Lecture 36: Entanglement at a distance (May 2nd, 2007), L.Motl

Entanglement at a distance is probably the most shocking feature of quantum mechanics. This bizarre feature of quantum mechanics was first pointed out by Einstein, and later in 1935 by Einstein and his collaborators Boris Podolsky and Nathan Rosen (EPR). They hypothesized that quantum mechanics had to be wrong because in various contexts, it was predicting correlations between measurements that looked like voodoo to EPR. Meanwhile, the proponents of quantum mechanics insisted that the predictions of quantum mechanics were right.

Additional reading if you wish: Griffiths A.1-A2; Mermin - American Journal of Physics 49, 940, 1981; or, for a more popular presentation of the same experiment, chapter 4 of "The Fabric of the Cosmos" by Brian Greene

The EPR arguments were improved and quantified by others who had similar preconceptions as Einstein, especially by John Bell – who discovered Bell's inequalities. Who was right? In the last decades, the experiments have verified the strange predictions of quantum mechanics. Einstein was wrong and there cannot be any classical "deeper" explanation of the wavefunction in quantum mechanics, except for some very contrived non-local models. The wavefunction in quantum mechanics simply must be probabilistic and remarkably, we can prove this statement experimentally.

Entangled spins

Imagine that we have a source of spin-1/2 fermions. It emits pairs of particles to the left and to the right. Draw a picture: a source in the middle and two detectors on the left and on the right. The source can be such that the wavefunction of the two fermions is in the singlet state:

$$|\psi\rangle = \frac{|\uparrow\rangle\,|\downarrow\rangle\,-\,|\downarrow\rangle\,|\uparrow\rangle}{\sqrt{2}}$$

It is an entangled state because it cannot be written in a factorized form, $|\psi_1\rangle|\psi_2\rangle$. In fact, it is "maximally entangled" because if you compute the reduced density matrix for the first (or the second) particle by tracing over the second particle's degrees of freedom, you obtain the density matrix of "complete ignorance", namely $(1/2) \times \mathbf{1}$, which is "maximally mixed" because it has the maximal entropy among all matrices of the same size.

The entanglement implies that if the first particle is spinning up, the other particle is spinning down, and vice versa. The two experimentalists will always find the opposite spins. Moreover, quantum mechanics guarantees that the same conclusion holds for the spin with respect to any axis. For example, choose the x-axis. We know that

$$|\uparrow\rangle\ = \frac{|\rightarrow\rangle + |\leftarrow\rangle}{\sqrt{2}}, \qquad |\downarrow\rangle\ = \frac{|\leftarrow\rangle - |\rightarrow\rangle}{\sqrt{2}}$$

where $|\rightarrow\rangle$ and $|\leftarrow\rangle$ are eigenstates of σ_x with eigenvalues $+1$ and -1, respectively. If you express $|\psi\rangle$ in terms of these states, you will get

$$|\psi\rangle = \frac{|\rightarrow\rangle|\leftarrow\rangle - |\leftarrow\rangle|\rightarrow\rangle}{\sqrt{2}}$$

Indeed, the spins of the two particles are always opposite whatever axis you choose. This $|\psi\rangle$ is the only antisymmetric wavefunction of the two spins (up to a normalization), and because this definition is axis-independent, all of its properties must be axis-independent, too.

For Einstein, this correlation with respect to all axes was already hard to swallow. He was imagining that if the spins with respect to the z-axis are anticorrelated – if there is a 50 percent probability that it is "up-down" and 50 percent that the spins are "down-up" – then the particles must actually be at (with a solid, classical meaning of "be") one of these two configurations of the spins before we measure them. If we decide to measure the spin with respect to the x-axis, things change. Because there is a 50 percent probability to get "spin left" and 50 percent to get "spin right" and because the electrons are independent "pieces of objective reality", using Einstein's words, EPR believed that for the measurements of the spin with respect to the x-axes, all four combinations had to have the probability of 25 percent. Quantum mechanics gives us a different result. Only two configurations are possible and they always have 50 percent each.[1]

Let's emphasize from the beginning that quantum mechanics was correct and Einstein was wrong. Experiments show that it is the case.

Adding switches to the detectors

In order to make our discussion even more interesting, let us improve the detectors in our experiment. Each of the new two detectors will have a switch that can be set to the values $1, 2, 3$. These switches determine the axis with respect to which we measure the spin. We will choose three axes in a plane whose relative angles are 120 degrees. To be completely specific, let us decide that the detectors will measure the following components of the spin depending on the switch:

$$(1): \quad \hat{J}_z, \qquad\qquad (2): \quad \frac{-\hat{J}_z + \sqrt{3}\hat{J}_x}{2}, \qquad (3): \quad \frac{-\hat{J}_z - \sqrt{3}\hat{J}_x}{2}$$

The eigenvalues of either of these operators are $\pm\hbar/2$; they're just like $\hat{J}_z$ in *some* system of coordinates. Moreover, the detectors are constructed to flash whenever they determine the spin with respect to the axis you chose.

If the eigenvalue found by the left detector is $+\hbar/2$, it flashes the green light; if the eigenvalue found by the left detector is $-\hbar/2$, it flashes the red light. For later convenience, let us use the opposite rules for the right detector. If the eigenvalue found by the right detector is $-\hbar/2$, it flashes the green light; if the eigenvalue found by the right detector is $+\hbar/2$, it flashes the red light.

