NON-RENEWABLE RESOURCES, EXTRACTION TECHNOLOGY, AND ENDOGENOUS GROWTH*

Martin Stuermer †

Gregor Schwerhoff‡

July 16, 2016

Abstract

We develop a theory of innovation in non-renewable resource extraction and economic growth. Firms increase their economically extractable reserves of non-renewable resources through R&D investment in extraction technology and reduce their reserves through extraction. Our model allows us to study the interaction between geology and technological change, and its effects on prices, total output growth, and the resource intensity of the economy. The model accommodates long-term trends in non-renewable resource markets – namely stable prices and exponentially increasing extraction – for which we present data extending back to 1792. The paper suggests that over the long term, development of new extraction technologies balances the increasing demand for non-renewable resources. (JEL codes: O30, O41, Q30)

* We thank Klaus Desmet, Martin Hellwig, Jürgen von Hagen, Anton Cheremukhin, Michael Sposi, Gordon Rausser, Thomas Covert, Maik Heinemann, Dirk Krüger, Pietro Peretto, Sjak Smulders, Lars Kunze, Florian Neukirchen, Gert Pönitzsch, Salim Rashid, Luc Rouge, Sandro Schmidt, and Friedrich-Wilhelm Wellmer for useful comments. We are highly grateful for comments of participants at the NBER Summer Institute 2016, AEA annual meeting in Philadelphia, AERE summer meeting in Breckenridge, EAERE 2013 conference in Toulouse, SURED 2012 conference in Ascona, AWEEE 2012 workshop in La Toja, SEEK 2012 conference in Mannheim, USAEE annual meeting in Pittsburgh, and SEA meeting in New Orleans, and at seminars at the U Chicago, U Cologne, U Bonn, MPI Bonn, and the Federal Reserve Bank of Dallas. We thank Mike Weiss for editing and Navi Dhaliwal, Achim Goheer and Ines Gorywoda for research assistance. All remaining errors are ours. An online-appendix is available from the authors upon request. An earlier version was published as a working paper with the title "Non-renewable but inexhaustible: Resources in an endogenous growth model." The views in this paper are those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

†Federal Reserve Bank of Dallas, Research Department, Email: martin.stuermer@dal.frb.org.

‡Mercator Research Institute on Global Commons and Climate Change, Email: schwerhoff@mcc-berlin.net

1 Introduction

In his seminal paper, Nordhaus (1974) estimates that the crustal abundance of non-renewable resources is sufficient to continue consumption for hundreds of thousands of years. He also emphasizes that prohibitively high extraction costs make a very large share of mineral deposits not recoverable. Proven reserves – those non-renewable resources that are economically recoverable with current technologies – are a far smaller share.

However, innovation in extraction technology helps overcome scarcity by turning mineral deposits into economically recoverable reserves (Nordhaus, 1974; Simon, 1981, and others). There is empirical evidence for such technological change across a broad variety of non-renewable resources (see, e.g., Managi et al., 2004; Mudd, 2007; Simpson, 1999).

In the literature on growth and natural resources, models rarely consider technological change in extraction. Scarcity is primarily overcome by technological change involving the efficient use of resources and substitution of capital for non-renewable resources (see Groth, 2007; Aghion and Howitt, 1998). These models typically predict decreased non-renewable resource extraction, and increasing prices in the long run, which is not in line with empirical evidence of increasing production and non-increasing prices (see Krautkraemer, 1998; Livernois, 2009; Von Hagen, 1989).

This paper develops a theory of technological change in non-renewable resource extraction in an endogenous growth model. Modeling technological change in resource extraction in a growth model is technically challenging because it adds a layer of dynamic optimization to the model. We boil down the investment and extraction problem to a static problem, which makes our model both simple enough to solve and rich enough to potentially connect to long-run data as a next step.

To our knowledge, our model is the first that allows the study of the interaction between technological change and geology, and its effects on prices, total output growth, and its use in the economy. Learning about these effects is important for making predictions of long-run development of resource prices and for understanding the impact of resource production on aggregate output. For example, distinguishing between increasing and constant resource prices in the long run is key to the results of a number of recent papers on climate economics (Acemoglu et al., 2012; Golosov et al., 2014; Hassler and Sinn, 2012; van der Ploeg and Withagen, 2012).

We add an extractive sector to a standard endogenous growth model of expanding varieties and directed technological change by Acemoglu (2002), such that aggregate output is produced from non-renewable resources and intermediate goods.

Modeling the extractive sector has four components: First, we assume that there is a continuum of deposits of declining grades. The quantity of the non-renewable resource is distributed such that it increases exponentially as the ore grades of deposits decrease, as a local approximation to Ahrens (1953, 1954) fundamental law of geochemistry. Although we recognize that non-renewable resources are ultimately finite in supply, we make the assumption that the underlying resource quantity goes to infinity for all practical economic purposes as the grade of the deposits approaches zero. Without innovation in extraction technology, the extraction cost is assumed to be infinitely high.

Second, we build on Nordhaus' (1974) idea that reserves are akin to working capital or inventory of economically extractable resources. Firms can invest in grade specific extraction technology to subsequently convert deposits of lower grades into economically extractable reserves. We assume that R&D investment exhibits decreasing returns in making deposits of lower grades extractable, as historical evidence suggests. Once converted into a reserve, the firm that developed the technology can extract the resource at a fixed operational cost.

Third, new technology diffuses to all other firms. As each new technology is specific to a deposit of a certain grade, it cannot be used to extract resources from deposits of lower grades. However, all firms can build on existing technology when they invest in developing new technology for deposits of lower grades. The idea is that firms can, for example, use the shovel invented by another firm but have a cost to train employees to use it for a specific deposit of lower grade. As technology diffuses, firms only maximize current profits in their R&D investment decisions in equilibrium.

Finally, the non-renewable resource is a homogeneous good. Despite a fully competitive resource market in the long run, firms invest in extractive technology because it is grade specific. Most similar to this understanding of innovation is Desmet and Rossi-Hansberg (2014). We abstract from other possible features like uncertainty about deposits, negative externalities from resource extraction, recycling, and short-run price fluctuations.

Our model accommodates historical trends in the prices and production of major non-renewable resources, as well as world real GDP for which we present data extending back to 1792. It implies a constant resource price equal to marginal cost over the long run. Extraction firms face constant R&D costs in converting one unit of the resource into a new reserve. This is due to the offsetting interaction between technological change and geology: (i) new extraction technology exhibits decreasing returns in making deposits of lower grades extractable; (ii) the resource quantity is geologically distributed such that it increases exponentially as the grade of its deposits decreases.

The resource price depends negatively on the average crustal concentration of the resource. For example, our model predicts that iron ore prices are on average lower than copper prices, because iron is more abundant (5 percent of crustal mass) than copper (0.007 percent). The price is also negatively affected by the average effect of technology in terms of making lower grade deposits extractable. For example, the average effect might be larger for deposits that can be extracted in open pit mines (e.g. coal) than for deposits requiring underground operations (e.g. crude oil). This implies that coal prices are lower than crude oil price in the long term.

The resource intensity of the economy, defined as the resource quantity used to produce one unit of aggregate output, is positively affected by the average geological abundance and the average effect of extraction technology, while the elasticity of substitution has a strong negative effect. If the resource and the intermediate good are complements, the resource intensity of the economy is relatively high, while it is significantly lower in the case of the two being substitutes. As the resource intensity is constant in equilibrium, firms extract the non-renewable resource at the same rate as aggregate output.

Aggregate output growth is constant on the balanced growth path. Our model predicts that a higher abundance of a particular resource or a higher average effect of extractive technology in terms of lower grades positively impact aggregate growth in the long run.

The extractive sector features only constant returns to scale. In contrast to the intermediate goods sector, where firms can make use of the entire *stock* of technology for production, firms in the extractive sector can only use the *flow* of new technology to convert deposits of lower grades into new reserves. Earlier developed technologies are grade specific and the related deposits are exhausted. The stock of extraction technology therefore grows proportionally to output, while technology in the intermediate goods sectors increases at the same rate as aggregate output.

The paper contributes to a literature that mostly builds on the seminal Hotelling (1931) optimal depletion model. Heal (1976) introduces a non-renewable resource, which is inexhaustible, but extractable at different grades and costs. Extraction costs increase with cumulative extraction, but then remain constant when a "backstop technology" (Heal, 1976, p. 371) is reached. Slade (1982) adds exogenous technological change in extraction technology to the Hotelling (1931) model and predicts a U-shaped relative price curve. Cynthia-Lin and Wagner (2007) use a similar model with an inexhaustible non-renewable resource and exogenous technological change. They obtain a constant relative price with increasing extraction.

There are three papers, to our knowledge, that like ours include technological change in the extraction of a non-renewable resource in an endogenous growth model. Fourgeaud et al. (1982) focuses on explaining sudden fluctuations in the development of non-renewable resource prices by allowing the resource stock to grow in a stepwise manner through technological change. Tahvonen and Salo (2001) model the transition from a non-renewable energy resource to a renewable energy resource. Their model follows a learning-by-doing approach as technological change is linearly related to the level of extraction and the level of productive capital. It explains decreasing prices and the increasing use of a non-renewable energy resource over a particular time period before prices increase in the long term. Hart (2012) models resource extraction and demand in a growth model with directed technological change. The key element in his model is the depth of the resource. After a temporary "frontier phase" with a constant resource price and consumption rising at a rate only close to aggregate output, the economy needs to extract resources from greater depths. Subsequently, a long-run balanced growth path is reached with constant resource consumption and prices that rise in line with wages.

In Section 2, we document stylized facts on the long-term development of non-renewable resource prices, production, and world real GDP. We also provide evidence for the major assumptions of our model regarding geology and technological change. Section 3 presents the setup of the growth model and its extractive sector. In section 4, we present and discuss the model's theoretical implications. In Section 5 we draw conclusions.

2 Stylized facts

2.1 Prices, Resource Production, and Aggregate Output over the Long Term

Annual data for major non-renewable resource markets going back to 1792 indicate that real prices are roughly trend-less and that worldwide primary production as well as world real GDP grow roughly at a constant rate.

Figure 1 presents data on the real prices of five major base metals and crude oil. Real prices exhibit strong short-term fluctuations. At the same time, the growth rates of all prices are not significantly different from zero (see Table 1 in the appendix). The real prices are, thus, trend-less. This is in line with evidence over other time periods provided by Krautkraemer (1998), Von Hagen (1989), Cynthia-Lin and Wagner (2007), Stuermer (2016) and references therein. The real price for crude oil exhibits structural breaks, as shown in Dvir and Rogoff (2010). Overall, the literature is certainly not conclusive (see Pindyck, 1999; Lee et al., 2006; Slade, 1982; Jacks, 2013; Harvey et al., 2010), but we believe the evidence is sufficient to take trend-less prices as a motivation for our model.

