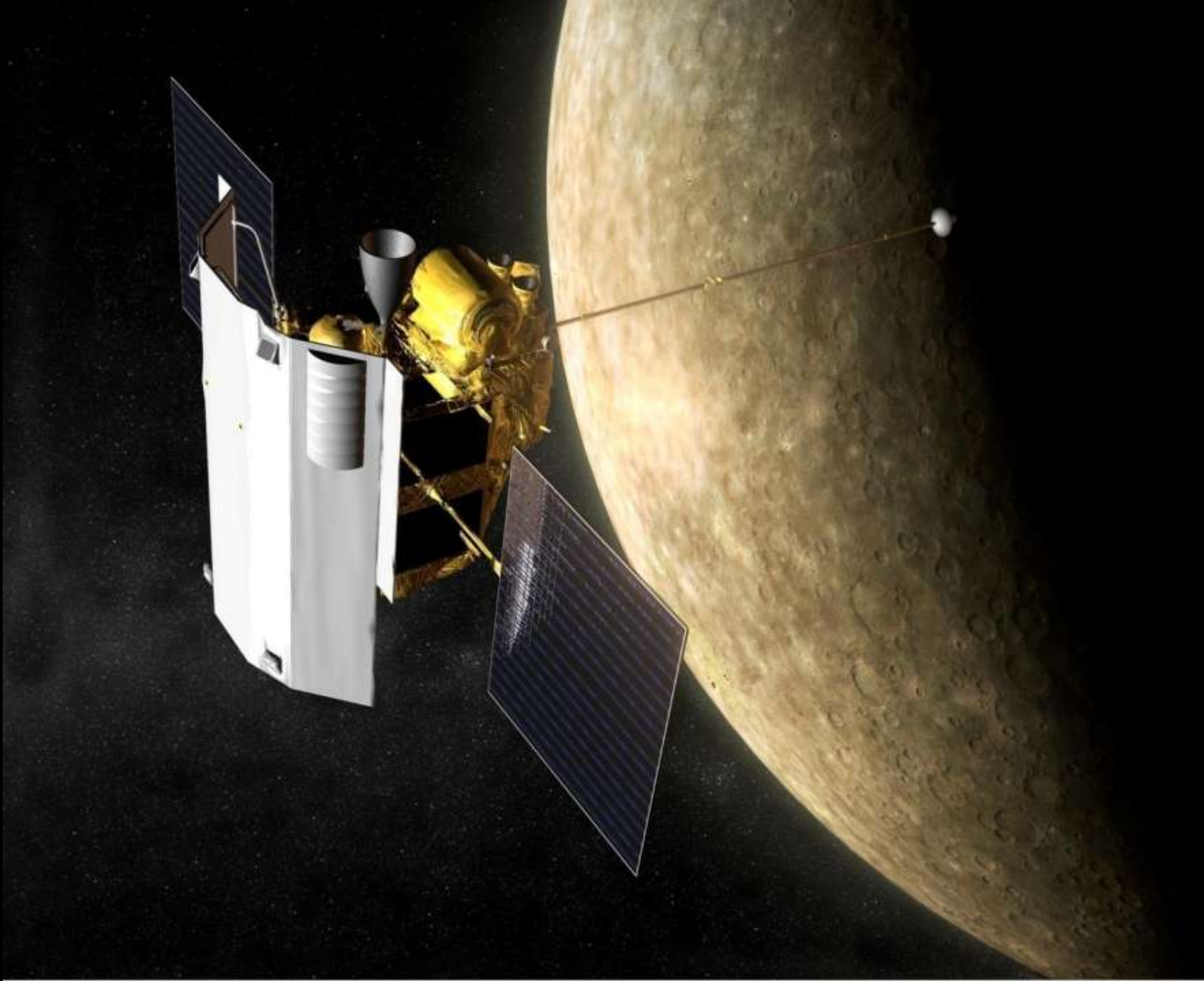
9 May 2012

Saturn's Satellites and Ring Structure

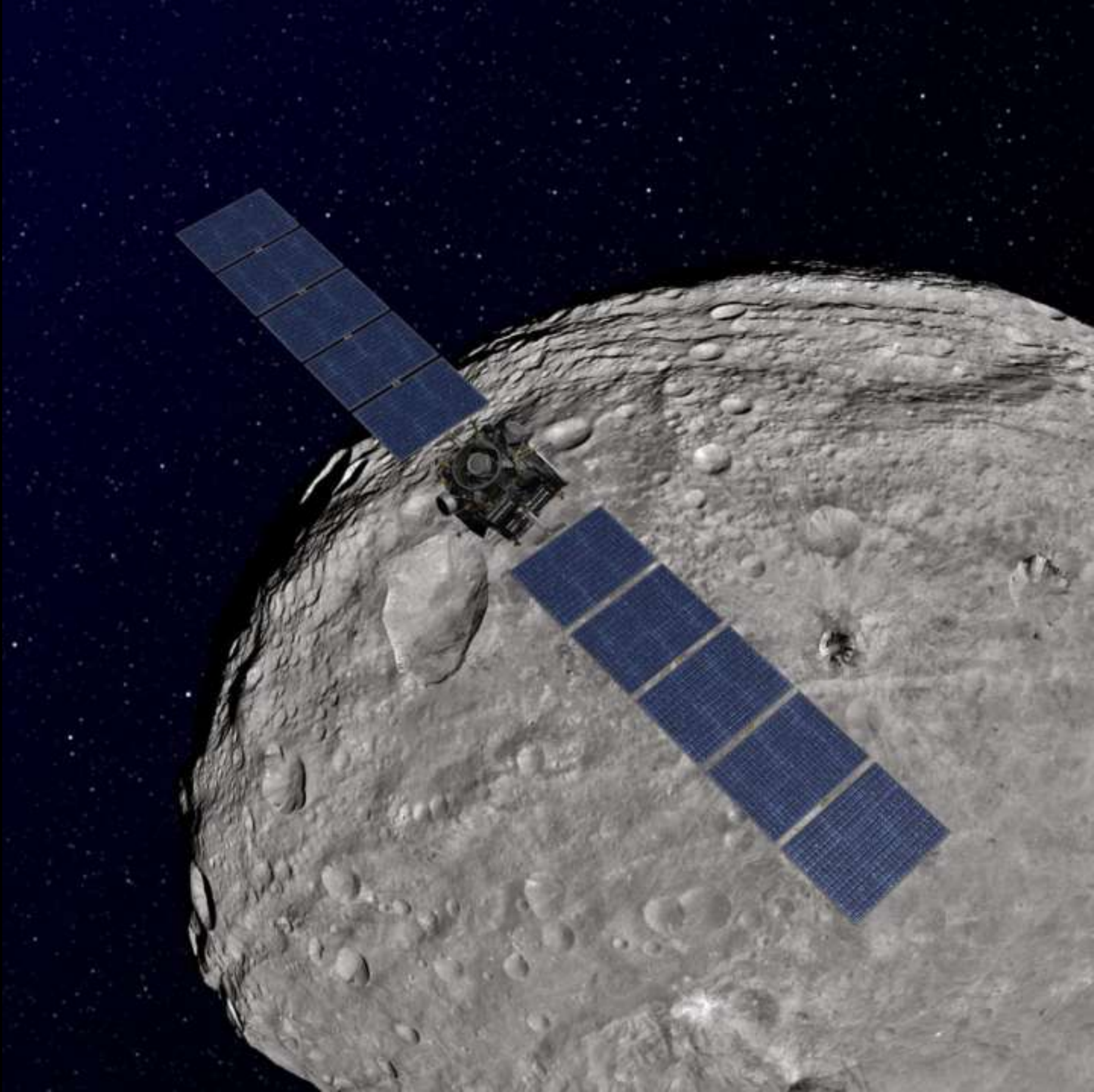


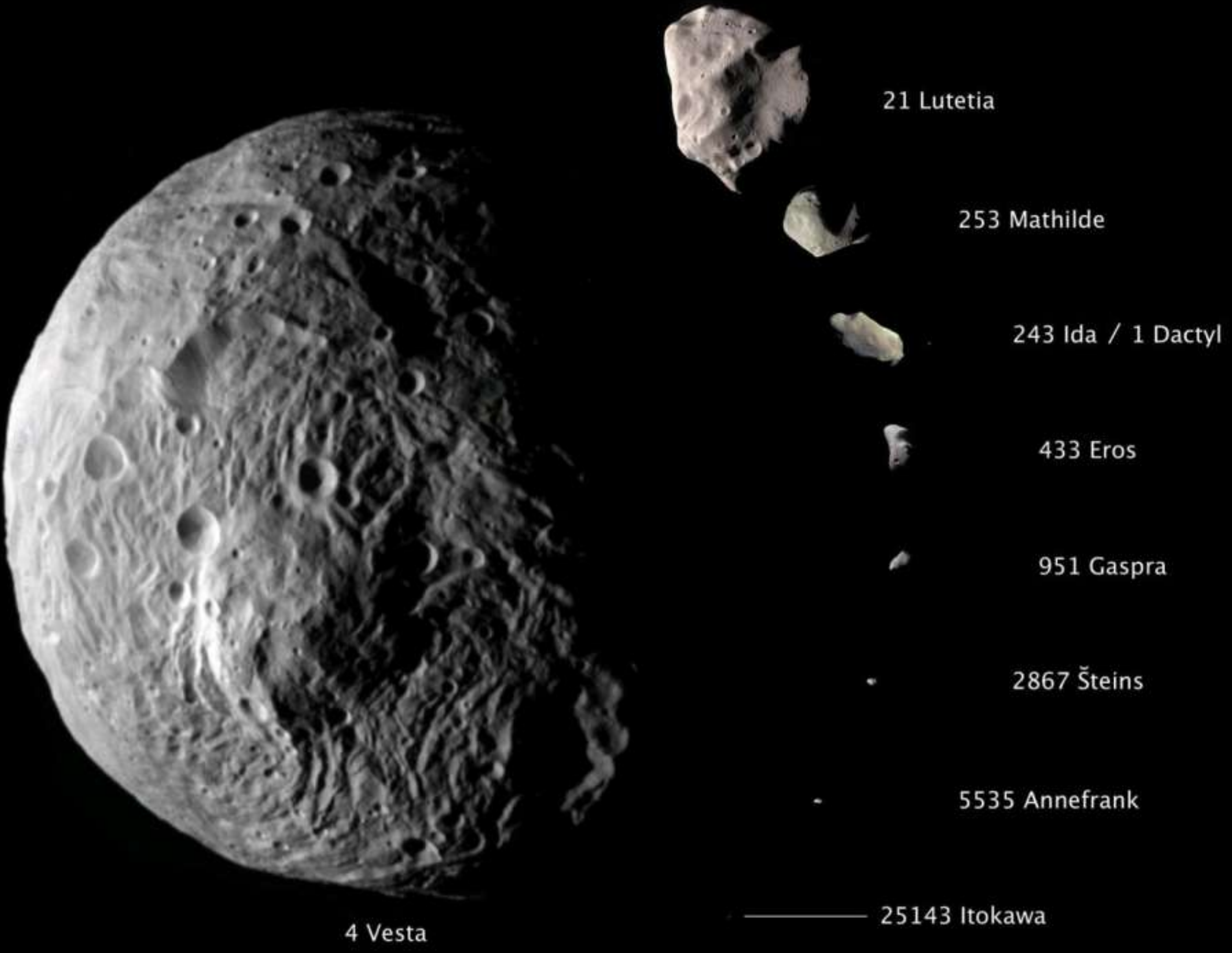
All bodies are to scale except for Pan, Atlas, Telesto, Calypso, and Helene, whose sizes have been exaggerated by a factor of 5 to show rough topography.

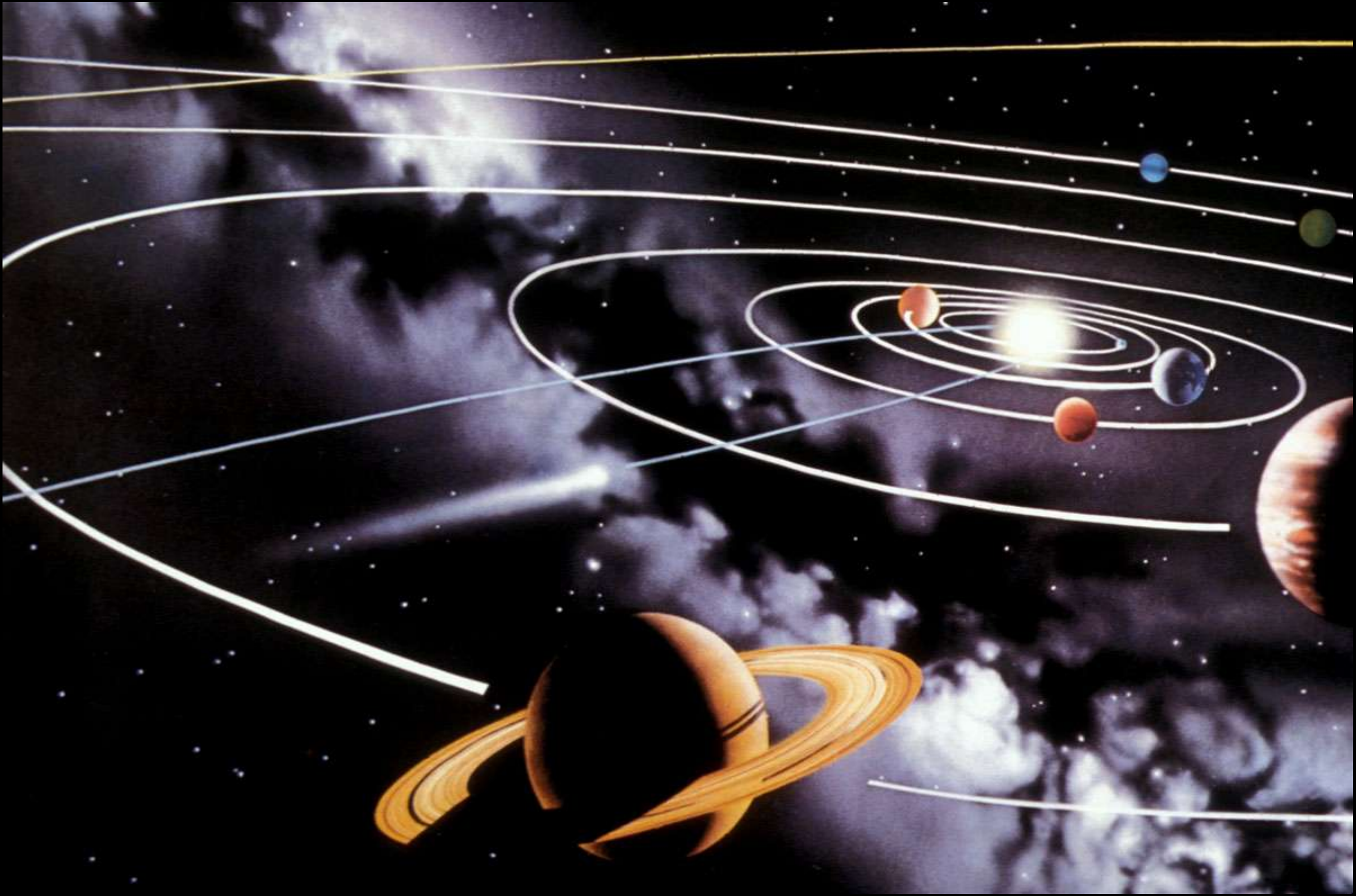





Wm K HARTMANN




21 Lutetia
253 Mathilde
243 Ida / 1 Dactyl
433 Eros
951 Gaspra
2867 Šteins
5535 Annefrank
25143 Itokawa
4 Vesta

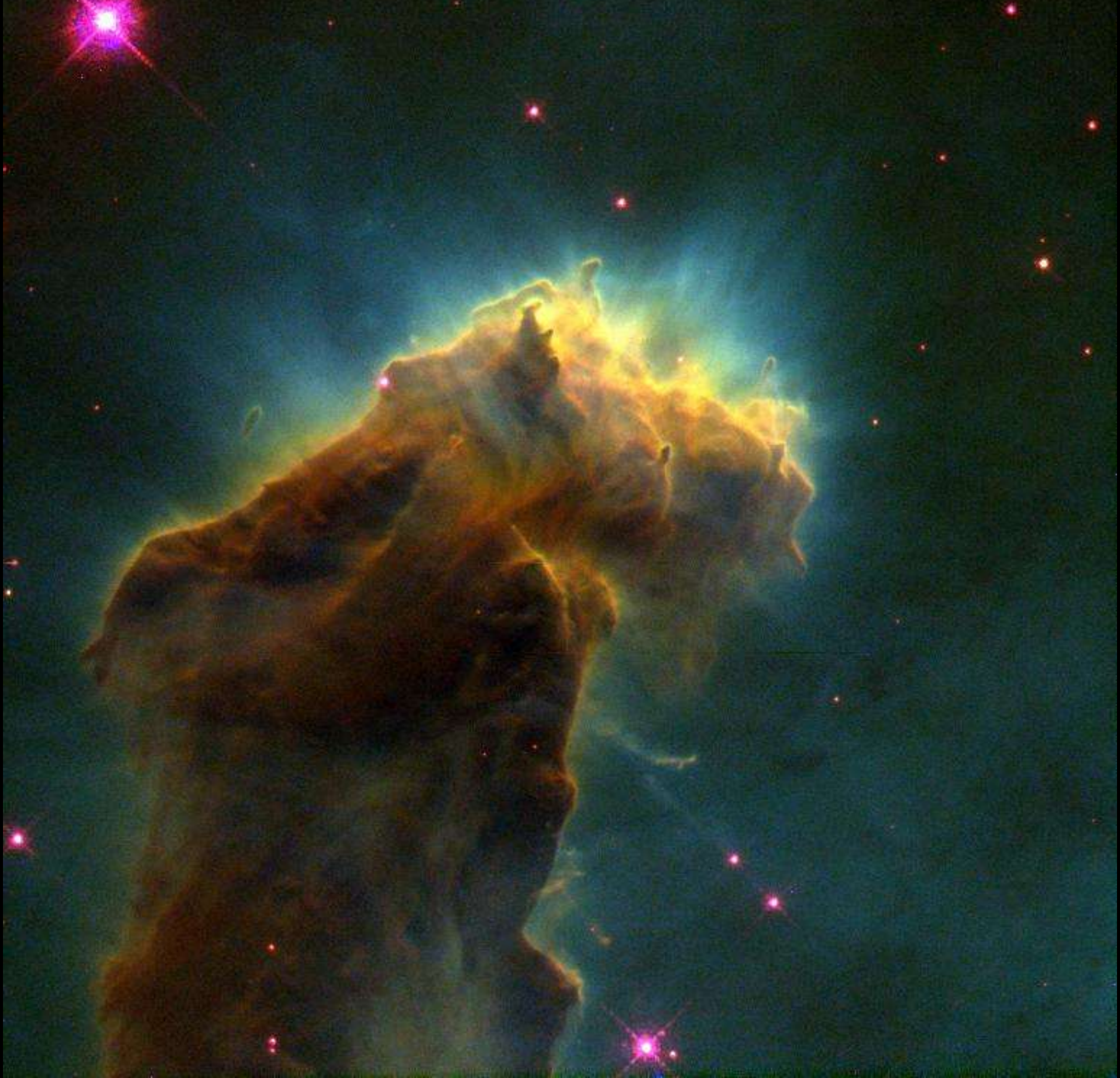


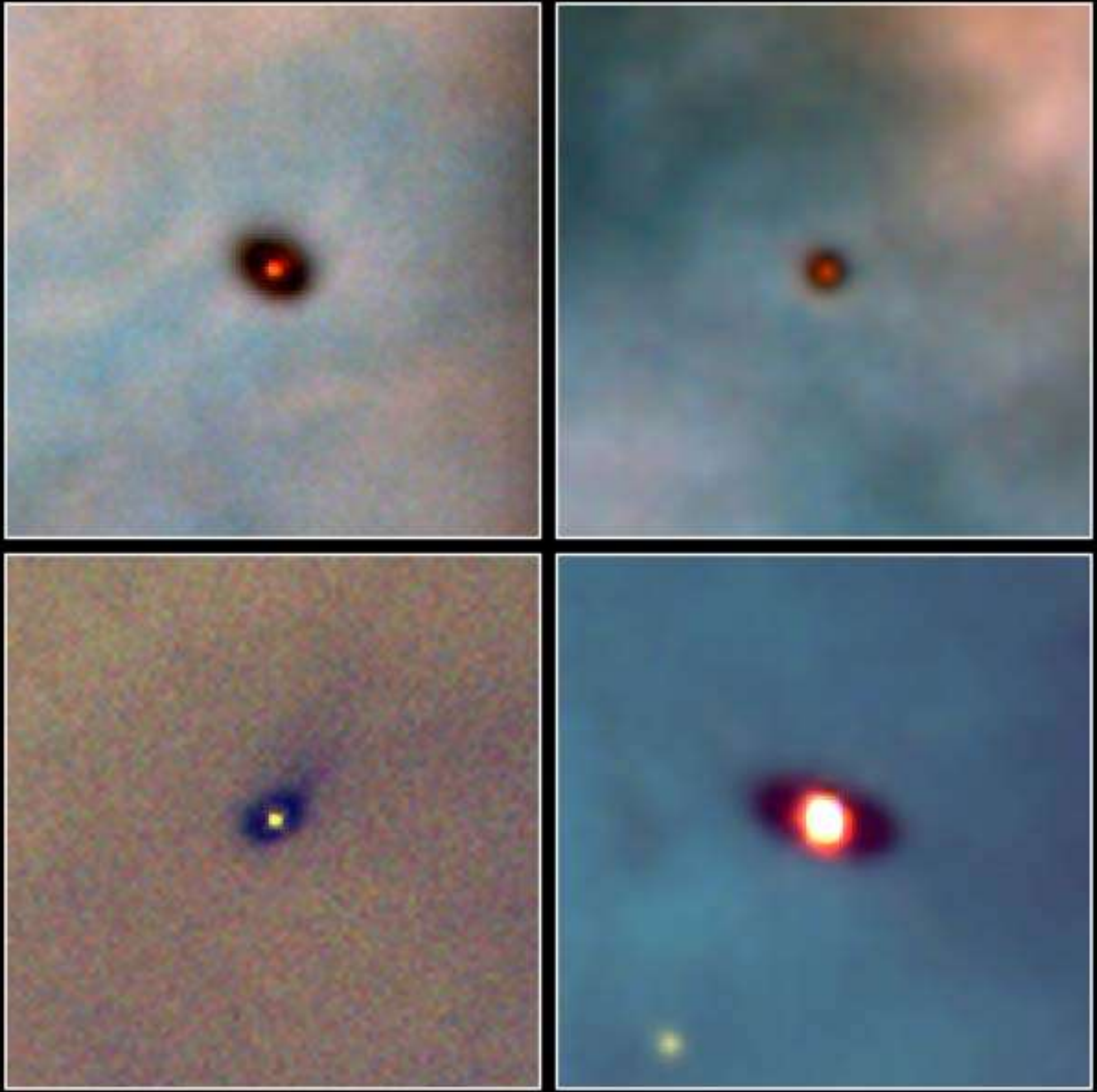
© Anglo-Australian Observatory

Eagle Nebula
M16

Hubble Heritage

NASA, ESA and The Hubble Heritage Team (STScI/AURA) • Hubble Space Telescope ACS • STScI-PRC05-12b

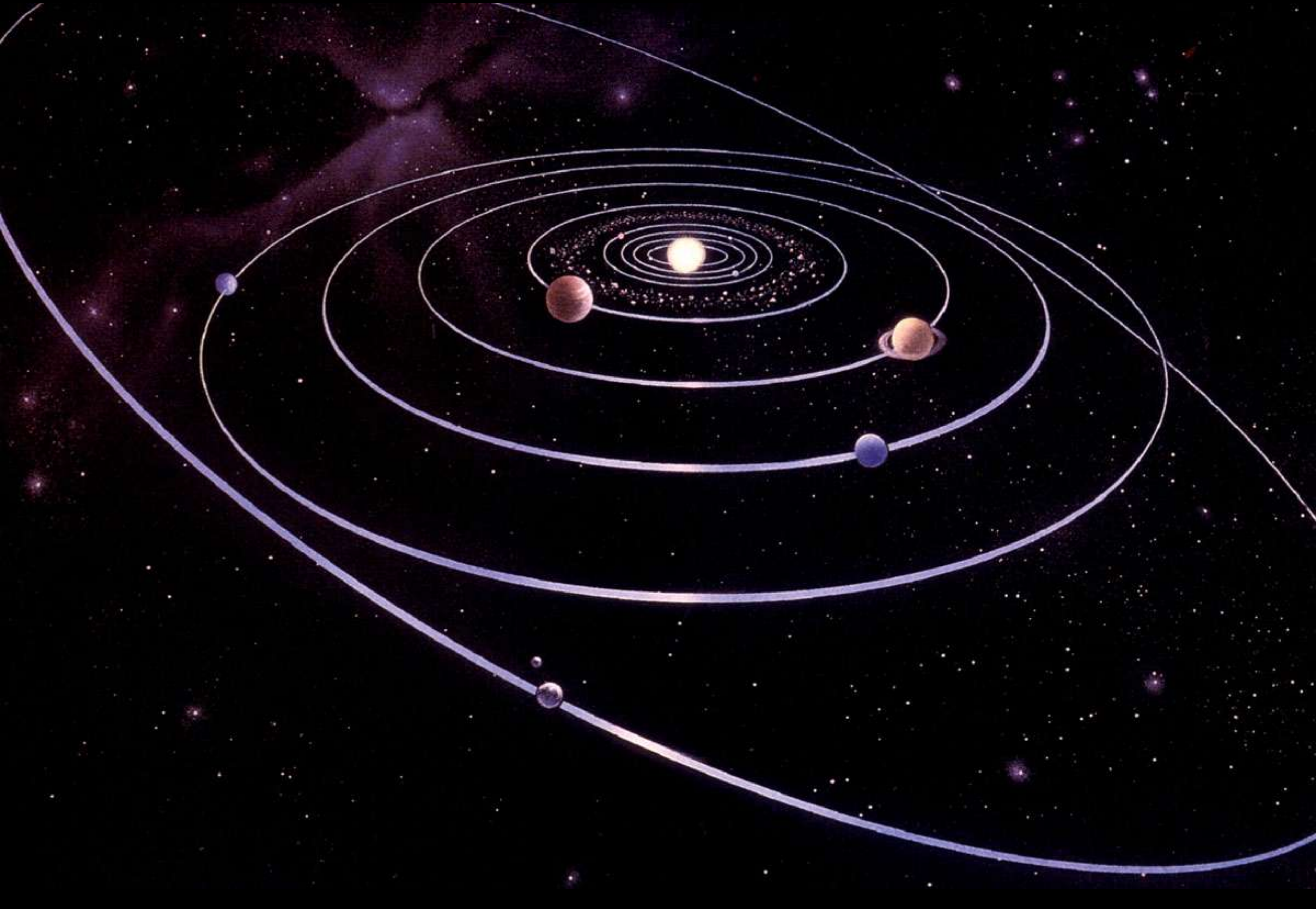


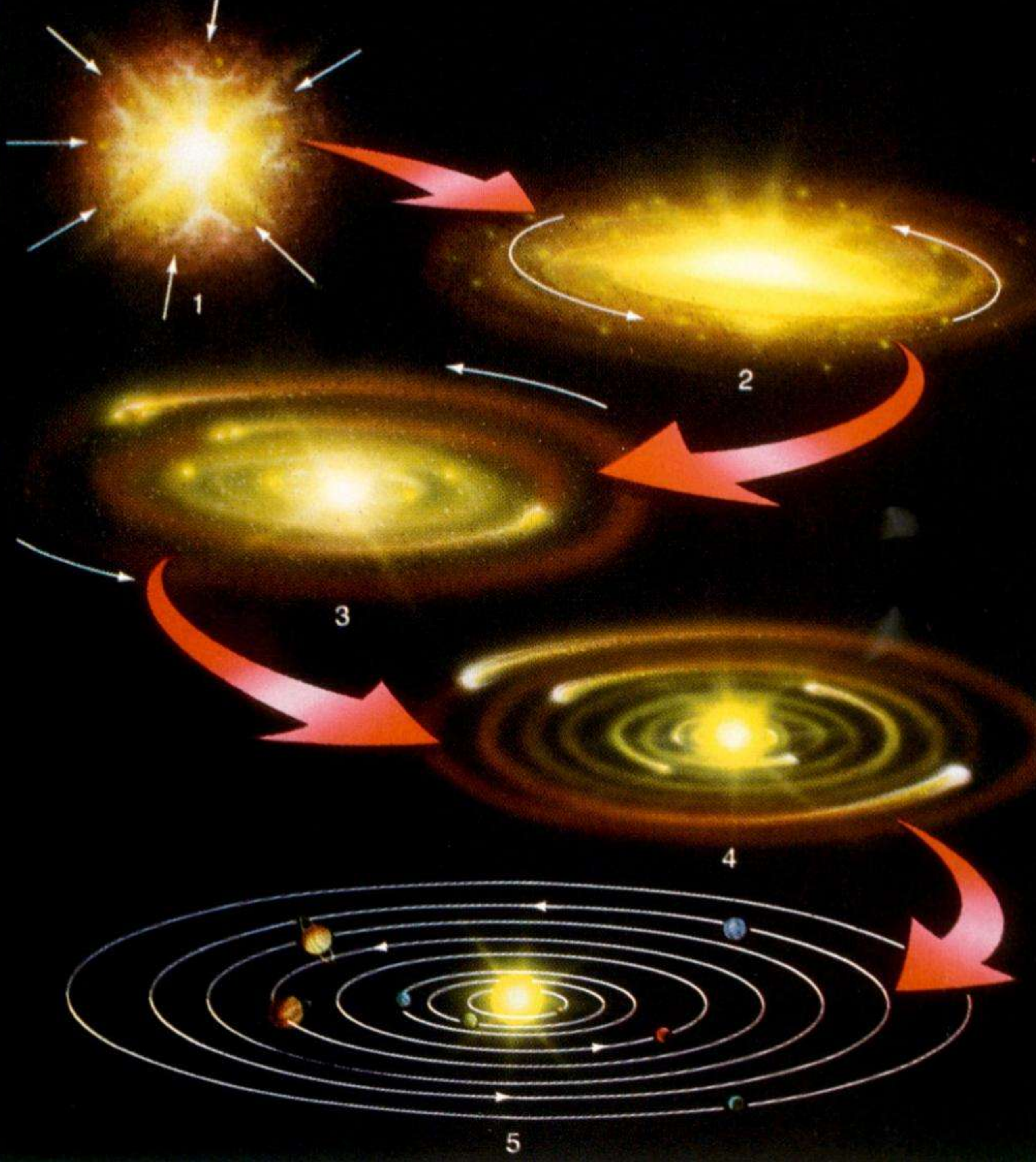


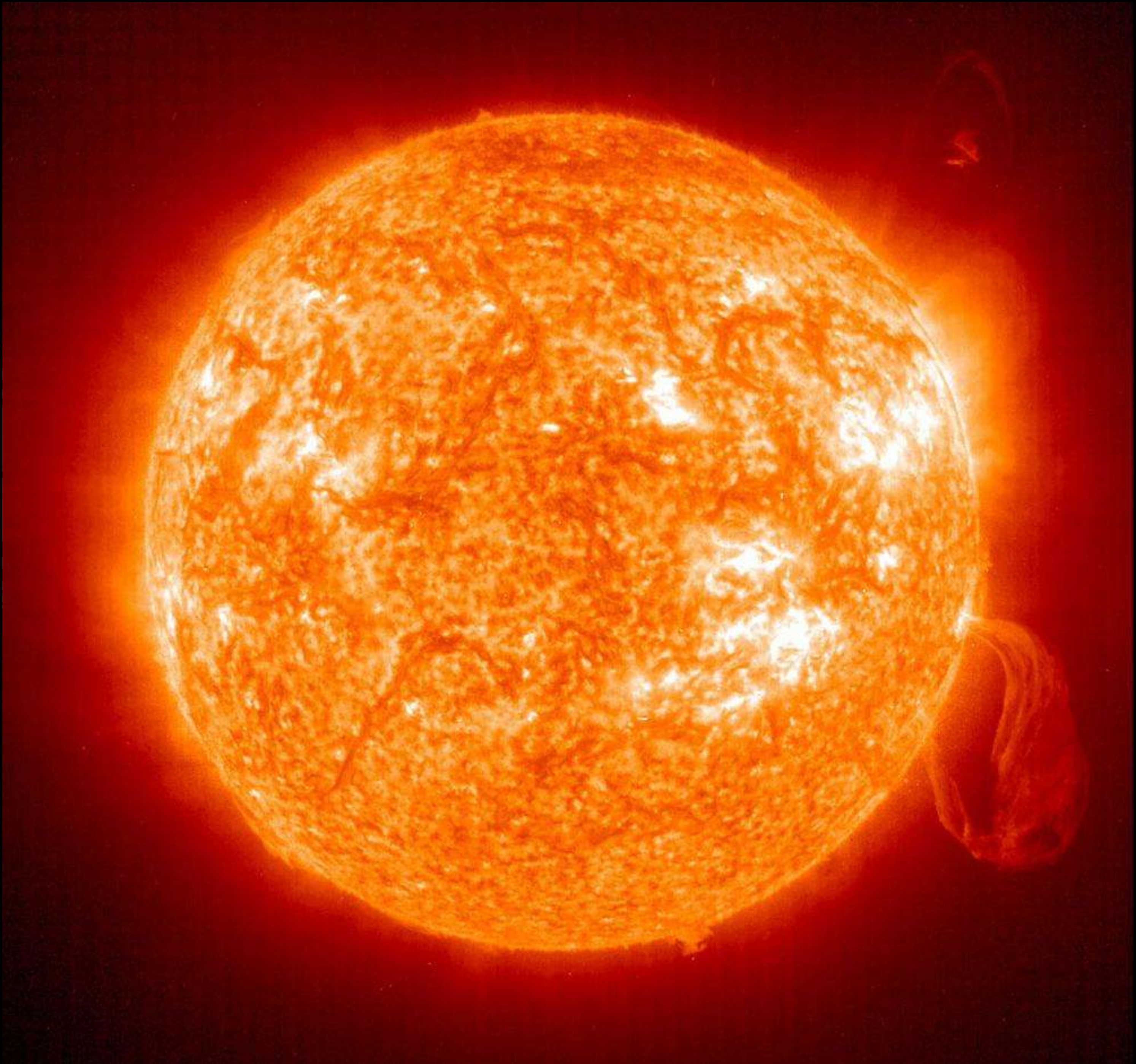
Protoplanetary Disks
Orion Nebula

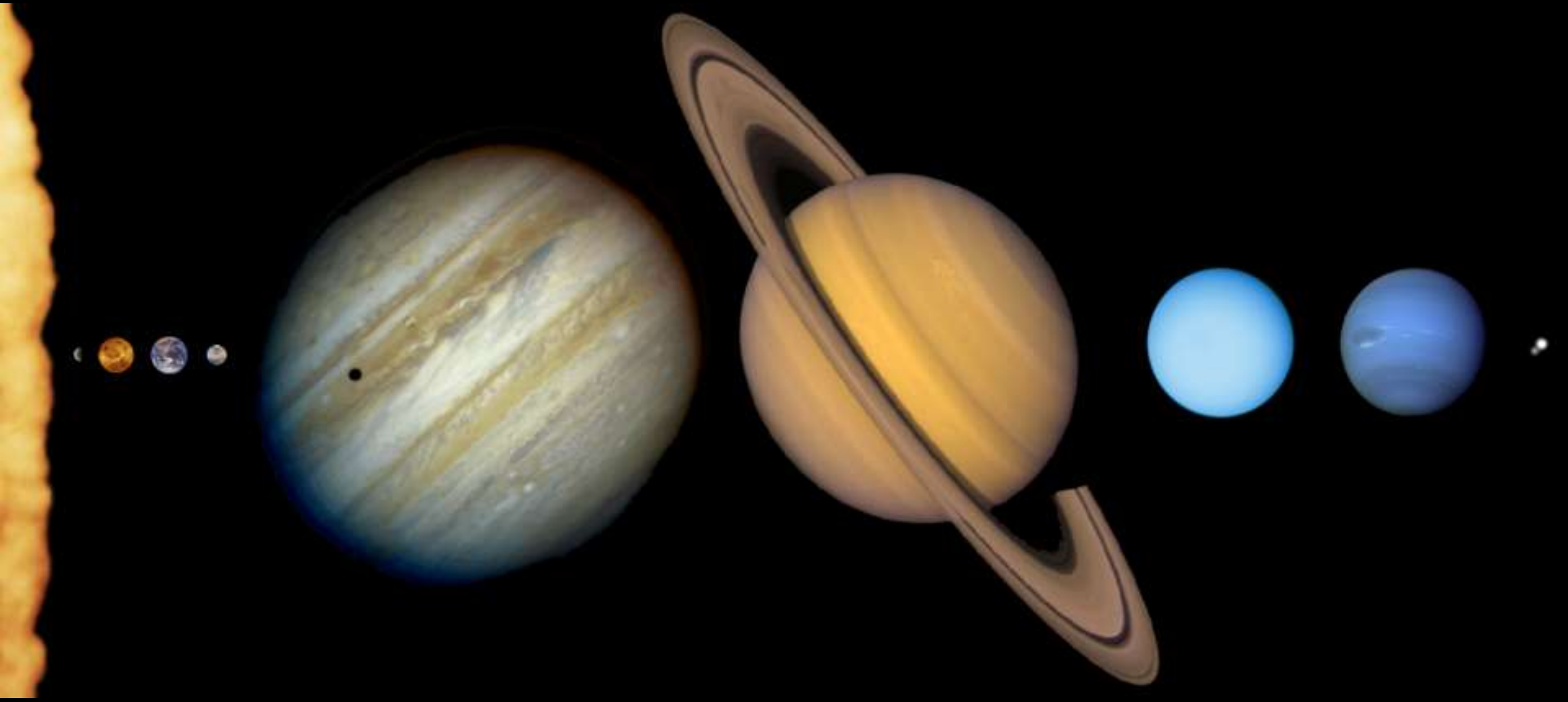
HST • WFPC2

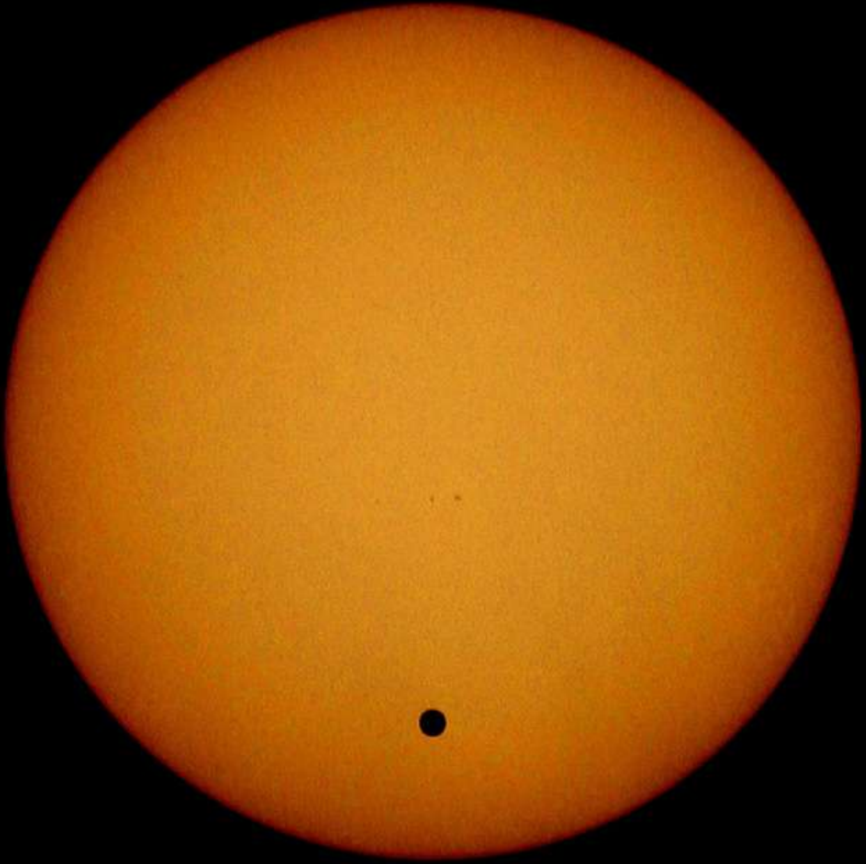
PRC95-45b • ST ScI OPO • November 20, 1995
M. J. McCaughrean (MPIA), C. R. O'Dell (Rice University), NASA

