Calculating Forces in the Pulley Mechanical Advantage Systems Used in Rescue Work

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Introduction

If you have not read the companion article: *Understanding Mechanical Advantage in the Single Sheave Pulley Systems Used in Rescue Operations*, it is recommended that you do so before beginning this one. It explains the means by which a pulley can double its input force, examines common examples of both simple and compound rope and pulley MA (mechanical advantage) systems, and explains how to calculate mechanical advantage ratios in those systems.

One of the key points emphasized in that piece is that pulleys are force magnifiers *wherever* they are attached, be it the rescue load or the anchor system. It is crucial that rescue personnel understand this point. When deciding on how to configure a rescue MA system, it is too easy to allow one's focus to be directed to the working end of the system without remembering to give due consideration to the stresses that will be applied to the anchor system. In other words, the advantages gained by using a pulley MA system come at a cost, and that cost is highly variable, depending on the MA system selected and how it is configured.

Remember too, that the forces generated by a mechanical advantage system are transferred to the rescue load through every piece of equipment in the system, any of which could fail. In particular, we would do well to worry about the lead, 3-wrap Prusik that serves as the interface between the hauling system and the main haul rope. It is probably the weak-link in the system. This Prusik can also serve as a force governor for the whole system, the canary-in-the-coalmine so to speak. Tests indicate that an 8 mm, 3-wrap Prusik, on a 1/2" rescue rope will begin to slip at between 7 and 9.5 kN (1,575- 2,125 lb.). For this reason, it is wise to task someone with keeping a watchful eye on that Prusik and instruct them to call a halt at the first sign of slippage.

While it is fun to contemplate MA systems that produce outputs such as 5:1, 6:1, 7:1, and even 9:1, in actual practice, the use of such systems should be accompanied by a sense of extreme trepidation. Remember: keep an eye on that lead Prusik!

Preliminaries

When tracking forces through a system, it is common practice to assign a symbol, such as "T" for *Tension,* to the input force, and then multiply it to T2, T3, etc. as the effects of pulleys are factored in. While this practice has great value, it fails to reveal the actual stresses that might be generated in rescue applications.

In this article, we will run some numbers based on a reasonable, theoretical rescue load of 2 kN (450 lb.). In this manipulation, the value of "T" becomes the amount of actual input force required to hold the 2 kN load in equilibrium with the rope routed through the MA system in question. We will track this input force through the system and calculate stresses at every point.

As you read along and consider the numbers, remember that this *equilibrium* force is the amount required to hold the rescue load in stationary suspension. If we want to raise the load, we must unbalance the system—by *any* amount. Remember: when we increase the input force, all of the numbers within the system go up. Furthermore, It is not just the amount of the input force that is important, but the *rate* at which that force is applied. In his first law of motion (inertia), rescue guru par excellence, Sir Isaac Newton, reminded us that it takes more force to put an object into motion than to keep it in motion. If, in the worst-case scenario, we jerk on the haul rope, we send

a spike of high force to all critical points. Therefore, we are well advised to apply that force gradually with each pull.

<u>Review of the Basics</u>

<u>Fundamental Concept 1:</u> According to the familiar paraphrase, Newton's "Law of Reaction" states: "For every action, there is an equal, and opposite reaction." When we pull on a rope, the rope pulls back, and tension runs through the rope in both directions. Both ends are loaded equally! (See: *Understanding Mechanical Advantage in the Single Sheave Pulley Systems Used in Rescue Operations*)

<u>Fundamental Concept 2:</u> In a theoretical, frictionless system, a pulley equalizes the tension in both legs of the rope that passes through it. In the real world, the same *principle* applies, minus the effect of friction within the workings of the pulley in question.

<u>Fundamental Concept 3:</u> Pulleys are force magnifiers! When the two weighted legs of the rope passing through a pulley are maintained in a parallel configuration, the forces brought to bear on the pulley's point of attachment are **doubled!**

<u>Fundamental Concept 4:</u> Within a pulley hauling system, a pulley can function in one of two ways: as a "Standing Pulley", or as a "Traveling Pulley."

- The Standing Pulley:
  - Attaches to the anchor system
  - Does <u>not</u> move
  - Changes direction only (directional), and
  - <u>Does not</u> contribute to mechanical advantage.
- The Traveling Pulley:
  - Attaches to either the resistance or to the interior of the system
  - Does move
  - Changes direction, and
  - It <u>does</u> contribute to mechanical advantage.

Tracking Stresses Through the Pulley MA System

For the moment, let's revert to use of the "T" system, forgoing actual numbers in order to illustrate some key principles by the simplest means available. Consider the following:
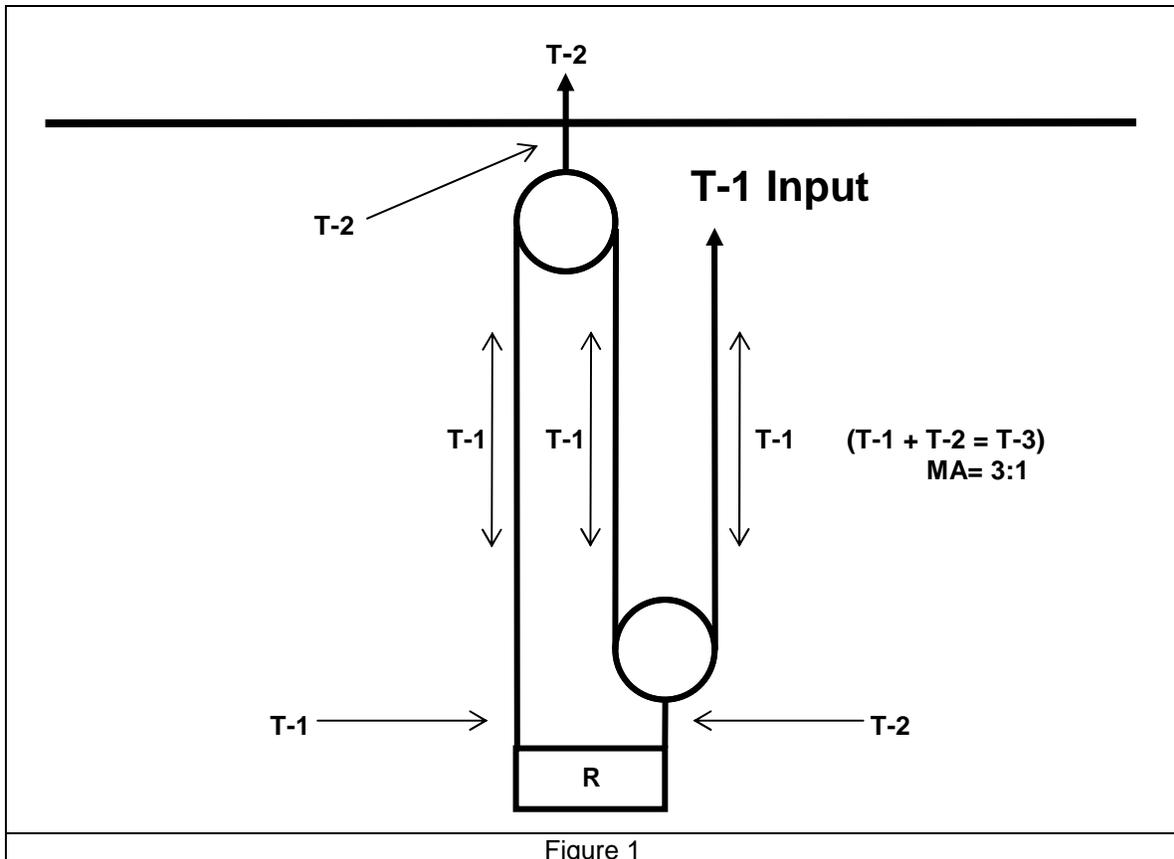
Figure 1