Figure 2 shows that the world primary production of the examined non-renewable resources and world real GDP approximately exhibit constant positive growth rates since 1792. A closer statistical examination confirms that the production of non-renewable resources exhibits significantly positive growth rates in the long term (see table 2 in the appendix).[1]

The crude oil production follows this pattern up to 1975. Inclusion of the time period from 1975 until 2009 reveals a statistically significant negative trend and, therefore, declining growth rates over time due to a structural break in the oil market (Dvir and Rogoff, 2010; Hamilton, 2009). In the case of primary aluminum production, we also find declining growth rates over time and hence, no exponential growth of the production level. This might be attributable to the increasing importance of recycling (see data by U.S. Geological Survey, 2011a).

Insert Figure 1 about here.

Insert Figure 2 about here.

Overall, we take these stylized facts as motivation to build a model that exhibits trend-less resource prices and constant growth in the worldwide production of non-renewable resources and in world aggregate output.

2.2 Geological Abundance of Non-Renewable Resources

We update earlier computation of the total abundance (or quantity) of non-renewable resources by Nordhaus (1974). Table 4 shows the ratios of the quantities of reserves, resources, and geological abundance with respect to annual mine production for several

[1] As our model does not include population growth, we run the same tests for the per capita data as a robustness check. The results are roughly in line with the results described above. See table 3 in the appendix.

important non-renewable resources.[2] It provides evidence supporting the validity of Nordhaus' statement that "the future will not be limited by sheer availability of important materials"(Nordhaus, 1974, p. 23) As most metals are recyclable, the extractable stock in the techno-sphere even increases (Wellmer and Dalheimer, 2012).

We also add numbers for hydrocarbons. Even though conventional oil resources may be exhausted someday, resources of unconventional oil, natural gas, and coal, which could substitute for conventional oil in the long run, are abundant. Aguilera et al. (2012) state that conventional and unconventional resources "are likely to last far longer than many now expect" (p. 59). Rogner (1997) concludes that "fossil energy appears almost unlimited" (p. 249) given a continuation of historical technological trends.

Insert table 4 about here.

2.3 Geological Distribution of Non-Renewable Resources

Non-renewable resources are not uniformly concentrated in the earth's crust, reflecting variations in geochemical processes over time. Ahrens (1953, 1954) states in the fundamental law of geochemistry that the elements exhibit a log-normal grade-quantity distribution in the earth's crust, as he postulates a decided positive skewness.

Geologists do not fully agree on a log-normal distribution, especially regarding very low concentrations of metals, which might be mined in the distant future. Skinner

[2]Table 5 in the appendix illustrates that the assumption of exponentially increasing extraction of non-renewable resources does not alter the overall conclusion of table 4.

(1979) and Gordon et al. (2007) propose a discontinuity in the distribution due to the so-called "mineralogical barrier," the approximate point below which metal atoms are trapped by atomic substitution.

Gerst (2008) concludes in his geological study of copper deposits that he can neither confirm nor refute these two hypotheses. However, based on worldwide data on copper deposits over the past 200 years, he finds evidence for a log-normal relationship between copper production and ore grades. Mudd (2007) analyzes the historical evolution of extraction and grades of deposits for different base metals in Australia. He finds that production has increased at a constant rate, while grades have consistently declined.

We recognize that there remains uncertainty about the geological distribution, especially regarding hydrocarbons with their distinct formation processes. However, we believe that it is reasonable to assume that a non-renewable resource is distributed according to a log-normal relationship between the grade of deposits and quantity.

2.4 Technological Change in the Extractive Sector

Technological change in resource extraction offsets the depletion of economically extractable reserves of non-renewable resources (Simpson, 1999, and others). Hence, reserves are drawn down by extraction, but increase by technological change in extraction technology. The reason for this phenomenon is that non-renewable resources such as copper, aluminum, and hydrocarbons are extractable at different costs due to varying grades, thickness, depths, and other characteristics of mineral deposits. Technological change makes deposits economically extractable that, due to high costs, have not been previously extractable (see Simpson, 1999; Nordhaus, 1974, and others).

There is ample empirical and narrative evidence for this phenomenon (see for example Lasserre and Ouellette, 1991; Mudd, 2007; Simpson, 1999; Wellmer, 2008). For example, Radetzki (2009) and Bartos (2002) describe how technological changes in mining equipment, prospecting, and metallurgy have gradually made possible the extraction of copper from lower grade deposits. Figure 4 shows that copper reserves[3] have increased by more than 700 percent over the last couple of decades. As a consequence, the average ore grades of copper mines, for example, have decreased from about twenty percent 5,000 years ago to currently below one percent (Radetzki, 2009). Figure 3 illustrates this development using the example of U.S. copper mines.

Gerst (2008) and Mudd (2007) come to similar results for worldwide copper mines and the mining of different base-metals in Australia. The evidence also shows that decreases in average mined ore grades have slowed as technological development progressed. Under the assumption that global R&D investment has stayed constant or increased in real terms, this suggests that there are decreasing returns to R&D in terms of making mining from deposits of lower grades economically feasible.

Insert Figure 3 about here.

Insert Figure 4 about here.

We observe similar developments for hydrocarbons. Using the example of the offshore oil industry, Managi et al. (2004) show that technological change has offset the

[3]Reserves are those resources for which extraction is considered economically feasible (U.S. Geological Survey, 2011c).

cost-increasing degradation of resources. Crude oil has been extracted from ever deeper sources in the Gulf of Mexico, as Figure 5 in the appendix shows. Furthermore, technological change and high prices have made it profitable to extract hydrocarbons from unconventional sources, such as light tight oil, oil sands, and liquid natural gas (International Energy Agency, 2012). As a result, oil reserves have doubled since the 1980s (see figure 6 in the appendix).

Overall, empirical evidence suggests that technological change offsets resource depletion by increasing economically extractable reserves. History shows that average ore grades of mines declined while technological development progressed. Evidence suggests that the effect of technological change has slowed down in terms of making deposits of lower grades economically extractable.

3 The Model

We build an endogenous growth model with two sectors, an extractive sector and an intermediate goods sector, and take the framework by Acemoglu (2002) as a starting point.

3.1 The Setup

We consider a standard setup of an economy with a representative consumer that has constant relative risk aversion preferences:[4]

$$\int_0^\infty \frac{C_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \ . \tag{1}$$

[4]For a social planner version of the model, see the online appendix.

The variable C_t denotes the consumption of aggregate output at time t, ρ is the discount rate, and θ is the coefficient of relative risk aversion. To ease notation, we drop time indexes, whenever possible. The budget constraint of the representative consumer is:

$$C + I + M \leq Y \equiv \left[\gamma Z^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) R^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \ . \tag{2}$$

On the left hand side, C denotes consumption, I is aggregate investment in production, and M denotes aggregate R&D investment both in the extractive sector and the intermediate goods sector, where $M = M_Z + M_R$. The usual no-Ponzi game conditions apply.

On the right hand side, aggregate output production uses the non-renewable resource R, produced by the extractive sector, and an intermediate good Z, produced by the intermediate goods sector. The distribution parameter $\gamma \in (0, 1)$ indicates their respective importance in producing aggregate output Y. The elasticity of substitution between the resource and the intermediate good is denoted by $\epsilon \in (0, \infty)$.

3.2 The Extractive Sector

There is an infinite number of infinitely small extractive firms i, which produce the non-renewable resource from a geological environment.[5]

[5] For an alternative version of the extraction sector (using an approach analogous to the machine types in the intermediate sector) see the online appendix.

3.2.1 Geology

The geological environment consists of a continuum of deposits of declining grades $d \in [0, 1]$, which contain the non-renewable resource R. Mineral deposits exhibit a multitude of characteristics that affect extraction cost. However, we concentrate on the grade of deposits. Grade d refers to a measure of quality of the deposits, for example, ore grades in the case of metals. It may also point to different types of deposits of hydrocarbons. For example, we could say that conventional crude oil is extracted from high grade deposits, while unconventional crude oil is produced from low grade deposits. Grade $d = 1$ corresponds to a deposit, which is extractable without any technology. For example, 7,000 years ago humans picked-up high-grade copper nuggets from the ground. A deposit of grade zero does not contain any of the non-renewable resource.

We take a local approximation of the log-normal grade-quantity distribution proposed by Ahrens (1953, 1954) (see chapter 2). We assume that the resource quantity D is a function of the grade d of its deposits according to:

$$D(d) = -\delta_1 \ln(d), \quad \delta_1 \in \mathbb{R}_+ \ , \tag{3}$$

This functional form is also in line with above described historical evidence that the total quantity of non-renewable resource production has been inversely proportional to the grades of the deposits. We assume that this relationship is continuous to hold in a time-frame that is relevant to economic decision-making. The functional form therefore implies that the quantity of the resource becomes infinitely large for deposits of small grades.

Parameter δ_1 controls the curvature of the function. If δ_1 is high, the marginal effect on the quantity of the non-resources from shifting to deposits of lower grades is high. It implies that the average concentration of the non-renewable resource is high in the crustal mass (see also figure 7).

Insert Figure 7 about here.

3.2.2 Extraction Technology

Firms fully know about the deposits and the geological distribution of the resource quantity by grade. Firms invest M_R in terms of total output to develop grade specific extraction technology and to convert deposits of a specific grade into economically extractable reserves. Firms can extract the non-renewable resource from reserves at a constant operational cost. We assume that the operational cost for deposits that are not classified as reserves is infinitely high.

Each deposit of a certain grade d is associated with a unique state of the accumulated extraction technology N_R. Technology in the extractive sector is thus vertical. The grade specific technology diffuses directly after the invention, but allows a firm to claim all of the non-renewable resource in the respective deposit. The assumption of technology diffusion is reasonable for the long-run and for the global perspective of our model, as patent protection is often not available or enforced. In oil production, for example, firms are obliged by law to make partially public the ingredients of fracking new wells.

Based on the stylized facts in chapter 2, we assume that technological development makes deposits economically extractable, but that there are decreasing returns in terms of ore grades. We therefore assume the following extraction technology function h:

$$h(N_R) = e^{-\delta_2 N_R}, \ \delta_2 \in \mathbb{R}_+ \ . \tag{4}$$

Figure 8 shows the function. It shows how an increase in the stock of mining technology N_R makes deposits of certain grades extractable. The curve starts with deposits of 100 percent ore grade, which represents the state of the world several thousand years ago. The functional form is roughly in line with historical data for U.S. copper mining over the last 100 years, as presented in Figure 3. We assume this relationship holds continuous in a time-frame that is relevant to economic decision making. We presume that ore grades only get closer to zero in the long term.

The marginal effect of the extraction technology on converting deposits to economically extractable reserves declines as grades decrease. δ_2 is the curvature parameter of the extraction technology function. If, for example, δ_2 is high, the average effect of new technology on converting deposits to reserves in terms of grades is relatively high.

Insert Figure 8 about here.

We make use of a simple extraction cost function. Each deposit of a certain grade d_N is associated with a unique state of the technology, above which the deposit can be extracted at cost $\phi_{N_R} = E$. Extraction is impossible at grades lower than d_N, because the cost is assumed to be infinite.

