What is the Expected Return on the Market?

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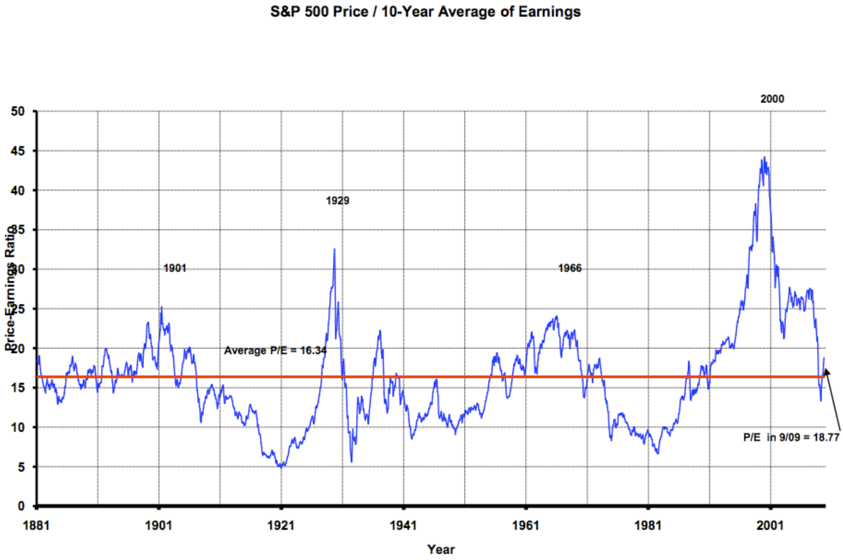
Returns on the stock market are predictable

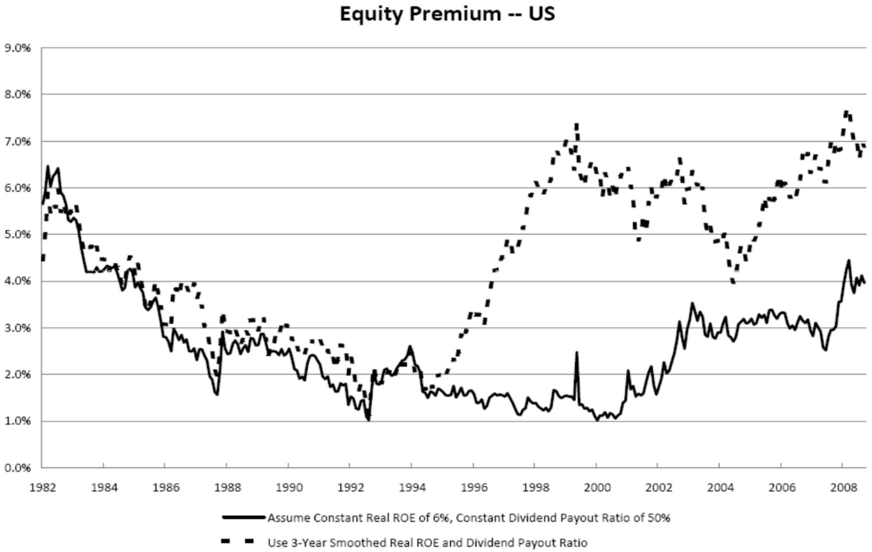
$$\text{return}_{t+1} = \frac{\text{price}_{t+1} + \text{dividend}_{t+1}}{\text{price}_t} = \underbrace{\frac{\text{price}_{t+1}}{\text{price}_t}}_{\text{capital gain}} + \underbrace{\frac{\text{dividend}_{t+1}}{\text{price}_t}}_{\text{dividend yield}}$$

- *Naive investor*: If I buy when the dividend yield is high, I will have a high return on average
- *'Sophisticated' investor*: No! The high dividend yield—that is, low price—is a sign that the market anticipates that *future* dividends will be disappointing. I therefore expect that a low capital gain will offset the high dividend yield
- Empirically, it appears that the naive investor is right


S&P 500 Price / 10-Year Average of Earnings
Price-Earnings Ratio
50
45
40
35
30
25
20
15
10
5
0
2000
1929
1901
1966
Average P/E = 16.34
P/E in 9/09 = 18.77
1881
1901
1921
1941
1961
1981
2001
Year

The equity premium

Figure from John Campbell's Princeton Lecture in Finance

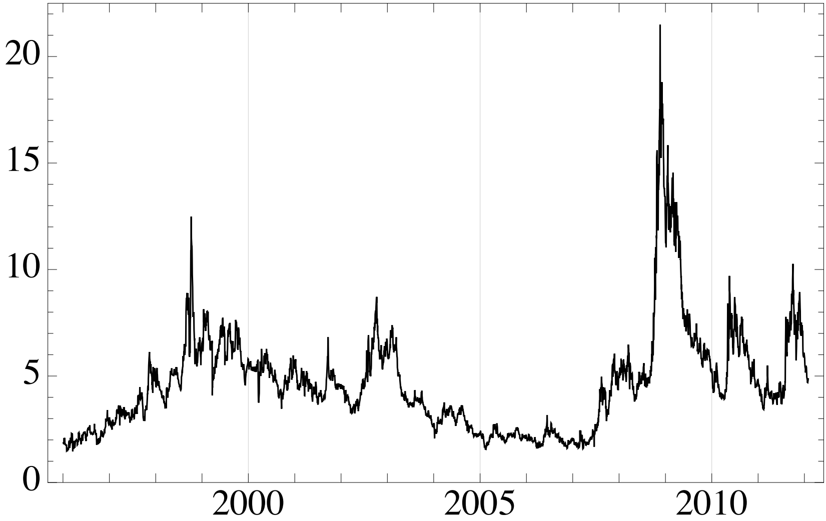


Motivation

- Find an asset price that forecasts expected returns
  - without using accounting data
  - without having to estimate any parameters
  - imposing minimal theoretical structure
  - and in real time

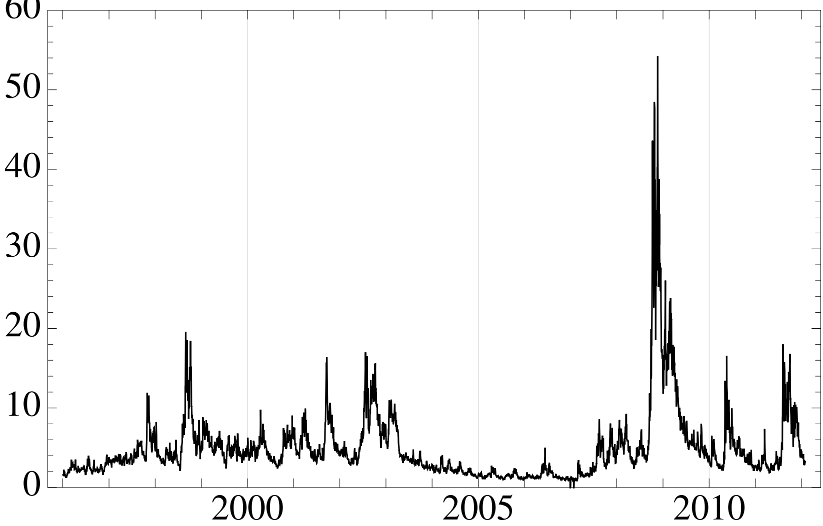
A lower bound on the equity premium

1 year horizon, in %



A lower bound on the equity premium

1 month horizon, annualized, in %



Outline

1. A volatility index, SVIX, gives a lower bound on the equity premium
2. SVIX and VIX
3. SVIX as a predictor variable
4. What is the probability of a 20% decline in the market?