Figure 1: A Simple 3:1 System

- The "R" in the rectangular box at the bottom of the figure represents the resistance.
- The pulley at the top of the figure is the Standing Pulley.
- The bottom pulley is the Traveling Pulley.
- The big **"T-1"** (upper right) represents an input force of 1.
- These theoretical, friction-free pulleys equalize the tension throughout the system.
- Thus, the T-1 force runs throughout the system, and each leg of the rope carries a T-1 force.
- The law of reaction reminds us that, even though the rope moves through an active system in one direction only, none-the-less, there is equal tension at each end of the rope; hence, the double-ended arrows.
- Each pulley carries two, T-1 loaded rope segments, and therefore, transfers a T-2 (2:1) force to its point of attachment.
    - The Standing Pulley (top) applies a T-2 force to the anchor
    - The Traveling Pulley (bottom) applies a T-2 force to the resistance ("R")
- In addition to the T-2 pulley force, the resistance is attached to a single T-1 loaded rope-leg for a total force of T-3. (T-2 + T-1= T-3; MA= 3:1)
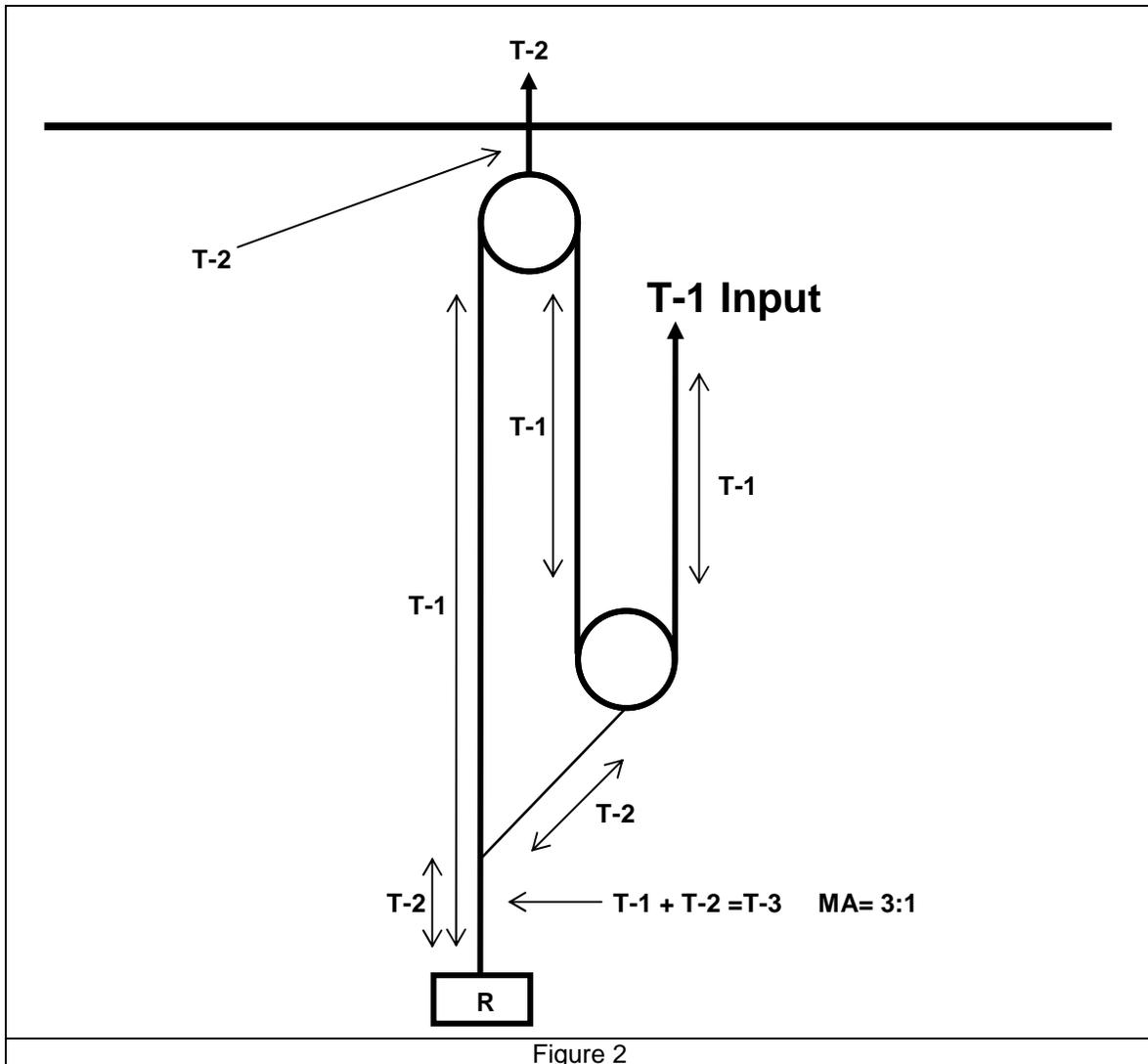
Figure 2

Figure 2: The Simple 3:1 System Rigged with a Prusik

- This is the same simple 3:1 system from Figure 1, but this time rigged with a 3-wrap Prusik connecting the Traveling Pulley to the main line coming from the resistance (represented by the thin, diagonal line). This is consistent with standard practice in rescue applications.
- The T-1 tension continues to run through the entire system.
- Because the Traveling Pulley is rigged within the system, it <u>adds</u> its T-2 force to the T-1 force already running through the system.
- The forces within the system are as shown.
- Ultimately, there is a T-2 force applied to the anchor and a T-3 force applied to the resistance, just as in Figure 1. (T-2 + T-1= T-3; MA= 3:1)

Real Numbers, Real Forces

In the remaining diagrams, we'll replace the "T-1," symbolic load with a 2 kN (450 lb.) load held in equilibrium.
- We'll follow this force through the system and calculate the stresses at all critical points.

- In order to remind the reader of the forces transferred to the anchor, the stress on the anchor system will be expressed as a percentage of the main load.
- Because it's not uncommon to route the haul line through a directional pulley, we'll calculate that stress as well. In doing so, we'll assume that the two legs of the rope passing through the directional pulley run parallel to one-another. In the real world this may not be the case. When the rope angle at the pulley is greater than 0°, the stress on the pulley and anchor is <u>reduced</u>—more later (p. 19).

T-1= 225   T-2= 450

T-2= 450

Directional Pulley

T-1= 225

T-1= 225

T-1= 225 Input

T-1= 225

T-1= 225

T-1 (225) + T-1 (225)= T-2 (450)