The extraction cost function takes the degenerate form of

$$\phi_{N_R}(d) = \begin{cases} E, & \text{if} \quad d \geq d_{N_R}, \\ \\ \infty, & \text{if} \quad d < d_{N_R}. \end{cases} \tag{5}$$

The underlying idea is that firms always need to innovate or train employees to extract the non-renewable resource from a deposit of lower grade. This very stylized form allows us to obtain an analytical solution in the growth model of Section 3, while preserving key features, namely: (i) extraction cost depends on the ore grade and (ii) innovations in extraction technology reduce extraction cost for a deposit of certain grade.

3.2.3 Firms' Innovation Process

While the model is in continuous time, we explain the intuition for the innovation process based on discrete time. We define time as the interval between two successive innovations. In the early period of t, firm i spends on R&D in terms of the final product M_{Rt} and develops a new extraction technology based on the level of extraction technology in the previous period N_{Rt-1}. The new extraction technology is specific to a deposit of a certain grade. It allows the firm to claim ownership of the respective deposit and to declare it to be part of its reserves.

In the middle of period t, the new technology diffuses and becomes available to all other firms. New technology in the extractive sector is produced according to:

$$\dot{N}_R = \eta_R M_R \ , \tag{6}$$

where η_R is a cost parameter of technological innovation. One unit of final good spent on R&D will produce η_R new technologies. Technology is derived from R&D investment in a deterministic way. This reflects a long-term perspective.

In the late period of t, the firm decides whether to extract the non-renewable resource from its reserves and sell it. Firms take the price of the non-renewable resource, p_R, as given. There is perfect competition due to free market entry and a linear production technology.[6] One firm extracts the resource at any given point in time.

3.2.4 Firms' Optimization Problem

Firms in the extractive sector invest in new extraction technology despite the fully competitive environment, because technology is grade specific. As soon as the cost of developing a lower-grade technology can be paid for by revenues, firms will invest.

As new technology is specific to a deposit of a certain grade and benefits diffuse within one period, firms maximize only current profits when making their technology investment decisions. They generate just enough revenue from selling the resource R at the given price p_R to cover the R&D spending on extraction technology M_R. Firms have operational costs ϕ_{N_R} for extracting one unit of the non-renewable resource. Firms make zero profits while innovating. Each extraction firm solves a static optimization problem, where the firm chooses investment into extraction technology optimally:

[6] We believe that perfect competition is a sensible assumption for non-renewable resource markets in the long run. There is ample evidence that this is the case when looking at these markets over a long period, as supply is elastic in the long run (see e.g. Radetzki, 2008). Perfect competition in the extraction sector results from the inexhaustible character of the resource. If one firm demands a price above marginal cost, another firm can develop additional technology, extract the resource from lower ore grades and sell it at a lower price in the long run.

$$\max_{R} p_R R - \phi_{N_R} R - M_R \ . \tag{7}$$

Most similar to this understanding of innovation is Desmet and Rossi-Hansberg (2014), where non-replicable factors of production ensure technological development in a perfectly competitive environment. In our case these non-replicable factors are deposits of a specific grade.

Boiling down a dynamic optimization problem to a static one is key to our theory. It allows us to make the model solvable and computable. At the same time, the model is rich enough to derive meaningful theoretical predictions about the relationship between technological change, geology and economic growth.

Each extractive firm's reserves evolve according to:

$$\dot{S}_t = X_t - R_t \ , \qquad S_t \geq 0, X_t \geq 0, R_t \geq 0 \ , \tag{8}$$

A change in the resource quantity in reserves $\dot{S}$ is the result of two flows: (i) inflows of new resource quantities X due to technology investment that convert mineral deposits into reserves, and (ii) outflows of resource quantities R due to extraction and marketing.

Rearranging equation (8), we obtain the production function of the extractive sector:

$$R_t = X_t - \dot{S}_t \ . \tag{9}$$

Note that for $X_t = 0$, this formulation is the standard Hotelling (1931) setup.

3.3 The Intermediate Goods Sector

The intermediate goods sector follows the basic setup of Acemoglu (2002). Firms produce an intermediate good Z according to the following production function:[7]

$$Z = \frac{1}{1-\beta}\left(\int_0^{N_z} x_z(j)^{1-\beta}dj\right)L^{\beta} \ , \tag{10}$$

where $\beta \in (0,1)$. Firms use labor L, which is in fixed supply, and machines as inputs to production. $x_Z(j)$ refers to the number of machines used for each machine variety j. Machines depreciate fully within one period. We denote the range of machines that can be used for production by N_z. The intermediate good market is competitive. The maximization problem of firms in the intermediate goods sector is:

$$\max_{L,\{x_Z(j)\}} p_Z Z - wL - \int_0^{N_Z} \chi_Z(j)x_Z(j)dj \ , \tag{11}$$

where p_Z denotes the price of the intermediate good, w is the wage rate, and $\chi_Z(j)$ refers to the rental rate for machine type j.

Sector-specific technology firms invent new technologies for which they hold a fully enforceable patent. They exploit the patent by becoming the sole supplier of the machine that corresponds uniquely to their technology. The uniqueness provides market power that they can use to set a price $\chi_Z(j)$ above marginal cost. This allows them to derive profits and to finance innovation. The marginal cost of producing a patent for a new machine in terms of the final good is the same for all machines.

[7] Like Acemoglu (2002) we assume that the firm level production functions exhibit constant returns to scale, so there is no loss of generality in focusing on the aggregate production functions.

Technology firms in the intermediate goods sector invent new varieties of machines $\dot{N}_Z$ according to

$$\dot{N}_Z = \eta_Z M_Z \ , \tag{12}$$

where M_Z is R&D investment by the technology firms in terms of the final product, and η_Z is a cost parameter. For more details, the reader is referred to the appendix and Acemoglu (2002).

4 Theoretical Implications

We derive six propositions on the interaction between geology and technology and its impact on the resource price, total output, and the resource intensity of the economy.[8]

First, the geological function, equation (3), and the technological function, equation (equation 4), have offsetting effects. Extraction firms therefore face constant R&D costs in converting one unit of the resource from a deposit into a new reserve.

Proposition 1 *The total resource quantity converted to economically extractable reserves develops proportionally to the level of extraction technology N_R:*

$$D(h(N_{Rt})) = \delta_1 \delta_2 N_{Rt} \ . \tag{13}$$

The marginal return on extraction technology, $\dot{N}_R$, in terms of resource quantities in new reserves, X, is constant due to offsetting interaction between technological change

[8] Proofs for this section are in the Appendix.

and geology: (i) new extraction technology exhibits decreasing returns in terms of making lower grade deposits extractable; (2) the resource quantity is geologically distributed such that it implies increasing returns in terms of resource quantities as the grade of its deposits declines.

$$X_t = \frac{\partial D(h(N_{Rt}))}{\partial t} = \delta_1 \delta_2 \dot{N}_{Rt} \ . \tag{14}$$

Equation 14 depends on the shapes of the geological function and the technology function. If the respective parameters δ_1 and δ_2 are high, the marginal return on new extraction technology will also be high.

The marginal effect of R&D investment on new reserves in terms of resource quantities is also constant. Using equations (6) and (14), we can show that new reserves X develop in proportion to R&D investment in extraction technology $M_R = \frac{1}{\eta_R}\dot{N}_R = \frac{1}{\eta_R \delta_1 \delta_2} X_t$.

Second, since the marginal effect of R&D investment on new reserves is constant, extractive firms' optimization yields the typical result of stock management: inflows and outflows into reserves balance over time.

Proposition 2 *Firms increase the quantity of resources in their reserves due to R&D in extraction technology X. This equals resource extraction R: $X_t = R_t$.*

If we assume stochastic technological change, extractive firms will keep a positive stock of reserves S_t to insure against a series of bad draws in R&D. Reserves will grow over time in line with aggregate growth. Proposition 2 would, however, remain the same: In the long term, resource extraction equals resources in new reserves.

Third, the resource price equals marginal production costs due to perfect competition in the resource market. Firms' marginal production costs consist of their R&D cost for converting one unit of the resource into new reserves, if we set operational cost to zero $\phi_{N_R} = 0$.

Proposition 3 *The resource price depends negatively on the average crustal concentration of the non-renewable resource and the average effect of extraction technology:*

$$p_{Rt} = \frac{1}{\eta_R \delta_1 \delta_2} \ . \tag{15}$$

The intuition is as follows: If, for example, δ_1 increases, the average crustal concentration of the resource increases (see equation (3)) and the price decreases. If δ_2 rises, the average effect of new extraction technology on converting deposits of lower grades to reserves increases (see equation (4)). This implies lower prices. The resource price level also depends negatively on the cost parameter of R&D development η_R.

Fourth, substituting equation (15) into the resource demand equation (18) in the appendix, we obtain the ratio of resource consumption to aggregate output.

Proposition 4 *The resource intensity of the economy is positively affected by the average crustal concentration of the resource and the average effect of extraction technology:*

$$\frac{R}{Y} = \left[(1-\gamma)\eta_R \delta_1 \delta_2\right]^{\varepsilon} \ .$$

The elasticity of substitution has a strong negative impact on the resource intensity. If the resource and the intermediate good are complements, $\epsilon < 1$, the resource intensity of the economy is relatively high and the resource price is relatively low compared to the intermediate goods price. If they are substitutes, $\epsilon > 1$, the resource intensity is significantly lower and the relative resource price is high. The resource intensity also depends positively on parameter γ, which governs the importance of the resource in the aggregate production function.

Fifth, we turn to the solution of the model. Adding the extractive sector to the standard model by Acemoglu (2002), changes the interest part of the Euler equation, $g = \theta^{-1}(r - \rho)$.[9] Instead of two exogenous production factors, the interest rate r in our model only includes labor, but adds the resource price.

Proposition 5 *The growth rate on the balanced growth path of the economy is constant and given by*

$$g = \theta^{-1}\left(\beta\eta_Z L\left[\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{1}{\eta_R\delta_1\delta_2}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}\frac{1}{\beta}} - \rho\right).$$

The growth rate of the economy depends positively on the average geological concentration of the resource, δ_1, and the average effect of extractive technology in terms of lower grades, δ_2.[10]

[9]There is no capital in this model, but agents delay consumption by investing in R&D as a function of the interest rate.

[10]When the elasticity of substitution is low and the resource price is high (such that $\gamma^{-\varepsilon} < \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{1}{\eta_R\delta_1\delta_2}\right)^{1-\varepsilon}$ a "limits to growth" situation occurs. There is no solution to equation (28) in this case. The intuition is that developing new technologies to make new reserves extractable is extremely difficult and substituting the resource by intermediate goods is only limited. Aggregate output production may therefore be impossible (see also Dasgupta and Heal, 1979, p. 196).

Finally, we derive the growth rates of technology in the two sectors. The stock of technology in the intermediate goods sector grows at the same rate as the economy.

Proposition 6 *The stock of extraction technology grows proportionally to output according to:*

$$\dot{N}_R = \eta_R M_{Rt} = (\delta_1 \delta_2)^{\varepsilon - 1}(1-\gamma)^{\varepsilon}\eta_R Y \ .$$

In contrast to the intermediate goods sector, where firms can make use of the entire *stock* of technology, firms in the extractive sector can only use the *flow* of new technology to convert deposits of lower grades into new reserves. Previously developed technology cannot be employed because it is grade specific, and deposits of that particular grade have already been depleted. Note also that firms in the extractive sector need to invest a larger share of total output to attain the same rate of growth in technology in comparison as firms in the intermediate goods sector.