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Notation

- S_T: level of S&P 500 index at time T
- R_T: gross return on the S&P 500 from time t to time T
- $R_{f,t}$: riskless rate from time t to time T
- M_T: SDF that prices time-T payoffs from the perspective of time t
- We can price any time-T payoff X_T either via the SDF or by computing expectations with risk-neutral probabilities:

$$\text{time-}t \text{ price of a claim to } X_T = \mathbb{E}_t(M_T X_T) = \frac{1}{R_{f,t}} \mathbb{E}_t^* X_T$$

- Asterisks indicate risk-neutral quantities

Risk-neutral variance and the risk premium

- As an example, we can write conditional risk-neutral variance as

$$\text{var}_t^* R_T = \mathbb{E}_t^* R_T^2 - \left(\mathbb{E}_t^* R_T\right)^2 = R_{f,t}\, \mathbb{E}_t \left(M_T R_T^2\right) - R_{f,t}^2 \qquad (1)$$

- We can decompose the equity premium into two components:

$$\begin{aligned} \mathbb{E}_t R_T - R_{f,t} &= \left[\mathbb{E}_t(M_T R_T^2) - R_{f,t}\right] - \left[\mathbb{E}_t(M_T R_T^2) - \mathbb{E}_t R_T\right] \\ &= \frac{1}{R_{f,t}} \text{var}_t^* R_T - \text{cov}_t(M_T R_T, R_T) \end{aligned}$$

- The first line adds and subtracts $\mathbb{E}_t(M_T R_T^2)$
- The second exploits equation (1) and the fact that $\mathbb{E}_t M_T R_T = 1$

Risk-neutral variance and the risk premium

$$\mathbb{E}_t R_T - R_{f,t} = \frac{1}{R_{f,t}} \operatorname{var}_t^* R_T - \underbrace{\operatorname{cov}_t(M_T R_T, R_T)}_{\leq 0, \text{ under the NCC}}$$

- The decomposition splits the risk premium into two pieces
- Risk-neutral variance can be computed from time-t asset prices
- The covariance term can be controlled: it is negative in theoretical models and in the data
- Formalize this key assumption as the *negative correlation condition*:

$$\operatorname{cov}_t(M_T R_T, R_T) \leq 0$$

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   - This holds because $R_T u'(W_t R_T)$ is decreasing in R_T: its derivative is $u'(W_t R_T) + W_t R_T u''(W_t R_T) = -u'(W_t R_T)\left[\gamma(W_t R_T) - 1\right] \leq 0$.

Whose equity premium?

$$\mathbb{E}_t R_T - R_{f,t} \geq \frac{1}{R_{f,t}} \operatorname{var}_t^* R_T$$

- Does not require that everyone holds the market
- Does not assume that all economic wealth is invested in the market
- Simply ask: What is the equity premium perceived by a rational one-period investor who holds the market and whose risk aversion is at least 1?
- This question is a sensible benchmark even in the presence of constrained and/or irrational investors

Comparison to Merton (1980)

- Merton (1980) suggested estimating the equity premium from

$$\text{equity premium} = \text{risk aversion} \times \text{return variance}$$

- Holds if marginal investor has power utility and the market follows a geometric Brownian motion
- No distinction between risk-neutral and real-world variance in a diffusion-based model (Girsanov's theorem)
- The appropriate generalization relates the equity premium to *risk-neutral* variance
  - Bonus: Risk-neutral variance is directly measurable from asset prices

Comparison to Hansen–Jagannathan (1991)

$$\frac{1}{R_{f,t}} \operatorname{var}_t^* R_T \leq \mathbb{E}_t R_T - R_{f,t} \leq R_{f,t} \cdot \sigma_t(M_T) \cdot \sigma_t(R_T)$$

- Left-hand inequality is the new result
  - Good: relates unobservable equity premium to an observable quantity
  - Bad: requires the negative correlation condition
- Right-hand inequality is the Hansen–Jagannathan bound
  - Good: no assumptions
  - Bad: neither side is observable

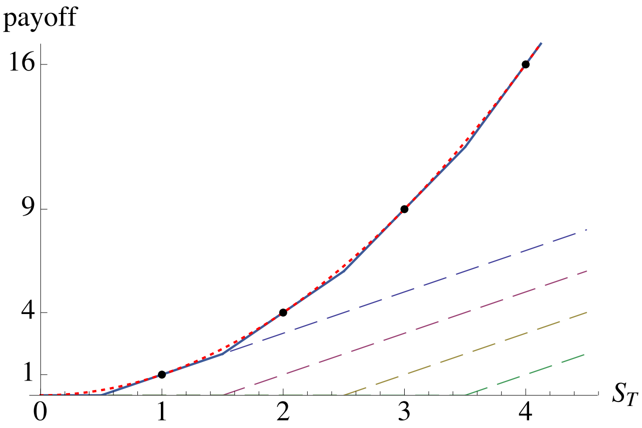
How to measure risk-neutral variance

- We want to measure $\frac{1}{R_{f,t}} \operatorname{var}_t^* R_T = \frac{1}{R_{f,t}} \mathbb{E}_t^* R_T^2 - \frac{1}{R_{f,t}} \left(\mathbb{E}_t^* R_T\right)^2$
- Since $\mathbb{E}_t^* R_T = R_{f,t}$, this boils down to calculating $\frac{1}{R_{f,t}} \mathbb{E}_t^* S_T^2$
- That is: how can we price the 'squared contract' with payoff S_T^2?