450

Figure 3
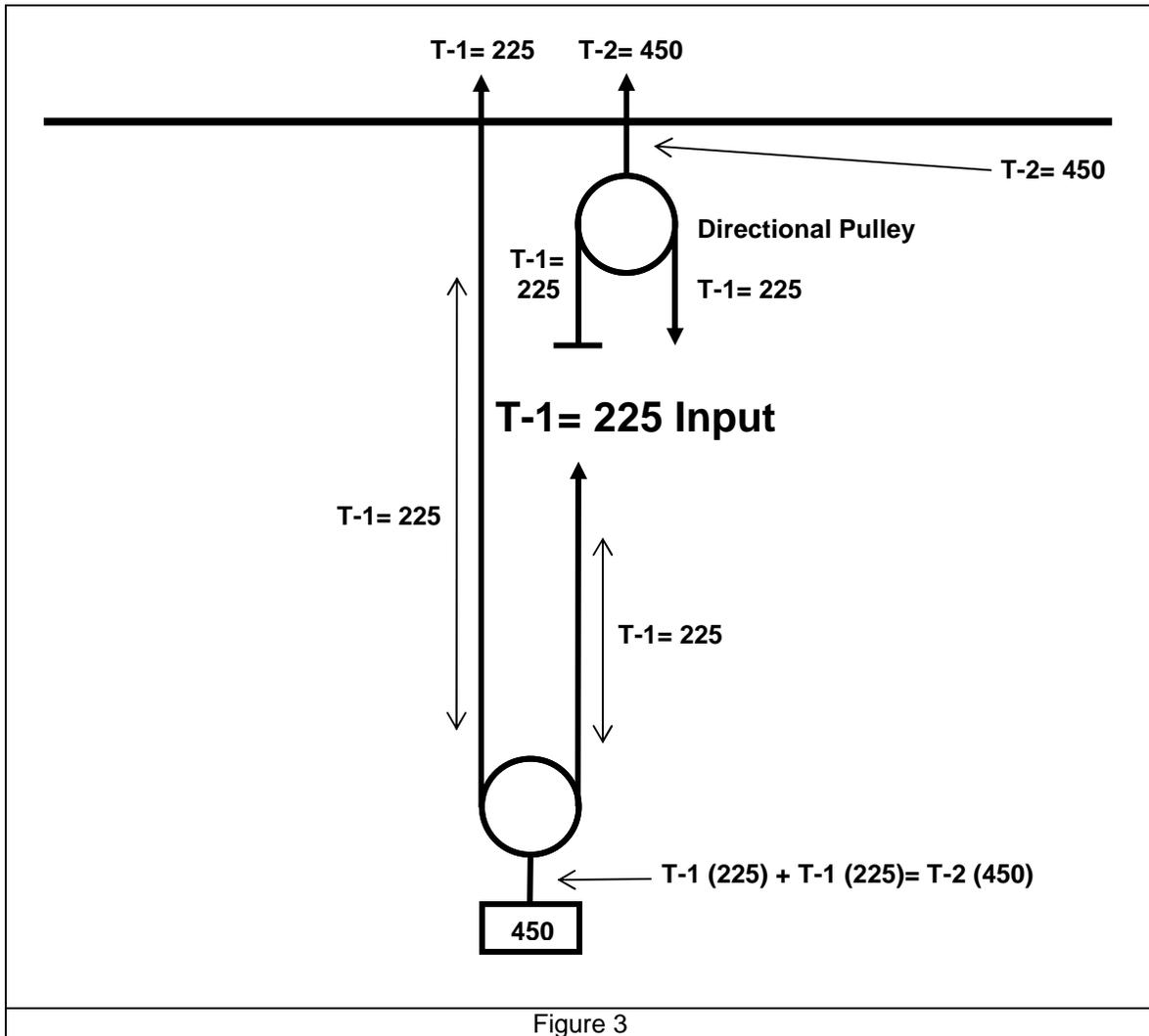
- **Stress on the anchor = 225 lb., or 50% of the 450 lb. output force**
- **If the main and directional anchors are shared, the stress on the combined anchors= 675 lb., or 150% of the 450 lb. output force**

- Fig. 3 shows a 2:1 MA system
- The T-1 input force is 225 lb.
- Each of the two rope legs carries 225 lb. of force
- The stress on the anchor is 225 lb.
- In practice, it is likely that this system will be set up separately from the main haul-rope and connected to it by a 3-wrap Prusik

- Notice that if a directional is used, and configured as shown here, the stress on that anchor is the same as the stress brought to bear on the main load

T-2= 300   **T-2= 300**

T-2= 300

T-2= 300

Directional Pulley

T-1= 150

T-1= 150

T-1= 150 Input

T-1= 150

T-1= 150

T-1= 150

T-2= 300

T-2= 300   **T-1 (150) + T-2 (300)= T-3 (450)**

450
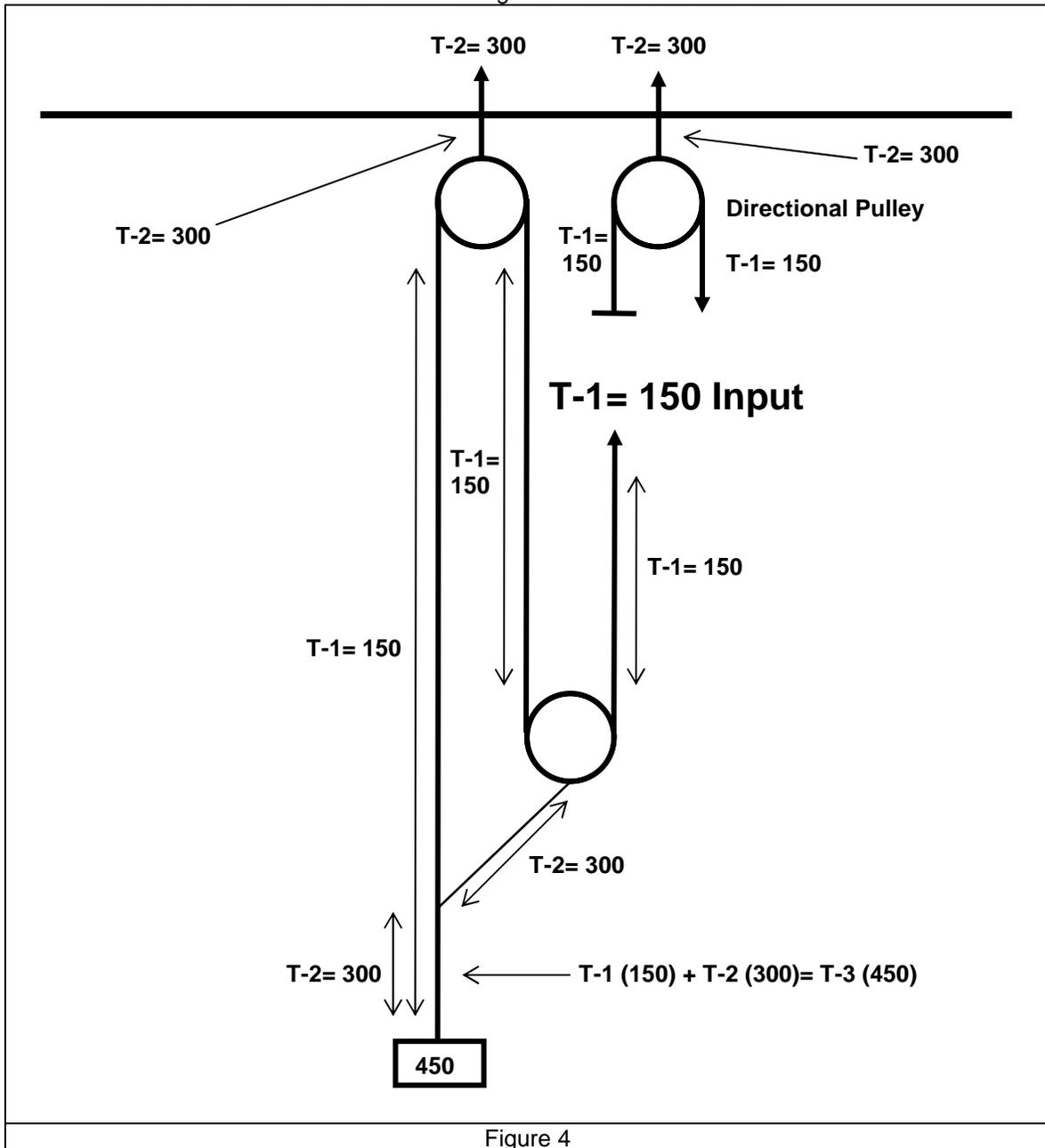
Figure 4

- **Stress on the anchor = 300 lb., or 66% of the 450 lb. output force**
- **If the main and directional anchors are shared, the stress on the combined anchors= 600 lb., or 133% of the 450 lb. output force**

- Fig. 4 illustrates the same 3:1 MA system diagrammed in Figure 2 with the addition of an optional directional
- The T-1, 150 lb. input force runs through all 3 legs of the system
- The Traveling Pulley doubles the force to T-2= 300 lb.
- This is <u>added</u> to the running T-1= 150 lb. force in the rope segment running between the Prusik and the load, to produce a 450 lb. force.