Imagine that Scully is playing with the detector on the left and Mulder is playing with the detector on the right. (In Greene's book, Scully and Mulder are actually playing with boxes they received from the aliens, but the physics is fully equivalent.) What does quantum mechanics tell us about the flashes? One observation has already been done:

> **When the switches of the two detectors are set to the same value (1-1 or 2-2 or 3-3), the flashes have the same color (red-red or green-green) for both detectors.**

Why is it so? It's because if we choose the same values for the switches, we measure the spins of the two fermions with respect to the same axis. Because $|\psi\rangle$ is in a singlet state, we are guaranteed to obtain the opposite values of the given component of the spin (an easier explanation: the total spin must be zero), and because we have configured the colors of the detector flashes in the opposite way, different eigenvalues from the two detectors translate to the same color of the flashes.

Everyone agrees. Quantum mechanics, Scully, Mulder, experiments – all of them agree that in this particular experiment, the colors will be identical whenever the switches are set to the same value.

[1] A mathematically isomorphic discussion applies to experiments with photons that are right-handed or left-handed – the counterpart of the spin with respect to one axis; or x-linearly-polarized or y-linearly-polarized – the counterpart of the spin with respect to another axis. The maximally entangled state remains entangled in all bases you choose – circularly polarized bases or linearly polarized bases with respect to any pair of orthogonal axes.

Bell's inequality

But what Einstein did not like was the idea that quantum mechanics could only predict the probabilities but it did not actually say what the spins were prior to the measurements. He was convinced that "God did not play dice" and he also asked:

> **Is the Moon there only when you look at it?**

Einstein argued that either

- quantum mechanics was complete and objective reality such as the position of particles or something that replaces it did not exist
- or quantum mechanics was incomplete and objective reality exists

Einstein clearly preferred the second alternative; the standard approach to quantum mechanics – the approach of this course – prefers the first one.

Although the example of the Moon is exaggerated because its wavefunction of its center-of-mass may be thought of to be almost completely localized so that its position obeys the laws of classical physics with a great accuracy, it is true that quantum mechanics says exactly what irritated Einstein so much, at least about the microparticles. The position or the spin of the particles is fuzzy, described by a wavefunction, and it only becomes sharp when we measure these quantities. Does it mean that these quantities are not well-defined before we measure them? According to quantum mechanics, this is exactly how the world works. For Einstein, this was unacceptable.

It was equally unacceptable for John Bell at CERN. Moreover, Bell was the first one who realized in 1964 that the question whether the quantities were well-defined was defining more than just a difference between two religions that could never be decided. Bell realized that the dispute between Einstein and quantum mechanics – whether some quantities have well-defined, objective values even before you measure them – can be decided by an experiment! This experiment has eventually shown that Einstein and Bell were wrong and quantum mechanics was correct, but this fact makes Einstein's and Bell's insights just a little bit less ingenious. Incidentally, the quantities that Einstein and Bell (incorrectly) believed to be deciding about the outcome of the experiments – that looked random according to quantum mechanics – were called "hidden variables"; they were "hidden" because quantum mechanics did not see them.

What's Bell's argument? Imagine that we measure the spins of millions of particles and the switches of the two detectors are set to completely random and uncorrelated values; the probability of each arrangement of the switches is 1/9. The quantity we want to measure is the following:

> **What is the probability that the colors match when the switches are set to random values?**

Surprisingly, we will see that quantum mechanics gives different, incompatible answers from any theory that assumes that there exists "local objective reality" before the measurement. Let us start with John Bell who will try to defend Einstein's viewpoint.

Bell's testimony about the correlation of the colors

Prof. Bell says: Because it has been established experimentally that the colors have to match whenever the switches are set to the same value, there must exist a mechanism that guarantees that the colors will match. Because I believe that the colors of the flashes are determined by some "reality" that objectively exists even before the measurements (Wolfgang Pauli the quantum mechanician, who sits in the audience, vehemently turns his head to show his disagreement), there must exist a simple "program" before the measurements that generates the colors as a function of the values we set for the switches. Such a program assigns the colors $C_1, C_2(S_1, S_2)$ as functions of the switches $S_1, S_2 = 1, 2, 3$. Because the detectors are spatially separated, the program must actually decide about the color C_1 according to S_1 only and similarly for C_2, S_2. If it were not the case, physics near the first detector would be directly affected by the decisions about the switch of the other detector that can be many light years away. A decision that can be made instantly and just before the measurement. Such an action at a distance would violate basic intuition about locality as well as relativity.

This implies that the left detector immediately before the measurement must have a program in it, like the following:

1 – red, 2 – green, 3 - red

This precise colors depend on the detailed properties of the fermions and the detectors and their surroundings, including any hypothetical "hidden variables". The program informs the detector which color should be flashed for different choices of the switch. The other detector on the right has a similar program. Because we know that the colors have to match when the switches are set to the same value, the other detector must actually have the same program:

1 – red, 2 – green, 3 - red

For this particular program, the colors will agree in 5/9 of cases:

11, 22, 33, 13, 31

They must agree if the switches are set to the same value (11 or 22 or 33), but they also agree in the cases 13 and 31 because both 1 and 3 are programmed to flash the red light.

The same probability 5/9 of the same color holds for all programs that use one color for two values of the switch and the other color for the last value. For the two remaining programs (red-red-red and green-green-green) that generate the color independently of the switch, the probability is 9/9 that the colors will agree for random configurations of the switches. At any rate, the overall probability will be somewhere in between 5/9 and 9/9 for a general combination of "diverse programs" and "uniform programs":

$$\frac{5}{9} \leq p_{\text{same color}} \leq 1$$

Because I just proved this inequality to you, let us call this inequality and all analogous inequalities *Bell's inequalities*. These inequalities tell you that the probability of a correlation between two quantities (such as the color) is bounded unless we want to sacrifice common sense. Consequently, I predict that the experiments will show that the colors will agree in more than 55 percent of the cases, and if my quantum colleagues predict something else, they will be proved wrong, I think. (Pauli gets really upset.)