How to measure risk-neutral variance

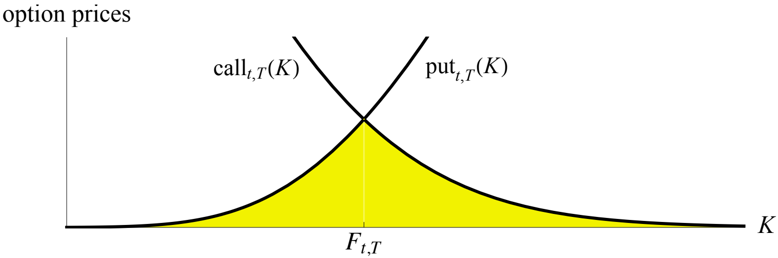
- How can we price the 'squared contract' with payoff S_T^2?
- Suppose you buy:
  - 2 calls with strike $K = 0.5$
  - 2 calls with strike $K = 1.5$
  - 2 calls with strike $K = 2.5$
  - 2 calls with strike $K = 3.5$
  - etc ...

How to measure risk-neutral variance



- So, $\frac{1}{R_{f,t}} \mathbb{E}_t^* S_T^2 \approx 2 \sum_K \text{call}_{t,T}(K)$
- In fact, $\frac{1}{R_{f,t}} \mathbb{E}_t^* S_T^2 = 2 \int_0^\infty \text{call}_{t,T}(K)\, dK$

How to measure risk-neutral variance



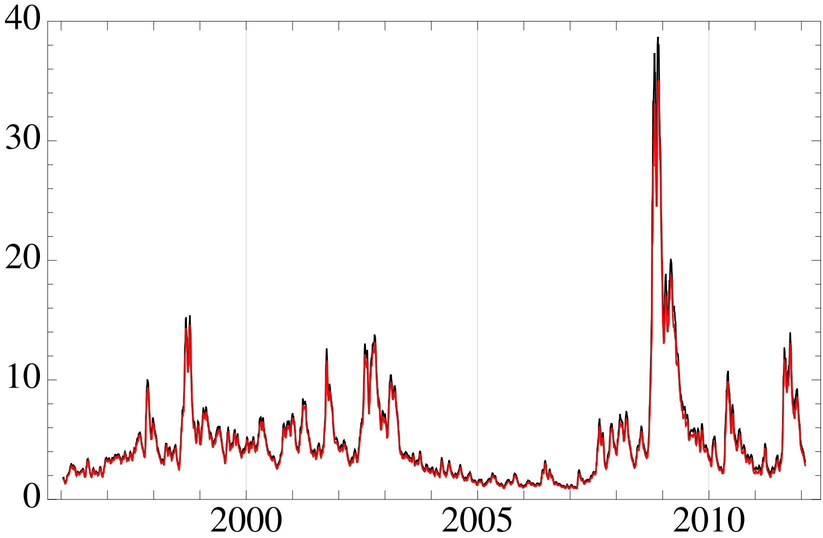
- Using put-call parity, we end up with a simple formula:

$$\frac{1}{R_{f,t}} \operatorname{var}_t^* R_T = \frac{2}{S_t^2} \left\{ \int_0^{F_{t,T}} \operatorname{put}_{t,T}(K)\, dK + \int_{F_{t,T}}^{\infty} \operatorname{call}_{t,T}(K)\, dK \right\}$$

- $F_{t,T}$ is the forward price of the underlying, which is known at time t

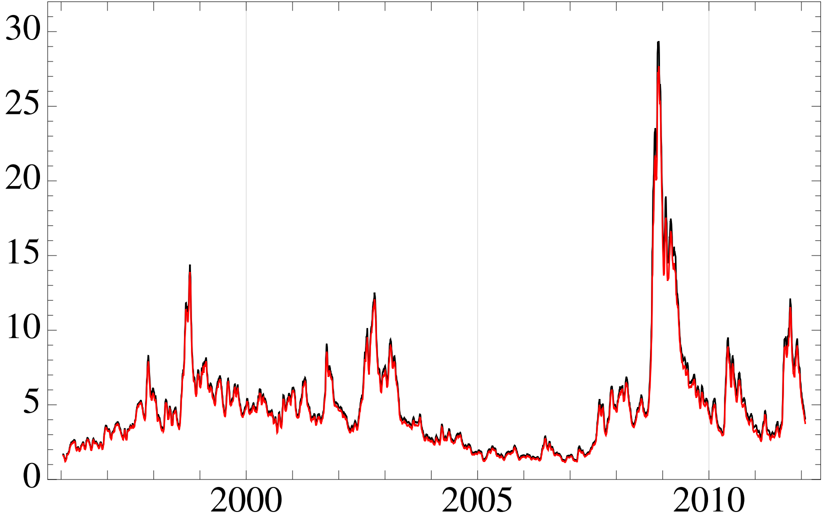
A lower bound on the equity premium

1mo horizon, annualized, 10-day moving avg. Mid prices in black, bid prices in red



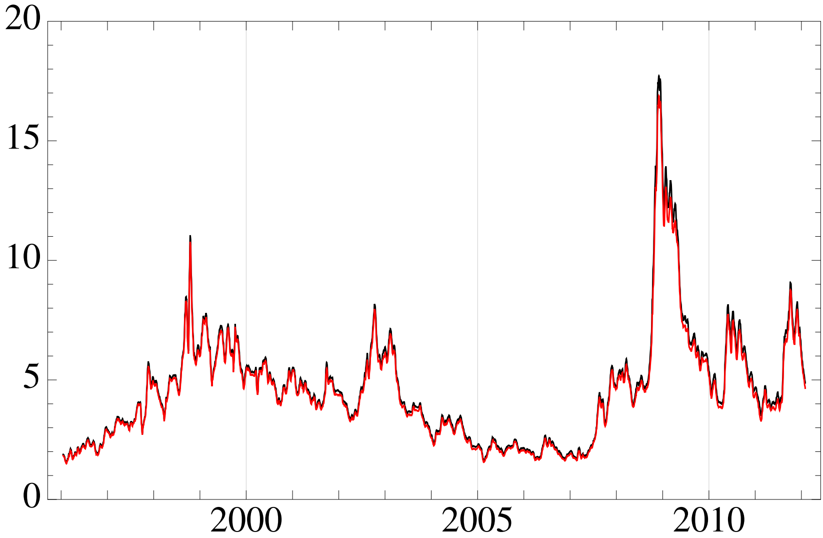
A lower bound on the equity premium

3mo horizon, annualized, 10-day moving avg. Mid prices in black, bid prices in red



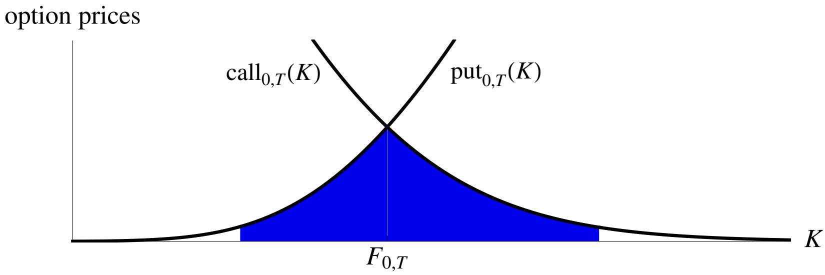
A lower bound on the equity premium

1yr horizon, annualized, 10-day moving avg. Mid prices in black, bid prices in red