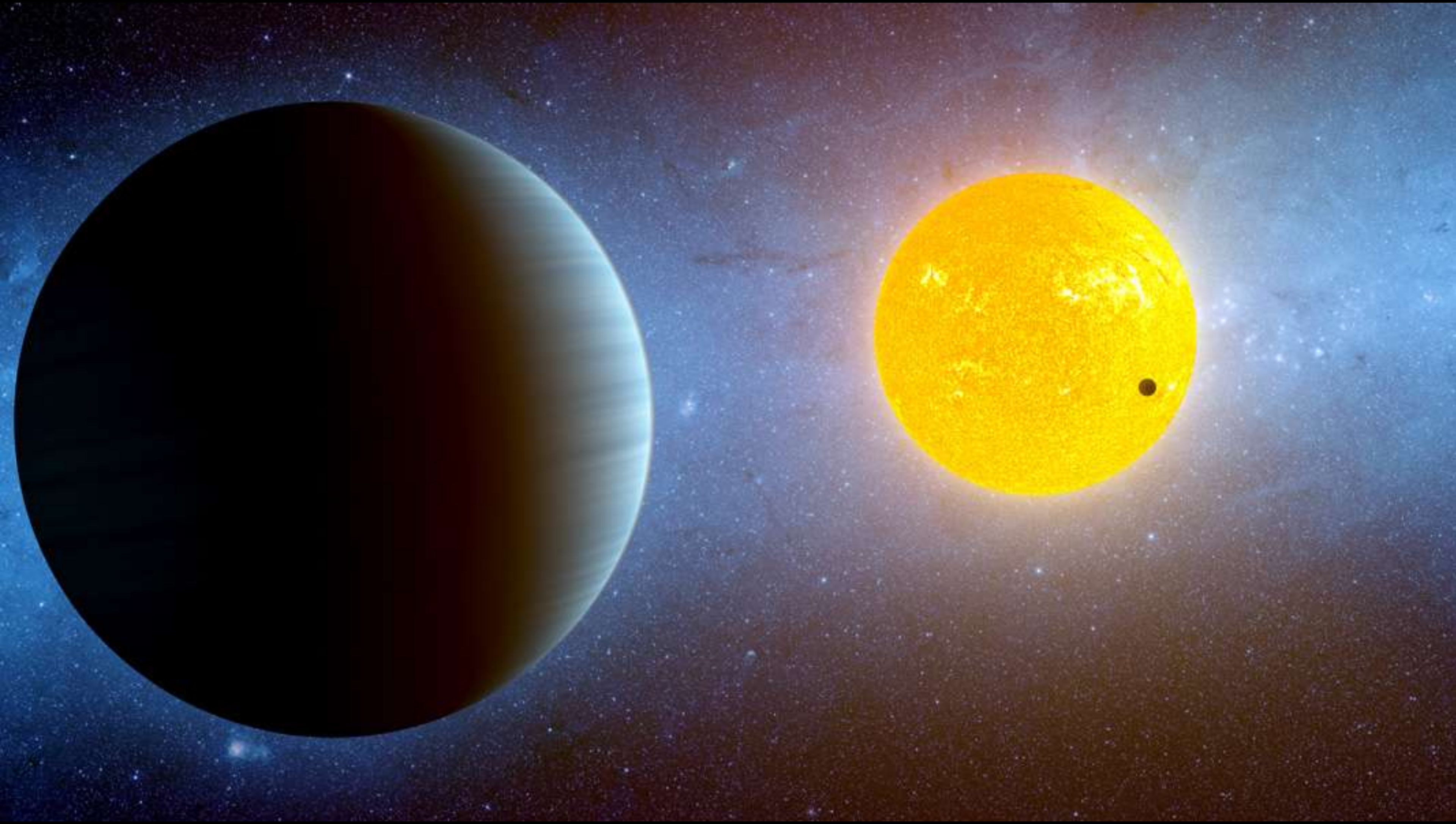


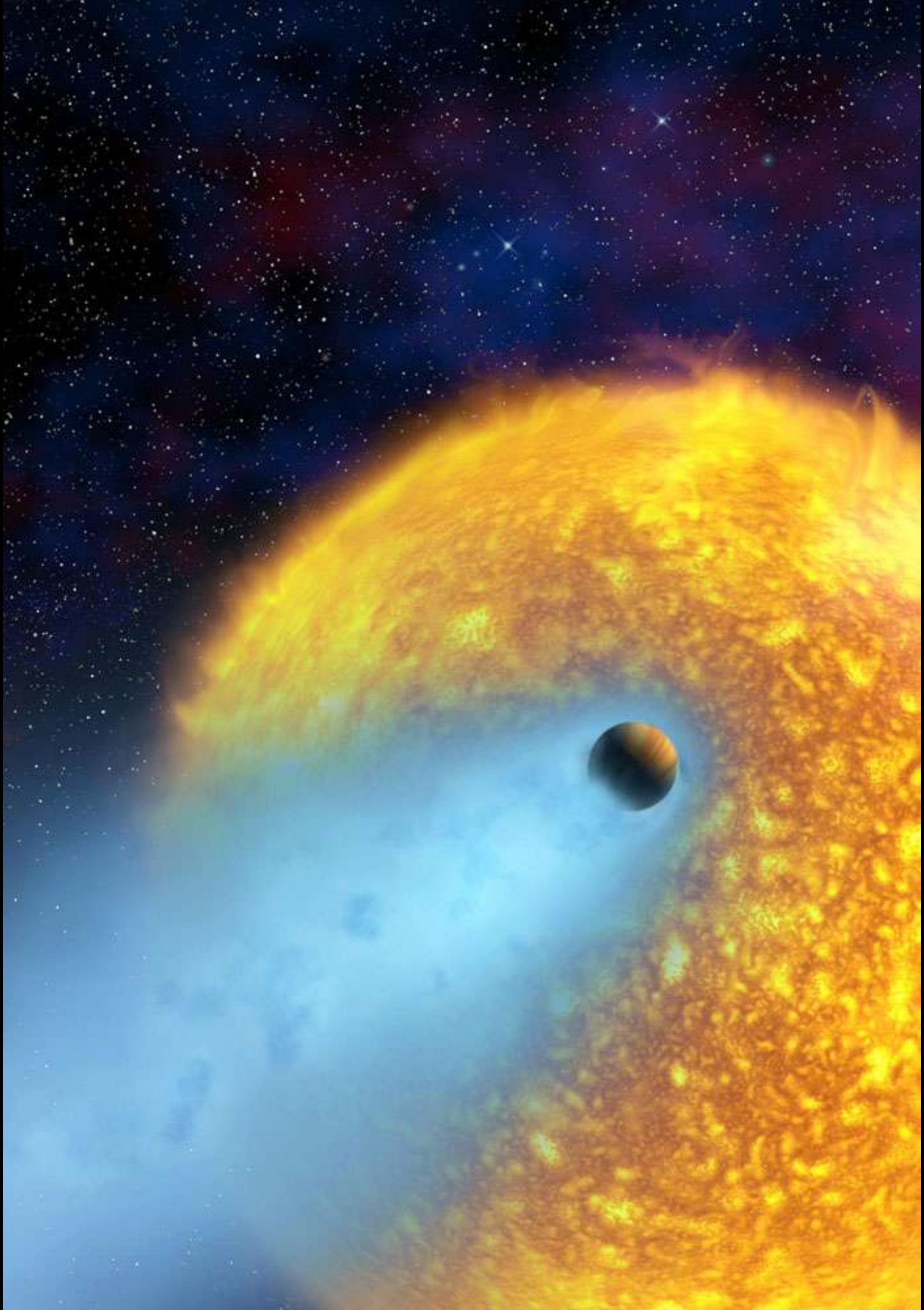

1
2
3
4
5

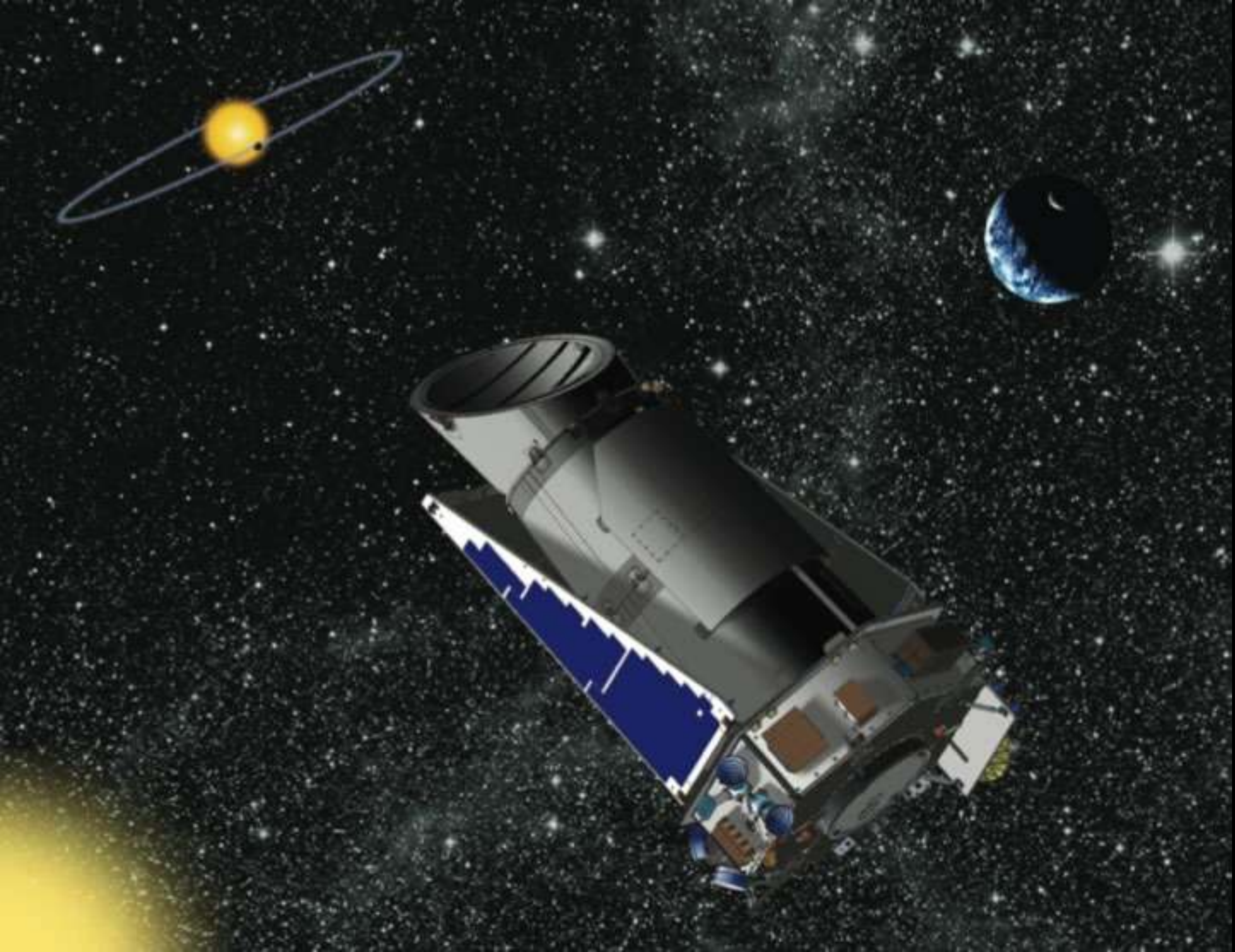




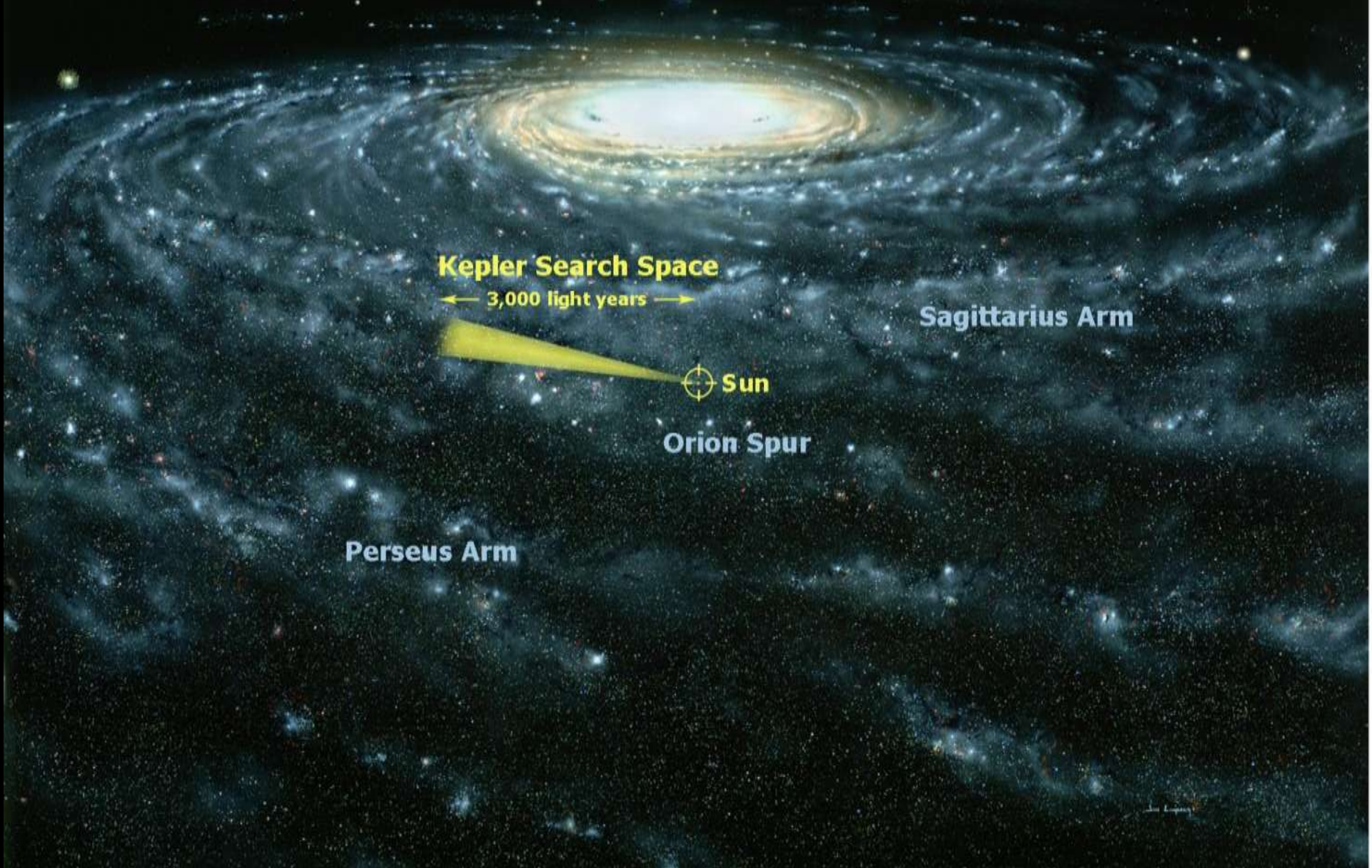






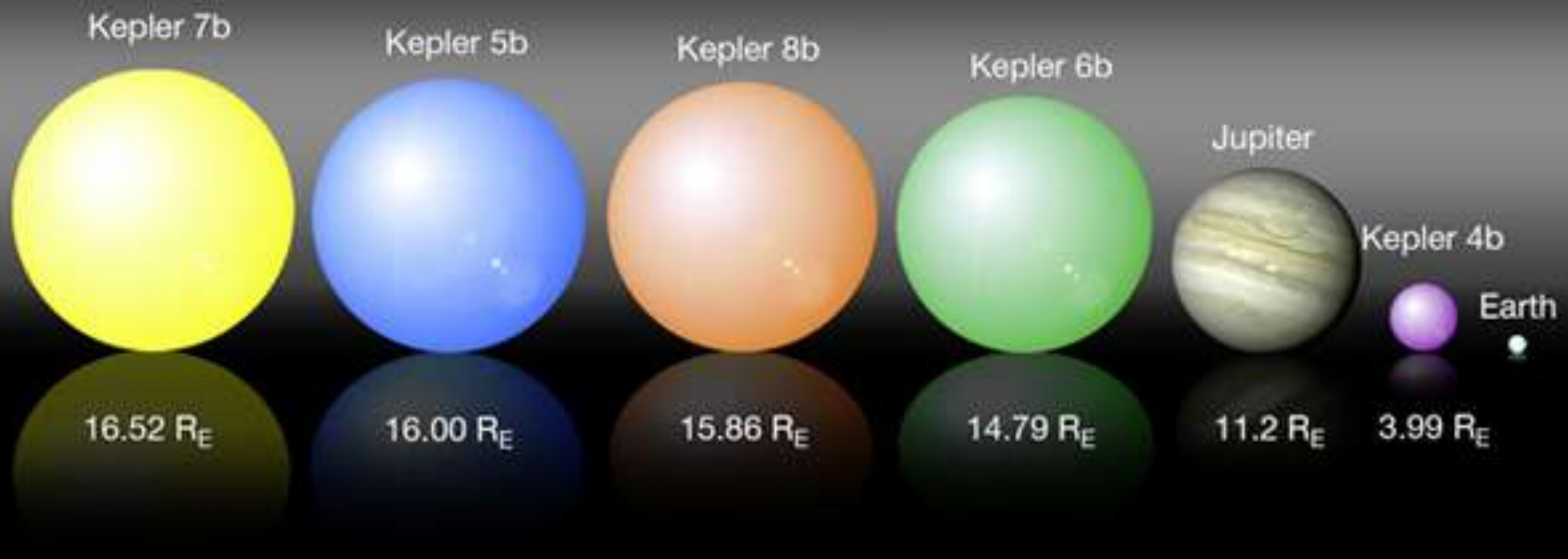


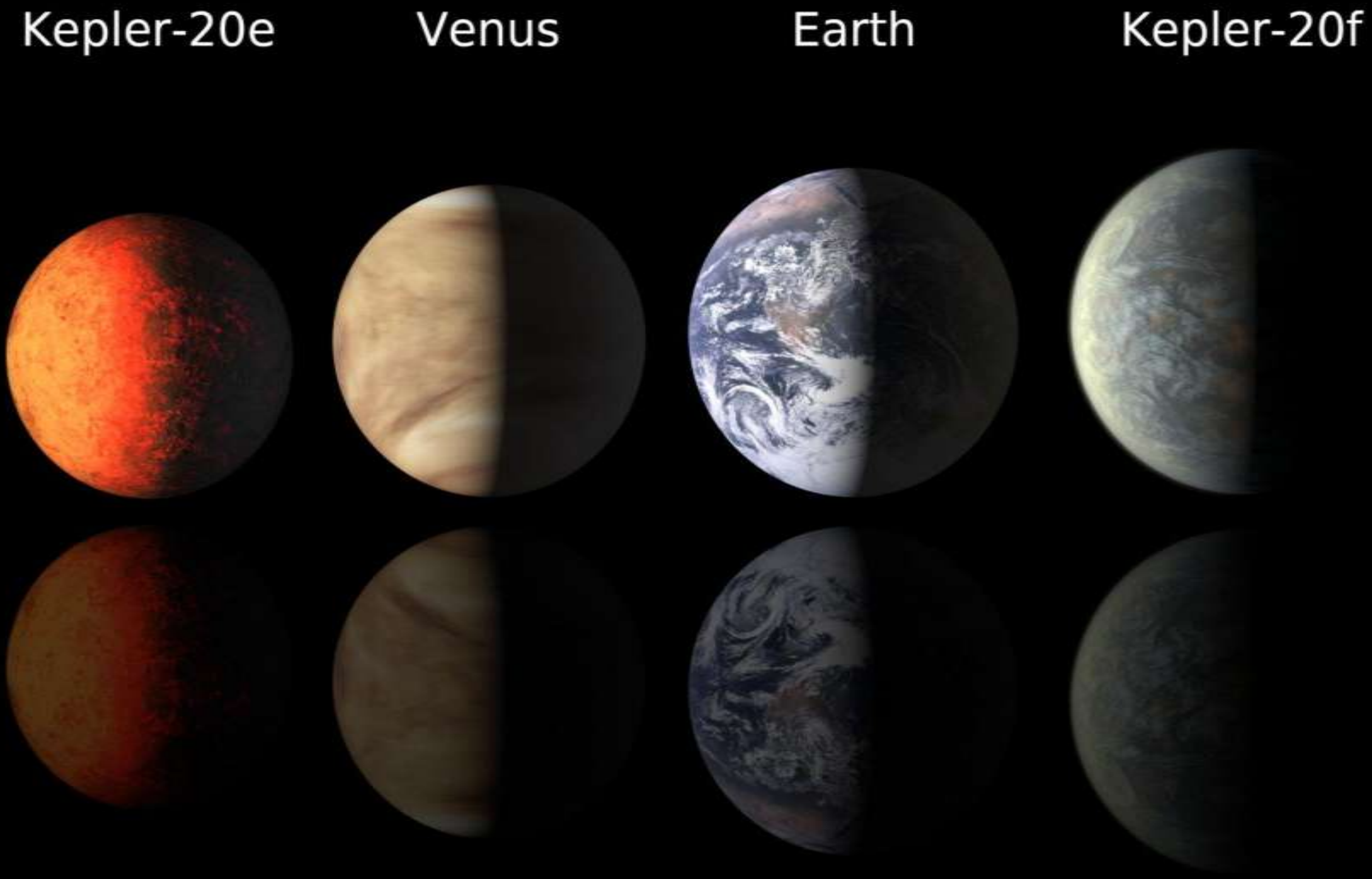
Milky Way Galaxy

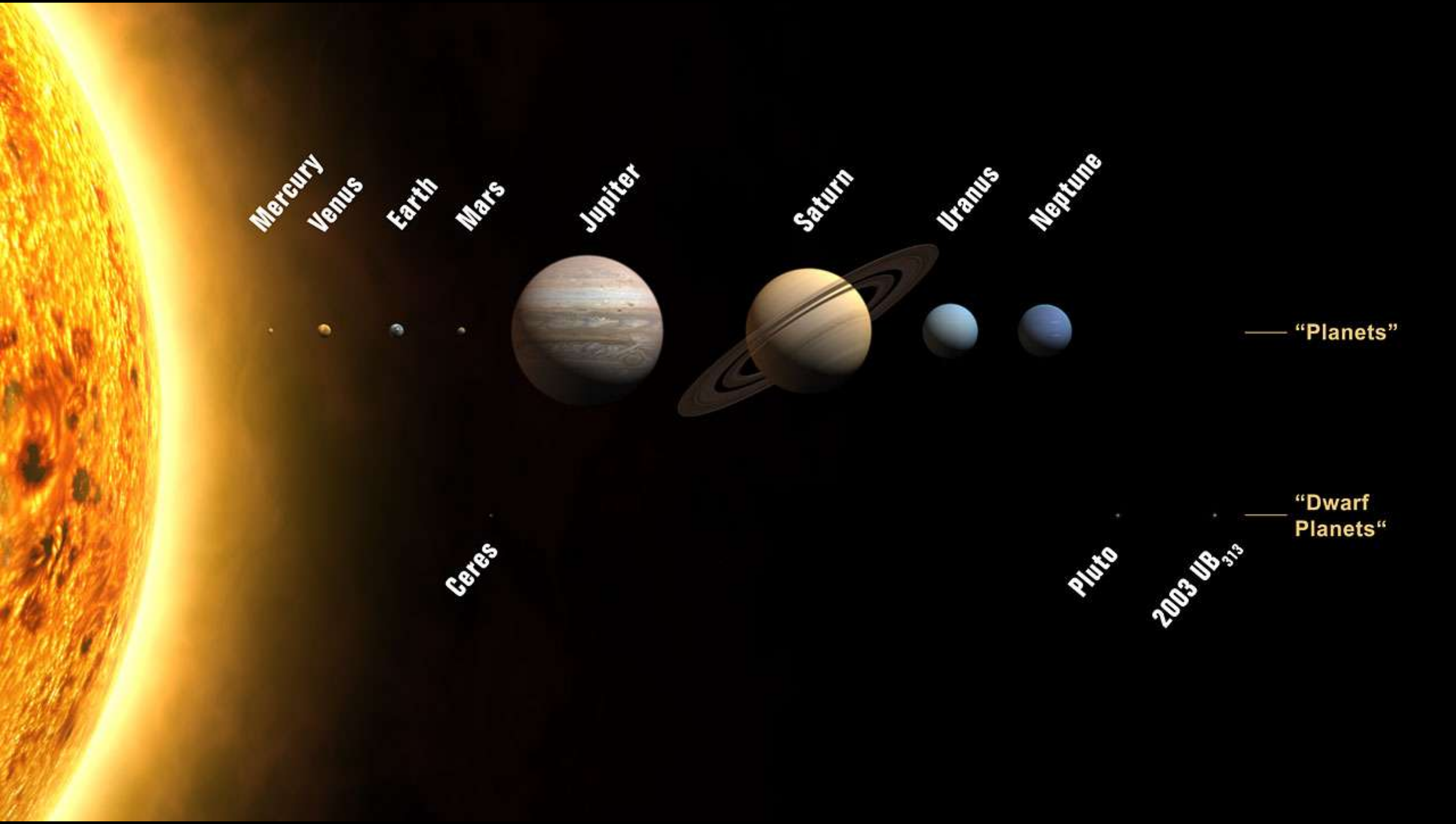


Portrait of the Milky Way © Jon Lomberg www.jonlomberg.com

Planet Size


Kepler 7b
Kepler 5b
Kepler 8b
Kepler 6b
Jupiter
Kepler 4b
Earth
16.52 R_E
16.00 R_E
15.86 R_E
14.79 R_E
11.2 R_E
3.99 R_E


Kepler-20e
Venus
Earth
Kepler-20f

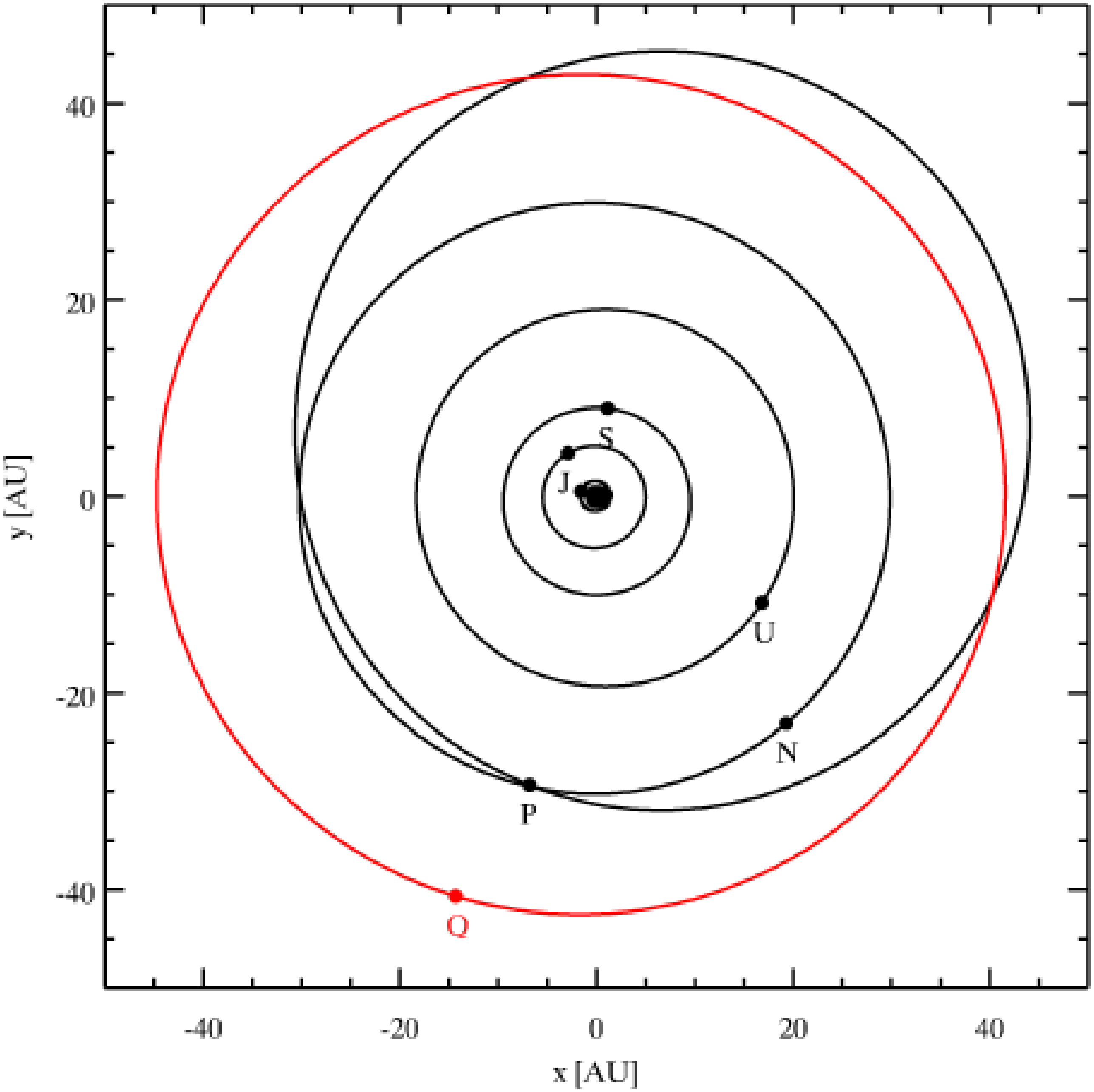

Mercury
Venus
Earth
Mars
Jupiter
Saturn
Uranus
Neptune
"Planets"
"Dwarf Planets"
Ceres
Pluto
2003 UB$_{313}$

Pluto:
1400
miles

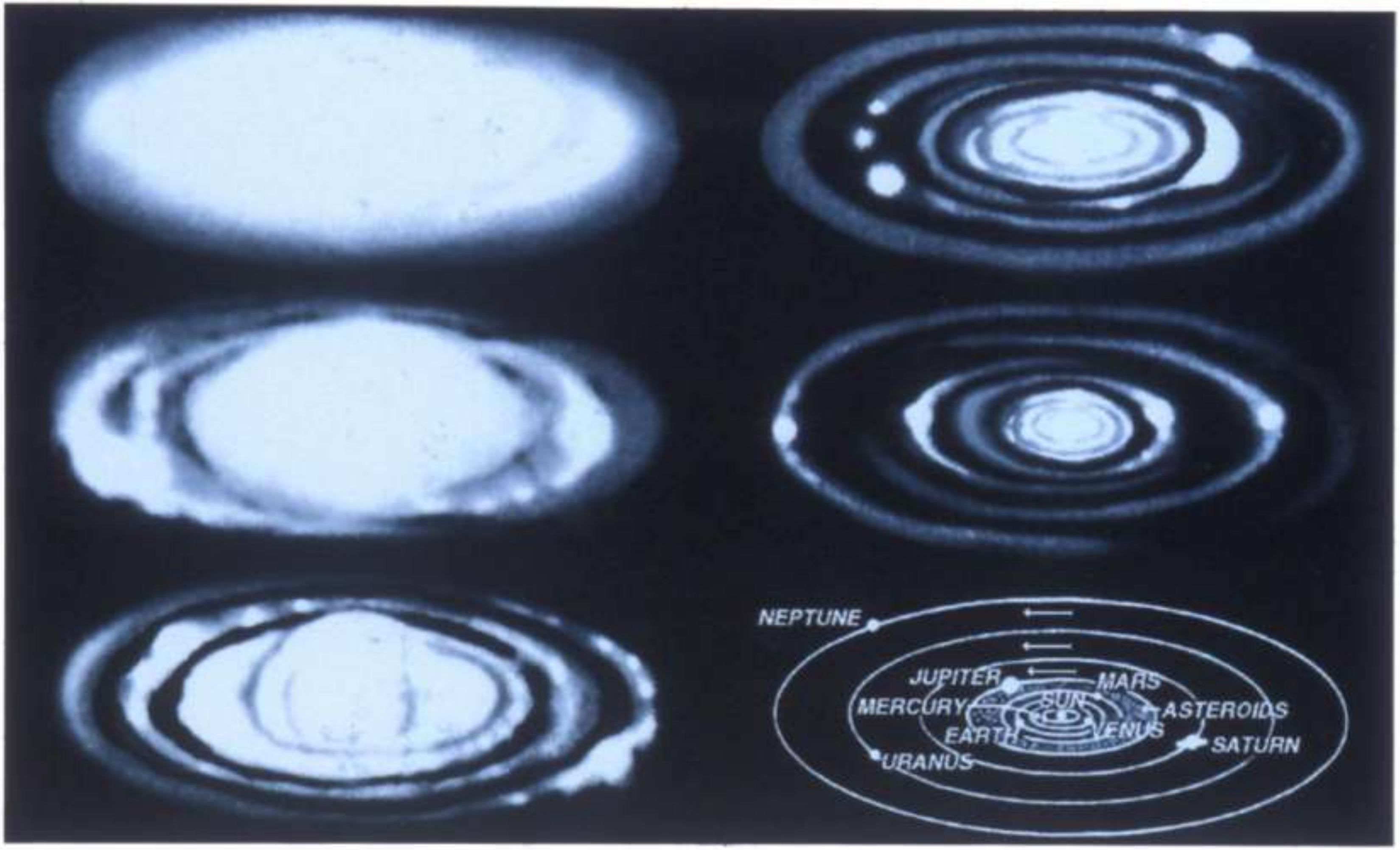
**Quaoar:
800
miles**

Earth's
moon:
2100
miles

Earth:
8000
miles

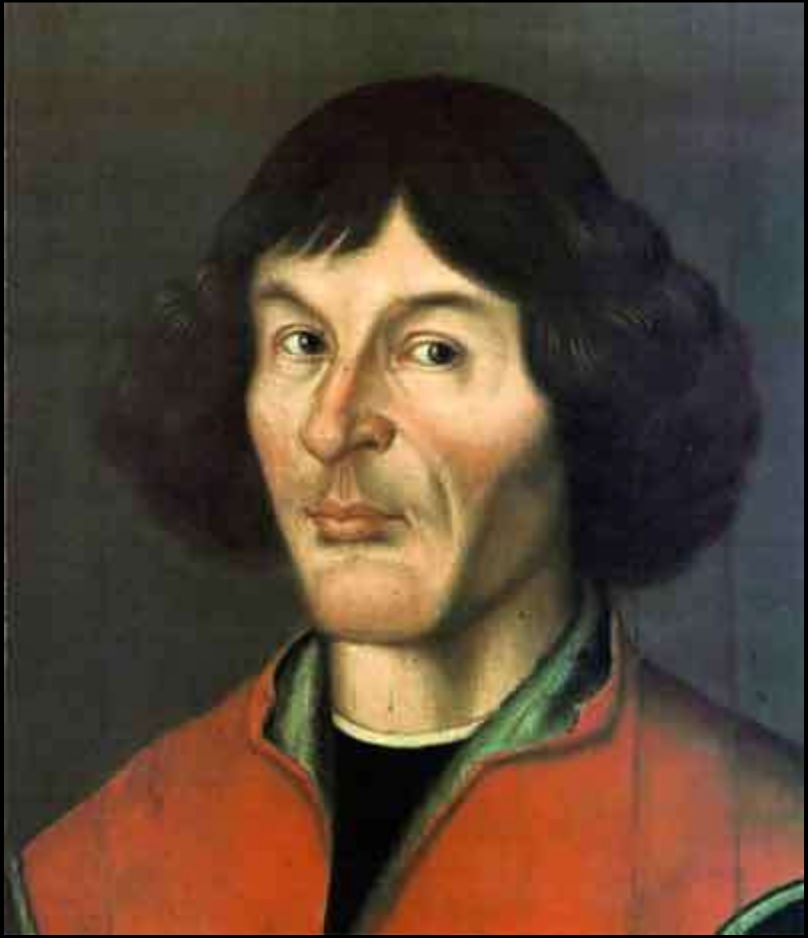


40
20
0
-20
-40
y [AU]
-40
-20
0
20
40
x [AU]
J
S
U
N
P
Q



Artist's impression of Laplace's nebula hypothesis. The young, contracting, and rotating proto-solar cloud sheds a system of orbiting gas rings, from which the planets later condense [from drawings by Scriven Bolton, F.R.A.S., Figure 172 of Whipple 1968].

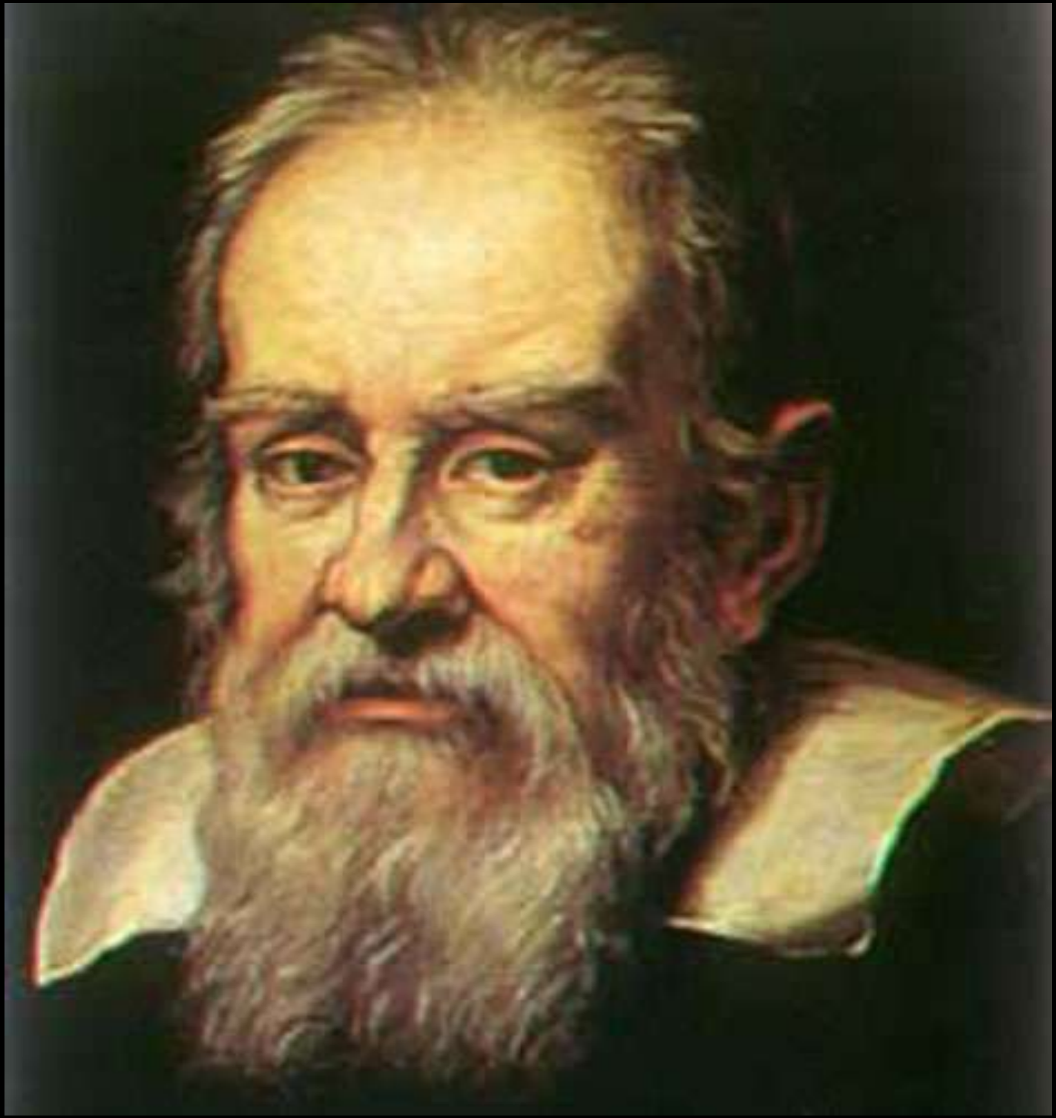
Pierre-Simon Laplace
[1749 - 1827]



Nicolaus Copernicus

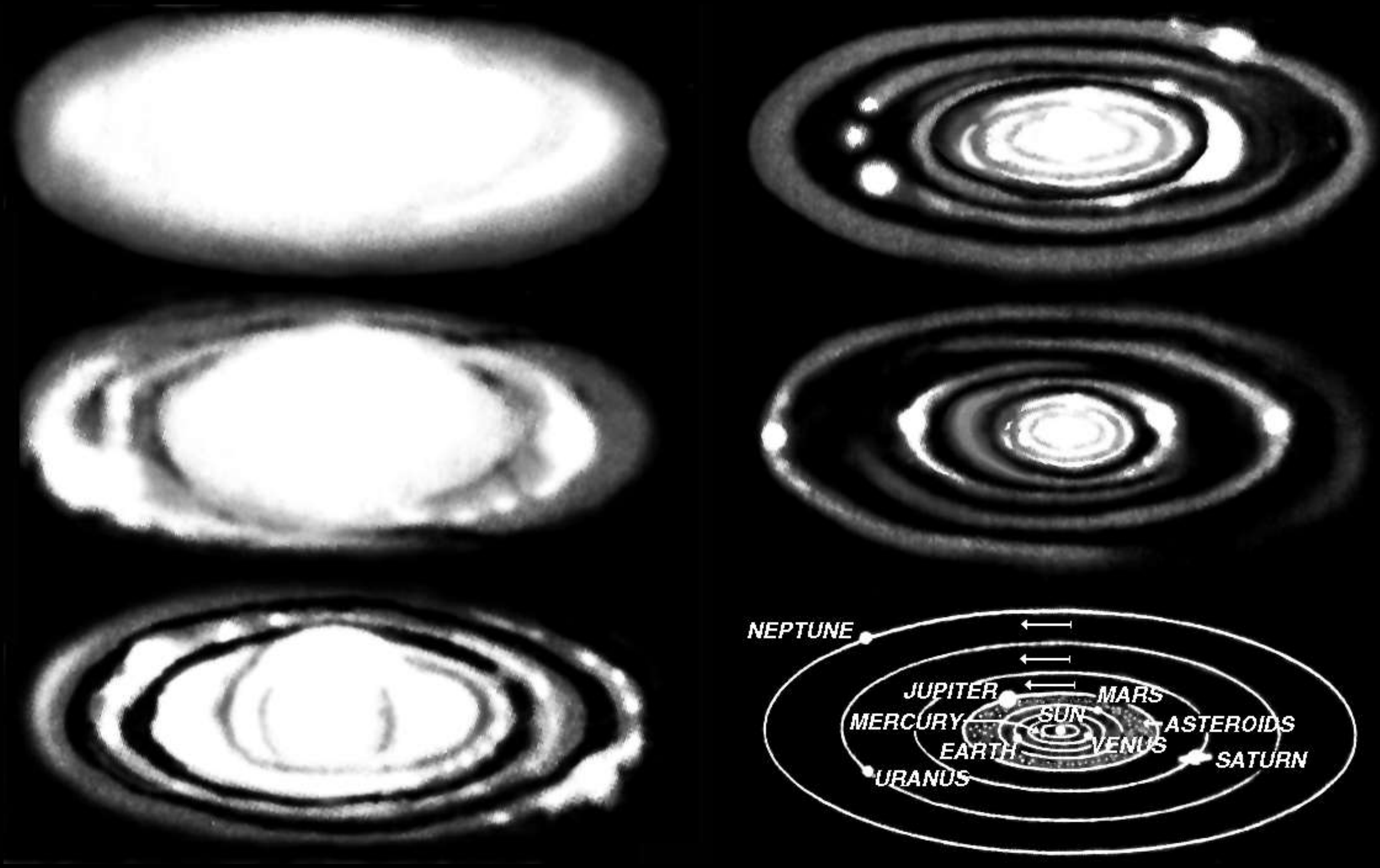
(1473 – 1543)

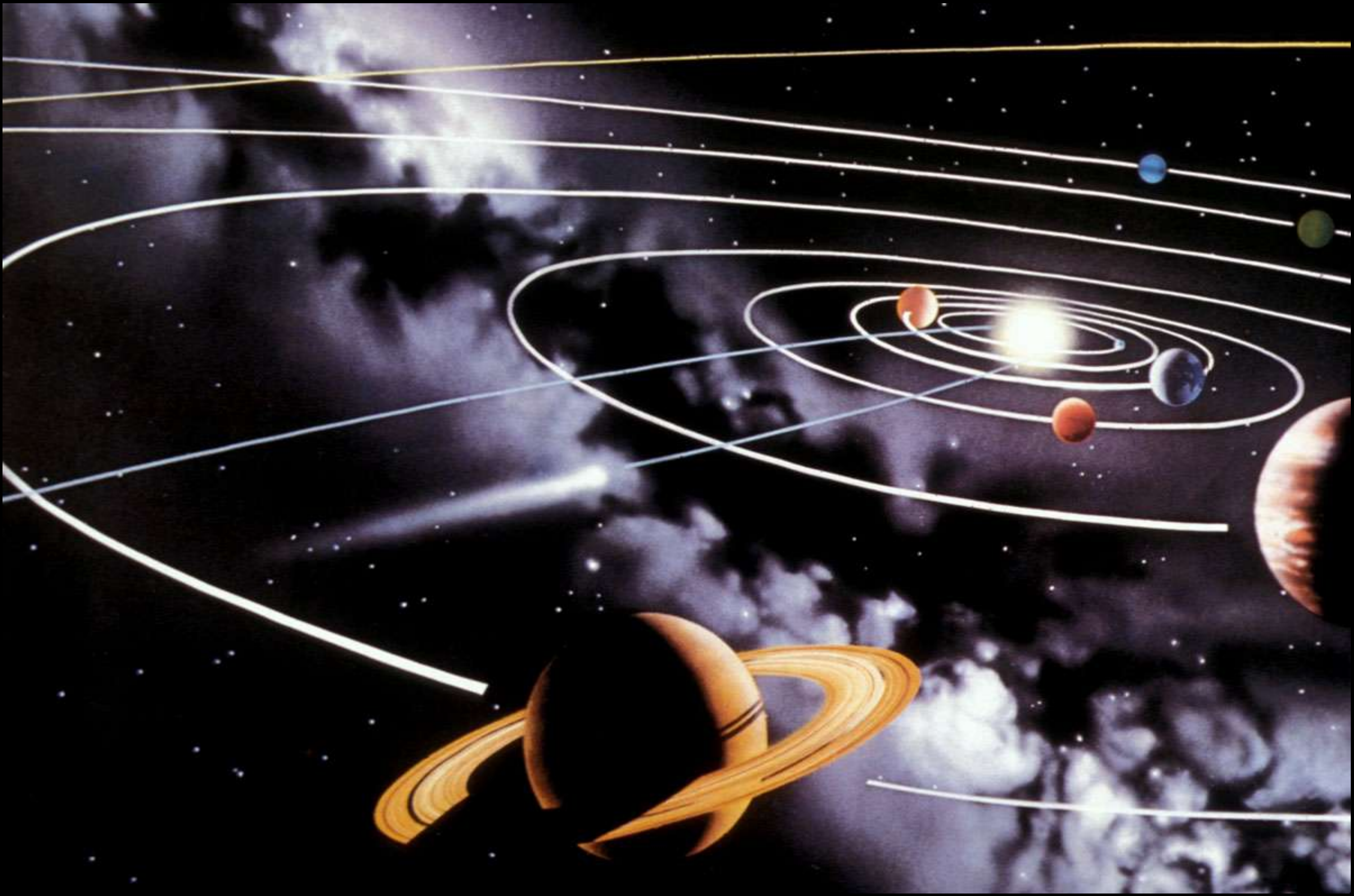
Johannes Kepler
(1571-1630)

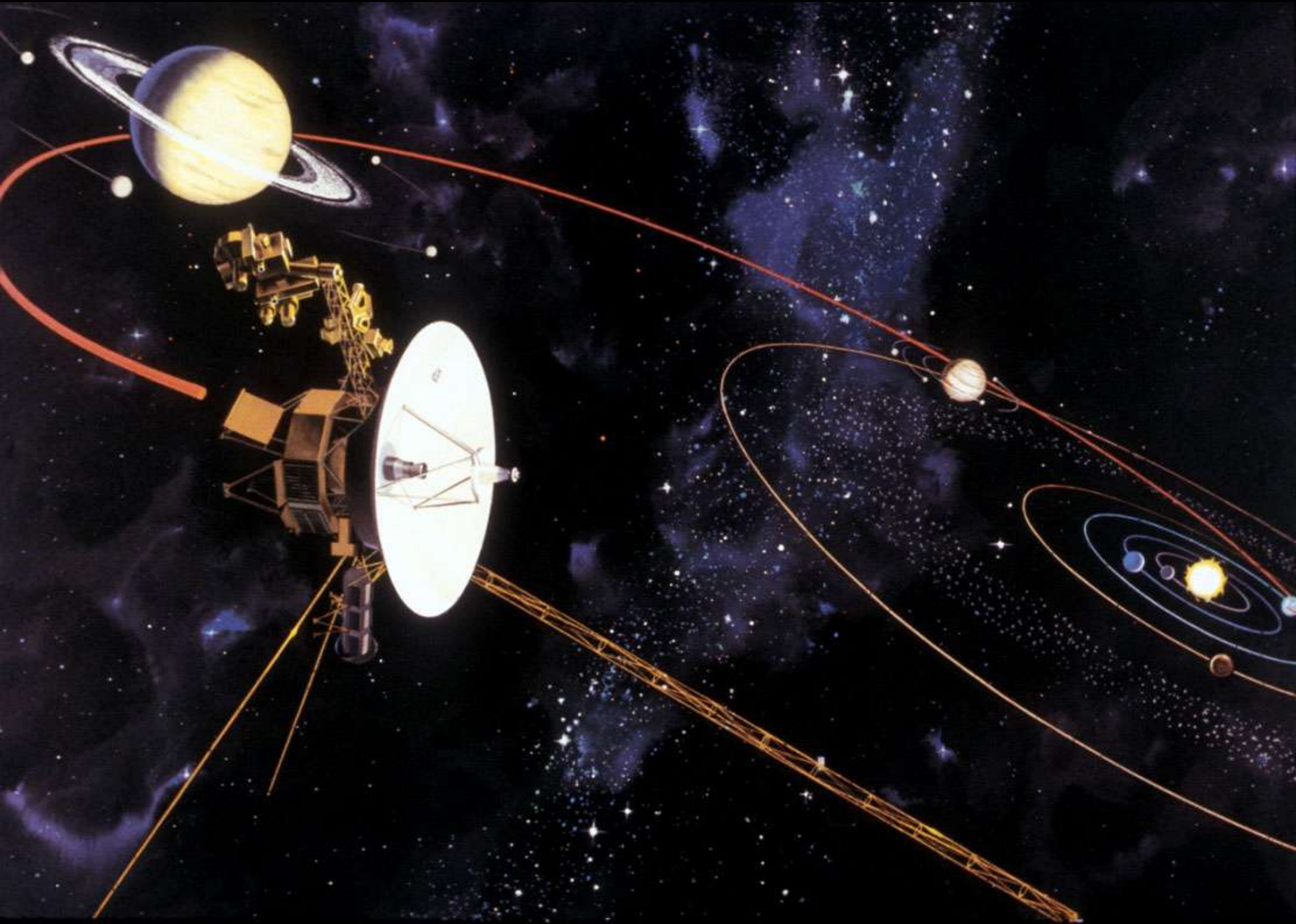


Galileo Galilei
(1564-1642

Sir Isaac Newton
[1642 – 1727]

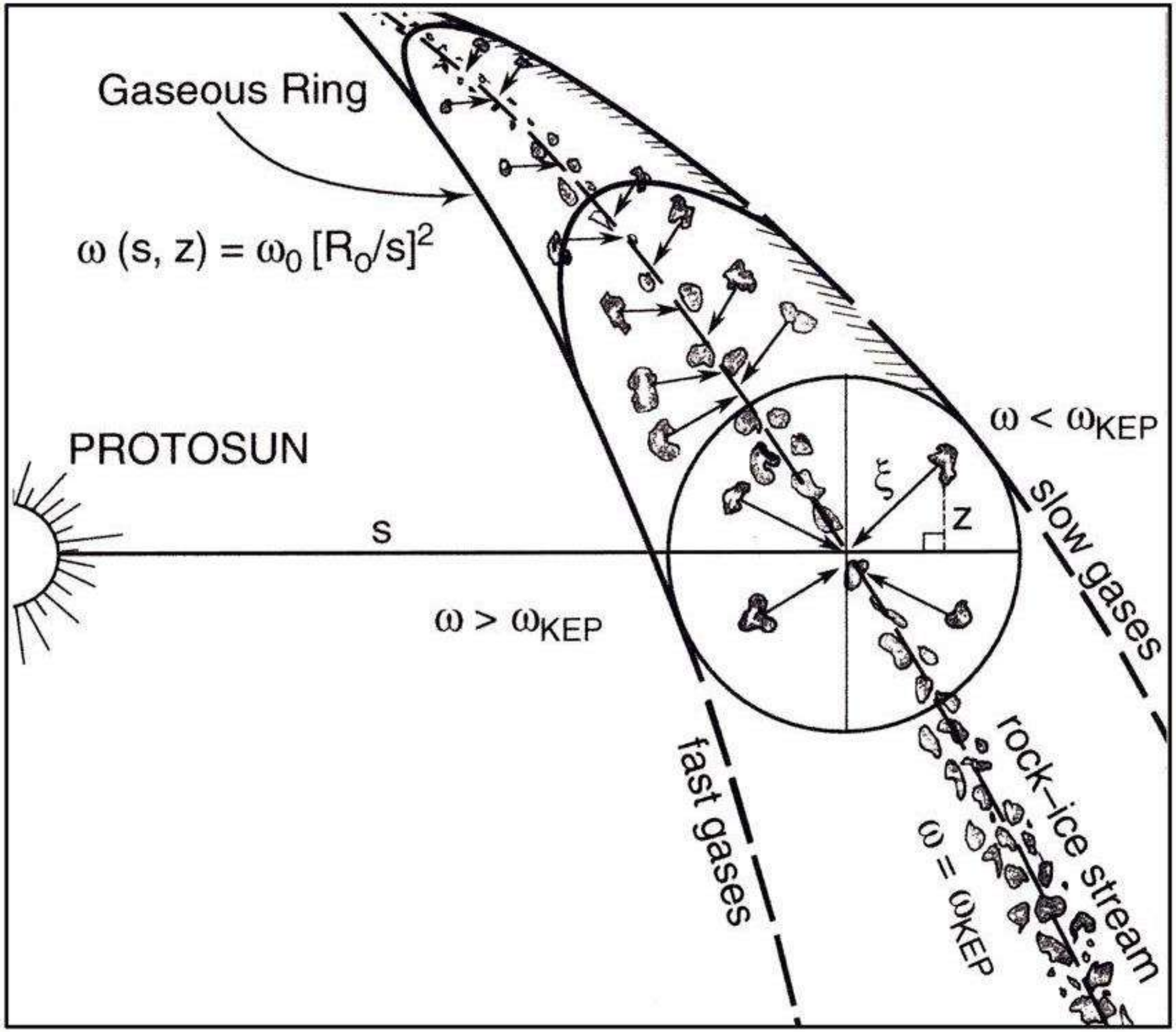

NEPTUNE
JUPITER
MARS
MERCURY
SUN
ASTEROIDS
EARTH
VENUS
SATURN
URANUS

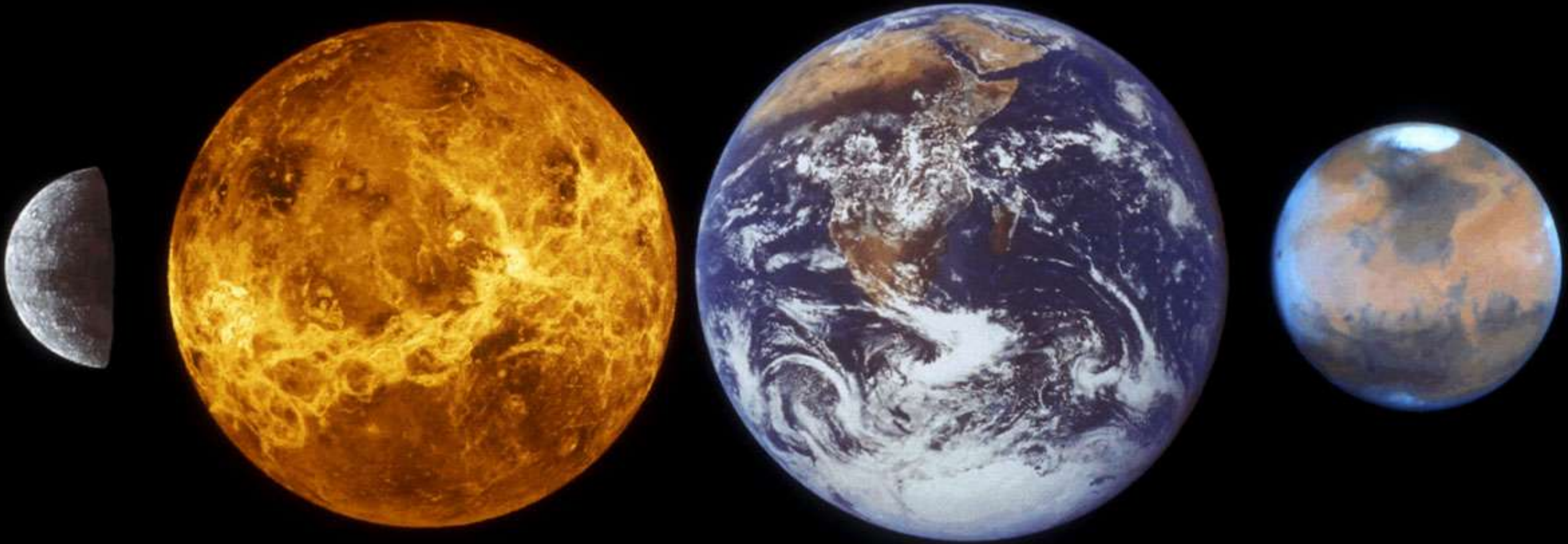




THE MODERN LAPLACIAN THEORY

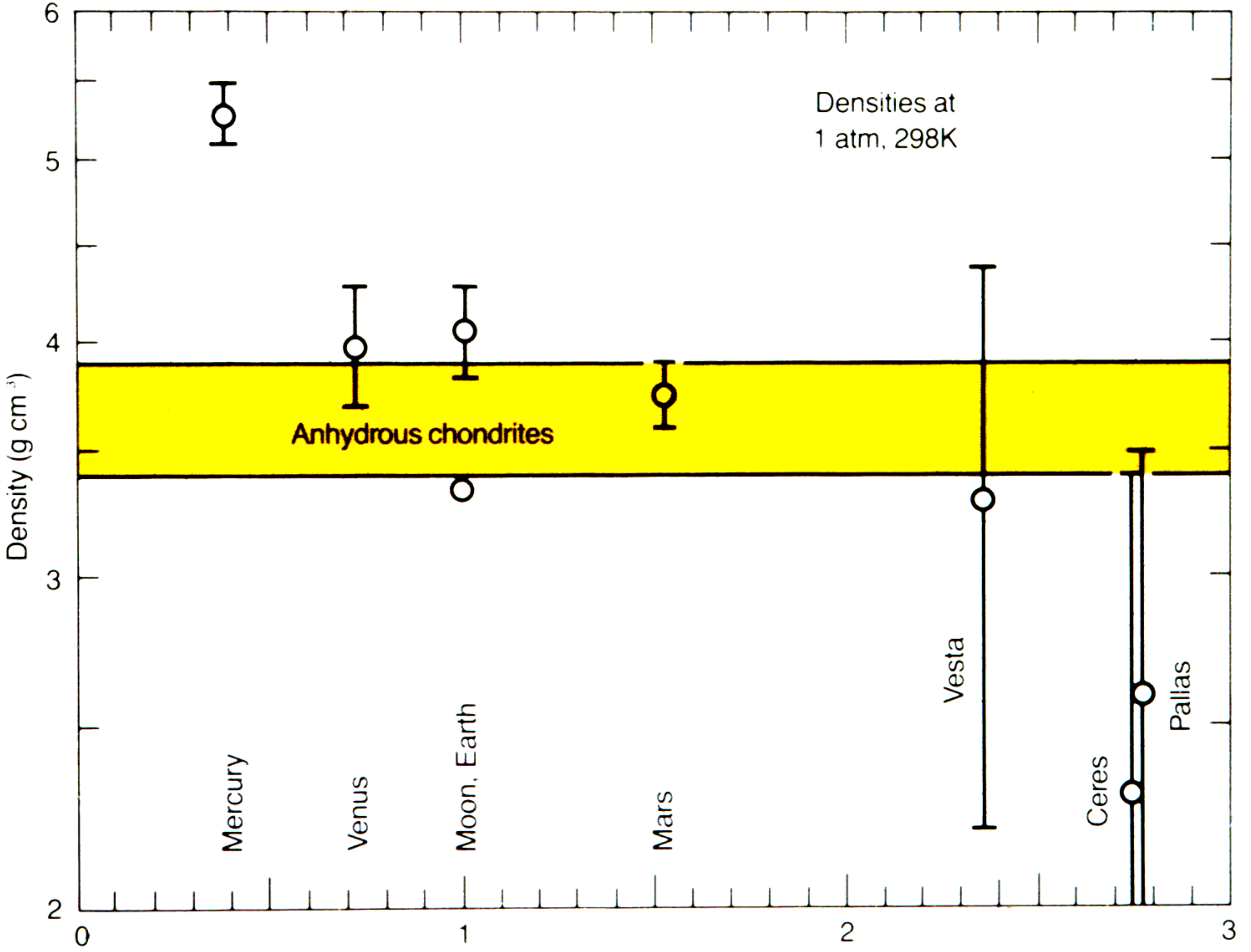
GRAVITATION SETTLING OF THE CONDENSATE GRAINS ONTO THE MEAN ORBIT OF THE GASEOUS RING

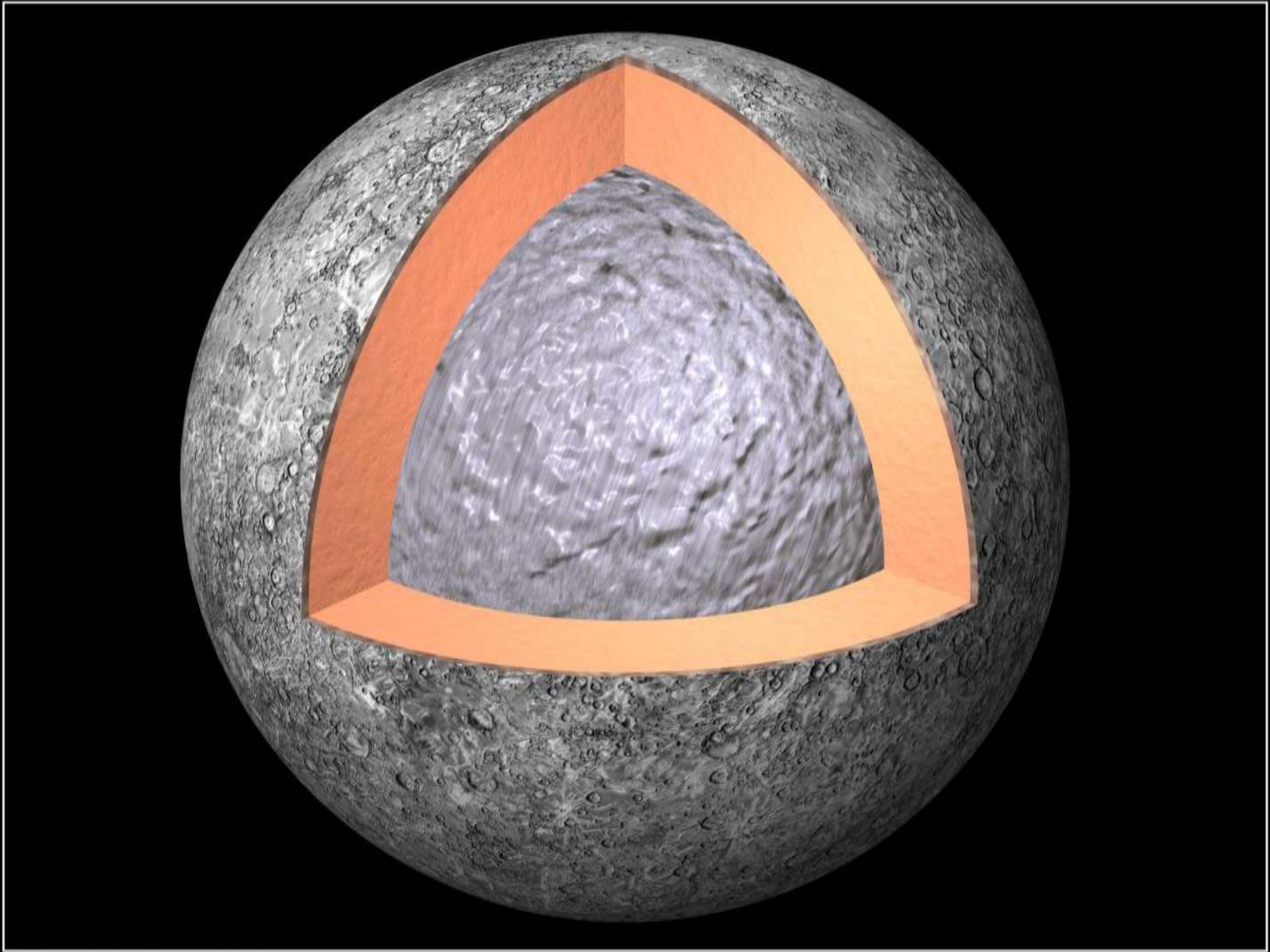




UNCOMPRESSED PLANETARY DENSITIES

[from John T. Wasson, 1985]