Quantum mechanical prediction for the correlation

Well, the quantum colleagues predict 50 percent probability for the same colors if the switches are set to random values. Why is it so? The switches will be set to the same value in 1/3 of the cases. In these cases, the probability that the colors match is $p = 1$. In the remaining 2/3 of the cases, the probability that the colors agree will be 1/4, as we show below. The overall probability is therefore the weighted average

$$p \;=\; \frac{1}{3} \times 1 + \frac{2}{3} \times \frac{1}{4} \;=\; \frac{1}{2}.$$

Why is the probability equal to 1/4 in the case that we choose different values of the switches? Start with the initial singlet state

$$|\psi\rangle = \frac{|\uparrow\rangle\,|\downarrow\rangle\, - \,|\downarrow\rangle\,|\uparrow\rangle}{\sqrt{2}}$$

and make first a measurement of the first particle (without a loss of generality). Without a loss of generality, you obtain $|\uparrow\rangle$. This means that you may forget about all the contributions to the wavefunction that disagree with your current measurement. Only the contributions that start with $|\uparrow\rangle$ are to be kept. In other words, the wavefunction after the first measurement "collapses" to

$$|\psi\rangle_{after} = |\uparrow\rangle\,|\downarrow\rangle\,.$$

You should not ask too many questions about the collapse itself. The only role of the wavefunction is to predict the probabilities of different outcomes of the measurement, and what you imagine that the wavefunction is doing before the measurement or after the measurement is unphysical and unmeasurable.

Now you also make the measurement of the second particle's spin with respect to another axis that differs from the z-axis by $\vartheta = 120°$. You should remember that the probability that you measure "up" with respect to this axis, if the spin of the second particle is "down" with respect to the z-axis, is

$$p = |d^{1/2}_{1/2,-1/2}(\vartheta)|^2 = \ldots = \sin^2(\vartheta/2)$$

For $\vartheta = 120°$, you obtain $p = \sin^2 60°$ i.e. $p = 3/4$. This is the probability that both particles will generate the same eigenvalue of the spin with respect to the two axes that differ by 120 degrees. Because of the different color conventions for the two detectors, the probability of getting the same colors is 1/4 which completes a missing step in the calculation above.

Experimental resolution

This looks like a serious disagreement between Prof. Bell on one side and quantum mechanics on the other side. What do the experiments say? Well, the experiments answer a resounding "Yes" to the quantum mechanical predictions including the 50 percent overall probability that the colors match for random values of the switches. In other words, the experiments tell us that the hidden variables cannot exist, and even in the case that we want to believe that they exist, they would have to behave in a very non-local way. Einstein, the author of relativity, would probably dislike such a nonlocality.

Remaining questions?

The discussion in the lecture today should have convinced you that the entanglement is a real and experimentally verified prediction of quantum mechanics. Moreover, this phenomenon is completely incompatible with any classical picture that describes the world in terms of "objectively existing" classical quantities that only interact locally. In some sense, quantum mechanics seems to act non-locally and this non-locality is experimentally proved. In terms of the "programs" we discussed previously, such a non-locality allows us to consider more general programs that determine the colors to be different in the cases 13 and/or 31 so that the overall probability that the colors agree may decrease below 55 percent.

> **The canonical conclusion is quantum mechanics is correct, it predicts certain things that look non-local, but the non-locality does not allow us to send any actual information faster than light because the wavefunction has a purely probabilistic interpretation. We must give up not only determinism but any idea of**

an objective, uniquely determined "classical" state of the system prior to the measurement.

A possible loophole is that there exists a classical, non-probabilistic description of the real world and the correlations in the experiments we discussed are caused by some superluminal, material, instant communication between the detectors. These would be real signals faster than light and they would contradict special theory of relativity. They would indeed allow you to send actual information faster than light to the opposite side of our galaxy, at least in principle. From the viewpoint of other reference frames, such faster-than-light signals would go backward in time, and they would allow you to kill your grandfather before he met your grandmother – which would make the Universe inconsistent.

Such things should not happen and indeed, they do not happen in the real world. It is because in the real world, the wavefunction is probabilistic. No real signals ever propagate faster than light; just illusionary signals based on wrong preconceptions about reality may seem to propagate superluminally but these can't be used to transmit any information. The predictions of quantum mechanics work and they are compatible with special relativity. In fact, quantum field theory (and its ramification, string theory) is a framework that incorporates both relativity as well as quantum mechanics.

Absence of superluminal signals

You should realize that even though the quantum mechanical entanglement shows that there is some subtle correlation between detectors that may be very far away from each other, there is no actual information being sent in between the detectors that would guarantee that the correlation will work in the cases $11, 22, 33$ even though it will be smaller in the remaining cases than Einstein would have thought. More precisely, there is no way to send actual "packets" of information faster than light. It is because all the quantum mechanical predictions for the detector 1 are completely independent of the decisions what to do with the switch of the detector 2. Recall that the initial state is

$$|\psi\rangle = \frac{|\uparrow\rangle\,|\downarrow\rangle - |\downarrow\rangle\,|\uparrow\rangle}{\sqrt{2}}.$$

One can easily show that all predictions for the measurement done with the first system are fully contained in the reduced density matrix of the first system, and this density matrix is simply the "ignorance" density matrix

$$\rho_1 = \frac{|\uparrow\rangle\,\langle\uparrow| + |\downarrow\rangle\,\langle\downarrow|}{2}$$

which implies that all measurements of the spin with respect to any axis will give the 50 percent probabilities for both alternatives, regardless of the decisions what to do with the other detector. Scully and Mulder who play with these detectors may use the correlations to figure out what happened on the other side assuming that the other character chose the same value of the switch. But these correlations cannot be used to send information from one place to the other. It is because Scully cannot order her detector to behave in one way or another. Even her own detector behaves randomly, and therefore the correlations are not helpful in sending information.