Compound Systems

The next four diagrams show "Compound," or "Piggy-back" systems in which MA rigs are joined together in series.
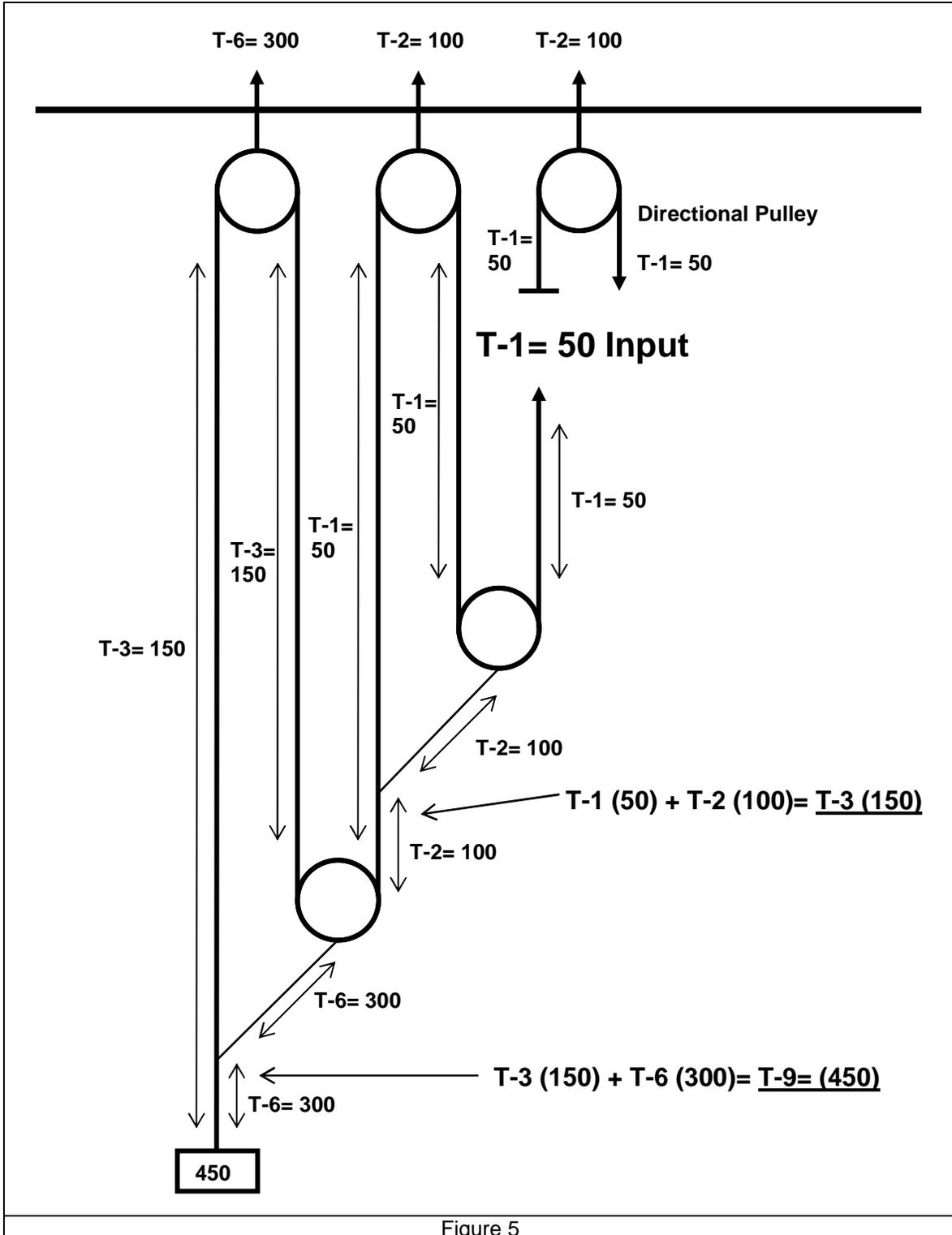
T-6= 300   **T-2= 100**   **T-2= 100**

Directional Pulley

T-1= 50

T-1= 50

T-1= 50

T-1= 50 Input

T-1= 50

T-1= 50

T-3= 150

T-1= 50

T-3= 150

T-2= 100

T-1 (50) + T-2 (100)= T-3 (150)

T-2= 100

T-6= 300

T-3 (150) + T-6 (300)= T-9= (450)

T-6= 300

450

Figure 5

- **Stress on the main anchor = 300 lb., or 66% of the 450 lb. output force**
- **Stress on the piggy-back anchor = 100 lb., or 22% of the 450 lb. output force**
- **If the main and piggy-back anchors are shared, the stress on the combined anchors= 400 lb., or 89% of the 450 lb. output force**

- Fig. 5 is a 3:1 piggy-backed on a 3:1 to create a 9:1 MA system
- The (T-1) 50 lb. input force runs throughout all three legs of the piggy-backed 3:1 MA system
- The first Traveling Pulley (upper right) doubles the input force (T-2= 100 lb.), and adds it to the (T-1) 50 lb. running force, creating a (T-3) 150 lb. input force which is passed into the second 3:1 MA system
- The (T-3) 150 lb. force enters the second 3:1 system and is equalized through all three legs
- The second Traveling Pulley (lower left) doubles the T-3, 150 lb. force (T-6, 300 lb.) which is added to the T-3, 150 lb. running force, for a total of T-9= 450 lb.
- **Warning:** Remember that these high force outputs increase the stresses throughout the system. Of particular concern is the Prusik nearest the load.
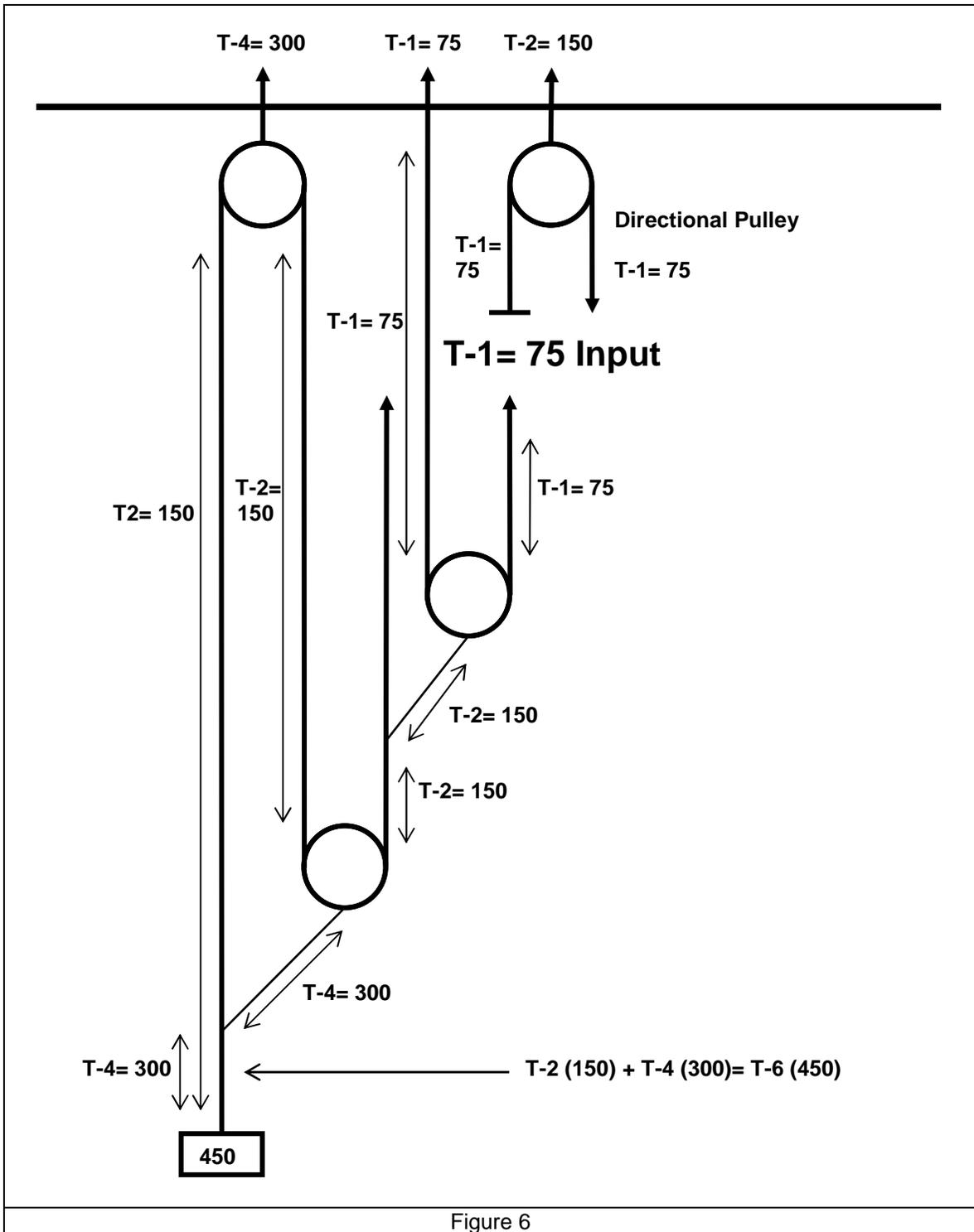
Figure 6