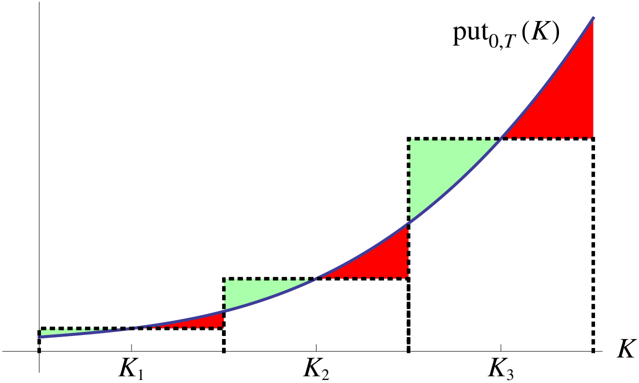
Robustness

- Can't observe deep-OTM option prices



Robustness

- Even near-the-money, can't observe a continuum of strikes



Robustness

- Both these effects mean that the true lower bound is even higher
- By ignoring deep-OTM options, we underestimate the true area under the curve
- Discretization in strike also leads to underestimating the true area, because $\text{call}_{t,T}(K)$ and $\text{put}_{t,T}(K)$ are both convex in K
- Maybe option markets were totally illiquid in November 2008?
- If so, we should expect to see wide bid-ask spread
- Is lower bound much lower if bid prices are used for options, rather than mid prices? No. And volume was high

A lower bound on the equity premium

horizon	mean	s.d.	min	1%	10%	25%	50%	75%	90%	99%	max
1 mo	5.00	4.60	0.83	1.03	1.54	2.44	3.91	5.74	8.98	25.7	55.0
2 mo	5.00	3.99	1.01	1.20	1.65	2.61	4.11	5.91	8.54	23.5	46.1
3 mo	4.96	3.60	1.07	1.29	1.75	2.69	4.24	5.95	8.17	21.4	39.1
6 mo	4.89	2.97	1.30	1.53	1.95	2.88	4.39	6.00	7.69	16.9	29.0
1 yr	4.64	2.43	1.47	1.64	2.07	2.81	4.35	5.72	7.19	13.9	21.5

Table: Mean, standard deviation, and quantiles of EP bound (in %)

- The time series average of the lower bound is about 5%
- It is volatile and right-skewed, particularly at short horizons

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SVIX and VIX

- By analogy with VIX, define

$$\text{SVIX}_t^2 = \frac{2R_{f,t}}{(T-t)\cdot F_{t,T}^2}\left\{\int_0^{F_{t,T}} \text{put}_{t,T}(K)\,dK + \int_{F_{t,T}}^{\infty} \text{call}_{t,T}(K)\,dK\right\}$$

- In this notation, equity premium $\geq R_{f,t}\cdot \text{SVIX}_t^2$
- Compare SVIX with

$$\text{VIX}_t^2 = \frac{2R_{f,t}}{T-t}\left\{\int_0^{F_{t,T}} \frac{1}{K^2}\,\text{put}_{t,T}(K)\,dK + \int_{F_{t,T}}^{\infty} \frac{1}{K^2}\,\text{call}_{t,T}(K)\,dK\right\}$$

- These are *definitions*, not statements about pricing

SVIX and VIX

- VIX is similar to SVIX, but is more sensitive to left tail events
- SVIX measures risk-neutral variance, $\text{SVIX}^2 = \text{var}_t^*(R_T/R_{f,t})$
- VIX measures risk-neutral entropy,
  $\text{VIX}^2 = \log \mathbb{E}_t^*(R_T/R_{f,t}) - \mathbb{E}_t^* \log(R_T/R_{f,t})$
- What VIX does *not* measure: $\text{VIX}^2 \neq \frac{1}{T-t}\,\mathbb{E}_t^*\left[\int_t^T \sigma_\tau^2\,d\tau\right]$

VIX and SVIX

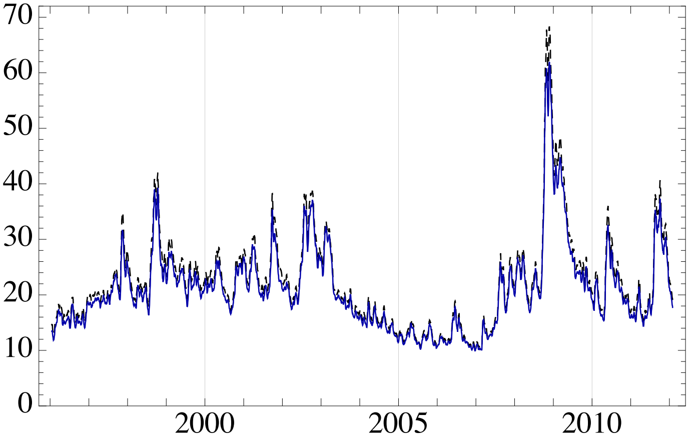


Figure: VIX (dotted) and SVIX (solid). Jan 4, 1996–Jan 31, 2012
Figure shows 10-day moving average. $T = 1$ month

VIX minus SVIX

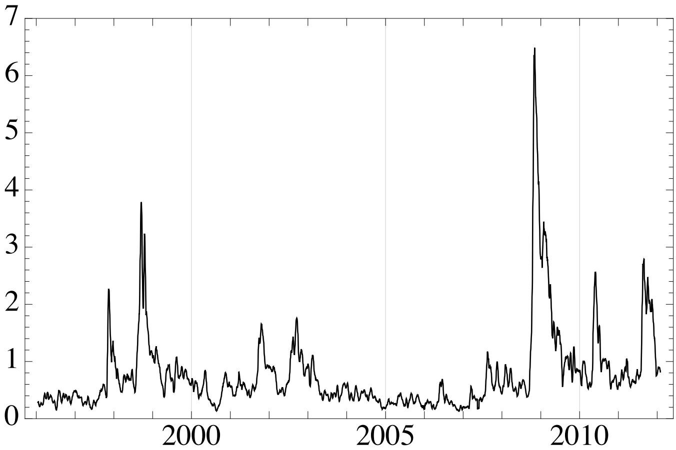


Figure: VIX minus SVIX. Jan 4, 1996–Jan 31, 2012

Figure shows 10-day moving average. $T = 1$ month

No conditionally lognormal model fits option prices

- If returns and the SDF are conditionally lognormal with return volatility $\sigma_{R,t}$ then we can calculate VIX and SVIX in closed form:

$$\begin{aligned} \text{SVIX}_t^2 &= \frac{1}{T-t}\left(e^{\sigma_{R,t}^2(T-t)} - 1\right) \\ \text{VIX}_t^2 &= \sigma_{R,t}^2 \end{aligned}$$

- VIX would be lower than SVIX—which it *never* is in my sample
- No conditionally lognormal model is consistent with option prices

Outline

Might the lower bound hold with equality?