If there were – at least in principle – a way to order your detector to show one result or another (by playing with some "hidden variables"), a result that would no longer be probabilistic, and if this method kept the correlations between the detectors, you could indeed send information faster than light. That would be a real contradiction with locality and relativity. This can't happen which means that the wavefunction must be probabilistic and the mortal beings can't look into the "algorithm" how the random outcomes of the experiments are decided; not even in principle.

Bell's original setup and Aspect's experiments

I was cheating a bit concerning the history. The example with the detectors with 3 switches and 2 colors was not one that John Bell actually proposed in 1964. He has originally proposed a slightly less entertaining arrangement but it also involved a spin-0 state made of two spin-1/2 fermions.

Let us say in advance that the French physicist Alain Aspect has performed experiments very similar to Bell's original description – but with the electrons replaced by photons and the Stern-Gerlach apparata replaced by polarizers – in 1981 and 1982. His detectors were 13 meters apart and the polarizers were adjusted when the photons were already in flight. This guaranteed that no conventional information slower than light could have propagated between the detectors. In the last decade, Swiss and Austrian groups have extended the experiments using fiber optics and other gadgets to make the distance between the detectors to be around 10 kilometers. All these experiments confirmed the predictions of quantum mechanics.

Quantum calculation of Bell's original problem

OK, back to Bell's original picture. Two fermions start in the singlet state $|\psi\rangle$ we described previously and they again fly in the opposite directions. Two Stern-Gerlach detectors detect the fermions and measure the spins with respect to the directions $\hat{a}$ and $\hat{b}$ that are transverse to the direction of motion. The question is what is

$$P(\hat{a},\hat{b}) = \langle \sigma_{\hat{a}}\sigma_{\hat{b}}\rangle$$

i.e. the average value of the product of the two spins in units of $\hbar^2/4$. Let us start with the quantum answer. Let us choose the coordinate axes so that the fermions move along the y direction and the directions for the spin measurements are, without a loss of generality, along the z axis and along another axis in the xz-plane. This means that

$$\sigma_{\hat{a}} = \sigma_z, \qquad \sigma_{\hat{b}} = \cos\vartheta\ \sigma_z + \sin\vartheta\ \sigma_x$$

where ϑ is the angle in between $\hat{a},\hat{b}$. The expectation value in our state is easy to calculate:[2]

$$\begin{aligned} \langle \sigma_{\hat{a}}\sigma_{\hat{b}}\rangle &= \frac{\langle\uparrow|\,\langle\downarrow| - \langle\downarrow|\,\langle\uparrow|}{\sqrt{2}}\sigma_z^{(a)}(\cos\vartheta\ \sigma_z^{(b)} + \sin\vartheta\ \sigma_x^{(b)})\frac{|\uparrow\rangle\,|\downarrow\rangle - |\downarrow\rangle\,|\uparrow\rangle}{\sqrt{2}} \\ &= -\frac{\cos\vartheta}{2} + 0 + 0 - \frac{\cos\vartheta}{2} = -\cos\vartheta = -\hat{a}\cdot\hat{b} \end{aligned}$$

We have evaluated the expectation value of the operator in each of the four bra-ket combinations that follow from the distribution law. And we have also used the well-known matrix elements of the Pauli matrices with respect to $|\uparrow\rangle$ and $|\downarrow\rangle$: $\langle\uparrow|\,\sigma_z\,|\uparrow\rangle = 1 = -\langle\downarrow|\,\sigma_z\,|\downarrow\rangle$. The expectation value of the term proportional to $\sin\vartheta$ never contributed.

Classical, hidden-variable calculation of Bell's original problem

OK, quantum mechanics gives us $P(\hat{a},\hat{b}) = -\cos\vartheta$. What about the local theories of hidden variables? If we measure the spin of the particle A, we obtain a result ± 1 that depends on the vector $\hat{a}$ defining the axis and on some extra hidden variables that we call λ. This value of the spin is called $A(\hat{a},\lambda)$ and it does not depend on $\hat{b}$ because the choice of the axis in the other measurement can't influence the result of the measurement A in a local theory. In a similar way, the spin $B(\hat{b},\lambda)$ only depends on the axis $\hat{b}$ and the hidden variables λ but not on $\hat{a}$.

[2] We now use the convention in which the first $\langle\uparrow|$ vector represents the first, a-particle, both in the case of ket-vectors as well as bra-vectors.

Now we know that if we choose the axes identical $\hat{a} = \hat{b}$, we must obtain the opposite spins by the conservation of angular momentum which means that

$$A(\hat{a}, \lambda) = -B(\hat{a}, \lambda).$$

The average product of the spins is then given by

$$P(\hat{a}, \hat{b}) = \int d\mu(\lambda) A(\hat{a}, \lambda) B(\hat{b}, \lambda) = -\int d\mu(\lambda) A(\hat{a}, \lambda) A(\hat{b}, \lambda)$$

where $d\mu(\lambda) = \rho(\lambda) d\lambda$ is an integration measure that determines the probability distribution for the hidden variables. We can also replace $\hat{b}$ by another axis $\hat{c}$ to write the same formula for $P(\hat{a}, \hat{c})$ as well as the following difference:

$$\begin{aligned} P(\hat{a}, \hat{b}) - P(\hat{a}, \hat{c}) &= \int d\mu(\lambda) \left[-A(\hat{a}, \lambda) A(\hat{b}, \lambda) + A(\hat{a}, \lambda) A(\hat{c}, \lambda) \right] \\ &= -\int d\mu(\lambda) A(\hat{a}, \lambda) A(\hat{b}, \lambda) \left[1 - A(\hat{b}, \lambda) A(\hat{c}, \lambda) \right] \end{aligned}$$