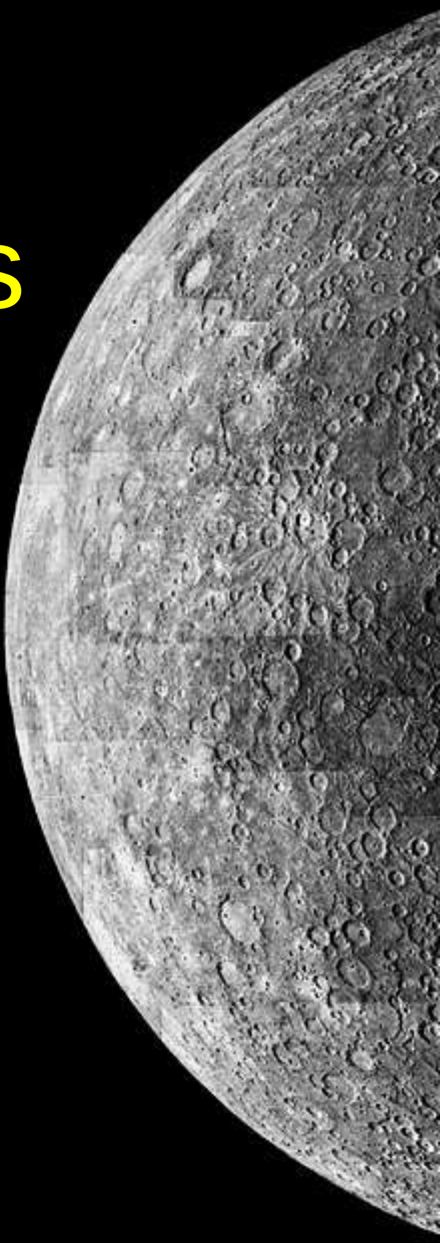


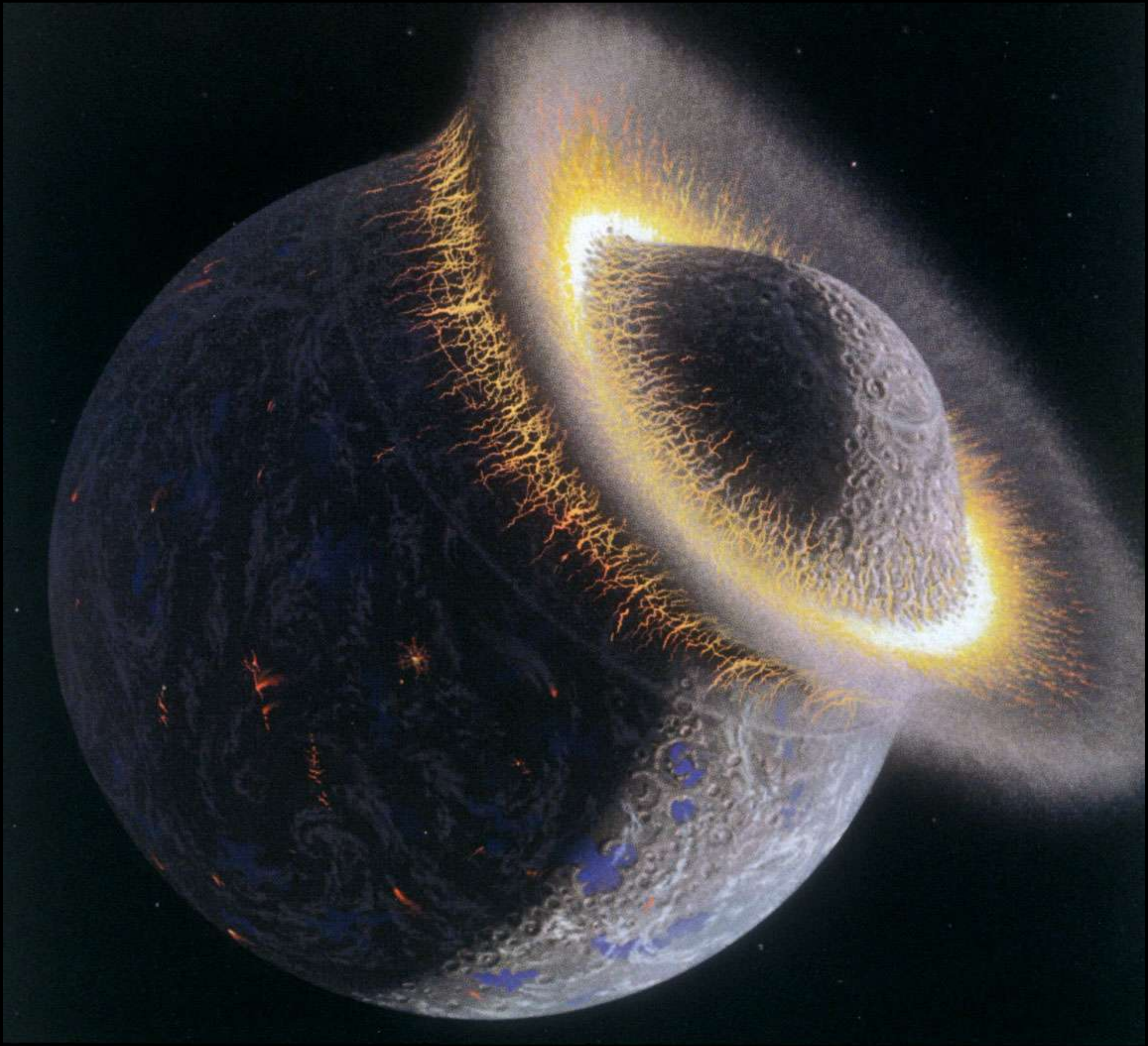
The Interior of Mercury

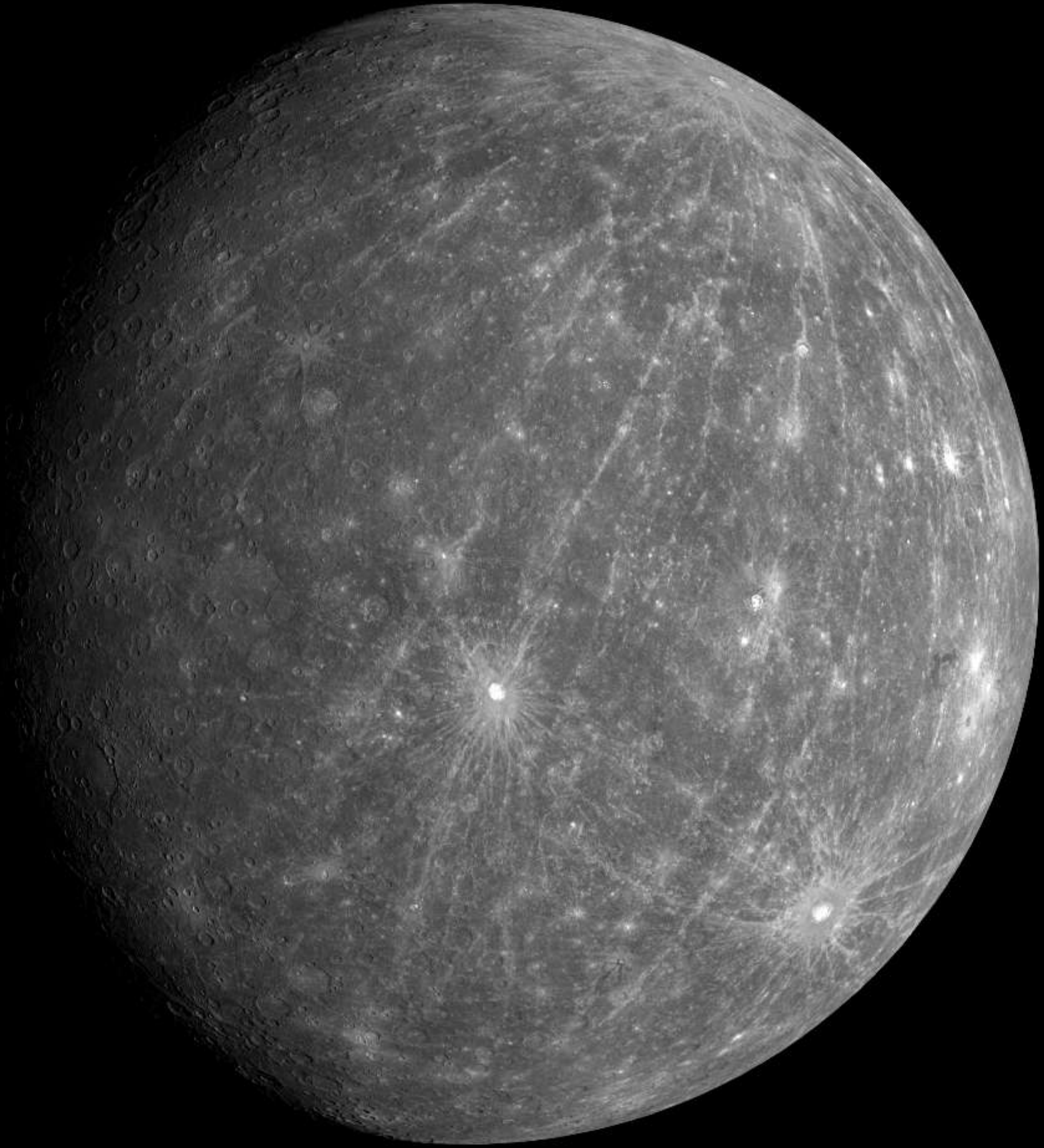
© Copyright Calvin J. Hamilton

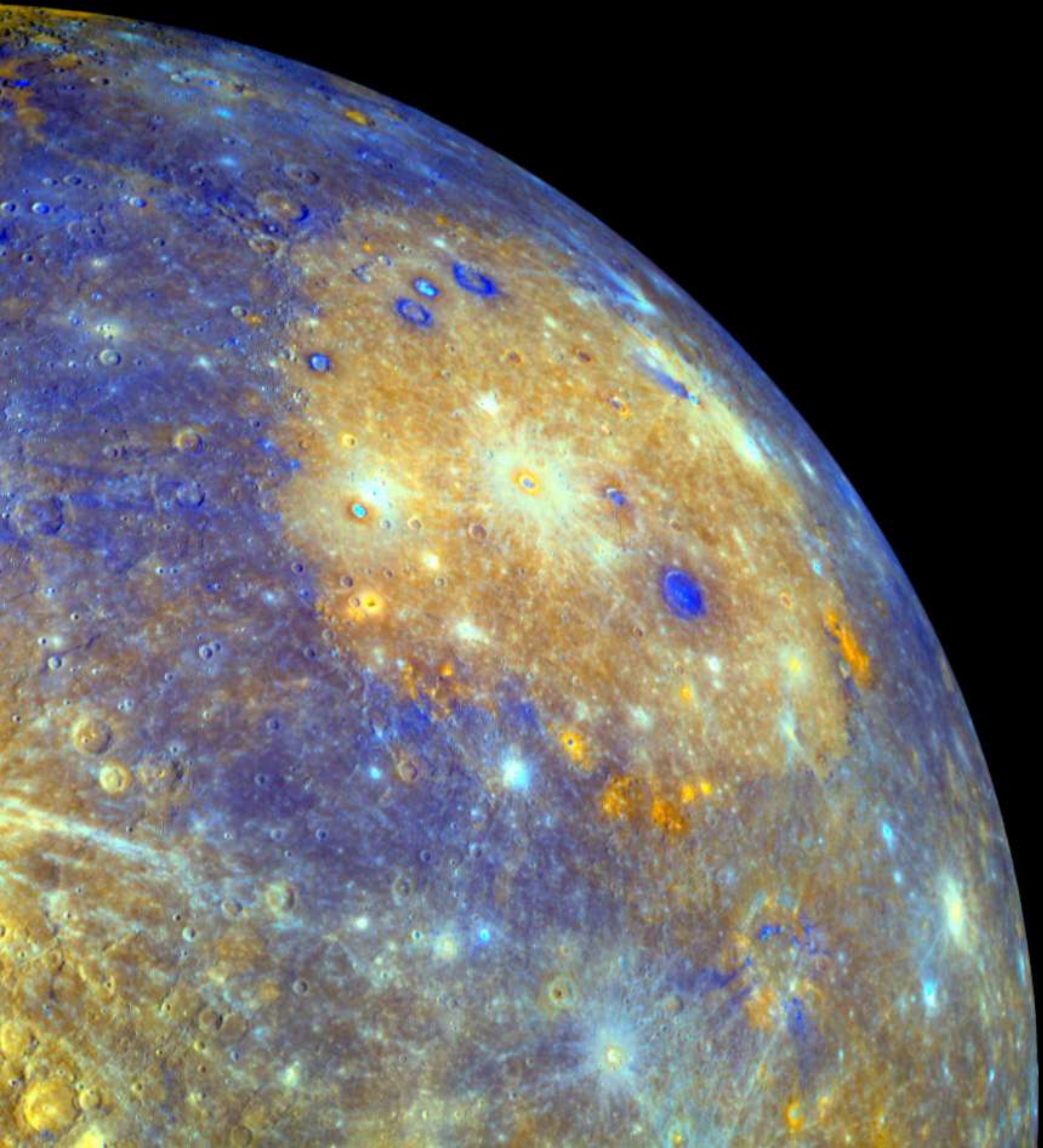
MERCURY

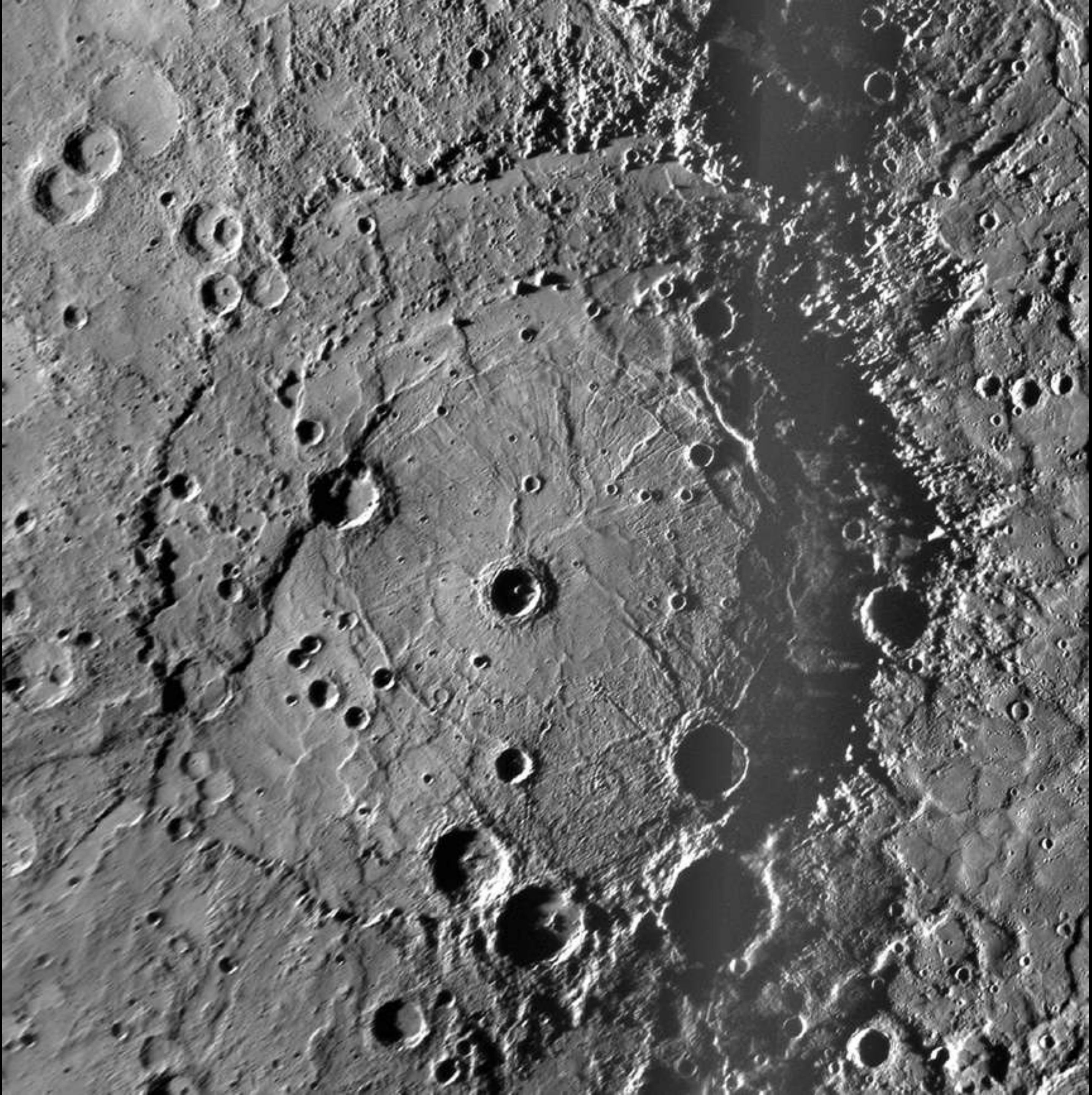
Chemical Species	% Mass
Al_2O_3	1.2
$CaTiO_3$	0.8
Ca_2AlSiO_7	19.0
$MgAl_2O_4$	3.8
$MgSiO_3$-Mg_2SiO_4	8.1
Fe-Ni-Cr-Co-V	67.1
	100.0



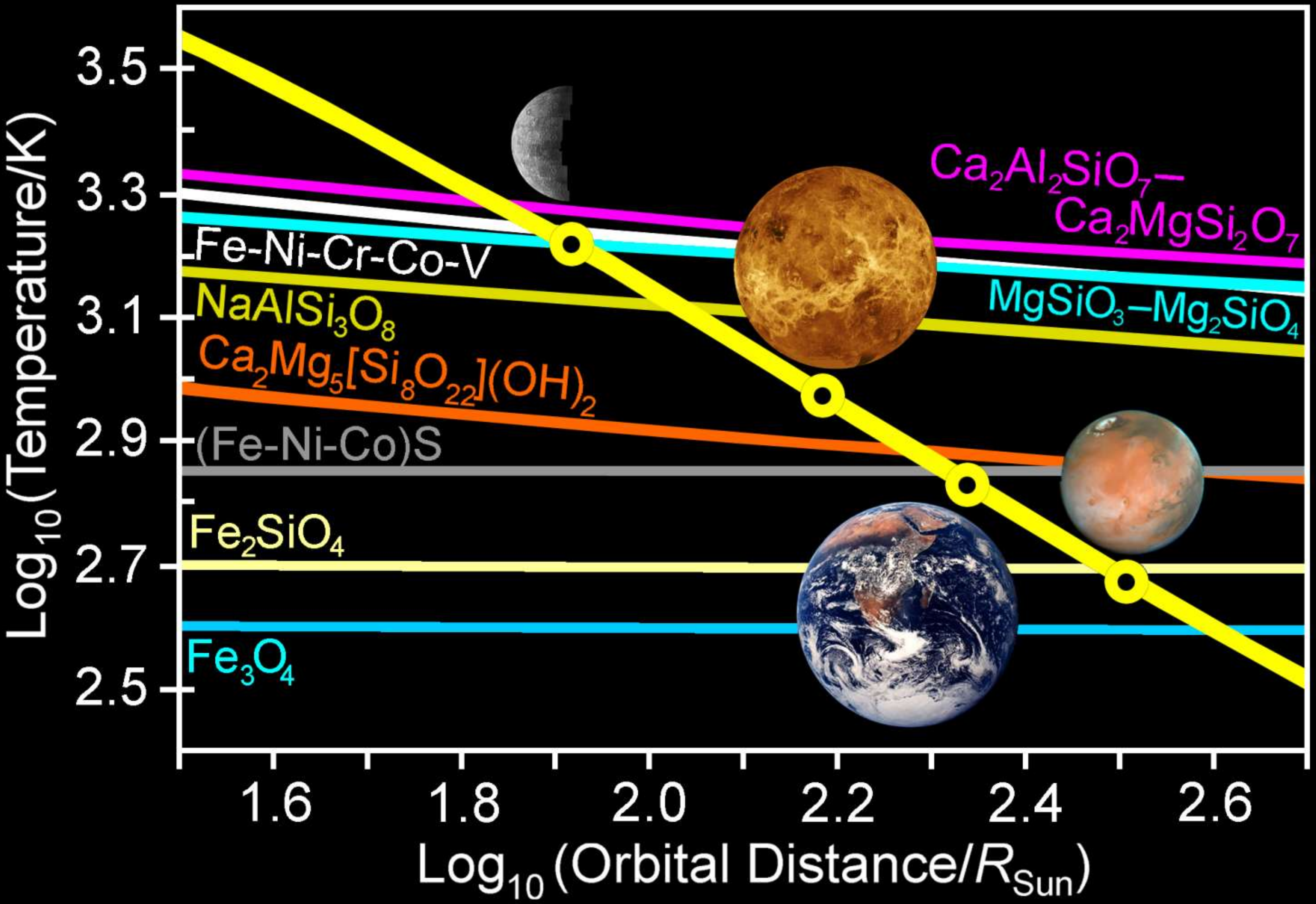








CHEMICAL CONDENSATION SEQUENCE



TEMPERATURE – DISTANCE RELATION

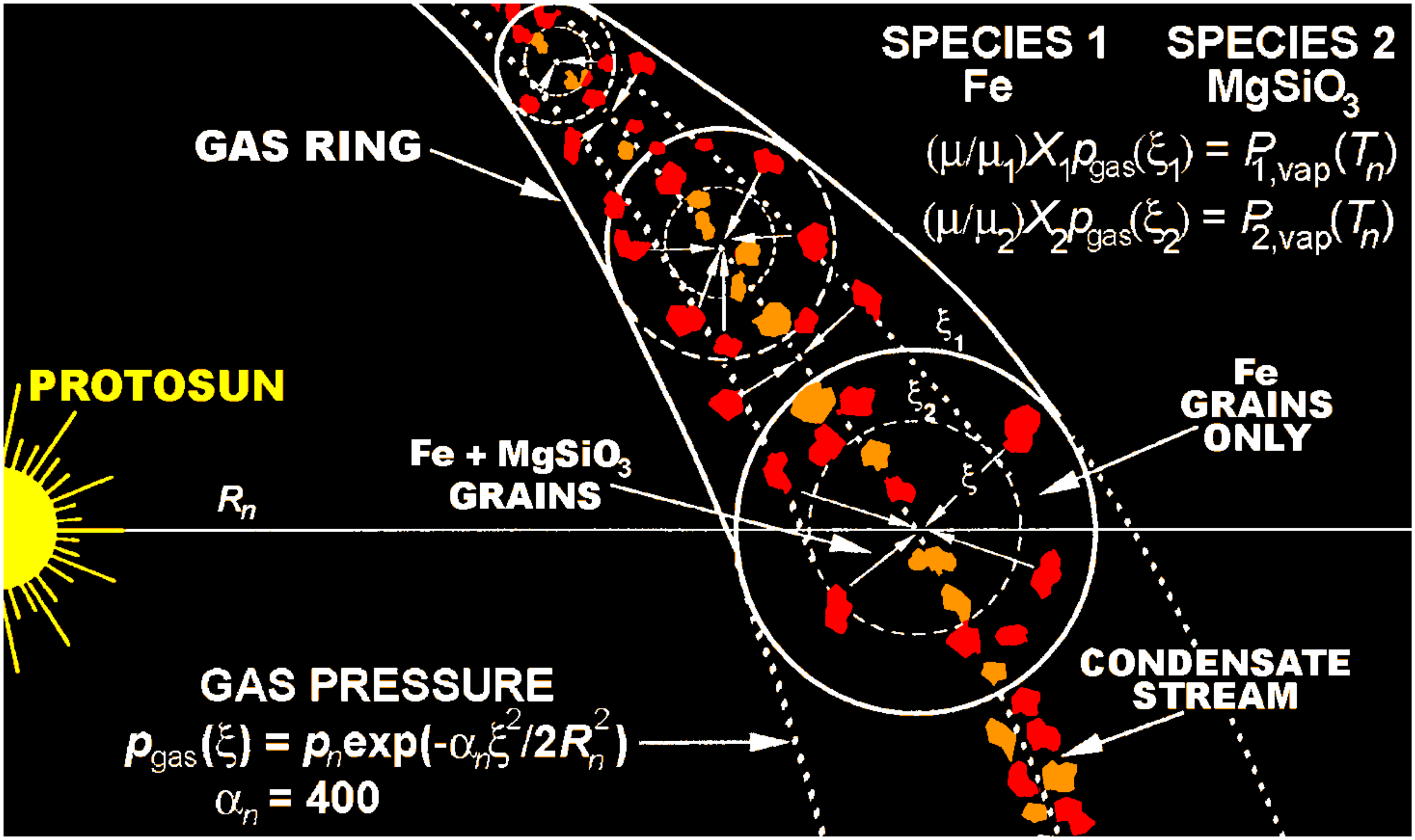
$$\frac{\nu}{2}\,\frac{\mathcal{R}T_e}{\mu} \propto \frac{GM}{R_e}$$

Thermal energy /gram at equator

gravitational potential energy/gram

Hence temperature T_e at equator of cloud varies with equatorial radius R_e as

$$T_e \propto 1/R_e$$



SPECIES 1
Fe

SPECIES 2
$MgSiO_3$

$$(\mu/\mu_1)X_1 p_{\mathrm{gas}}(\xi_1) = P_{1,\mathrm{vap}}(T_n)$$

$$(\mu/\mu_2)X_2 p_{\mathrm{gas}}(\xi_2) = P_{2,\mathrm{vap}}(T_n)$$

GAS RING

PROTOSUN

Fe + $MgSiO_3$ GRAINS

R_n

ξ_1

ξ_2

ξ

Fe GRAINS ONLY

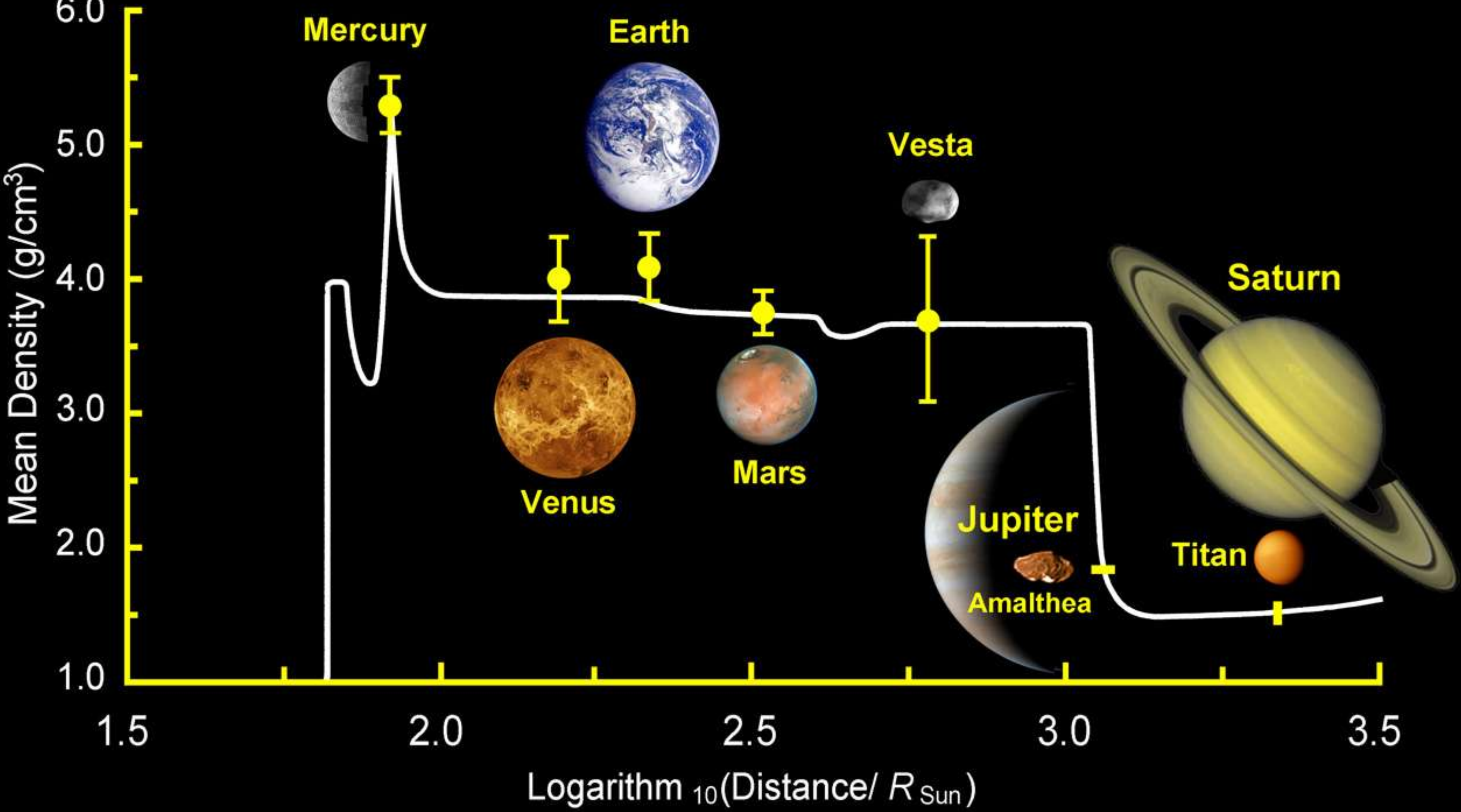
GAS PRESSURE

$$p_{\mathrm{gas}}(\xi) = p_n \exp(-\alpha_n \xi^2 / 2R_n^2)$$

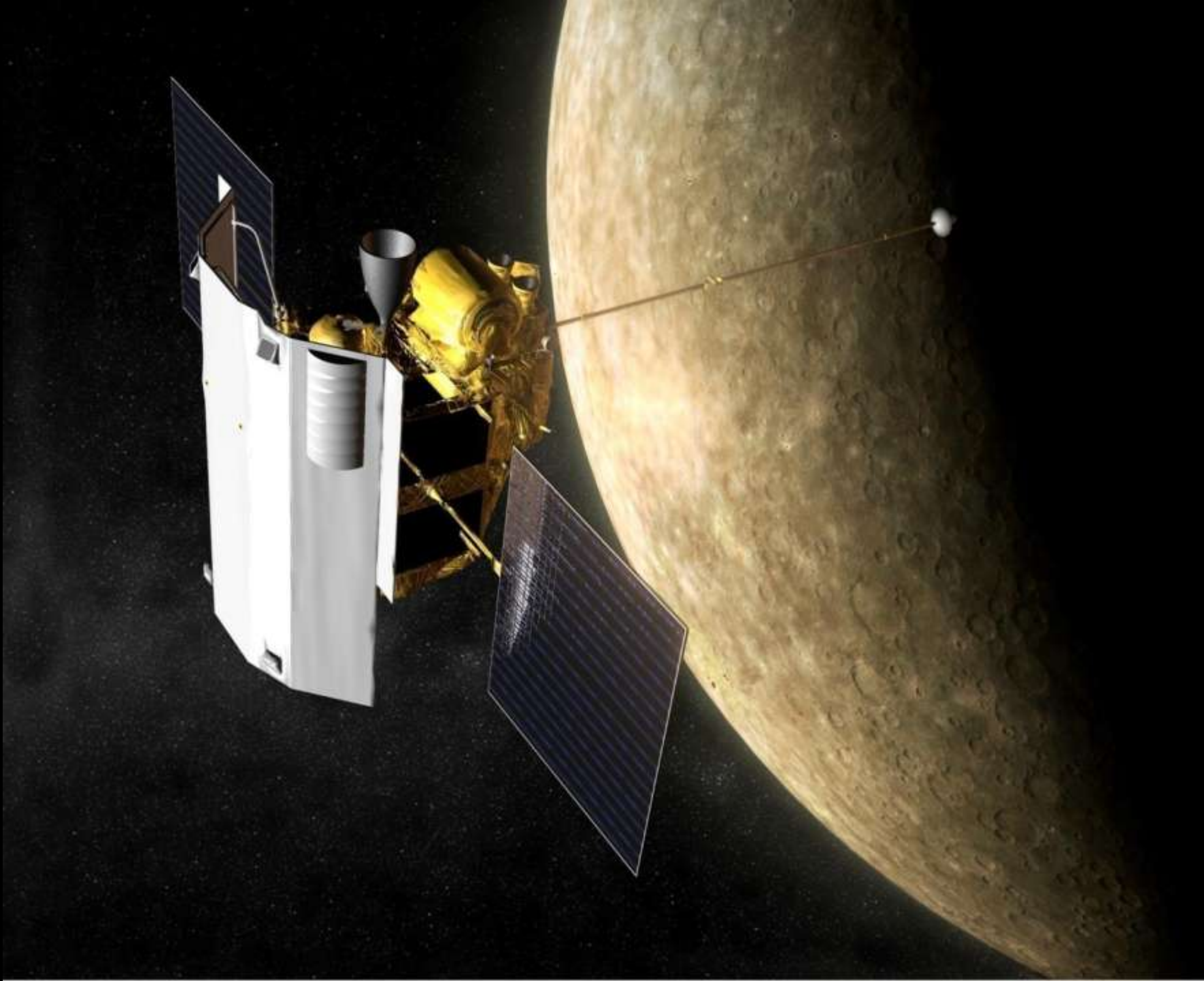
$$\alpha_n = 400$$

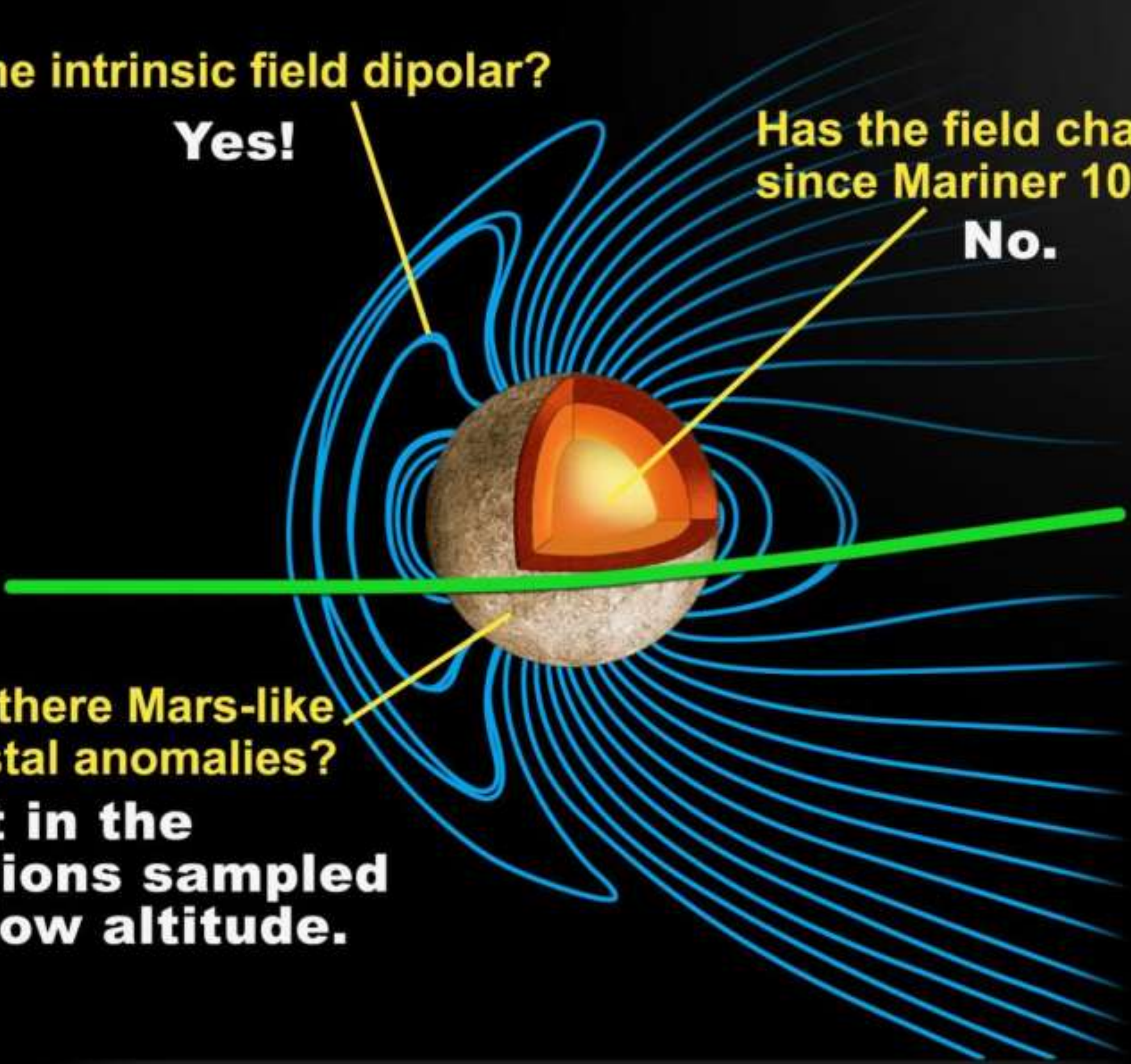
CONDENSATE STREAM

Condensate Mean Density vs Orbital Distance



THE MODERN LAPLACIAN THEORY





Is the intrinsic field dipolar?

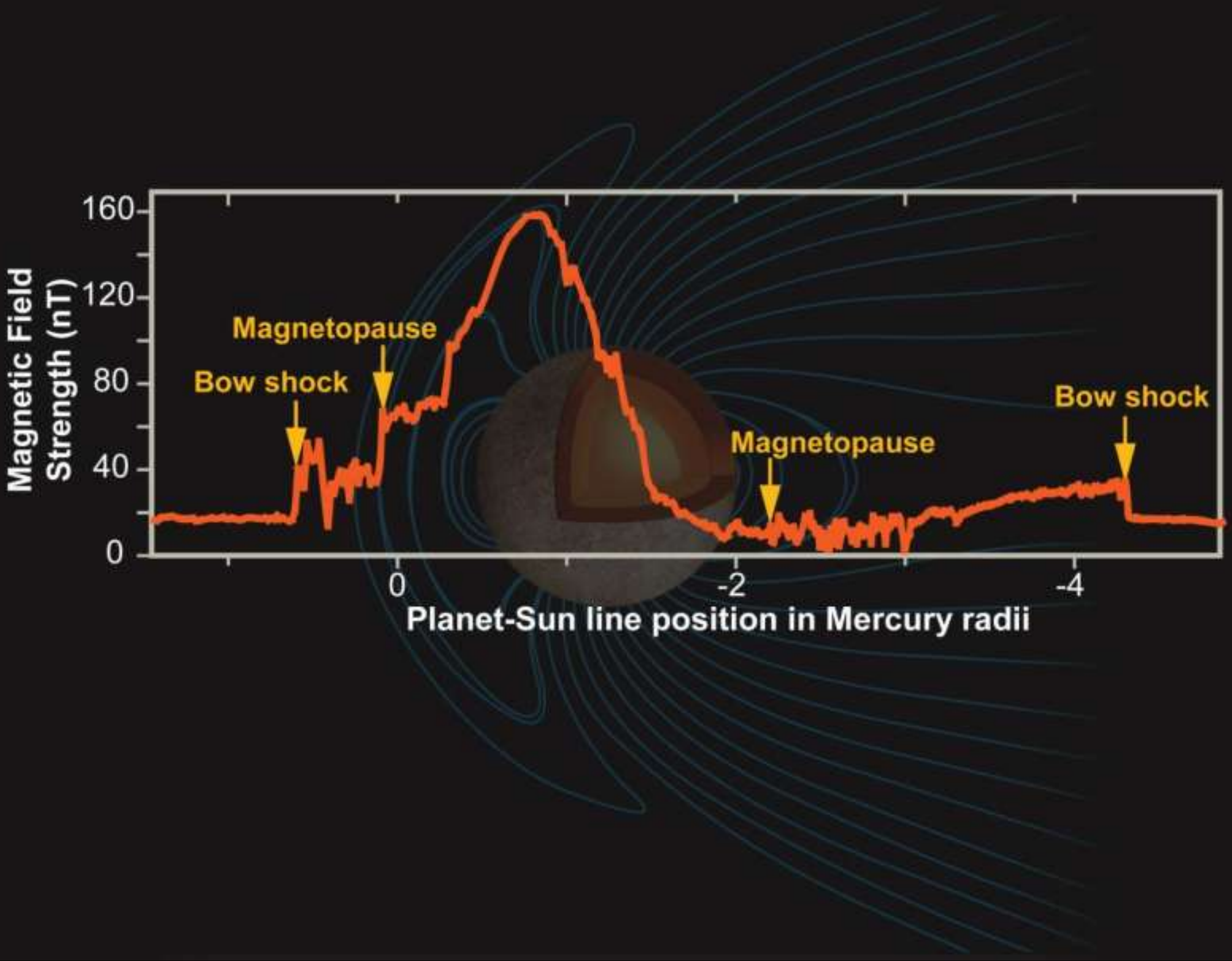
Yes!

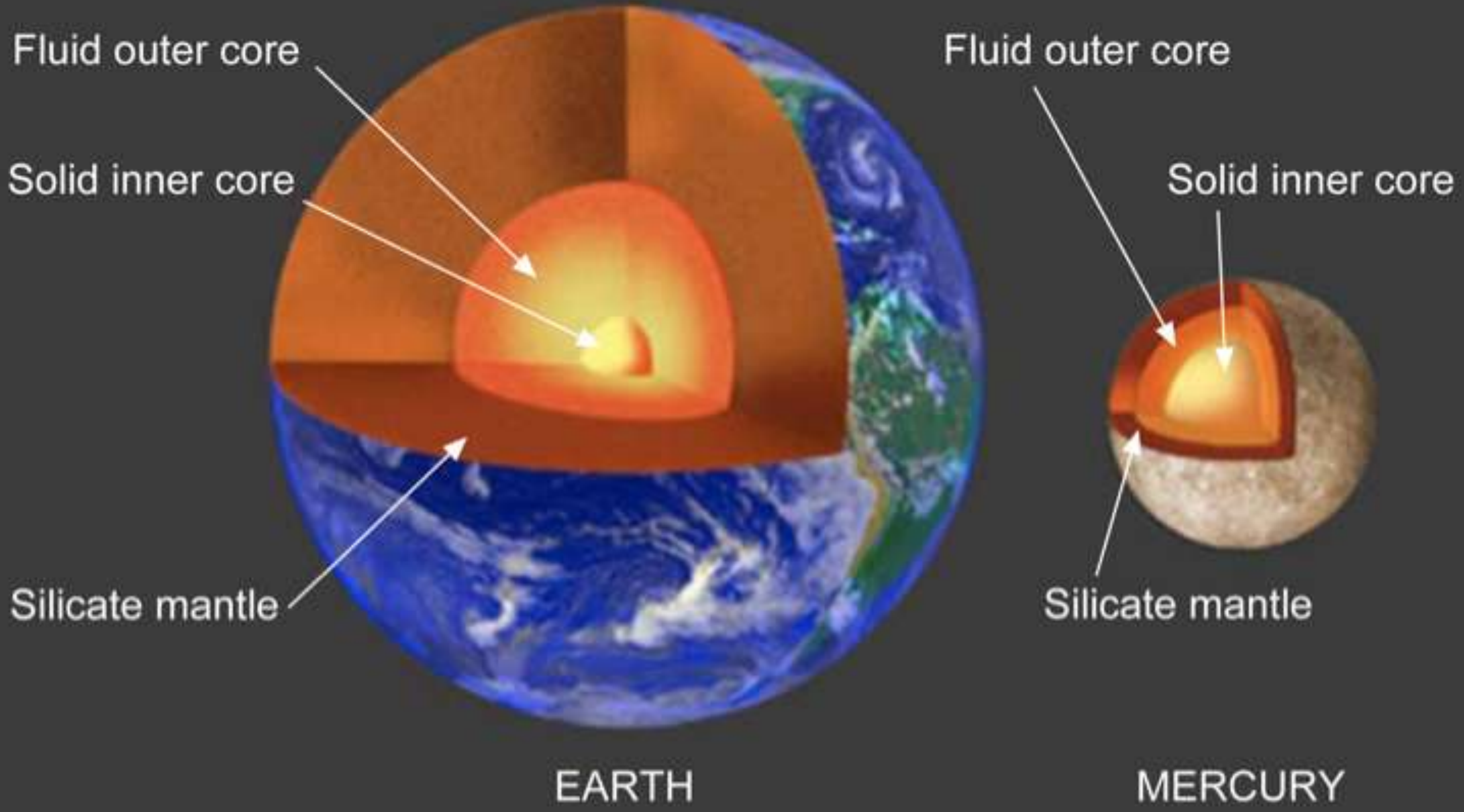
Has the field changed since Mariner 10?

No.

Are there Mars-like crustal anomalies?

Not in the regions sampled at low altitude.


Magnetic Field Strength (nT)
160
120
80
40
0
Bow shock
Magnetopause
Magnetopause
Bow shock
0
-2
-4
Planet-Sun line position in Mercury radii



Fluid outer core

Solid inner core

Silicate mantle

EARTH

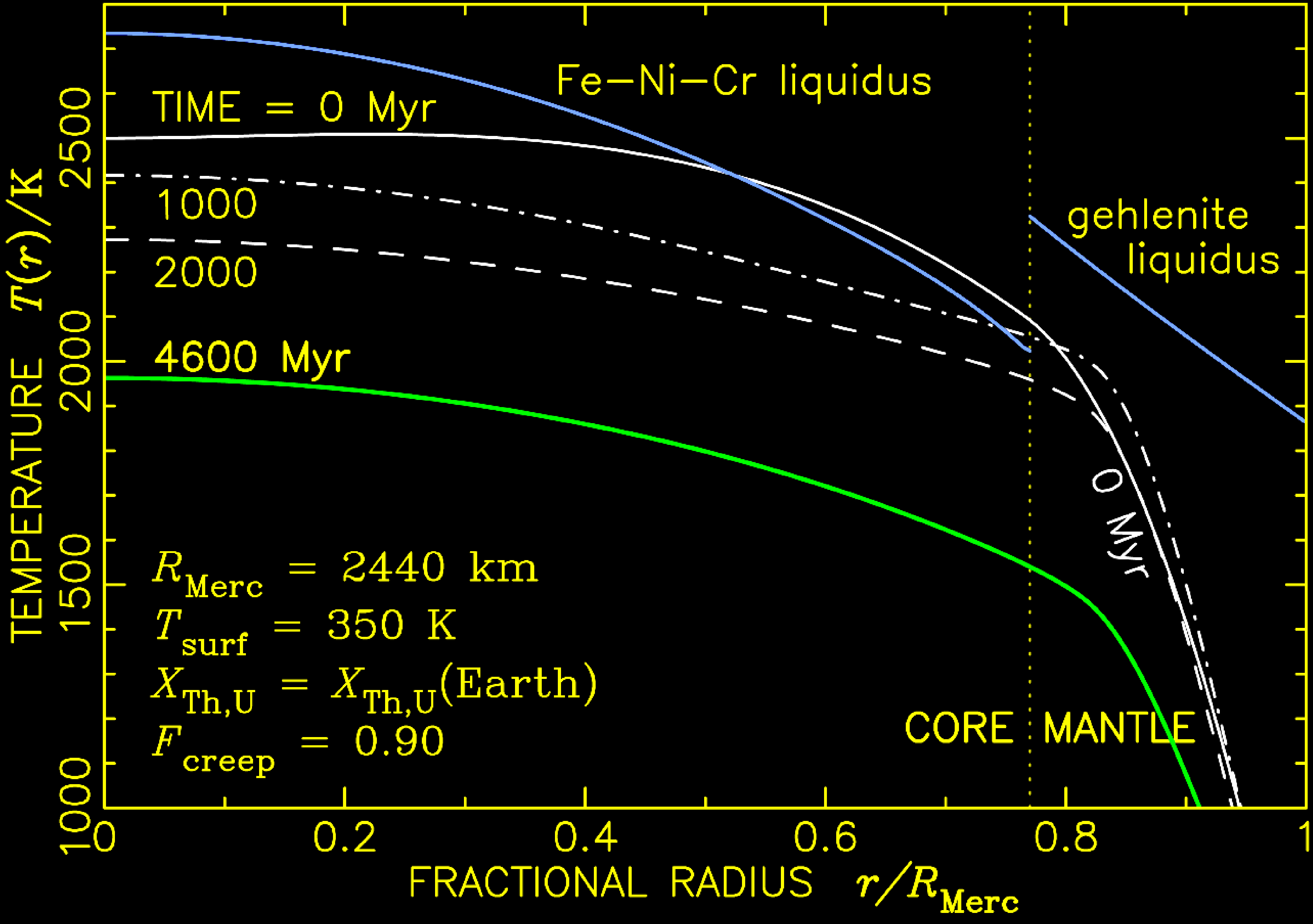
Fluid outer core

Solid inner core

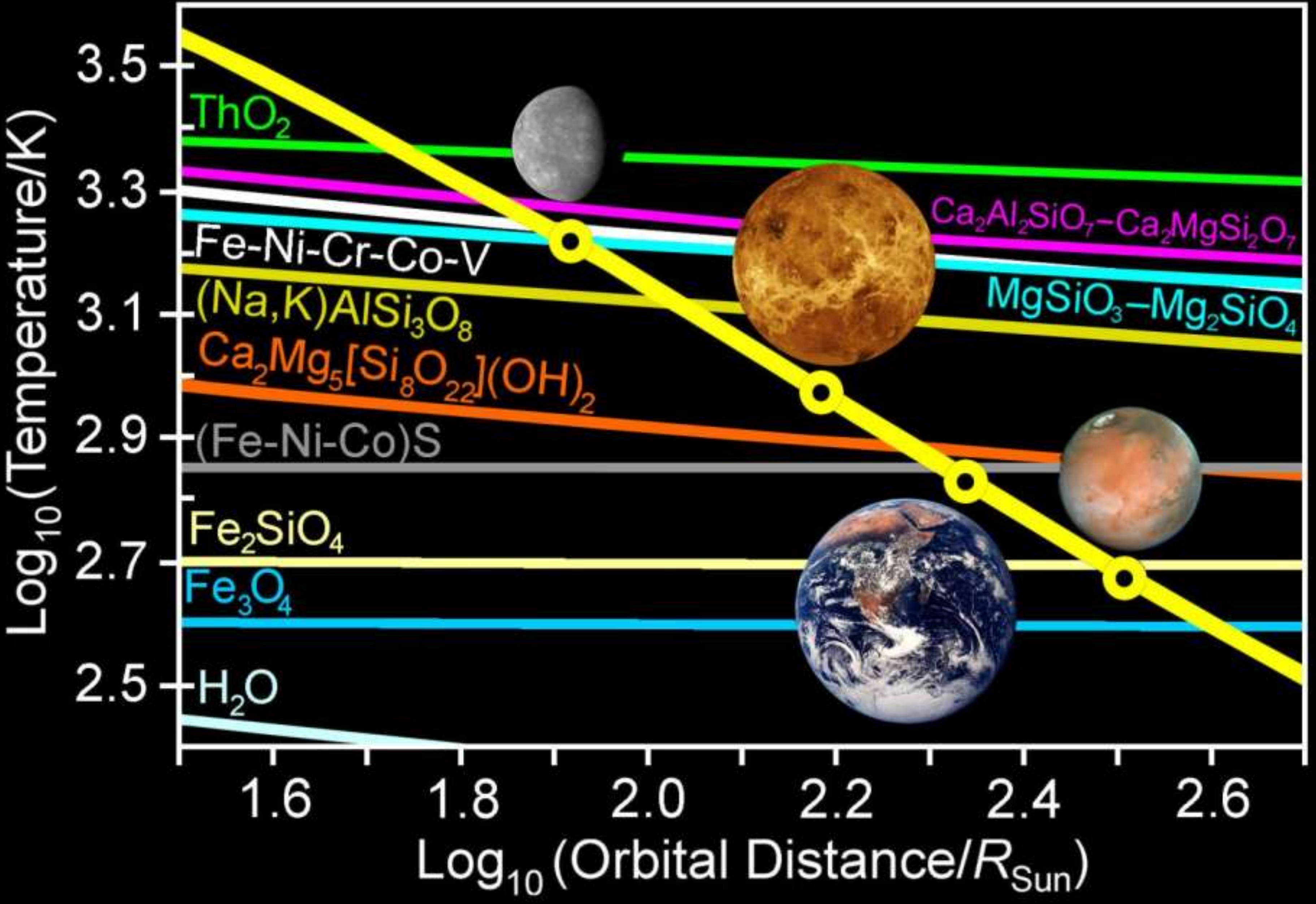
Silicate mantle

MERCURY

STANDARD MERCURY THERMAL MODEL



CHEMICAL CONDENSATION SEQUENCE

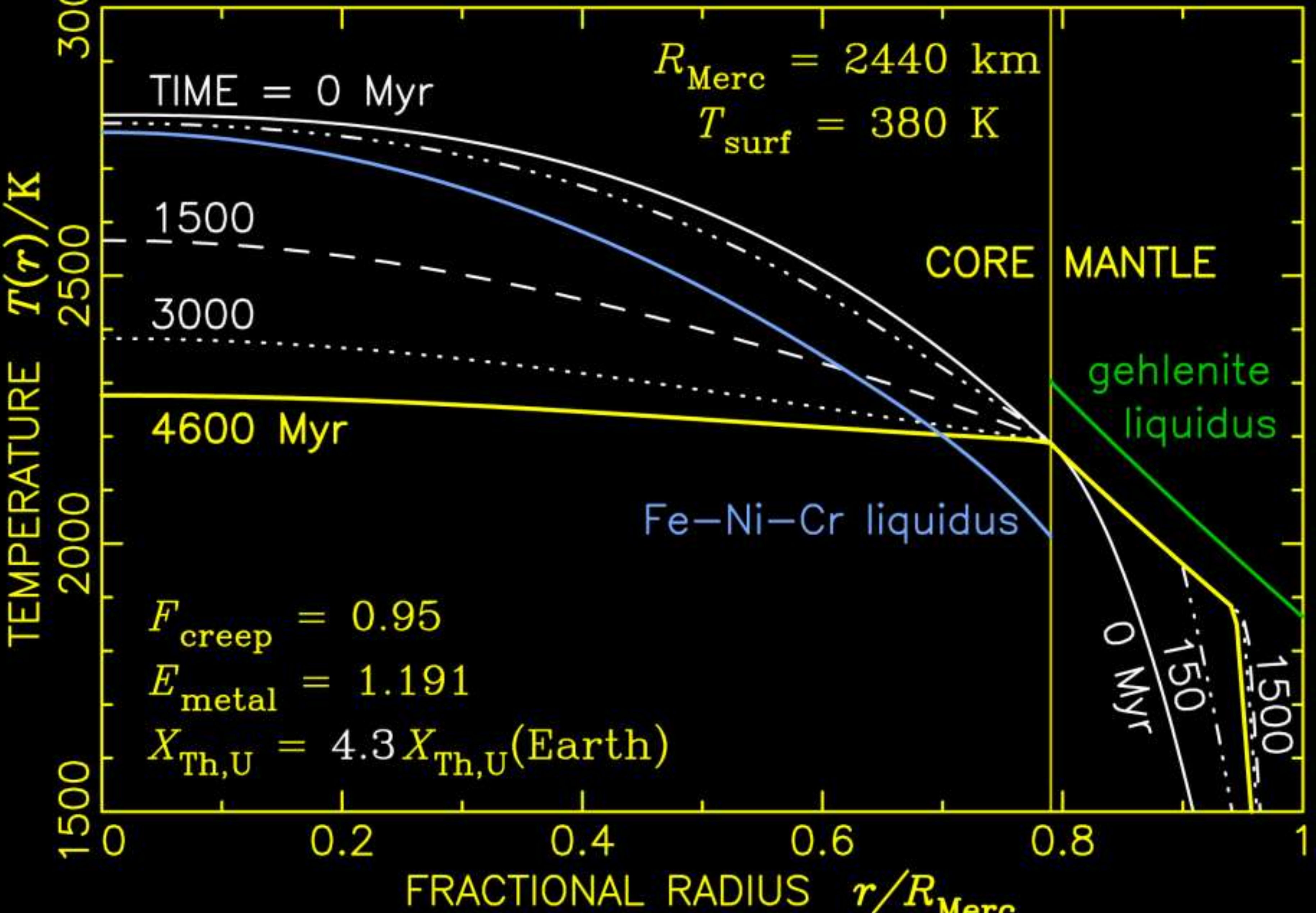


GLOBAL ABUNDANCES OF THORIUM AND URANIUM

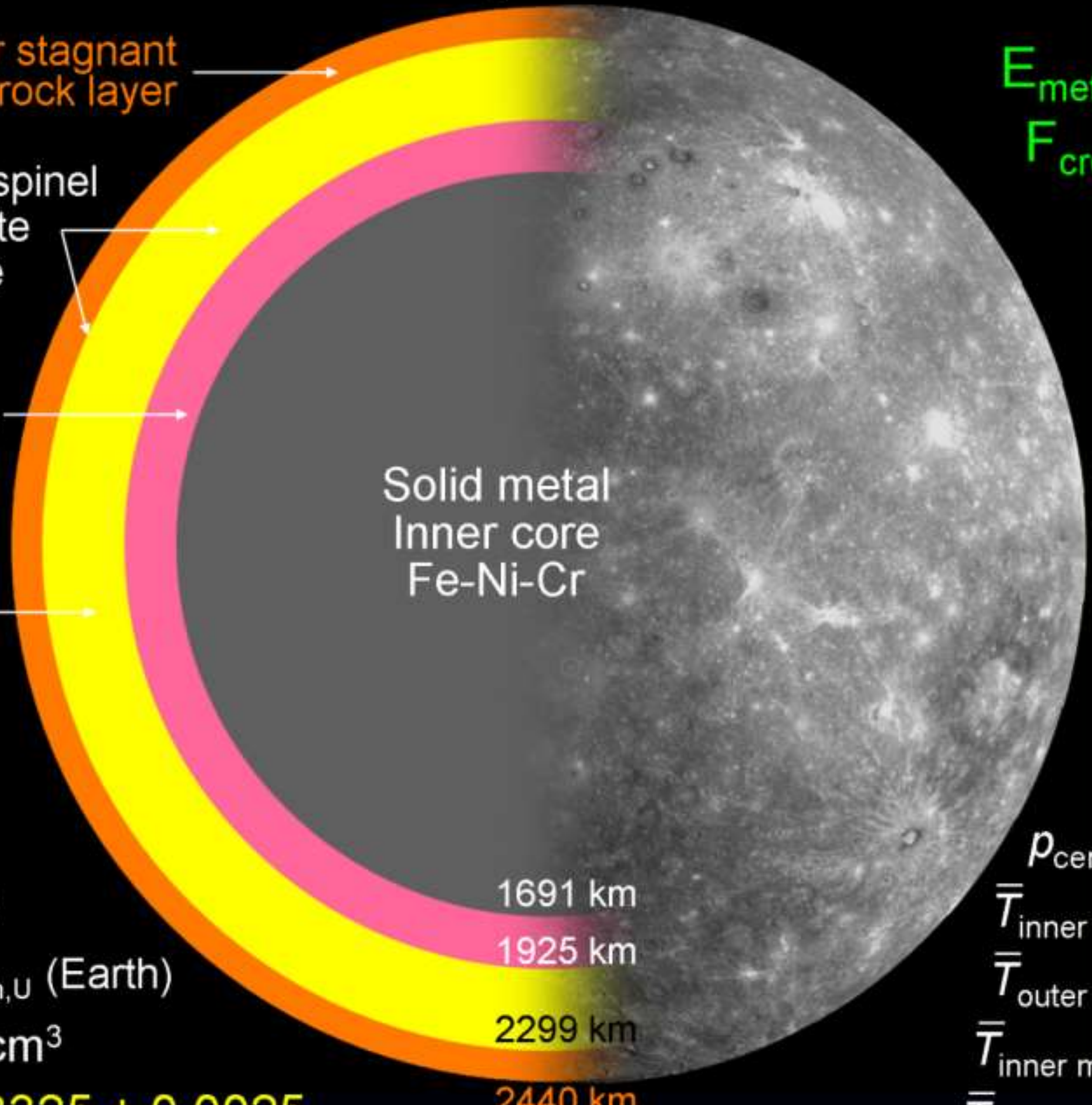
(Prentice, A.J.R, 2008, LPSC XXXIX, 1945.pdf)

Element	Earth	Mercury	Abundance ratio
Thorium	4.84×10^{-8}	2.08×10^{-7}	4.3
Uranium	1.315×10^{-8}	5.66×10^{-8}	4.3

2–ZONE MERCURY THERMAL MODEL



4-Zone Mercury Structure Model



Outer stagnant rock layer

Gehlenite, spinel + Mg-Silicate rock mantle

Liquid metal outer core

Convective inner rock layer

Solid metal Inner core Fe-Ni-Cr

1691 km

1925 km

2299 km

2440 km

$E_{\mathrm{metal}} = 1.191$

$F_{\mathrm{creep}} = 0.95$

$X_{\mathrm{core}} = 0.708$

$X_{\mathrm{liquid}} = 0.218$

$X_{\mathrm{Th,U}} =$ **4.3**$X_{\mathrm{Th,U}}$ (Earth)

$\overline{\rho} = 5.426\ \mathrm{g/cm^3}$

$C/MR^2 = 0.3325 \pm 0.0025$

$p_{\mathrm{centre}} = 397$ kbar

$\overline{T}_{\mathrm{inner\ core}} = 2230$ K

$\overline{T}_{\mathrm{outer\ core}} = 2195$ K

$\overline{T}_{\mathrm{inner\ mantle}} = 2020$ K

$\overline{T}_{\mathrm{outer\ mantle}} = 1080$ K

30 SEPTEMBER 2011 VOL 333 **SCIENCE**
www.sciencemag.org

Radioactive Elements on Mercury's Surface from MESSENGER: Implications for the Planet's Formation and Evolution

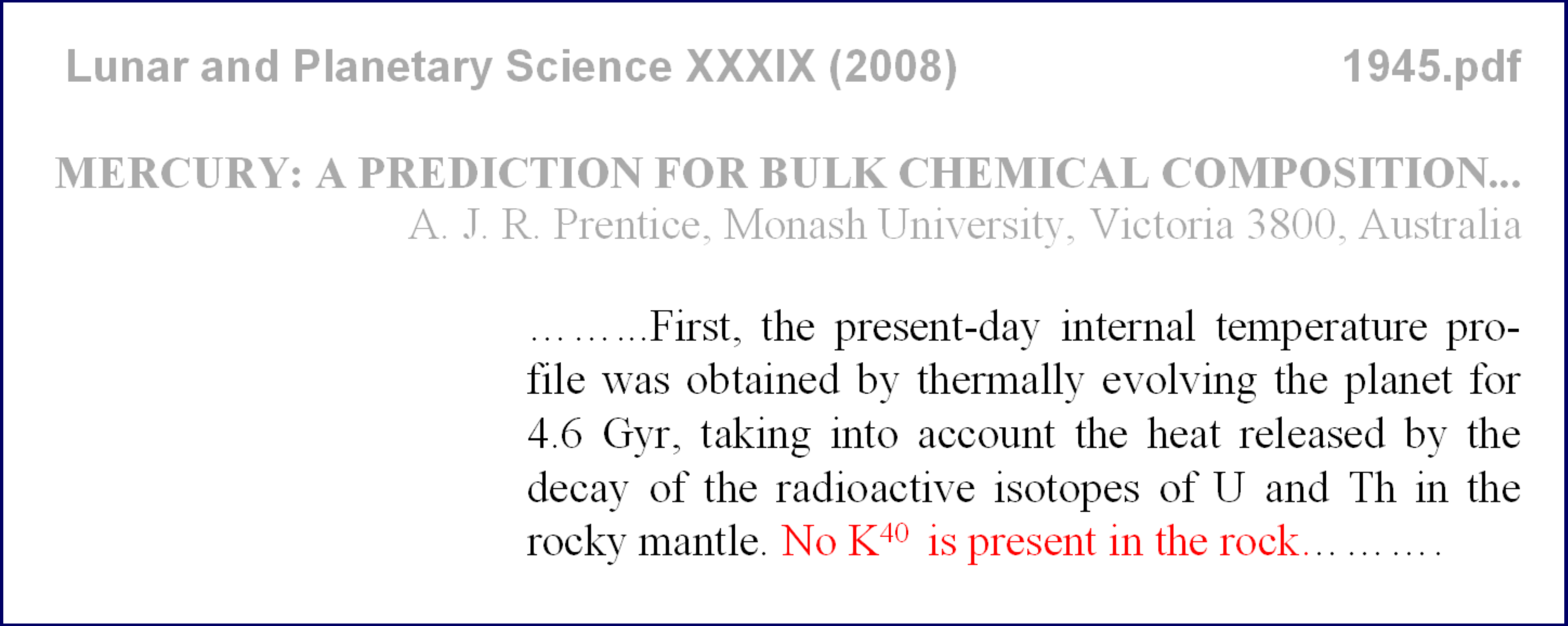
Patrick N. Peplowski,[1*] Larry G. Evans,[2] Steven A. Hauck II,[3] Timothy J. McCoy,[4] William V. Boynton,[5] Jeffery J. Gillis-Davis,[6] Denton S. Ebel,[7] John O. Goldsten,[1] David K. Hamara,[5] David J. Lawrence,[1] Ralph L. McNutt Jr.,[1] Larry R. Nittler,[8] Sean C. Solomon,[8] Edgar A. Rhodes,[1] Ann L. Sprague,[5] Richard D. Starr,[9] Karen R. Stockstill-Cahill[4]

The MESSENGER Gamma-Ray Spectrometer measured the average surface abundances of the radioactive elements potassium (K, 1150 ± 220 parts per million), thorium (Th, 220 ± 60 parts per billion), and uranium (U, 90 ± 20 parts per billion) in Mercury's northern hemisphere.