The effects of the two parameters δ_1 from the geological function and δ_2 from the extraction technology function on $\dot{N}_R$ depend on the elasticity of substitution ε. If $\epsilon < 1$, the resource and the intermediate good are gross complements, and increases in δ_1 or δ_2 affect $\dot{N}_R$ negatively. If the two goods are gross substitutes, higher parameter values for δ_1 or δ_2 lead to more R&D investment in the extractive sector and hence a higher rate of technological development in the extractive sector.

4.1 Discussion

We discuss the assumptions made in section 3, the comparison to the other models with non-renewable resources, and the question of the ultimate finiteness of the resource.

How would other functional forms of the geological function in equation (3) affect the predictions of the model? First, the predictions are valid for all parameter values $\delta_1 \in \mathbb{R}_+$. Secondly, if D is discontinuous with an unanticipated break at d_0, at which the parameter changes to $\delta_1' \in \mathbb{R}_+$, there would be two balanced growth paths: one for the period before, and one for the period after the break. Both paths would behave according to the model's predictions. The paths would differ in the extraction cost of producing the resource, level of extraction, and use of the resource in the economy. To see this, recall from proposition 1 that X_t is a function of δ_1. A non-exponential form of D would produce results that differ from ours. It could feature a scarcity rent as in the Hotelling (1931) model, as a non-exponential form of D would cause a positive trend in resource prices. The extraction of deposits of lower ore grade might also become infeasible. In these cases, the extractive firms consider the opportunity cost of extracting the resource in the future, in addition to extraction and innovation cost. Note that such a scarcity rent has not yet been found empirically (see e.g. Hart and Spiro, 2011).

How does our model compare to other models with non-renewable resources? We make the convenient assumption that the quantity of non-renewable resources is for all practical economic purposes infinite. As a consequence, resource availability does not limit growth. Substitution of capital for non-renewable resources, technological change in the use of the resource, and increasing returns to scale are therefore not necessary for sustained growth as in Groth (2007) or Aghion and Howitt (1998). If the resource were finite in our model, the extractive sector would behave in the same way as in standard models with a sector based on Hotelling (1931). As Dasgupta and Heal (1980) point out, in this case the growth rate of the economy depends strongly on the degree of substitution between the resource and other economic inputs. For $\varepsilon > 1$, the resource is non-essential; for $\varepsilon < 1$, the total output that the economy is capable of producing is finite. The production function is, therefore, only interesting for the Cobb-Douglas case.

Our model suggests that the non-renewable resource can be thought of as a form of capital: if the extractive firms invest in R&D in extraction technology, the resource is extractable without limits as an input to aggregate production. This feature marks a distinctive difference from models such as the one of Bretschger and Smulders (2012). They investigate the effect of various assumptions about substitutability and a decentralized market on long-run growth, but keep the assumption of a finite non-renewable resource. Without this assumption, the elasticity of substitution between the non-renewable resource and other input factors is no longer central to the analysis of limits to growth.

Some might argue that the relationship described in proposition 1 cannot continue to hold in the future as the amount of non-renewable resources in the earth's crust is ultimately finite. Scarcity will become increasingly important, and the scarcity rent will be positive even in the present. However, for understanding current prices and consumption patterns, current expectations about future developments are important. Given that the quantities of available resources indicated in table 4 are very large, their ultimate end far in the future should not affect behavior today. The relationship described in proposition 1 seems to have held in the past and looks likely to hold for the foreseeable future. Since in the long term, extracted resources equal the resources added to reserves due to R&D in extraction technology, the price for a unit of the resource will equal the extraction cost plus the per-unit cost of R&D and hence, stay constant in the long term. This may explain why scarcity rents cannot be found empirically.

5 Conclusion

This paper examines interaction between geology and technology and its impact on the resource price, total output growth, and the resource intensity of the economy. We argue that economic growth causes the production and use of a non-renewable resource to increase at a constant rate. Marginal production costs stay constant in the long term. Economic growth enables firms to invest in extraction technology R&D, which makes resources from deposits of lower grades economically extractable. We help explain the long-term evolution of non-renewable resource prices and world production for more than 200 years. If historical trends in technological progress continue, it is possible that non-renewable resources are, within a time frame relevant for humanity, practically inexhaustible.

Our model makes strong simplifying assumptions, which render our model analytically solvable. However, we believe that a less simple model would essentially provide the same results. There are four major simplifications, which should be examined in more detail in future extensions. First, there is no uncertainty in R&D development, and therefore no incentive for firms to keep a positive amount of the non-renewable resource in their reserves. If R&D development is stochastic as in Dasgupta and Stiglitz (1981), there would be a need for firms to keep reserves.

Second, our model features perfect competition in the extractive sector. We could obtain a model with monopolistic competition in the extractive sector by introducing explicitly privately-owned deposits. A firm would need to pay a certain upfront cost or exploration cost in order to acquire a mineral deposit. This upfront cost would give technology firms a certain monopoly power as they develop machines that are specific to a single deposit.

Third, extractive firms could face a trade-off between accepting high extraction costs due to a lower technology level and investing in R&D to reduce extraction costs. A more general extraction technology function would provide the basis to generalize this assumption.

Fourth, our model does not include recycling. Recycling has become more important for metal production over time due to the increasing abundance of recyclable materials and the comparatively low energy requirements (see Wellmer and Dalheimer, 2012). Introducing recycling into our model would further strengthen our argument, as it increases the economically extractable stock of the non-renewable resource.

Appendix 1 Proofs

Proof of Proposition 1

$$\begin{aligned} D(h(N_{Rt})) &= -\delta_1 \ln(d_{N_{Rt}}) \\ &= -\delta_1 \ln(e^{-\delta_2 N_{Rt}}) \\ &= \delta_2 \delta_1 N_{Rt} \end{aligned}$$

$\square$

Proof of Proposition 2 and 3

The final good producer demands the resource for aggregate production. The price of the final good is the numeraire. The first order condition with respect to the resource in equation (2) is

$$Y^{\frac{1}{\varepsilon}}(1-\gamma)R^{-\frac{1}{\varepsilon}} - p_R = 0 \ , \tag{16}$$

so that the demand for the resource is

$$R = \frac{Y(1-\gamma)^{\varepsilon}}{p_R^{\varepsilon}} \ . \tag{17}$$

Assume that initially, economically extractable reserves available to extractive firms are zero, $S_t = 0$. Since reserves S cannot be negative, new reserves cannot be less than resources sold to the final good producer: $X_t \geq R_t$. In a world without uncertainty holding reserves would not be profitable. The resource price therefore equals marginal cost:

$$p_R = \frac{1}{\eta_R \delta_1 \delta_2} \ . \tag{18}$$

It remains to consider the case of a positive initial stock of reserves, $S_t > 0$. Under perfect competition, these reserves are immediately sold off to the final good producer such that the case of $S_t = 0$ returns. □

Proof of Proposition 5

The first order conditions of the final good producer for the optimal inputs of Z and R are $Y^{\frac{1}{\varepsilon}} \gamma Z^{-\frac{1}{\varepsilon}} - p_Z = 0$ and $Y^{\frac{1}{\varepsilon}}(1-\gamma)R^{-\frac{1}{\varepsilon}} - p_R = 0$, where the final good is the numeraire. From this the relative price is

$$p = \frac{p_R}{p_Z} = \frac{1-\gamma}{\gamma} \left(\frac{R}{Z} \right)^{-\frac{1}{\varepsilon}} \ . \tag{19}$$

Setting the price of the final good as the numeraire gives (for the derivation of the price index see the derivation of equation (12.11) in Acemoglu (2009)):

$$\left[\gamma^{\varepsilon} p_Z^{1-\varepsilon} + (1-\gamma)^{\varepsilon} p_R^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} = P = 1 \ . \tag{20}$$

The Intermediate Goods Sector

The maximization problem in the intermediate goods sector is provided in equation 11 in the main body of the paper. The FOC with respect to $x_Z(j)$ is $p_Z x_Z(j)^{-\beta} L^{\beta} - \chi_Z(j) = 0$ so that

$$x_Z(j) = \left(\frac{p_Z}{\chi_Z(j)} \right)^{\frac{1}{\beta}} L \ . \tag{21}$$

From the FOC with respect to L we obtain the wage rate

$$w = \frac{\beta}{1-\beta} p_Z \left(\int_0^{N_Z} x_Z(j)^{1-\beta} dj \right) L^{\beta-1} \ . \tag{22}$$

Profits of the technology firm supplying machine j equal:

$$\pi_Z(j) = (\chi_Z(j) - \psi) x_Z(j) \ . \tag{23}$$

Substituting equation (21) into equation (23) we calculate the FOC with respect to the price of a machine $\chi_Z(j)$: $\left(\frac{p_Z}{\chi_Z(j)}\right)^{\frac{1}{\beta}} L - (\chi_Z(j) - \psi) p_Z^{\frac{1}{\beta}} \frac{1}{\beta} \chi_Z(j)^{\frac{1}{\beta}-1} L = 0$. Solving this for $\chi_Z(j)$ yields $\chi_Z(j) = \frac{\psi}{1-\beta}$. Following Acemoglu (2002) we normalize $\psi = 1 - \beta$ so that $\chi_Z(j) = 1$. Combining this result with equations (21) and (23) we express profits as

$$\pi_Z(j) = \beta p_Z^{\frac{1}{\beta}} L \ . \tag{24}$$

The present discounted value of profits is:

$$r V_Z - \dot{V}_Z = \pi_Z \ , \tag{25}$$

where r is the interest rate.

The steady state ($\dot{V} = 0$) is:

$$V_Z = \frac{\beta p_Z^{\frac{1}{\beta}} L}{r} \ . \tag{26}$$

Substituting equation (21) into equation (10) yields

$$Z = \frac{1}{1-\beta} p_Z^{\frac{1-\beta}{\beta}} N_Z L \ . \tag{27}$$

Solving equation (20) for p_Z yields the price of the intermediate good:

$$p_Z = \left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon} p_R^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \ . \tag{28}$$

This can be used, together with the expression for R from equation (17) and the expression for p_R from equation (18) to determine Z as a function of Y from equation (19). We obtain the range of machines N_Z as a function of Y from equation (27).