In the last step, we used $A^2(\hat{b}, \lambda) = +1$. Next you should notice that

$$\left| A(\hat{a}, \lambda) A(\hat{b}, \lambda) \right| \leq 1, \qquad \left| A(\hat{b}, \lambda) A(\hat{c}, \lambda) \right| \leq 1 \qquad \Rightarrow \qquad \left[1 - A(\hat{b}, \lambda) A(\hat{c}, \lambda) \right] \geq 0$$

which implies that

$$\left| P(\hat{a}, \hat{b}) - P(\hat{a}, \hat{c}) \right| \leq \int d\mu(\lambda) \left[1 - A(\hat{b}, \lambda) A(\hat{c}, \lambda) \right]$$

By identifying the last term, we can finally write down "the" Bell's inequality

$$\left| P(\hat{a}, \hat{b}) - P(\hat{a}, \hat{c}) \right| \leq 1 + P(\hat{b}, \hat{c})$$

Is it satisfied by the result from quantum mechanics? That would mean that

$$|\cos \vartheta_{ab} - \cos \vartheta_{ac}| \leq 1 - \cos \vartheta_{bc}$$

However, this inequality (Bell's inequality) is easily violated in quantum mechanics (as well as by the experiments). For example, choose $\hat{a}, \hat{b}, \hat{c}$ in the same plane and $\hat{a}, \hat{b}$ orthogonal so that $\cos \vartheta_{ab} = 0$. Then the inequality says that

$$|\cos \vartheta_{ac}| \leq 1 - \sin \vartheta_{ac}$$

which is actually violated for any $0 < \vartheta_{ac} < \pi/2$; for example, for $\vartheta_{ac} = 45^\circ$ we obtain a wrong inequality $.707 < .293$. This means that the experimentally verified quantum mechanical prediction cannot be computed from a "classical" theory with local hidden variables – and most likely, other hidden variable theories fail, too.

Next week: interpretation of quantum mechanics and decoherence

We will now continue with the foundational questions of quantum mechanics. The insights about entanglement we discussed so far constrain our freedom to give the wavefunction various interpretations and so forth. Nevertheless, the history of interpretation of quantum mechanics is interesting and some moderate uncertainties remain. We will look at them.

Second big part of the lecture: Decoherence and foundations of QM

Quantum mechanics significantly differs from classical physics and its interpretation has been a controversial issue since the very beginning.

Additional reading if you wish: Griffiths A.3; Omnes chapters 17-19

We will discuss the major interpretational frameworks today and the questions they want to address. One of the most important recent insights from the 1980s that every up-to-date treatment of the foundations of quantum mechanics must take into account is the so-called decoherence, and we will dedicate special time to this phenomenon. Our discussion at the beginning of the lecture will be mostly non-quantitative.

List of interpretations

The most important interpretations are the following ones:

- "Shut up and calculate" interpretation by Richard Feynman
- Copenhagen interpretation (whose wavefunction is not real)
- Copenhagen interpretation (whose wavefunction is real)
- Interpretation with a special role of consciousness
- Transactional interpretation
- Bohm-de Broglie interpretation
- Everett's many-worlds interpretation
- Consistent histories

The differences between them are sometimes subtle and philosophical in character. Among the interpretations above, the Bohmian interpretation has become very awkward after Bell's inequalities were understood and experimentally falsified: it seems to contradict special theory of relativity that prohibits actual superluminal signals that would be necessary for the Bohmian picture to agree with observations of the entanglement. The interpretations with a special role of consciousness are obsolete today because of our understanding of decoherence. The motivation for the transactional interpretation remains obscure to nearly all physicists and we won't discuss it at all.

Your instructor thinks that the Consistent Histories are the most satisfactory and modern interpretation. In reality, they are just a small update of the old Copenhagen interpretation where some of the subtle details are explained and decoherence is appreciated. The many-worlds interpretation is popular with many active physicists, too, and Everett – who pioneered it – was also an early pioneer of decoherence. The Copenhagen interpretation is still called the "orthodox" or "canonical" approach. Let us look at all of these interpretations.

Shut up and calculate

Feynman's dictum "Shut up and calculate" captures the most favorite approach of most active, technically oriented physicists. It is important to know how to use the mathematical tools, how to extract the predicted probabilities, and how to compare them with observations because these are the only physically meaningful and testable questions. A physicist should leave all other questions to philosophers, artists, and crackpots, Feynman argued.

Copenhagen interpretation

The Copenhagen interpretation – named after the city from which Niels Bohr influenced all of his soulmates – is the classical interpretation. Max Born was the first one who figured out that the wavefunctions are interpreted probabilistically. The Copenhagen interpretation states that "quantum objects" (usually particles and microscopic systems) are described by quantum mechanics. "Classical objects" (usually macroscopic ones) essentially follow the laws of classical physics. They can be used to measure the properties of the "quantum objects". The measurement is an interaction between a "quantum object" and a "classical object" in which the wavefunction of the "quantum object" collapses to one of the final states with a well-defined value of the measured quantity (such as the position of the electron that reaches the screen). The probabilities of different outcomes are determined from the wavefunction. The wavefunction can either be thought of as a "state of our knowledge" or an actual "wave" in a configuration space that suddenly collapses, but asking about the physical origin of the collapse is unphysical. Nevertheless, you may imagine that the evolution of the wavefunction has two stages: the smooth evolution according to the Schrödinger equation; and the sudden collapse induced by the act of measurement. This strange collapse is a topic that is generally studied as the **measurement problem**.