- **Stress on the main anchor = 300 lb., or 66% of the 450 lb. output force**
- **Stress on the piggy-back anchor = 75 lb., or 16% of the 450 lb. output force**
- **If the main and piggy-back anchors are shared, the stress on the combined anchors= 375 lb., or 83% of the 450 lb. output force**
- Figure 6 shows a discreet 2:1 system, piggy-backed on a 3:1 by means of a 3-wrap Prusik, to produce a 6:1 system

- The (T-1) 75 lb. input force runs through both legs of the piggy-backed 2:1 MA system
- The Traveling Pulley of the 2:1 doubles the input force (T-2= 150 lb.) into the 3:1 MA system
- The (T-2) 150 lb. force enters the 3:1 system and is equalized throughout all three legs
- The Traveling Pulley doubles the T-2, 150 lb. force (T-4, 300 lb.) which is added to the T-2, 150 lb. running force, for a total of T-6= 450 lb.
- **Warning:** Remember that these high force outputs increase the stresses throughout the system. Of particular concern is the Prusik nearest the load.
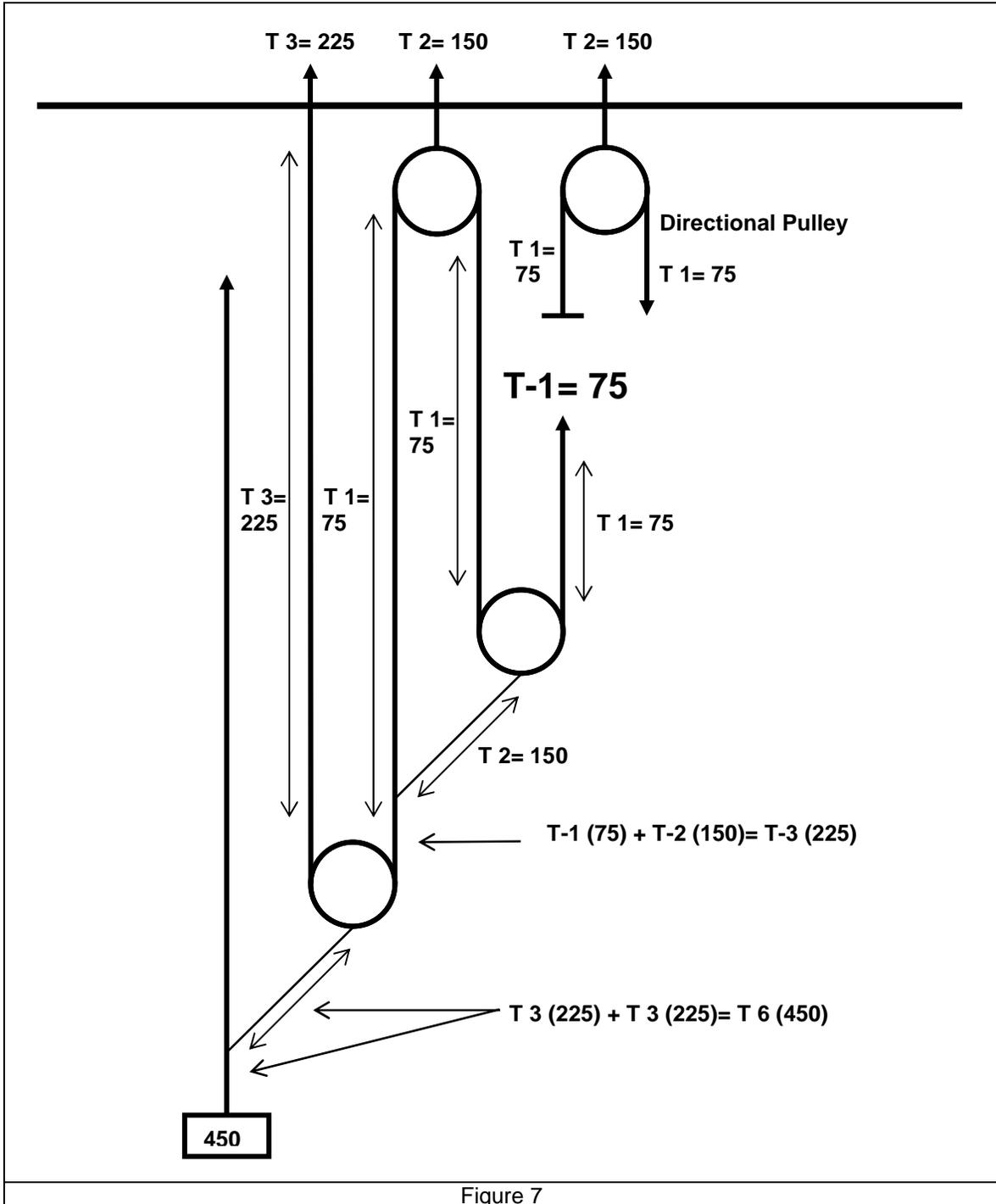
| Figure 7 |

- **Stress on the main anchor= 225 lb., or 50% of the 450 lb. output force**
- **Stress on the piggy-back anchor= 150 lb., or 33% of the 450 lb. output force**
- **If the main and piggy-back anchors are shared, the stress on the combined anchors= 375 lb., or 83% of the 450 lb. output force**

- Figure 7 shows a 3:1 system, piggy-backed on a discreet 2:1 to produce a 6:1 system; all of which is attached to a separate, main haul-line, by means of a 3-wrap Prusik,
- The (T-1) 75 lb. input force runs through all three legs of the piggy-backed 3:1 MA system
- The first Traveling Pulley (upper right) of the 3:1 doubles the input force (T-2= 150 lb.)
- This (T-2) 150 lb. force is added to the 75 lb. force running through all three legs of the 3:1 system to create a (T-3) 225 lb. force
- The (T-3) 225 lb. force enters the 2:1 system and is equalized through both legs
- The second (bottom) Traveling Pulley doubles the T-3, 225 lb. input force (T-3 [225] + T-3 [225]= T-6 [450]), which is transferred to the separate, main haul-line by means of a 3-wrap Prusik to produce a cumulative 450 lb. force
- **Warning:** Remember that these high force outputs increase the stresses throughout the system. Of particular concern is the Prusik nearest the load.