The Growth Rate

The consumer earns wages from working in the sector which produces good Z and earns interest on investing in technology M_Z. The budget constraint thus is $C = wL + rM_Z$. Maximizing utility in equation (1) with respect to consumption and investments yields the first order conditions $C^{-\theta} e^{-\rho t} = \lambda$ and $\dot{\lambda} = -r\lambda$ so that the growth rate of consumption is

$$g_c = \theta^{-1}(r - \rho) \ . \tag{29}$$

This will be equal to output growth on the balanced growth path. We can thus solve for the interest rate and obtain $r = \theta g + \rho$. The free entry condition for the technology firms imposes that profits from investing in patents must be zero. Revenue per unit of R&D investment is given by V_Z, cost is equal to $\frac{1}{\eta_Z}$. Consequently, we obtain $\eta_Z V_Z = 1$. Making use of equation (26), we obtain $\frac{\eta_Z \beta p_Z^{\frac{1}{\beta}} L}{r} = 1$. Solving this for r and substituting it into equation (29) we obtain:

$$g = \theta^{-1}(\beta \eta_Z L p_Z^{\frac{1}{\beta}} - \rho) \; . \tag{30}$$

Together with Equations (18) and (28) this yields the growth rate on the balanced growth path. □

Proof of Proposition 6

The expression for $\dot{N}_R$ follows from equation (14), proposition 2 and equation (17).

□

References

Acemoglu, D. (2002). Directed technical change. *The Review of Economic Studies*, 69(4):781–809.

Acemoglu, D. (2009). *Introduction to modern economic growth*. Princeton University Press, Princeton, N.J.

Acemoglu, D., Aghion, P., Bursztyn, L., and Hemous, D. (2012). The environment and directed technical change. *American Economic Review*, 102(1):131–66.

Aghion, P. and Howitt, P. (1998). *Endogenous growth theory*. MIT Press, London.

Aguilera, R., Eggert, R., Lagos C.C., G., and Tilton, J. (2012). Is depletion likely to create significant scarcities of future petroleum resources? In Sinding-Larsen, R. and Wellmer, F., editors, *Non-renewable resource issues*, pages 45–82. Springer Netherlands, Dordrecht.

Ahrens, L. (1953). A fundamental law of geochemistry. *Nature*, 172:1148.

Ahrens, L. (1954). The lognormal distribution of the elements (a fundamental law of geochemistry and its subsidiary). *Geochimica et Cosmochimica Acta*, 5(2):49–73.

Bartos, P. (2002). SX-EW copper and the technology cycle. *Resources Policy*, 28(3-4):85–94.

Bretschger, L. and Smulders, S. (2012). Sustainability and substitution of exhaustible natural resources: How structural change affects long-term R&D investments. *Journal of Economic Dynamics and Control*, 36(4):536 – 549.

British Petroleum (2013). BP statistical review of world energy. http://www.bp.com/ (accessed on March 14, 2014).

Cynthia-Lin, C. and Wagner, G. (2007). Steady-state growth in a Hotelling model of resource extraction. *Journal of Environmental Economics and Management*, 54(1):68–83.

Dasgupta, P. and Heal, G. (1979). Economic theory and exhaustible resources. *Cambridge Economic Handbooks (EUA)*.

Dasgupta, P. and Heal, G. (1980). *Economic theory and exhaustible resources*. Cambridge University Press, Cambridge, U.K.

Dasgupta, P. and Stiglitz, J. (1981). Resource depletion under technological uncertainty. *Econometrica*, 49(1):85–104.

Desmet, K. and Rossi-Hansberg, E. (2014). Innovation in space. *American Economic Review*, 102(3):447–452.

Dvir, E. and Rogoff, K. (2010). The three epochs of oil. *mimeo*.

Federal Institute for Geosciences and Natural Resources (2011). *Reserven, Resourcen, Verfügbarkeit von Energierohstoffen 2011*. Federal Institute for Geosciences and Natural Resources, Hanover, Germany.

Fourgeaud, C., Lenclud, B., and Michel, P. (1982). Technological renewal of natural resource stocks. *Journal of Economic Dynamics and Control*, 4(1):1–36.

Gerst, M. (2008). Revisiting the cumulative grade-tonnage relationship for major copper ore types. *Economic Geology*, 103(3):615.

Golosov, M., Hassler, J., Krusell, P., and Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88.

Gordon, R., Bertram, M., and Graedel, T. (2007). On the sustainability of metal supplies: a response to Tilton and Lagos. *Resources Policy*, 32(1-2):24–28.

Groth, C. (2007). A new growth perspective on non-renewable resources. In Bretschger, L. and Smulders, S., editors, *Sustainable Resource Use and Economic Dynamics*, chapter 7, pages 127–163. Springer Netherlands, Dordrecht.

Hamilton, J. (2009). Understanding crude oil prices. *The Energy Journal*, 30(2):179–206.

Hart, R. (2012). The economics of natural resources: understanding and predicting the evolution of supply and demand. *Working Paper of the Department of Economics of the Swedish University of Agricultural Sciences*, 1(3).

Hart, R. and Spiro, D. (2011). The elephant in Hotelling's room. *Energy Policy*, 39(12):7834–7838.

Harvey, D. I., Kellard, N. M., Madsen, J. B., and Wohar, M. E. (2010). The Prebisch-Singer hypothesis: four centuries of evidence. *The Review of Economics and Statistics*, 92(2):367–377.

Hassler, J. and Sinn, H.-W. (2012). The fossil episode. Technical report, CESifo Working Paper: Energy and Climate Economics.

Heal, G. (1976). The relationship between price and extraction cost for a resource with a backstop technology. *The Bell Journal of Economics*, 7(2):371–378.

Hotelling, H. (1931). The economics of exhaustible resources. *Journal of Political Economy*, 39(2):137–175.

International Energy Agency (2012). *World energy outlook 2012*. International Energy Agency, Paris.

Jacks, D. S. (2013). From boom to bust: A typology of real commodity prices in the long run. Technical report, National Bureau of Economic Research.

Krautkraemer, J. (1998). Nonrenewable resource scarcity. *Journal of Economic Literature*, 36(4):2065–2107.

Lasserre, P. and Ouellette, P. (1991). The measurement of productivity and scarcity rents: the case of asbestos in canada. *Journal of Econometrics*, 48(3):287–312.

Lee, J., List, J., and Strazicich, M. (2006). Non-renewable resource prices: deterministic or stochastic trends? *Journal of Environmental Economics and Management*, 51(3):354–370.

Littke, R. and Welte, D. (1992). *Hydrocarbon source rocks*. Cambridge University Press, Cambridge, U.K.

Livernois, J. (2009). On the empirical significance of the Hotelling rule. *Review of Environmental Economics and Policy*, 3(1):22–41.

Managi, S., Opaluch, J., Jin, D., and Grigalunas, T. (2004). Technological change and depletion in offshore oil and gas. *Journal of Environmental Economics and Management*, 47(2):388–409.

Mudd, G. (2007). An analysis of historic production trends in australian base metal mining. *Ore Geology Reviews*, 32(1):227–261.

Nordhaus, W. (1974). Resources as a constraint on growth. *American Economic Review*, 64(2):22–26.

Officer, L. (2011). *Dollar-Pound exchange rate from 1791*. MeasuringWorth.

Officer, L. and Williamson, S. (2011). The annual consumer price index for the united states, 1774-2010. http://www.measuringworth.com/uscpi/ (accessed November 2, 2011).

Perman, R., Yue, M., McGilvray, J., and Common, M. (2003). *Natural resource and environmental economics.* Pearson Education, Edinburgh.

Pindyck, R. (1999). The long-run evolution of energy prices. *The Energy Journal*, 20(2):1–28.

Radetzki, M. (2008). *A handbook of primary commodities in the global economy.* Cambridge Univ. Press, Cambridge, U.K.

Radetzki, M. (2009). Seven thousand years in the service of humanity: the history of copper, the red metal. *Resources Policy*, 34(4):176–184.

Rogner, H. (1997). An assessment of world hydrocarbon resources. *Annual Review of Energy and the Environment*, 22(1):217–262.

Scholz, R. and Wellmer, F. (2012). Approaching a dynamic view on the availability of mineral resources: what we may learn from the case of phosphorus? *Global Environmental Change*, 23(1):11–27.

Simon, J. (1981). *The ultimate resource.* Princeton University Press, Princeton, N.J.

Simpson, R., editor (1999). *Productivity in natural resource industries: improvement through innovation.* RFF Press, Washington, D.C.

Skinner, B. (1979). A second iron age ahead? *Studies in Environmental Science*, 3:559–575.

Slade, M. (1982). Trends in natural-resource commodity prices: an analysis of the time domain. *Journal of Environmental Economics and Management*, 9(2):122–137.

Stuermer, M. (2013). *What drives mineral commodity markets in the long run?* PhD thesis, University of Bonn. http://hss.ulb.uni-bonn.de/2013/3313/3313.pdf (accessed September 4, 2013).

Stuermer, M. (2016). 150 years of boom and bust: What drives mineral commodity prices? *Macroeconomic Dynamics*, forthcoming.

Tahvonen, O. and Salo, S. (2001). Economic growth and transitions between renewable and nonrenewable energy resources. *European Economic Review*, 45(8):1379–1398.

Tilton, J. and Lagos C.C., G. (2007). Assessing the long-run availability of copper. *Resources Policy*, 32(1-2):19–23.

U.S. Bureau of Labor Statistics (2010). Consumer price index. All urban consumers. U.S. city average. All items.

U.S. Bureau of Mines (1991). *Minerals yearbook. 1991. Metals and minerals.* U.S. Bureau of Mines, Washington, D.C.

U.S. Geological Survey (2010). *Gold statistics.* U.S. Geological Survey, Reston, V.A.

U.S. Geological Survey (2011a). *Historical statistics for mineral and material commodities in the United States.* U.S. Geological Survey, Reston, V.A.

U.S. Geological Survey (2011b). *Mineral commodity summaries 2011.* U.S. Geological Survey, Reston, V.A.

U.S. Geological Survey (2011c). *Minerals yearbook 2010.* U.S. Geological Survey, Reston, V.A.

U.S. Geological Survey (2012a). *Historical statistics for mineral and material commodities in the United States.* U.S. Geological Survey, Reston, V.A.

U.S. Geological Survey (2012b). *Mineral commodity summaries 2012.* U.S. Geological Survey, Reston, V. A.

van der Ploeg, F. and Withagen, C. (2012). Too much coal, too little oil. *Journal of Public Economics*, 96(1):62–77.

Von Hagen, J. (1989). Relative commodity prices and cointegration. *Journal of Business & Economic Statistics*, 7(4):497–503.

Wellmer, F. (2008). Reserves and resources of the geosphere, terms so often misunderstood. Is the life index of reserves of natural resources a guide to the future. *Zeitschrift der Deutschen Gesellschaft für Geowissenschaften*, 159(4):575–590.

Wellmer, F. and Dalheimer, M. (2012). The feedback control cycle as regulator of past and future mineral supply. *Mineralium Deposita*, 47:713–729.