GLOBAL ABUNDANCES OF POTASSIUM, THORIUM AND URANIUM

(Prentice 2008, AGU Fall Meeting, Abstract U21A-0030)
(MESSENGER: Peplowski *et al*. 2011 *Science* **333** 1850)

Element	Prentice	MESSENGER	Agreement
Potassium	**0 ppm**	**1150 ± 220 ppm**	**no!**
Thorium	**208 ppb**	**220 ± 60 ppb**	**yes**
Uranium	**57 ppb**	**90 ± 20 ppb**	**maybe**



Lunar and Planetary Science XXXIX (2008) 1945.pdf

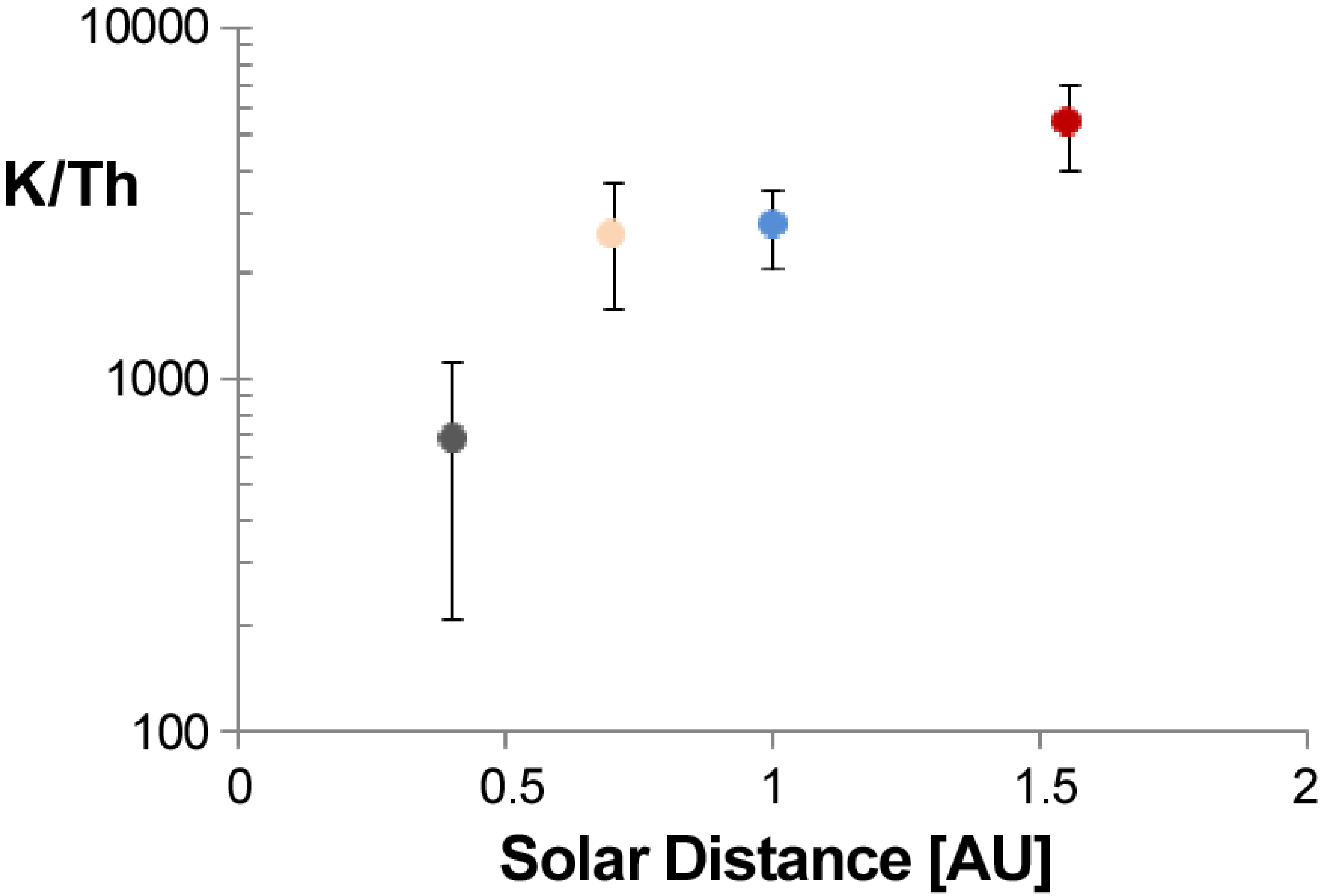
MERCURY: A PREDICTION FOR BULK CHEMICAL COMPOSITION...

A. J. R. Prentice, Monash University, Victoria 3800, Australia

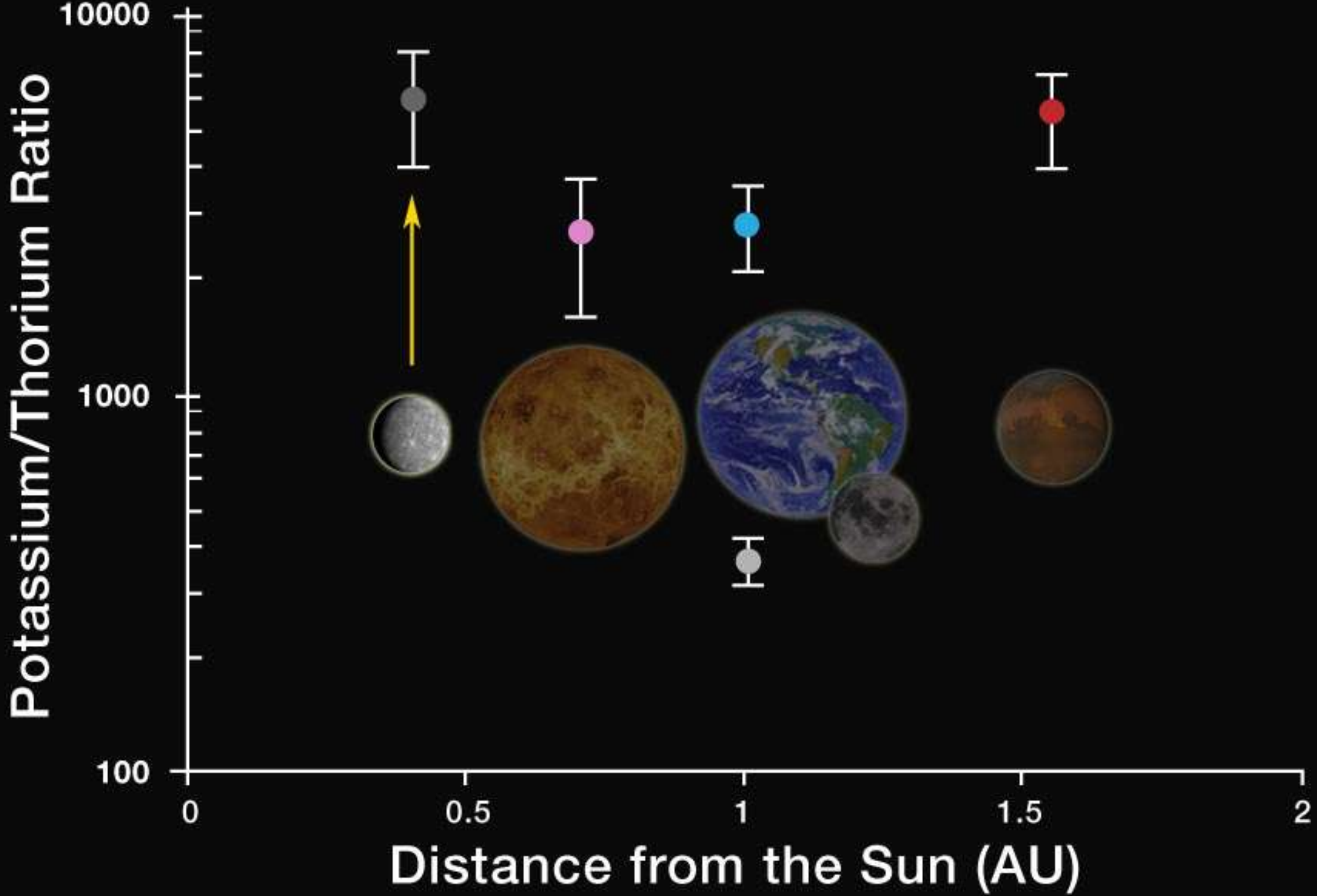
……...First, the present-day internal temperature profile was obtained by thermally evolving the planet for 4.6 Gyr, taking into account the heat released by the decay of the radioactive isotopes of U and Th in the rocky mantle. No K^{40} is present in the rock……….

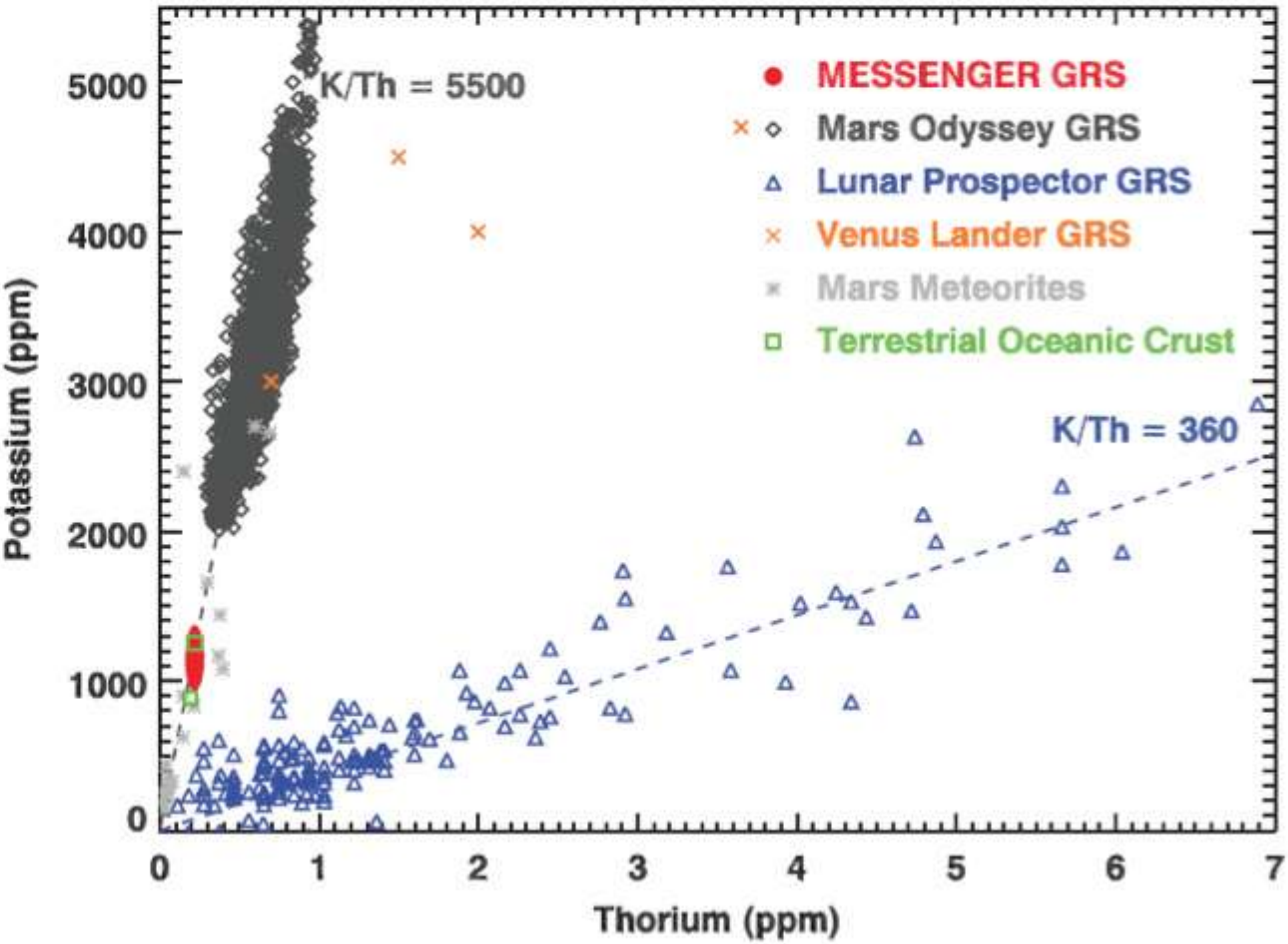
Gamma-ray spectrometer (GRS)

- Pre-flyby modeling indicated possibility to detect K if at highest model levels
- Flyby: 22 minutes below 2500 km altitude
  - Instrument worked well
  - Clearly detected Si γ-rays
  - No detection of K

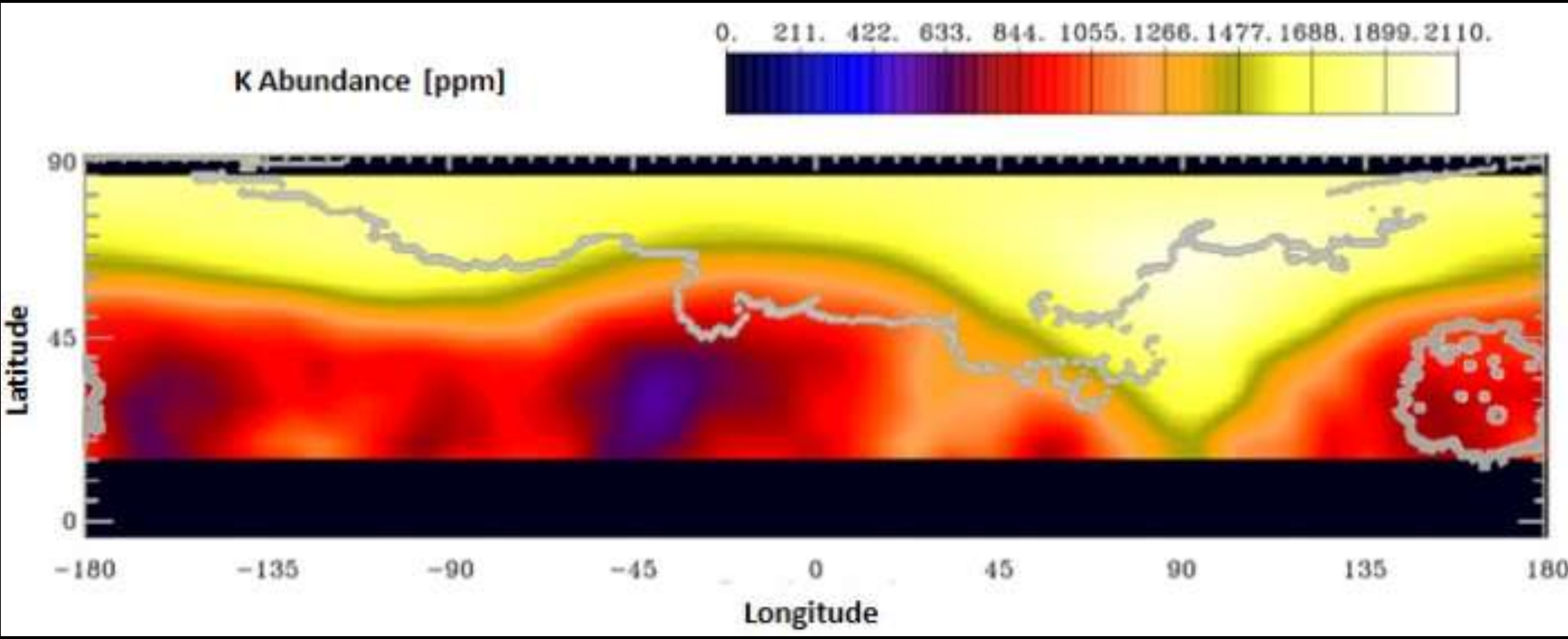


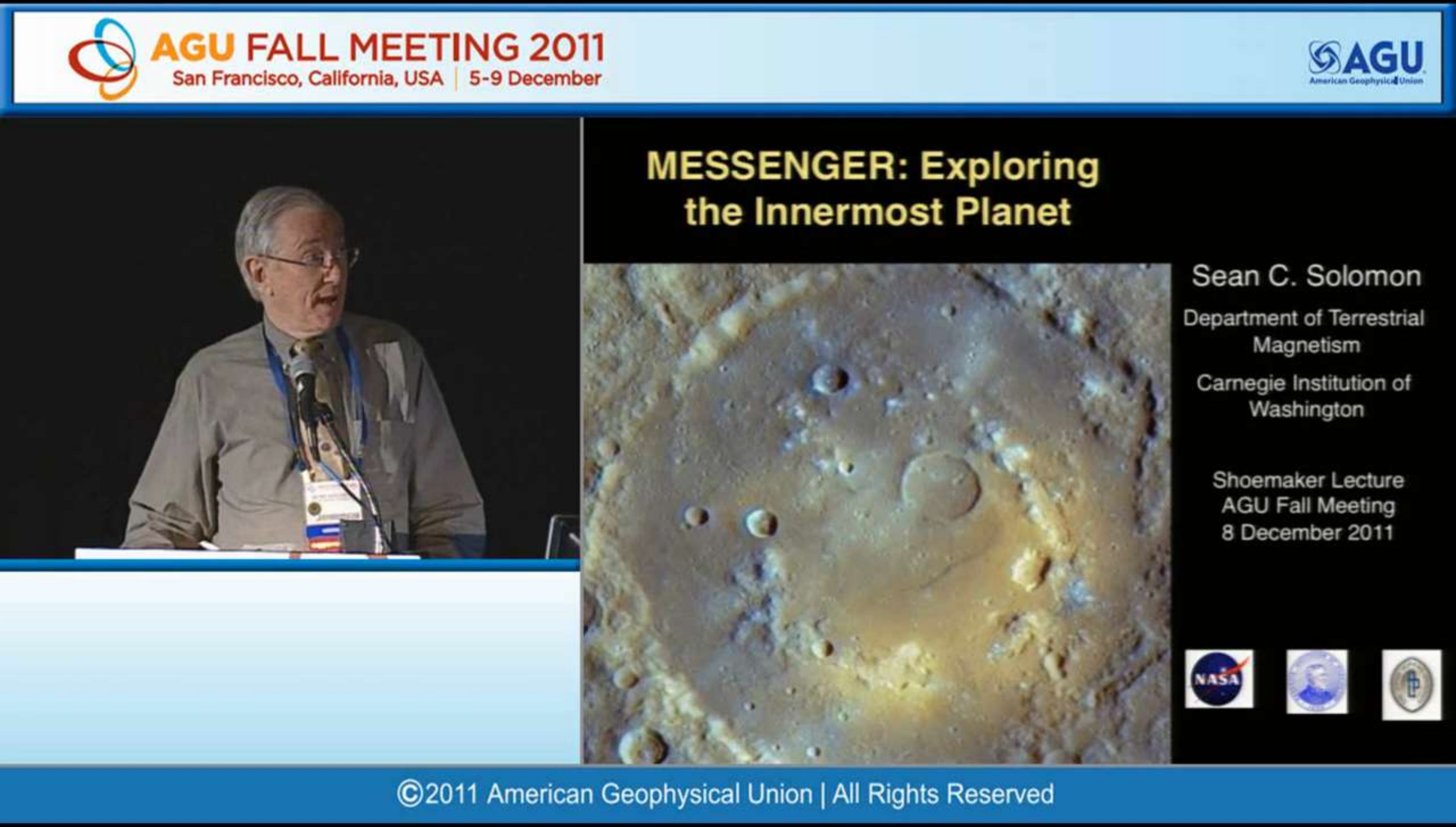
K/Th as a function of solar distance for the terrestrial planets: Mars – red, Earth – blue, Venus – tan, and Mercury – grey. Errors shown are 1-σ.


10000
1000
100
Potassium/Thorium Ratio
0
0.5
1
1.5
2
Distance from the Sun (AU)


MESSENGER GRS
Mars Odyssey GRS
Lunar Prospector GRS
Venus Lander GRS
Mars Meteorites
Terrestrial Oceanic Crust
K/Th = 5500
K/Th = 360
Potassium (ppm)
Thorium (ppm)
0
1000
2000
3000
4000
5000
0
1
2
3
4
5
6
7

Peplowski et al. (2012), 43rd LPSC, Abstract 1541.pdf





AGU FALL MEETING 2011
San Francisco, California, USA | 5-9 December

MESSENGER: Exploring the Innermost Planet

Sean C. Solomon

Department of Terrestrial Magnetism

Carnegie Institution of Washington

Shoemaker Lecture
AGU Fall Meeting
8 December 2011

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MESSENGER

Formation Models

- MESSENGER measurements:
  - K = 1150 ± 220 ppm (GRS)
  - 0.05 < S/Si < 0.15 (XRS)
  - Th/U = 2.5 ± 0.9 (GRS)
- Formation models calling for extended periods of high temperatures (e.g., evaporation in a hot solar nebula, high-temperature condensates, or some giant impact scenarios) are inconsistent with observed S and K abundances.
- Mercury's metal-rich, FeO-poor composition likely reflects highly reduced precursor materials.

MESSENGER

Constraints on Mantle, Core

- Bulk density, spin state, gravity field
  - Obliquity $i = 2.06 \pm 0.1$ arc min [*Margot*, 2011]
  - Degree-2 components of Mercury's gravity field
- Polar moment of inertia
  - $C/MR^2 = 0.353 \pm 0.017$
- Moment of inertia of solid outer shell
  - $C_m/C = 0.452 \pm 0.035$
- C/MR^2 and C_m/C combination
  - No previously published models fit C/MR^2 <u>and</u> C_m/C

AGU FALL MEETING 2011
San Francisco, California, USA | 5-9 December

AGU FALL MEETING 2011

Session U23B. *Dawn* Explores Vesta III

"Vesta and Mercury: Predictive Models for Bulk Chemical Composition and Thermal and Physical Structure"

Andrew J.R. Prentice
School of Mathematical Sciences,
Monash University, Clayton,Victoria, Australia

MONASH University

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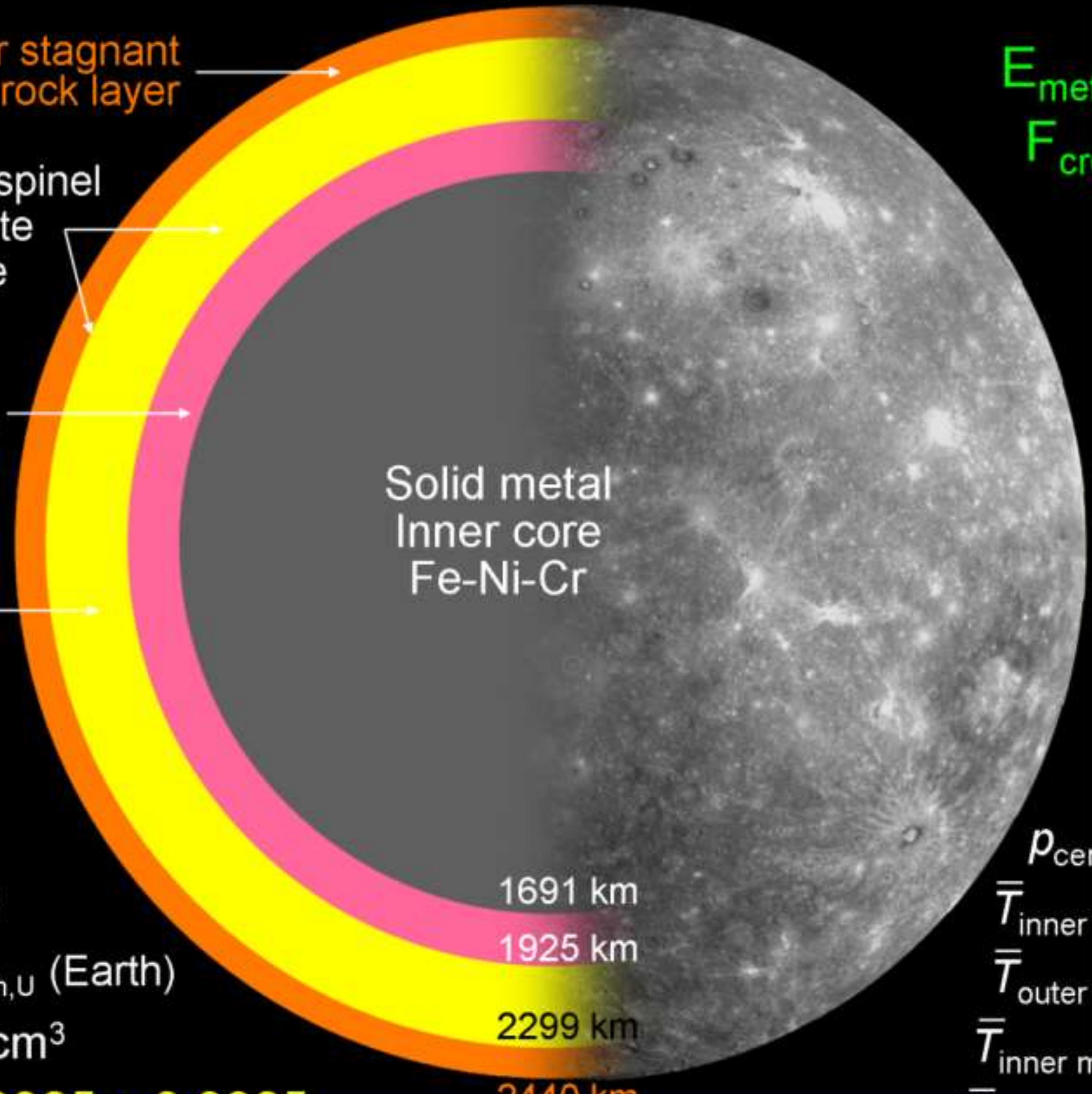
AGU Fall Meeting 2011 - AGU FM2011-U23B-III-Vesta-Prentice.wmv

4-Zone Mercury Structure Model



Outer stagnant rock layer

Gehlenite, spinel + Mg-Silicate rock mantle

Liquid metal outer core

Convective inner rock layer

Solid metal Inner core Fe-Ni-Cr

1691 km

1925 km

2299 km

2440 km

$E_{metal} = 1.191$

$F_{creep} = 0.95$

$X_{core} = 0.708$

$X_{liquid} = 0.218$

$X_{Th,U} = \mathbf{4.3} X_{Th,U}$ (Earth)

$\bar{\rho} = 5.426\ \mathrm{g/cm^3}$

$C/MR^2 = 0.3325 \pm 0.0025$

$p_{centre} = 397$ kbar

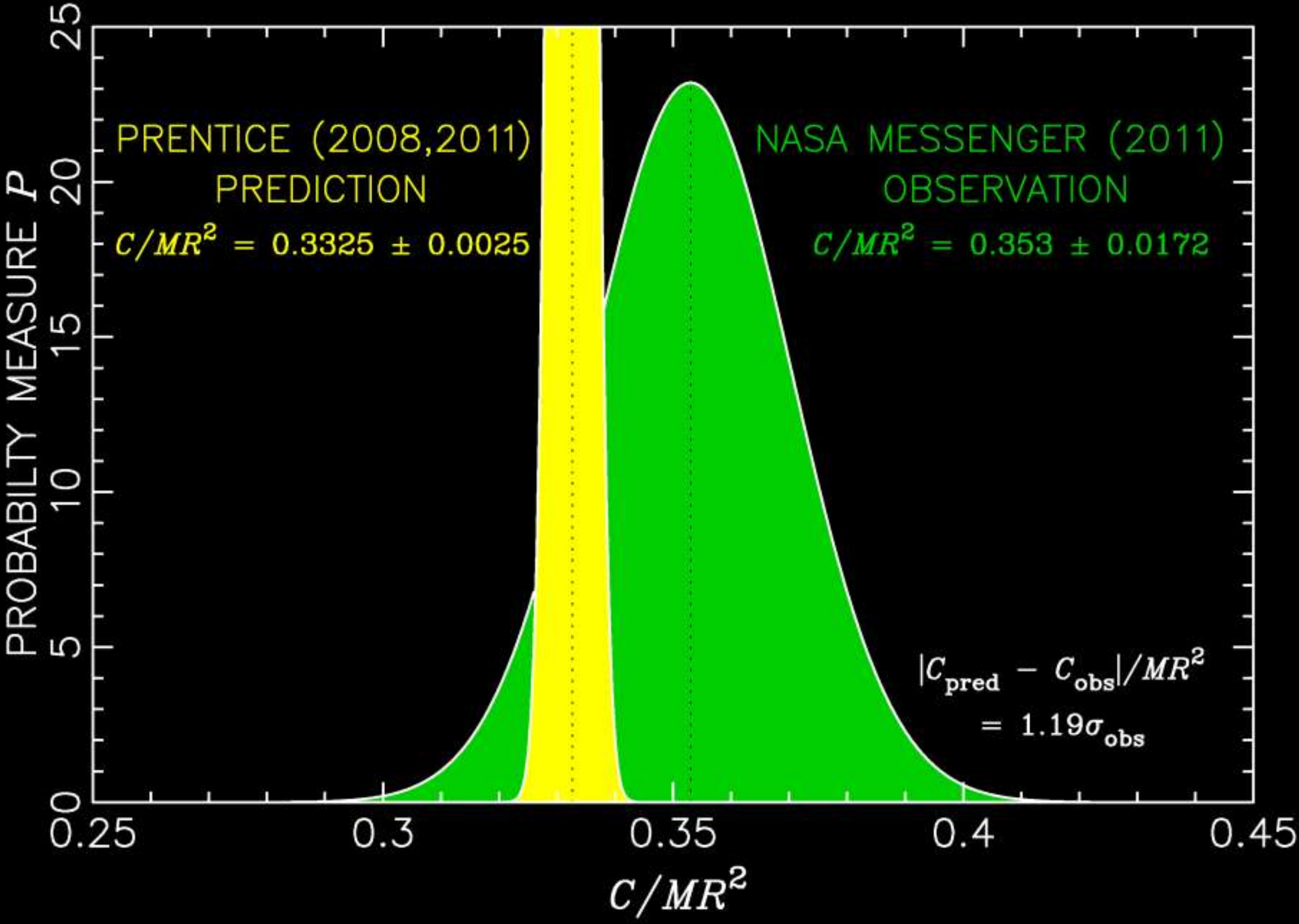
$\bar{T}_{inner\ core} = 2230$ K

$\bar{T}_{outer\ core} = 2195$ K

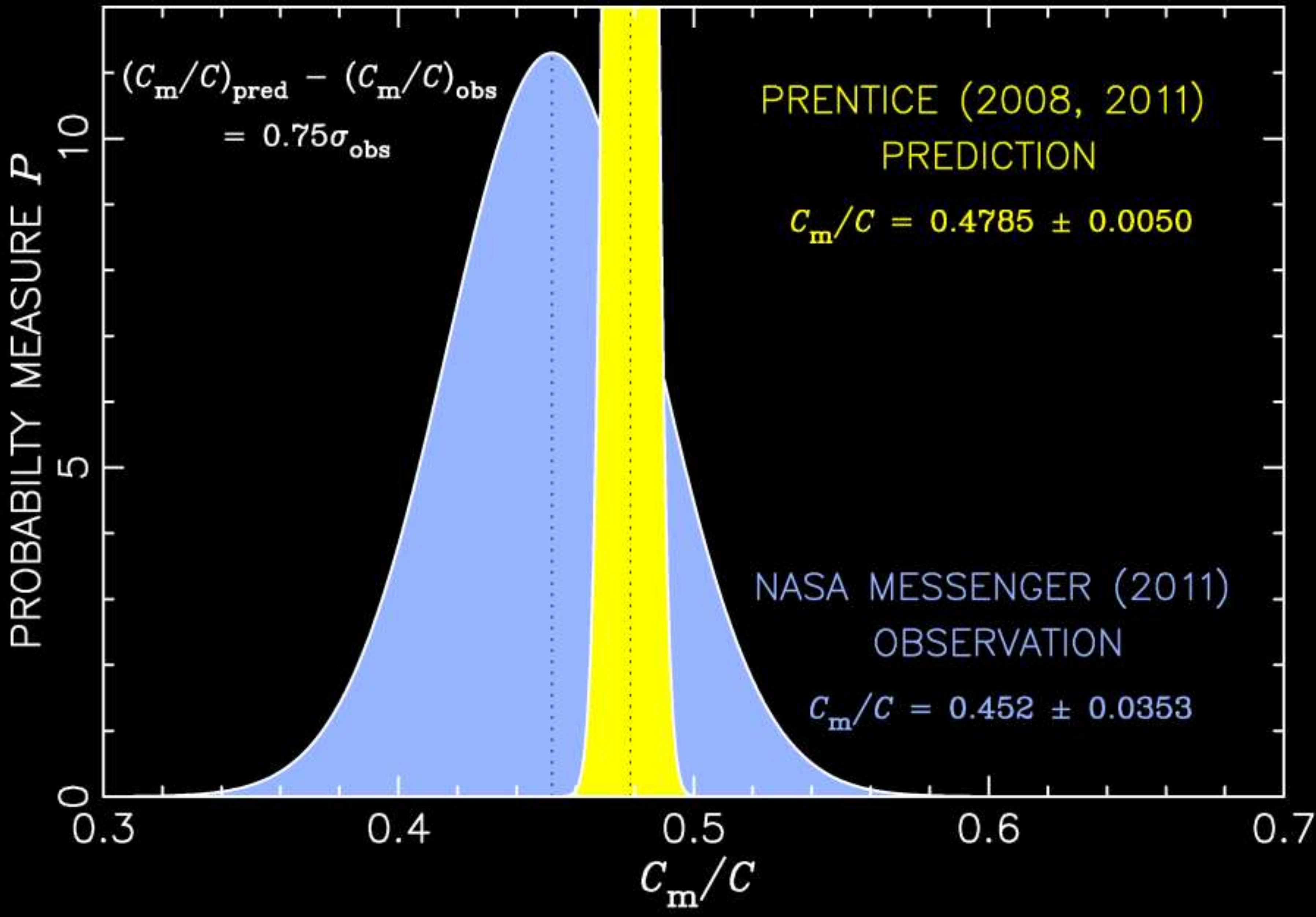
$\bar{T}_{inner\ mantle} = 2020$ K

$\bar{T}_{outer\ mantle} = 1080$ K

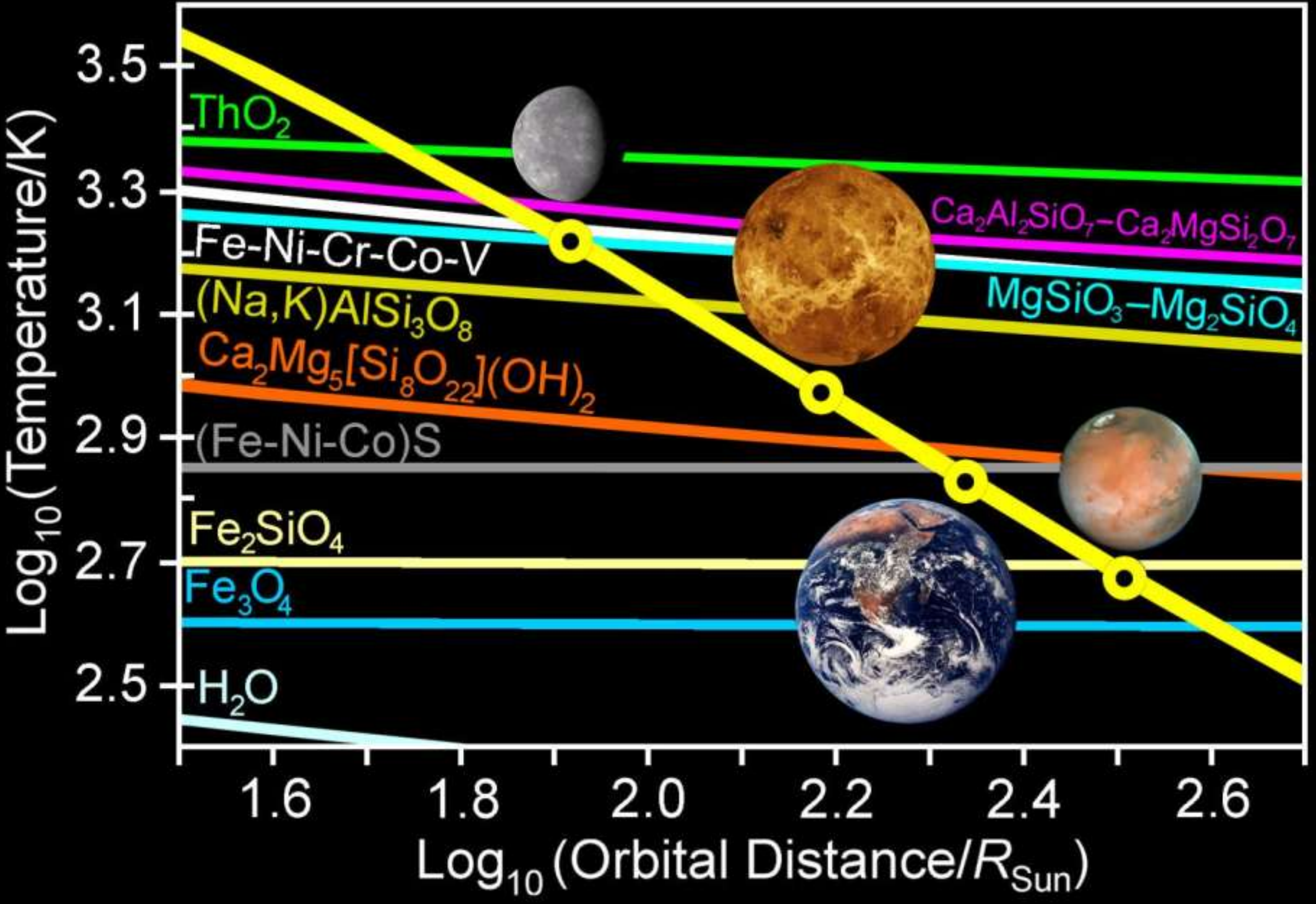
MERCURY: MOMENT-OF-INERTIA COEFFICIENT

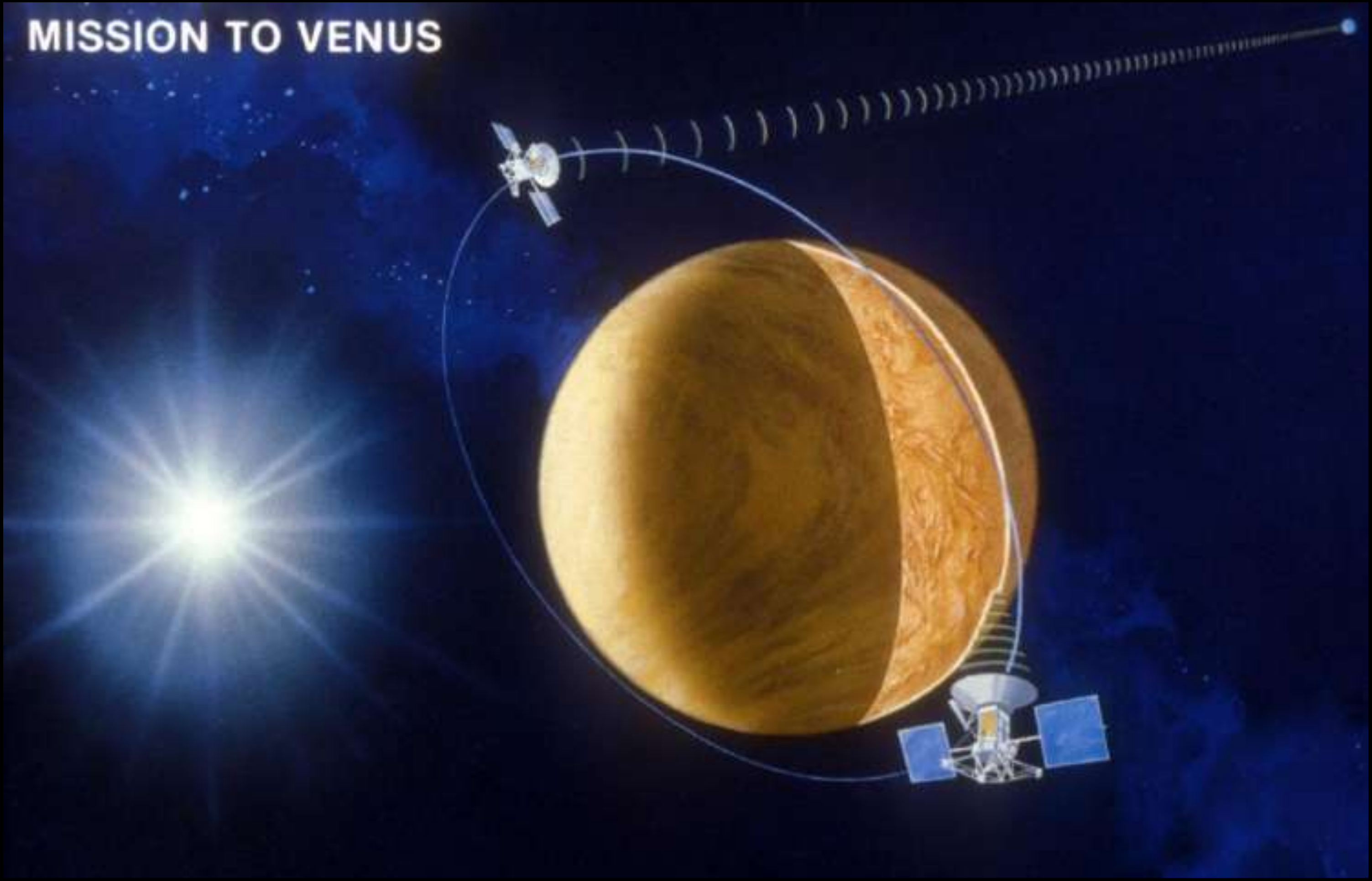


MERCURY: MANTLE MOMENT-OF-INERTIA FRACTION



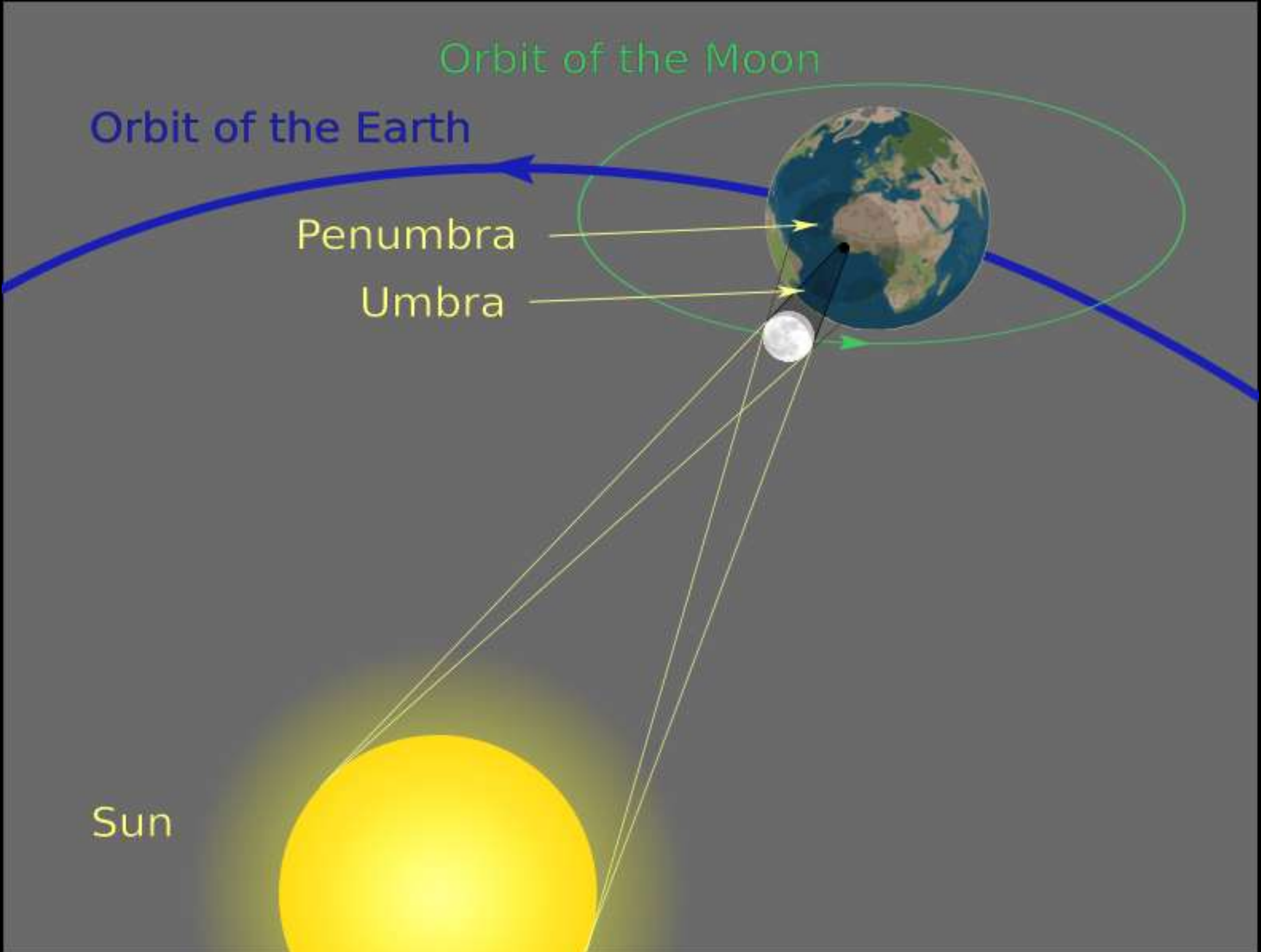
CHEMICAL CONDENSATION SEQUENCE

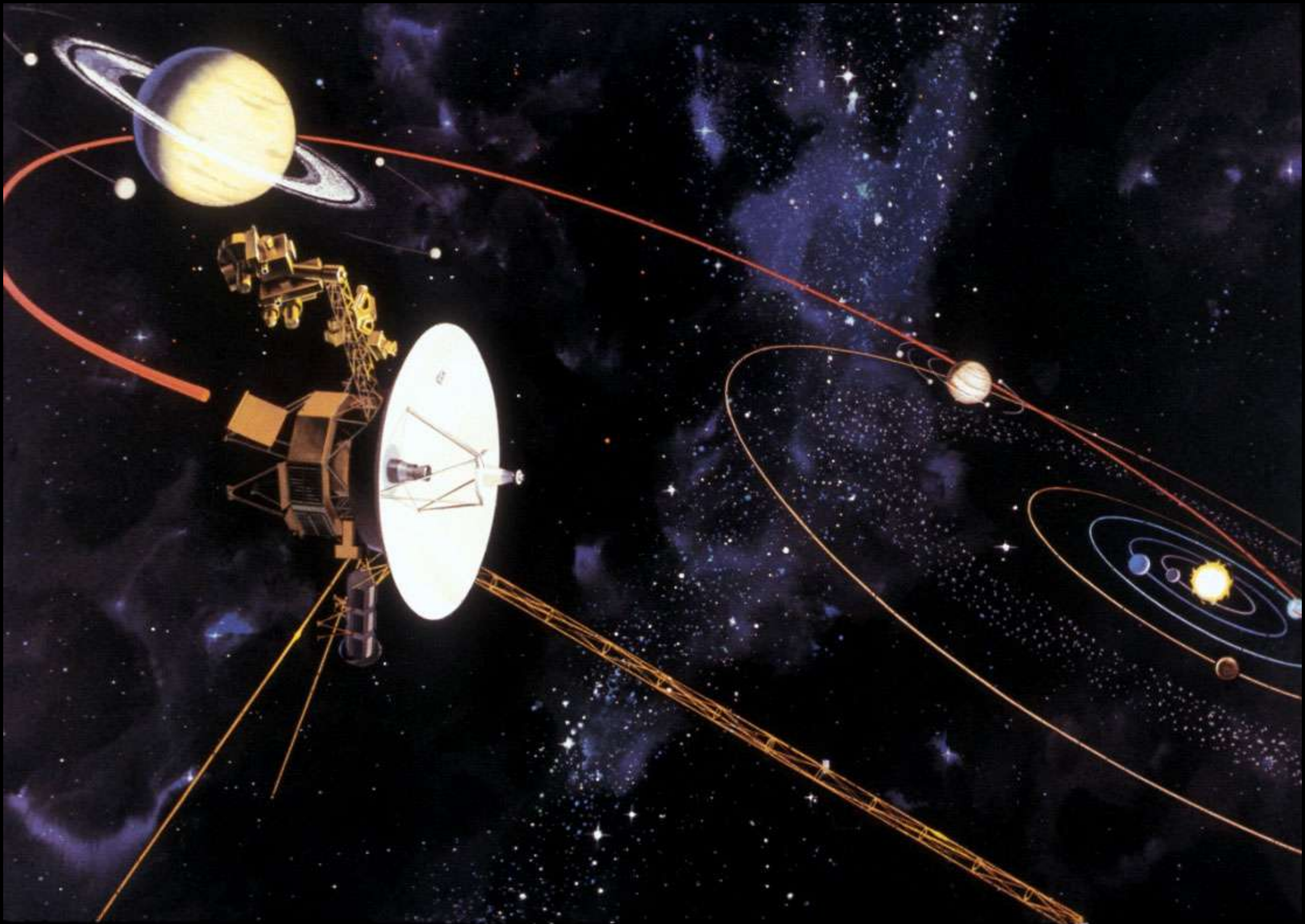



MISSION TO VENUS

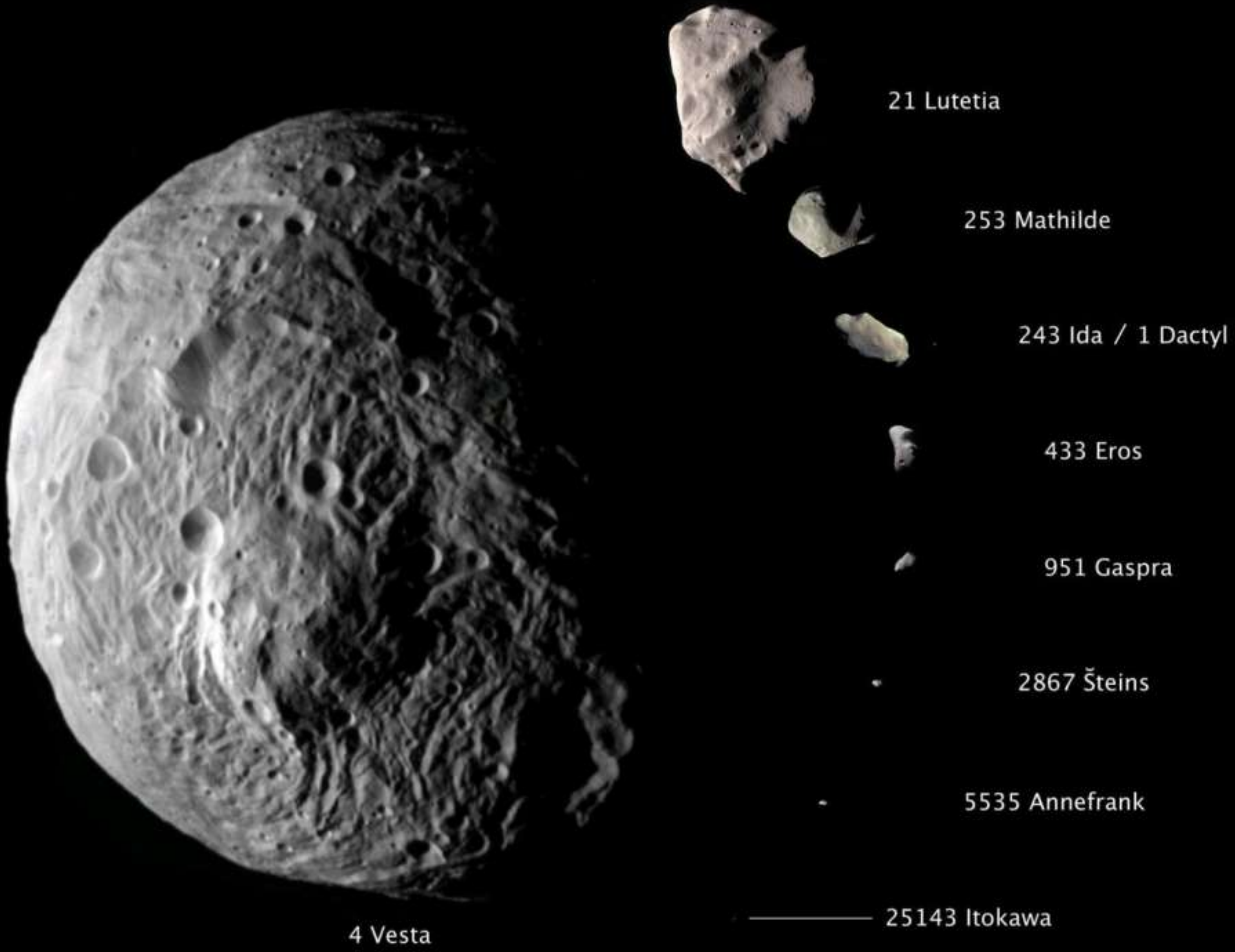
EARTH

Chemical Species	% Mass
$Ca_2MgSi_2O_7$	0.5
$MgAl_2O_4$	3.3
Fe-Ni-Cr-Co-V	23.0
$MgSiO_3$ - Mg_2SiO_4	44.0
SiO_2	6.2
$NaAlSi_3O_8$	1.1
$Ca_3(PO_4)_2$ - $Ca_5(PO_4)_3F$	0.8
(Fe-Ni-Co)S	10.8
MnS, ZnS	0.5
NaCl	0.2
TiO_2, Cr_2O_3	0.2
$Ca_2Mg_5[Si_8O_{22}](OH)_2$	9.4
	100.0