Figure 8

- **Stress on the main anchor= 337.5 lb., or 75% of the 450 lb. output force**
- **Stress on the anchor for the Directional Pulley= 225 lb., or 50% of the 450 lb. output force**
- **If the main, and directional anchors are combined, the force = 562.5 lb., or 1.25% of the 450 lb. output force**

- This discreet 4:1 piggy-back system connects to the main haul line by a 3-wrap Prusik
- It is rigged by using a knot, such as a figure-of-eight loop, tied near the middle of the rope, to create two working ropes from one
- The T-1 (112.5) input force is equalized between the two legs of the first 2:1 rig, through its pulley and up to the knot
- This upper-most of two Traveling Pulleys doubles the input force (T-2= 225)
- The T-2, 225 input force is equalized between the two legs running through the second (lower) Traveling Pulley, and up to the knot
- The second (lower) Traveling Pulley doubles the T-2, 225 force to create a total output force of 450
- The anchor stress is a composite of the T-1, 112.5 force from the first 2:1, and the T-2, 225 force from the second 2:1
- In use, the main haul-line is routed through a 3-wrap Prusik used as a Progress Capture Device and anchored separately, which offers an additional degree of redundancy
- In the real world, the rope legs entering and exiting the pulleys are parallel
- **Warning:** Remember that these high force outputs increase the stresses throughout the system. Of particular concern is the Prusik nearest the load.

Complex Systems

Notice, that in each of the *simple* MA systems considered so far, the Traveling Pulleys have moved in the same direction as the resistance. If the force generated by the *simple* system in question proves inadequate, this can be rectified by adding-on another similar system, in an in-line configuration, to create one of the *compound* MA systems discussed in the previous section.

Complex MA systems offer an alternative solution to this problem. In these systems, a Traveling Pulley is introduced into the *interior* of the *simple* system in order to increase the amount of force running through the equalized legs, thus increasing the final MA. As will be seen, the add-on Traveling Pulley can move in a direction opposite to that of the load, a most interesting situation, indeed. As one might expect, this configuration makes it more difficult to calculate the stresses within, and to understand the resulting MA.

Because *complex* systems are sometimes viewed as requiring less equipment, and being easier, and faster to set-up compared to *compound* systems, they may be chosen for reasons of expedience when the MA of a *simple* system proves to be less than desired. As is so often the case in life, such convenience can come at a cost. A closer look at our sample *complex* systems will reveal that it is good to have a thorough understanding of a tool before putting it to work.

T-6= 540

T-1 (90) + T-1 (90) + T-2 (180) + T-2 (180)= T-6 (540)

T-2= 180

T-2= 180

T-1= 90

T-1=
90

T-2= 180

T-1= 90

T-1= 90 Input

T-1= 90

T-2= 180

T-2= 180

T-1 (90) + T-2 (180) + T-2 (180)= T-5 (450)

450
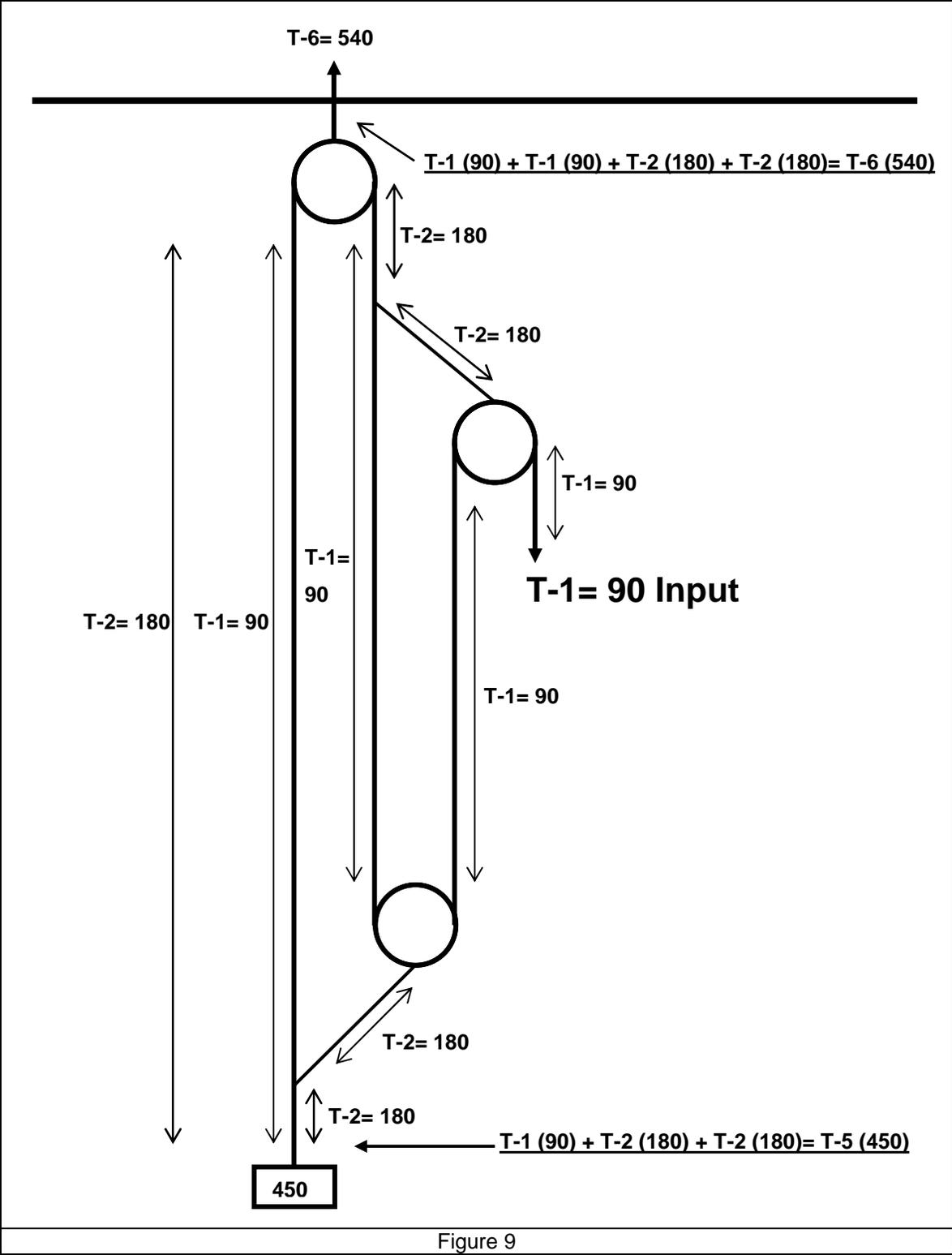
Figure 9

The stress on the anchor = T-6, 540 lb., or 120% of the 450 lb. output force!

Notice that this 5:1 system includes two Traveling Pulleys.

- Each doubles a T-1 (90) input force to T-2 (180), and applies that force to the system at separate locations
- The Traveling Pulleys direct their respective forces in <u>opposite</u> directions
- Because the first (upper) Traveling Pulley directs its T-2 (180) force *toward* the Standing Pulley, and this is added to the running T-1 (90) force, the T-1 (90) input force is <u>tripled</u> in both legs passing through that pulley, creating a T-6 (540 lb.) force at the anchor

It breaks down as follows

- The T-1 (90) input force is equalized by the various pulleys and runs through <u>all</u> legs of the system
- The first (upper) Traveling Pulley doubles the input force to T-2 (180), which is added to the T-1 (90) running tension via the 3-wrap Prusik.
- This (T-1 [90] + T-2 [180]= T-3 [270]) force passes through the Standing Pulley, and is equalized in both rope legs, doubling the T-3 (270) force and passing it on to the anchor T-6 (540 lb.).
- The second (lower) Traveling Pulley doubles the T-1 (90) input force to T-2 (180), and transmits it to the main haul Line via the (lower) 3-wrap Prusik
- The net force applied to the load combines the original T-1 (90) running tension, plus the T-2 (180) force contributed by the first (upper) Traveling Pulley, plus the T-2 (180) force added by the second (lower) Traveling Pulley:
  T-1 (90) + T-2 (180) + T-2 (180)= <u>T-5</u> (450)

Discussion

- **<u>Notice</u> that the force applied to the rescue load is T-5 (450 lb.), while the force applied to the *anchor system* is <u>T-6 (540)</u>!**
- Doesn't it seem that we might be better off turning this system 180°?
- If we do so, this becomes nothing more than a 3:1 on a 2:1= 6:1 *compound* system, requiring the same gear, plus an anchor (See: Figure 7)
- Alternatively, this could be re-configured into a 2:1 on a 3:1= 6:1 with the same equipment (See: Figure 6)
- In either case, the MA goes up from 5:1 to 6:1 while the anchor stress is reduced from 120% to 83% of the 450 lb. output force; this, in the worst case scenario, assuming that the *compound* system uses only one anchor
  - If a 2:1 on a 3:1= 6:1 *compound* system is rigged from separate anchors, the stresses drop to 16% (75 lb.) and 66% (300 lb.) of the 450 lb. output force respectively
  - If a 3:1 on a 2:1= 6:1 compound system is rigged off of separate anchors, the stresses drop to 33% (150 lb.) and 50% (225 lb.) of the 450 lb. output force respectively (<u>The preferred set-up:</u> this 3:1 on 2:1 configuration requires less frequent re-setting compared to the 2:1 on 3:1—Shokoples, 1994)
- **<u>Warning:</u>** Remember that these high force outputs increase the stresses throughout the system. Of particular concern is the Prusik nearest the load.