Appendix 2 Tables

		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil
Range		1905-2009	1792-2009	1792-2009	1792-2009	1824-2009	1862-2009
Constant	Coeff.	-1.774	0.572	0.150	1.800	1.072	8.242
	t-stat.	(-0.180)	(0.203)	(0.052)	(0.660)	(0.205)	(0.828)
Lin.Trend	Coeff.	0.008	0.009	0.016	0.001	0.014	-0.021
	t-stat.	(0.137)	(0.428)	(0.714)	(0.069)	(0.357)	(-0.317)
Range		1905-2009	1850-2009	1850-2009	1862-2009	1850-2009	1850-2009
Constant	Coeff.	-1.299	0.109	-0.268	2.439	1.894	7.002
	t-stat.	(-0.200)	(0.030)	(-0.073)	(0.711)	(0.407)	(1.112)
Lin.Trend	Coeff.	0.008	0.020	0.030	-0.004	0.013	-0.021
	t-stat.	(0.137)	(0.518)	(0.755)	(-0.109)	(0.267)	(-0.317)
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	-0.903	-1.428	-0.490	1.068	2.764	-1.974
	t-stat.	(-0.239)	(-0.332)	(-0.102)	(0.269)	(0.443)	(-0.338)
Lin.Trend	Coeff.	0.008	0.055	0.054	0.010	0.010	0.100
	t-stat.	(0.137)	(0.820)	(0.713)	(0.168)	(0.099)	(1.106)
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	2.269	1.556	-3.688	-0.061	-0.515	3.445
	t-stat.	(0.479)	(0.240)	(-0.505)	(-0.011)	(-0.062)	(0.354)
Lin.Trend	Coeff.	-0.055	0.041	0.198	0.049	0.103	0.090
	t-stat.	(-0.411)	(0.225)	(0.958)	(0.307)	(0.441)	(0.326)
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	-0.549	1.323	0.370	3.719	1.136	-1.111
	t-stat.	(-0.088)	(0.266)	(0.081)	(0.812)	(0.176)	(-0.176)
Lin.Trend	Coeff.	-0.003	0.011	0.030	-0.012	0.051	0.094
	t-stat.	(-0.033)	(0.135)	(0.383)	(-0.152)	(0.468)	(0.875)

Notes: The table presents coefficients and t-statistics for regressions of the growth rates on a constant and a linear trend.***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 1: Tests of the stylized fact that the growth rates of real prices of mineral commodities equal zero and do not follow a statistically significant trend.

		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil	World GDP
Range		1855-2009	1821-2009	1802-2009	1792-2009	1821-2009	1861-2009	1792-2009
Constant	Coeff.	48.464	4.86	16.045	4.552	30.801	35.734	0.128
	t-stat.	*** 3.810	*** 2.694	*** 3.275	* 2.231	** 2.58	*** 4.365	0.959
Lin.Trend	Coeff.	-0.221	-0.006	-0.087	-0.016	-0.174	-0.182	0.018
	t-stat.	** -2.568	-0.439	** -2.294	-0.999	* -1.975	*** -3.334	*** 16.583
Range		1855-2009	1850-2009	1850-2009	1850-2009	1850-2009	1861-2009	1850-2009
Constant	Coeff.	48.464	5.801	6.032	3.569	5.579	25.198	0.995
	t-stat.	*** 3.810	*** 3.461	***3.371	* 2.185	*** 3.774	*** 4.81	*** 5.49
Lin.Trend	Coeff.	-0.221	-0.018	-0.038	-0.015	-0.021	-0.182	0.019
	t-stat.	** -2.568	-1.007	-1.938	-0.833	-1.308	*** -3.334	*** 9.797
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	19.703	5.965	2.980	2.844	4.44	9.883	2.004
	t-stat.	*** 5.498	*** 2.651	* 2.043	1.361	* 2.225	*** 6.912	*** 7.8
Trend	Coeff.	-0.178	0.035	-0.019	-0.015	-0.018	-0.083	0.018
	t-stat.	*** 3.174	-0.995	-0.853	-0.464	-0.592	***-3.711	***4.549
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	10.781	5.043	13.205	0.051	5.675	9.897	4.729
	t-stat.	*** 7.169	*** 4.979	*** 2.936	0.028	*** 4.619	*** 9.574	*** 12.89
Lin.Trend	Coeff.	-0.171	-0.057	-0.48	0.04	-0.078	-0.196	-0.028
	t-stat.	*** -3.999	-1.978	-1.553	0.768	* -2.255	*** -6.64	*** -2.724
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	50.75	6.307	3.851	3.762	4.384	12.272	1.244
	t-stat.	*** 4.846	** 2.543	1.938	1.664	* 2.032	*** 4.060	*** 5.509
Lin.Trend	Coeff.	-0.53	-0.024	-0.018	-0.026	-0.005	-0.072	0.027
	t-stat.	*** -2.974	-0.566	-0.536	-0.66	-1.26	-1.403	***7.045

Notes: The table presents coefficients and t-statistics for regressions of the growth rates on a constant and a linear trend. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 2: Tests for the stylized facts that growth rates of world primary production and world real GDP are equal to zero and trendless.

		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil	World GDP
Range		1855-2009	1821-2009	1802-2009	1792-2009	1821-2009	1861-2009	1792-2009
Constant	Coeff.	48.301	5.474	20.57	4.427	30.7	35.689	0.032
	t-stat.	*** 3.824	*** 3.06	*** 3.845	* 2.181	** 2.584	*** 4.379	0.276
Lin.Trend	Coeff.	-0.229	-0.018	-0.125	-0.023	-0.182	-0.19	0.01
	t-stat.	*** -2.677	-1.367	*** -3.025	-1.457	* -2.071	*** -3.499	*** 11.066
Range		1855-2009	1850-2009	1850-2009	1850-2009	1850-2009	1861-2009	1850-2009
Constant	Coeff.	48.301	5.399	5.629	3.179	5.18	24.681	0.628
	t-stat.	*** 3.824	*** 3.254	***3.169	1.961	*** 3.541	*** 4.733	*** 4.052
Lin.Trend	Coeff.	-0.229	-0.027	-0.047	-0.024	-0.03	-0.19	0.01
	t-stat.	*** -2.677	-1.523	** -2.442	-1.348	-1.895	*** -3.499	*** 5.876
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	18.595	4.985	2.028	1.903	3.473	8.869	1.071
	t-stat.	*** 5.242	* 2.241	1.41	0.918	1.763	*** 6.306	*** 4.862
Trend	Coeff.	-0.184	-0.042	-0.027	-0.023	-0.026	-0.09	0.01
	t-stat.	*** -3.315	-1.214	-1.186	-0.694	-0.404	*** -4.084	*** 3.01
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	8.583	2.952	1.141	-1.954	3.578	7.716	2.632
	t-stat.	*** 5.742	*** 2.892	1.04	1.086	*** 2.87	*** 7.493	*** 7.444
Lin.Trend	Coeff.	-0.156	-0.044	-0.35	0.051	-0.065	-0.18	-0.016
	t-stat.	*** -3.667	-1.515	-1.129	0.997	-1.819	*** -6.14	-1.551
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	50.004	5.854	3.413	3.317	3.942	11.789	0.834
	t-stat.	*** 4.81	** 2.386	1.738	1.480	1.851	*** 3.933	*** 4.509
Lin.Trend	Coeff.	-0.542	-0.038	-0.032	-0.039	-0.019	-0.086	0.013
	t-stat.	*** -3.06	-0.908	-0.959	-1.028	-0.517	-1.691	***4.004

Notes: The table presents coefficients and t-statistics for regressions of the growth rates on a constant and a linear trend. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 3: Tests for the stylized fact that growth rates of world per capita primary production and world per capita real GDP are equal to zero and trendless.

	Reserves/ Annual production (Years)	**Resources/ Annual production** (Years)	**Crustal abundance/ Annual production** (Years)
Aluminum	139[1a]	263,000[1a]	48,800,000,000[bc]
Copper	43[a]	189[a]	95,000,000[ab]
Iron	78[a]	223[a]	1,350,000,000[ab]
Lead	21[a]	362[a]	70.000.000[ab]
Tin	17[a]	"Sufficient"[a]	144.000[ab]
Zinc	21[a]	158[a]	187.500.000[ab]
Gold	20[d]	13[d]	27,160,000[ef]
Rare earths[2]	827[a]	"Very large"[a]	n.a.
Coal[3]	129[g]	2,900[g]	
Crude oil[4]	55[g]	76[g]	} 1,400,000[6i]
Gas[5]	59[g]	410[g]	

Notes: Reserves include all material which can currently be extracted. The definition of resources can be found in Section 2.4. Sources: [a]U.S. Geological Survey (2012b), [b]Perman et al. (2003), [c]U.S. Geological Survey (2011c),[d]U.S. Geological Survey (2011b),[e]Nordhaus (1974),[f]U.S. Geological Survey (2010), [g]Federal Institute for Geosciences and Natural Resources (2011) [gi]Littke and Welte (1992). Notes: [1] data for bauxite, [2] rare earth oxide, [3] includes lignite and hard coal, [4] includes conventional and unconventional oil, [5] includes conventional and unconventional gas, [6] all organic carbon in the earth's crust.

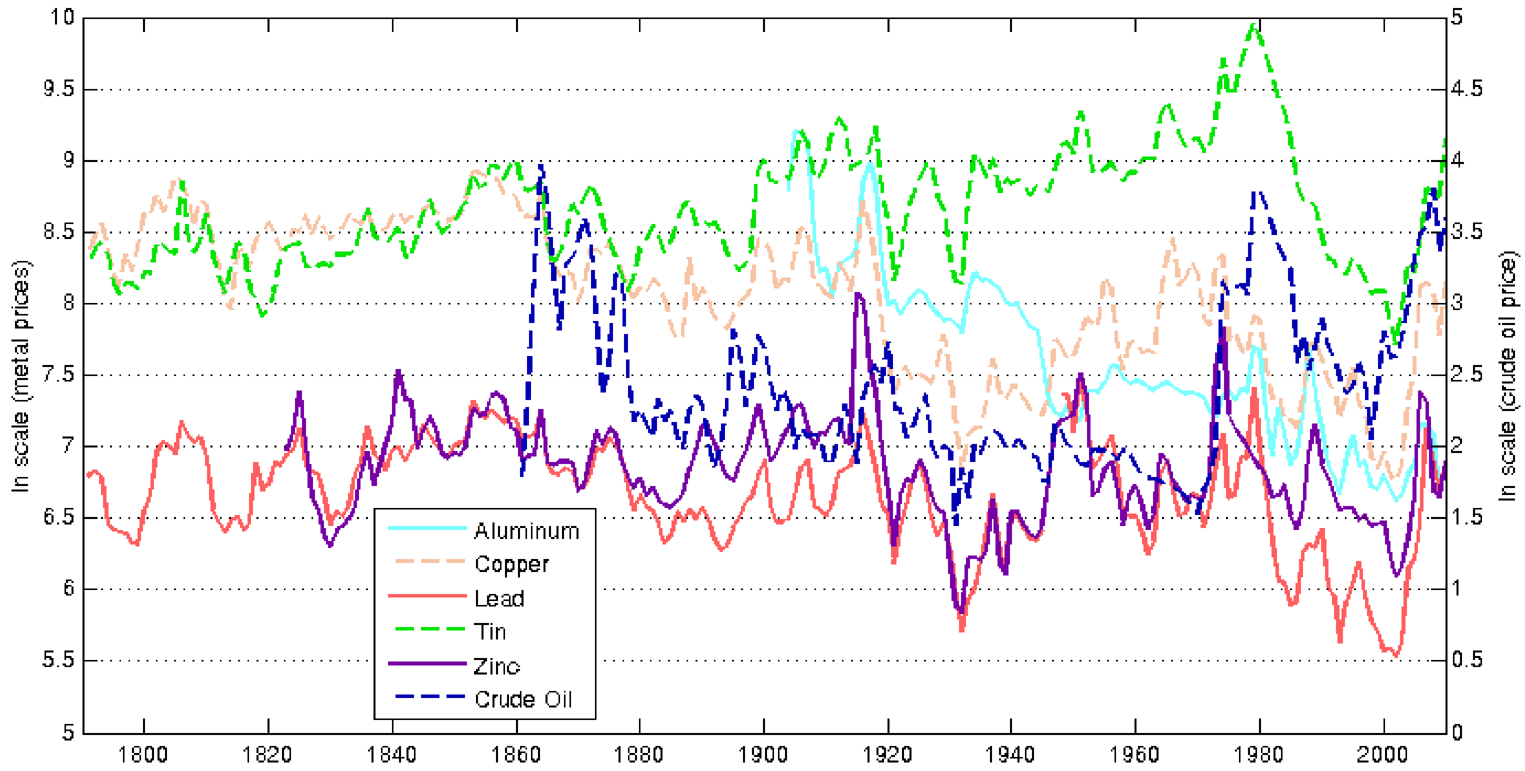
Table 4: Availability of selected non-renewable resources in years of production left in the reserve, resource and crustal mass based on current annual mine production.

	Reserves/ Annual production (Years)	Resources/ Annual production (Years)	Crustal abundance/ Annual production (Years)
Aluminum	65^{1ah}	419^{1ah}	838^{bch}
Copper	30^{ag}	77^{ag}	718^{abg}
Iron	44^{ah}	78^{ah}	744^{abh}
Lead	18^{ah}	181^{ah}	$1{,}907^{abh}$
Tin	18^{ah}	n.a.	$3{,}588^{abh}$
Zinc	17^{ah}	74^{ah}	842^{abh}
Gold	18^{dh}	11^{dh}	$2{,}170^{efh}$
Rare earths[2]	127^{ah}	n.a.	n.a.
Coal[3]	65^{gk}	215^{gk}	
Crude oil[4]	46^{gk}	60^{gk}	} 729^{6j}
Natural gas[5]	41^{gk}	123^{gk}	

Notes: The numbers for reserves and resources are not summable as in Table 4. We have used the following average annual growth rates of production from 1990 to 2010: Aluminum: 2.5%, Iron: 2.3%, Copper: 2%, Lead: 0.7%, Tin: 0.4%, Zinc: 1.6%, Gold: 0.6%, Rare earths: 2.6%, Crude oil: 0.7%, Natural gas: 1.7%, Coal: 1.9%, Hydrocarbons: 1.4%. Reserves include all material which can currently be extracted. The definition of resources can be found in Section 2.4. Sources: [a]U.S. Geological Survey (2012b), [b]Perman et al. (2003), [c]U.S. Geological Survey (2011c),[d]U.S. Geological Survey (2011b),[e]Nordhaus (1974),[f]U.S. Geological Survey (2010), [g]Federal Institute for Geosciences and Natural Resources (2011), [h]U.S. Geological Survey (2012a), [i]U.S. Bureau of Mines (1991), [j]Littke and Welte (1992), [k]British Petroleum (2013). Notes: [1] data for bauxite, [2] rare earth oxide, [3] includes lignite and hard coal, [4] includes conventional and unconventional oil, [5] includes conventional and unconventional gas, [6] all organic carbon in the earth's crust.

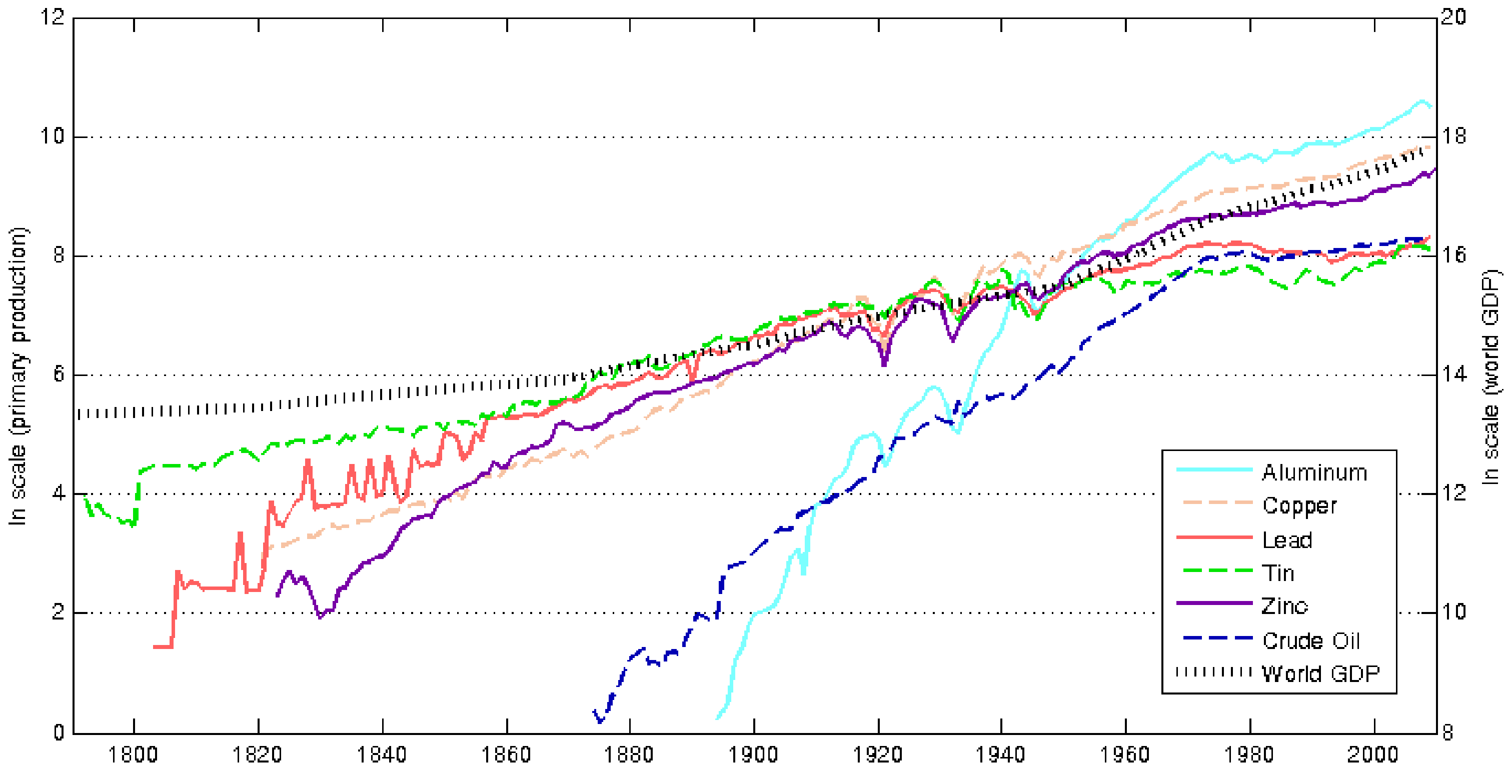
Table 5: Availability of selected non-renewable resources in years of production left in the reserve, resource and crustal mass based on an exponentially increasing annual mine production (based on the average growth rate over the last 20 years).

Appendix 3 Figures



Notes: All prices, except for the price of crude oil, are prices of the London Metal Exchange and its predecessors. As the price of the London Metal Exchange used to be denominated in Sterling in earlier times, we have converted these prices to U.S.-Dollar by using historical exchange rates from Officer (2011). We use the U.S.-Consumer Price Index provided by Officer and Williamson (2011) and the U.S. Bureau of Labor Statistics (2010) for deflating prices with the base year 1980-82. The secondary y-axis relates to the price of crude oil. For data sources and description see Stuermer (2013).

Figure 1: Real prices of major mineral commodities in natural logs.



For data sources and description see Stuermer (2013).

Figure 2: World primary production of non-renewable resources and world real GDP in logs.

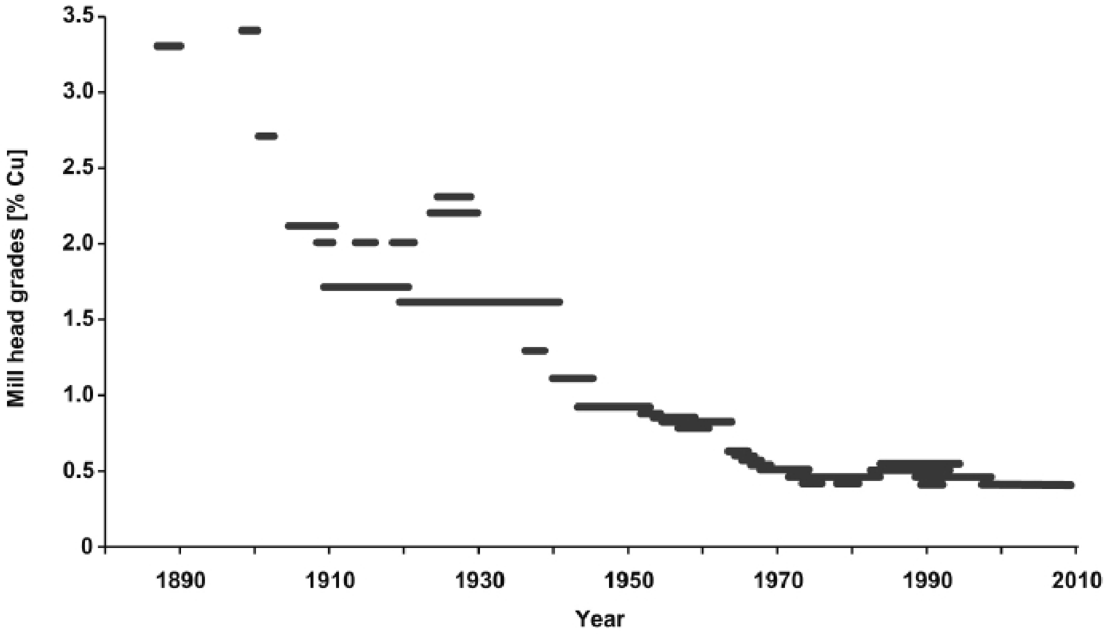


Figure 3: The historical development of mining of various grades of copper in the U.S. Source: Scholz and Wellmer (2012)

