Optimal Producer Behavior in the Presence of Area-Yield Crop Insurance

Robert G. Chambers[1] and John Quiggin[2]

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[1]Professor, Department of Agricultural and Resource Economics, University of Maryland, College Park and Adjunct Professor, Faculty of Agriculture, University of Western Australia.

[2]Australian Research Council Senior Fellow, Faculty of Economics, Australian National University

Multiple-risk crop insurance schemes have been introduced in a number of countries, including the United States, but their performance has been uniformly disappointing (Gardner and Kramer). Plagued by problems of adverse selection, moral hazard, and high administrative costs, multiple-risk crop insurance schemes have normally required substantial subsidies. These problems have led economists to suggest insurance schemes in which payouts are based on exogenous and easily observable variables which are likely to be closely correlated with yields for producers participating in the schemes. The two most popular suggestions have been area-yield insurance schemes (Halcrow; Industries Assistance Commission; Miranda; Smith, Chouinard, and Baquet; Skees, Black, and Barnett; Miranda and Glauber; Mahul) and rainfall insurance (Bardsley, Abey and Davenport; Quiggin), but temperature and frost insurance have also received attention (Lee). In recent years, this interest has culminated in the United States introducing a pilot area-yield insurance program referred to as the Group Risk Program (GRP) (Skees, Black, and Barnett).

This paper examines optimal producer behavior in the presence of area-yield insurance. Beginning with Miranda, much analytic attention has been focussed on the choice of an optimal coverage level by the individual participant (Miranda; Wang et al.) and on the optimal design of an area-yield insurance contract (Miranda; Smith, Chouinard, and Baquet; Mahul). Virtually no attention, however, has been given to the nexus between the producer's insurance choice and his farm-level production decisions. Rather, the tendency has been to model individual yield as a stochastic variable not subject to producer control, but which is decomposable into systemic and idiosyncratic risk. This decomposition has analytic advantages. Most importantly, it reinforces the close connection between the insurance decision and tools of modern portfolio analysis, such as the capital-asset pricing model and the arbitrage pricing theorem. But it also ignores an important reality. The choices of farmers affect their yield, and those choices are conditioned by the presence or absence of area-yield insurance. Thus, a complete theory of area-yield insurance has as an essential component, a complete theory of producer reaction to area-yield insurance, both in terms of her choice of a coverage level and her production decisions. This paper attempts to initiate that study.

In what follows, we first introduce our model. It is a variant of the Arrow-Debreu state-contingent model as recently extended by Chambers and Quiggin (2000a). We, therefore, analyze producer behavior under very general assumptions about both the producer's preferences towards uncertainty as well his stochastic production technology. Special cases of the model that follows include models based upon expected-utility preferences and the stochastic production function, models based upon mean-variance preferences, as well as models based upon generalized expected utility preferences. After introducing the model, we characterize the producer's optimal production *cum* insurance choices. Then we turn to an analysis of the impact that the presence of area-yield insurance has on the producer's production choices and on his input utilization. We isolate, among other results, several sets of sufficient conditions for the introduction of area-yield insurance to lead the individual producer to adopt a riskier production pattern. Our next focus of attention is the connection between area-yield insurance and other tools of risk management, such as futures contracts, forward contracts, and yield-based futures. We state sufficient conditions for area-yield insurance to be redundant in the presence of such financial instruments. We also derive a 'separation' result, which shows, under appropriate assumptions, that area-yield insurance can cause a separation between production and insurance decisions in the sense that the production choice is completely independent of the producer's attitudes toward risk.

1 The Model

1.1 Technology and preferences

Uncertainty is modelled by 'Nature' making a choice from a set of S alternatives referred to as states of nature , $\Omega = \{1, 2, ..., S\}$. The single-product stochastic technology is represented by an input correspondence, $X : \Re^S_+ \to \Re^N_+$, which maps a state-contingent output vector, $\mathbf{z} \in \Re^S_+$, into sets of inputs, $\mathbf{x} \in \Re^N_+$, that are capable of producing that state-contingent output vector. It is defined (Chambers and Quiggin, 2000a):

$$X(\mathbf{z}) = \{\mathbf{x} \in \Re^N_+ : \mathbf{x} \text{ can produce } \mathbf{z}\}.$$