T-6= 385.68

T-3 (192.84) + T-3 (192.84)= T-6 (385.68)

T-2= 128.56
(T-1 [64.28] + T-2 [128.56]= T-3 [192.84])

T-2=
128.56

T-1= 64.28

T-1= 64.28

C

T-1= 64.28

T-2=
128.56

T-1= 64.28

T-2=
128.56

T-2= 128.56

T-4= 257.12

T-2 (128.56) + T-1 (64.28) + T-4 (257.12)= T-7 (450)

450
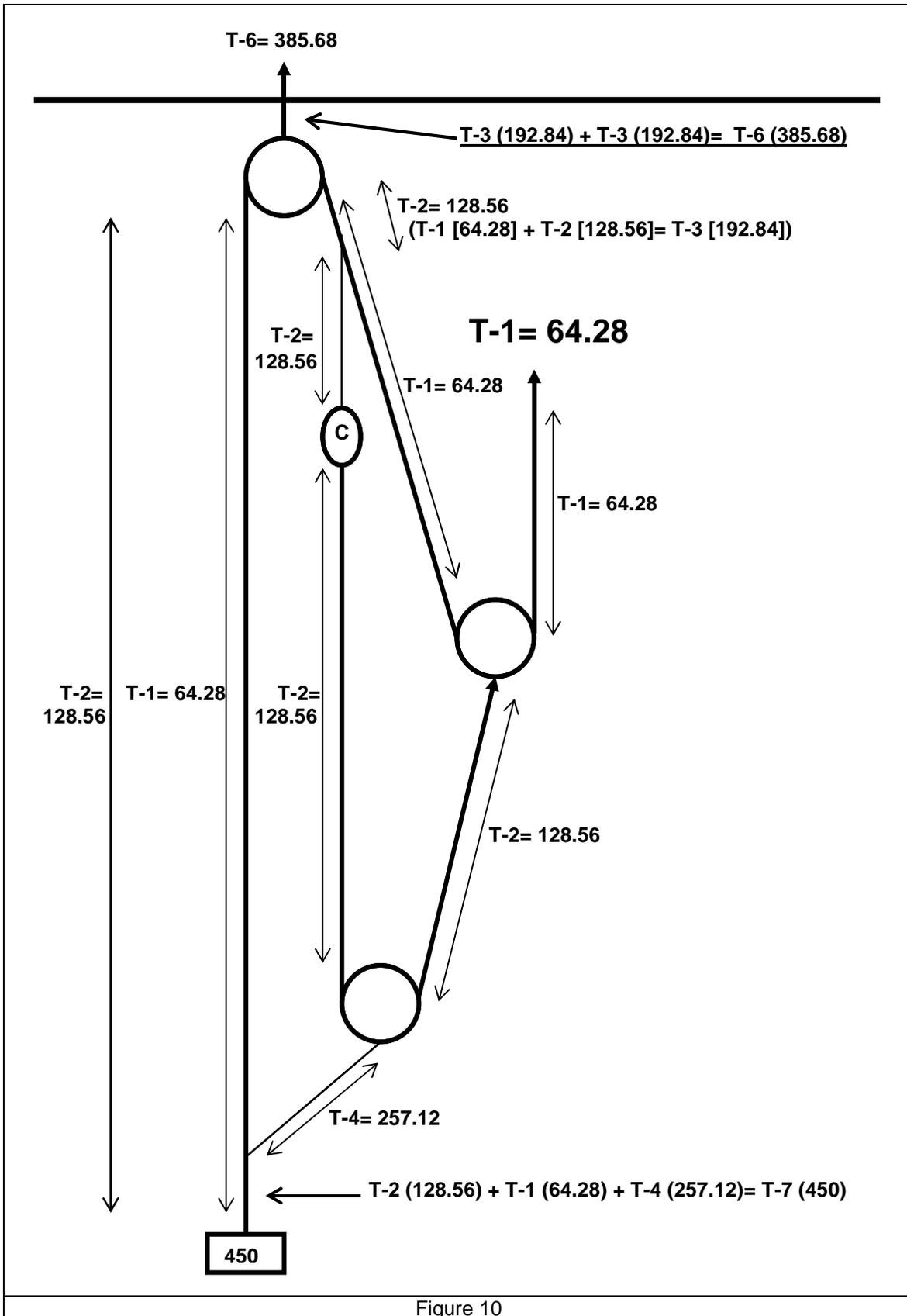
Figure 10

Stress on the anchor = 385.68 lb., or 85.7% of the 450 lb. output force

- This interesting 7:1 complex MA system features two Traveling Pulleys and a 2:1 MA system built into the interior of a simple, 3:1 rig to produce an impressive force output without subjecting the anchor system to excessively high stresses
- The T-1 (64.28) force is equalized throughout the three legs of the 3:1 core system
- The first Traveling Pulley (upper right) doubles the 64.28 lb. input to transfer a T-2 (128.56) force into the interior 2:1 system
- The T-2 (128.56) is equalized between the two legs of the interior 2:1, and passed on, through a carabiner (the oval- 'C') and Prusik (thin line), to add to the T-1 (64.28) running force in the 3:1 core system (T-1 [64.28] + T-2 [128.56]= T-3 [192.84])
- The T-3 (192.84) force passes through the Standing Pulley (top) undiminished, into the third leg of the core, 3:1 rig (creating a T-3 [192.84] + T-3 [192.84]= T-6 [385.68] stress on the anchor)
- The interior 2:1 system (lower Traveling Pulley) passes along a T-4 (257.12) force to the third leg of the core 3:1 system and the load, via another 3-wrap Prusik
- The final result is: T-2 (128.56) + T-1 (64.28) + T-4 (257.12)= T-7 (450)

Discussion

- This system produces a high output force at the working end with acceptable stresses at the anchor because both Traveling Pulleys direct their forces toward the resistance, rather than the anchor
- Because:
  - The length of the internal 2:1 system is fixed
  - And sized to fit inside the foundational 3:1 system
  - While the overall MA is high (7 ft. of rope must be pulled through the system to move the resistance 1 ft.)

  It is likely that this complex rig would require frequent re-setting, making it slow and cumbersome to use
- **Warning:** Remember that these high force outputs increase the stresses throughout the system. Of particular concern is the Prusik nearest the load.

