

1. Introduction

The (non-)equivalence of cost effectiveness analysis and cost benefit analysis has received much attention in the literature on the economic evaluation of health care (Phelps and Mushlin, 1991; Johannesson, 1995; Donaldson 1998). This attention emanates from a concern about the theoretical properties of cost effectiveness analysis. Notwithstanding the fact that it is currently the most common tool in the economic evaluation of health care, cost effectiveness analysis, unlike cost benefit analysis, has no foundation in economic welfare theory.

The most widely used outcome measure in cost effectiveness analysis are quality-adjusted life-years (QALYs). There is a well-established literature describing the conditions under which QALY based decision making is consistent with preferences over lifetime health profiles (Pliskin et al., 1980; Bleichrodt, 1995; Bleichrodt et al., 1997; Bleichrodt and Quiggin, 1997; Miyamoto et al., 1998). Much less is known about the consistency between QALY based decision making¹ and individual preferences when lifetime utility depends not only on health status, but also on consumption. In this paper we derive a set of conditions that is both necessary and sufficient for the consistency of QALY based decision making with life-cycle preferences over consumption and health status. Cost benefit analysis is always consistent with life-cycle preferences over consumption and health status. This follows because cost benefit analysis imposes no assumptions on the lifetime utility function over consumption and health status. Consequently, to show under which conditions cost effectiveness analysis is consistent with life cycle preferences over consumption and health also answers the question under which conditions cost effectiveness analysis is consistent with cost benefit analysis.

Two recent papers by Garber and Phelps (1997) and Meltzer (1997) also analyzed the allocation of health resources within a life cycle framework, where both health status and consumption are arguments of the utility function. These papers focus on the question when the cost side of cost effectiveness analysis is consistent with life-cycle

¹ Throughout the text we interchangeably use the terms cost effectiveness analysis and QALY based decision making. Our central result, derived in Section Two, carries over to other outcome measures as

preferences over consumption and health status. Their main concern is whether future consumption should be regarded as a cost of life-saving medical interventions. Although Garber and Phelps assume that the health argument of the utility function may be represented in terms of QALYs, neither they nor Meltzer consider the conditions under which an individual concerned about both health status and general consumption would seek to maximise QALYs. This paper complements the papers by Garber and Phelps and by Meltzer by focusing on the question when the outcome side of cost effectiveness analysis, i.e., QALY maximisation, is consistent with life cycle preferences. Throughout the paper, we assume that costs are measured in a consistent way and we restrict attention to the valuation of outcomes.

The paper is structured as follows. In Section 2, we derive the central result of this paper, that QALY maximisation is consistent with life cycle preferences only if the utility function over consumption and health status can be decomposed multiplicatively and the utility of consumption is constant. Section 3 provides an axiomatic analysis of the conditions under which the utility function over consumption and health status is multiplicative. The axiomatic analysis of the multiplicative model is given both under expected utility theory and under rank dependent utility theory, currently the most important nonexpected utility model. Even though this section is technical, we have tried to increase its accessibility by putting all technical assumptions and proofs in an appendix and by making ample use of examples to illustrate the conditions that are introduced. In Section 4 we derive constancy of consumption. We show that if the utility function over consumption and health status is multiplicative then consumption is constant if an individual's rate of time preference equals the interest rate. Section 4 further examines the implications of the multiplicative model for the valuation of longevity and the willingness to pay for a QALY gained. Johannesson (1995) has argued that cost effectiveness can only be a useful tool if information is available on the willingness to pay for a QALY gained. To date, such information is not available. We derive expressions for the willingness to pay for a QALY gained in several decision contexts and identify the principal factors that determine it. Section 5 concludes.

2. The central result

Our aim is to show under what conditions QALY based decision making is consistent with life-cycle preferences over consumption and health status. The general idea of our argument follows from the uniqueness properties of the additive utility function and a special property of health. Let (h_1, \dots, h_T) denote a sequence of health states, where h_t stands for the health state in time period t . Similarly $((c_1, h_1), \dots, (c_T, h_T))$ denotes a sequence of consumption levels and health states, where (c_t, h_t) denotes the consumption level and the health state in period t .

The first step in our derivation is to realize that according to the QALY model the utility of the sequence (h_1, \dots, h_T) is equal to $\sum_{t=1}^T q(h_t)$, where $q(h_t)$ is the quality weight or utility of the health state in period t . Two things should be noted about the QALY utility function. First, it is additive and second, the one-period utility functions are identical. It is well known from the literature on additive representations (e.g. Wakker, 1989) that if the one-period utility functions are additive then they are unique up to positive linear transformations. That is, if $q(h_t)$ and $q'(h_t)$ are both additive one-period utility functions then

$$q(h_t) = \alpha + \beta q'(h_t) \tag{1}$$

where α can be any real number and β is a strictly positive real number. Note that because the $q(h_t)$ are unique up to positive linear transformations, we are free to choose their scale and location. Throughout, we set the utility of death equal to zero.

Suppose now, that an individual is not only concerned about the sequence of health states, but also cares about consumption. That is, we now study preferences over the sequence $((c_1, h_1), \dots, (c_T, h_T))$. We assume that an individual always prefers more consumption to less as long as health status is better than death. By consequence, utility is

strictly increasing in consumption. If the utility function over $((c_1, h_1), \dots, (c_T, h_T))$ is consistent with the QALY utility function then we know by the argument of the preceding paragraph that it must be a positive linear transformation thereof. Hence, it must have the following form:

$$U[(c_1, h_1), \dots, (c_T, h_T)] = \sum_{t=1}^T [v(c)q(h_t) + w_t(c)] \quad (2)$$

where v is a utility function over consumption that must be strictly positive ($v(c)$ corresponds to β in Eq.(1)) and the w_t are period-specific utility functions over consumptions that are real-valued (the w_t correspond to α in Eq.(1)).

We next employ a special characteristic of the life cycle model for consumption and health, namely the fact that an individual will derive no more utility from consumption once he has died. Bequest motives may lead individuals to consume amounts less than their full wealth. However, it is reasonable to suppose, as is commonly done in economic analyses of bequest motives, that any utility of bequests is additively separable from $U[(c_1, h_1), \dots, (c_T, h_T)]$. Therefore, the existence and form of the utility of bequests has no implications for the form of $U[(c_1, h_1), \dots, (c_T, h_T)]$. Because $q(\text{death})$ is equal to zero, it follows from Eq. (2) that the $w_t(c)$ must be equal to zero to capture the fact that the individual derives no more utility from consumption after he has died. It follows that life cycle preferences over consumption and health status are consistent with QALY maximization only if :

$$U[(c_1, h_1), \dots, (c_T, h_T)] = \sum_{t=1}^T v(c)q(h_t) \quad (3)$$

Equation (3) has three interesting properties. First, the utility of consumption is constant over the individual's life cycle from which it follows as a consequence of the

assumption that utility is strictly increasing in consumption that consumption itself is constant over the individual's life cycle. Second, the utility of consumption is positive. If there exists a subsistence level of consumption beyond which additional life-years are negatively valued, then consumption has to be above this subsistence level. This is not a major restriction. Rosen (1988) has shown that if consumption falls below the subsistence level an individual will convexify his preferences by randomizing between death and survival at a consumption level which is higher than the subsistence level. Hence, Eq.(3) captures all cases of economic interest. Third, in Eq.(3) the utility of health status is multiplied by the utility of consumption. By consequence, a given gain in quality of life will be more appreciated at higher levels of consumption. This implies that in the allocation of health care resources, larger gains in QALYs can be obtained by devoting resources to those individuals who have a high level of general consumption. A comparable result was derived by Pratt and Zeckhauser (1996) for the valuation of risk reductions.

In the next section we present an axiomatic analysis of the multiplicative model

$$U[(c_1, h_1), \dots, (c_T, h_T)] = \prod_{t=1}^T v(c_t)q(h_t) .$$

This model is slightly more general than Eq.(3)

because consumption is allowed to vary over time. If consumption varies over the life cycle, an optimal allocation of health resources will assign more weight to health status in high consumption years. Conversely, other things equal, an individual with multiplicative preferences will choose higher levels of consumption in those years in which his health status is high. The selection of a constant consumption level has to follow from an optimization problem which requires specification of the budget constraint. We present such an optimization problem in Section 4.

3. Characterization of the multiplicative model

We focus on preferences under risk. Preferences under certainty follow by restricting attention to *degenerate lotteries*: lotteries giving one outcome with probability one. Life-cycle preferences are first analyzed under expected utility theory, the only

theory considered thus far in the literature on the valuation of life. However, it is by now widely accepted that people do not behave according to expected utility theory (Camerer, 1995). Several nonexpected utility theories have been developed among which rank dependent utility theory (Quiggin, 1982; Yaari, 1987; Quiggin and Wakker, 1994) is currently the most influential theory. We therefore also derive the multiplicative model under rank-dependent utility theory.

We start with the case where utility is not discounted. The treatment of discounting is similar in expected utility and in rank dependent utility and we therefore leave it to the end. We characterize two discounting models: constant rate discounting and a general discounting model which is consistent with the most important alternatives for constant rate discounted utility that have been proposed to date in the literature on intertemporal preferences.

3.1. Expected Utility Theory

3.1.1. Some notation

In this subsection we introduce the main concepts used in our axiomatic analysis. To improve accessibility, we have stated the technical assumptions regarding these concepts in the appendix.

We assume that there are T points in time. We express this by saying that there is a set $S = \{1, \dots, T\}$ of time points. The set of all outcomes, i.e. sequences $[(c_1, h_1), \dots, (c_T, h_T)]$, is denoted by X . For ease of notation, we sometimes refer to pairs (c_t, h_t) as y_t . The consumption levels c_t in each period are elements of a set C^+ , which consists of all attainable consumption levels. The plus sign serves as a reminder that we only consider consumption levels that are above the subsistence level. The health states h_t are elements of a set H , which consists of all attainable health states. We assume that H consists only of those health states that are at least as good as death. For expected utility this assumption can easily be relaxed. In fact our axiomatic analysis also holds if negative health states, health states worse than death, are included. For rank dependent utility

however, the generalization to negative health states is more arduous. We assume that the sets C^+ and H are equal in each time period. This assumption is made for convenience. It does not restrict our analysis; it is straightforward to extend the analysis to cases where C^+ and H vary over time.

3.1.2. Preference conditions and representation theorem

Let P be the set of all *lotteries* over X . A typical element of P is $[p_1, x^{(1)}; \dots; p_m, x^{(m)}]$ which gives outcome $x^{(1)} = [(c_1^1, h_1^1), \dots, (c_T^1, h_T^1)]$ with probability p_1 , outcome $x^{(2)} = [(c_1^2, h_1^2), \dots, (c_T^2, h_T^2)]$ with probability p_2 , etc. and m is any natural number. In medical decision making, lotteries can be interpreted as treatments, the outcomes of which are risky. We assume that the set P contains all *degenerate lotteries*: lotteries of the type $[1, x]$, that give outcome x with probability one, i.e., with certainty.

An individual is assumed to have preferences over P . We denote the individual's preference relation over P by \succsim , which stands for *at least as preferred as*. We write $x \succ y$ if it is true that $x \succsim y$ but not $y \succsim x$. That is, $x \succ y$ means that x is *strictly preferred to* y . We write $x \sim y$ if both $x \succsim y$ and $y \succsim x$ are true. That is, $x \sim y$ means that x is *indifferent to* y .

Even though \succsim is defined over lotteries over outcomes, we can derive a preference relation over outcomes from it by restricting attention to degenerate lotteries. We assume that the preference relation over outcomes satisfies *monotonicity with respect to consumption*: higher consumption levels are strictly preferred to lower levels. We also assume a sort of *monotonicity with respect to health status*: if health state h^1 is preferred to health state h^2 for a given level of consumption, then h^1 is preferred to h^2 for all levels of consumption.

A function V is said to *represent* the preference relation \succsim , if for any two lotteries P_1 and P_2 that belong to the set P it is true that the individual considers P_1 at least as preferred as P_2 if and only if the value of V at P_1 is at least as great as the value of

V at P_2 . We express this condition mathematically as: $P_1 \succ P_2$ if and only if $V(P_1) \geq V(P_2)$.

We assume that the expected utility axioms (von Neumann and Morgenstern, 1953; Jensen, 1967) hold. Then the preference relation is represented by the following function:

$$EU[p_1, x^{(1)}; \dots; p_m, x^{(m)}] = \sum_{i=1}^m p_i U(x^{(i)}) \quad (4)$$

where U is a utility function over outcomes. The function U in Eq.(4) is still entirely general. Our aim is to give a set of conditions that ensure the utility function $U[(c_1,$

$h_1), \dots, (c_T, h_T)] = \sum_{t=1}^T v(c_t) q(h_t)$ is representing. We arrive at this final result by imposing

four conditions.

The first condition, *marginality*, ensures that U is a weighted additive function

$$U[(c_1, h_1), \dots, (c_T, h_T)] = \sum_{t=1}^T U_t(c_t, h_t)$$

of one period utility functions U_t .

Definition 1: The preference relation over P satisfies *marginality* if for all $P_1, P_2 \in P$ with

$$\text{equal marginal probability distributions over } y_1, \dots, y_T : P_1 \sim P_2.^2$$

As an example of *marginality*, consider a simple model in which there are only two time periods. Let y' be the outcome (20,000, good health), i.e., a consumption level of 20,000 Euro and good health, and let y'' be the outcome (5,000, bad health). Let P_1 be a lottery that yields y' in both periods with probability $\frac{1}{2}$ and y'' in both periods also with

² Marginality can be weakened. For example, we can only impose it for gambles with two outcomes both with probability 0.5. We used this version of *marginality* because we believe it is the most

a probability of $\frac{1}{2}$, i.e. $P_1 = [\frac{1}{2}, (y', y'); \frac{1}{2}, (y'', y'')]$. P_2 is a lottery that yields a sequence (y', y'') , i.e., y' in the first period and y'' in the second period, with probability $\frac{1}{2}$ and a sequence (y'', y') with probability $\frac{1}{2}$. By marginality the individual should be indifferent between P_1 and P_2 . This follows because in both lotteries there is in each time period a probability of $\frac{1}{2}$ that the individual obtains y' and a probability of $\frac{1}{2}$ that the individual obtains y'' . This example illustrates the working of marginality. Marginality excludes all complementarity between time periods. It might well be that the individual prefers P_1 to P_2 because he dislikes variation in his consumption and health status levels. Such an aversion to variation is not permitted under marginality.

The next step in our derivation is to make the one-period utility functions identical and to exclude the differential weighting of time periods. That is, we have to impose a condition which makes it possible to represent by preferences by the utility

function $U(c_1, h_1, \dots, c_T, h_T) = \sum_{t=1}^T U(c_t, h_t)$. Before we introduce the condition that achieves

this end, symmetry, we have to define a new concept. A *permutation function* $\pi(t)$ is a function that specifies a rearrangement of the time periods. For example if $\pi(t)=s$ then the point t is moved to point s in the rearrangement of the time periods.

Definition 2: The preference relation over P satisfies *symmetry* if for all $x \in X$ and for all

permutation functions π it is true that $(x_1, \dots, x_T) \sim (x_{\pi(1)}, \dots, x_{\pi(T)})$.

Let y' and y'' be as in the previous example. Then by symmetry the individual is indifferent between the outcome x' which yields y' in the first period and y'' in the second period and the outcome x'' which yields y'' in the first period and y' in the second. This follows because x'' can be obtained from x' by applying the permutation $\pi(1) = 2$ and $\pi(2) = 1$.

If symmetry holds, the individual's preferences are unaffected if we turn a decreasing sequence, i.e., a sequence of which the outcomes become worse over time, into an increasing sequence, a sequence of which outcomes improve over time, by a permutation of the time periods. That is, symmetry renders the point in time at which a particular outcome occurs irrelevant. Symmetry is at odds with the assumption that people have positive time preference made in most economic evaluations. Later in this section we show how symmetry can be replaced by conditions that ensure positive discounting.

Marginality and symmetry ensure that the utility function over sequences of consumption and health status is additively decomposable over time and that the utility function is equal for each time period. We now have to establish that the one-period utility function can be multiplicatively decomposed, i.e., $U(c_t, h_t) = v(c_t)q(h_t)$. The next condition we introduce, standard gamble invariance, ensures that $U(c_t, h_t)$ can be written as $v(c_t)q(h_t) + w_t(c_t)$. The final condition, the zero condition, then allows us to set $w_t(c_t) = 0$ for all t and for all consumption levels.

Definition 3: The preference relation over P satisfies *standard gamble invariance* if for all $c, c' \in C^+$:

$(c, h) \succsim [p, (c', h'); 1-p, (c, h'')]$ if and only if $(c', h) \succsim [p, (c', h'); 1-p, (c', h'')]$

with all lotteries elements of P .

To illustrate, let $h, h',$ and h'' be three health states such that h' is better than h which is better than h'' . For example, h can be no asthma, h' a mild form of asthma, and h'' a severe form of asthma. Suppose that annual consumption is held fixed at a given level, say 20,000 Euro, and that at this consumption level the individual considers being in health state h for certain at least as good as undergoing a risky treatment which yields h' with probability p and h'' with probability $1-p$. Then standard gamble invariance says that the individual should still consider being in health state h for certain at least as good

as undergoing the risky treatment if consumption is held fixed at another level, say 5,000 Euro. This example illustrates the effect of standard gamble invariance. Standard gamble invariance enables consideration of preferences over health states irrespective of the level at which consumption is held fixed.

Standard gamble invariance is typically invoked in health utility measurement. In assessing the utility of a health state by the standard gamble, it is commonly assumed that life-years can be held fixed and that the value at which life-years are held fixed does not affect preferences. This idea is similar to standard gamble invariance as we use it here. The only difference is that we hold consumption instead of life-years fixed in the assessment of preferences over health states.

Because standard gamble invariance allows separate consideration of preferences over health status, it also allows the definition of a separate utility function over health status which does not depend on consumption. In the appendix we prove that imposing standard gamble invariance on top of marginality and symmetry implies that the one-period utility functions $U(c_t, h_t)$ can be written as $w_t(c_t) + v(c_t)q(h_t)$ with $w_t(c_t)$ real and $v(c_t)$ positive. This model differs from the multiplicative model that we seek to derive by the terms $w_t(c_t)$. To complete our characterization, we have to impose a condition that gives $w_t(c_t)=0$ for all t and for all c_t . As it turns out, the condition that ensures this, the zero condition, is a condition that is naturally satisfied in the medical context. In words, the zero condition says that a person derives no more utility from consumption once he has died. Formally, the zero condition is defined as:

Definition 4: The preference relation satisfies the *zero condition* if for all consumption

$$\text{levels } c, c' \in C^+: (c, \text{death}) \sim (c', \text{death}).$$

We summarize the derivation of the multiplicative representation in Theorem 1. A formal proof of Theorem 1 is given in the appendix.

Theorem 1: Under expected utility theory, the following two statements are equivalent:

- (i) Life-cycle preferences are consistent with the maximization of

$$U = \sum_{t=1}^T v(c_t)q(h_t) .$$

- (ii) The preference relation satisfies marginality, symmetry, standard gamble invariance, and the zero condition.

3.1.3. An assessment of the conditions of Theorem 1

Before moving on to the characterization of the multiplicative model in the rank dependent utility model, let us briefly comment on the conditions described in the previous subsection. Because the conditions are new in the context of life-cycle preferences over consumption and health, there exists no direct empirical evidence with respect to their descriptive validity. We believe that marginality and symmetry are the most restrictive conditions of the characterization. The zero condition is unobjectionable in the medical context. Further, even though standard gamble invariance has not been tested in the context of life-cycle preferences over consumption and health status, there is some evidence that preferences over health status and duration satisfy utility independence which implies standard gamble invariance (Miyamoto and Eraker, 1988, Bleichrodt and Johannesson, 1997). Utility independence is commonly assumed both in medical decision analyses (Torrance et al., 1982; Torrance et al., 1995; Torrance et al., 1996) and in general decision analysis (Keeney and Raiffa, 1976) and it is believed to be a reasonable condition in most decision contexts.

As was shown above, marginality excludes all complementarity between time periods. This is a strong restriction. In the example presented in subsection 3.2, marginality implies that people are indifferent with respect to variations in their consumption level and in their health state. However, empirical evidence shows that people have a tendency to overweight their status quo or endowment and are averse to

changes therein (Samuelson and Zeckhauser, 1988; Kahneman et al., 1990). Such a tendency is incompatible with marginality. In the next subsection we show that under rank dependent utility theory marginality need no longer be imposed and can be replaced by an alternative condition. This may enhance the descriptive validity of the multiplicative representation.

Finally, we have seen that symmetry excludes differential weighting of time periods and therefore symmetry excludes positive time preference. Economic theory posits and empirical evidence shows that people typically prefer to receive benefits sooner rather than later, i.e., they have positive time preference. The existence of positive time preference calls for the replacement of symmetry. In subsection 3.3., we show how symmetry can be relaxed to allow for (positive) time preference.

3.2. Rank Dependent Utility Theory

3.2.1. Some notation

Contrary to expected utility theory, rank dependent utility theory does not assume that the utility of a lottery is linear in probability. Under rank dependent utility theory, preferences over lotteries are represented by the functional

$$\text{RDU}[p_1, x^{(1)}; \dots; p_m, x^{(m)}] = \sum_{i=1}^m \pi_i U(x^{(i)}) \quad (5)$$

where the π_i are decision weights that depend on, but are in general not equal to the probabilities. depend on the cumulative probability distribution. If the decision weights are equal to the probabilities, i.e. $\pi_i = p_i$ for all i , then rank dependent utility theory is equal to expected utility theory. That is, rank dependent utility theory includes expected utility theory as a special case.

Compared with expected utility theory, there is one conceptual change. Remember that under expected utility we studied preferences over the set of lotteries P . Under rank-dependent utility theory, we consider preferences over a subset of the set of

lotteries P , the set of rank-ordered lotteries. A lottery $[p_1, x^{(1)}; \dots; p_m, x^{(m)}]$ is said to be a *rank-ordered lottery* if its outcomes are ranked in decreasing order of preference, i.e., $x^{(1)} \succ x^{(2)} \dots \succ x^{(m)}$. For example, the lottery $[p, \text{full health}; 1-p, \text{death}]$ is rank-ordered because full health is a better health state than death. However, the lottery $[p, \text{death}; 1-p, \text{full health}]$ is not rank-ordered. We denote the set of rank-ordered lotteries by P_{\downarrow} .

The set of rank ordered lotteries contains all degenerate probability distributions. This follows because each outcome is at least as preferred as itself: $x \succ x$ for all $x \in X$. By consequence, we can once again define preferences over outcomes by restricting attention to the degenerate probability distributions.

We also introduce one new notation. Let A be a subset $\{s, t\}$ of the set of time points $\{1, \dots, T\}$ which we denoted by S . Obviously, $1 \leq s \leq t \leq T$. By $a_A x$ we denote the outcome $(x_1, \dots, x_{s-1}, a_s, \dots, a_t, x_{t+1}, \dots, x_T)$, i.e., we replace the elements x_s, \dots, x_t of the sequence $x = (x_1, \dots, x_T)$ with the respective elements a_s, \dots, a_t of the sequence (a_1, \dots, a_T) . To give a simple example, let T be 40, let S be $\{25, \dots, 30\}$, let x be the constant sequence yielding a consumption level of 20,000 Euro and good health for 40 periods and let a be the constant sequence yielding a consumption level of 5,000 Euro and bad health for 40 periods. Then $a_A x$ is the outcome yielding a consumption level of 20,000 Euro and good health from period 1 to period 24, a consumption level of 5,000 Euro and bad health from period 25 to period 30, and a consumption level of 20,000 Euro and good health from period 31 to period 40.

In case the set A consistst of just one point in time, say point t , then we write $a_t x$ instead of $a_A x$.

3.2.2. Preference conditions and representation theorem

We use the same steps in the characterization of the multiplicative representation $\prod_{t=1}^T v(c_t)q(h_t)$ as in subsection 3.1.2. Hence, the first step is to find a condition that allows utility to be written as the additive sum of the one-period utility functions, i.e.,

$$U[(c_1, h_1), \dots, (c_T, h_T)] = \sum_{t=1}^T U_t(c_t, h_t) .$$

Under expected utility theory, marginality served

to ensure that the utility function could be written in this form. However, marginality is no longer available under rank dependent utility theory, because marginality only leads to an additive decomposition of utility if the evaluation function for lotteries is linear in the probabilities. As we have seen, this is not necessarily true under rank dependent utility theory. Therefore, marginality cannot be retained and we have to find a condition which achieves the additive decomposition under rank dependent utility theory. The following condition, generalized utility independence, has this effect.

Definition 5: The preference relation \succsim on $P \downarrow$ satisfies *generalized utility independence* if

for all subsets A of S:

$$[p, a_{AZ}; 1-p, b_{AY}] \succsim [p, c_{AZ}; 1-p, d_{AY}] \text{ if and only if}$$

$$[p, a_{AW}; 1-p, b_{AV}] \succsim [p, c_{AW}; 1-p, d_{AV}]$$

with all lotteries elements of $P \downarrow$.

An example may clarify the effect of generalized utility independence. For ease of notation, we assume that health status is constant, say at full health, and we will suppress it from our notation. Let there be two time periods and let $A=\{1\}$. Let $a = z = 20,000$, $c = 25,000$, $b = y = w = 15,000$, and $d = v = 10,000$. It can be verified that all lotteries are rank-ordered. The first preference then says that the lottery P_1 yielding the outcome (20,000, 20,000) with probability p and the outcome (15,000, 15,000) with probability $1-p$ is at least as preferred as the lottery P_2 yielding the outcome (30,000, 20,000) with probability p and the outcome (10,000, 15,000) with probability $1-p$. Generalized utility independence then implies that the lottery P_3 yielding the outcome (20,000, 15,000) with probability p and the outcome (15,000, 10,000) with probability $1-p$ should also be at least as preferred as the lottery P_4 yielding the outcome (30,000, 15,000) with probability p and the outcome (10,000, 10,000) with probability $1-p$. Note that the

probability distribution over what happens in the second period is identical both for P_1 and P_2 (a probability of $\frac{1}{2}$ of 20,000 and a probability of $\frac{1}{2}$ of 15,000) and for P_3 and P_4 (a probability of $\frac{1}{2}$ of 15,000 and a probability of $\frac{1}{2}$ of 10,000). Generalized utility independence therefore implies that if two lotteries have identical probability distributions over the outcomes in a given period, then the individual will ignore the outcomes occurring in this period and he will only focus on the periods in which the two lotteries have different probability distributions over the outcomes. Note that this excludes the possibility that an individual pays attention to complementarity of outcomes between the periods. For example, generalized utility independence excludes that the individual has a preference for variation.

In the absence of empirical evidence, we can only speculate about the descriptive validity of generalized utility independence. Note however that the more common condition of utility independence can be derived from the definition of generalized utility independence by setting $z = y$ and $v = w$. As remarked before, utility independence is widely believed to be a preference condition that describes preferences reasonably well, both in medical decisions and in other decision contexts. Generalized utility independence is only slightly stronger than utility independence and from this it may be inferred that generalized utility independence describes life-cycle preferences over consumption and health reasonably well. On the other hand, if the individual dislikes variation in his consumption level, then it may well be that he prefers P_1 to P_2 and P_4 to P_3 in the above example.

The other three conditions used in the characterization of the multiplicative model $\prod_{t=1}^T v(c_t)q(h_t)$ under expected utility theory can still be used under rank-dependent utility theory. Theorem 2 summarizes the derivation. A formal proof of Theorem 2 is given in the appendix.

Theorem 2: Under rank dependent utility theory the following two statements are equivalent:

- (i) Life-cycle preferences are consistent with the maximization of

$$U = \sum_{t=1}^T v(c_t)q(h_t) .$$

- (ii) The preference relation \succsim on $P \downarrow$ satisfies generalized marginality. Further \succsim satisfies symmetry, standard gamble invariance, and the zero-condition.

3.3 Discounting

3.3.1. The general discounting model

The characterizations presented in Theorems 1 and 2 imply that people give equal weight to each time period, i.e., they are *timing neutral*. As remarked, in economic evaluations it is more common to assume that people have positive time preference and discount future time periods. Empirical evidence concerning intertemporal choices is also supportive of the existence of positive time preference for health (Olsen, 1993a, Cairns, 1994, Chapman, 1996).

In this section, we characterize two models that incorporate time preference. In one model, we impose no restrictions on the discount weights. Therefore, this model is consistent with most discounting models that have been proposed in the literature on intertemporal preferences (Loewenstein and Prelec, 1992; Harvey, 1994). The other model is the constant rate discounting model, which commonly applied in cost effectiveness analysis and also underlies the analyses by Garber and Phelps (1997) and Meltzer (1997).

Timing neutrality is a consequence of imposing symmetry in the characterization, both under expected utility theory and under rank dependent utility theory. Therefore, to allow for time preference symmetry has to be replaced. However, symmetry also served to select identical one-period utility functions in Theorems 1 and 2. If symmetry is dropped the one-period utility functions are no longer necessarily identical. Hence we

have to replace symmetry by another condition that ensures that the one period utility functions are strategically equivalent and can be chosen identical. The following condition, trade-off consistency, serves this end.

Definition 6: The preference relation \succsim on X satisfies *trade-off consistency* if for all $s, t \in S$:

if $(a_s x \succ b_s y)$ and $(c_s x \succ d_s y)$ and $(a_t v \succ b_t w)$ then $(c_t v \succ d_t w)$, with all outcomes elements of X .

Trade-off consistency can be explained in terms of strength of preference. Suppose that a is strictly preferred to b , and that c is strictly preferred to d . The preference $(a_s x \succ b_s y)$ then tells us that getting the strictly preferred outcome a instead of b in period s is not sufficient to outweigh getting x instead of y in all periods other than s . The preference $(c_s x \succ d_s y)$, however, tells us that getting the strictly preferred outcome c instead of d in period s is sufficient to outweigh getting x instead of y in the other periods. These two preferences, therefore, indicate that in period s the strength of preference of c over d must be at least as great as the strength of preference of a over b . Now, trade-off consistency asserts that if the strength of preference of c over d is at least as great as the strength of preference of a over b in period s , then there does not exist another period in which the strength of preference of c over d is smaller than the strength of preference of a over b . That is, if we observe in another period t that getting a instead of b is sufficient to outweigh getting v instead of w in all periods other than t then getting c instead of d should also be sufficient.

Trade-off consistency thus ensures that utility differences are ordered similarly in different periods. This implies that the different one-period utility functions are cardinally equivalent and thus that they can be chosen identical. However, trade-off consistency does not imply that each period gets the same weight. In combination with marginality and generalized utility independence in the expected utility model respectively the rank-dependent utility model, trade-off consistency implies that the

utility function can be written as $\sum_{t=1}^T \lambda_t U(c_t, h_t)$. Imposing standard gamble variance and

the zero condition as well gives the general multiplicative discounting model $U =$

$$\sum_{t=1}^T \lambda_t v(c_t) q(h_t).$$

3.3.2. The constant rate discounted utility model

The constant rate discounted utility model, $U = \sum_{t=1}^T \beta^{t-1} v(c_t) q(h_t)$ can be

obtained by imposing one additional condition, stationarity, on the general multiplicative discounting model. It is easily verified that in the constant rate discounted utility model, the ratio between the weights assigned to utility in period t and to utility in period s is equal to β^{t-s} . Similarly, the ratio between the weights assigned to utility in period $t+e$ and to utility in period $s+e$ is equal to β^{t-s} . This implies that in the constant rate discounted utility model only the difference in timing between outcomes (e.g. $t-s$) affects preferences, but not the position in time at which the outcomes occur. For example, the constant rate discounted utility model implies that if a person is indifferent between an amount of money x now and an amount of money y in 5 years, then he should also be indifferent between x in 10 years and y in 15 years, because in both preference comparisons the difference in timing is 5 years. Stationarity captures the idea that preferences depend only on the difference in timing and not on the exact timing of the outcomes. The formal definition of stationarity is as follows.

Definition 7: The individual preference relation on X satisfies *stationarity* if there exists

a common outcome $q \in Y$ such that for all $x_t, y_t \in X$:

$(x_1, \dots, x_{T-1}, q) \succ (y_1, \dots, y_{T-1}, q)$ if and only if

$(q, x_1, \dots, x_{T-1}) \succ (q, y_1, \dots, y_{T-1})$.

In words, stationarity says that preferences over outcomes are unaffected if we move the common outcome from the last to the first period and delay of all other outcomes with one period. Note that the differences in timing between the x_t are unaffected by this permutation of outcomes in time.

Theorem 3 summarizes the previous two subsections. A formal proof is given in the appendix.

Theorem 3: If we replace symmetry by trade-off consistency in Theorems 1 and 2 then

$$\text{the general multiplicative discounting model } U = \prod_{t=1}^T \lambda_t v(c_t) q(h_t)$$

represents life-cycle preferences over consumption and health status.

If stationarity is imposed as well then the constant rate discounted utility

$$\text{model } U = \prod_{t=1}^T \beta^{t-1} v(c_t) q(h_t) \text{ is representing..}$$

3.3.3. *An assessment of the conditions*

To conclude the axiomatic analysis, let us briefly comment on the empirical content of the conditions used to characterize the two discounted utility models. No tests of trade-off consistency exist in the medical context. The main effect of trade-off consistency is to impose an additive representation. In an additive representation, what happens in one period is independent of what happens in all other periods. Therefore, trade-off consistency is most likely to hold in decision contexts where complementarity between periods does not affect preference.

Studies that have tested stationarity yield negative results (Cairns and van der Pol, 1997; Bleichrodt and Johannesson, 1998). These studies find that people do not only pay attention to differences in timing between outcomes, i.e. to their relative position in time, but also to the moment at which they occur, i.e., their absolute position in time.

The general pattern that emerges from the literature on intertemporal preferences for health is that people are more timing averse, in the sense that their implied rate of time preference is higher, for delays that occur in the near future than for delays that occur in the more distant future. That is, the difference between year s and year t is discounted more than the difference between year $s+e$ and year $t+e$. This finding suggests that other discounting models than the constant rate discounted utility model may be more appropriate. If trade-off consistency holds then these models can be derived as special cases of the general discounted utility model.

4. The valuation of longevity and willingness to pay for QALYs

Next we show when consumption will be constant over time. As was shown in Section 2, constant consumption implies together with the multiplicative model derived in Section 3 that cost effectiveness is consistent with life-cycle preferences over consumption and health status, or, which is equivalent, that cost effectiveness analysis is consistent with cost benefit analysis.

Given that consumption is constant and the multiplicative model holds we can derive tractable expressions for the valuation of longevity and the willingness to pay for QALYs. We extend the models proposed by Rosen (1988) by including health status in the analysis. We start with the deterministic case where the individual knows his life duration with certainty. The deterministic case illustrates the essential ideas while keeping the analysis relatively straightforward. We then turn to the more realistic stochastic case.

4.1. Certainty

4.1.1. The optimization problem

Consider an individual whose preferences can be described by the multiplicative model with constant rate discounting:

$$U = \sum_{t=1}^T v(c_t)q(h_t) \frac{1}{(1+a)^{t-1}} \quad (5)$$

where a is the individual's constant rate of time preference. We assume that v is strictly increasing and concave. The first derivative of v with respect to c is denoted by v_c .

The individual's wealth consists of initial wealth W and a fixed annual labor income w . The individual allocates his wealth between consumption and medical expenditures, which we denote by m . Medical expenditures are in each period a function of the sequence of quality of life levels $(q(h_1), \dots, q(h_T))$ and duration T : $m_t = g((q(h_1), \dots, q(h_T)), T)$. We assume that for all t the first derivative of medical expenditures with respect to $q(h_t)$, denoted g_{h_t} is positive. The first derivative of medical expenditures with respect to duration, g_T , is also positive. The individual faces a pure capital market at which he can borrow and invest at interest rate r . The individual cannot die in debt and has no heirs. Under these assumptions, the individual's budget constraint becomes:

$$W + w \sum_{t=1}^T \frac{1}{(1+r)^{t-1}} = \sum_{t=1}^T [c_t + m_t] \frac{1}{(1+r)^{t-1}} \quad (6)$$

The Lagrangian expression for this problem is:

$$L = \sum_{t=1}^T v(c_t)q(h_t) \frac{1}{(1+a)^{t-1}} + \lambda \left[W + w \sum_{t=1}^T \frac{1}{(1+r)^{t-1}} - \sum_{t=1}^T (c_t + m_t) \frac{1}{(1+r)^{t-1}} \right] \quad (7)$$

and the first order conditions are:

$$v_c(c_t)q(h_t)\frac{1}{(1+a)^{t-1}}=\lambda\frac{1}{(1+r)^{t-1}} \text{ for all } t\in[1,T] \quad (8a)$$

$$v(c_t)\frac{1}{(1+a)^{t-1}}=\lambda g_{h_t}\frac{1}{(1+r)^{t-1}} \text{ for all } t\in[1,T] \quad (8b)$$

$$v(c_T)q(h_T)\frac{1}{(1+a)^{T-1}}= \quad (8c)$$

$$\lambda\{c_T + m_T - w\}\frac{1}{(1+r)^{T-1}} + \sum_{t=1}^T \frac{1}{(1+r)^{t-1}}g_T$$

Under the assumption that the rate of time preference equals the interest rate ($a=r$) it follows from (8a) and (8b) that the individual will choose c_t and h_t such that they are constant throughout $[0,T]$. Constancy of health status in turn implies constancy of medical expenditures. This analysis answers the question when an individual with a multiplicative utility function will select a constant consumption profile: he will do so when his rate of time preference equals the interest rate. Hence, given the multiplicative utility model, cost effectiveness analysis is consistent with cost benefit analysis if the individual's rate of time preference is equal to the interest rate. This observation has important implications for the discussion whether the discount rate for health benefits can be different from the interest rate (the discount rate for costs) (Keeler and Cretin, 1983; Olsen, 1993b). Our analysis shows that if the purpose is to achieve equivalence between cost effectiveness analysis and cost benefit analysis then health benefits should be discounted at the interest rate, that is, health benefits should be discounted at the same rate as costs.

4.1.2. The willingness to pay for a QALY gained

Let us now derive the willingness to pay for a QALY gained if cost effectiveness analysis is consistent with cost benefit analysis. That is, we assume the multiplicative model and equality of the rate of time preference and the interest rate.

Because equality of the rate of time preference and the interest rate implies that $q(h_t)$ is constant for all t , g_{h_t} is constant for all t . Denote the constant value of $q(h_t)$ by q and the constant value of g_{h_t} by g_h . Rewriting (8b) gives:

$$\frac{v(c)}{g_h} = \lambda \quad (9)$$

which we substitute in (8a) to give:

$$g_h = \frac{v(c)}{v_c(c)q} = \frac{c}{q\varepsilon} \quad (10)$$

where $\varepsilon = v_c \frac{c}{v}$ is the elasticity of the utility function for consumption. The parameter ε reflects the possibilities for intertemporal substitution of consumption. The higher ε , the better are the possibilities for intertemporal substitution. As ε tends to unity, $v(c)$ becomes more linear in c and the individual is less concerned about the distribution of consumption over time than in the value of aggregate consumption. In the limiting case where ε is equal to one, the individual is not interested in the distribution of consumption over time, but only cares about total consumption.

The term g_q indicates how medical expenditures change as a result of a change in quality of life. Hence, g_q can be interpreted as the marginal cost of an additional unit of quality of life. A (discounted) QALY is gained if quality of life increases with $z =$

$$\frac{1}{\sum_{t=1}^{\infty} \frac{1}{(1+r)^{t-1}}} \text{ units. The higher the interest rate the greater the gain in quality of life has}$$

to be. Substitution of z in Eq. (10) defines the willingness to pay for a QALY gained. We observe that the willingness to pay for a QALY gained depends on four factors. It is increasing in consumption and the interest rate and decreasing in quality of life (ceteris

paribus, individuals in worse health are willing to pay more for improvements in quality of life), and the possibilities for intertemporal substitution.

In providing a numerical illustration, we will avoid the complications associated with discounting by focusing on immediate improvements in health. Empirical estimates of ϵ are in the range 0.20 to 0.40 (Thaler and Rosen, 1975; Rosen, 1988). We assume that ϵ is equal to 0.25. Consider a person whose annual consumption is US\$ 20,000 and whose quality of life initially is equal to 0.8. It follows that this individual will be willing to pay about US\$ 100,000 per QALY for immediate improvements.

4.1.3. The willingness to pay for longevity

Let us finally derive the willingness to pay for longevity. Substituting Eq.(9) in Eq.(8c) and rearranging gives:

$$g_T = \frac{c \frac{1-\epsilon}{\epsilon} + (w-m)}{A} \quad (11)$$

$$\text{with } A = (1+r)^{T-1} \sum_{t=1}^T \frac{1}{(1+r)^{t-1}} = \sum_{t=1}^{T-1} \frac{1}{(1+r)^{t-1}}$$

The term g_T reflects the responsiveness of medical expenditures to changes in longevity. That is, g_T indicates the marginal cost of additional units of life or the willingness to pay for longevity. Like the willingness to pay for a QALY gained, it increases in consumption and it decreases in the interest rate (the higher the interest rate the less valued are future life years) and in ϵ . The negative sign of the elasticity ϵ follows from the fact that ceteris paribus additional longevity does change the individual's total consumption throughout his life, but it changes the distribution of consumption. Because the individual's concern about the distribution of consumption decreases with ϵ , his willingness to pay for longevity decreases with ϵ .

The willingness to pay for longevity also increases in the surplus of the annual wage rate over the annual medical expenditures. During the additional time that the individual lives he is able to create more wealth. The higher his wage rate the more wealth he creates and the greater his possibilities for increasing consumption.

4.2 Risk

4.2.1. The optimization problem

We now turn to the more realistic case where the individual is uncertain about his life duration. Let f_t be the probability of living for t time periods and let F_t be the cumulative probability of surviving until time period t at most. That is, $F_t = \prod_{s=1}^{t-1} f_s$. Then the probability of being alive at the beginning of time period t , denoted S_t , is equal to $1 - F_t$. The period-specific death rate ρ_t is defined as the probability of dying during time period t given that one has survived up to time period t . The period-specific death rate is a conditional probability defined as $\frac{f_t}{S_t}$. Hence, S_t and ρ_t are related as follows:

$$S_t = \prod_{s=1}^{t-1} (1 - \rho_s) \quad (12)$$

We assume that the individual maximizes expected utility. For each t , if he lives exactly t periods, then his utility is defined by the multiplicative model with constant rate discounting and $T=t$. The individual's expected utility is equal to:

$$EU = \sum_{t=1}^{\infty} S_t v(c_t) q(h_t) \frac{1}{(1+a)^{t-1}} \quad (13)$$

We assume that the individual participates with a cohort of identical individual in an actuarially fair annuity system [Yaari, 1965; Rosen, 1988]. Each individual hands over his wealth to an insurance company in exchange for a contract that ensures them their optimal consumption and medical expenditure bundles until death. Obviously, individual choices are restricted by the total wealth available. The budget constraint facing each individual is based on overall life expectancy. If initial wealth is positive then those individuals who die early effectively subsidize the insurance pool and the claims of the individuals who live longer than expected are financed out of these subsidies. Under these assumptions the individual's budget constraint becomes:

$$W + w \sum_{t=1}^{\infty} S_t \frac{1}{(1+r)^{t-1}} = \sum_{t=1}^{\infty} S_t (c_t + m_t) \frac{1}{(1+r)^{t-1}} \quad (14)$$

where the medical expenditures are in each period a function of the infinite sequences of the quality of life levels $(q(h_1), q(h_2), \dots)$ and the age-specific death rates (ρ_1, ρ_2, \dots) : $m_t = g((q(h_1), q(h_2), \dots), (\rho_1, \rho_2, \dots))$. The first derivative of medical expenditures with respect to $q(h_t)$ and ρ_t are denoted g_{h_t} and g_{ρ_t} respectively.

Maximization of expected utility, Eq.(13), subject to the budget constraint, Eq. (14), yields the following first order conditions, in which common terms S_t have been cancelled.

$$v_c(c_t) q(h_t) \frac{1}{(1+a)^{t-1}} = \lambda \frac{1}{(1+r)^{t-1}} \quad \text{for all } t \in [1, T] \quad (15a)$$

$$v(c_t) \frac{1}{(1+a)^{t-1}} = \lambda g_{h_t} \frac{1}{(1+r)^{t-1}} \quad \text{for all } t \in [1, T] \quad (15b)$$

$$\begin{aligned} & \times \sum_{t=1}^{\infty} \frac{S_t}{1-\rho_s} \frac{1}{(1+a)^{t-1}} v(c_t) q(h_t) = \\ & \lambda \left\{ \sum_{t=1}^{\infty} \frac{S_t}{1-\rho_s} \frac{1}{(1+r)^{t-1}} (c_t + m_t - w) - g_{\rho_s} \sum_{t=1}^{\infty} \frac{S_t}{(1+r)^{t-1}} \right\} \text{ for all } s \in [1, T] \end{aligned} \quad (15c)$$

The first order conditions in the decision problem under risk, Eqs. (15a) and (15b) are identical with the conditions under certainty, Eqs. (8a) and (8b), because the term S_t occurs both in Eq(13) and Eq(14) and cancels out. Hence, the problem under risk becomes similar to that under certainty and it follows immediately that the optimal paths for consumption and quality of life are constant if the rate of time preference equals the interest rate. Hence, the conclusion of decision under certainty that cost effectiveness analysis will only be equivalent to cost benefit analysis if the individual's rate of time preference is equal to the interest rate is still valid if his life duration is uncertain.

4.2.2. The willingness to pay for a QALY gained

Even though our conclusions about the consistency between cost effectiveness analysis and cost benefit analysis are unaffected by the introduction of risk, there is a change in the willingness to pay for a QALY gained. Let the rate of time preference be equal to the interest rate. Denote the constant value of $q(h_t)$ by q and the constant value of g_{h_t} by g_h . It then follows from Eqs. (15a) and (15b) that g_h , the willingness to pay for a QALY gained is still equal to:

$$g_h = \frac{v(c)}{v'_c(c)q} = \frac{c}{q\epsilon} \quad (16)$$

However, under uncertainty quality of life has to increase with

$$S_t \times \frac{1}{(1+r)^{t-1}} \quad (17)$$

to generate an additional QALY, which exceeds the increase in the certainty case by the factor S_t . Because future life-years are uncertain, the survival probabilities act as an additional discount factor of life-years in the future. Alternatively stated, the willingness to pay for a given increase in quality of life will be lower in the uncertainty case because the risk of premature death leads to a lower weight for future periods.

4.2.3. The willingness to pay for longevity

Expected life-duration changes if the period-specific death rates change. The willingness to pay for longevity therefore follows under uncertainty from the willingness to pay for changes in the age-specific death rates. We determine the latter expression. Because expected life-duration is negatively related to changes in the period-specific death rates, the willingness to pay for (expected) longevity follows from the willingness to pay for reductions in the period-specific death rates.

Substitution of $\lambda = \frac{v(c)}{g_h}$ and Eq. (16) in Eq.(15c) gives after some rearranging:

$$\rho_s = - \frac{(c \frac{1-\varepsilon}{\varepsilon} + w - m)}{1-\rho_s} \quad (18)$$

Obviously, Eq.(18) is negative, because individuals will desire compensation for increases in the age-specific death rates. Eq.(18) shows that the willingness to pay for reductions in the age-specific death rates are positively related with consumption and wealth and negatively with medical expenditures, and the possibilities for intertemporal substitution. Finally, the willingness to pay for reductions in the period-specific death

rates increases with the death rate. The higher the death rate, the higher the individual's willingness to pay for reductions in the death rate.

5. Discussion

This paper has shown under what conditions QALY maximisation is consistent with life cycle preferences over consumption and health. By implication, this answers the question under what conditions cost effectiveness analysis will give the same results as cost benefit analysis. We have shown that cost effectiveness is equivalent to cost benefit analysis if the lifetime utility function over consumption and health status is additive over time, multiplicative in consumption and health status, and the utility of consumption is constant over time. We have derived that the utility function has this form under expected utility if the preference relation satisfies marginality, symmetry, standard gamble invariance, and the zero condition and if the individual's rate of time preference is equal to the interest rate. The latter condition has an interesting implication for the debate about the appropriate rate of discount for health benefits: our analysis shows that if the aim is to achieve consistency between the results of cost effectiveness analysis and individual preferences then health benefits must be discounted at the same rate as health costs.

Expected utility is now widely believed to be descriptively invalid and we therefore have also provided an axiomatic analysis of the above utility function under the most influential nonexpected utility theory, rank dependent utility theory. Under rank dependent utility theory, marginality has to be replaced by generalized utility independence.

If cost effectiveness analysis is consistent with cost benefit analysis, i.e., the above utility function is representing, then it becomes possible to derive tractable expressions for the willingness to pay for a QALY gained and for the willingness to pay for longevity. This analysis answers a research question posed by Johannesson (1995a), who argued that it is important to have information on the factors that determine the willingness to pay for a QALY gained if cost effectiveness is to be a useful tool in societal

decisions about the allocation of health care resources. We show that the willingness to pay for a QALY is determined by four factors: wealth, life expectancy, health status and the possibilities for intertemporal substitution of consumption. The willingness to pay for a QALY gained increases with wealth and with life-expectancy and decreases with health status and the possibilities for intertemporal substitution of consumption.

Even though we have focused on QALY based decision making, our central result is also valid for over outcome measures. For example if healthy-years equivalents (HYEs) are used the utility function over consumption and health still has to be multiplicative and the utility of consumption still has to be constant. That is, standard gamble invariance, the zero condition, and equality between the interest rate and the individual's time preference all have to hold. A problem with using HYE in this context is that the HYE as intended by Mehrez and Gafni (e.g. Gafni and Birch, 1997) is not a utility and that additional conditions have to be imposed to use HYE in life cycle problems involving both consumption and health. In particular, the utility function over years in full health has to be linear (Johannesson, 1995b).

Our analysis is based on the view that economic evaluation should have a foundation in welfare economics. There is a different conception of economic evaluation which places cost effectiveness analysis outside the realm of welfare economics (e.g. Culyer, 1991; Williams, 1993; Donaldson, 1998). This extra-welfarist or decision-making approach posits that principles of optimization theory coupled with an exogenously specified objective function and an exogenously specified resource constraint suffice as a foundation for cost effectiveness analysis. As was noted by Johannesson (1995a) and Weinstein and Manning (1997), the decision-making approach provides little guidance if the object of cost effectiveness analysis is to compare the efficiency of different programs and may well lead to problems of suboptimization. It is not our intention to resolve the controversy about the role of cost effectiveness analysis. We only observe that there exists a perception of cost effectiveness which requires a foundation of cost effectiveness analysis in welfare economics. This perception provides the rationale for this paper.

The aim of our axiomatic analysis is to reveal the conditions under which cost effectiveness analysis is equivalent to cost benefit analysis. Let us emphasize, that we do not intend to argue that these conditions have descriptive or normative force. In fact, as already indicated in Section 3, we believe that several of the conditions are unlikely to hold. For example, marginality is a strong condition, and empirical evidence indicates that marginality does not hold in medical decision making (Maas and Wakker, 1994). Under rank dependent utility theory, marginality is replaced by generalized utility independence, which may be more realistic being a strengthening of utility independence. There exists some empirical support for utility independence in the medical context. Symmetry also seems too restrictive given that empirical research (e.g. Cairns, 1994; Chapman, 1996) indicates that people have time preference, i.e., they give different decision weights to different time periods. However, as was shown in Section 3, symmetry can be replaced by conditions that allow differential weighting of time points. Empirical evidence on the validity of stationarity in medical decision making is negative, but the more general discounting model can be a good description (e.g. Cairns and van der Pol, 1997; Bleichrodt and Johannesson, 1998).

Future research should test the validity of the conditions identified in this paper. Difficulties in the empirical estimation of willingness to pay for changes in health status have spurred the use of cost effectiveness analysis as a tool in the allocation of health care resources. If the viewpoint is accepted that economic evaluation of health care should have a foundation in welfare economic theory and if the conditions identified in this paper do not hold, then the way to advance for methodological research in economic evaluation is to try and solve the empirical problems surrounding cost benefit analysis instead of resorting to cost effectiveness analysis.

Appendix: Technical assumptions and proofs

Structural assumptions:

The set X is a *Cartesian product* of the one-period outcome sets Y , which are assumed identical. The one-period outcome set Y is a Cartesian product of C^+ and H . C^+ is a subset of the set of nonnegative real numbers, which is a convex subset of a linear space over \mathbb{R} and hence endowed with the Euclidean topology. We assume that H is a connected topological space and that X and X^T are both endowed with the product topology.

The set P consists of all *simple lotteries*: lotteries with finite support. The preference relation over P satisfies the von Neumann Morgenstern axioms (Jensen, 1967). Preferences over X are derived by restricting attention to degenerate lotteries. Preferences over Y are derived by restricting attention to constant outcomes: $(c^{(1)}, h^{(1)}) \succ (c^{(2)}, h^{(2)})$ iff the sequence that yields the pair $(c^{(1)}, h^{(1)})$ in each time period is at least as preferred as the sequence that yields the pair $(c^{(2)}, h^{(2)})$ in each time period. Preferences over consumption and over health status are derived from the preference relation over Y by restricting attention to those pairs (c, h) in which one of the attributes is held constant. That is, the preference relation over C^+ is defined as: for all $c^{(1)}, c^{(2)} \in C^+$ and for all $h \in H$, $c^{(1)} \succ c^{(2)}$ iff $(c^{(1)}, h) \succ (c^{(2)}, h)$. The preference relation over C^+ is assumed to satisfy *monotonicity*: for all $c^1, c^2 \in C^+$ such that $c^1 > c^2$ and for all $h \in H$, it is true that $(c^1, h) \succ (c^2, h)$. The preference relation over H is defined as: for all $h^{(1)}, h^{(2)} \in H$ and for all $c \in C^+$, $h^{(1)} \succ h^{(2)}$ iff $(c, h^{(1)}) \succ (c, h^{(2)})$. We assume *preferential independence* of H from C^+ : if $(c, h^{(1)}) \succ (c, h^{(2)})$ for one $c \in C^+$ then $(c, h^{(1)}) \succ (c, h^{(2)})$ for all $c \in C^+$. Loosely speaking, preferential independence can be interpreted as a monotonicity condition for the preference relation over H .

We say that consumption is *essential* if there exist $c, c' \in C^+$ and $h \in H$ such that $(c, h) \succ (c', h)$. Similarly we say that health status is essential if there exist $c \in C^+$ and $h, h' \in H$ such $(c, h) \succ (c, h')$. Let $a_t x$ denote the outcome $x \in X^T$ with x_t replaced by a : $(x_1, \dots, x_{t-1}, a, x_{t+1}, \dots, x_T)$. A point in time $t \in S$ is essential if there exist $a_t x, b_t x \in X^T$ such that $a_t x \succ b_t x$. Essentiality of either consumption or health status implies that at least one point in time must be essential. We assume that both consumption and health status are

essential (otherwise our problem would become trivial) and that at least two points in time are essential.

Proof of Theorem 1

By Theorem 4 in Fishburn (1965) marginality implies that $U(x) = \prod_{i=1}^T U_i(x_i)$.

The proof that symmetry implies that all utility functions can be chosen identical has been given in Bleichrodt and Quiggin (1997).

Fix a $c^1_t \in C^+$ and define $q(h_t)$ as $U(c^1_t; h_t)$. By standard gamble invariance, for all $c_t \in C^+$, $U(c_t, h_t)$ is strategically equivalent to $q(h_t)$. Hence for all $c_t \in C^+$, $U(c_t, h_t)$ is a positive linear transform of $q(h_t)$: $U(c_t, h_t) = w_t(c_t) + v(c_t)q(h_t)$ with $w_t(c_t)$ real and $v(c_t)$ positive. Positivity of $v(c_t)$ follows because we used weak preference () in the definition of standard gamble invariance. If we would have used indifference instead $v(c_t)$ would have been real and preference reversals would have been possible. Denote death by $h=0$ and scale $q(h_t)$ such that $q(0)=0$. By the zero condition, for all $c^2_t, c^3_t \in C$: $w_t(c^2_t) + v(c^2_t) \cdot 0 = w_t(c^3_t) + v(c^3_t) \cdot 0$. Hence $w_t(c_t)$ is constant. By the uniqueness properties of the von Neumann Morgenstern utility function we may subtract a constant to give $U(c_t, h_t) = v(c_t)q(h_t)$. By monotonicity $v(c_t)$ is increasing.

Proof of Theorem 2:

Generalized utility independence implies utility independence of all subsets $A \subseteq S$ by setting $y=z$ and $v=w$. Miyamoto and Wakker (1996) have shown for two attributes that utility independence implies that $U(x)$ is either additive or multiplicative. Their argument can easily be generalized to more than two attributes. We illustrate the case where $U(x)$ is multiplicative. The case where $U(x)$ is additive can be derived in a similar fashion. By utility independence of x_1 from x_2, \dots, x_T and by utility independence of x_2, \dots, x_T from x_1 we have $U(x) = f_1(x_1)f_{2..T}(x_2, \dots, x_T)$, with f_1 and $f_{2..T}$ utility functions over Y and Y^{T-1} respectively. Applying utility independence on Y^{T-1} gives $f_{2..T}(x_2, \dots, x_T) = f_2(x_2) \dots f_T(x_T)$. Repeating this procedure gives the multiplicative utility function

Now, we use generalized utility independence to distinguish between the multiplicative and the additive utility function. We give a proof by contradiction that generalized utility independence implies that the utility function over life-years must be additive. Suppose the utility function is multiplicative instead and let generalized utility independence hold. Then we have

$$w(p)Av_t(a) + [1-w(p)]Bv_t(b) \geq w(p)Av_t(c) + [1-w(p)]Bv_t(d) \quad (A1)$$

if and only if

$$w(p)Cv_t(a) + [1-w(p)]Dv_t(b) \geq w(p)Cv_t(c) + [1-w(p)]Dv_t(d) \quad (A2)$$

with $A = v_1(z_1) \cdot \dots \cdot v_{t-1}(z_{t-1})v_{t+1}(z_{t+1}) \cdot \dots \cdot v_T(z_T)$, $B = v_1(y_1) \cdot \dots \cdot v_{t-1}(y_{t-1})v_{t+1}(y_{t+1}) \cdot \dots \cdot v_T(y_T)$, $C = v_1(w_1) \cdot \dots \cdot v_{t-1}(w_{t-1})v_{t+1}(w_{t+1}) \cdot \dots \cdot v_T(w_T)$, and $D = v_1(v_1) \cdot \dots \cdot v_{t-1}(v_{t-1})v_{t+1}(v_{t+1}) \cdot \dots \cdot v_T(v_T)$.

From (A1) and (A2) we derive that

$$v_t(a) - v_t(c) \geq \frac{[1-w(p)]B}{w(p)A} \{v_t(d) - v_t(b)\} \quad (A3)$$

if and only if

$$v_t(a) - v_t(c) \geq \frac{[1-w(p)]D}{w(p)C} \{v_t(d) - v_t(b)\} \quad (A4)$$

which is clearly not always true. We derive a contradiction by assuming the multiplicative utility function and hence the additive utility function must be true. The rest of the proof is similar to the proof of Theorem 1.

Proof of Theorem 3:

Wakker (1989) has shown that the structural assumptions and trade-off consistency ensure that there exists an additive representation $\sum_{t=1}^T \lambda_t U(x_t)$ over X .

Because both P and $P\downarrow$ contain all degenerate probability distributions, the preference relations over P and $P\downarrow$ are also representing under certainty. Further, there exist additive representations over P and $P\downarrow$ by marginality and generalized marginality

respectively. It follows that $U(x) = \sum_{i=1}^T \lambda_i U(x_i)$ represents preferences over P and $P\downarrow$.

Fishburn (1970) has shown that if stationarity is imposed as well then $U(x) =$

$\sum_{i=1}^T \beta^{t-1} U(x_i)$. The rest of the proof is similar to the proof of Theorem 1.

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