So, if state $s \in \Omega$ is realized (picked by 'Nature'), and the producer has chosen $\mathbf{x} \in X(\mathbf{z})$, then the realized or *ex post* output is z_s corresponding to the sth element of $\mathbf{z}$.[1]

State-contingent incomes are denoted by $\mathbf{y} \in \Re^S$. We consider a producer with a strictly increasing and strictly generalized Schur concave certainty-equivalent function over state-contingent incomes, $e(\mathbf{y})$, who is risk-averse for probabilities $\boldsymbol{\pi}$ (Quiggin and Chambers) and faces state-contingent output (spot) prices $\mathbf{p} \in \Re^S_{++}$. Thus, the producer faces both price and production uncertainty. Typically, we shall presume that $e(\mathbf{y})$ is smoothly differentiable in all arguments, but at certain points it will prove advantageous to relax that assumption. Examples of certainty-equivalent functions that provide useful intuition for what follows are expected utility

$$e(\mathbf{y}) = u^{-1}\left(\sum_s \pi_s u(y_s)\right)$$

where u is a concave von Neumann-Morgenstern utility function and π_s is the probability of state s occuring, and the mean-variance class of certainty equivalents

$$e(\mathbf{y}) = \phi(\sum_s \pi_s y_s, \sigma^2(\mathbf{y}))$$

where $\sigma^2(\mathbf{y})$ is the variance of $\mathbf{y}$ given the probabilities $\boldsymbol{\pi}$ and ϕ is increasing in its first argument and decreasing in its second.

Because preferences are risk-averse, the producer will be willing to trade increases in mean income off against reductions in risk. Conversely, the producer's revealed preference may be used to rank the riskiness of income vectors $\mathbf{y}, \mathbf{y}'$ with the same mean More precisely, for given preferences e and vectors $\mathbf{y}, \mathbf{y}'$, such that $\sum_s \pi_s y_s = \sum_s \pi_s y'_s$ and $e(\mathbf{y}) \geq e(\mathbf{y}')$ we say that $\mathbf{y} \preceq_e \mathbf{y}'$ (stated as '$\mathbf{y}$ is regarded as less risky than $\mathbf{y}'$ under e). The risk ordering induced by the certainty equivalent is said to be *translation invariant* if for any $\delta \in \Re$

$$\mathbf{y} \preceq_e \mathbf{y}' \Leftrightarrow (\mathbf{y} + \delta\mathbf{1}) \preceq_e (\mathbf{y}' + \delta\mathbf{1})$$

where $\mathbf{1}$ is the vector with all entries equal to 1. Because we operate with a net returns model, which always involves a nonstochastic cost level, we restrict attention to risk orderings which are translation invariant.

Let $\mathbf{r} \in \Re_+^S$ denote the vector of corresponding state contingent revenues with typical element

$$r_s = p_s z_s.$$

Input prices, which are non-stochastic, are denoted by $\mathbf{w} \in \Re_{++}^S$.

1.2 Area yield insurance

Area yield insurance for the individual is modeled by a vector of state-contingent indemnities of the form

$$I_s = \max\left\{v - \frac{1}{N}\sum_{n=1}^{N} v_{sn}, 0\right\}$$

where N is the number of farmers in the risk pool, v_{sn} is yield by individual n in state of nature s, and v is the threshold level of yield which triggers actual payments. We consider two cases. In the first, v is not the subject of farmer choice. In the second, the trigger level depends upon the 'coverage level', denoted c, adopted by the farmer. We assume that the risk pool is large enough that each individual farmer treats $\frac{1}{N}\sum_{n=1}^{N} v_{sn}$ as exogenous. By enrolling a acres in the area-yield insurance program, the individual, therefore, effectively purchases a risky asset, which is equivalent to a put option, with state-contingent yields $a\mathbf{I}$. The enrolment price per acre enrolled is denoted by q. When the farmer's indemnity depends on c, the enrolment price is also taken to depend functionally on c.[2]

The farmer enrolling a acres in the area-yield insurance program, therefore, has net returns in state s equalling

$$y_s = r_s + (p_s I_s - q)\, a - C(\mathbf{w}, \mathbf{r}, \mathbf{p}),$$

where the revenue-cost function is defined

$$C(\mathbf{w}, \mathbf{r}, \mathbf{p}) = \min\left\{\mathbf{w} \cdot \mathbf{x} : \mathbf{x} \in X(\mathbf{z}), \sum_m p_{ms} z_{ms} \geq r_s, s \in \Omega\right\},$$

if there exists a feasible state-contingent output array capable of producing $\mathbf{r}$ and ∞ otherwise. $C(\mathbf{w}, \mathbf{r}, \mathbf{p})$ satisfies[3]

Properties of the Revenue-Cost Function (CR):

CR.1$C(\mathbf{w}, \mathbf{r}, \mathbf{p})$ is positively linearly homogeneous, non-decreasing, concave, and continuous in $\mathbf{w} \in \Re^N_{++}$.

CR.2 Shephard's Lemma.

CR.3 $C(\mathbf{w}, \mathbf{r}, \mathbf{p}) \geq \mathbf{0}$ with equality if and only if $\mathbf{r} = 0$.

CR.4 $\mathbf{r}' \geq \mathbf{r} \Rightarrow C(\mathbf{w}, \mathbf{r}', \mathbf{p}) \geq C(\mathbf{w}, \mathbf{r}, \mathbf{p})$.

CR.5 $\mathbf{p}' \geq \mathbf{p} \Rightarrow C(\mathbf{w}, \mathbf{r}, \mathbf{p}') \leq C(\mathbf{w}, \mathbf{r}, \mathbf{p})$.

CR.6 $C(\mathbf{w}, \mathbf{r}_{-s}, \theta r_{s,}, \mathbf{p}_{-s}, \theta \mathbf{p}_s) = C(\mathbf{w}, \mathbf{r}_{-s}, \theta r_{s,}, \mathbf{p}_{-s}, \theta \mathbf{p}_s), \quad \theta > 0$.

CR.7$C(\mathbf{w}, \mathbf{r}, \mathbf{p}) = C(\mathbf{w}, \mathbf{r}/k, \mathbf{p}/k), \quad k > 0$.

CR.8 $C(\mathbf{w}, \mathbf{r}, \mathbf{p})$ is convex in $\mathbf{r}$.

We also assume that $C(\mathbf{w}, \mathbf{r}, \mathbf{p})$ is smoothly differentiable in all state-contingent revenues and input prices.

Visually, therefore, one can interpret the area-yield insurance program as giving the farmer the opportunity of choosing between two risky assets and a safe asset in an attempt to optimally smooth his net returns distribution. As discussed by Chambers and Quiggin (2000a), the producer, in choosing to produce, creates a risky asset that itself is a linear combination of a risky asset with state-contingent returns equalling $\mathbf{r}$ and a safe asset with state-invariant return equalling $-C(\mathbf{w}, \mathbf{r}, \mathbf{p})$. We illustrate this constructed asset in Figure 1 by the vector $(r_1 - C(\mathbf{w}, \mathbf{r}, \mathbf{p}), r_2 - C(\mathbf{w}, \mathbf{r}, \mathbf{p}))$, which we have labelled as point A. As drawn this vector is nonnegative. It need not be. Under the presumption that $I_2 = 0$ and $I_1 > 0$, for fixed c the net returns for the risky asset associated with area-yield insurance program are illustrated by the ray through the point $(p_1 I_1 - q, -q)$ in Figure 1. By choosing the magnitude of a, the producer invests in a risky asset. His holding of that risky asset can be visualized as a point on the ray through A in the direction of $(p_1 I_1 - q, -q)$. Because we do not allow short sales of area-yield insurance, i.e., the farmer effectively cannot sell area-yield insurance, we restrict ourselves to move in the direction of the arrow emanating from point A. The presence of area-yield insurance, thus, allows a producer producing at A to expand his state-contingent consumption set from point A in the direction of the arrow as far as he or she would like.[4]

The area-yield insurance contract is said to be *commercially viable* if

$$\sum_{s\in\Omega} \pi_s I_s p_s \leq q,$$

and *arbitrage-free* if

$$\sum_{s\in\Omega} \pi_s I_s p_s = q.$$

Any commercially offered area-yield insurance contract must be commercially viable. In the absence of transactions costs, area-yield insurance must be arbitrage free in equilibrium if there is free entry and insurers are risk-neutral or if short-selling is permitted and risk-neutral speculators are present. Because area-yield insurance is usually seen as a commercial risk-management tool and not as an implicit means of income support, we restrict ourselves to the case where it is commercially viable. Moreover, because of the absence of adverse selection and moral hazard problems, it seems reasonable to assume that transactions costs will be small and therefore to consider the properties of arbitrage-free insurance contracts. The low level of transactions costs is often cited as one of the primary strengths of area-yield insurance contracts (Skees, Black, and Barnett).

A very similar analysis is applicable to rainfall insurance. Supposing that θ is some measure of rainfall, consider an insurance contract that offers a positive payment ρ whenever θ falls short of some drought level θ^*. Then, setting

$$I_s = max\{\rho(\theta^* - \theta), 0\}$$

and letting a be the number of units of insurance purchased, the analysis above applies, and may be extended to allow for variable trigger levels, more complex climate indexes and so on.

2 Fixed Trigger Levels

We first consider producer behaviour under the assumption that v and q are independent of the coverage level chosen.

2.1 Characterizing Producer Equilibrium

The farmer's problem in the presence of area-yield insurance with a fixed trigger level is to choose an enrolment level and a production level to

$$Max_{\mathbf{r},a}\left\{e\left(\mathbf{r}+\left(\mathbf{pI}-q\mathbf{1}\right)a-C\left(\mathbf{w},\mathbf{r},\mathbf{p}\right)\right)\right\}.$$

Letting subscripts on functions denote partial derivatives, her first-order conditions are given by

$$\begin{aligned} e_s\left(\mathbf{y}\right)-C_s\left(\mathbf{w},\mathbf{r},\mathbf{p}\right)\sum_{t\in\Omega}e_t\left(\mathbf{y}\right) &\leq 0, \quad r_s\geq 0, \quad s\in\Omega, \\ \sum_{s\in\Omega}e_s\left(\mathbf{y}\right)\left(p_sI_s-q\right) &\leq 0, \quad a\geq 0, \end{aligned} \tag{1}$$

in the notation of complementary slackness.

The first S conditions in (1) require that the producer's marginal rate of substitution between state-contingent incomes equal his marginal rate of transformation between state-contingent revenues. Dividing both sides of these S conditions by $\sum_{t\in\Omega}e_t\left(\mathbf{y}\right)$ yields

$$\frac{e_s\left(\mathbf{y}\right)}{\sum_{t\in\Omega}e_t\left(\mathbf{y}\right)}\leq C_s\left(\mathbf{w},\mathbf{r},\mathbf{p}\right), \quad s\in\Omega.$$

Obviously, $\frac{e_s(\mathbf{y})}{\sum_{t\in\Omega}e_t(\mathbf{y})}>0$, and $\sum_{s\in\Omega}\frac{e_s(\mathbf{y})}{\sum_{t\in\Omega}e_t(\mathbf{y})}=1$. Thus, these normalized marginal utilities of state-contingent income, as evaluated at the optimal solution to (1), are interpretable as the 'virtual' or 'shadow' probabilities that will lead a risk-neutral individual facing these subjective probabilities and the same technology as the risk-averse individual to make the same production choice as the risk averter (Chambers and Quiggin 2000a). In what follows, we shall denote these virtual probabilities by $\boldsymbol{\pi}^*\in\Re^S_+$.

The final condition in (1) can now be rewritten

$$\sum_{s\in\Omega}\pi^*_s p_s I_s\leq q, \quad a\geq 0.$$

Hence, the area-yield insurance contract must be commercially viable at the virtual probabilities. And for an interior solution, the area-yield insurance contract must be arbitrage free at the virtual probabilities. Perhaps, more intuitively, an interior solution requires that the enrolment price of the insurance contract be a subjectively discounted martingale of the state-contingent returns from the area-yield insurance contract.

Hence, the conditions necessary for a producer equilibrium are that a risk-neutral individual facing the virtual probabilities would choose the same production equilibrium as the risk-averter and have no strong disincentive to take a positive position in the insurance market. These are standard arbitrage results familiar from portfolio analysis. Notice, in particular that if the area-yield insurance contract is not commercially viable at the virtual probabilities, risk-neutral individuals with these probabilities would be willing to take arbitrarily large positions in the insurance contract. This is inconsistent with equilibrium.