The Copenhagen interpretation has been enough to explain and understand the results of all experiments that have ever been done (except for some experiments involving decoherence that we will mention at the end; decoherence was only appreciated since the 1980s). However, it has the following basic aesthetic flaws:

- no clear criterion how to divide the objects into "classical" and "quantum" one is provided
- if someone provides a "cutoff" that divides the objects in this way, it is arbitrary and unnatural
- a related problem: even macroscopic objects should follow the laws of Nature, i.e. quantum mechanics, and it is not clear how a priviliged basis is chosen (see the Schrödinger cat problem below)
- a very related problem: it is not really explained why the classical physics is a limit of quantum physics even for the process of measurement
- if the wavefunction is thought to be real, the origin of collapse is unexplained and no non-linearity that would be required for the collapse are found in the theory
- such a collapse looks non-local

Most of these problems are explained by decoherence. Let us look at the classic paradox.

Schrödinger cat

Schrödinger considered a cat under a system of hammers that were turned on whenever a decay of a neutron was detected. The fate of the cat depends on a random process described by quantum mechanics (a decaying particle). The wavefunction of the neutron is, after several minutes, described as a combination of $|not - decayed\rangle$ and $|decayed\rangle$ and consequently, because these two states imply what will happen with the hammers, the cat is, according to Schrödinger equation, found in the state

$$\psi = \alpha|dead\rangle + \beta|alive\rangle.$$

In reality there are very many different states that describe the cat in detail but these two are enough. When you look at the cat, you will either see it is alive or dead. You will never see that it is $0.6|alive\rangle + 0.8|dead\rangle$ even though it is a perfectly nice and normalized state. Does it mean that before you look, the cat was actually in the superposition, with a chance to be alive? When you see it is dead, did you actually kill the cat just by looking at it? Most importantly, what tells you that the states $|dead\rangle$ and $|alive\rangle$ may be results of a measurement while other combinations of these vectors can't? The last problem is addressed by decoherence. The previous problems are probably left to your classes on moral reasoning.

Consciousness and quantum mechanics

Let us return to the problem of the difference between the macroscopic and microscopic objects. Von Neumann and Wigner actually argued that everything – including the macroscopic objects – evolves according to the Schrödinger equation, and the "collapse" is only done when you actually want to observe something which requires consciousness. Alternatively, you may imagine that even other people evolve into strange linear superpositions and it is just you who has the right to make the wavefunction collapse. (The philosophical direction that "I" is the only real entity is called sollipsism.) At any rate, you see that such an interpretation based on a special role of consciousness looks religious, and decoherence made it obsolete.

Transactional interpretation

This proposal by John Cramer tries to recycle the ideas of Wheeler and Feynman about particles going forward and backward in time and promote it into an interpretation of quantum mechanics, but no one else understands how such a statement solves the measurement problem or any other problem discussed above, and we won't say anything else about the picture.

de Broglie-Bohm interpretation

In 1927, Prince Louis de Broglie proposed a non-probabilistic possible interpretation of quantum mechanics based on the "pilot wave". These ideas were re-invented and updated by David Bohm in 1952. Although the framework is trying to be deterministic and Einstein would like this feature, it is mathematically ugly (and does not offer any new predictions beyond orthodox quantum mechanics) so that even Einstein declared the Bohmian framework to be an "unnecessary superstructure".

According to the Bohmian mechanics, there objectively exists BOTH a point-like particle as well as the wavefunction – interpreted as an actual wave. Because the particle is localized at a point, there is no problem to see that it will be observed at one point. On the other hand, the wavefunction is a "pilot wave" that adds new forces acting on the particle. The corresponding "quantum potential" can be defined in such a way that the particle will be "repelled" from the interference minima, and the probabilistic distribution of the particle at time T will agree with $|\psi|^2$ if it agreed in the past. The precise "guiding potential" acts as follows:

$$\frac{d\vec{x}_{clas}}{dt} = \frac{\hbar}{m}\mathrm{Im}\frac{\psi^*_{clas}\nabla\psi_{clas}}{\psi^*_{clas}\psi_{clas}}$$

where we emphasized that both the wave ψ as well as the position x are ordinary classical degrees of freedom. You see that the right hand side is nothing else than the "velocity" computed from the probability current you should remember from the discussion about the continuity equation. The initial position of the particle must however be chosen to be random, according to the $|\psi|^2$ distribution, and no one has explained how this occurs.

It is not explained what happens with the wavefunction or the pilot wave when the particle is absorbed. The framework has some problems to describe multi-particle quantum mechanics, even bigger problems to describe the spin, and very serious problems to be compatible with special relativity (which essentially requires locality). These are the main reasons why the Bohmian interpretation is not treated seriously by most physicists. On the other hand, it is the most popular interpretation among many philosophers who have philosophical reasons to believe that the world can't be probabilistic, and no amount of experiments will convince these philosophers that their assumption is incorrect.

Everett's many-worlds interpretation

Everett wrote a provoking thesis in 1956.

If you measure a particle and you obtain the result A, although there was a non-zero probability to obtain the result B, you should imagine, according to this interpretation, that there exists another Universe where you obtained B. Every time Nature makes a probabilistic decision, it actually branches the Universe into several parallel Universes. In some of them, you are living a slightly different live. In others, you don't live at all because your parents made some different decisions. We live in a "typical" Universe from this collection in the sense that the results of repeated experiments follow the quantum probability predictions. However, we can never observe the other Universes because we are a part of ours. Many physicists find this picture intriguing, but I personally find the branching structure ill-defined.