Orbit of the Moon
Orbit of the Earth
Penumbra
Umbra
Sun

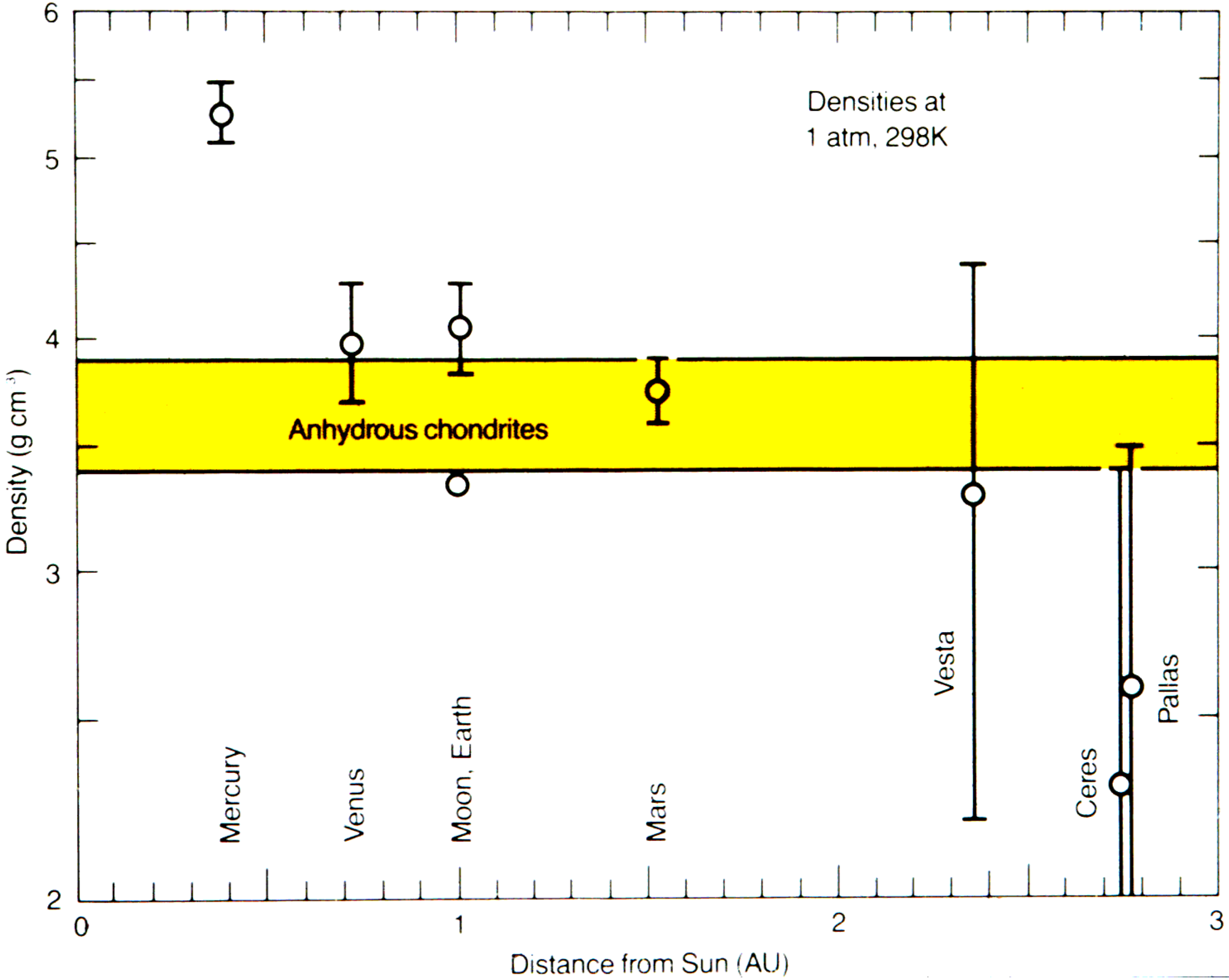


Discover of the
two asteroids
Vesta and Pallas


21 Lutetia
253 Mathilde
243 Ida / 1 Dactyl
433 Eros
951 Gaspra
2867 Šteins
5535 Annefrank
25143 Itokawa
4 Vesta

UNCOMPRESSED PLANETARY DENSITIES

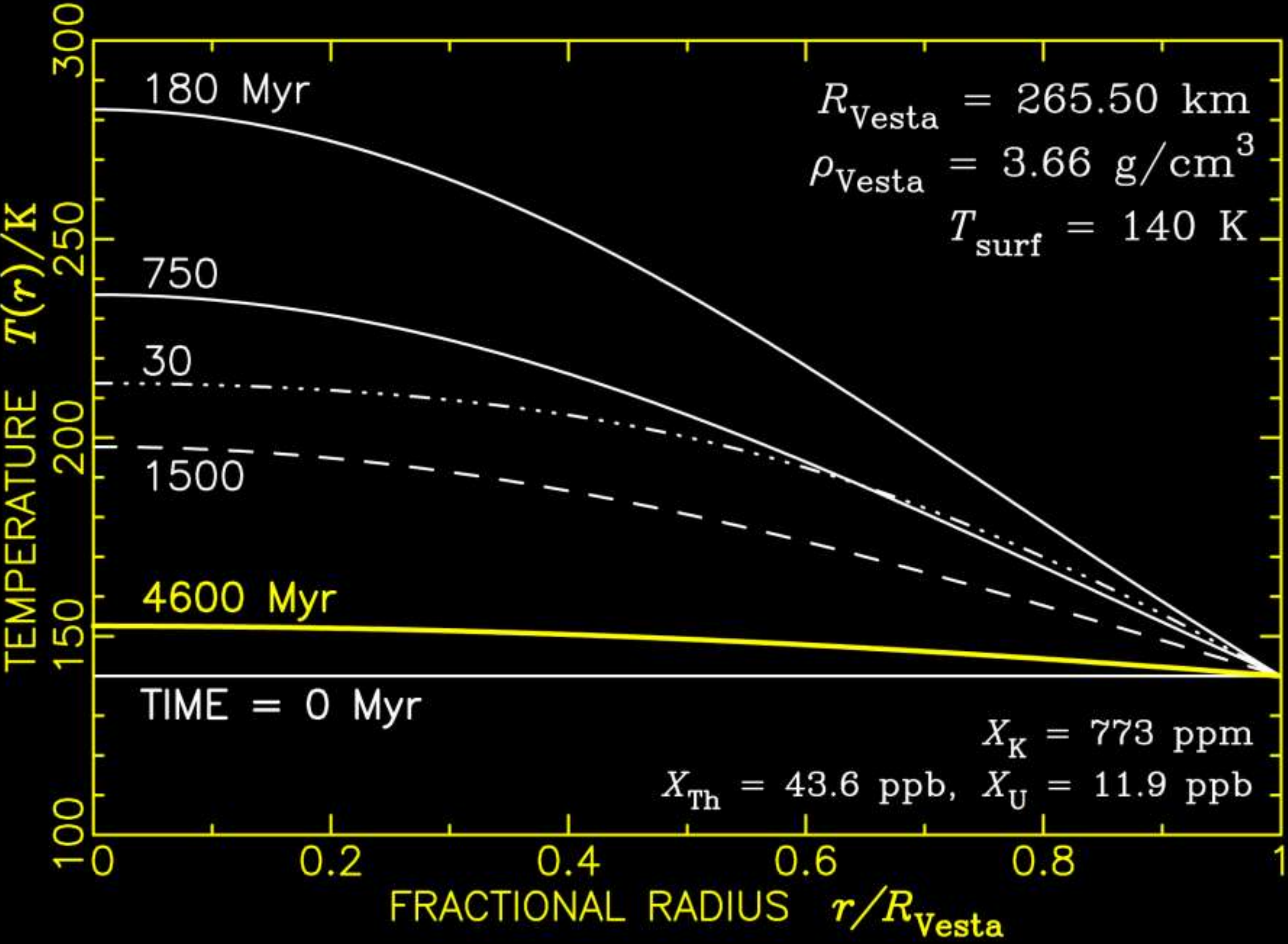
[from John T. Wasson, 1985]



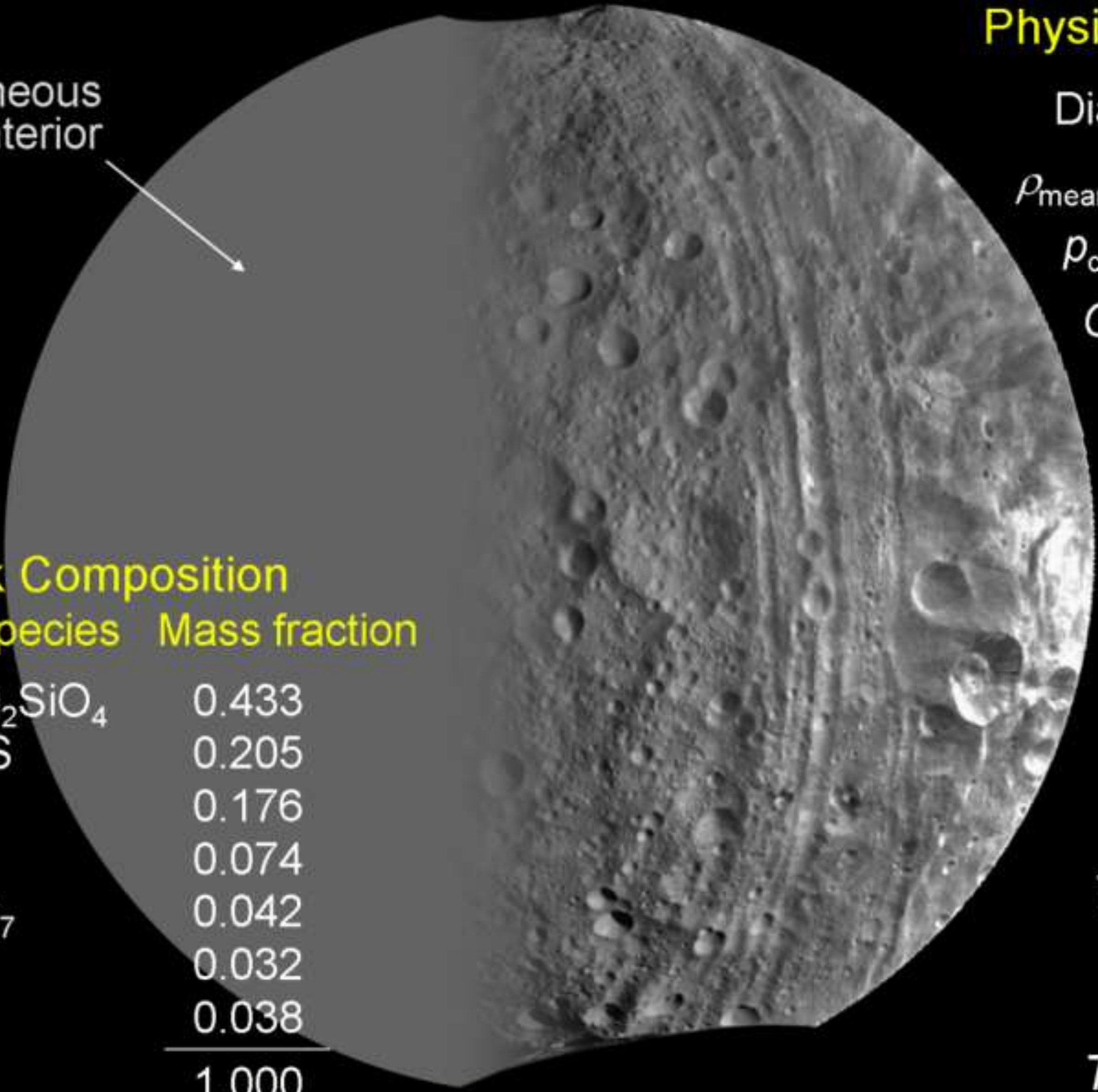
VESTA

Chemical Species	% Mass
$Ca_2MgSi_2O_7$	4.2
$MgAl_2O_4$	3.2
Fe-Ni-Co	0.9
(Fe-Ni-Co)S	20.5
Fe_3O_4	17.6
$MgSiO_3$ - Mg_2SiO_4	43.3
SiO_2	7.4
MnS, ZnS	0.5
Cr_2O_3, TiO_2	0.6
P_2O_5	0.3
NaCl	0.2
NaOH, KOH	1.3
	100.0

HOMOGENEOUS VESTA THERMAL MODEL



Homogeneous Vesta Structural Model



Physical Properties

Diameter: 531 km

$\rho_{\mathrm{mean}} = 3.664\ \mathrm{g/cm^3}$

$p_{\mathrm{centre}} = 1320$ bar

$C/MR^2 = 0.3999$

Bulk Composition

Chemical species	Mass fraction
$MgSiO_3$-Mg_2SiO_4	0.433
(Fe-Ni-Co)S	0.205
Fe_3O_4	0.176
SiO_2	0.074
$Ca_2MgSi_2O_7$	0.042
$MgAl_2O_4$	0.032
Remainder	0.038
	1.000

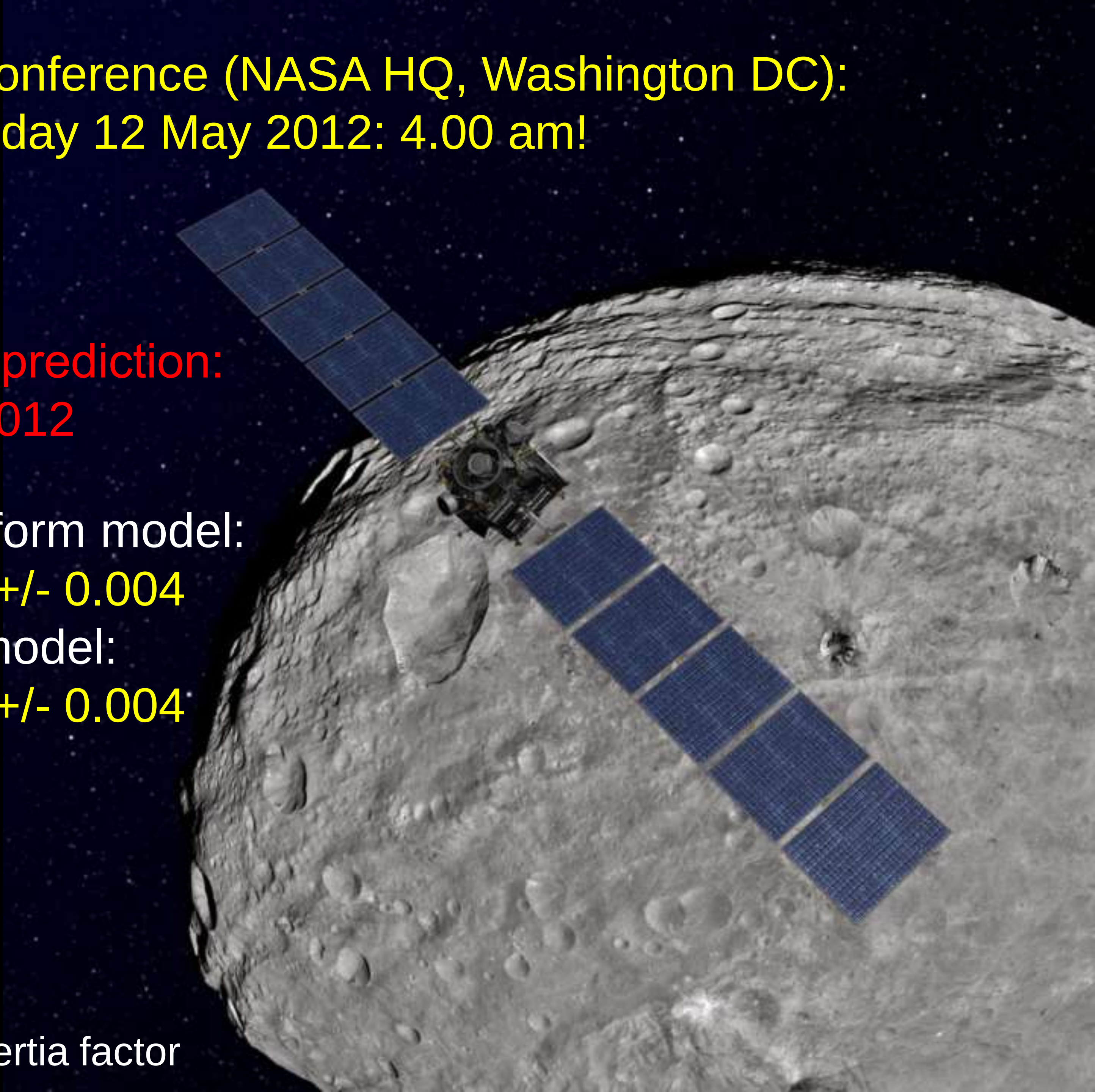
Thermal Properties

$T_{\mathrm{centre}} = 153$ K

$T_{\mathrm{mean}} = 145$ K

$T_{\mathrm{surface}} = 140$ K

Dawn Press Conference (NASA HQ, Washington DC):
Friday 12 May 2012: 4.00 am!

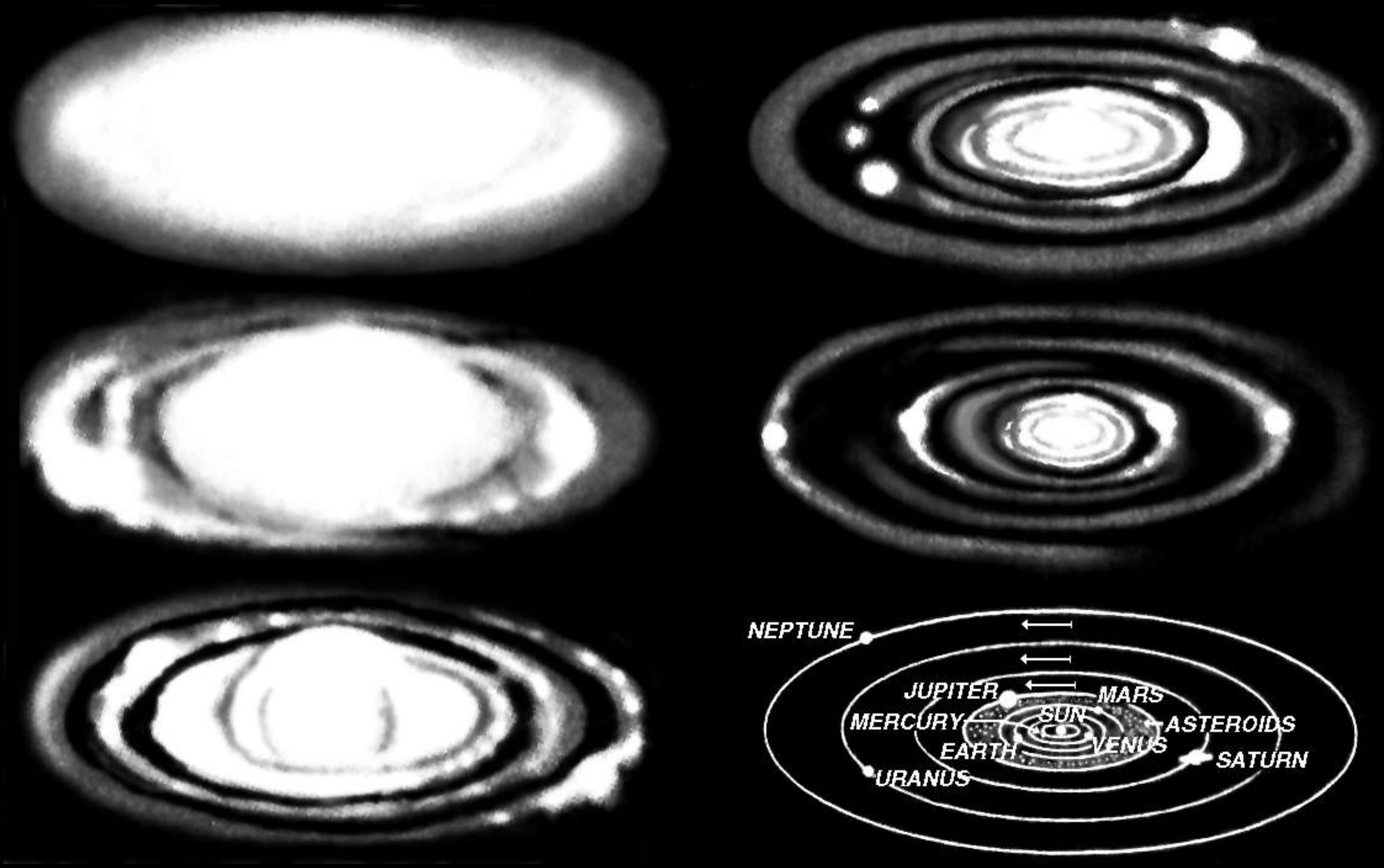


Prentice's final prediction:
9 May 2012

Chemically-uniform model:
MOI = 0.388 +/- 0.004
Differentiated model:
MOI = 0.369 +/- 0.004

?

MOI: moment-of-inertia factor


NEPTUNE
JUPITER
MARS
MERCURY
SUN
ASTEROIDS
EARTH
VENUS
SATURN
URANUS

Dirk ter Haar [1919-2002]

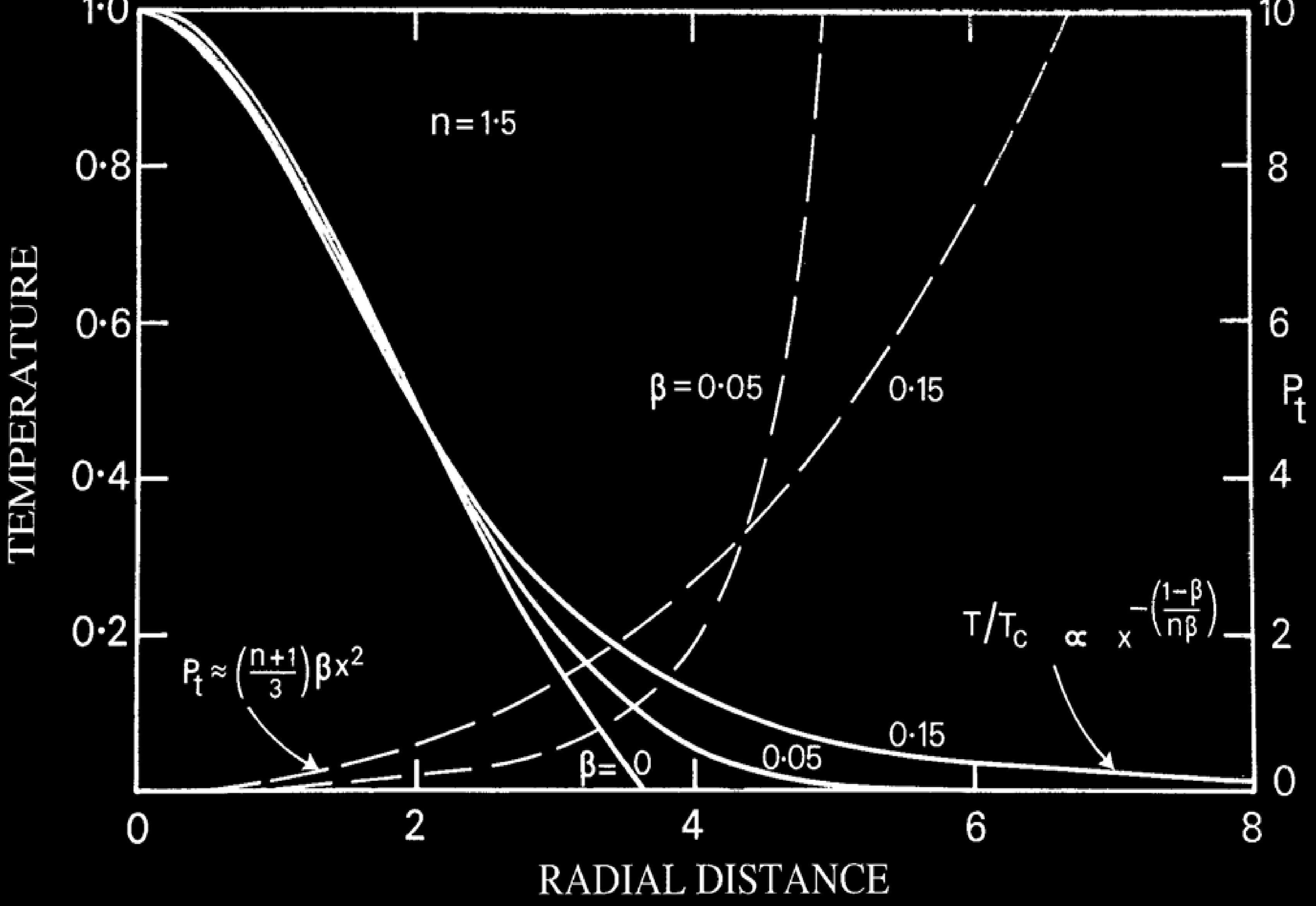
A KEY CONCEPT of the MLT is supersonic turbulent stress. Rising convective jets create a turbulent stress $<\rho_t v_t^2> = \beta\rho GM(r)/r$. Here ρ is the local gas density and $M(r)$ the total mass interior to radius r. $\beta \sim 0.1$ is the turbulence parameter (A.J.R. Prentice, *Astron. & Astrophys.* **27**, 237, 1973).

The total pressure in the contracting gas cloud is

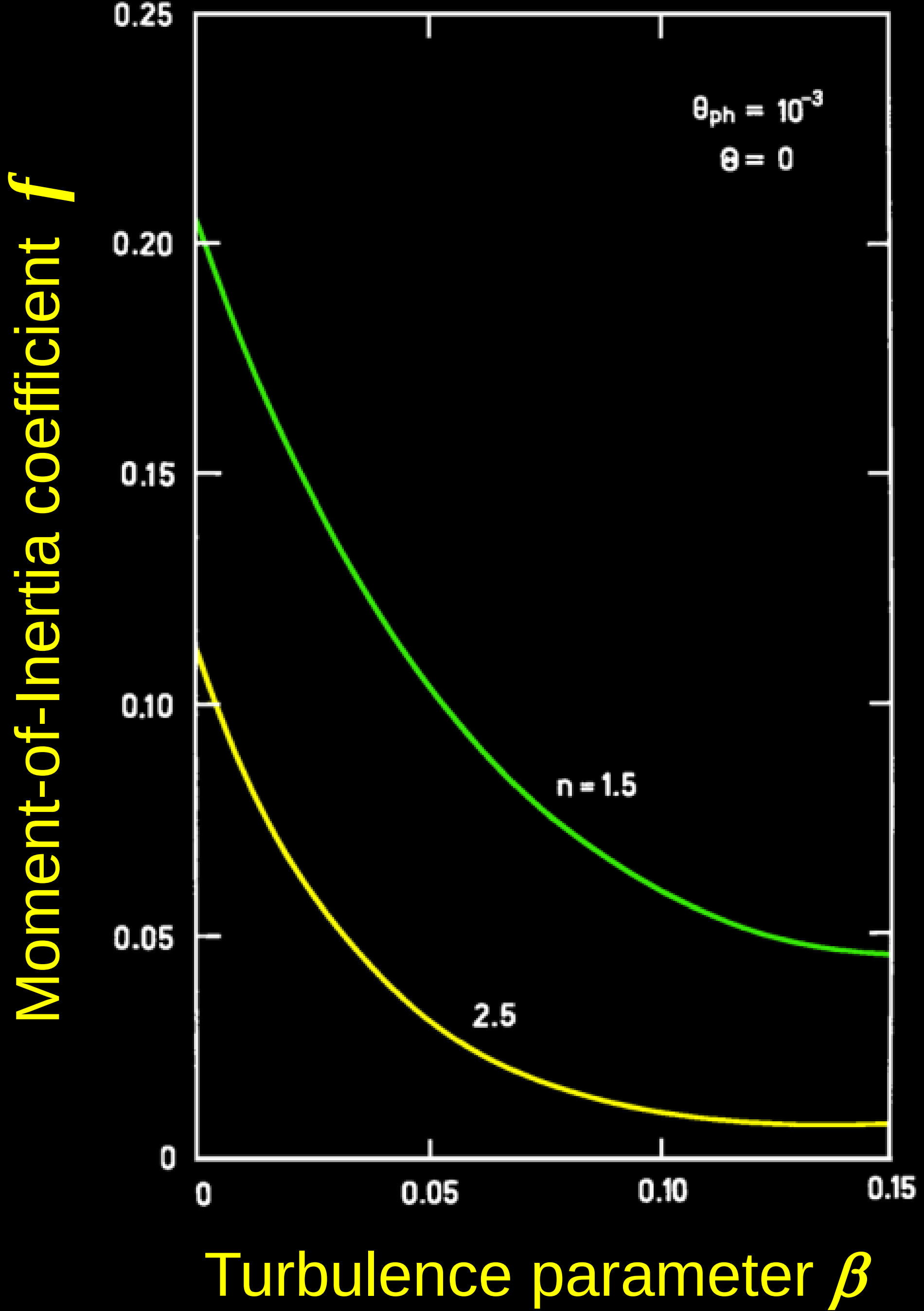
$$p_{\text{total}} = \frac{\rho \Re T}{\mu} + \frac{\beta \rho G M(r)}{r}$$

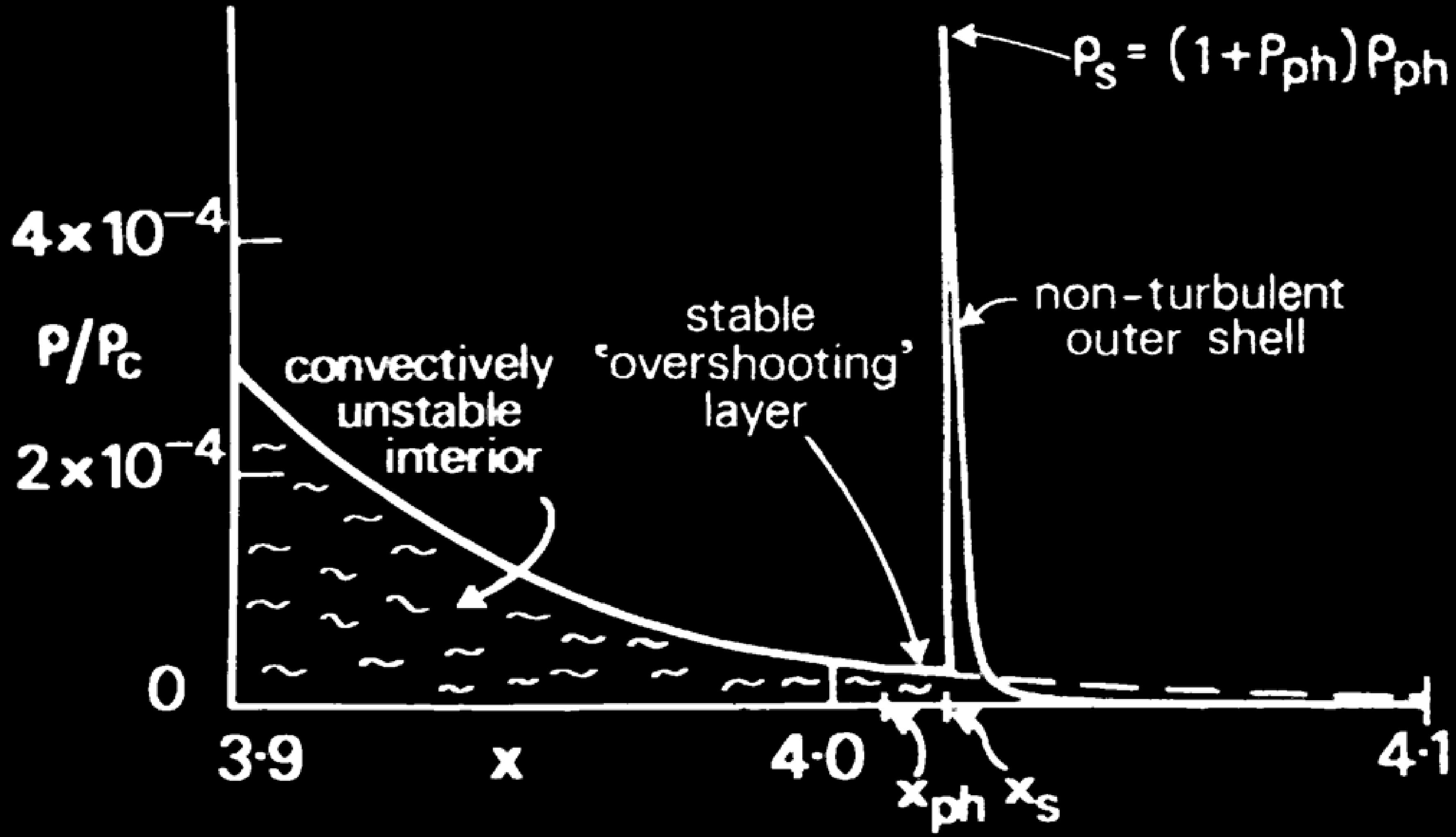
gas pressure		turbulent stress
1	:	35

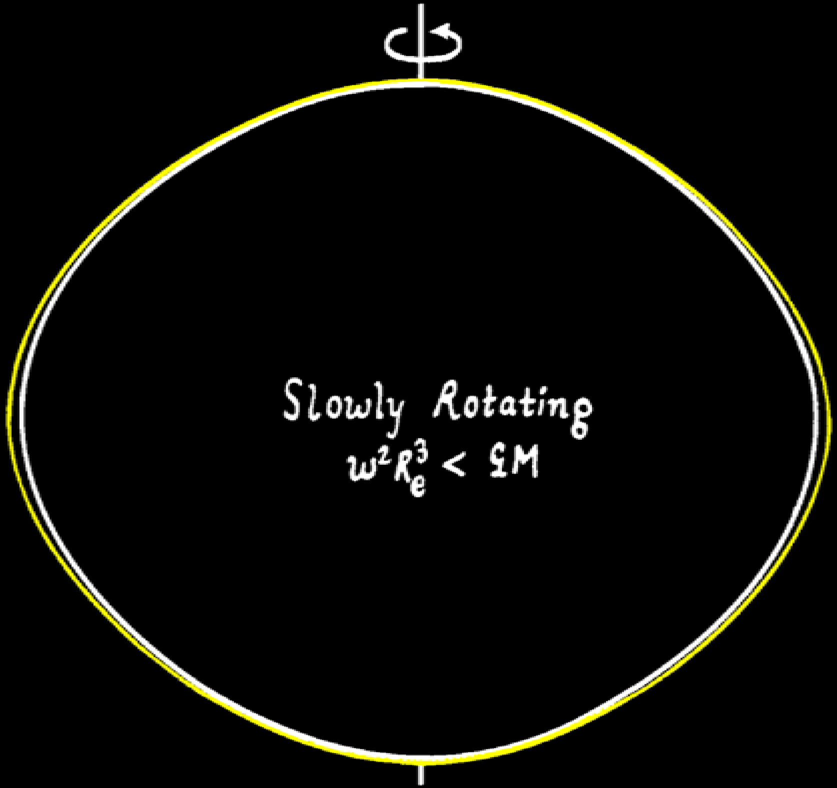
The turbulent stress rises to 30 - 40 times the gas pressure in the outer layers of the cloud.

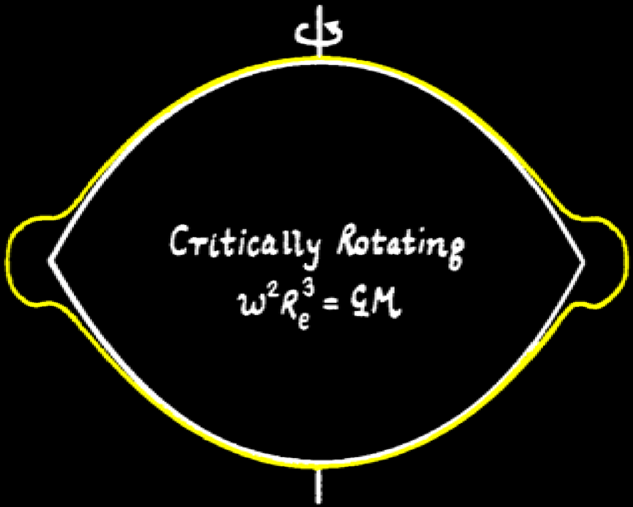


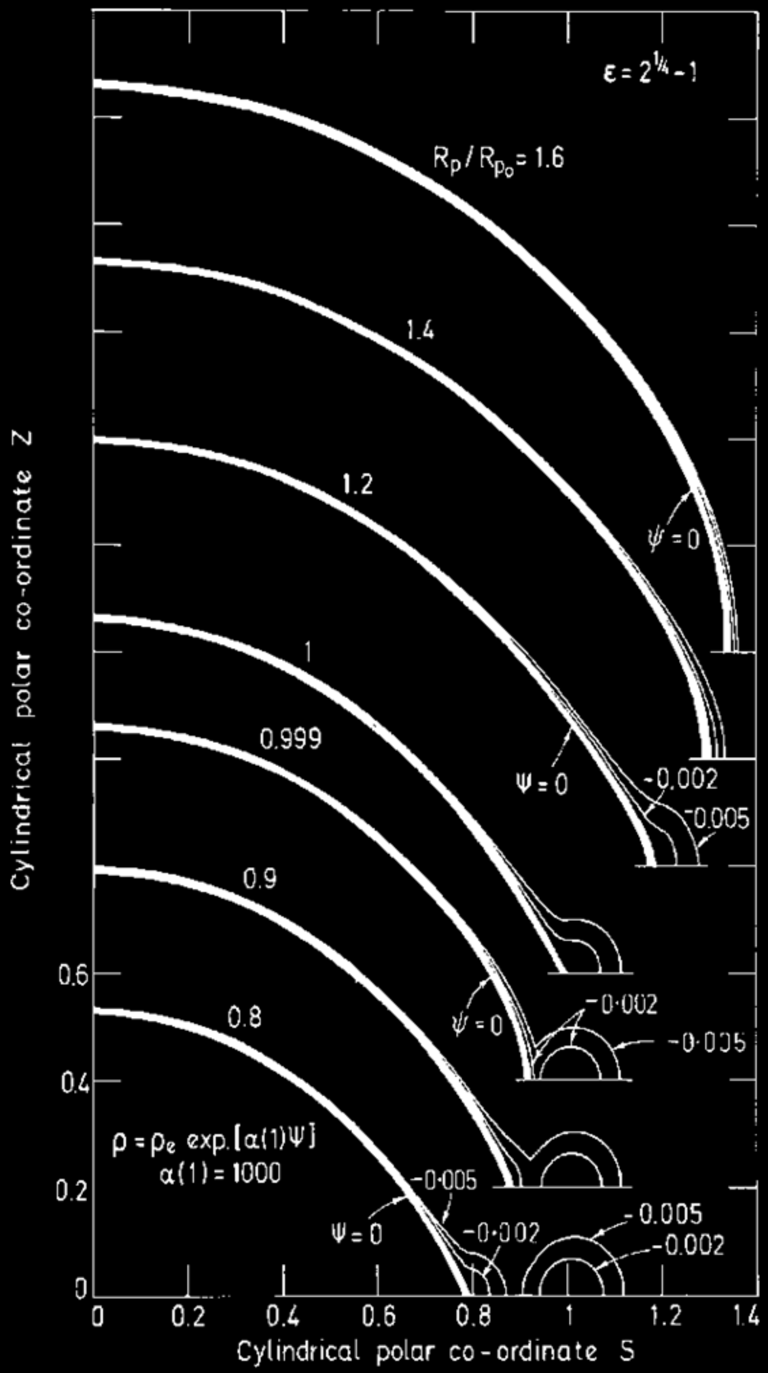
TEMPERATURE
RADIAL DISTANCE
P_t
n = 1·5
β = 0·05
0·15
β = 0
0·05
0·15
$P_t \approx \left(\frac{n+1}{3}\right)\beta x^2$
$T/T_c \propto x^{-\left(\frac{1-\beta}{n\beta}\right)}$
1·0
0·8
0·6
0·4
0·2
0
2
4
6
8
10
8
6
4
2
0

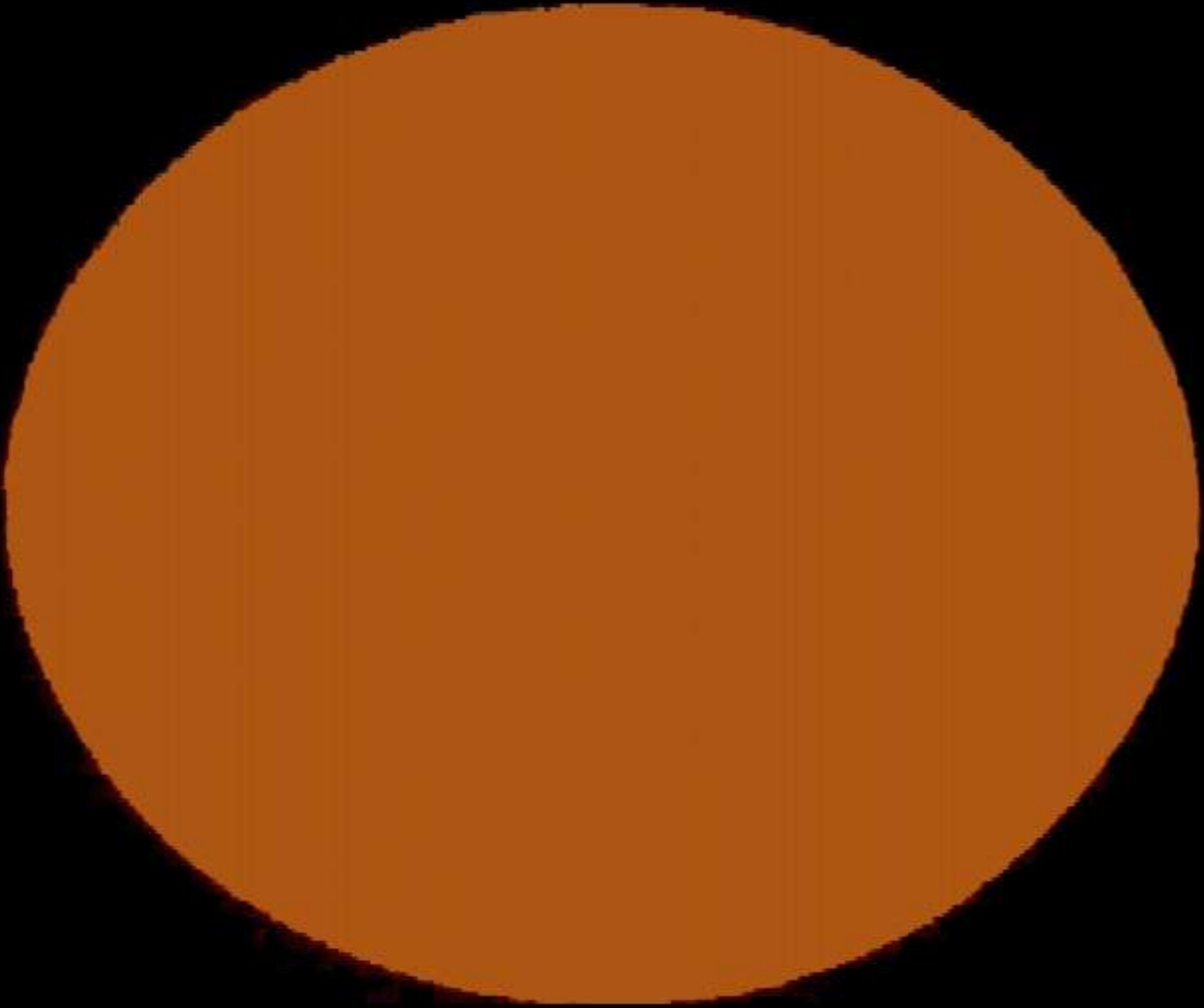

Moment-of-Inertia coefficient f
0.25
0.20
0.15
0.10
0.05
0
$\theta_{ph} = 10^{-3}$
$\theta = 0$
n = 1.5
2.5
0
0.05
0.10
0.15
Turbulence parameter β

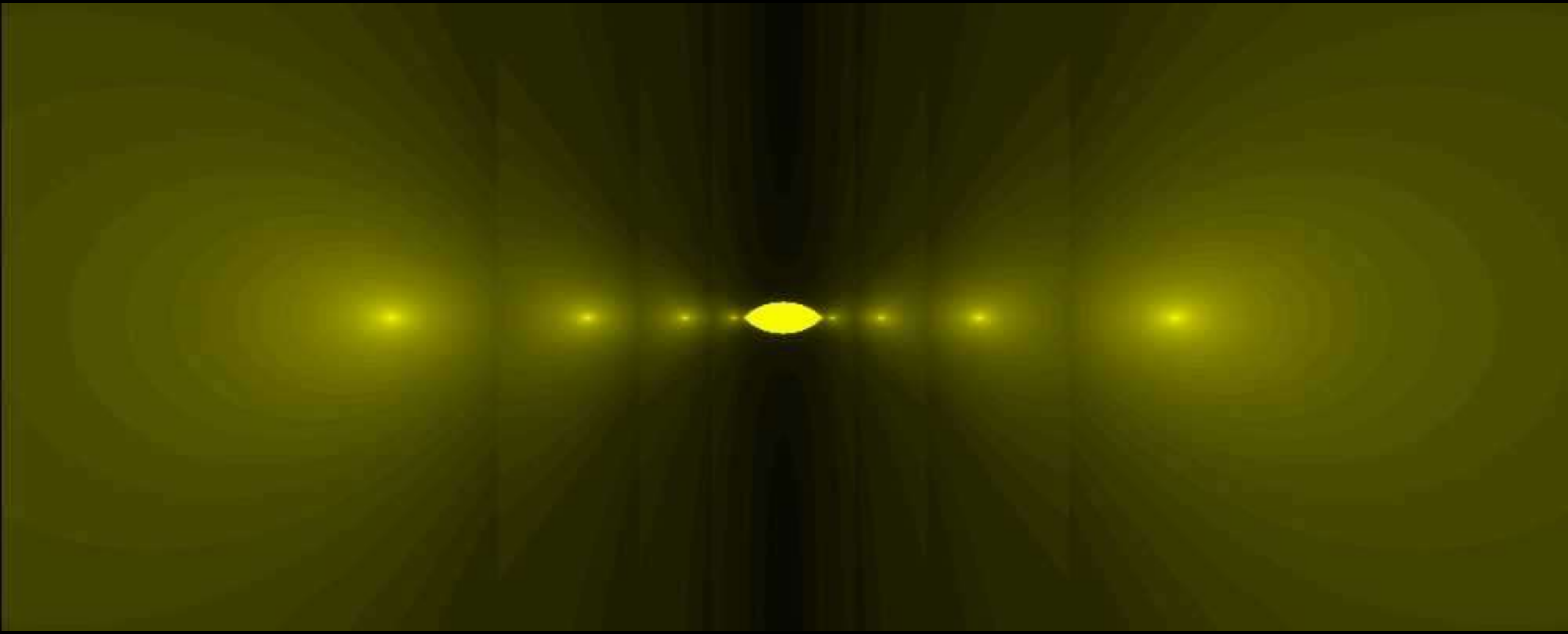

$\rho_s = (1+P_{ph})\rho_{ph}$
4×10^{-4}
ρ/ρ_c
non-turbulent outer shell
stable 'overshooting' layer
convectively unstable interior
2×10^{-4}
0
3·9
x
4·0
x_{ph}
x_s
4·1


Slowly Rotating
$\omega^2 R_e^3 < GM$


Critically Rotating
$\omega^2 R_e^3 = GM$


$\varepsilon = 2^{1/4} - 1$
$R_p / R_{p_0} = 1.6$
1.4
1.2
1
0.999
0.9
0.8
$\psi = 0$
$\Psi = 0$
-0.002
-0.005
-0·002
-0·005
$\rho = \rho_e \exp.[\alpha(1)\Psi]$
$\alpha(1) = 1000$
Cylindrical polar co-ordinate Z
Cylindrical polar co-ordinate S
0
0.2
0.4
0.6
0.8
1
1.2
1.4





ORBITAL DISTANCE EQUATION

$$\frac{R_n}{R_{n+1}} = \left[1 + \frac{m_n}{Mf}\right]^2$$

m_n = gaseous ring mass

R_n = orbital radius

M = cloud mass

f = moment of inertia coefficient

THE EQUATIONS OF THERMAL COMPRESSIBLE CONVECTION

The main physical quantities which define the convective flow at each position $\boldsymbol{r} = (x_1, x_2,)$ in the 2-D atmosphere at time t are the mass density $\rho(\boldsymbol{r},t)$, the velocity $\boldsymbol{u}(\boldsymbol{r},t) = (u_1,u_2)$, and the pressure $p(\boldsymbol{r},t)$. The 4 equations linking these quantities are

Continuity:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x_j}\left(\rho u_j\right)$$

Momentum: $(i = 1, 2)$:

$$\frac{\partial}{\partial t}(\rho u_i) = -\frac{\partial}{\partial x_j}\left(\rho u_i u_j\right) - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\sigma_{ij} + \rho g_i$$

where

$$\sigma_{ij} = \rho \nu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k} \right)$$

Energy:

$$\frac{\partial E_s}{\partial t} = -\frac{\partial}{\partial x_j}\left(E_s + p\right)u_j + \frac{\partial}{\partial x_j}\left(\rho \kappa \frac{\partial T}{\partial x_j}\right) + \frac{\partial}{\partial x_j}\left(u_i \sigma_{ij}\right) + \rho u_j g_j$$

where

$$E_s = \frac{1}{2}\rho u^2 + p/(\gamma - 1)$$

Here $u^2 = u_i u_i$, $\gamma = c_p / c_v$ is the ratio of specific heats, ν is the kinematic viscosity, κ is the thermal diffusivity, σ_{ij} is the shear tensor, g_i is the gravitational acceleration (assumed to be steady and directed along the axis Ox_2) and E_s is called the stagnation energy. Lastly, the temperature T is linked to the density ρ and pressure p via the ideal gas equation $p = \rho \Re T/\mu$, where μ is the mean molecular weight.

THE SUB-GRID SCALE TURBULENCE APPROXIMATION

(*after* Smagorinsky, I., *Mon. Weath. Rev.* **91**, 99–164, 1963)

Turbulent kinematic viscosity ν_t

$$\nu_t = (C_\nu h)^2 \sqrt{2S^2}$$

$$C_\nu = 0.4$$

$$h^2 = \Delta x \cdot \Delta z$$

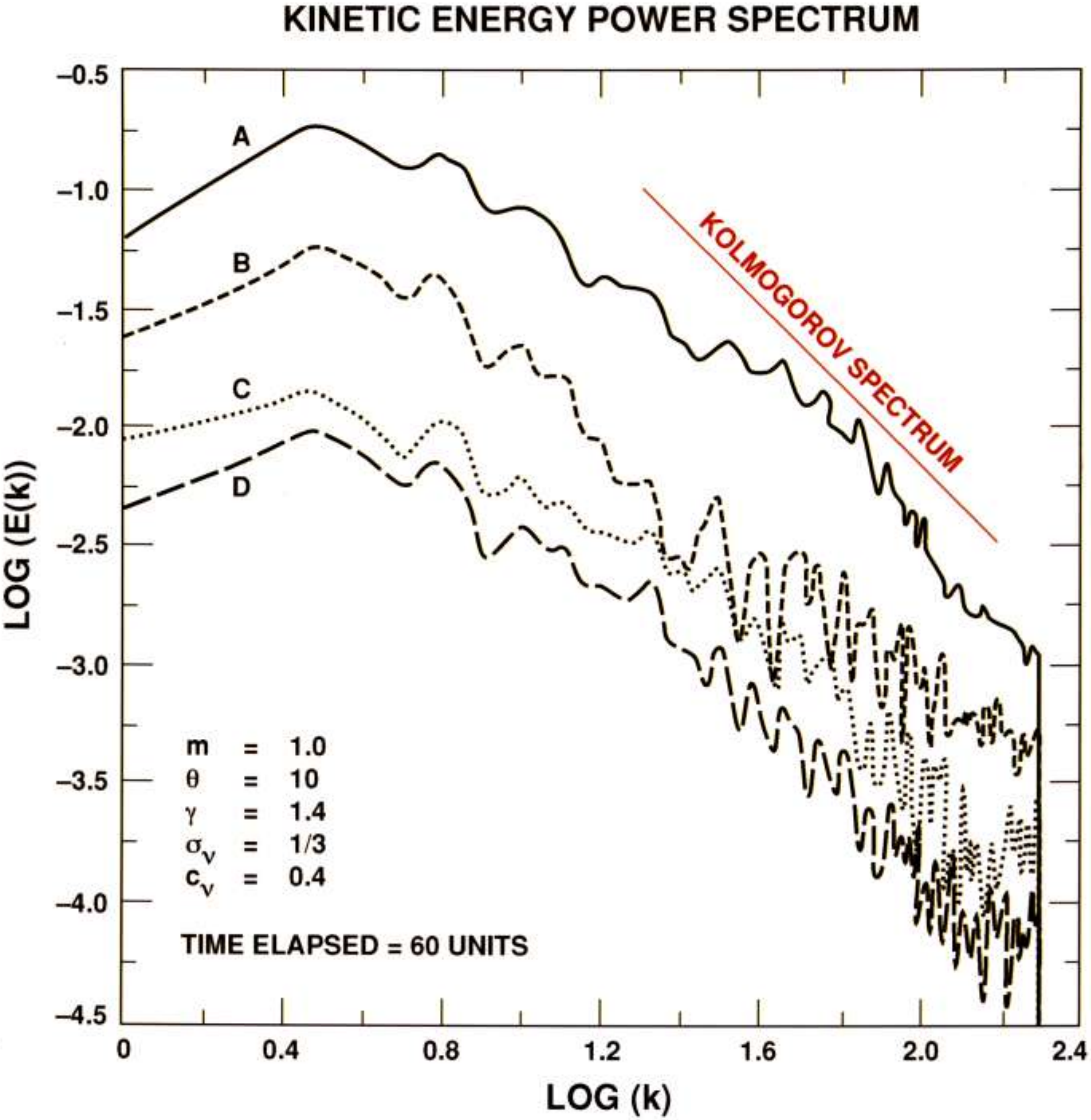
$$S^2 = S_{ij} : S_{ij}$$

$$S_{ij} = \frac{1}{2}(\partial u_i / \partial x_j + \partial u_j / \partial x_i)$$

Turbulent diffusivity κ_t

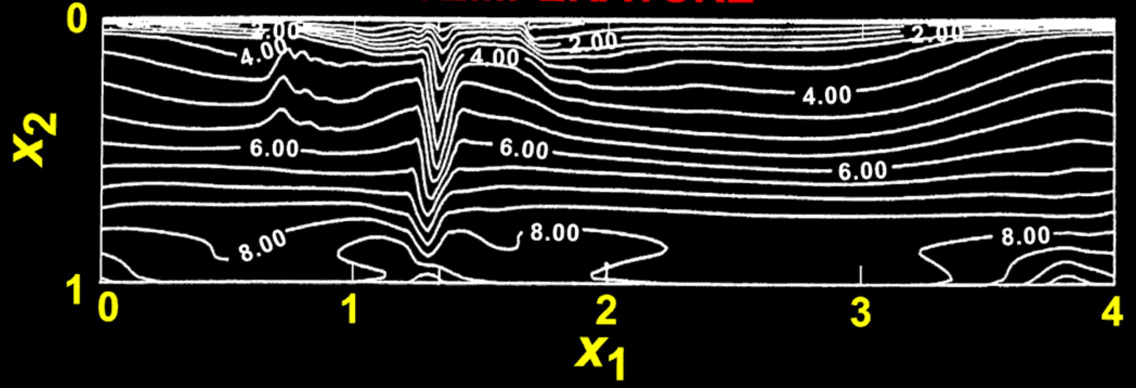
$$\kappa_t = \nu_t / \sigma_t$$

σ_t = Turbulent Prandtl number = 0.3

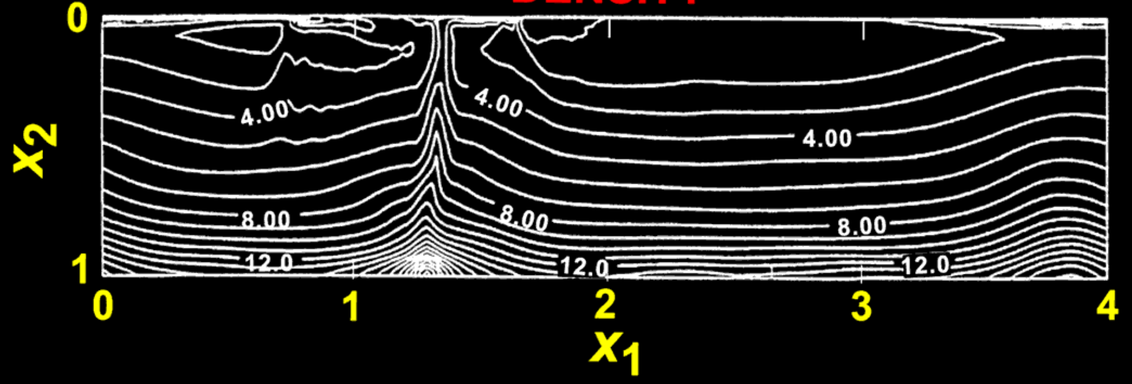


KINETIC ENERGY POWER SPECTRUM

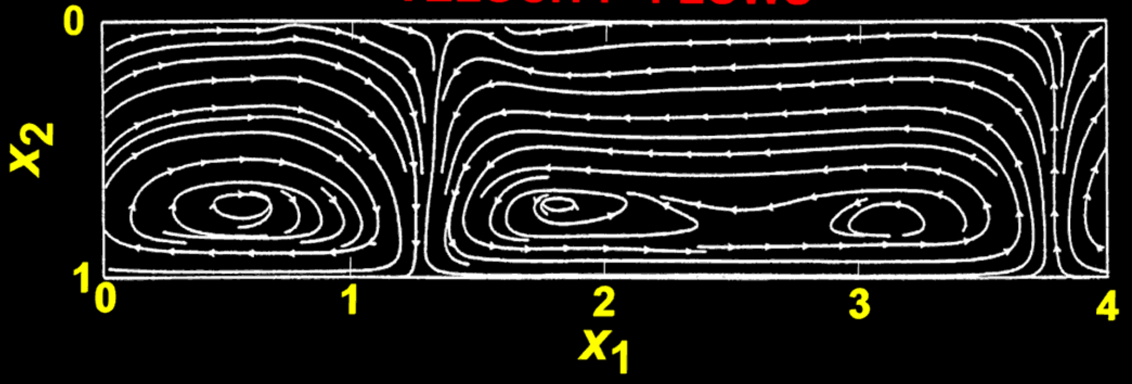
TEMPERATURE

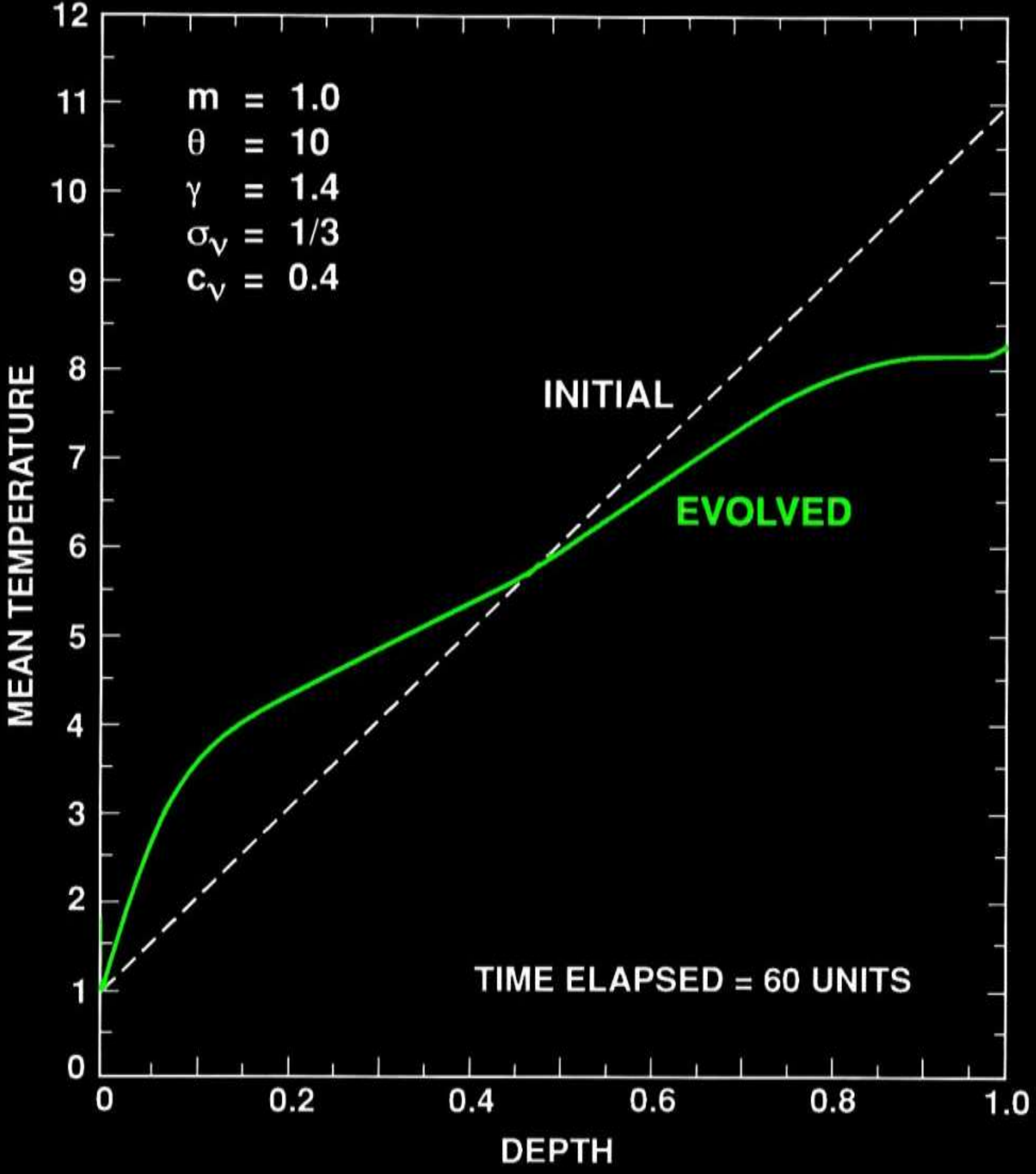


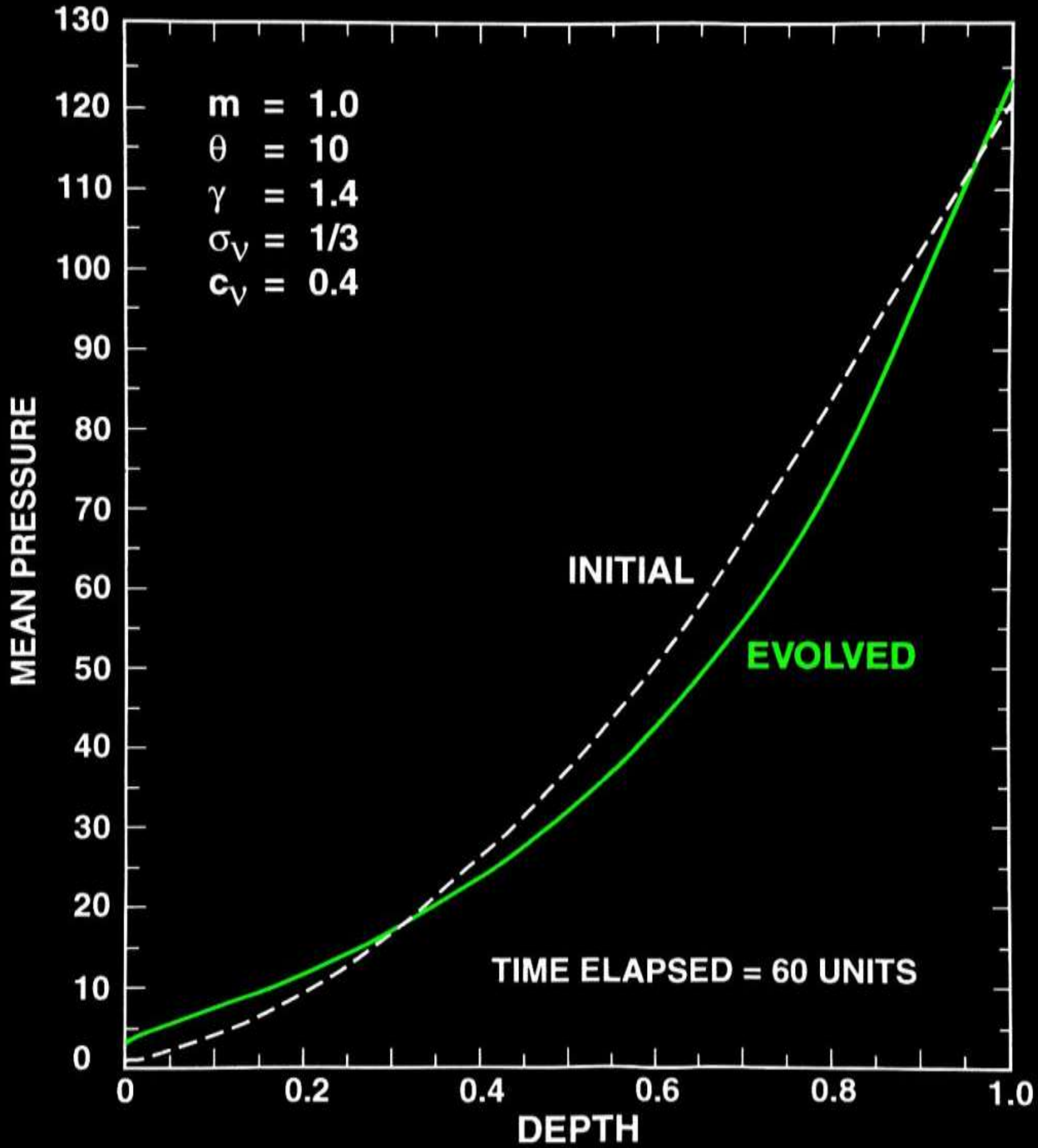
DENSITY



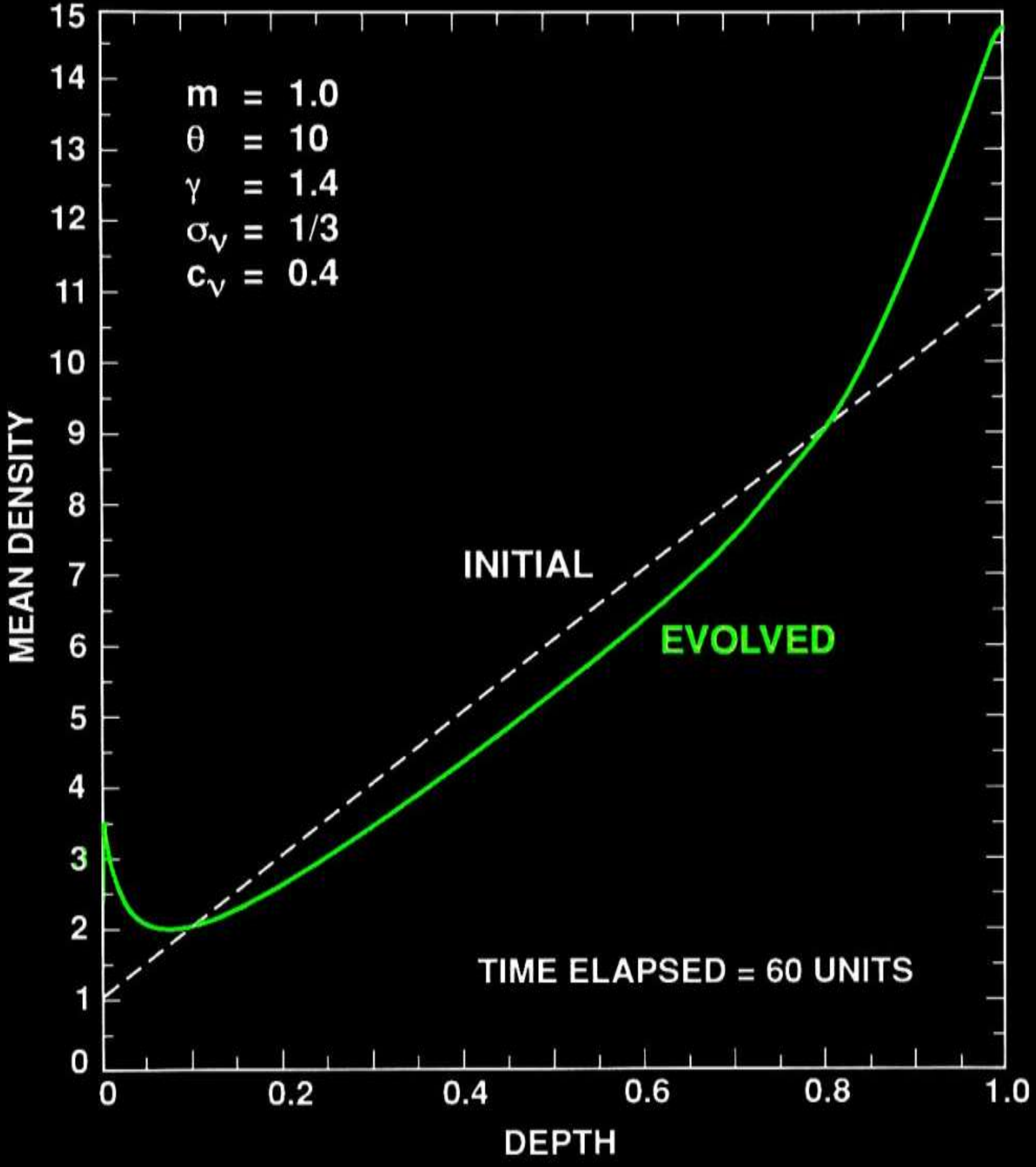
VELOCITY FLOWS

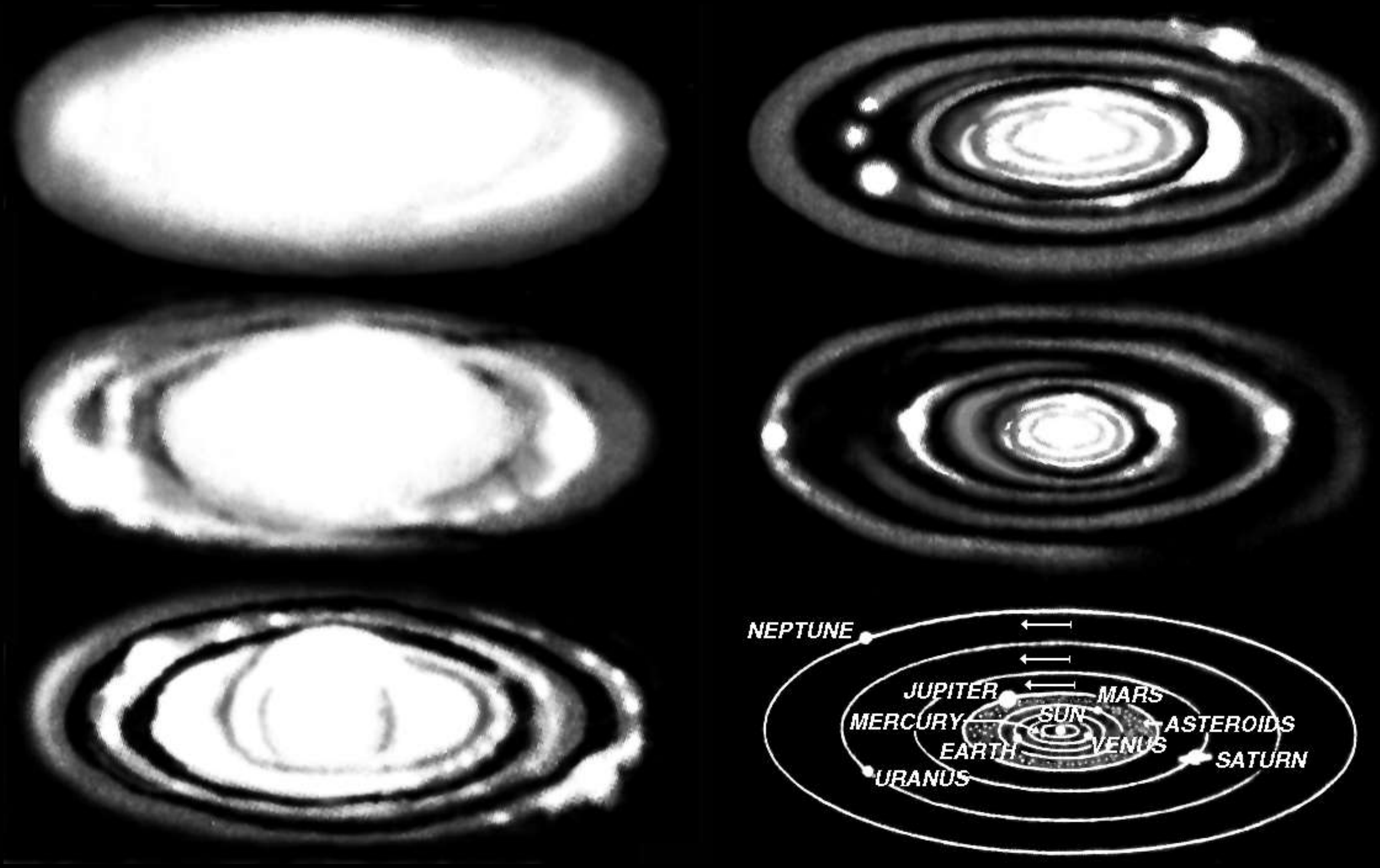



m = 1.0
θ = 10
γ = 1.4
σ_ν = 1/3
c_ν = 0.4
INITIAL
EVOLVED
TIME ELAPSED = 60 UNITS
MEAN TEMPERATURE
DEPTH
0
1
2
3
4
5
6
7
8
9
10
11
12
0.2
0.4
0.6
0.8
1.0



m = 1.0
θ = 10
γ = 1.4
σ_ν = 1/3
c_ν = 0.4
INITIAL
EVOLVED
TIME ELAPSED = 60 UNITS
MEAN PRESSURE
DEPTH
0
10
20
30
40
50
60
70
80
90
100
110
120
130
0
0.2
0.4
0.6
0.8
1.0


m = 1.0
θ = 10
γ = 1.4
σ_ν = 1/3
c_ν = 0.4
INITIAL
EVOLVED
TIME ELAPSED = 60 UNITS
MEAN DENSITY
DEPTH
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
0
0.2
0.4
0.6
0.8
1.0


NEPTUNE
JUPITER
MARS
SUN
MERCURY
ASTEROIDS
EARTH
VENUS
SATURN
URANUS

TEMPERATURE – DISTANCE RELATION

$$\frac{\nu}{2}\,\frac{\mathcal{R}T_e}{\mu} \quad \propto \quad \frac{GM}{R_e}$$

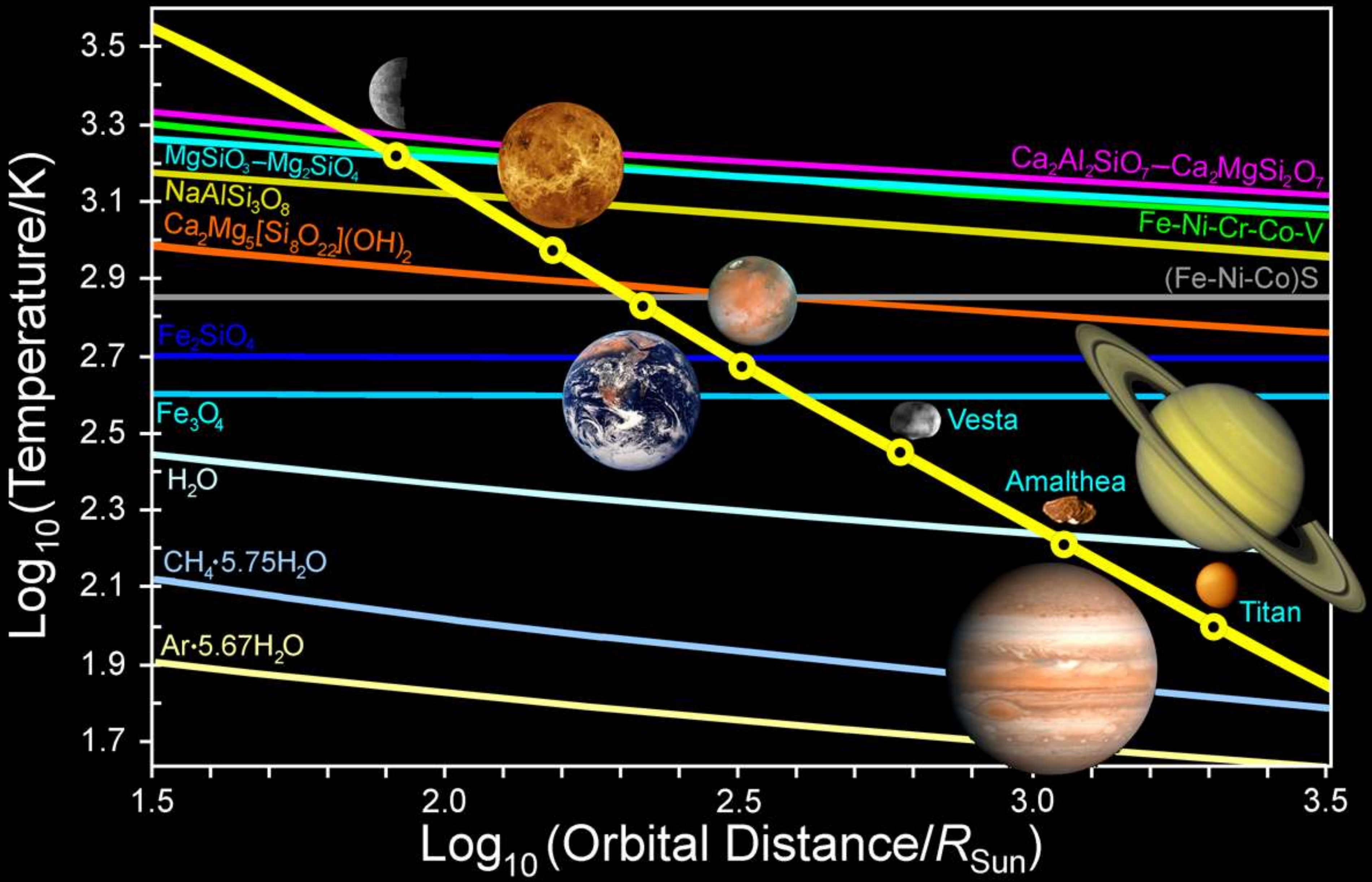
Thermal energy /gram at equator

gravitational potential energy/gram

Hence temperature T_e at equator of cloud varies with equatorial radius R_e as

$$T_e \propto 1/R_e$$

CHEMICAL CONDENSATION SEQUENCE

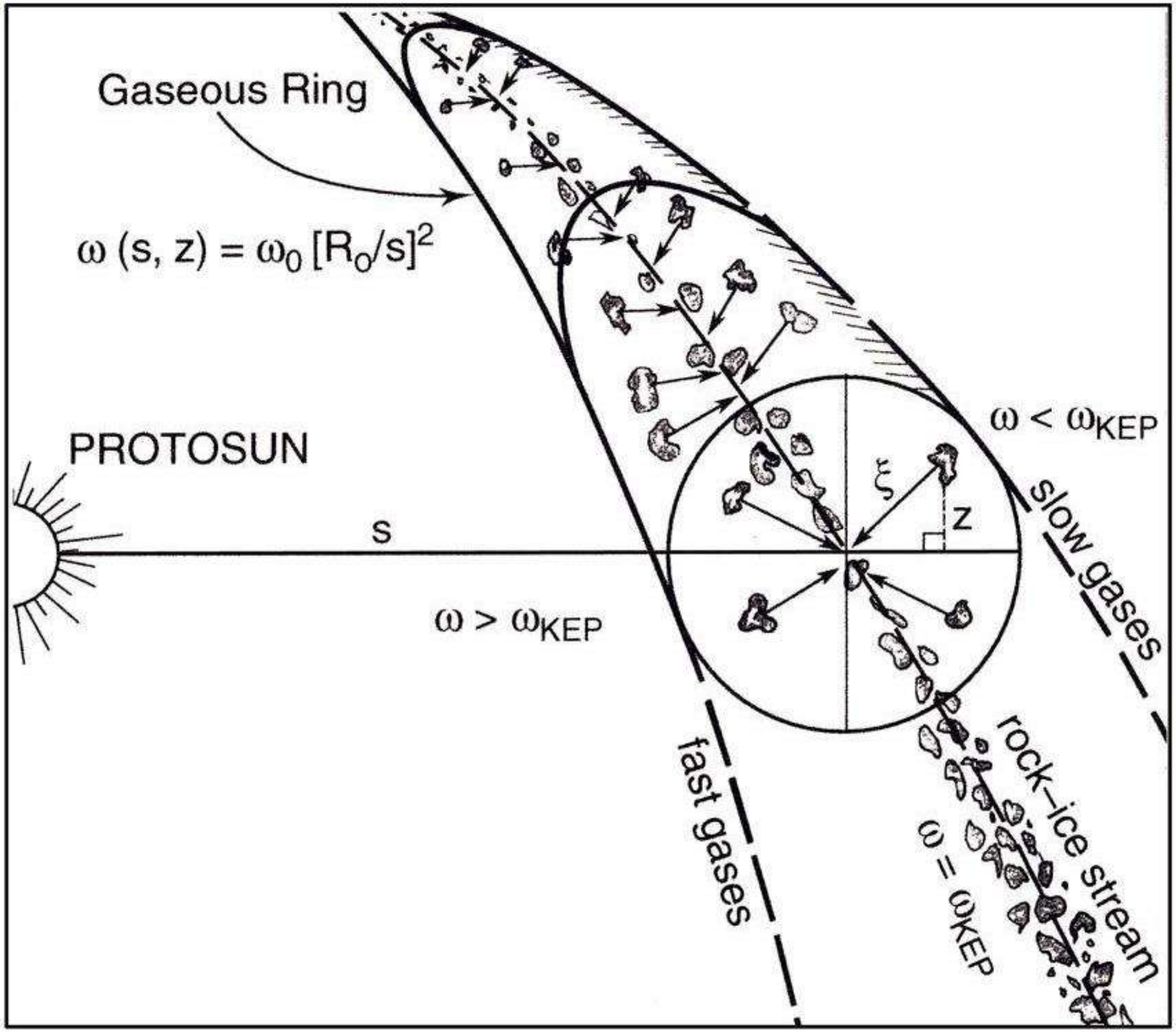


Physical properties of the gas rings

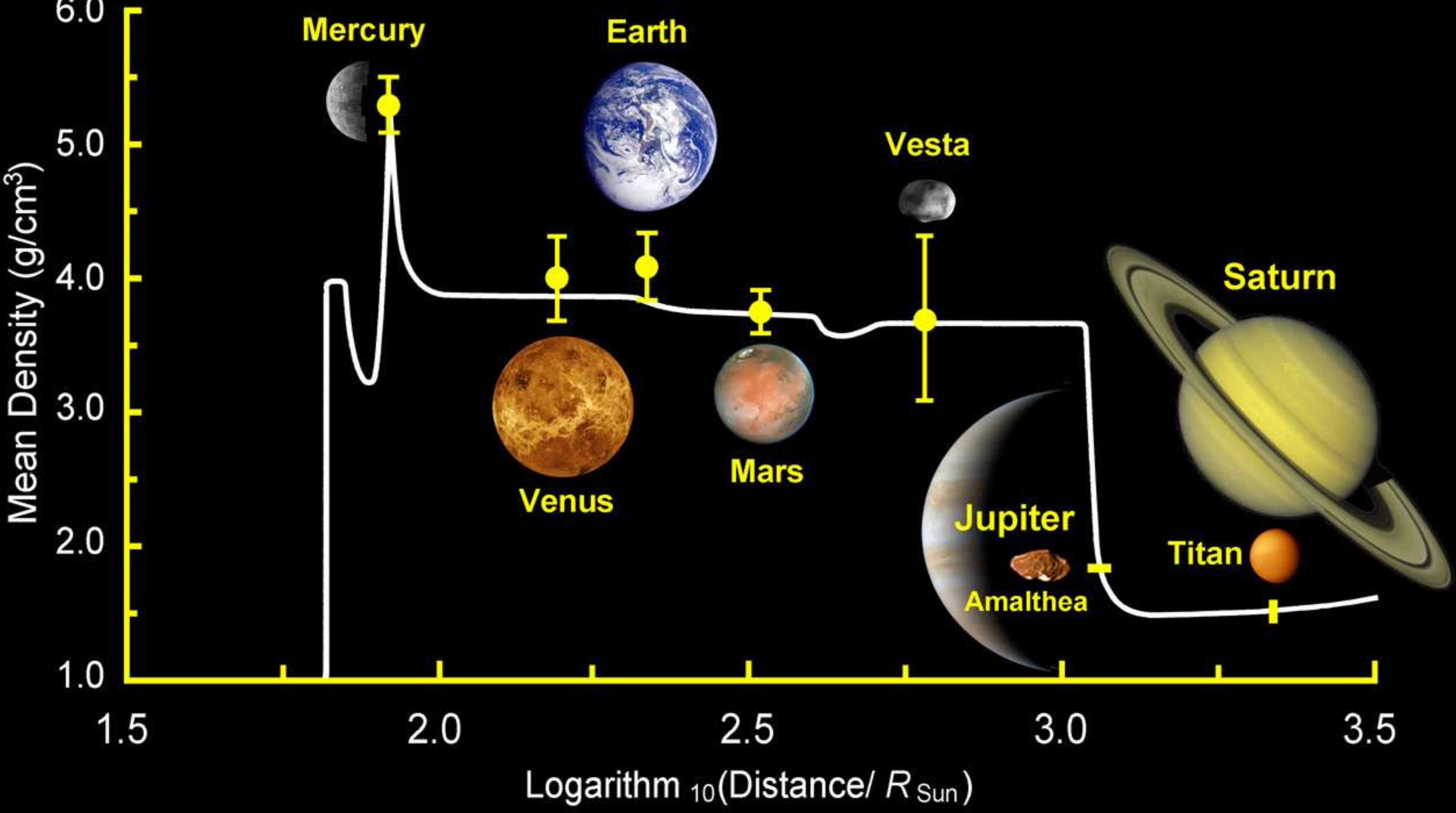
Planet	R_n (R_{Sun})	t_n (10^5 yr)	T_n (K)	p_n (bar)	X_{metal}	$\langle\rho_{\text{cond}}\rangle$ (g cm^{-3})
Mercury	83	3.54	1628	0.168	0.671	5.30
Venus	155	3.46	910	0.0157	0.313	3.85
Earth	215	3.40	673	$4.6\text{x}10^{-3}$	0.230	3.80
Mars	328	3.32	454	9.3×10^{-4}	0.033	3.70
Asteroids	605	3.14	267	7.4×10^{-5}	0.008	3.66
Jupiter	1118	2.90	158	5.9×10^{-6}	0.008	1.67
Saturn	2050	2.55	94	4.9×10^{-7}	0.008	1.52

THE MODERN LAPLACIAN THEORY

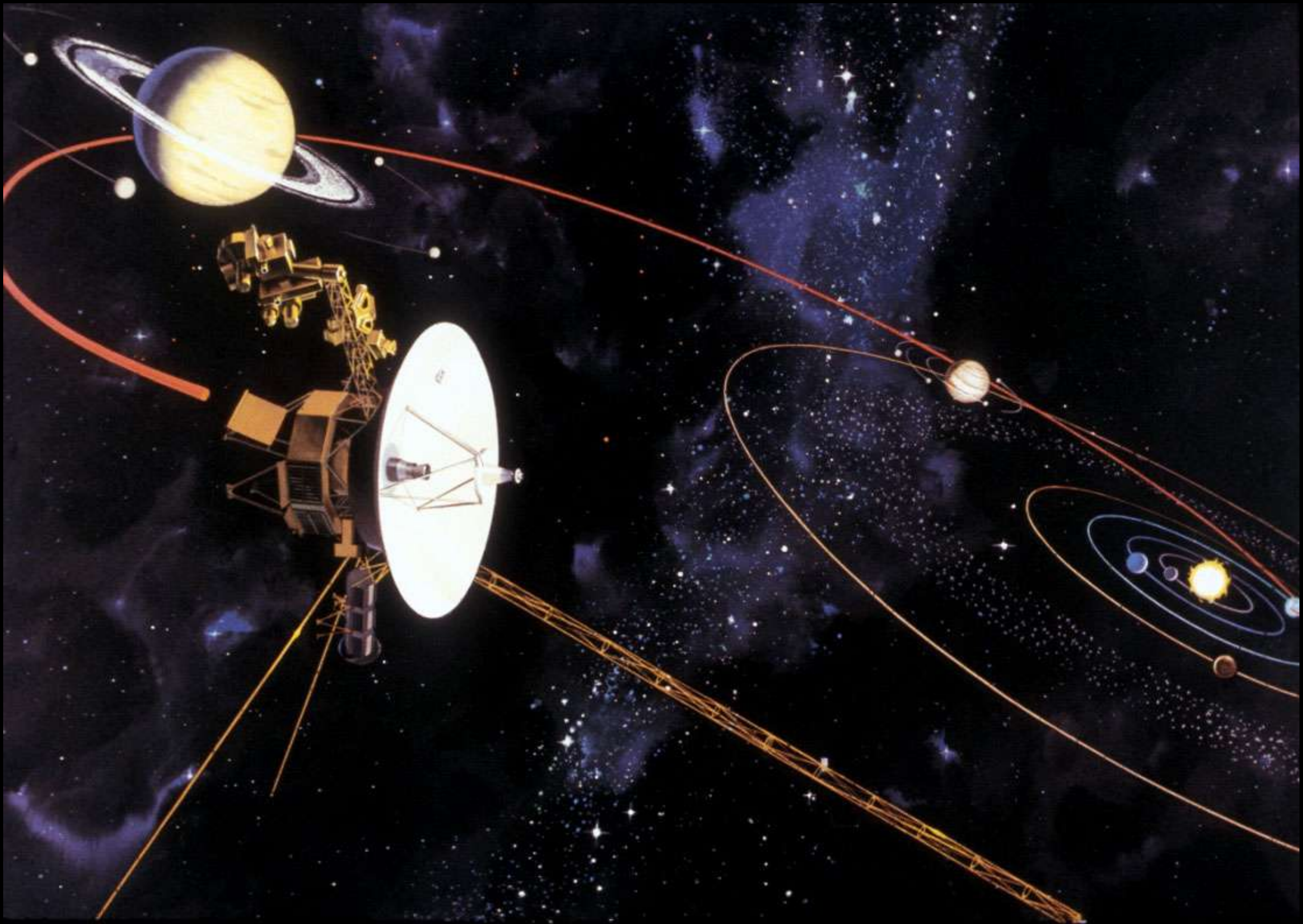
GRAVITATION SETTLING OF THE CONDENSATE GRAINS
ONTO THE MEAN ORBIT OF THE GASEOUS RING

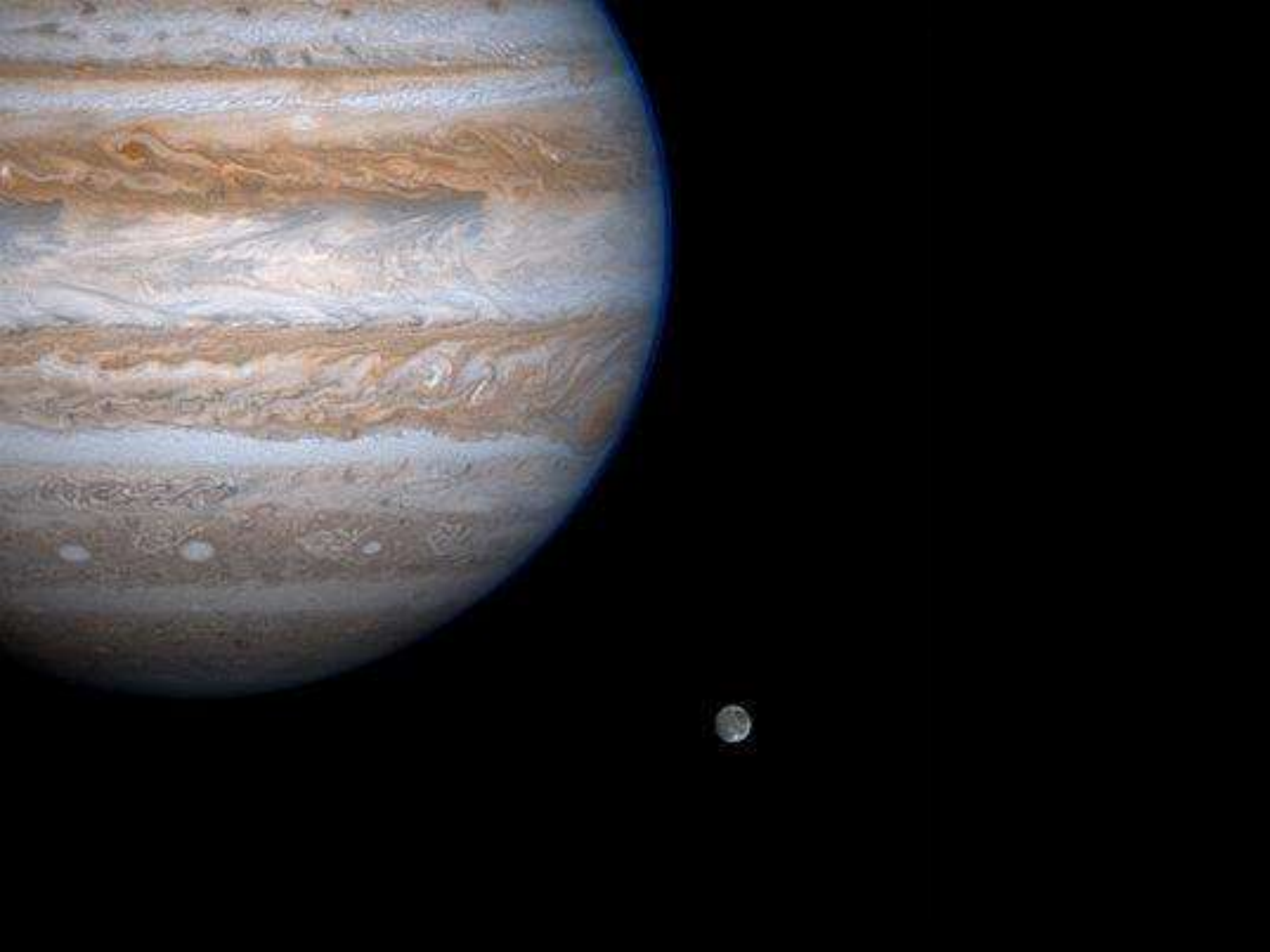


Condensate Mean Density vs Orbital Distance

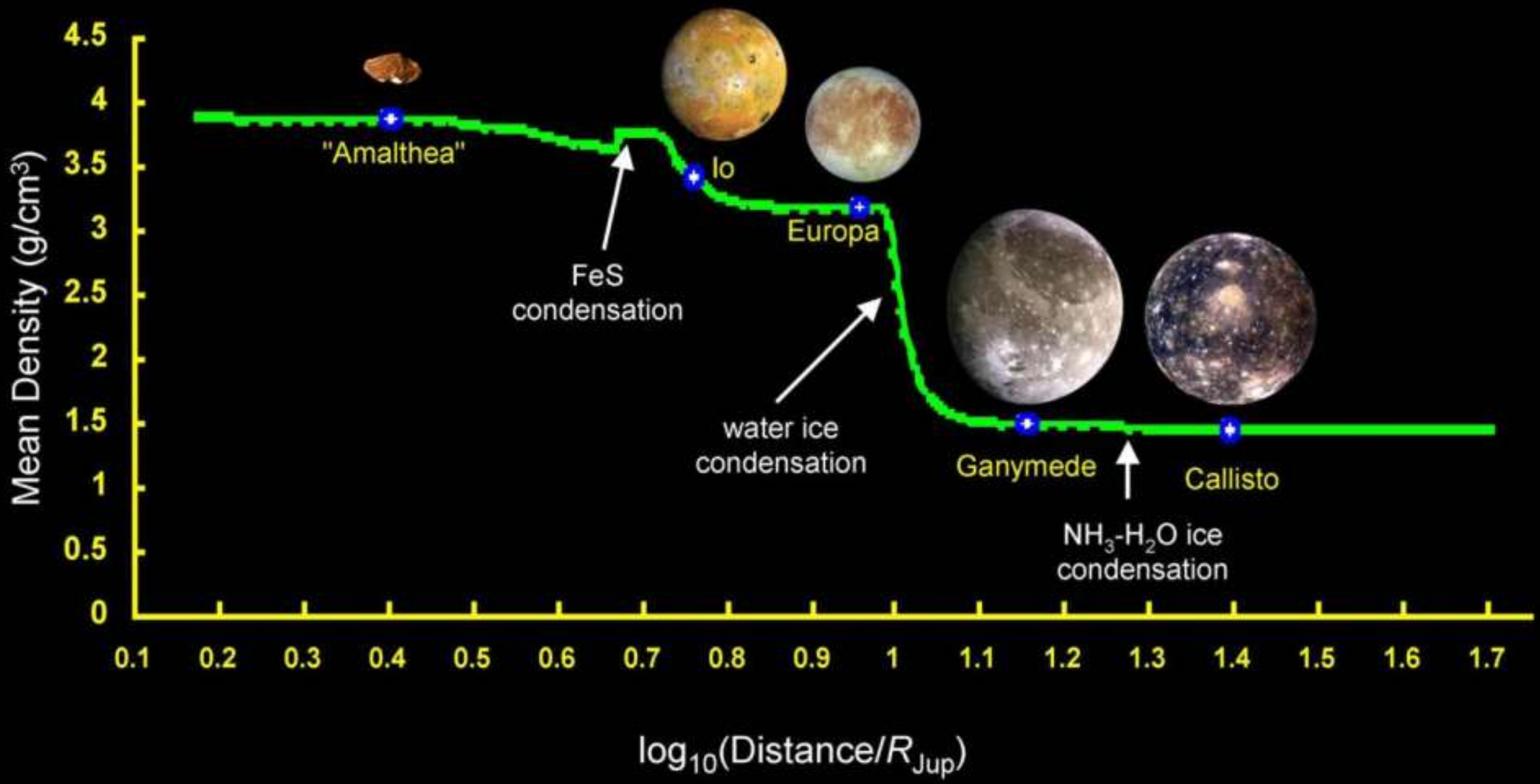


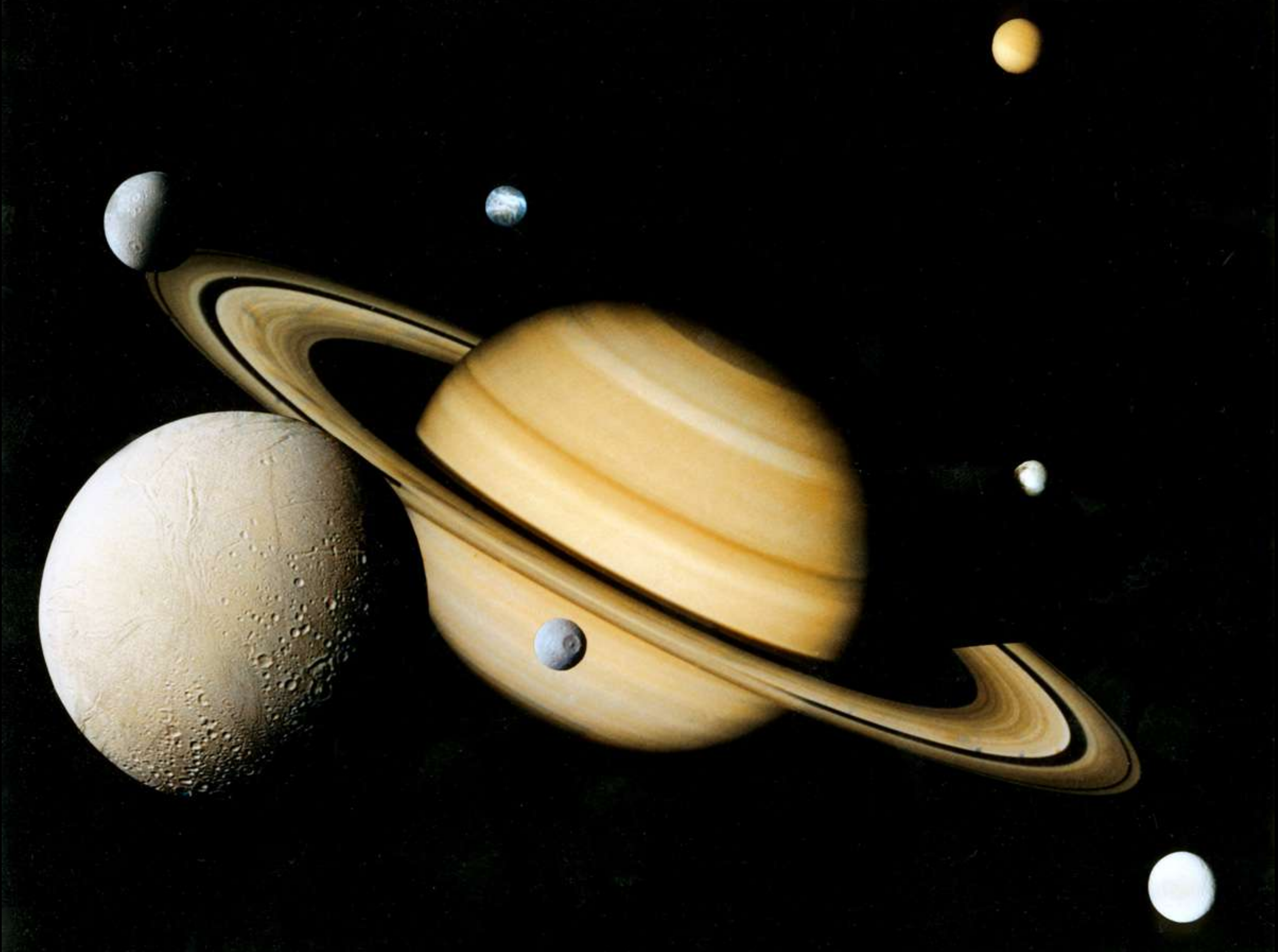
THE MODERN LAPLACIAN THEORY

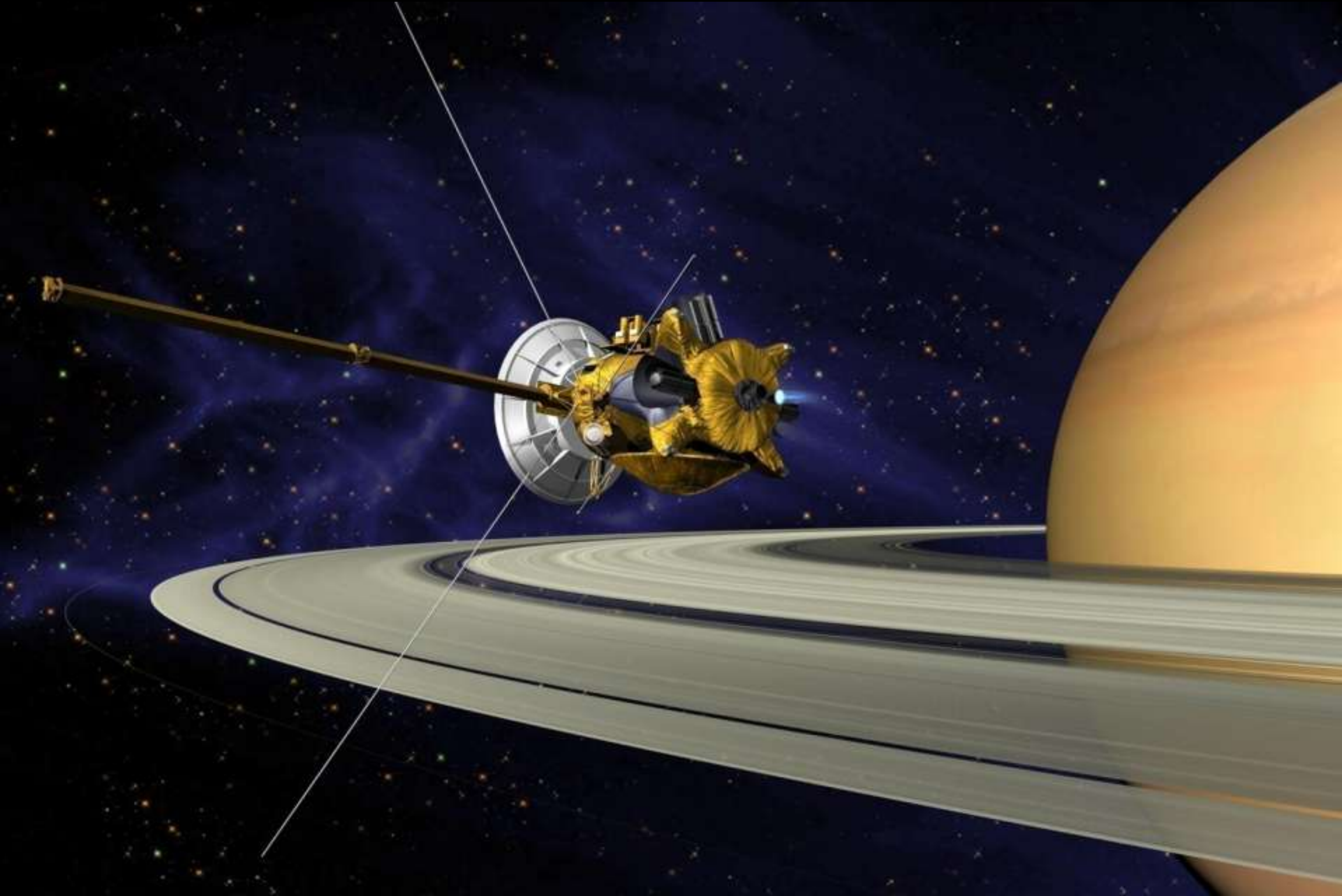




Jupiter System: Condensate Density vs Initial Orbital Radius







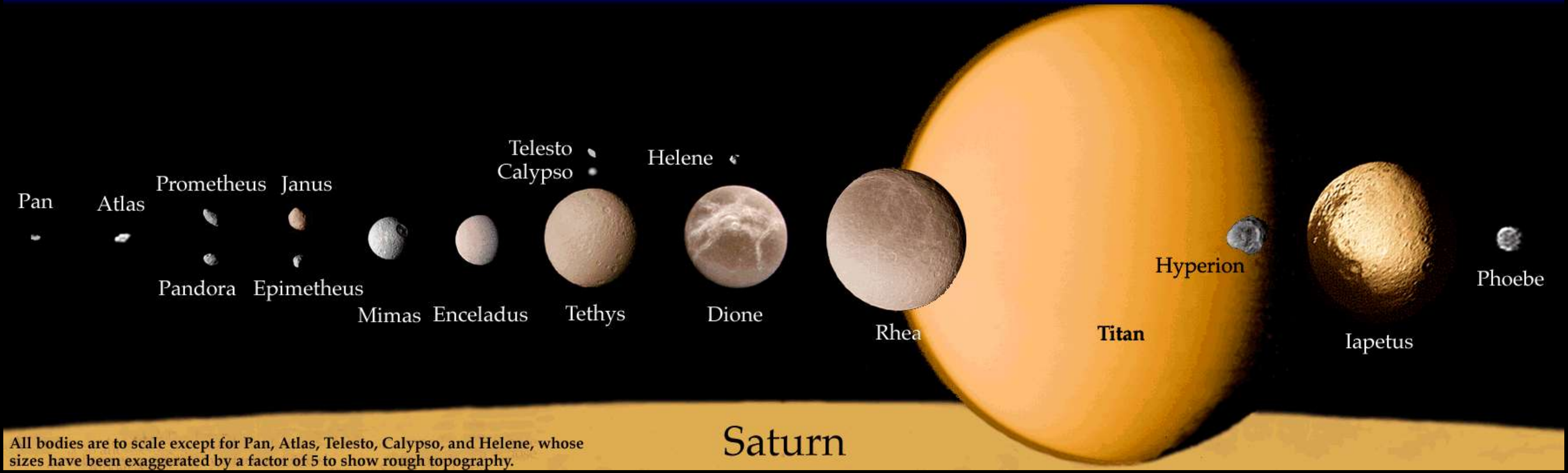
CASSINI

Gio Dom.co Cassini

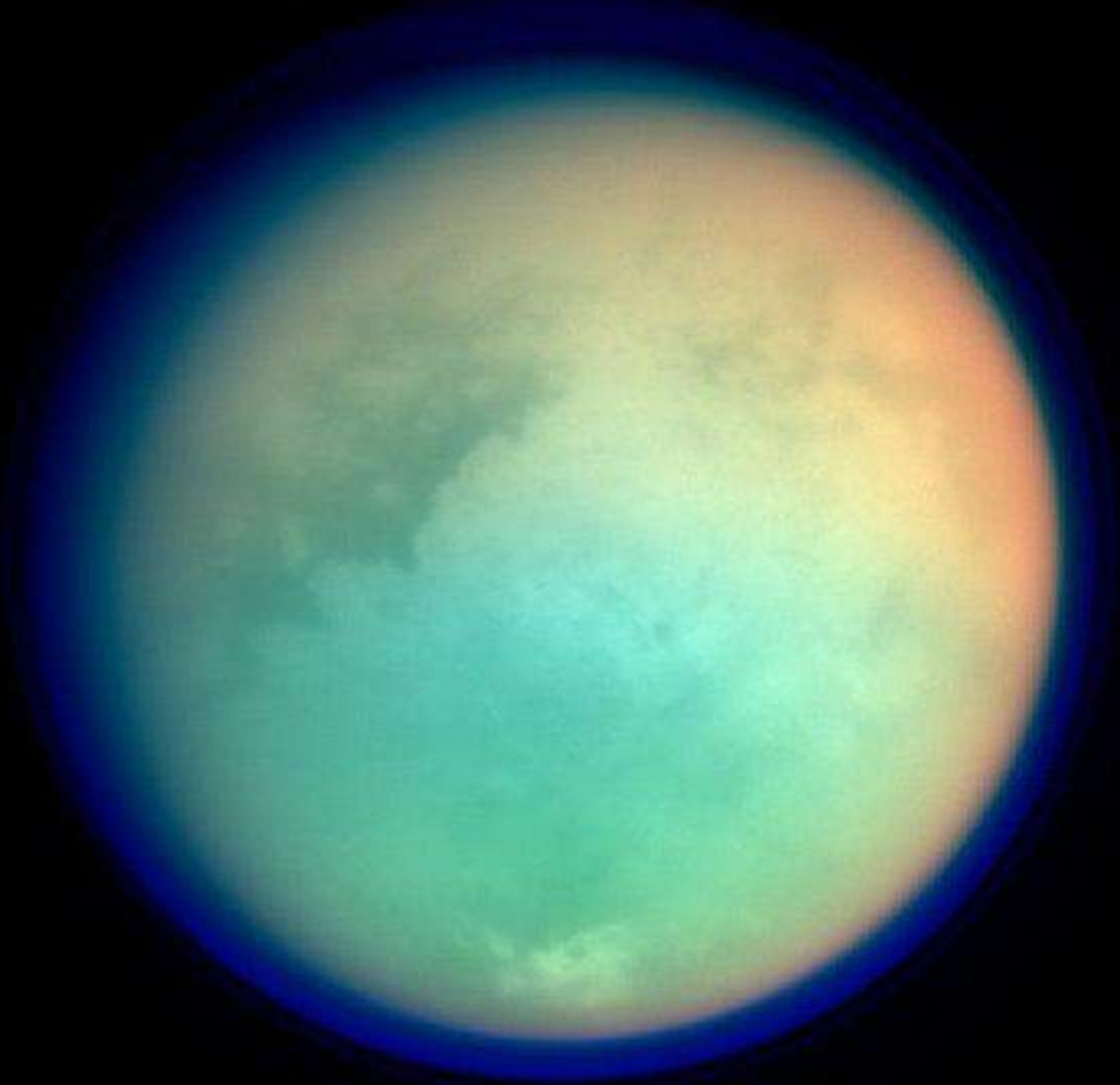
Cartographer,

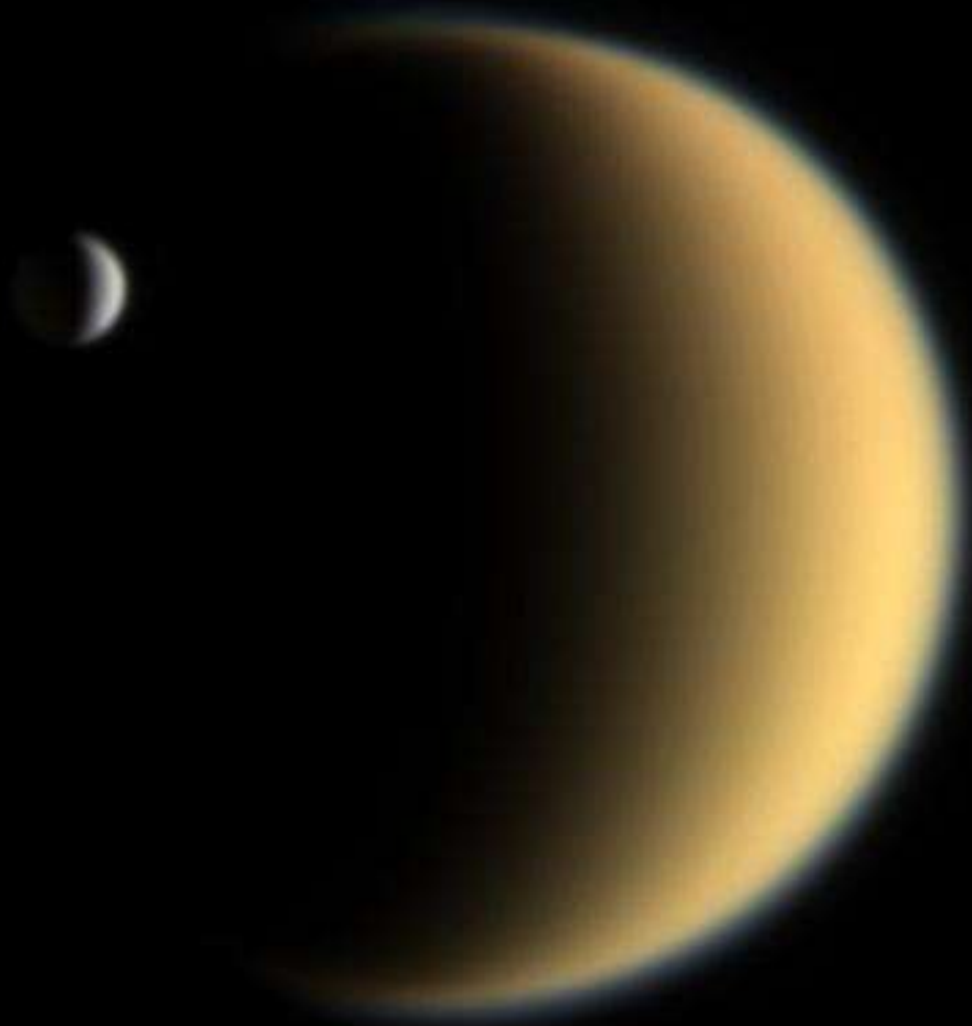

Astronomer HUYGENS

Saturn's Satellites and Ring Structure



All bodies are to scale except for Pan, Atlas, Telesto, Calypso, and Helene, whose sizes have been exaggerated by a factor of 5 to show rough topography.





ORBITAL DISTANCE EQUATION

$$\frac{R_n}{R_{n+1}} = \left[1 + \frac{m_n}{Mf}\right]^2$$

m_n = gaseous ring mass

R_n = orbital radius

M = cloud mass

f = moment of inertia coefficient

Prentice's Planetary Predictions

JUPITER	**Predicted (1977)** Rocky moon belt at 4 planetary radii from Jupiter's centre	**Found (1979)** Rocky ring at 1.8 Jupiter radii
SATURN	**Predicted (May 1981)** Moon Tethys 'some 25% larger than the . . . accepted value'	**Found (August 1981)** Mass of Tethys is 21.4% higher than the Earth-based value
URANUS	**Predicted (1977)** 2 new moons (or moonlet streams), 3.5 and 2.5 planetary radii from Uranus centre	**Found (1986)** New moon (Puck) at 3.4 radii, and family of 9 moonlets centred around 2.5 Uranian radii
NEPTUNE	**Predicted (May 1989)** New family of 4 large, jet-black moons at 5.0, 3.5, 2.5 & 1.8 planetary radii in Neptune's equatorial plane	**Found (1989)** 4 new large moons at 4.7, 3.0, 2.5 & 2.1 planet radii, plus 2 smaller moonlets at 2.01 and 1.94 radii. All very dark & in equatorial plane
TRITON	**Predicted (1989)** Dry ice (frozen CO_2) is the major (20% by mass) carbon-bearing chemical on Triton	**Found (1992)** Infrared measurements show that Triton's 'bedrock' composition is CO_2 ice
JUPITER'S CLOUDS	**Predicted (1995)** Sulphur content of Jovian atmosphere to be twice that of Sun, with ratio of H_2S to H_2 = 74 parts per million	**Found (1996)** H_2S to H_2 concentration = 77 ± 5 parts per million – a dramatic result for the Prentice's Modern Laplacian Theory
	Water vapour deficient relative to H_2S by 34%, ie. atmosphere dry relative to other heavy elements	Atmosphere very dry – 85% deficiency relative to H_2S (ie. different to Prentice's prediction but in right direction)

JUPITER'S MOON, IO	**Predicted (Jan 1996)** Core = molten iron sulphide (FeS) Moment of inertia = 0.390 ± 0.002	**Found (May 1996)** Molten FeS + pure Fe - mostly FeS Moment of inertia = 0.378 ± 0.008
	Prentice just outside error limit. 'This is very close to Galileo's findings,' according to Galileo Gravity Team Leader, Dr John Anderson.	
GANYMEDE	**Predicted (May 1996)** A solid-rock & water ice moon, in which half the rock has now settled to the centre, forming a core of 28% the moon's mass Moment of inertia = 0.354 ± 0.008	Galileo made one close fly-by of on June 27. A second close fly-by is due on September 6, 1996 – results to be released later in the year
CALLISTO	**Predicted (May 1996)** A solid, uniform mixture of rock, water ice and ammonia ice Moment of inertia = 0.384 ± 0.004	Galileo to make a close fly-by of Callisto on November 4 - results to released in 1997
TITAN	**Prediction (1980)** Titan is a captured moon, which condensed at Saturn's distance from the Sun prior to capture. Predictions of make-up and structure will be made before 2004.	Cassini mission to be launched in October 1997; arrival at Saturn in July 2004

PROPERTIES OF SATURN'S MOONS

Moon	Orbital Radius (R_{Sat})	Physical Radius (km)	Density (g/cm^3)
Mimas	3.08	198	1.15
Enceladus	3.95	252	1.61
Tethys	4.89	533	0.97
Dione	6.26	562	1.48
Rhea	8.75	764	1.23
Titan	20.27	2575	1.88
Hyperion	24.29	133	0.57
Iapetus	59.08	736	1.08

TEMPERATURE – DISTANCE RELATION

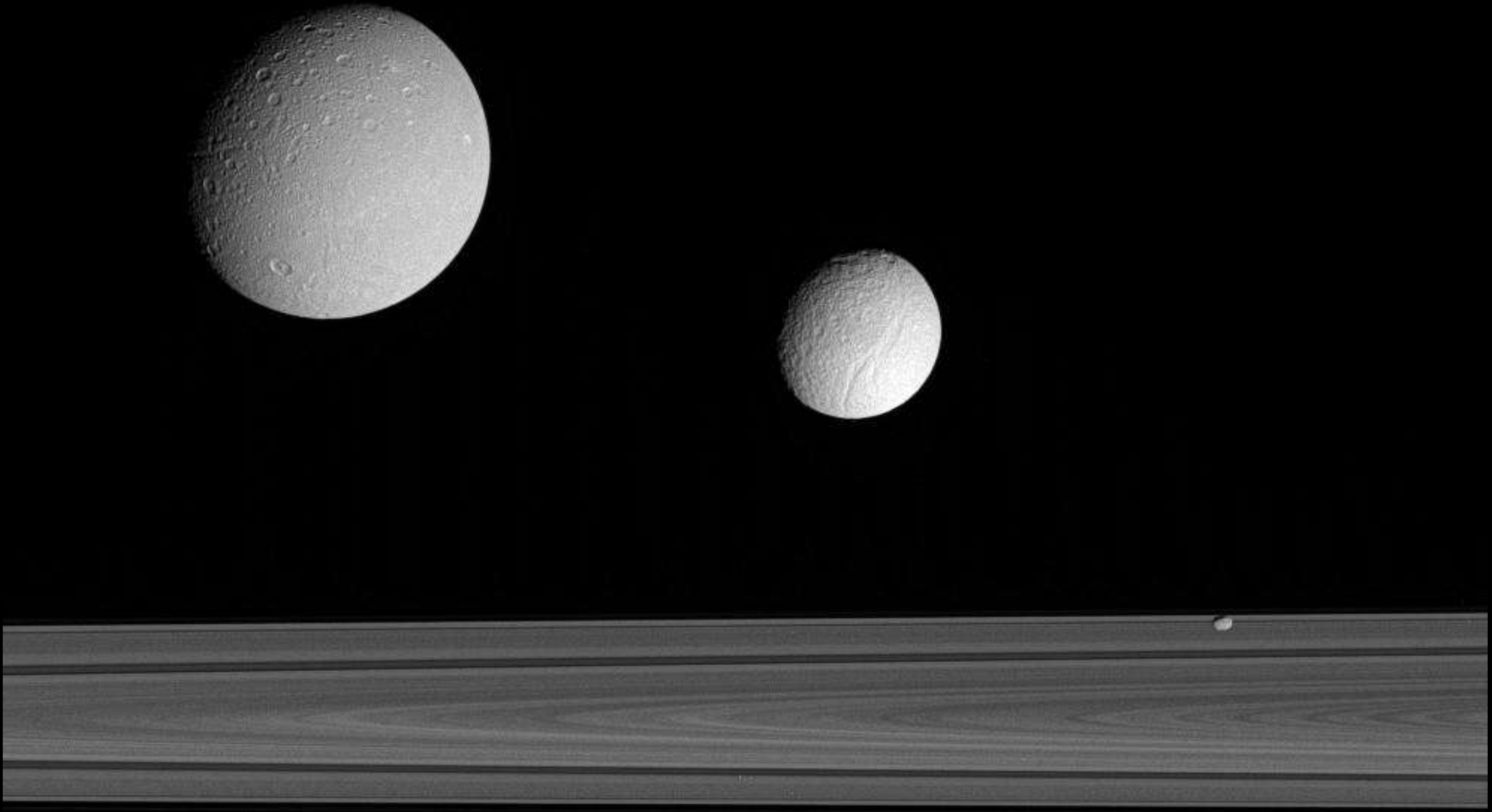
$$\frac{\nu}{2}\,\frac{\mathcal{R}T_e}{\mu} \quad \propto \quad \frac{GM}{R_e}$$

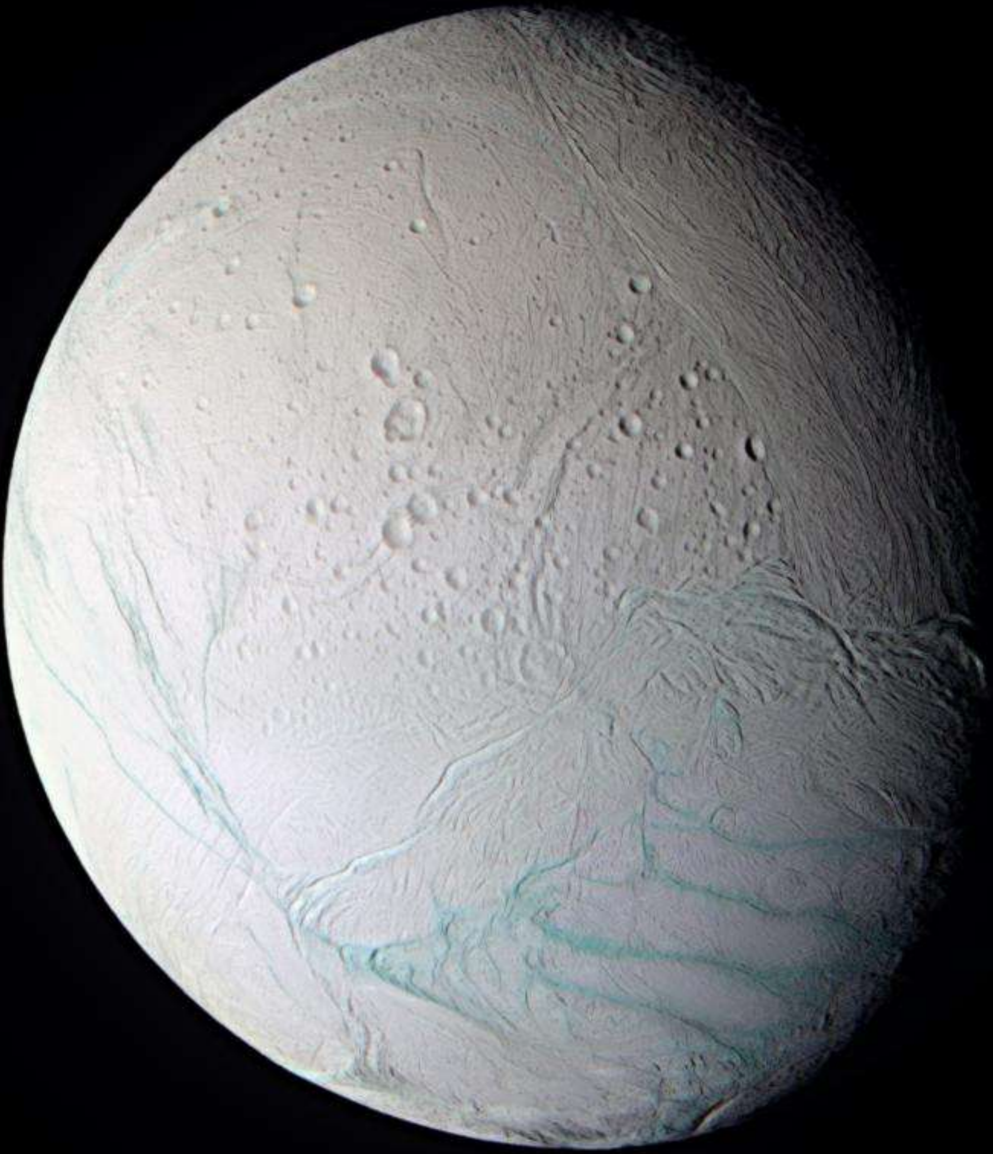
Thermal energy /gram at equator

gravitational potential energy/gram

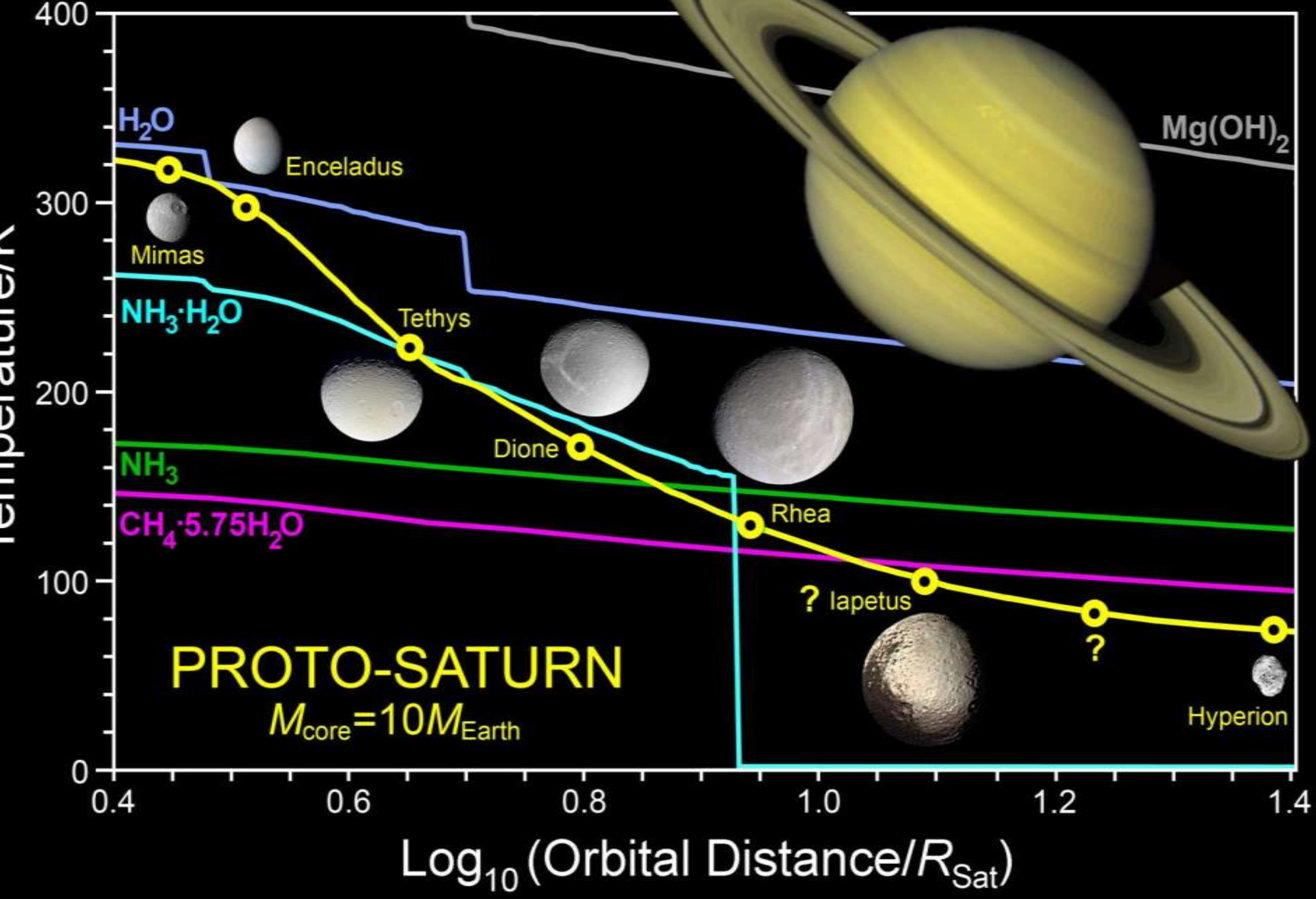
Hence temperature T_e at equator of cloud varies with equatorial radius R_e as

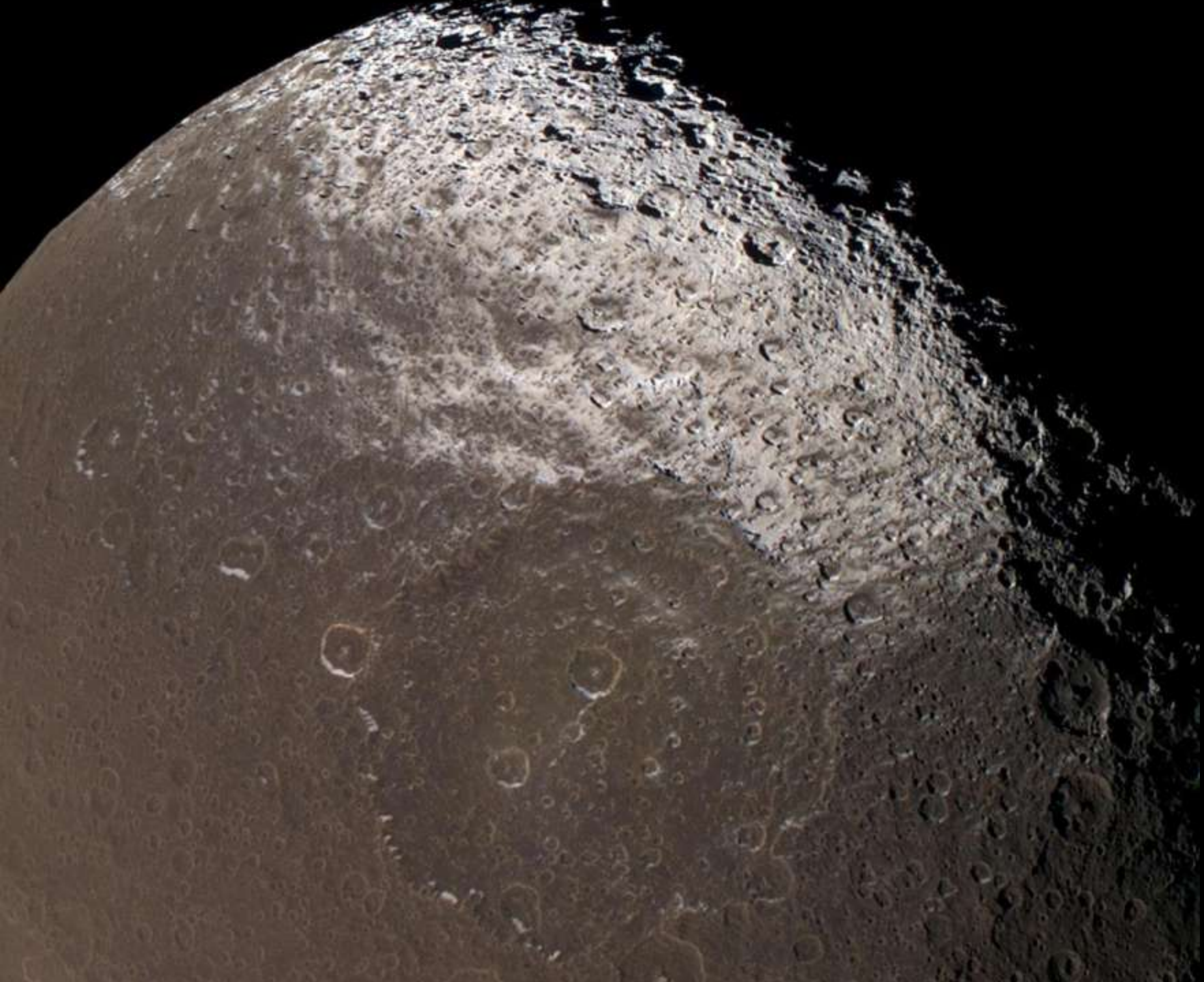
$$T_e \propto 1/R_e$$

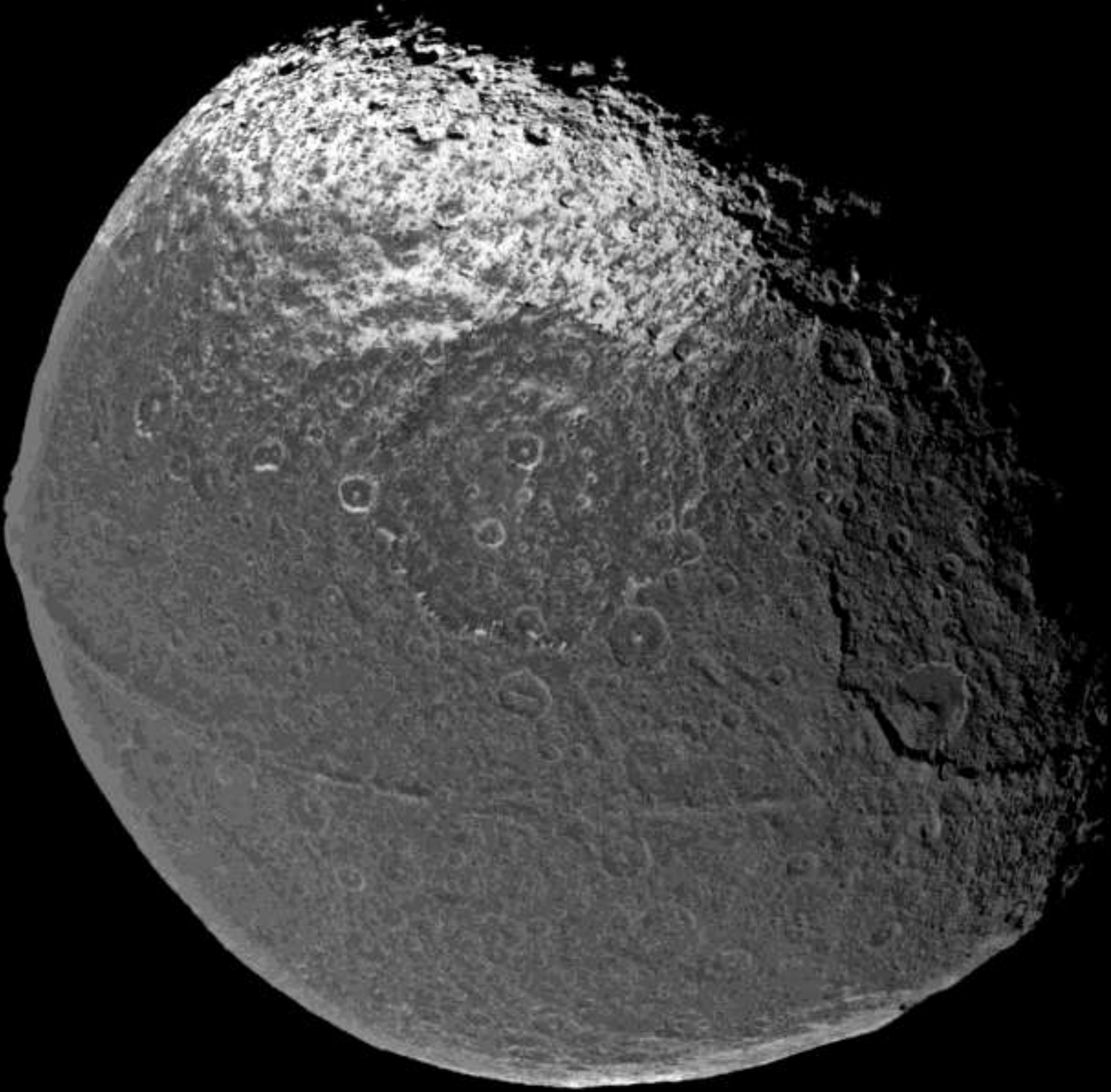




CHEMICAL CONDENSATION SEQUENCE

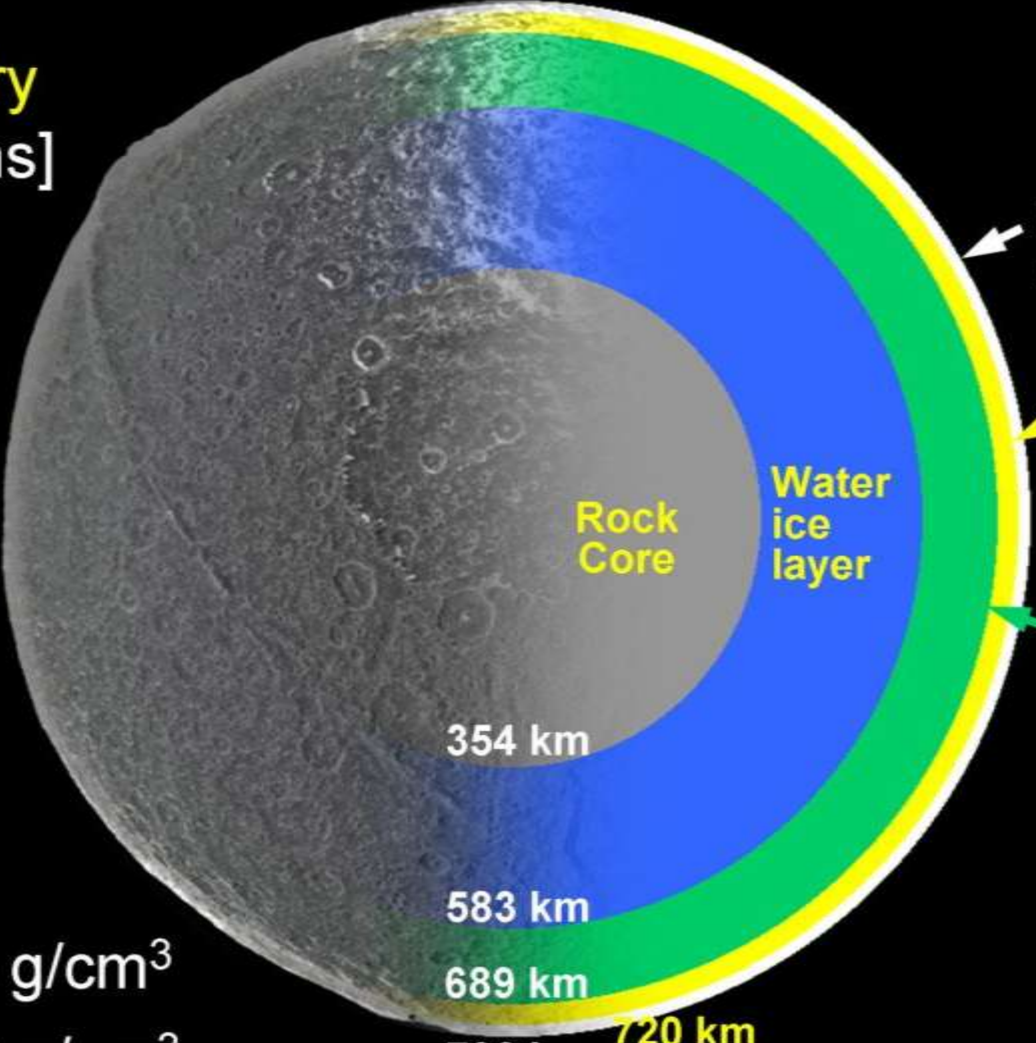






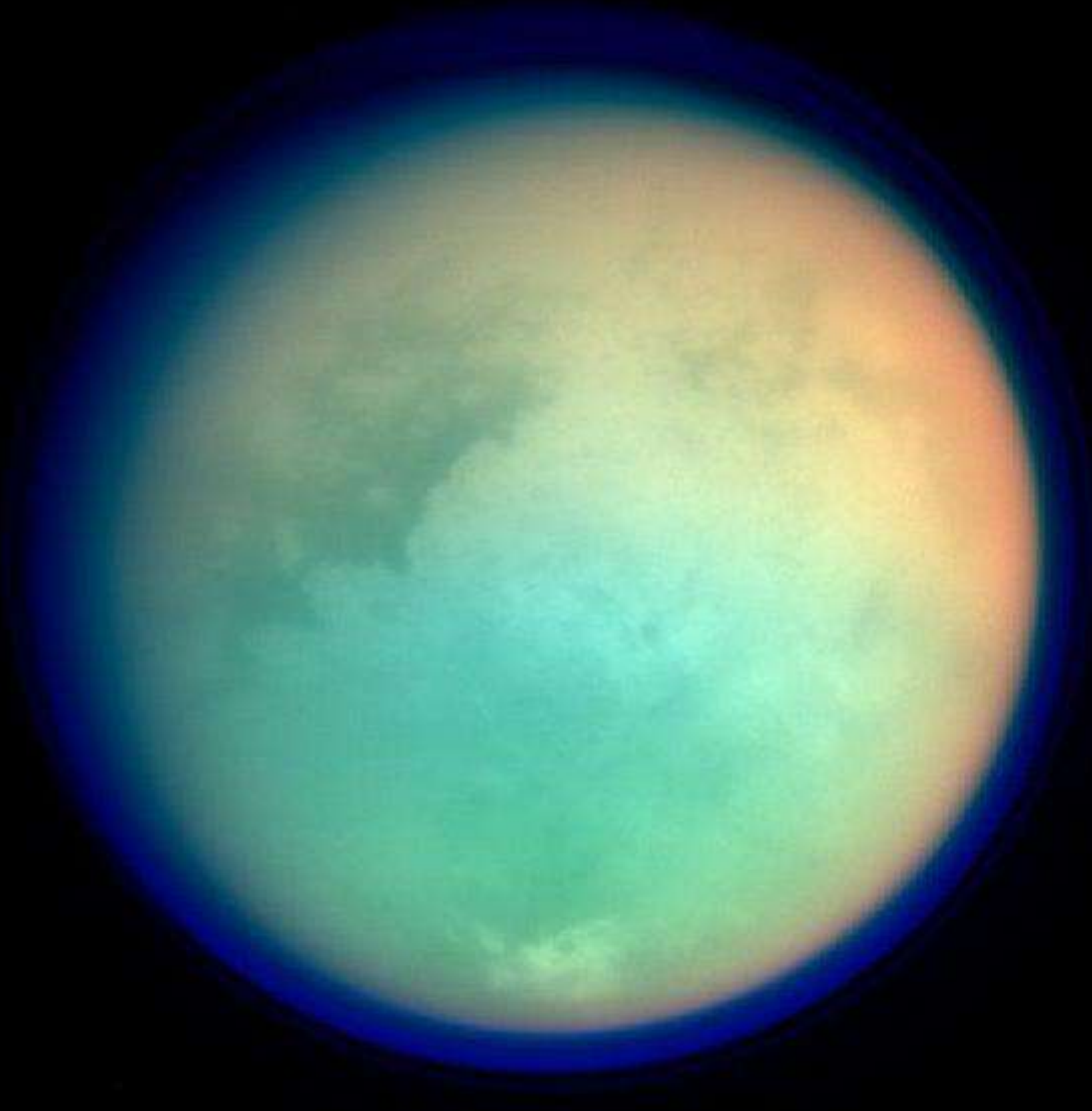
Predicted Iapetus Structure Model

Bulk Chemistry
[mass fractions]
Rock: 0.336
H_2O: 0.344
NH_3: 0.267
CH_4: 0.053



$\bar{\rho}_{model}$ = 1.083 g/cm^3
ρ_{obs} = 1.083 g/cm^3
C/MR^2 = 0.311

T_{core} = 94 K
T_{ice} = 80 K
T_{surf} = 76 K
p_c = 257 Mpa



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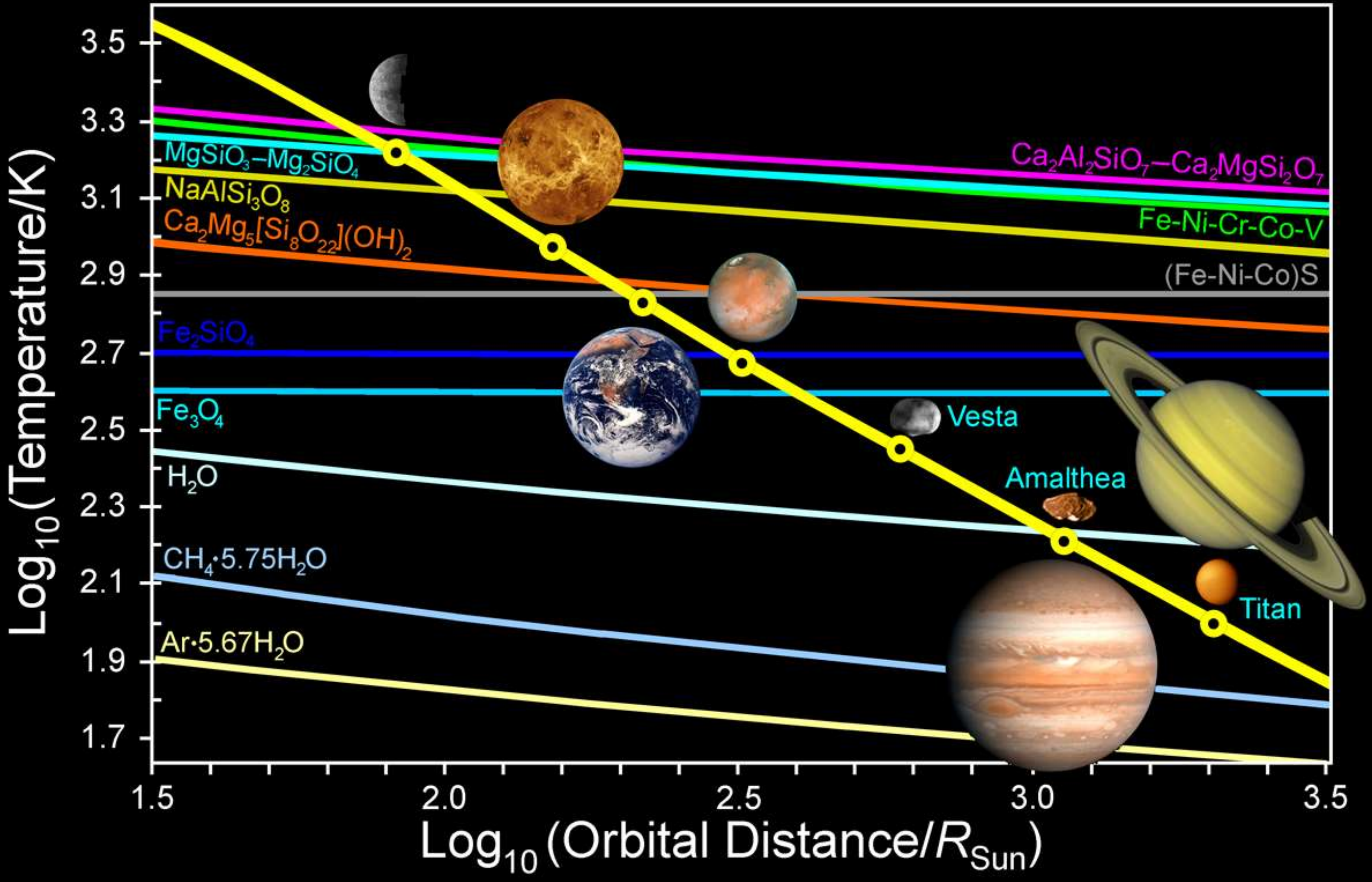
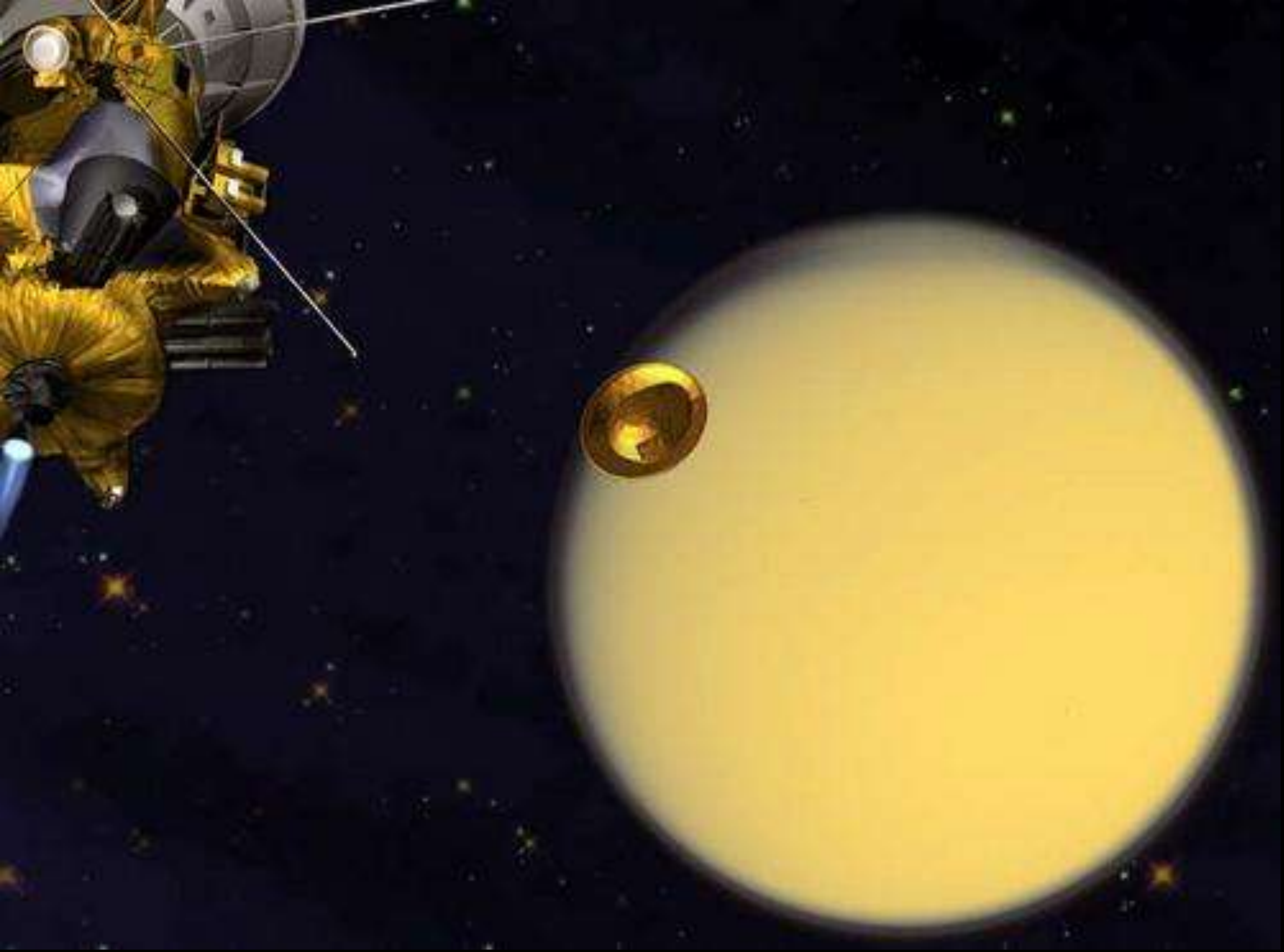




Post-Cassini/Huygens Composition of Titan's Atmosphere

[H.B. Niemann et al., 2005, Nature, **438**, 779]

Gas	Mole abundance fraction
Nitrogen (N_2)	0.95
Argon (Ar)	$(2.8 \pm 0.3) \times 10^{-7}$
Methane (CH_4)	0.0492

Missing Noble Gases Hint How Titan Got Its Dense Atmosphere

While the Huygens probe was discovering the weird yet familiar landscape on Titan (*Science*, 21 January, p. 330), it was failing to make some much-anticipated discoveries. The spacecraft's atmospheric analyzer never did detect the noble gases argon, krypton, or xenon, which cosmochemists expected to find lingering from the formative days of Saturn's lone big moon. "That's rather surprising," says physicist Robert Pepin of the University of Minnesota, Twin Cities, "and a bit of a disappointment."

Researchers had hoped to use the abundance of Titan's noble gases as a guide to how volatile elements essential to life, such as carbon and nitrogen, were divvied up among solar system bodies, including Earth, as the gases hitched a ride with water ice. The absence of detectable primordial noble gases puts a crimp in those plans.

Scientists are reasonably sure that Huygens would have detected primordial noble gases if they were there in the anticipated amounts. Huygens science team member Tobias Owen of the University of Hawaii, Manoa, noted at last week's press conference in Paris that the probe's gas chromatograph/mass spectrometer had detected argon-40, produced by radioactive decay of potassium-40 in the moon's rock. But there is as yet no sign of argon-38 or argon-36. This means the argon-to-nitrogen ratio must be on the order of 1000 times lower on Titan than on Earth, says Owen.

No show. Huygens's failure to detect certain noble gases suggests that a moon gets an atmosphere only if it forms at a low enough temperature.

The new upper limits for titanian primordial noble gases may be frustrating, but they at least point toward an explanation for Titan's uniquely massive nitrogen atmosphere. Its surface pressure is 1.5 times that of Earth, whereas Jupiter's large moons Ganymede and Callisto have no atmospheres to speak of. This, despite their being as massive as Titan—and therefore capable of gravitationally retaining an atmosphere—and just as ice-rich, suggests that all three moons would have started with similar allotments of ice-borne gases.

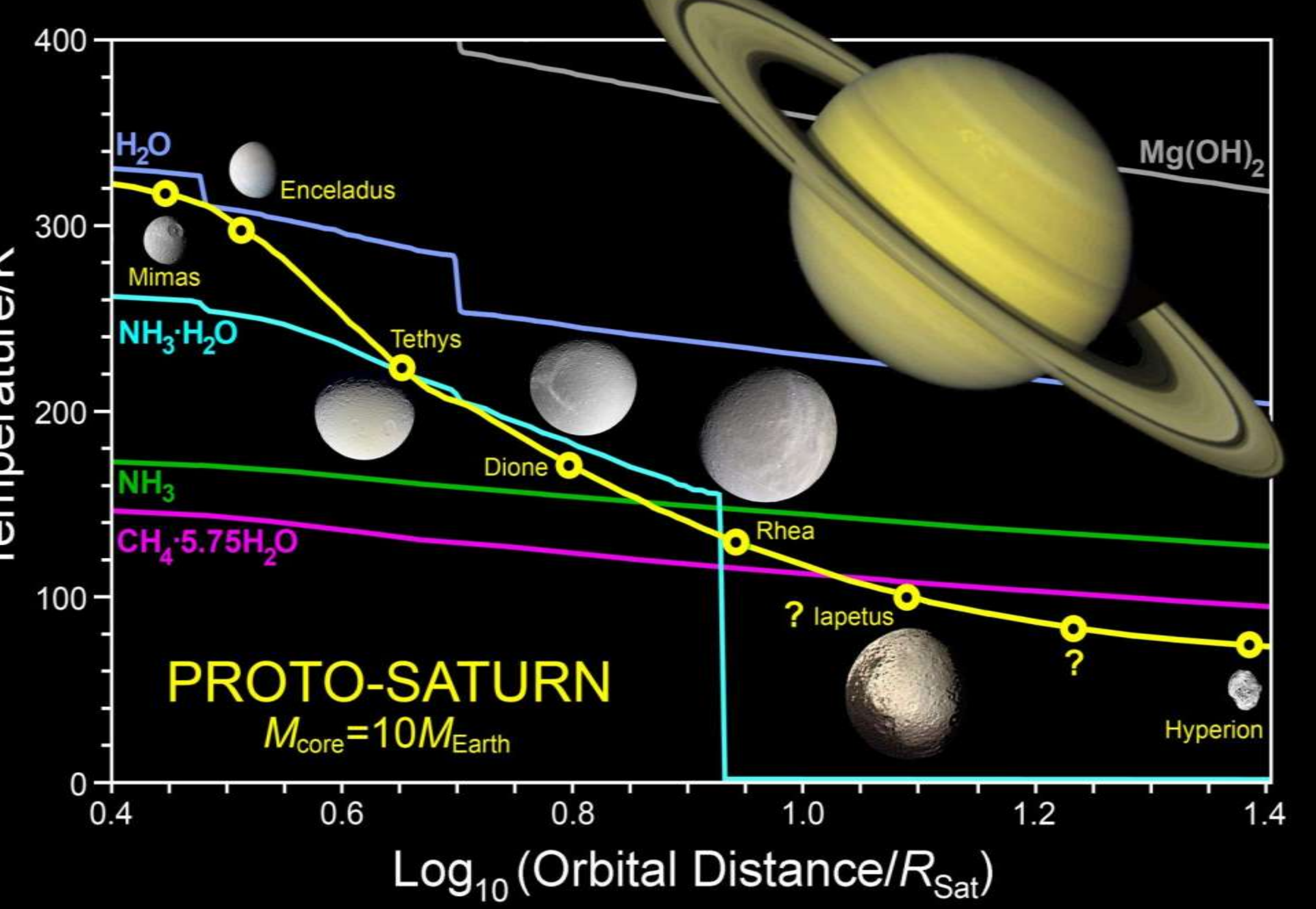
Titan's dearth of noble gases suggests that the nascent Saturn system was too warm for its ices to retain the notoriously inert noble gases through adsorption or trapping within their crystalline structure, says Owen. Laboratory experiments indicate they would not be retained above 50 K, he says. But it was evidently cold enough to retain nitrogen. The Jupiter system, being little more than half Saturn's distance from the sun, was warmer still—perhaps warm enough for the ice that formed its big moons to lose not only noble gases but also the ammonia that forms a nitrogen atmosphere. So, for even a big, icy moon to have a massive atmosphere, it had best keep its distance from the sun.

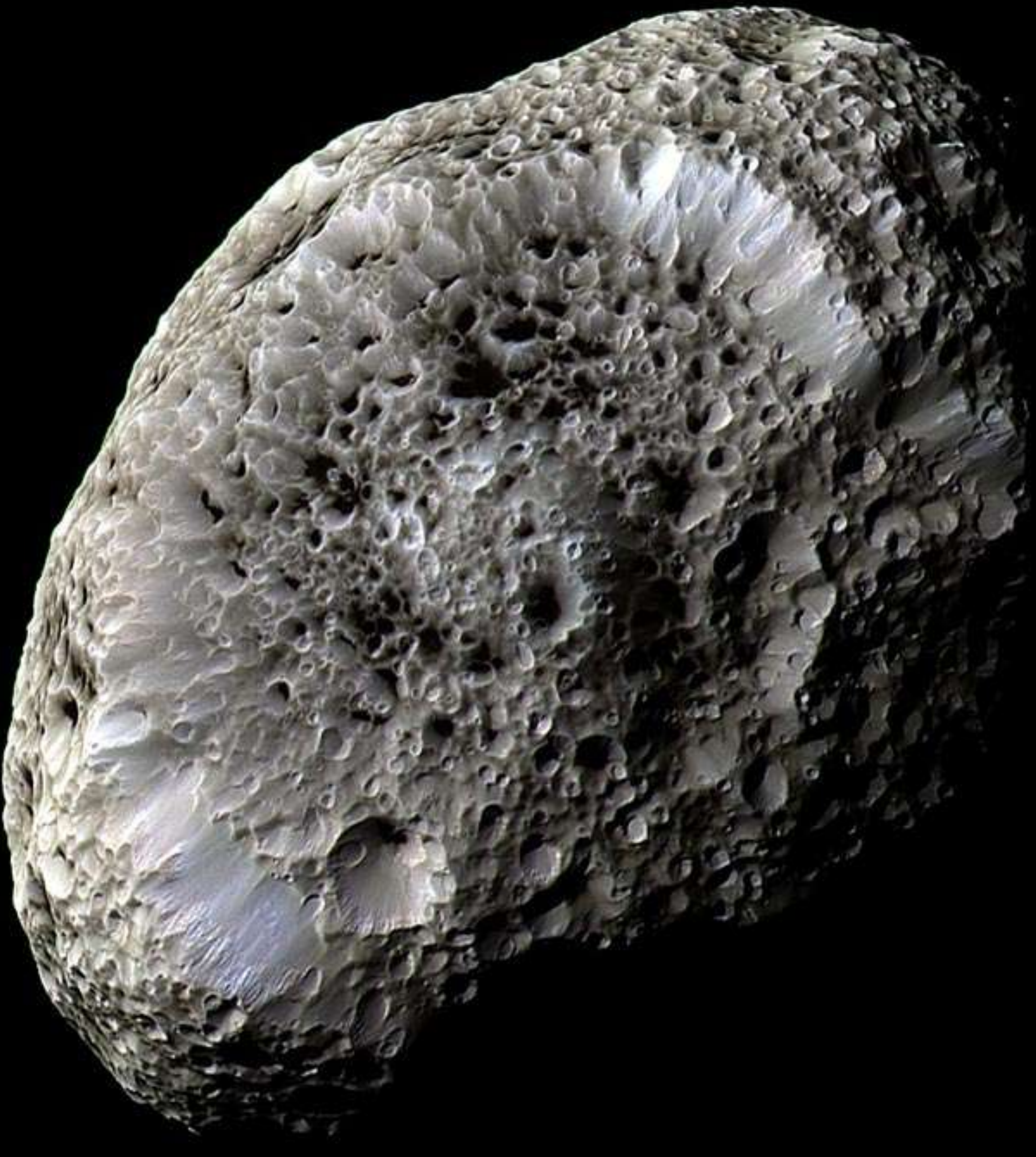
–RICHARD A. KERR

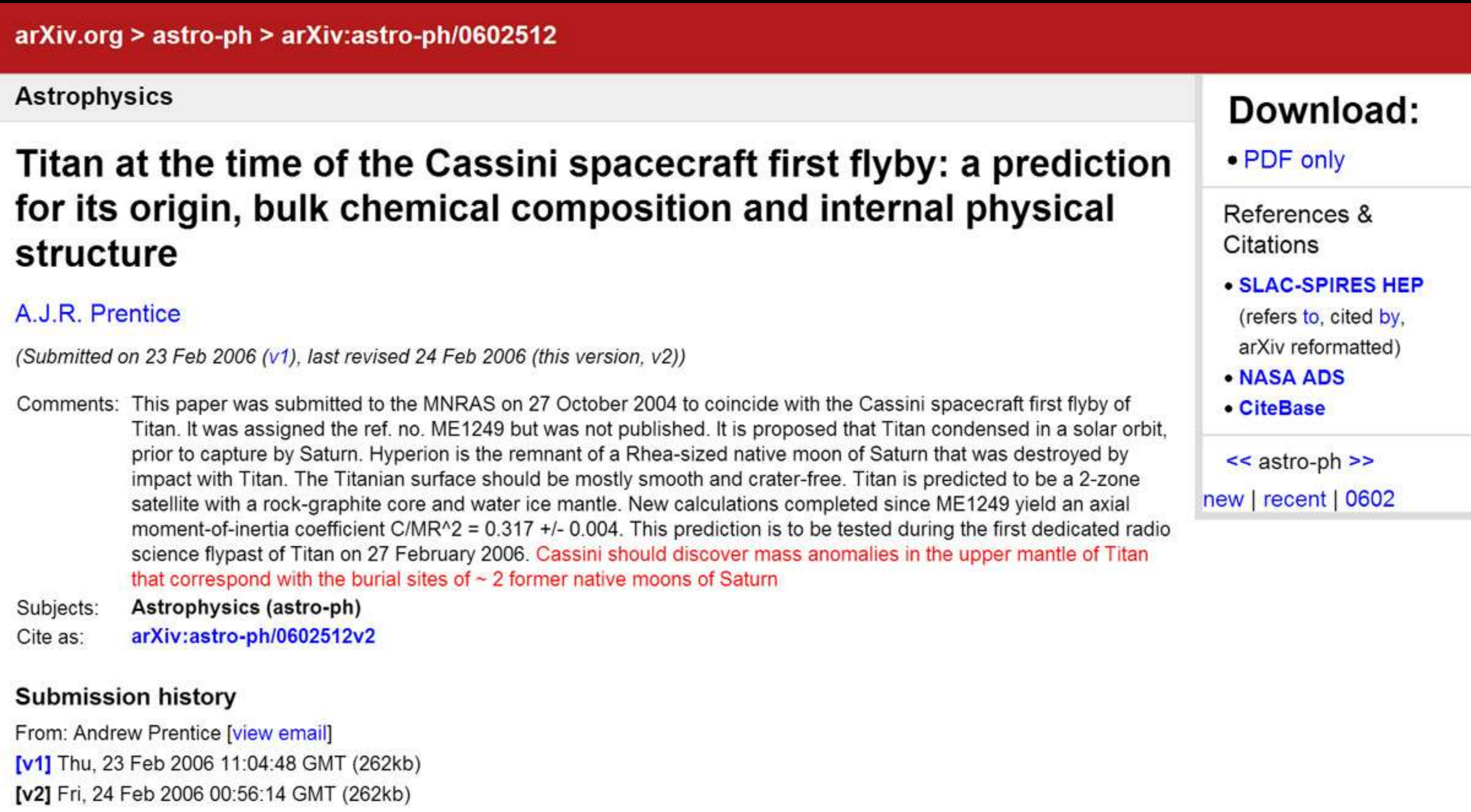
CREDIT: ESA

Published by AAAS

CHEMICAL CONDENSATION SEQUENCE







Astrophysics

Titan at the time of the Cassini spacecraft first flyby: a prediction for its origin, bulk chemical composition and internal physical structure

A.J.R. Prentice

(Submitted on 23 Feb 2006 (v1), last revised 24 Feb 2006 (this version, v2))

Comments: This paper was submitted to the MNRAS on 27 October 2004 to coincide with the Cassini spacecraft first flyby of Titan. It was assigned the ref. no. ME1249 but was not published. It is proposed that Titan condensed in a solar orbit, prior to capture by Saturn. Hyperion is the remnant of a Rhea-sized native moon of Saturn that was destroyed by impact with Titan. The Titanian surface should be mostly smooth and crater-free. Titan is predicted to be a 2-zone satellite with a rock-graphite core and water ice mantle. New calculations completed since ME1249 yield an axial moment-of-inertia coefficient C/MR^2 = 0.317 +/- 0.004. This prediction is to be tested during the first dedicated radio science flypast of Titan on 27 February 2006. Cassini should discover mass anomalies in the upper mantle of Titan that correspond with the burial sites of ~ 2 former native moons of Saturn

Subjects: **Astrophysics (astro-ph)**

Cite as: **arXiv:astro-ph/0602512v2**

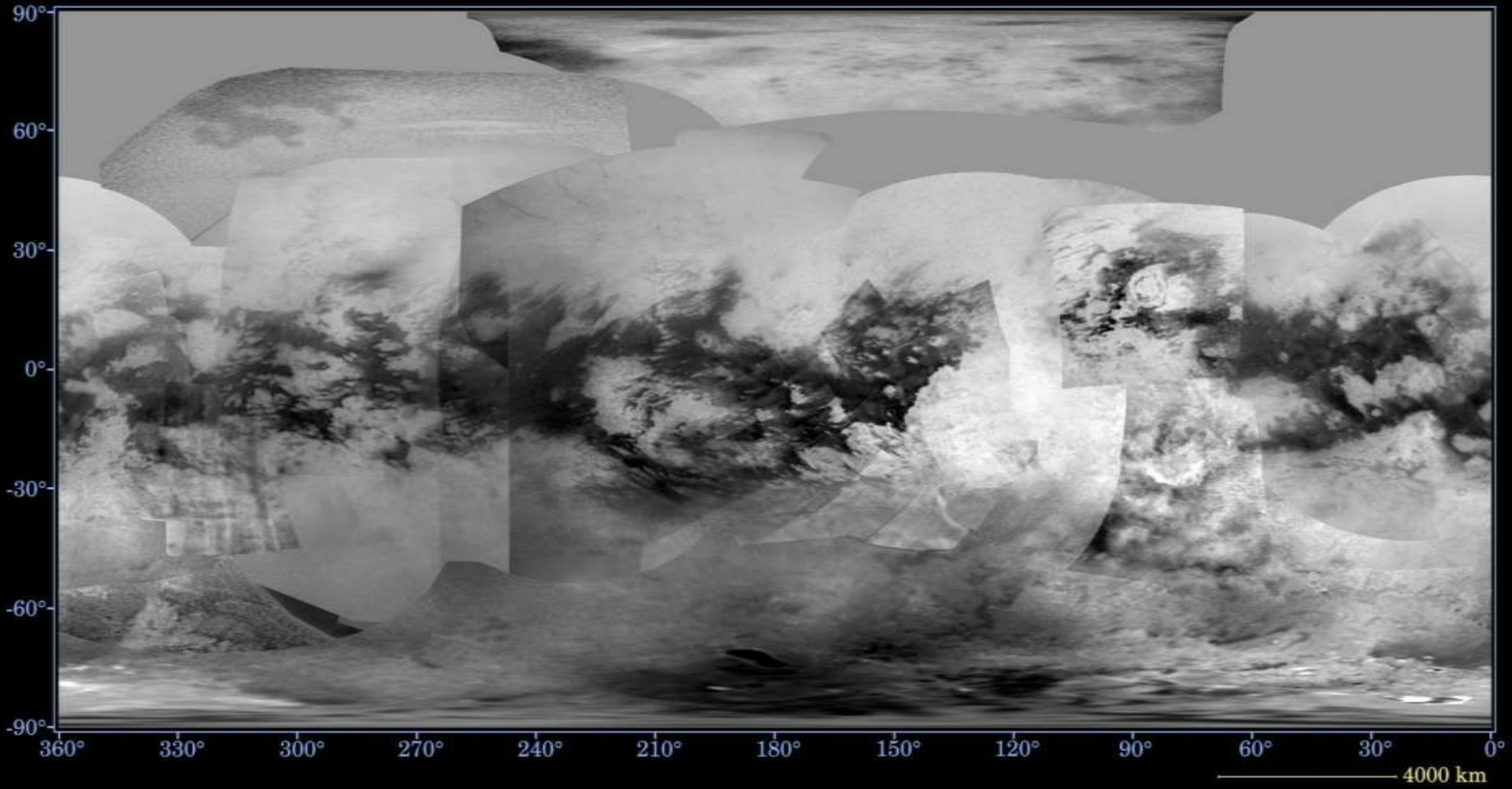
Submission history

From: Andrew Prentice [view email]

[v1] Thu, 23 Feb 2006 11:04:48 GMT (262kb)

[v2] Fri, 24 Feb 2006 00:56:14 GMT (262kb)

Map of Saturn's Moon Titan - February 2009



Title: Titan and Enceladus: Numerical Models for Internal Structure, Bulk Chemical Composition and Origin
Authors: Prentice, Andrew
Affiliation: AA(Monash University, Australia)
Publication: American Astronomical Society, DPS meeting #38, #56.12; Bulletin of the American Astronomical Society, Vol. 38, p.587
Publication Date: 09/2006
Origin: AAS
Abstract Copyright: (c) 2006: American Astronomical Society
Bibliographic Code: 2006DPS....38.5612P

Abstract

Titan is too massive to be a native moon of Saturn. It was captured from solar orbit (Prentice 2004 Earth Moon Planets 30 209; http://arxiv.org/abs/astro-ph/0602512). Rhea and the other mid-sized Saturnian moons have masses that are consistent with the values expected for condensation within a family of gas rings that were shed by the proto-Saturnian cloud during its gravitational contraction from an initial size $30R_{Sat}$ (Prentice 2006 PASA 23 1). Here R_{Sat} = 60268 km. Titan destroyed 2 Rhea-sized native moons that existed at orbital radii $17R_{Sat}$ and $24R_{Sat}$. Much of those moons, which contain 24% by mass of NH_3 ice and 5.7% of CH_4 as clathrate hydrate, are now buried in Titan's mantle. This is the source of Titan's atmosphere. Maybe Xanadu is the burial site of one such moon.

Titan 22 Flyby

Dec. 28, 2006

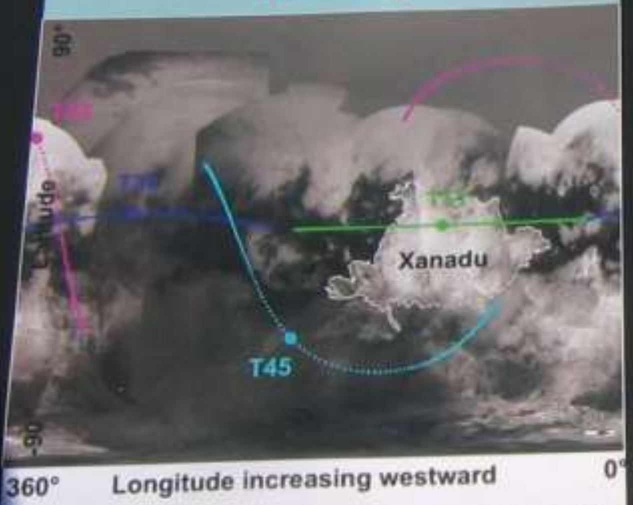
Cassini's Search for Subsurface Oceans

The Gravity Field of Titan from Four Cassini Flybys

N.J. Rappaport[1], R.A. Jacobson[1*], L. Iess[2], P. Racioppa[2], J.W. Armstrong[1], S. W. Asmar[1],
D.J. Stevenson[3], P. Tortora[4], M. Di Benedetto[2], A. Graziani[4], R. Meriggiola[2]

[1]Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, USA *NAV
[2]Università La Sapienza, Rome, Italy
[3]California Institute of Technology, Pasadena, California, USA
[4]Università di Bologna, Forlì, Italy

Geometry

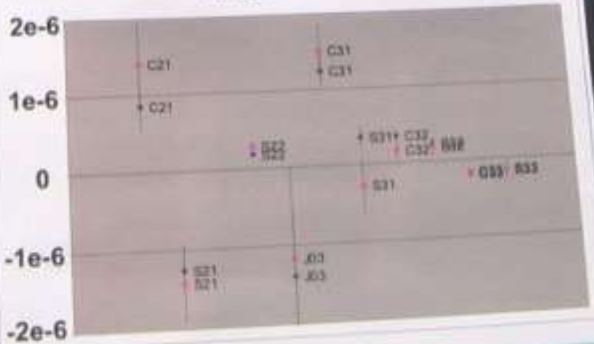


	Date	Altitude	SEP	M_{Titan}
T11	Feb. 27, 2006	1812 km	147°	173°
T22	Dec. 28, 2006	1297 km	132°	197°
T33	Jun. 29, 2007	1960 km	45°	15°
T45	Jul. 31, 2008	1591 km	29°	-36°

Methods

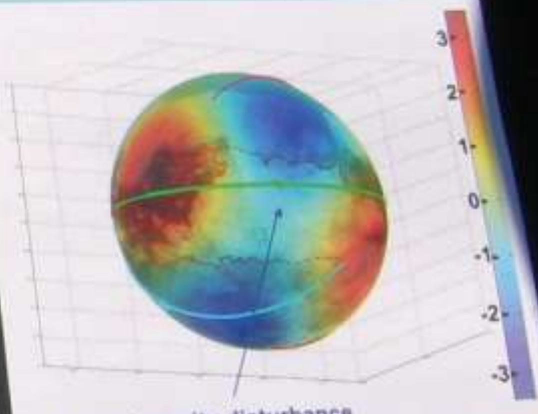
Titan's Gravity Solutions

Dark blue = RSS
Magenta=NAV



Titan's Reference Ellipsoid

Gravity Disturbances in mgal (RSS)



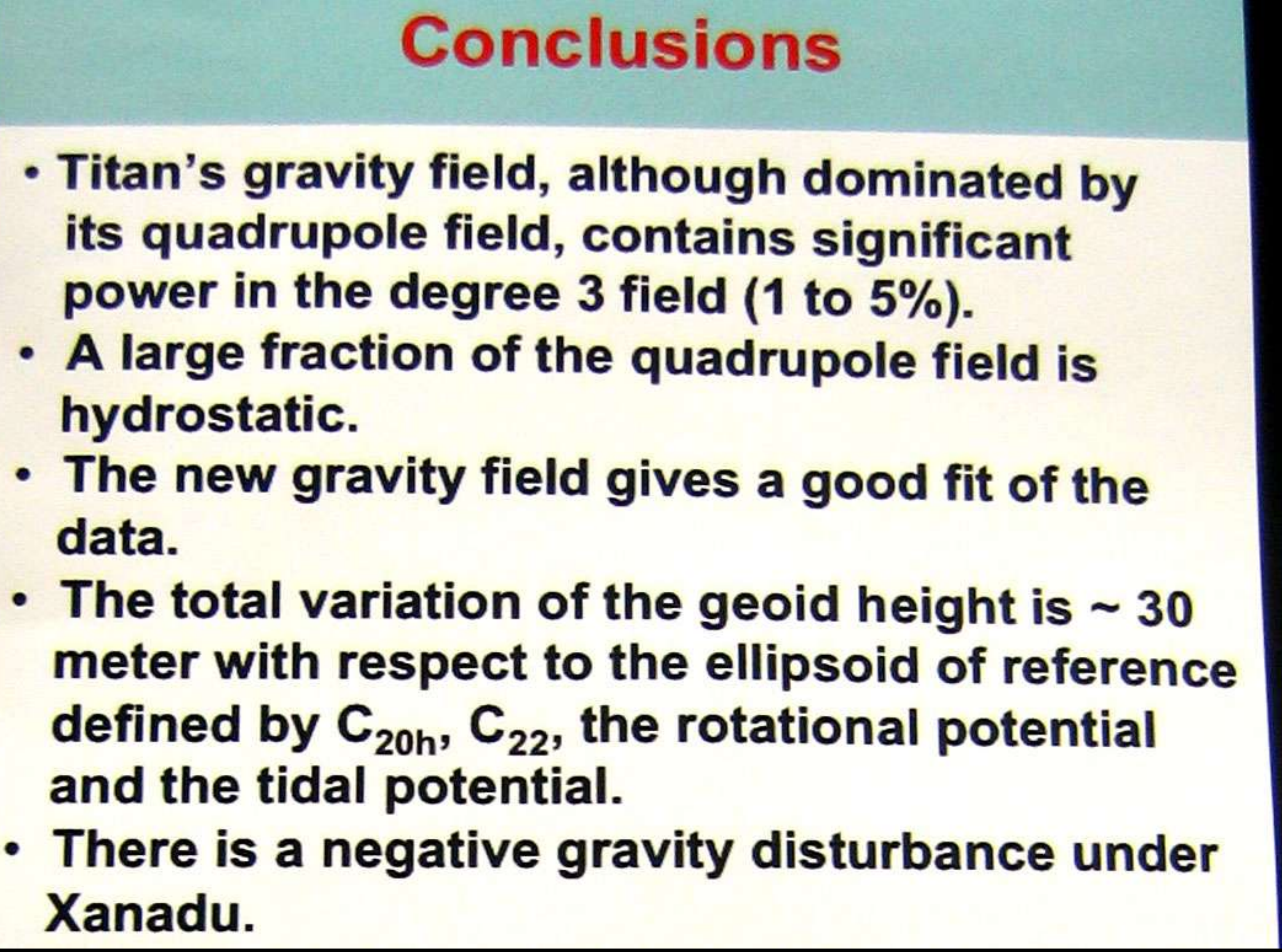
-2 mgal gravity disturbance

Titan Moment of Inertia

The fluid Love number kf describes the hydrostatic response to the tidal or rotational potentials as expressed in the external gravity field. Assuming hydrostaticit

$k_f = \frac{4C_{22}}{q_r} = 0.99$ (it is 1.5 for a uniform densi

liquid body). The nearly hydrostatic J2/C22 close to 10/3, consistent with hydrostatic. F application of Radau-Darwin equation one i



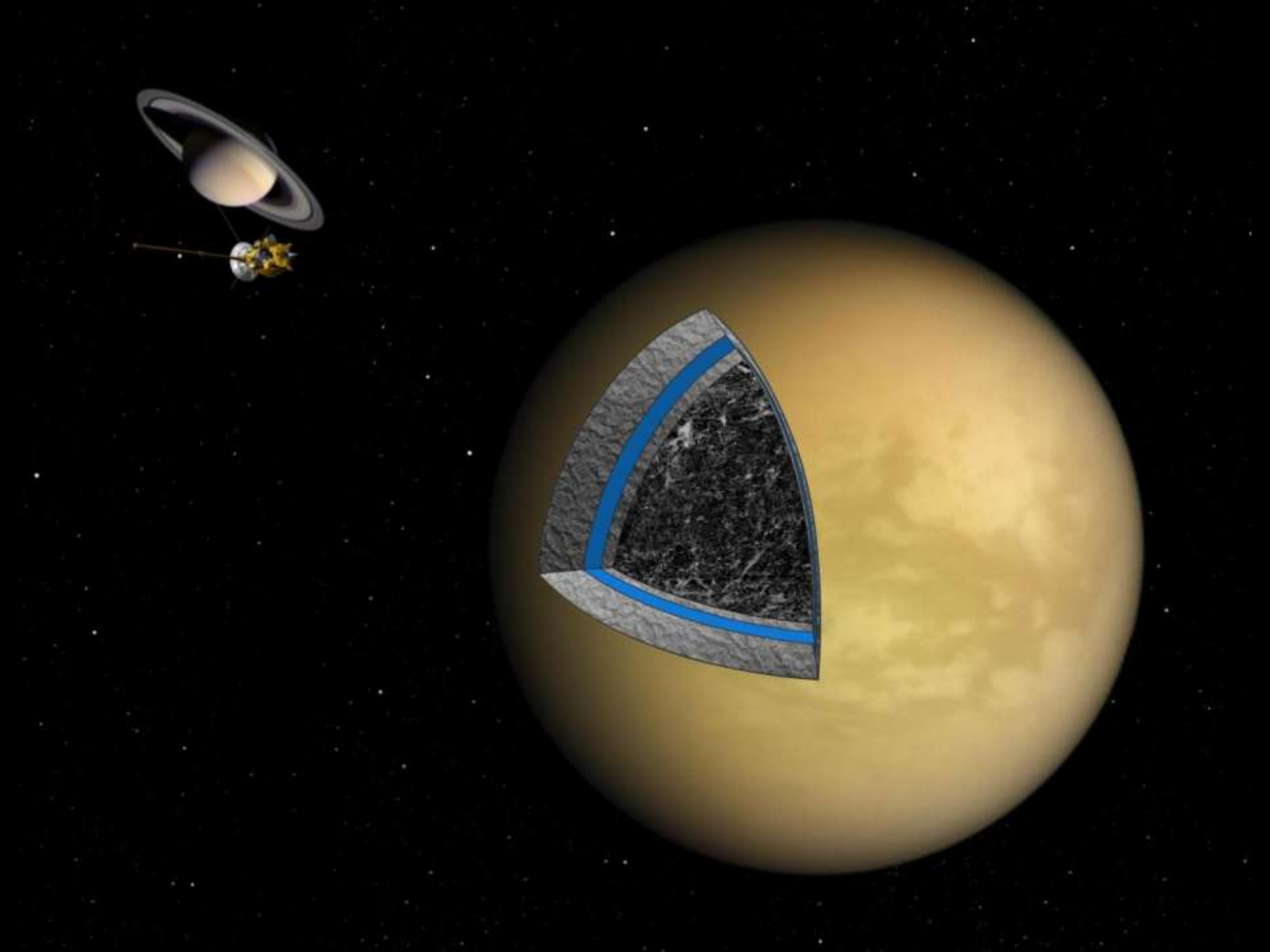
Conclusions

- Titan's gravity field, although dominated by its quadrupole field, contains significant power in the degree 3 field (1 to 5%).
- A large fraction of the quadrupole field is hydrostatic.
- The new gravity field gives a good fit of the data.
- The total variation of the geoid height is ~ 30 meter with respect to the ellipsoid of reference defined by C_{20h}, C_{22}, the rotational potential and the tidal potential.
- There is a negative gravity disturbance under Xanadu.

Titan 68 Flyby

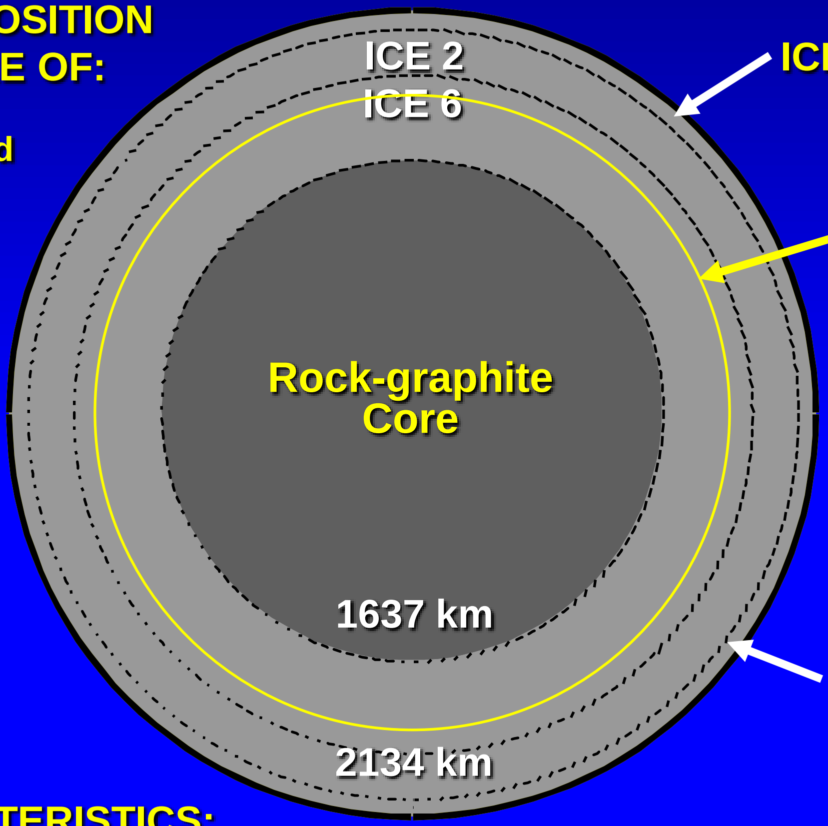
Gravity #5

May 20, 2010



PRE-CASSINI TITAN MODEL

CHEMICAL COMPOSITION
UNIFORM MIXTURE OF:
Rock (46.2%)
Water Ice (50.6%), and
Graphite (3.2%)



ICE 1h

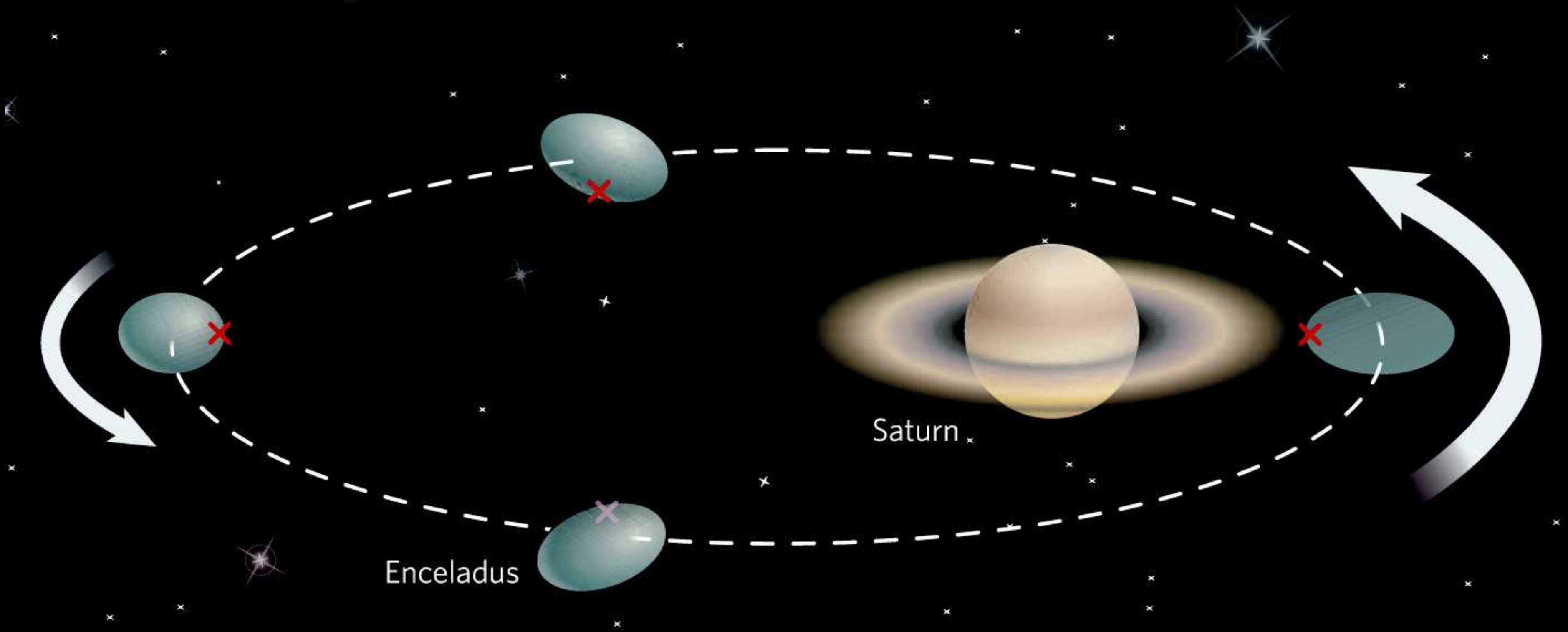
ICE CONVECTION BOUNDARY
2016 km

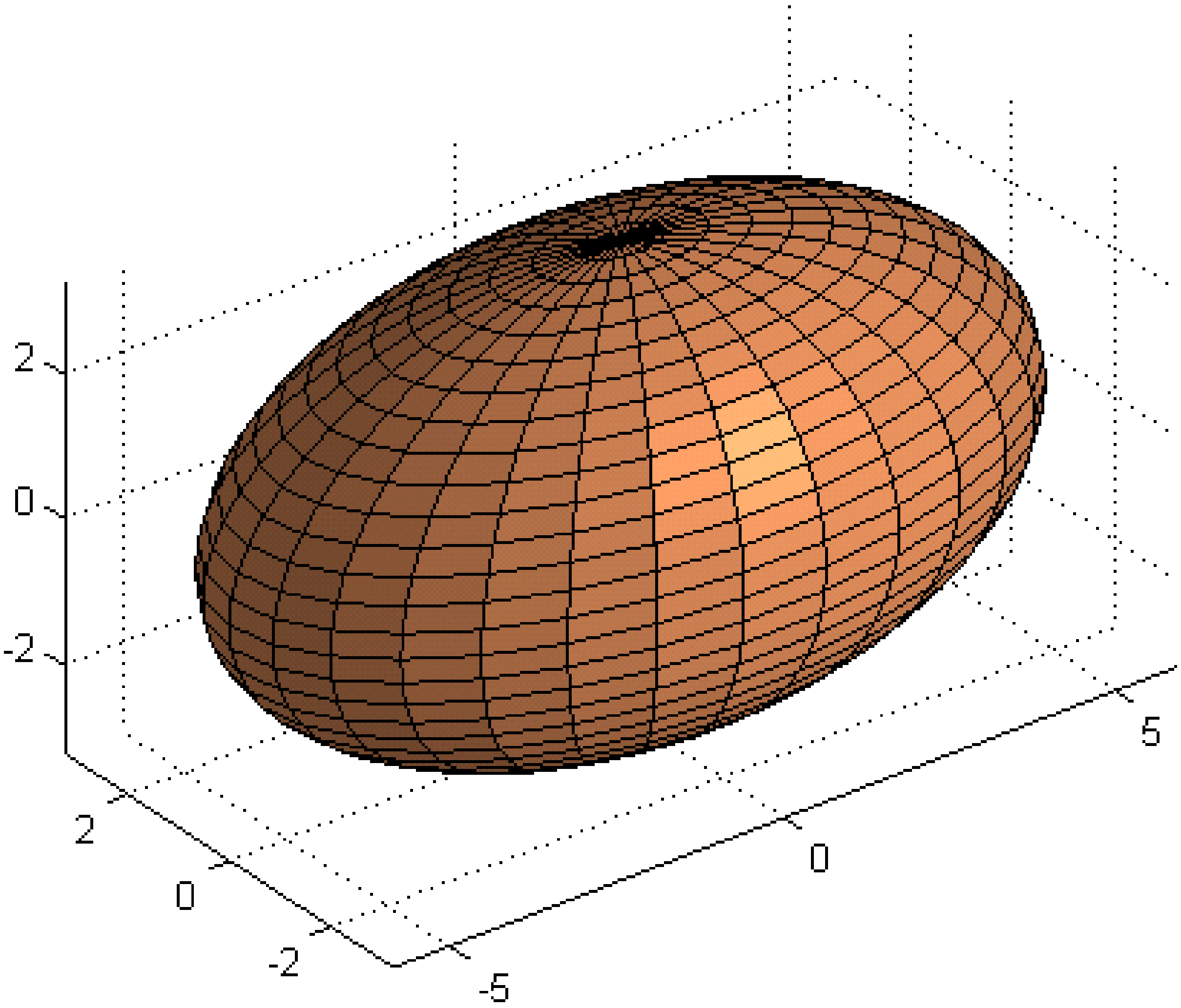
ICE 1h/2 BOUNDARY
2501 km

2575 km

MODEL CHARACTERISTICS:
Mean Density: 1.881 g/cm^3
Rock Density: 3.667 g/cm^3
Central Temperature: 238 K
Central Pressure: 68.1 kbar
Surface Temperature: 94 K

Predicted Moment-of-inertia factor: 0.317 ± 0.004


Saturn
Enceladus


2
0
-2
2
0
-2
-5
0
5

Titan's moments of inertia

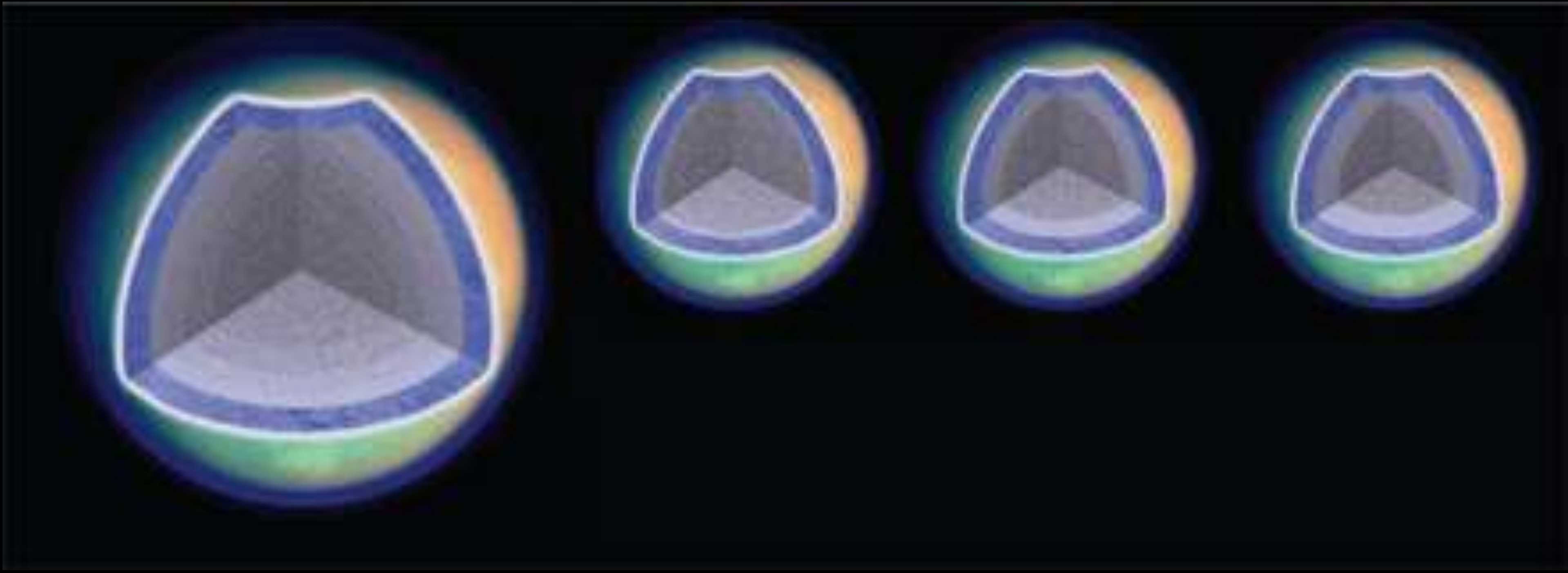
- The difference in the principal equatorial moments of inertia (A and B) of a fluid body in synchronous rotation about a planet is related to the rotation parameter q_{rot} and the fluid Love number k_2 by:

$$B - \mathrm{A} = q_{\mathbf{rot}} k_2 MR^2$$

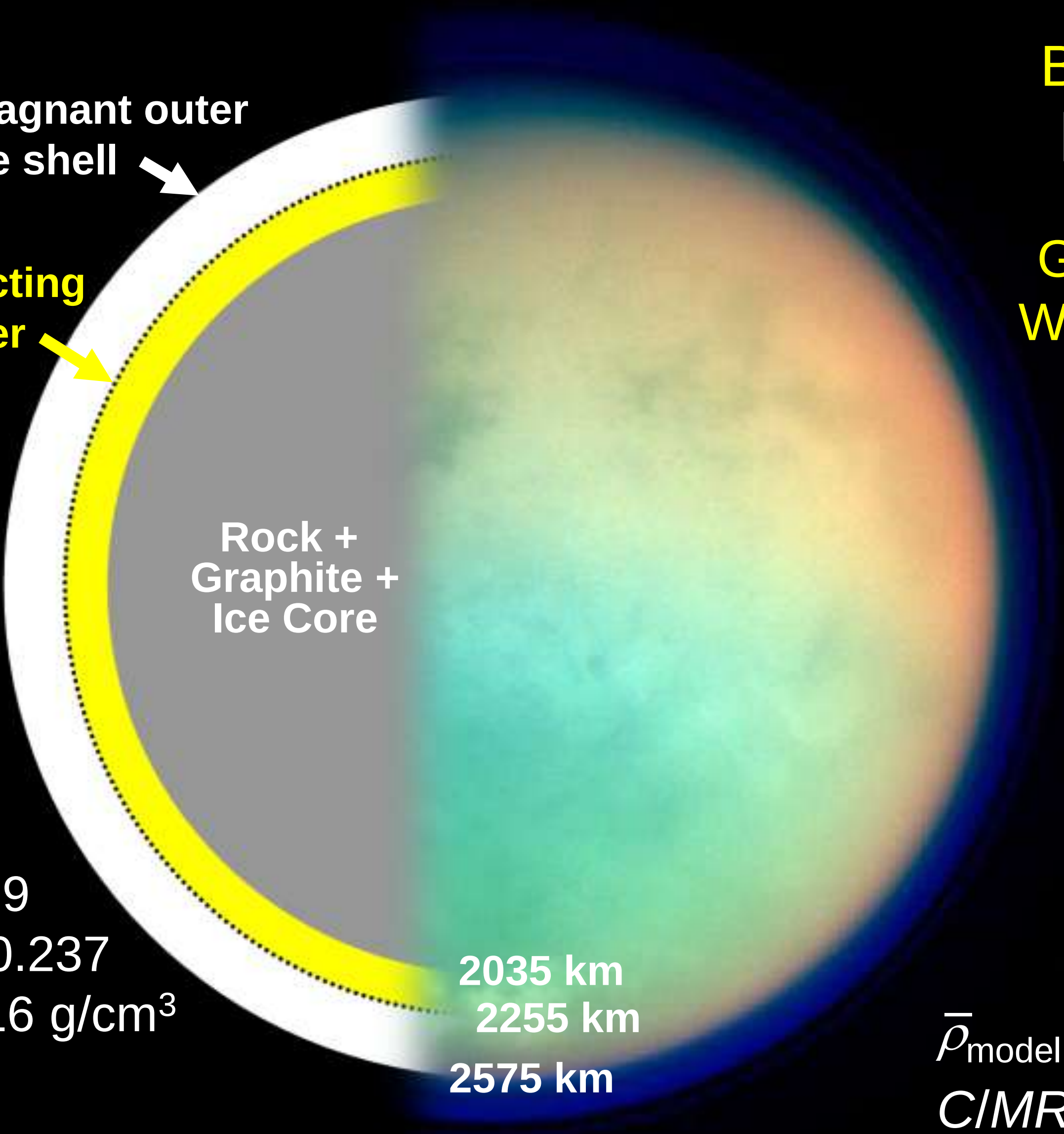
Dimensionless moment of inertia

- The dimensionless axial moment of inertia coefficient C/MR^2 is related to the fluid Love number k_2 by:

$$\frac{C}{MR^2} = \frac{2}{3}\cdot\left[1-\frac{2}{5}\cdot\left(\frac{4-k_2}{1+k_2}\right)^{\frac{1}{2}}\right]$$



Partly-differentiated Titan Model



Bulk Chemistry
[mass fractions]
Rock: 0.4925
Graphite: 0.0336
Water ice: 0.4739

Core properties
Mass fraction: 0.689
Ice mass fraction: 0.237
Mean Density: 2.116 g/cm^3
T_{centre} = 295 K
T_{edge} = 189 K
p_{centre} = 31.5 kbar

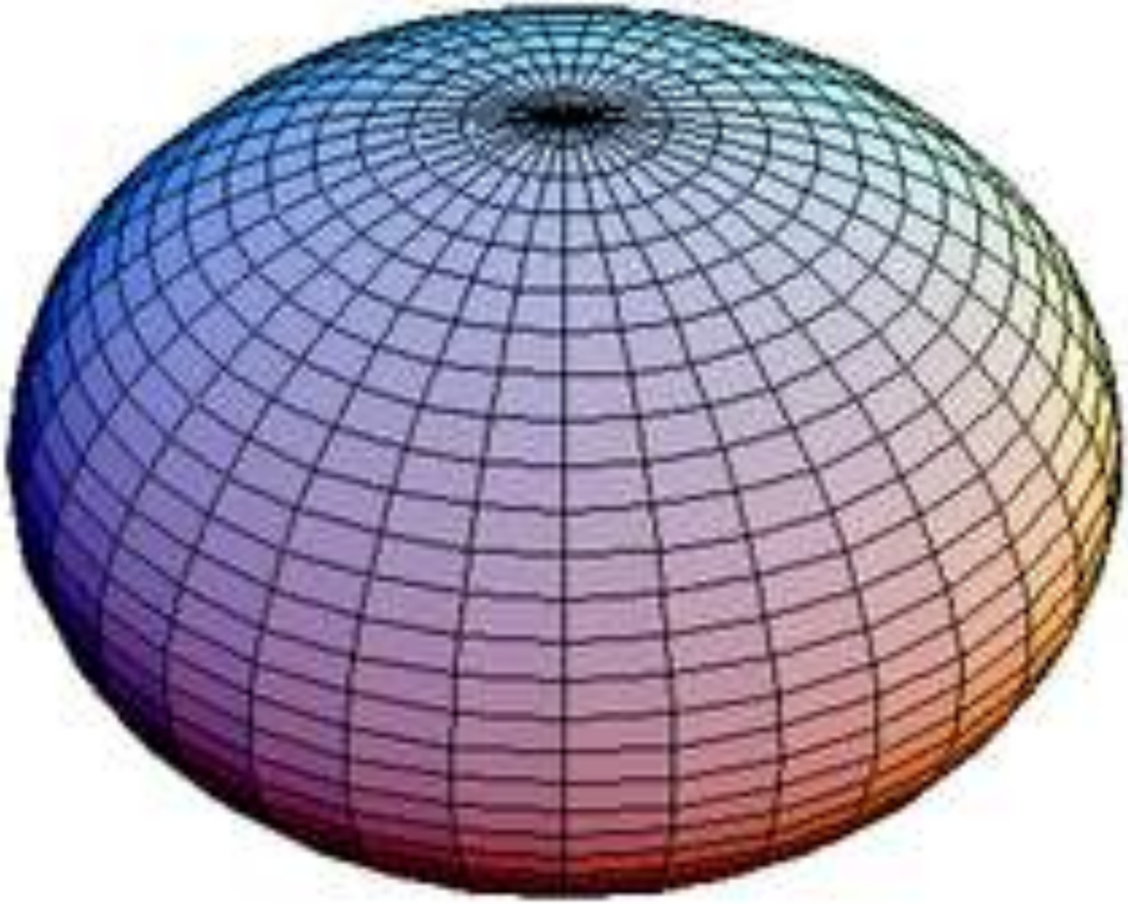
$\bar{\rho}_{\text{model}} = 1.512$ g/cm^3
$C/MR^2 = 0.342$
T_{surf} = 94 K

Size and Shape of Saturn's Moon Titan

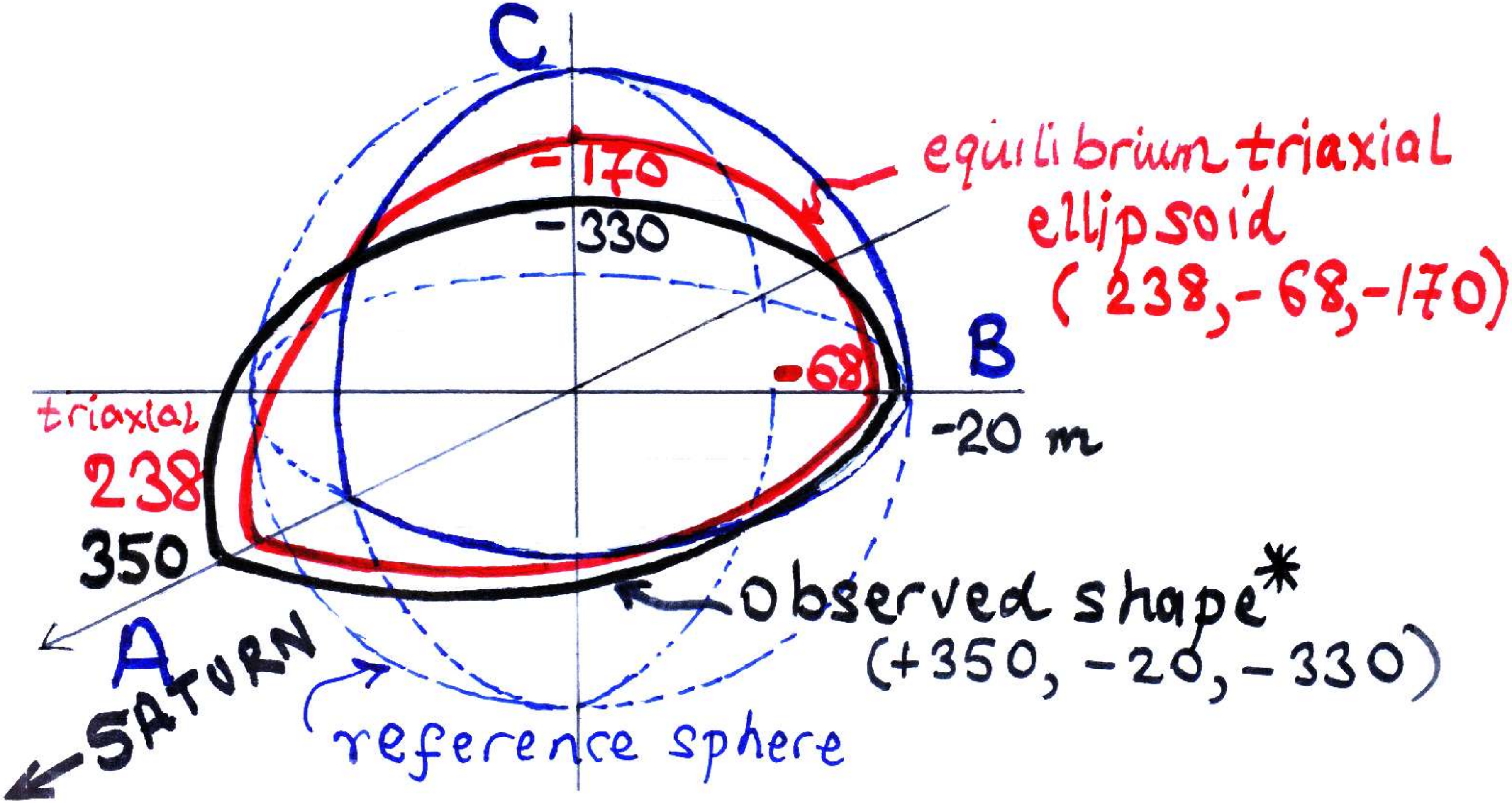
Howard A. Zebker,[1]* Bryan Stiles,[2] Scott Hensley,[2] Ralph Lorenz,[3]
Randolph L. Kirk,[4] Jonathan Lunine[5]

Cassini observations show that Saturn's moon Titan is slightly oblate. A fourth-order spherical harmonic expansion yields north polar, south polar, and mean equatorial radii of 2574.32 ± 0.05 kilometers (km), 2574.36 ± 0.03 km, and 2574.91 ± 0.11 km, respectively; its mean radius is 2574.73 ± 0.09 km. Titan's shape approximates a hydrostatic, synchronously rotating triaxial ellipsoid but is best fit by such a body orbiting closer to Saturn than Titan presently does. Titan's lack of high relief implies that most—but not all—of the surface features observed with the Cassini imaging subsystem and synthetic aperture radar are uncorrelated with topography and elevation. Titan's depressed polar radii suggest that a constant geopotential hydrocarbon table could explain the confinement of the hydrocarbon lakes to high latitudes.

www.sciencemag.org **SCIENCE** VOL 324 15 MAY 2009



TITAN'S SHAPE



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*Zebker et al., EOS Trans AGU, (2008) 89(53), abstract P11D-02

TITAN'S SHAPE

1. Observed shape

Zebker et al (2009) obtain general triaxial ellipsoid with semi-axes:

$$a_{obs} = R_o + 350\,m \quad (\pm 20m)$$
$$b_{obs} = R_o - 20\,m \quad (\pm 60)$$
$$c_{obs} = R_o - 330\,m \quad (\pm 60)$$

with $R_o = 2574.80$ km

2. Theoretical shapes

(a) Sync. rotating ellipsoid

with $k_2 = 0.827$ has semi-axes

$$a_e = R_o(1 + \tfrac{7}{6}\alpha_o) = R_o + 265\,m$$
$$b_e = R_o \cdot (1 - \tfrac{1}{3}\alpha_o) = R_o - 75.7\,m$$
$$c_e = R_o \cdot (1 - \tfrac{5}{6}\alpha_o) = R_o - 189.3\,m$$

where $\alpha_o = (1 + k_2) q_{rot,i} = 8.82 \times 10^{-5}$

(b) Sync. rot. ellipsoid with added oblateness

New semi-axes become

$$a_o = R_o \cdot (1 + \tfrac{7}{6}\alpha_o + \gamma_o) = R_o + 265 + \delta_o$$
$$b_o = R_o \cdot (1 - \tfrac{1}{3}\alpha_o + \gamma_o) = R_o - 75.7 + \delta_o$$
$$c_o = R_o (1 - \tfrac{5}{6}\alpha_o - 2\gamma_o) = R_o - 189.3 - 2\delta_o$$

where $\delta_o = R_o \gamma_o$

3. Rotating oblate-ellipsoid of best fit

Choose δ_0 to minimize the sum of squares

$$S(\delta_0) = \frac{1}{(20)^2}(350-265-\delta_0)^2 + \frac{1}{(60)^2}(-20+75.7-\delta_0)^2 + \frac{1}{(60)^2}(-330+189.3+2\delta_0)^2$$

$S'(\delta_0) = 0$ yields $\delta_0 = 78.7$ m

$$\gamma_0 = \delta_0 / R_0 = 3.0564 \times 10^{-5} = \frac{1}{6}(1+k_2)\, q_{rot}(\text{obl. sph})$$

Best fit semi-axes	Obs.
$a_0 = R_0 + 343.7$ m	350
$b_0 = R_0 + 3.0$ m	-20
$c_0 = R_0 - 346.7$ m	-330

CONCLUSION : The rotating oblate-ellipsoid provides an excellent fit to the Zebker radar data

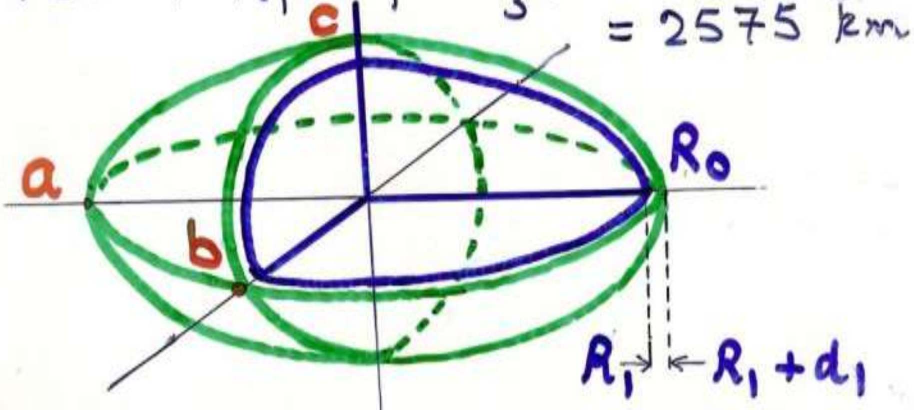
FINAL MODEL

Titan's shape & gravity data can be explained if the satellite has a **frozen** interior consisting of a massive **ellipsoid** defined by a former rotational frequency **ω_1 (ell.)** & mean radius **R_1** and **capped** by a thin outer **oblate spheroidal** shell of mean thickness **d_1** & rot'l freq. **ω_1 (obl.)**

- Semi-axes are

$$a = R_1\left(1 + \tfrac{7}{6}\alpha_1\right) + d_1(1+\gamma_1)$$
$$b = R_1\left(1 - \tfrac{1}{3}\alpha_1\right) + d_1(1+\gamma_1)$$
$$c = R_1\left(1 - \tfrac{5}{6}\alpha_1\right) + d_1(1-2\gamma_1)$$

Also : $R_1 + d_1 = \frac{1}{3}(a+b+c) = R_0$
$= 2575$ km



POST-CASSINI TITAN MODEL

CORE PROPERTIES

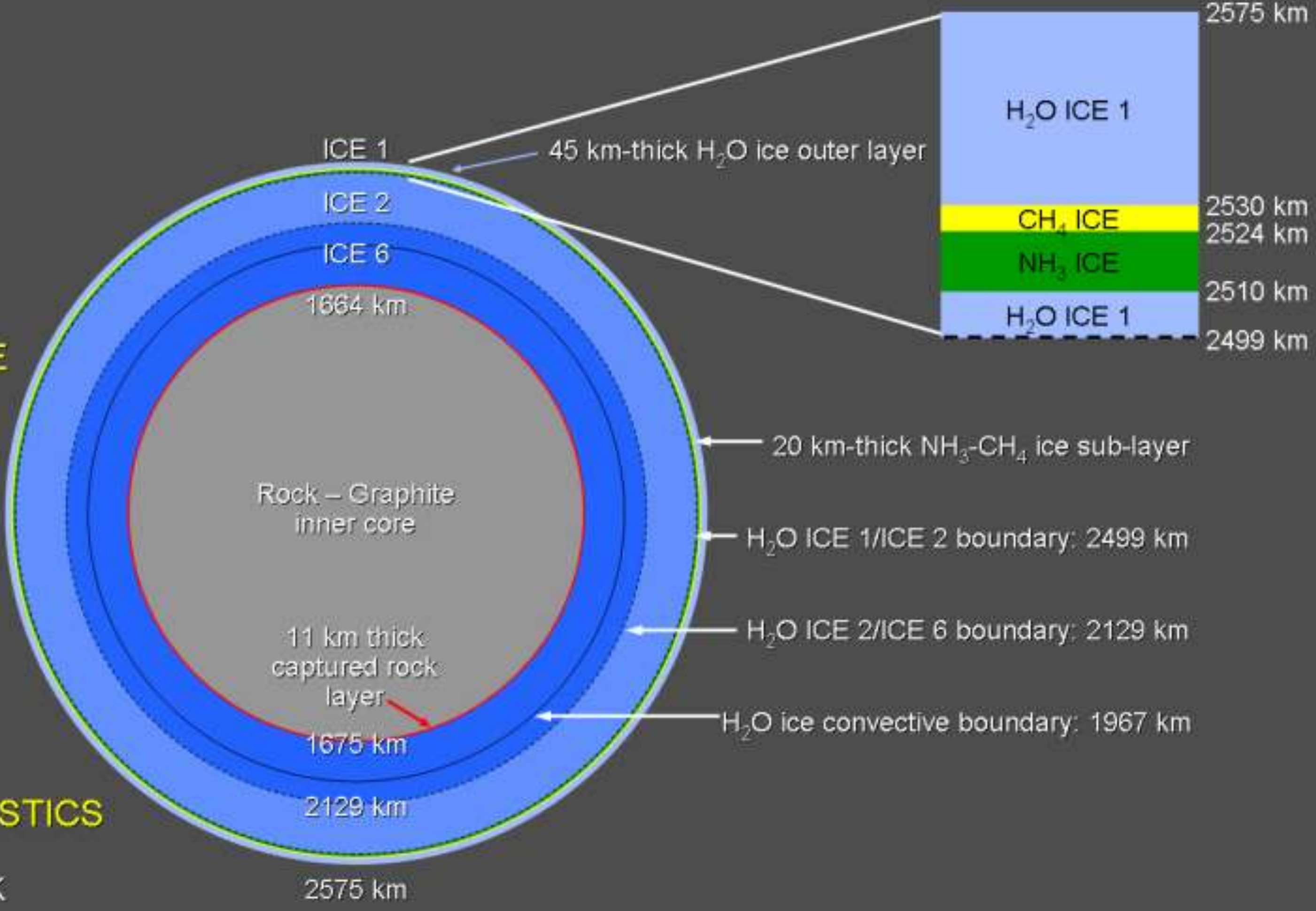
Mass fraction: 0.5103
Mean Temperature: 584 K
Mean density: 3.62 g/cm^3

CAPTURED ROCK-ICE PROPERTIES

Model mass fraction: 0.03
Constituent mass fractions:
- rock: 0.3396
- H_2O: 0.3651
- NH_3: 0.2388
- CH_4: 0.0565

MODEL CHARACTERISTICS

Mean density: 1.916 g/cm^3
Central temperature: 1450 K
Central pressure: 70.2 kbar
Surface temperature: 94 K
Moment-of-inertia coefficient: $C/MR^2 = 0.315$



45 km-thick H_2O ice outer layer

20 km-thick NH_3-CH_4 ice sub-layer

H_2O ICE 1/ICE 2 boundary: 2499 km

H_2O ICE 2/ICE 6 boundary: 2129 km

H_2O ice convective boundary: 1967 km


Does Titan's Slightly Oblate Shape suggest a Capture Origin?
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1529
1528
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