- Time-series average of lower bound in recent data is around 5%
- Fama and French (2002) estimate unconditional equity premium of 3.83% (from dividend growth) or 4.78% (from earnings growth)
- Fama interviewed by Roll: "I always think of the number, the equity premium, as five per cent."
- Estimates of $\text{cov}(M_T R_T, R_T)$ in linear factor models are statistically and economically close to zero

$\widehat{\text{cov}}(M_T R_T, R_T)$ is negative and close to zero

	constant	$R_M - R_f$	SMB	HML	MOM	$\widehat{\text{cov}}(M_T R_T, R_T)$
Full sample	1.072	−2.375	−0.648	−5.489	−5.572	−0.0018
	(0.020)	(0.746)	(1.011)	(1.131)	(1.033)	(0.0020)
Jan '27–Dec '62	1.071	−2.355	−0.587	−3.882	−5.552	−0.0021
	(0.029)	(1.034)	(1.747)	(2.163)	(1.565)	(0.0041)
Jan '63–Dec '13	1.092	−3.922	−2.400	−9.020	−5.152	−0.0020
	(0.029)	(1.272)	(1.475)	(1.795)	(1.427)	(0.0022)
Jan '96–Dec '13	1.047	−3.231	−2.327	−5.789	−2.548	−0.0017
	(0.034)	(1.981)	(2.224)	(2.491)	(1.637)	(0.0036)

Table: Estimates of coefficients in the 4-factor model, and of $\text{cov}(M_T R_T, R_T)$.

- Test assets: market, riskless asset, 5×5 portfolios sorted on size and B/M, 10 momentum portfolios; monthly data from Ken French's website
- Estimate M and $\text{cov}(M_T R_T, R_T)$ by GMM

Forecasting returns with risk-neutral variance

- We want to test the null hypothesis that $\mathbb{E}_t R_T - R_{f,t} = R_{f,t} \cdot \text{SVIX}_t^2$
- Run regressions

$$R_T - R_{f,t} = \alpha + \beta \times R_{f,t} \cdot \text{SVIX}_t^2 + \varepsilon_T$$

- Sample period: January 1996–January 2012
- Robust Hansen–Hodrick standard errors account for heteroskedasticity and overlapping observations

Forecasting returns with risk-neutral variance

horizon	$\widehat{\alpha}$	s.e.	$\widehat{\beta}$	s.e.	R^2
1 mo	0.012	[0.064]	0.779	[1.386]	0.34%
2 mo	−0.002	[0.068]	0.993	[1.458]	0.86%
3 mo	−0.003	[0.075]	1.013	[1.631]	1.10%
6 mo	−0.056	[0.058]	2.104	[0.855]	5.72%
1 yr	−0.029	[0.093]	1.665	[1.263]	4.20%

Table: Coefficient estimates for the forecasting regression.

- Cannot reject the null at any horizon

Forecasting returns with risk-neutral variance

horizon	$\widehat{\alpha}$	s.e.	$\widehat{\beta}$	s.e.	R^2
1 mo	−0.095	[0.061]	3.705	[1.258]	3.36%
2 mo	−0.081	[0.062]	3.279	[1.181]	4.83%
3 mo	−0.076	[0.067]	3.147	[1.258]	5.98%
6 mo	−0.043	[0.072]	2.319	[1.276]	4.94%
1 yr	0.045	[0.088]	0.473	[1.731]	0.27%

Table: Coefficient estimates excluding Aug ’08–Jul ’09

- Predictability is not driven by the crisis

Realized variance doesn't predict reliably

horizon	$\widehat{\alpha}$	s.e.	$\widehat{\beta}$	s.e.	R^2
1 mo	0.049	[0.045]	−0.462	[0.784]	0.27%
2 mo	0.044	[0.043]	−0.341	[0.586]	0.26%
3 mo	0.035	[0.046]	−0.173	[0.722]	0.09%
6 mo	−0.025	[0.050]	1.182	[0.430]	5.45%
1 yr	−0.042	[0.068]	1.293	[0.499]	8.13%

Table: Regression $R_T - R_{f,t} = \alpha + \beta \times SVAR_t + \varepsilon_T$, full sample.

Realized variance doesn't predict reliably

horizon	$\widehat{\alpha}$	s.e.	$\widehat{\beta}$	s.e.	R^2
1 mo	−0.007	[0.049]	1.478	[1.125]	0.71%
2 mo	−0.006	[0.050]	1.429	[1.272]	1.13%
3 mo	−0.004	[0.049]	1.342	[1.265]	1.32%
6 mo	0.028	[0.049]	0.299	[1.424]	0.09%
1 yr	0.034	[0.064]	−0.348	[2.469]	0.15%

Table: Regression $R_T - R_{f,t} = \alpha + \beta \times SVAR_t + \varepsilon_T$, excluding Aug '08–Jul '09.

Forecasting returns with valuation ratios

- Goyal–Welch (2008): Conventional predictor variables fail out-of-sample
- Campbell–Thompson (2008) response: Gordon growth model suggests a forecast

$$\mathbb{E}_t R_T = D/P_t + G$$

- Important: coefficient on D/P_t is not estimated but fixed *a priori*
- A good comparison for the risk-neutral variance approach

R^2 from Campbell and Thompson (2008)

	Sample: 1927–1956			Sample: 1956–1980			Sample: 1980–2005		
	Unconstrained	Pos. Intercept, Bounded Slope	Fixed Coefs	Unconstrained	Pos. Intercept, Bounded Slope	Fixed Coefs	Unconstrained	Pos. Intercept, Bounded Slope	Fixed Coefs
A: Monthly Returns									
Dividend/price	−0.86%	0.21%	0.63%	0.88%	0.57%	0.67%	−1.30%	−0.21%	−0.54%
Earnings/price	0.16	0.28	1.04	0.56	0.45	0.30	−0.53	−0.09	0.07
Smooth earnings/price	0.56	0.53	1.33	0.80	0.48	0.51	−1.06	−0.06	0.01
Dividend/price + growth	−0.15	0.18	0.78	0.18	0.18	0.59	0.11	0.11	0.14
Earnings/price + growth	−0.06	0.12	0.73	−0.12	−0.12	0.33	0.05	0.05	0.16
Smooth earnings/price + growth	0.09	0.25	0.93	0.19	0.19	0.47	0.06	0.06	0.16
Book-to-market + growth				−0.62	−0.73	0.73	−0.12	−0.02	0.00
Dividend/price + growth − real rate	−0.01	0.30	0.45	−0.24	−0.24	0.76	0.11	0.11	−0.08
Earnings/price + growth − real rate	0.06	0.20	0.41	−0.34	−0.34	0.66	0.06	0.06	0.03
Smooth earnings/price + growth − real rate	0.27	0.39	0.60	−0.28	−0.28	0.74	0.04	0.04	0.02
Book-to-market + growth − real rate				−0.82	−0.91	0.89	−0.14	−0.02	−0.27
B: Annual Returns									
Dividend/price	9.95	4.53	3.67	9.46	5.99	6.88	−16.19	−1.38	−7.98
Earnings/price	7.45	5.34	7.58	5.08	3.25	2.56	−6.06	0.88	1.47
Smooth earnings/price	12.51	8.22	10.49	4.93	3.71	3.71	−8.86	1.33	1.33
Dividend/price + growth	2.77	3.05	4.83	1.76	1.74	6.61	1.87	1.82	0.28
Earnings/price + growth	2.21	2.38	4.37	−0.85	−0.85	3.97	1.63	1.63	1.60
Smooth earnings/price + growth	3.73	3.87	6.38	1.27	1.19	4.65	2.30	2.23	1.81
Book-to-market + growth				−7.09	−5.16	10.34	−0.24	0.14	−2.43
Dividend/price + growth − real rate	4.40	4.51	1.67	−3.57	−3.56	8.28	2.19	2.19	−2.95
Earnings/price + growth − real rate	3.44	3.49	1.25	−4.68	−4.68	7.16	1.88	1.88	−0.64
Smooth earnings/price + growth − real rate	5.34	5.37	3.19	−4.91	−4.84	7.32	2.36	2.36	−0.47
Book-to-market + growth − real rate				−3.36	−4.22	11.85	−0.25	0.35	−6.20

Out-of-sample R^2

Fixed coefficients $\alpha = 0$, $\beta = 1$

horizon	R^2_{OS}
1 mo	0.42%
2 mo	1.11%
3 mo	1.49%
6 mo	4.86%
1 yr	4.73%

Table: R^2 using SVIX_t^2 as predictor variable with $\alpha = 0$, $\beta = 1$

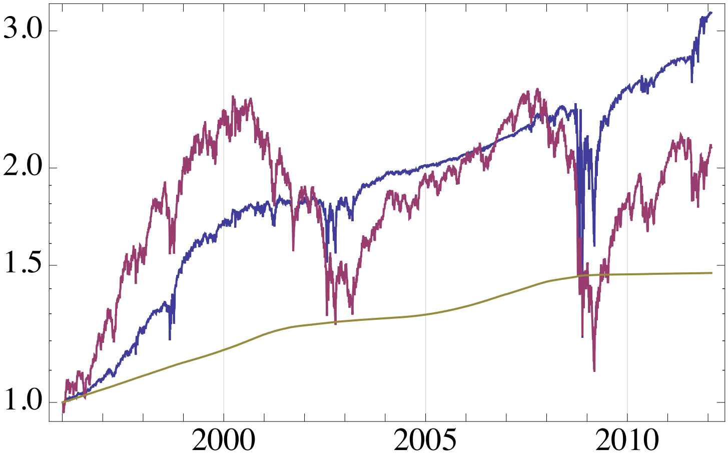
Are the R^2 too low?

No. Small R^2 $\longrightarrow$ high Sharpe ratios

- We can use the predictor in a market-timing strategy
- On day t, invest α_t in the S&P 500 index and $1 - \alpha_t$ in cash
- Choose α_t proportional to 1-mo SVIX_t^2
- Earns a daily Sharpe ratio of 1.97% in sample
- For comparison, the daily Sharpe ratio of the index is 1.35%
- The point is not that Sharpe ratios are necessarily the right metric, but that apparently small R^2 can make a big difference

The value of a dollar invested

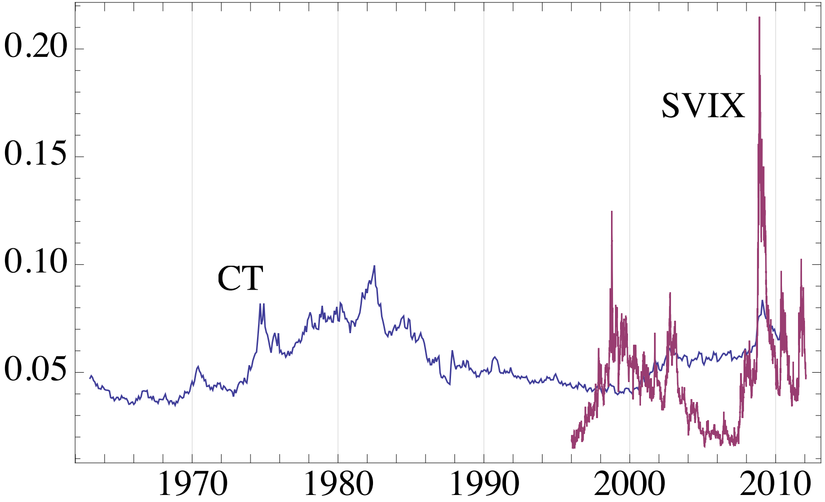
In cash (yellow), in the S&P 500 (red), and in the market-timing strategy (blue)



- Mean: 35% S&P 500, 65% cash. Median: 27% S&P 500, 73% cash.

Risk-neutral variance vs. valuation ratios

Blue: earnings yield (Campbell and Thompson (2008)). Red: risk-neutral variance

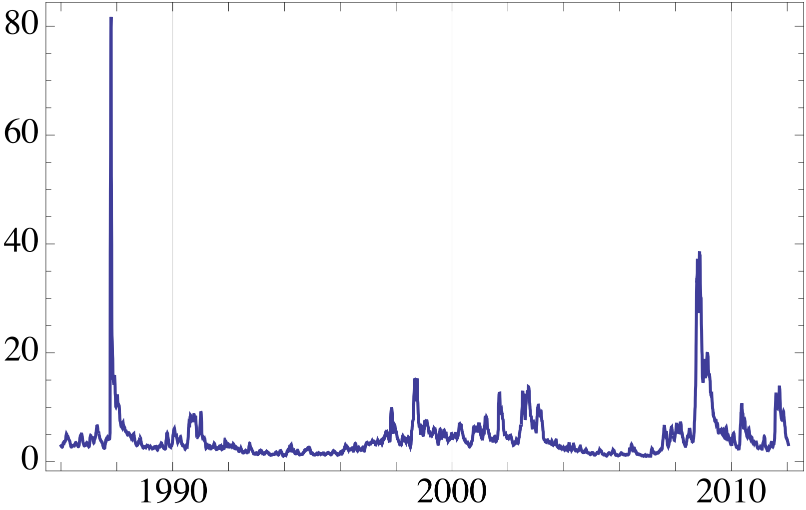


Black Monday, 1987

- It is interesting to identify points at which my claims contrast most starkly with the conventional view based on valuation ratios
- **In particular**: what happened to the equity premium during and immediately after Black Monday in 1987, which was by far the worst day in stock market history?
- Valuation ratios: it moved from about 5% to about 6%
    - Suppose $D/P = 2\%$ and then market *halves* in value. D/P only increases to 4%
- Options: it exploded
    - Implied risk premium about twice as high as in the recent crisis