T-2= 180   **T-2= 180**

T-1 (90) +
T-1 (90)= T-2 (180)   T-2= 180

T-1= 90

T-1=
90   T-1= 90   T-2= 180

T-1= 90

T-2= 180

T-4= 360

T-1 (90) + T-4 (360)= T-5 (450)

450
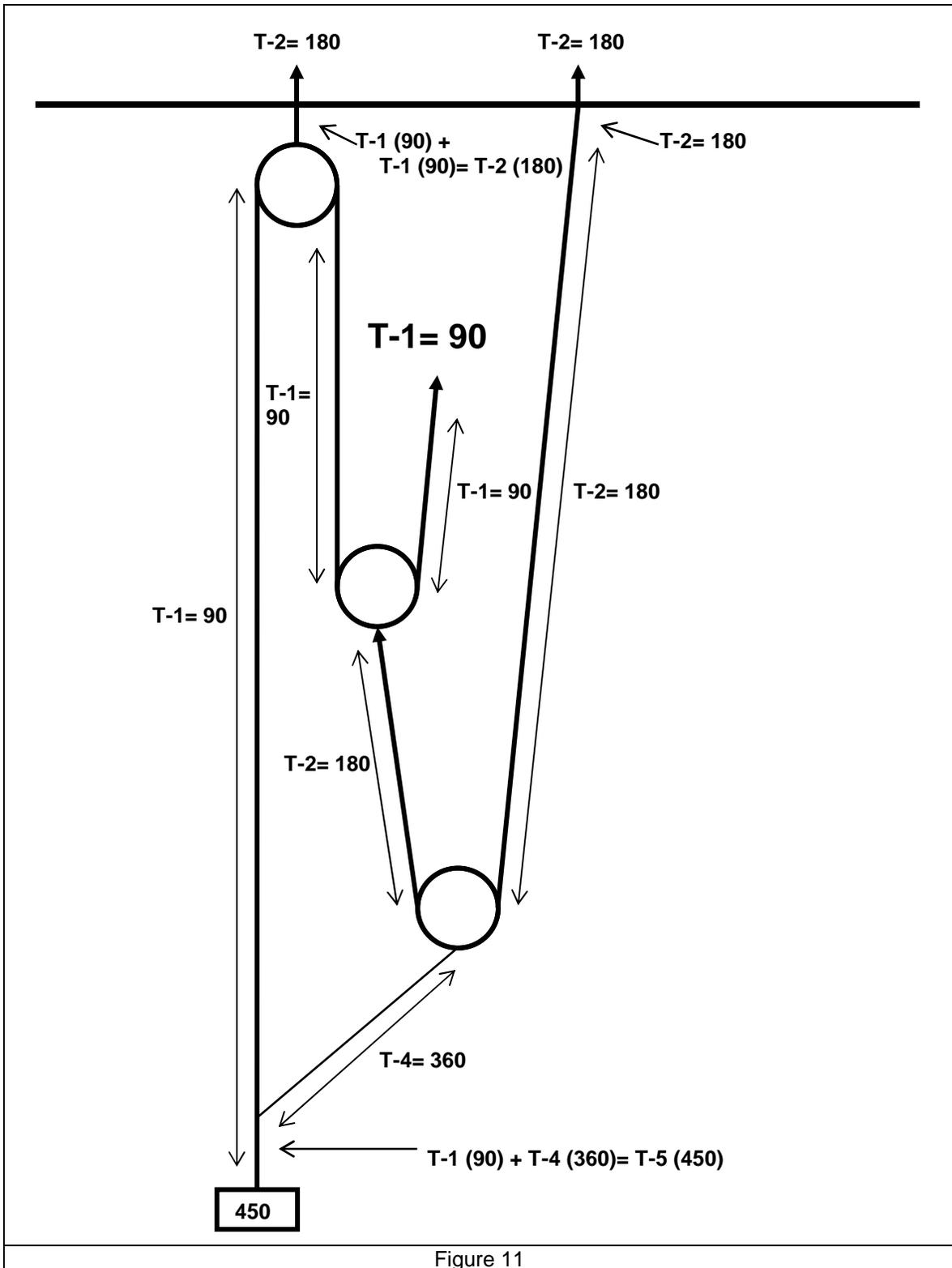
Figure 11

- **Stress on the primary anchor= 180 lb., or 40% of the 450 lb. output force**
- **Stress on the piggy-back anchor= 180 lb., or 40% of the 450 lb. output force**

- This promising 5:1, complex system is created by interposing a 2:1 set-up between the Traveling Pulley and the Prusik of a simple 3:1 rig
- The fortuitous combination of strong mechanical advantage and low anchor stress is the result of dividing the load between two anchors and directing the forces of both Traveling Pulleys toward the resistance, instead of the anchors
- As always, the T-1 (90) force is shared, undiminished, by all three legs of the 3:1 rig
- The Traveling Pulley of the 3:1 doubles the T-1 (90) force to pass on a T-2 (180) input force to the internal 2:1 system
- In the internal 2:1 system, the T-2 (180) running force is doubled by the second (bottom) Traveling Pulley to pass on a T-2 (180) + T-2 (180)= T-4 (360) force to the third leg of the 3:1
- In the final analysis, the T-1 (90) running force of the 3:1 is added to the T-4 (360) force generated by the internal 2:1 to produce a final force of T-1 (90) + T-4 (360)= T-5 (450)
- This diagram has been expanded horizontally for the sake of clarity; in practice, the rope legs should be set-up as close to parallel as possible

Directional Pulleys and Interior Angles

Rescue bosses may decide to append a directional pulley onto a hauling system in order to accommodate the local vegetation and terrain, or to direct the pulling crew away from the edge in order to create a safer working environment. Thus far, directional pulleys have been illustrated with the two rope legs maintained in a parallel configuration. In practice, this is fairly unusual, and this is a good thing.

- When creating a pulley mechanical advantage system, it is crucial to keep those rope legs parallel, or the force brought to bear on the resistance is degraded.
- When we attach a pulley to an anchor system, we *want* the force to be degraded. This is exactly what happens when we open up the rope angles.

Let's take a look at some examples. The numbers are accurate; the angles in the diagrams are approximations.

0° Yields 2:1
(Multiply the running force X 2 to calculate the anchor stress)
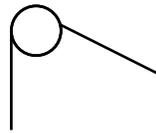
60° Yields 1.73:1
(Multiply the running force X 1.73 to calculate the anchor stress)
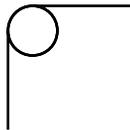
90° Yields 1.41:1
(Multiply the running force X 1.41 to calculate the anchor stress)
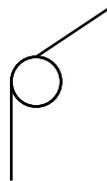
120° Yields 1:1
(The running force equals the anchor stress)

A Summary of Costs and Rewards

In the Table Below:

- Anchor stresses are listed in both pounds and percentage of the sample 450 lb. (2 kN) rescue load
- The shared anchor stresses apply to situations in which:
    - The main and secondary anchors are shared, or
    - The main and directional anchors are shared with parallel rope-legs in all pulleys

Fig.	MA	Anchor Stresses						Stresses on Shared Anchors	
		Main		Secondary		Directional			
		lb.	%	lb.	%	lb.	%	lb.	%
Simple Systems									
3	**2:1**	225	50	NA	NA	450	100	675	150
4	**3:1**	300	66	NA	NA	300	66	600	133
Compound Systems									
5	3:1/3:1=**9:1**	300	66	100	22	NA	NA	400	89
6	2:1/3:1=**6:1**	300	66	75	16	NA	NA	375	83
7	3:1/2:1=**6:1**	225	50	150	33	NA	NA	375	83
8	2:1/2:1=**4:1**	337.5	75	NA	NA	225	50	562.5	1.25
Complex Systems									
9	**5:1**	540	120	NA	NA	NA	NA	NA	NA
10	**7:1**	385.68	85.7	NA	NA	NA	NA	NA	NA
11	**5:1**	180	40	180	40	NA	NA	NA	NA

Conclusion: The Take-Home Messages

- Pulleys double their input forces wherever they attach, as long as the rope legs are parallel
- Forces move in both directions throughout the system
- These forces are transferred to all parts of the system
- The lead, 3-wrap Prusik is a probable weak link: watch it for slippage!
- Systems that produce high MA ratios should be used with extreme caution, or not at all
- Load the system smoothly when hauling
- Avoid combining main and directional anchors when using 2:1 systems
- Avoid complex systems in which a traveling pulley aims its force at the anchor

Sincere Thanks to: