# The Great Pyramid's Footprint: Results from Our 2015 Survey by Glen Dash

*In the last issue of AERAGRAM the author presented a brief overview of the survey of the Great Pyramid's base that he and his team undertook this past Season 2015.* Here Glen discusses the results of that work.*

What is the exact size and orientation of the Great Pyramid? Archaeologists, scientists, engineers, and mystics have sought answers for centuries. In an effort to finally and definitively answer these questions, at least to the extent that the current condition of the pyramid permits, my foundation and Ancient Egypt Research Associates (AERA) undertook a comprehensive survey of the pyramid's base in February of 2015.[1] In this article, I report on the findings of that survey, the Glen Dash Foundation Survey of 2015 (GDFS 2015).[2]

## Our Past Work

This was not our first attempt at determining the exact size and orientation of the Great Pyramid's footprint. In the fall of 2012 we published a study which used data assembled by Mark Lehner and David Goodman in 1984.[3] While that study provided new, more accurate estimates of the Great Pyramid's size and orientation, it also underscored the need for a new, more comprehensive survey, one which used the latest available instruments. In 2015, we completed the new work.

## Tracing the Base

Originally, the Great Pyramid was clad in more than 21 acres of hard, white casing stones that the Egyptians had hauled over from quarries at Tura across the Nile. Most of those casing stones were removed centuries ago for building material, leaving the pyramid as we see it today, without most of its original shell. The photo below was taken along the pyramid's north side. In it, we see some of the pyramid's few remaining casing stones still in place. These sit on a platform that originally extended out 39 to 47 centimeters (15–19 inches) beyond the outer, lower edge (the "foot") of the casing. Behind the casing stones in the photo we can see the rougher masonry that makes up the bulk of the pyramid as it stands today.

Our mission's first task was to locate any traces that remain of the pyramid's original casing baseline, which we define as the place where the foot of the casing stones once met the platform. However, along the Great Pyramid's 920-meter (3,018-foot) periphery, we now find only 54 meters (177 feet) of casing stone in place, and much of that is badly damaged. To determine the pyramid's baseline, therefore, we needed more information than we could get by just examining the casing stones themselves. We needed also to carefully examine the top of the platform for signs as to where missing casing stones had once stood.

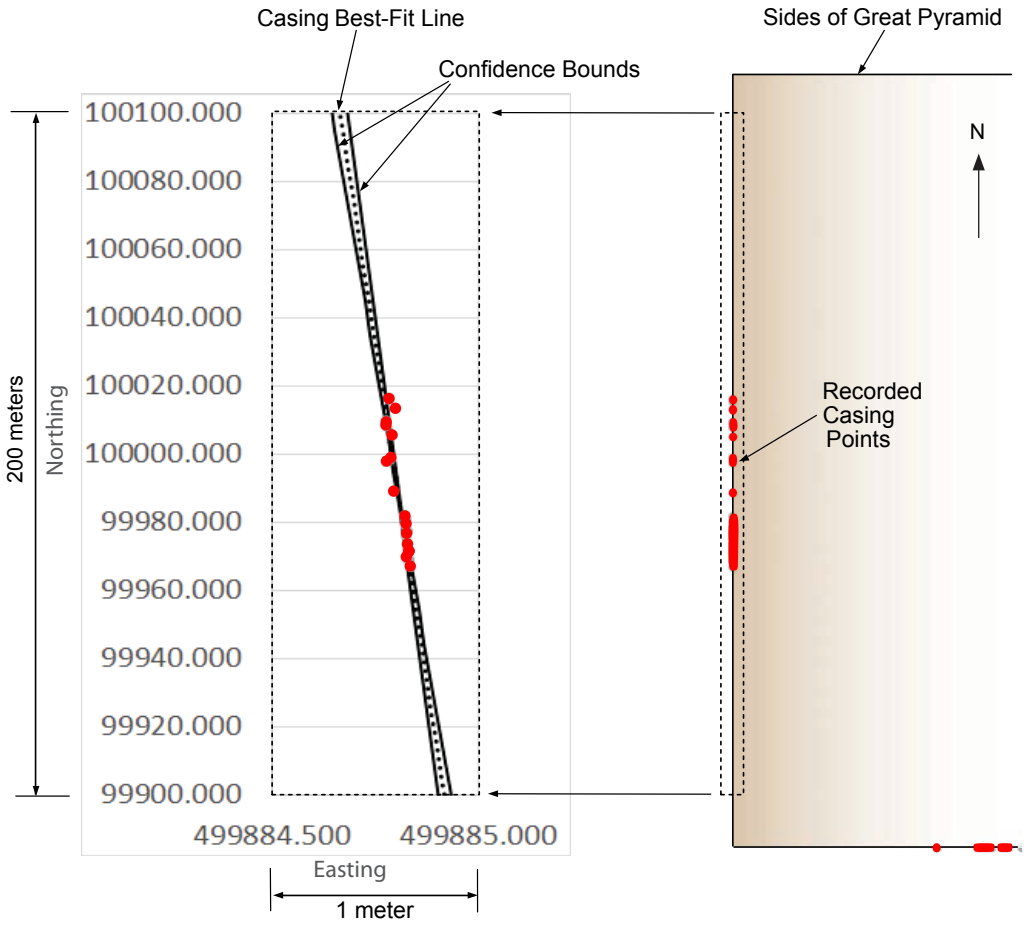
Initially, the task of finding traces of the original baseline fell to Mark Lehner. Lehner started the process by examining the casing stones that did remain. In most cases, he found the casing stone's leading edge worn back, so he looked for an etched or cut line in front of the casing stone to locate its original edge (photo facing page). Lehner also looked for telltale markings on the platform, including places where the surface of the platform had been subtly worn or eroded by the now missing casing stones.

In total, Lehner identified 84 points along 155 meters (508 feet) of the pyramid's 920-meter (3,018-foot) periphery where he

* "What Was the Original Size of the Great Pyramid's Footprint?" by Glen Dash, *AERAGRAM* 16-1, pages 8–11, Spring 2015. All back issues of *AERAGRAM* are available for free download at our website: aeraweb.org.

Casing and platform stones. The angled casing stones sit upon platform stones. The lower, outer edge of the casing and the top, outer edge of the platform provided the best places to measure the pyramid's lines. Photo by Mark Lehner.

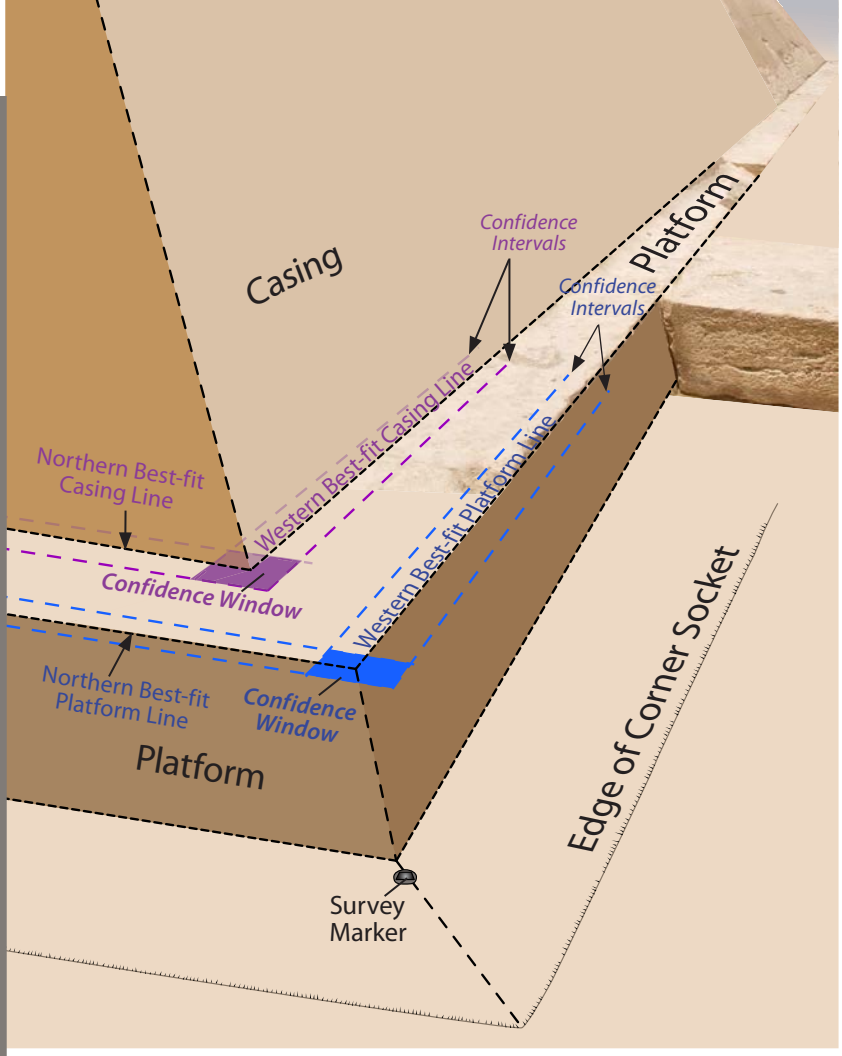
Above: Searching for the pyramid's baseline. Left: An etched line in front of a damaged casing stone indicates where the edge of the casing stone once met the platform. Photo by Rebecca Dash. Right: Mark Lehner photographs a subtle line on the top of the platform revealing where a now missing casing stone once stood. Photo by Glen Dash.




Right: Computing best-fit lines and confidence bounds. At the right is a portion of the pyramid's base showing the casing baseline points recorded on the pyramid's west side. We expand the area within the dotted lines and show that at the left. We computed a best-fit line for the data along with error bounds, known as confidence bounds. There is a 95% probability that the original casing baseline on the west side fell within these confidence bounds. The angle of the lines at the left are exaggerated due to the scale of axes being unequal.

found evidence of the original baseline. Along the remaining 765 meters (2,510 feet) of the periphery (83% of its total length), he found the pyramid too damaged to provide useful data. Nearly all the points Lehner identified were located near the center of each side. No direct evidence of the original corners remains.

Our mission also recorded the top outer edge of the pyramid's platform. In some places we found this edge well preserved, and we could record it directly. In other places, however, the top outer edge of the platform was eroded and worn. In those places we recorded two points on the sloping face of the platform, one above the other, and used those two points to project where the top, outer edge once was. In all, we identified 176 places along 262 meters (860 feet) of the pyramid's periphery (28% of its total) where we found direct evidence of the platform's original top outer edge or were able to derive its original position.