Risk-neutral variance exploded on Black Monday

1mo horizon, annualized and using VXO as a proxy for true measure



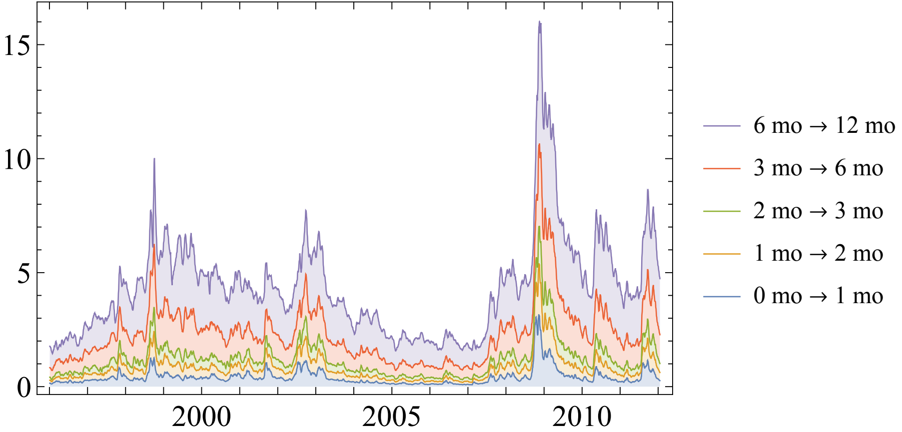
Risk-neutral variance vs. valuation ratios

- Campbell–Shiller: $d_t - p_t = k + \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \left(r_{t+1+j} - \Delta d_{t+1+j} \right)$
- If dividend growth is unforecastable,

$$d_t - p_t = k + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \, r_{t+1+j}$$

- Dividend yield measures expected returns *over the very long run*
- Difference between SVIX_t^2 and $d_t - p_t \approx$ gap between short-run expected returns and long-run expected returns
    - Consider the late 1990s: 1-year expected returns (SVIX_t^2) were high, very long-run expected returns (D/P) were low

The term structure of the equity premium



- In bad times, high equity premia can mostly be attributed to very high short-run premia

What's the equity premium right now?

- Annualized 1-month equity premium $\approx 20.77\%^2 = 4.3\%$

Outline

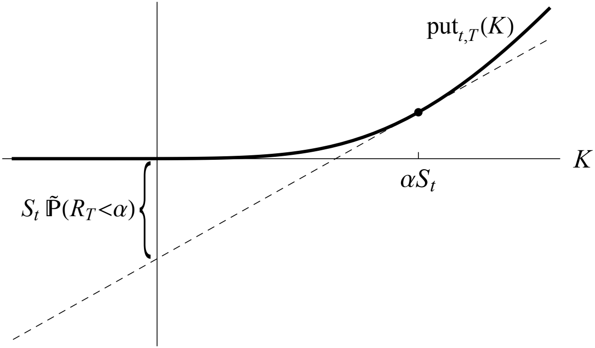
What is the probability of a 20% decline?

- Take the perspective of an investor with log utility whose portfolio is fully invested in the market
- Expectations of such an investor obey the following relationship:

$$\widetilde{\mathbb{E}}_t X_T = \frac{1}{R_{f,t}} \mathbb{E}_t^* \left[X_T R_T \right]$$

- So if we can price a claim to $X_T R_T$ then we know the log investor's expectation of X_T
- Interpretation: "What a log investor would have to believe about X_T to make him or her happy to hold the market"

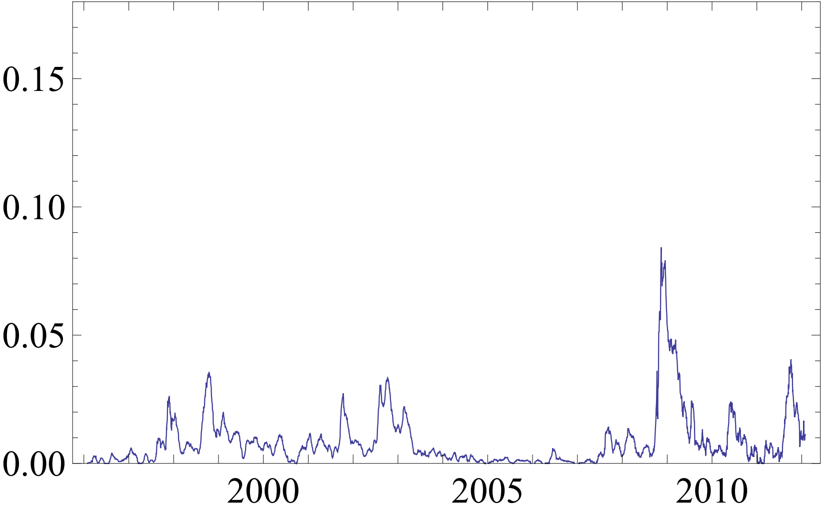
What is the probability of a 20% decline?



- $\widetilde{\mathbb{P}}\left(R_T < \alpha\right) = \alpha \left[\text{put}'_{t,T}(\alpha S_t) - \frac{\text{put}_{t,T}(\alpha S_t)}{\alpha S_t}\right]$

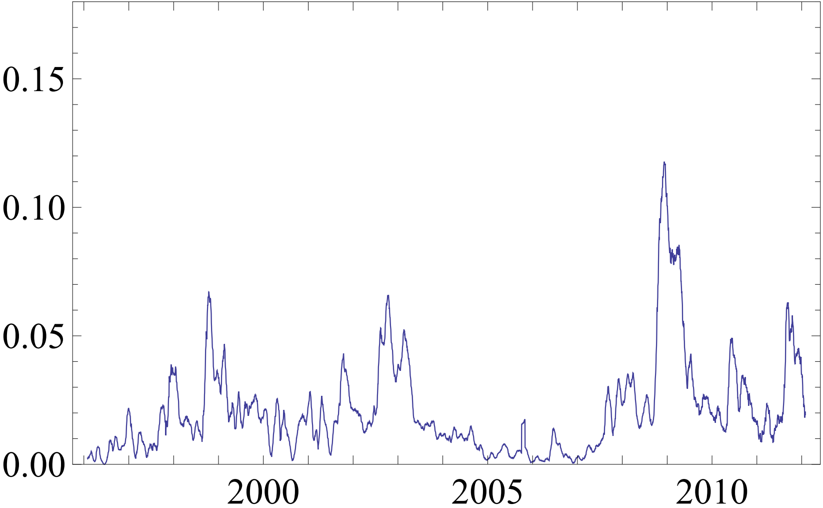
What is the probability of a 20% decline?