Consistent Histories

Gell-Mann, Hartle, Omnes, and others promoted the interpretation based on consistent histories. It states that all types of questions that are meaningful in physics may be phrased as a question what are the probabilities of different alternative histories. A history is a sequence of projectors P_i that require that a quantity at time t_i satisfied a certain condition. If the history H_A, $A = 1 \dots N$ is defined as

$$H_A = P_1(t_1)P_2(t_2)\dots P_n(t_n), \qquad t_1 < t_2 < \dots < t_n$$

i.e. as a product of projection operators ($P^2 = P$) expressing that a condition (P_1) was satisfied at time t_1, another condition was satisfied at time t_2, and so forth, its probability to occur, given the initial density matrix ρ, is computed as

$$Prob(A) = \mathrm{Tr}(H_A^\dagger \rho H_A)$$

where we have adopted the Heisenberg picture (the operators, including the projection operators, depend on time). Note that if you write $\rho = |\psi\rangle\langle\psi|$, you essentially get the usual expression $|\langle\psi|\phi\rangle|^2$ where $|\phi\rangle$ is a basis vector determined by the projectors. Two alternative histories must be consistent which essentially means that they are orthogonal in the following sense:

$$\mathrm{Tr}(H_B^\dagger \rho H_A) = 0 \quad \text{for} \quad A \neq B$$

This condition is necessary for the probability of

$$Prob(A \text{ or } B) = Prob(A) + Prob(B) - Prob(A \text{ and } B)$$

and similar identities. Because of decoherence explained below, we are only allowed to define "meaningful" histories including the states "dead cat" or "alive cat" but not their combinations.

Decoherence

The primary question that decoherence answers are

- where is the boundary between the quantum world described by interfering wavefunctions and the classical world that follows our intuition?
- how are the preferred basis vectors of macroscopic objects – such as the dead cat and the alive cat – chosen?

Decoherence is somewhat analogous to friction and it introduces an "arrow of time" (an asymmetry between the past and the future). It requires the system to be "open". In other words, we must look not only at the object itself with the Hamiltonian H_c (collective), but also the environment with H_e and their interaction Hamiltonian H_i.

$$H = H_c + H_e + H_i$$

Decoherence is the loss of the information about the relative phases – or, equivalently, the vanishing of off-diagonal elements of the density matrix ρ_c traced over the "uninteresting" environmental degrees of freedom.

These environmental degrees of freedom are delicate because the spectrum is essentially continuous. For example, if you have N atoms and each of them has n states, then there are n^N states in total, and if the energy difference between the lowest and the highest energy state is NE_0 and we assume that the distribution is essentially random, then the spacing is of order

$$\langle \Delta E \rangle = \frac{NE}{n^N}$$

which is usually very tiny.

Finally: we're going to decohere

Assume that we start with our micro-object being in the quantum mechanical state

$$|\psi\rangle = c_1|\psi_1\rangle + c_2|\psi_2\rangle$$

and the apparatus in a neutral state $|\phi_0\rangle$. Immediately after the measurement, the apparatus is affected by the result and the final state is

$$|\psi(t=0)\rangle = c_1|\psi_1\rangle|\phi_1\rangle + c_2|\psi_2\rangle|\phi_2\rangle$$

Actually, our particle was probably absorbed and we should treat it as a part of the apparatus:

$$|\psi(t=0)\rangle = c_1|\phi_1\rangle + c_2|\phi_2\rangle$$

So far we have still neglected the environment. They are not yet coupled to our apparatus and we may assume their state to be a universal $|\phi_e\rangle$ so that the total state is

$$|\psi(t=0)\rangle = [c_1|\phi_1\rangle + c_2|\phi_2\rangle]\,|\phi_e\rangle$$

What's important is that there are interactions between the apparatus and the environment, governed by the term H_i of the Hamiltonian. This will cause the environment to evolve depending on the state of the apparatus. The apparatus and the environment start to be entangled

$$|\psi(t=\varepsilon)\rangle = c_1|\phi_1\rangle|\phi_{e,1}\rangle + c_2|\phi_2\rangle|\phi_{e,2}\rangle$$

The process is analogous to dissipative forces (friction). When and if the states of the apparatus $|\phi_1\rangle$ and $|\phi_2\rangle$ are sufficiently different – for example they contain the absorbed particle located at two different places that differ by several atomic radii – they will also affect the environment differently and the states $|\phi_{e,1}\rangle$ and $|\phi_{e,2}\rangle$ will actually become (almost) orthogonal; for example, they contain an infrared photon (that carries heat) in two different regions which guarantees that

$$\langle\phi_{e,1}|\phi_{e,2}\rangle = 0$$

The full density matrix is

$$\begin{aligned}\rho = |\psi\rangle\langle\psi| &= |c_1|^2|\phi_1\rangle|\phi_{e,1}\rangle\langle\phi_1|\langle\phi_{e,1}| \\ &+ c_1c_2^*|\phi_1\rangle|\phi_{e,1}\rangle\langle\phi_2|\langle\phi_{e,2}| \\ &+ c_2c_1^*|\phi_2\rangle|\phi_{e,2}\rangle\langle\phi_1|\langle\phi_{e,1}| \\ &+ |c_2|^2|\phi_2\rangle|\phi_{e,2}\rangle\langle\phi_2|\langle\phi_{e,2}|\end{aligned}$$

Just as in classical statistical physics, we rightfully assume that we can't measure all environmental degrees of freedom. So we must partially trace over them. Thus

$$\rho_c = \mathrm{Tr}_e(\rho) = |c_1|^2|\phi_1\rangle\langle\phi_1| + |c_2|^2|\phi_2\rangle\langle\phi_2|$$

What is this density matrix? Last time we understood that this density matrix is not pure because its eigenvalues differ from $0, 1$. It does not describe a superposition of quantum states $|\phi_1\rangle$ and $|\phi_2\rangle$. Instead, it describes a system that has the probability $p_1 = |c_1|^2$ to be in the state $|\phi_1\rangle$, and the probability $p_2 = |c_2|^2$ to be in the state $|\phi_2\rangle$. Here, p_1, p_2 are the eigenvalues of the density matrix.