This schematic drawing of a hypothetical pyramid corner illustrates in three dimensions the location of best-fit lines, confidence bounds, confidence windows, and the corner socket in the diagrams on pages 12–13.

## Analyzing the Data

The first step in analyzing this data set was to place it on a master grid. The grid we used was the Giza Plateau Mapping Project (GPMP) control grid established by Lehner and David Goodman in 1984 and 1985. The grid assigns every point on the plateau an address, like houses on a city map. The origin of the map is at the center of the Great Pyramid as computed by Goodman, and everything is measured from that point. For example, there is a bronze survey marker off the northeast corner of the pyramid, 115.803 meters north of the center of the pyramid and 115.610 meters to its east. By convention, surveyors do not like to work with negative numbers, so instead of making the center of the Great Pyramid ($y$=0, $x$=0) as one might expect, Goodman arbitrarily assigned the center a coordinate of ($y$=100000, $x$=500000).[4] Since the y-axis is our north-south coordinate (the "northing") and the x-axis is our east-west component (the "easting"), we can express the center of the Great Pyramid as (N=100,000, E=500,000), or simply as N100,000, E500,000. That places the northeast survey marker at N100,115.803, E500,115.610.[5]

Once we placed all our data on the GPMP control grid, we could use a standard statistical method known as linear regression analysis to "best-fit" lines to the data. In the graphic on the previous page, we show the casing points we recorded on the pyramid's west side. The left side of the image expands a portion of the one on the right, showing an area one meter wide by 200 meters in length. We used Excel's Data Analysis Package to calculate a "best-fit" line through the data, which we show as a dotted line in the figure. We also calculated error bounds around this line, known as "confidence bounds." In theory, there is a 95% chance that the original casing baseline fell within these confidence bounds.

Once we derived best-fit lines and confidence bounds for all four sides of the casing and platform, we could find the original corners of the Great Pyramid by extrapolating those lines to see where they crossed. The schematic diagram on the left illustrates this method using the pyramid's northwest corner as an example.

We extrapolated the north and west best fit lines and confidence bounds to the corners where they crossed, creating "confidence windows." In theory, there is a 95% probability that the original casing and platform corners fell within these windows. On the left we also show the location of the survey marker we used and the outlines of an enigmatic cutting just outside the platform known as the "corner socket." The corner sockets were once thought to have braced the cornerstones of the pyramid. They did not, but their actual function is still the subject of some debate.

The centerfold (pages 12 and 13) shows plans for all four corners of the Great Pyramid. Here the coordinates and dimensions for the features shown in the schematic on the left are presented for the northeast, southeast, and southwest corners, in addition to those for the northwest corner. This includes our derived GPMP coordinates of the platform and the casing corners. Around each corner point we show the associated confidence window. The window at the northwest platform corner is 6.0 × 5.3 centimeters (2.4 × 2.1 inches) and at the casing corner, 2.7 × 4.4 centimeters (1.1 × 1.7 inches). At this corner, the platform extended from the casing baseline 41.2 centimeters (16.2 inches) on the north and 41.9 centimeters (16.5 inches) on the west. We also show the casing corner coordinates as reported by Finders Petrie and J. H. Cole.[6] Finally we show the coordinates for the survey control marker at the northwest corner, G1.4.

We can use the data from these four figures to calculate the dimensions of the base of the Great Pyramid and its platform. We show these in Tables 1 and 2 on the facing page. We used the confidence bounds to calculate minimum and maximum lengths for each line (95% probability).

The average length of the four sides of the casing is 230.363 meters (755.783 feet). Petrie estimated the Egyptian cubit to be

20.62 inches (0.5237 meters) plus or minus 0.01 inch.[7] Assuming he was correct, that makes the average side length somewhere between 440.05 and 439.62 cubits.

Table 3 shows the orientation of the sides relative to cardinal points in minutes and seconds.† The minus sign indicates a counterclockwise rotation from cardinal points.

Table 4 shows that the mean angle of the casing is -3 minutes and 54 seconds, plus or minus 44 seconds (-3′ 54″ ± 44″). This is consistent with Petrie's estimate of -3′ 43.″[8]

We also examined the pyramid's "diagonals." We define the diagonals as the lines connecting the opposite corners of the casing, shown in the figure on page 14. Where the diagonals cross is the center of the base. We calculated the center of the pyramid to be N100,000.023 and E499,999.987 plus or minus 4.9 centimeters north to south or east to west.[9] Remarkably, as shown in the figure, the diagonals crossed to form a nearly perfect right angle. The error was just -12″ of arc ± 1′ 27.″ That means that, to a 95% probability, the angle formed by the pyramid diagonals is somewhere between 89° 58′ 21″ and 90° 01′ 15,″ with the most probable angle being the mean of these two, 89° 59′ 48.″‡

We can only speculate as to how the Egyptians could have laid out these lines with such precision using only the tools they had.[10]

We also calculated the angle of the line that runs from the center of the base of the Great Pyramid to the center of the doorway to the Pyramid Temple and compared that to the pyramid's meridian, shown in the figure on page 14. The meridian is the pyramid's north-south axis and, by definition, it bisects the diagonals. The meridian's angle is 3′ 54″ counterclockwise from due north. The remains of the Pyramid Temple's doorway sit about 168 meters to the east of the center of the pyramid and 52.5 meters from the pyramid's eastern casing, about 100 cubits. The line between the center of the pyramid and the center of the doorway runs at an angle of 3′ 51″ counterclockwise of due east. That line and the meridian crossed at the center of the pyramid to form an angle of 90° 0′ 3″ ± 1′ 44.″ If not a perfect right angle, it was something very close to it.

*(continued on page 14)*

† There are 60 geographical minutes (60′) in a degree, and 60 geographical seconds (60″) in a minute.

‡ One minute of arc (1′) is about equal to the angle subtended by two fingers viewed from across the length of a football field.

**Table 1: Lengths of the Sides of the Casing Base**

| Casing Side | Minimum Length (meters) | Mean Length (meters) | Maximum Length (meters) |
|---|---|---|---|
| North | 230.256 | 230.329 | 230.402 |
| East | 230.295 | 230.334 | 230.373 |
| South | 230.329 | 230.384 | 230.439 |
| West | 230.378 | 230.407 | 230.436 |
| Average | | 230.363 | |

**Table 2: Lengths of the Sides of the Platform**

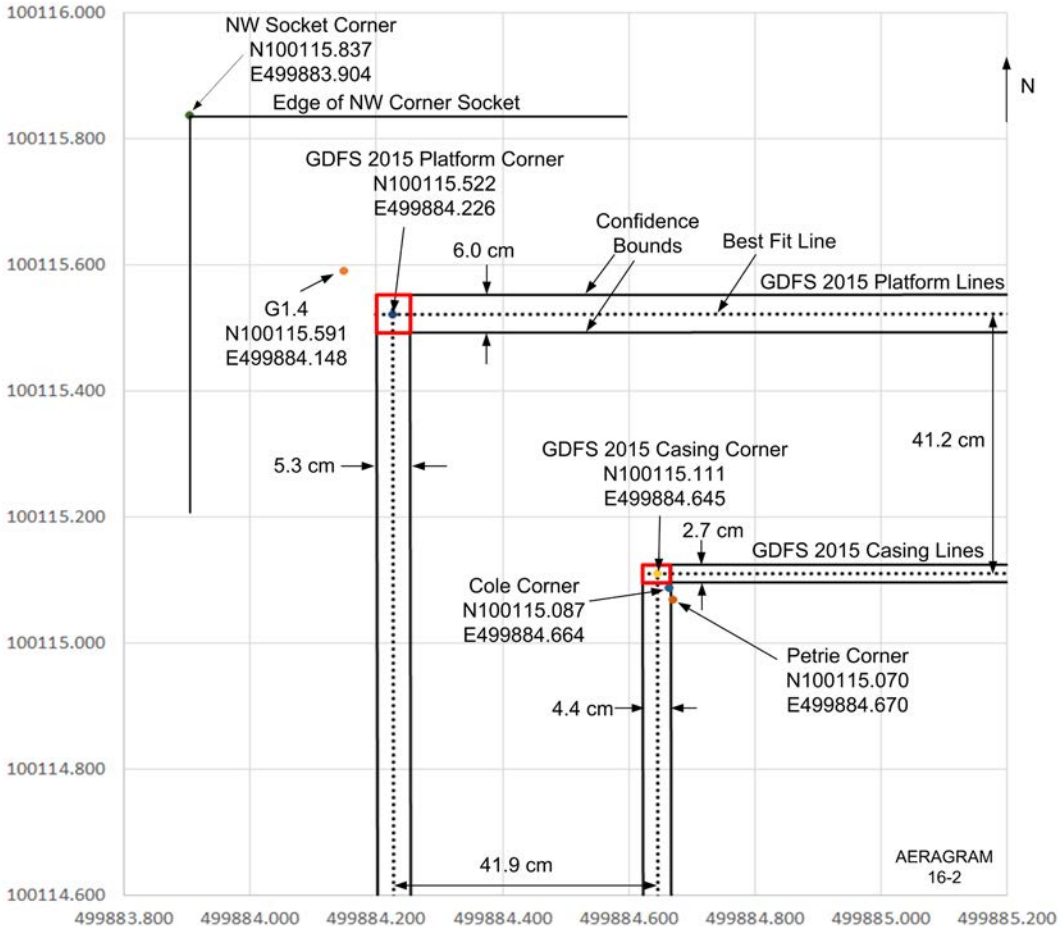
| Platform Side | Minimum Length (meters) | Mean Length (meters) | Maximum Length (meters) |
|---|---|---|---|
| North | 231.160 | 231.214 | 231.267 |
| East | 231.081 | 231.215 | 231.350 |
| South | 231.105 | 231.174 | 231.244 |
| West | 231.156 | 231.204 | 231.252 |
| Average | | 231.202 | |

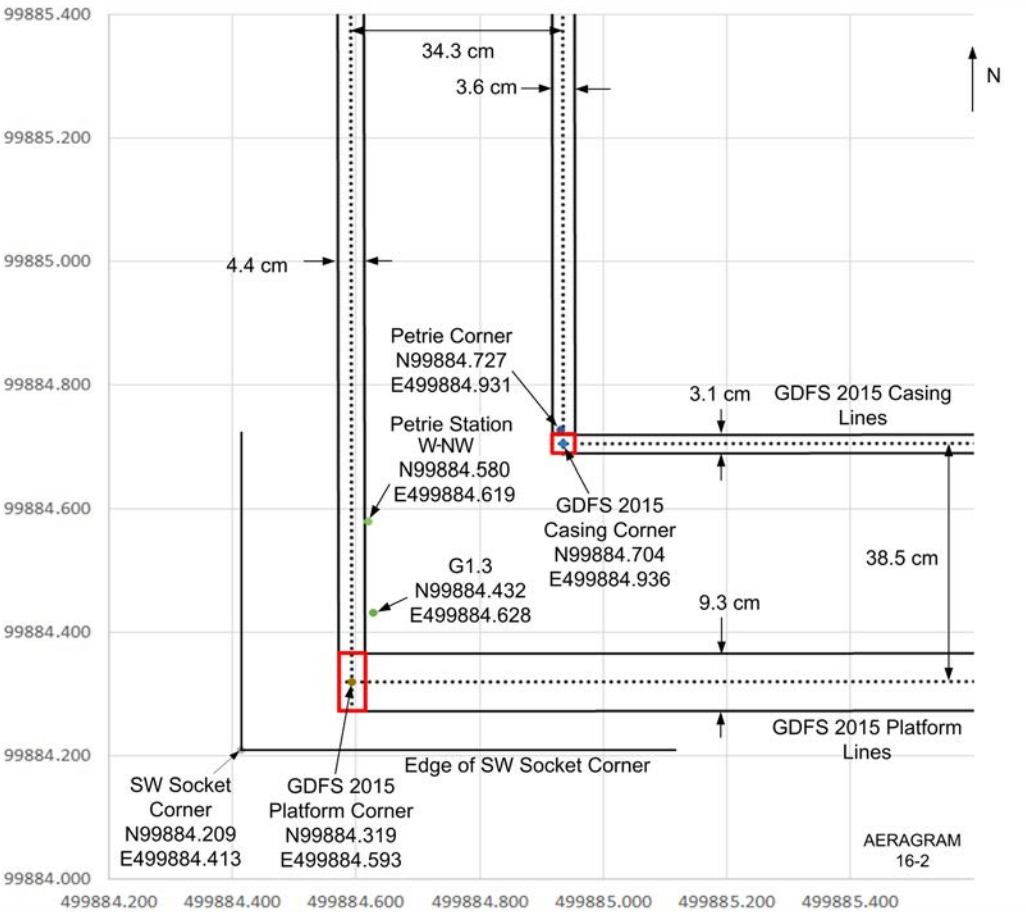
**Table 3: Angles of the Sides of the Casing Base**

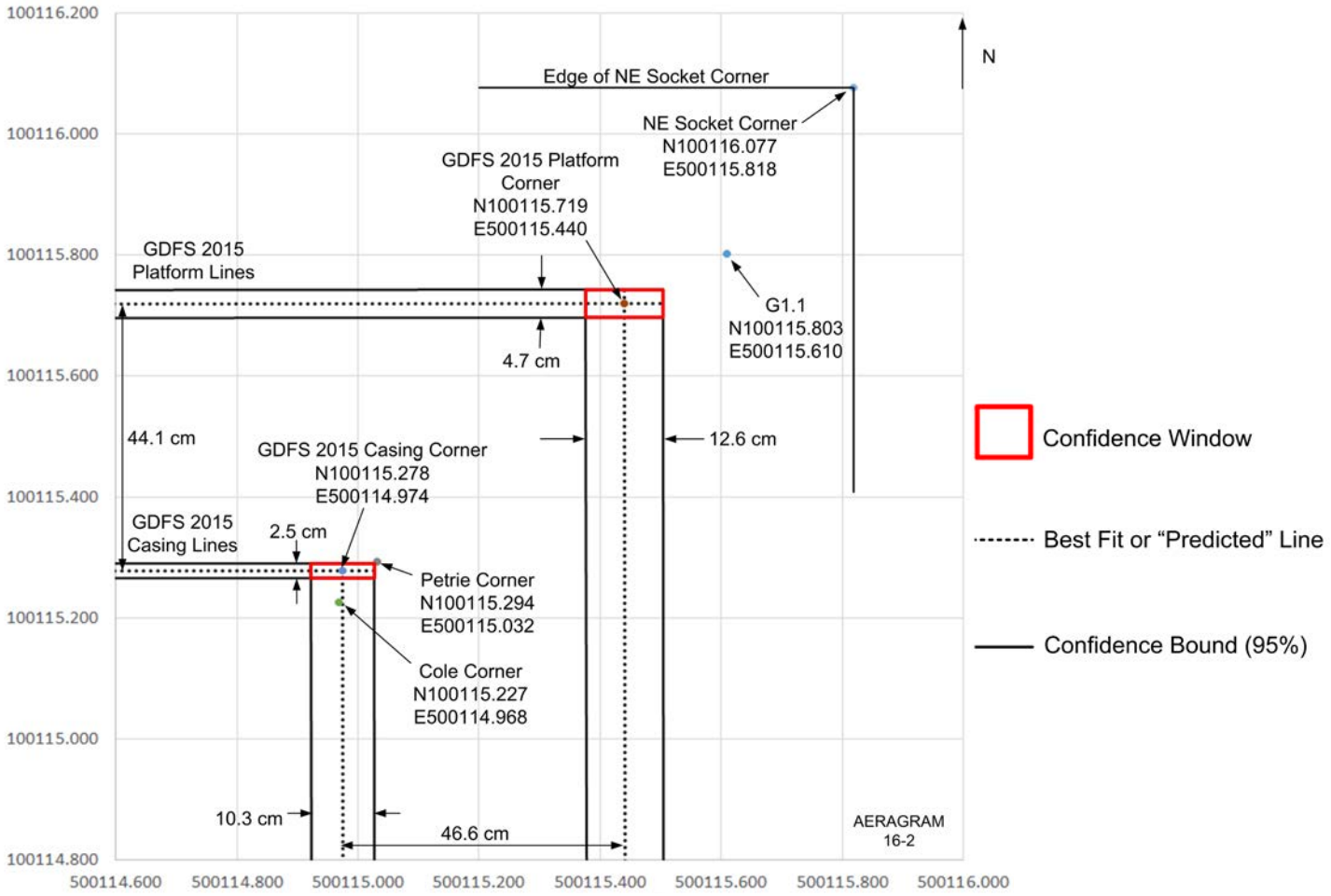
| Casing Side | Angle (Deviation from Cardinal Points) | Confidence Bound |
|---|---|---|
| North | -2′ 30″ | +/- 23″ |
| East | -5′ 10″ | +/- 1′ 19″ |
| South | -3′ 35″ | +/- 38″ |
| West | -4′ 21″ | +/- 35″ |
| Average | -3′ 54″ | +/- 44″ |

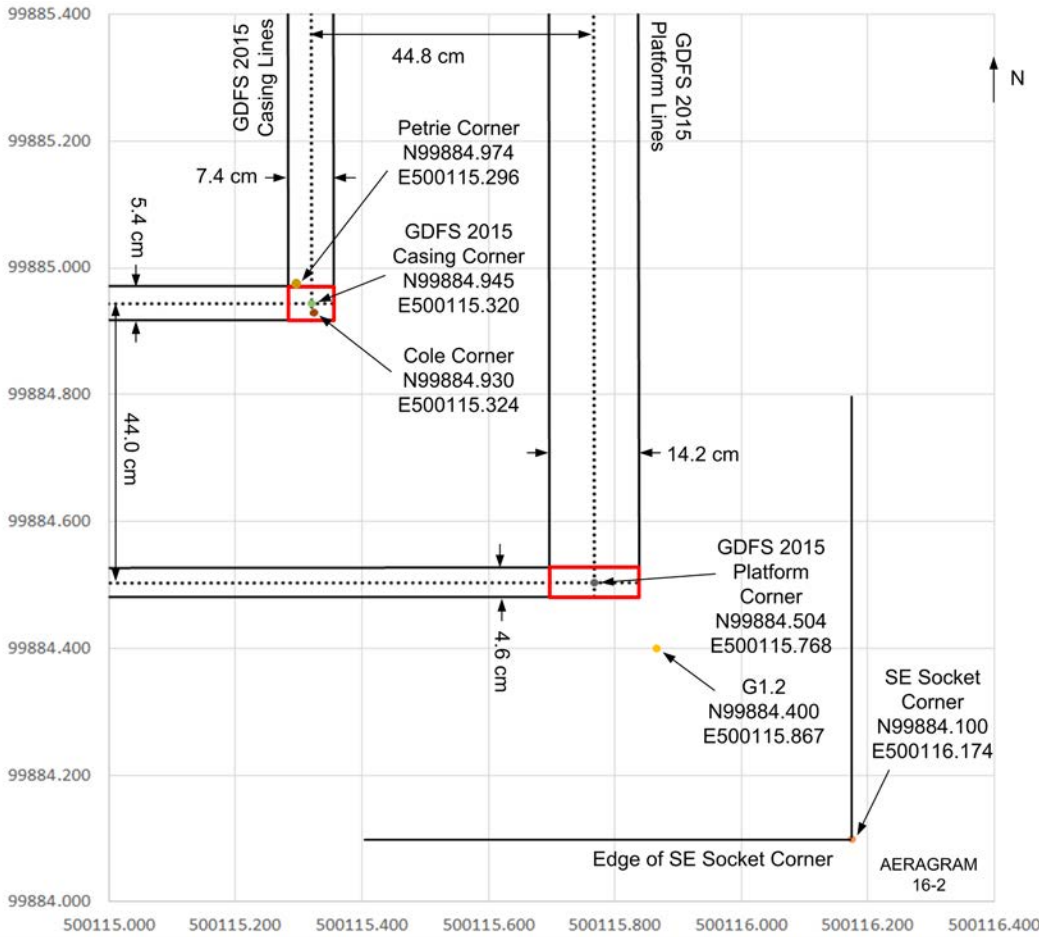
**Table 4: Angles of the Platform**

| Platform Side | Angle (Deviation from Cardinal Points) | Confidence Bound |
|---|---|---|
| North | -2′ 56″ | +/- 24″ |
| East | -4′ 53″ | +/- 43″ |
| South | -2′ 45″ | +/- 1′ 0″ |
| West | -5′ 28″ | +/- 43″ |
| Average | -4′ 0″ | +/- 43″ |


NW Socket Corner
N100115.837
E499883.904
Edge of NW Corner Socket
GDFS 2015 Platform Corner
N100115.522
E499884.226
6.0 cm
Confidence Bounds
Best Fit Line
GDFS 2015 Platform Lines
G1.4
N100115.591
E499884.148
5.3 cm
GDFS 2015 Casing Corner
N100115.111
E499884.645
41.2 cm
2.7 cm
GDFS 2015 Casing Lines
Cole Corner
N100115.087
E499884.664
Petrie Corner
N100115.070
E499884.670
4.4 cm
41.9 cm
AERAGRAM
16-2
N
100116.000
100115.800
100115.600
100115.400
100115.200
100115.000
100114.800
100114.600
499883.800
499884.000
499884.200
499884.400
499884.600
499884.800
499885.000
499885.200


34.3 cm
3.6 cm
N
4.4 cm
Petrie Corner
N99884.727
E499884.931
3.1 cm
GDFS 2015 Casing Lines
Petrie Station
W-NW
N99884.580
E499884.619
GDFS 2015
Casing Corner
N99884.704
E499884.936
38.5 cm
G1.3
N99884.432
E499884.628
9.3 cm
GDFS 2015 Platform Lines
Edge of SW Socket Corner
SW Socket
Corner
N99884.209
E499884.413
GDFS 2015
Platform Corner
N99884.319
E499884.593
AERAGRAM
16-2
99885.400
99885.200
99885.000
99884.800
99884.600
99884.400
99884.200
99884.000
499884.200
499884.400
499884.600
499884.800
499885.000
499885.200
499885.400


N
Edge of NE Socket Corner
NE Socket Corner
N100116.077
E500115.818
GDFS 2015 Platform Corner
N100115.719
E500115.440
GDFS 2015 Platform Lines
G1.1
N100115.803
E500115.610
4.7 cm
44.1 cm
12.6 cm
GDFS 2015 Casing Corner
N100115.278
E500114.974
GDFS 2015 Casing Lines
2.5 cm
Petrie Corner
N100115.294
E500115.032
Cole Corner
N100115.227
E500114.968
10.3 cm
46.6 cm
AERAGRAM 16-2
Confidence Window
Best Fit or "Predicted" Line
Confidence Bound (95%)
100116.200
100116.000
100115.800
100115.600
100115.400
100115.200
100115.000
100114.800
500114.600
500114.800
500115.000
500115.200
500115.400
500115.600
500115.800
500116.000


N
GDFS 2015 Casing Lines
44.8 cm
GDFS 2015 Platform Lines
Petrie Corner
N99884.974
E500115.296
7.4 cm
5.4 cm
GDFS 2015 Casing Corner
N99884.945
E500115.320
Cole Corner
N99884.930
E500115.324
44.0 cm
14.2 cm
GDFS 2015 Platform Corner
N99884.504
E500115.768
4.6 cm
G1.2
N99884.400
E500115.867
SE Socket Corner
N99884.100
E500116.174
Edge of SE Socket Corner
AERAGRAM 16-2
99885.400
99885.200
99885.000
99884.800
99884.600
99884.400
99884.200
99884.000
500115.000
500115.200
500115.400
500115.600
500115.800
500116.000
500116.200
500116.400

*(continued from page 11)*

## Conclusions

Our survey has produced new estimates for the size and orientation of the Great Pyramid. We also continue to analyze the data for new insights, and we have not been disappointed. The data show that the Egyptians possessed quite remarkable skills for their time. We hope to eventually figure out how the Egyptians laid out the pyramid with such precision, and in doing so hope to learn much about the tools and technology they had at their disposal.

1. Permission for the pyramid survey was granted to Mark Lehner and AERA as part of AERA's broader survey across the plateau. The project's chief surveyor was Joel Paulson of NV5, Inc. (San Diego, CA), who was assisted by Mohammed Abd el-Basset and Amr Zakaria of the Ministry of Antiquities. AERA's Mark Lehner oversaw the archaeological aspects of the project. He was assisted in his work by Ashraf Abd el-Aziz. Joan and Rebecca Dash of the Glen Dash Foundation assisted with the survey. I was the principal investigator. For making this survey possible, we extend our deep gratitude to the Ministry of Antiquities, Dr. Mahmoud el-Damati, Minister of Antiquities; Dr. Mustafa Amin, Chairman of the Supreme Council of Antiquities; Yusuf Khalifa, Director of Pharaonic Monuments; Dr. Mahmoud Affifi, Director of Central Administration and Middle Egypt; Shaaban Abd el-Gawad, Director of the Department of Egyptology and Museums in the Minister's Office; Hani Abu Azm, Director of Foreign Missions and Secretary of Permanent Committees; the late Kamel Waheed, General Director for Cairo and Giza; Sayeed Hassan, Director of Giza; Fedai Helmi, Chief Inspector of Giza; Giza Inspectors Mohamed Saidi and Ahmed Ezz, and Chief Inspector of the Solar Boat Project, Afifi Rohim Afifi. The author would also like to thank AERA Egypt's Executive Director Mohsen Kamel for his assistance in arranging permissions and AERA's Field Director for Season 2015, Ana Tavares, for her role in achieving a successful GDFS 2015.

2. We invite comments on this article. Any future corrections to this article will be found at http://www.DashFoundation.org/Aeragram-16-2-erratta.pdf

3. "New Angles on the Great Pyramid," by Glen Dash, *AERAGRAM* 13-2, pages 10-19, Fall 2012. The 1984 data, in addition to having been taken without the benefit of modern total stations, had three weaknesses when used to derive the pyramid's lines. First, no measurements of the casing's actual baseline were taken on the south side of the Great Pyramid in 1984 because the base of the casing no longer exists there. In this study, we derived data for the south side by measuring the top of the casing and extrapolating where the base once fell. Second, without the benefit of south side data, our 2012 study was based on the assumption that the corners of the pyramid fell on the "pyramid diagonals," lines that connect the opposing sockets. The sockets are cuttings just outside the pyramid's four corners. In this study, we did not have to make that assumption. Third, the 2012 study had too few points to provide for narrow confidence windows.

4. As designed, the GPMP system can be used to map features up to 100 kilometers south of the Pyramid, and 500 kilometers to its west, with unlimited range to its north and east.

5. These coordinates are slightly different than those reported in *Giza Reports 1* (Boston: Ancient Egypt Research Associates, Inc., 2007). We reestablished the exact location of the survey markers as part of the GDFS 2015 effort.

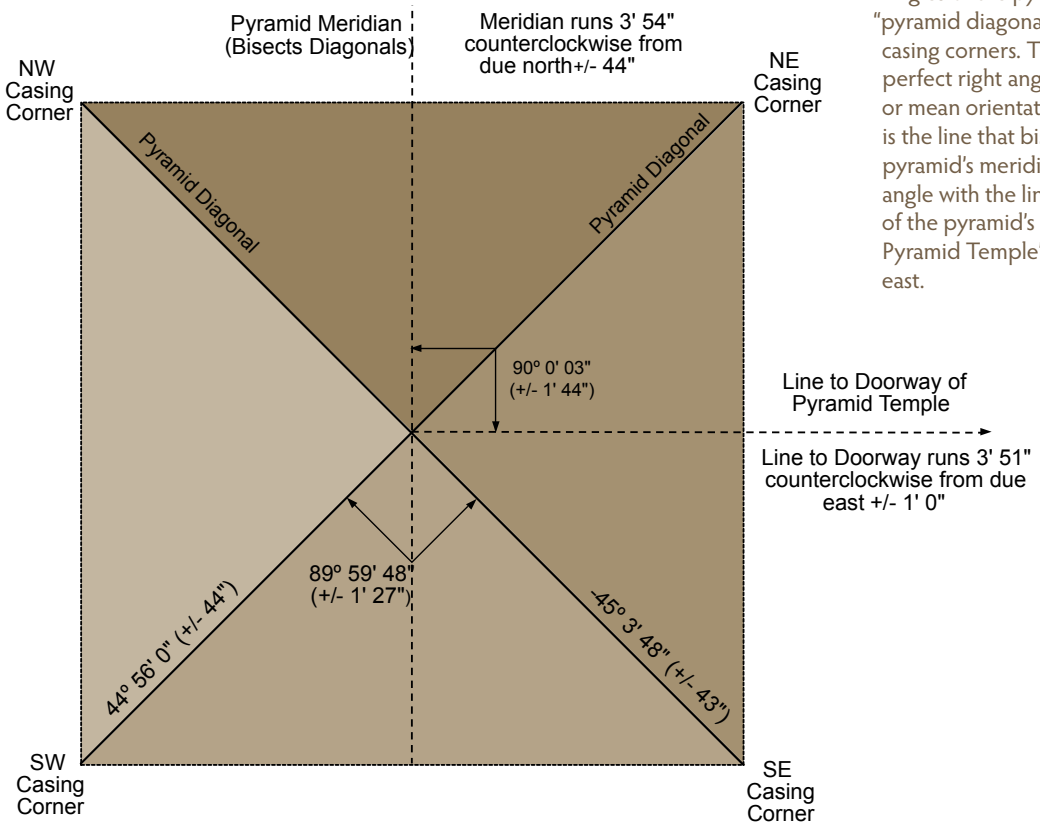
6. See W. M. F. Petrie, *The Pyramids and Temples of Gizeh*, (London: Field and Tuer, 1883), Plate X, and J. H. Cole, "Determination of the Exact Size and Orientation of the Great Pyramid of Giza," (Cairo: Government Press, 1925), page 9. Both Petrie and Cole report the position of the casing corners by offset from other features such as the corner sockets. I have converted these reported positions to GPMP coordinates.

7. Petrie 1883, page 181.

8. Petrie 1883, Plate X.

9. This estimate is slightly different than Goodman's estimate because he did not have the advantage of our new findings.

10. For some of the speculation on this topic, see http://glendash.com/blog/2014/12/03/the-great-pyramid-diagonals-do-they-point-to-a-hidden-inner-platform-within-the-pyramid/.




Angles of the pyramid's internal lines. The "pyramid diagonals" connect the opposing casing corners. They cross to form a nearly perfect right angle. The pyramid's meridian, or mean orientation relative to due north, is the line that bisects the diagonals. The pyramid's meridian forms a near perfect right angle with the line that connects the center of the pyramid's base with the center of the Pyramid Temple's entrance, 168 meters to the east.