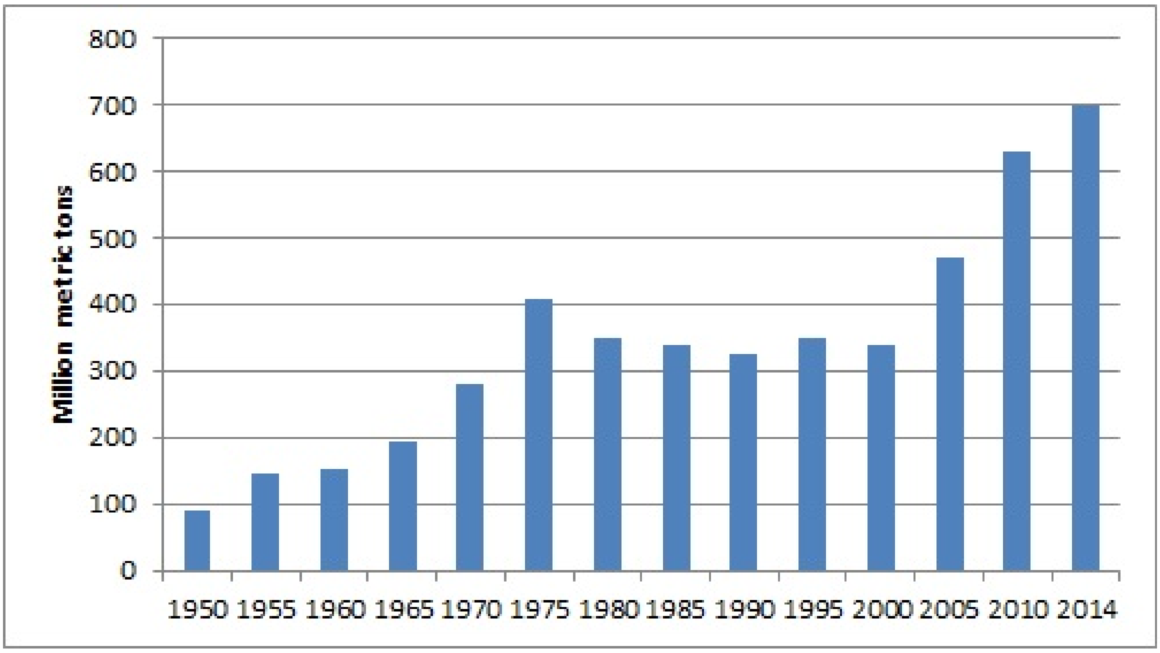


Figure 4: Historical evolution of world copper reserves from 1950 to 2014. Sources: Tilton and Lagos C.C. (2007), USGS.

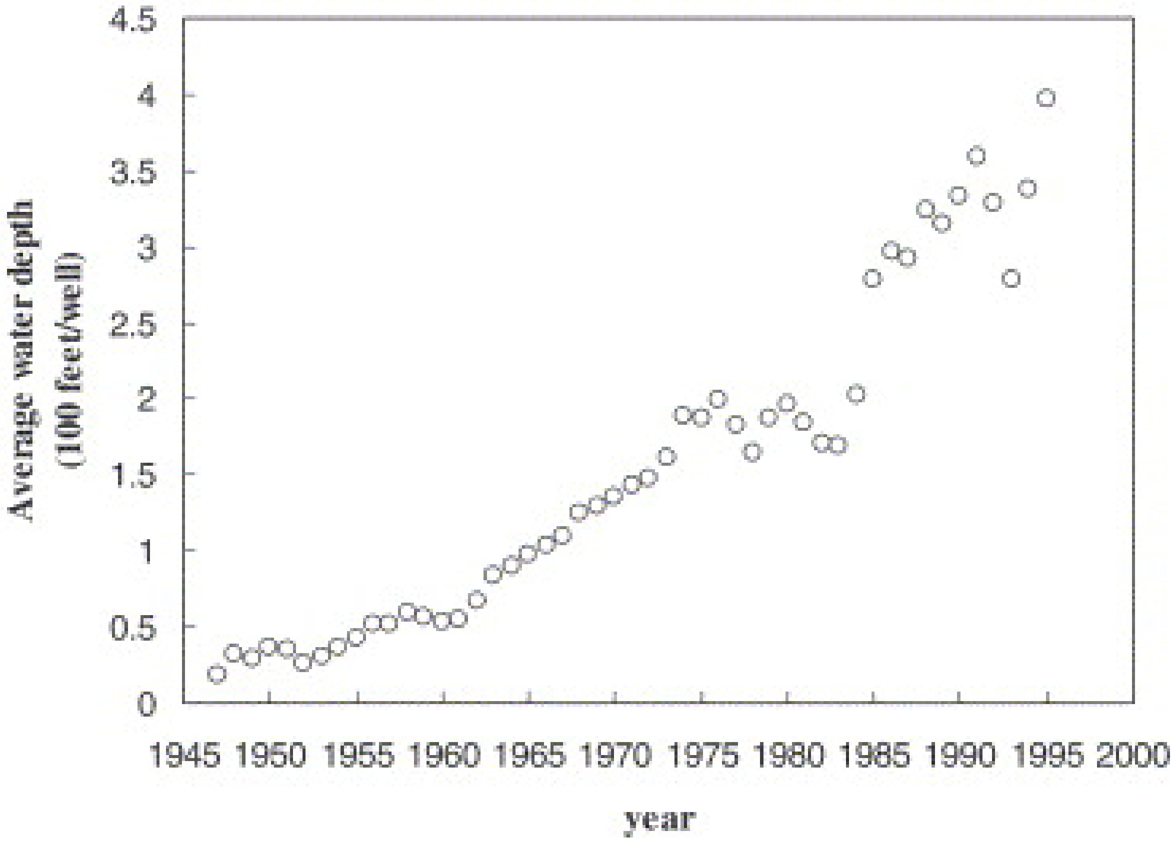


Figure 5: Average water depth of wells drilled in the Gulf of Mexico. Source: Managi et al. (2004).

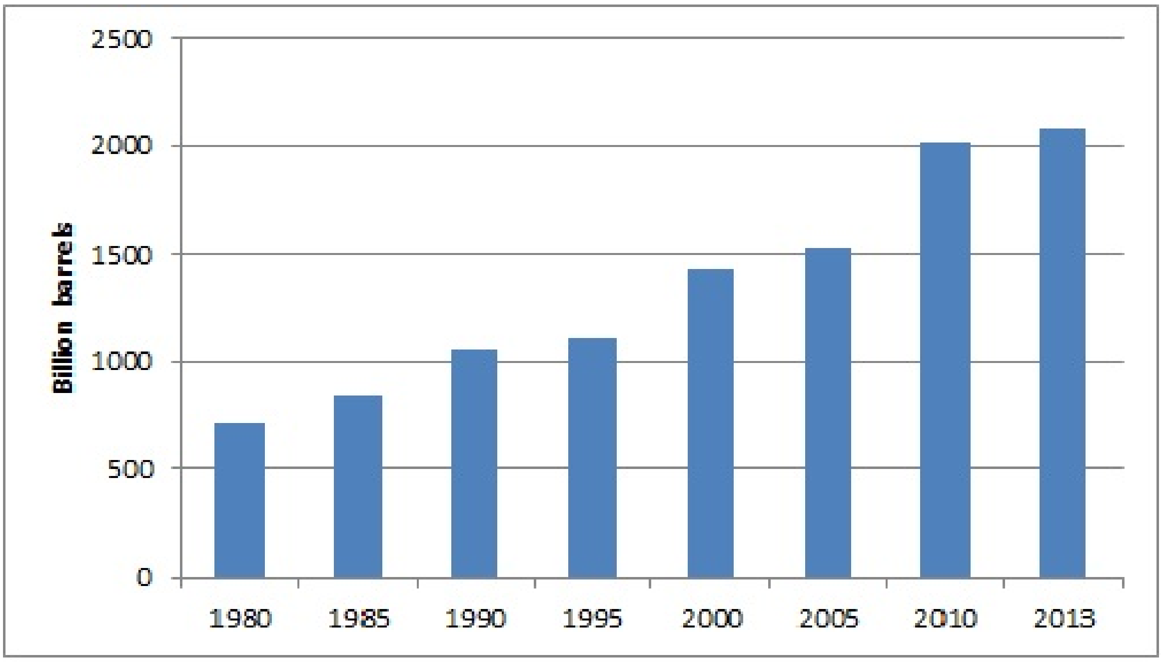


Figure 6: Historical evolution of oil reserves, including Canadian oil sands from 1980 to 2013. Source: BP, 2015.

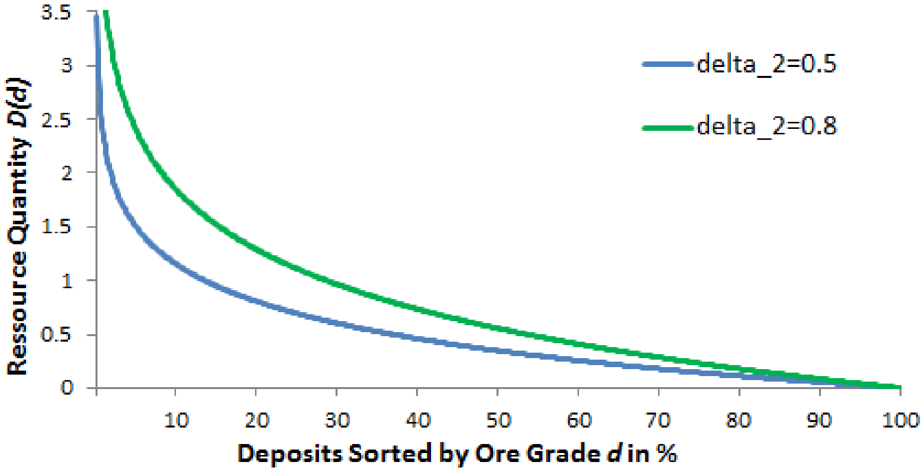


Figure 7: Geological function.

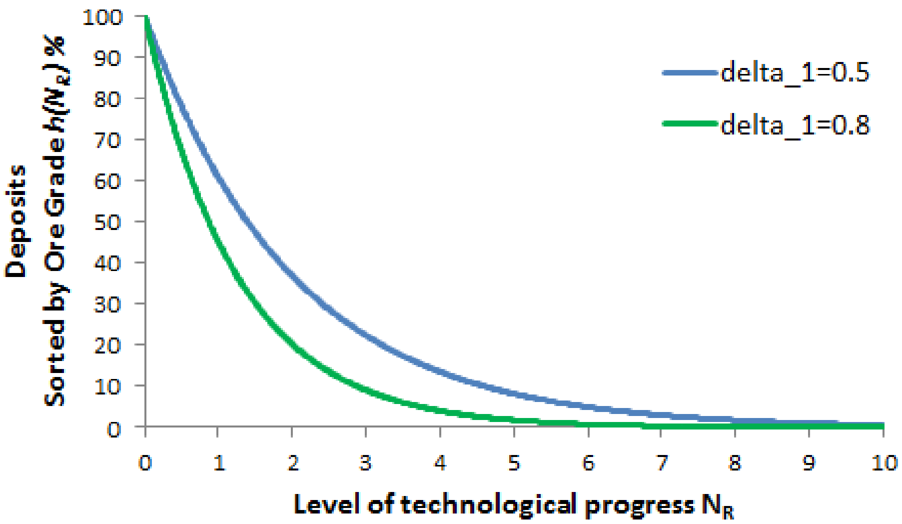


Figure 8: Technological function.