$T = 1$ mo



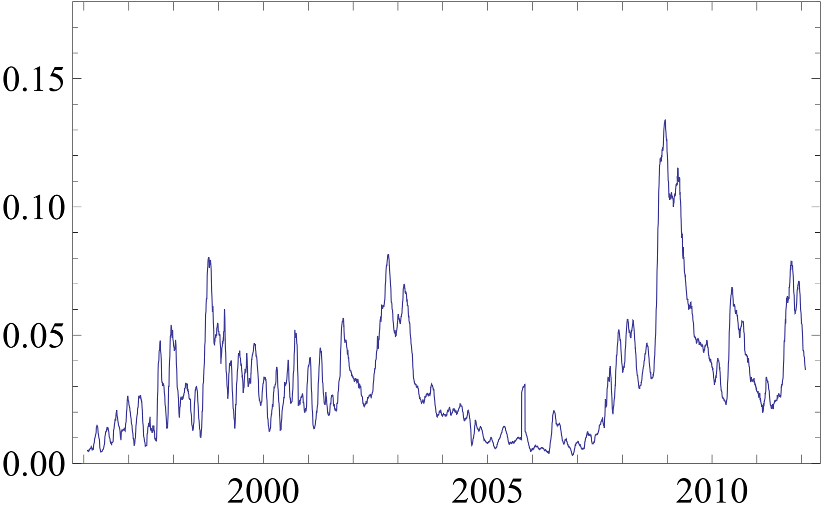
What is the probability of a 20% decline?

$T = 2$ mo



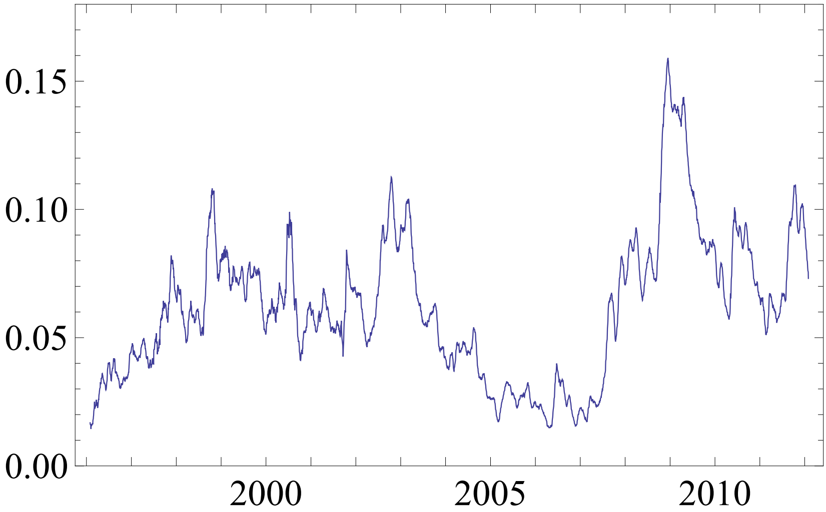
What is the probability of a 20% decline?

$T = 3$ mo



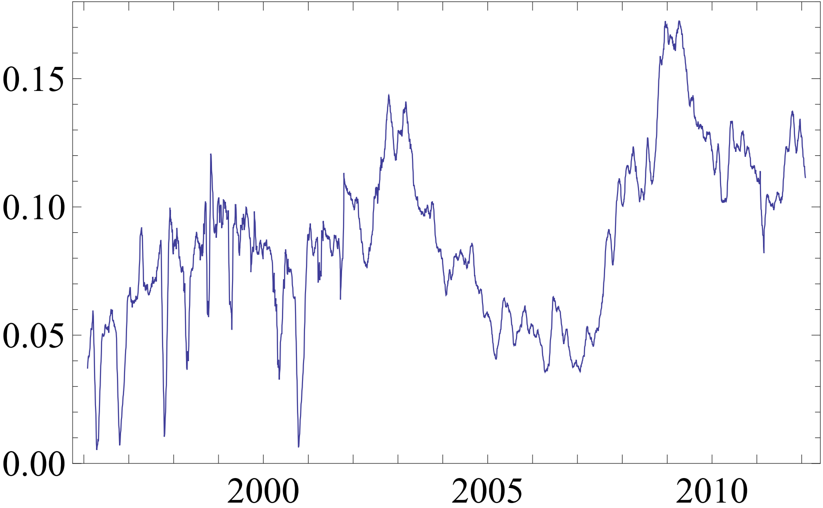
What is the probability of a 20% decline?

$T = 6$ mo

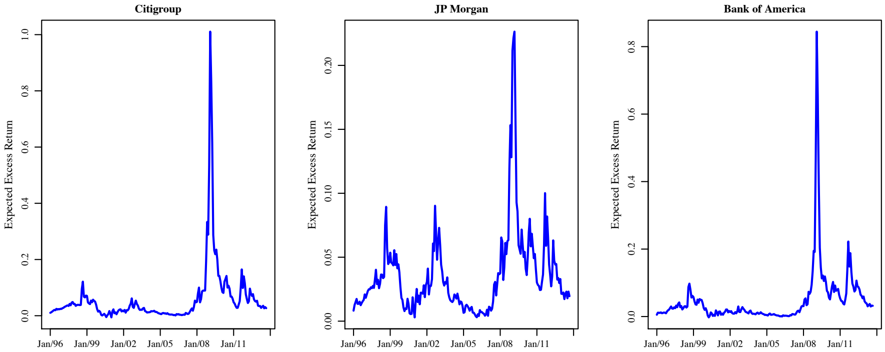


What is the probability of a 20% decline?

$T = 1$ yr



New directions



- What is the expected return on an individual stock? (joint work with **Christian Wagner**, Copenhagen Business School)
- Our approach outperforms conventional predictors

Conclusions

- Have shown how to measure the equity premium in real time
- The results point to a new view of the equity premium
  - Extremely volatile, at faster-than-business-cycle frequency
  - Right-skewed, with occasional opportunities to earn exceptionally high expected excess returns in the short run
- Black Monday, October 19, 1987, provides the starkest illustration
  - D/P: annual equity premium moved from 4% to 5%
  - SVIX: equity premium was $\sim 8\%$ over the next one month