You see that for $p_1 \neq p_2$, there is a unique basis of eigenvectors (up to a phase/normalization of each of them, of course). For $p_1 = p_2$ you actually see that the density matrix is diagonal in all bases. In reality, it is difficult to guarantee that the density matrix will be exactly a multiple of the identity matrix. But you can try to design an experiment in which the identity of the preferred states will remain ambiguous by making all the probabilities exactly equal.

You see that by taking the interaction with the environment into account, we created a diagonal density matrix whose entries p_1, p_2 have a probabilistic interpretation – but a probabilistic interpretation similar to classical physics. Interference and coherence is gone. A priviliged basis of vectors that are "accessible to consciousness" is picked. The priviliged basis vectors are, in a sense, the states that are able to "imprint themselves" faithfully to the environment: the states able to "self-reproduce". And you see that dynamics and the interactions with the environment determine how this occurs. No longer you need philosophy about consciouness or artificial bureaucratic rules that determine the boundary between quantum mechanics and the range of validity of classical intuition. The detailed properties of the actual Hamiltonian encode this information.

Time needed for decoherence

So how much time it takes for realistic systems before the coherence is lost, the priviliged states are chosen, and the classical interpretation of their probabilities becomes applicable? Assume that the inner product of the different environmental states goes like

$$\langle \phi_{e,1} | \phi_{e,2} \rangle \sim \exp(-\frac{t}{t_d})$$

To be specific, consider a pendulum with momentum p that is affected by the force F as well as the friction γp where $1/\gamma$ is the typical time of damping. The equation for the momentum says

$$\frac{dp}{dt} = F - \gamma p$$

With these definitions, it turns out that the time for decoherence is

$$t_d = \frac{\hbar^2}{\gamma m\, kT (\Delta x)^2}.$$

The more friction (the stronger interaction with the environment) you have, the faster the decoherence operates. The higher temperature you have, the faster it becomes. The more separated in space the states become, the faster your system decoheres. The more massive system you have, the faster the process goes. The formula above is rather universal; it does not really matter what kind of environment you consider. Incidentally, once the off-diagonal elements decrease to a small fraction of the original value, they continue to decrease at a fascinating rate

$$\rho_{off} \sim \exp(-A \exp(t/t'_d))$$

where A, t'_d are constants. It's because even one emitted photon or another element of the environment would be enough to make the system decohere exponentially, but because the number of such "photons" grows with time, the decrease is really exponentially exponential. It is superfast.

A few numbers

For a pendulum of mass $m = 10$ grams, the friction time $1/\gamma = 1$ minute, $\Delta x = 1$ micron, the time of decoherence turns out to be

$$t_d \approx 1.6 \times 10^{-26} \text{ sec.}$$

The case we considered – in which we measure the position and the states are separated spatially – is the most typical one but not the only one possible. You may replace space by voltage, for example. The analogy works as follows:

$$\left.\begin{array}{ccl} x & \to & q = CV \\ p & \to & j \\ m & \to & L \\ \gamma & \to & R/L \end{array}\right\} \quad \Rightarrow \quad t_d = \frac{\hbar^2}{RkTC^2V^2}$$

Plugging some reasonable numbers:

$$\left.\begin{array}{ccl} V & = & 100 \text{ mV} \\ C & = & 10 \text{ pF} \\ R & = & 100\ \Omega \end{array}\right\} \quad \Rightarrow \quad t_d = 2.4 \times 10^{-26} \text{ sec}$$

You could calculate the time for a piece of dust colliding with something as pathetic as the cosmic microwave background, and you would still need a nanosecond for it to decohere. The lesson is completely clear. Decoherence occurs almost instantly for all ordinary macroscopic (and even mesoscopic) systems. There are three basic exceptions in which the coherence may be preserved for macroscopic systems:

- superfluids
- superconductors
- photons

In the first cases, the system is macroscopically described by a field that looks much like the wavefunction: we "see" quantum mechanics in the macroscopic world.

Quantum computers – to be discussed later – are meant to be following the laws of quantum mechanics, with all of its complex amplitudes and interference, for long periods of time. Decoherence is a killer. The technological and, indeed, also the physical challenge is to minimize the decoherence while preserving the necessary interactions between the pieces of the quantum computer.

Decoherence and experiments

In 1996, S. Haroche and J. Raimond et al. confirmed the decoherence calculations using a special experiment where the decoherence was slow enough so that it could be measured. A rubidium atom in a superposition of $|n = 50\rangle$ and $|n = 51\rangle$ states was measured by a resonant cavity with 0,1,2,3,4 or 5 photons. The dissipation comes from the interaction of the photons with the walls of their cavity. The pointer is the phase of the cavity. Everything worked as expected.

Who decides what we measure?

If there are two possible outcomes, $|1\rangle$ and $|2\rangle$, who decides what the "random" result will be? We have seen that if the decision were made by "hidden variables", they would have to be non-local. Many philosophers keep on asking the question. Most physicists, especially Roland Omnes, argue that the question is not a scientific one. Other physicists argue that the many-worlds interpretation gives an answer. Despite their religious preconceptions, most physicists finally agree about the measurable quantities and their predictions from quantum mechanics.