An interior equilibrium for $\mathbf{r}$ and a in (1) is illustrated in Figure 2a presuming the same indemnity structure as in Figure 1. The producer produces at point A in that figure and then chooses the optimal enrolment level in the area-yield insurance program so that moving from A in the direction of the net-indemnity vector, as illustrated by the dotted line segment, brings him to point B where the slope of his indifference curve is the same as his isocost curve at A. As drawn, this movement moves the producer in the direction of the equal-income vector (the bisector) in Figure 2a. This happens, of course, because the high revenue state in Figure 2a corresponds to the zero indemnity state while the low revenue state there corresponds to the positive indemnity state. Hence, the producer's preference and technology lead him to choose a state-contingent revenue vector that positively covaries with the value of the risk pool's production. In terms of the capital-asset pricing model, the individual's β is positive.

Figure 2b illustrates the case of an interior solution for state-contingent revenues, but a corner solution in the area-yield insurance enrolment. It is drawn for a producer possessing the same technology as in Figure 2a but with different state-contingent preferences. This producer's preferences, heavily tilted towards state-1 income, lead him to choose state 1 as the high-revenue state and state-2 as the low-revenue state. Thus, he produces at point A in that figure where the marginal rate of substitution between state-contingent incomes is equalized to his marginal rate of transformation between state-contingent revenues. Now by the restriction that the producer cannot short the area-yield insurance market, the feasible insurance contracts for this producer move him the direction of the dotted arrow illustrated there which always leads to a lower welfare level. Hence, he sets the enrolment level to zero. It is an obvious consequence of this discussion that the producer always weakly gains from being allowed the flexibility to 'short' the insurance market.

Figures 2a and 2b illustrate another important fact that has not been emphasized in previous studies: The producer facing an area-yield insurance contract chooses whether his β is positive or negative. From these figures, it is clear that even if all producers face an identical stochastic technology, differences in preferences can lead them to choose revenue vectors that covary either negatively or positively with the value of the risk pool's production. A direct corollary is that the producer optimally chooses the amount of idiosyncratic risk that he will face.

What do the different preference structures in Figures 2a and 2b reflect, and what is the economic rationale underlying the different positions that these two producers take? Following Savage, Yaari, and Quiggin and Chambers, we first note that the farmer's subjective probabilities are given by

$$\pi_s = \frac{e_s(c\mathbf{1})}{\sum_t e_t(c\mathbf{1})} \quad s \in \Omega, c \in \Re.$$

(Performing the indicated differentiation upon the expected utility certainty equivalent provides an illustration.) Geometrically, these probabilities are depicted in Figures 2a and 2b by the slope of the individual's indifference curve at the point where it intersects the equal-incomes line (the bisector). Visually, therefore, it is apparent that the individual depicted in Figure 2b has a near zero subjective probability of state 2 occuring. Therefore, even though his production structure is the same as that in Figure 2a, he devotes more of his effort to preparing for state 1 than for state 2 and accordingly his revenue in state 2 is lower than in state 1.

Perhaps, a simple example illustrates. Suppose that the main source of production uncertainty is the level of moisture, and that natural moisture is much higher in state 2 than in state 1. Figure 2a might, therefore, be associated with an individual who remains fairly optimistic about the possibility of rainfall during the growing season. And, on this basis, he takes relatively few precautions in preparing for drought. Figure 2b, on the other hand, might be an individual who is excessively pessimistic about the possibility of rainfall, and who thus devotes the bulk of his effort to providing for irrigation and other opportunities to replace rainfall. If enough effort is devoted to this kind of activity, his output in the case where it rains will be adversely affected. The underlying point, however, is that ultimately whether a state is 'good' or a 'bad' state depends upon how the farmer prepares for it. As these examples illustrate, his preparation will depend critically upon his subjective evaluation of the world as embedded in his preferences. If his subjective evaluation differs enough from the other members of the risk pool, one might expect his β to be negative.

>From (1) and our definition of commercial viability, we obtain:

Proposition 1 *If the producer enrolls in a commercially viable area-yield insurance contract*

$$\sum_{s\in\Omega}\left(\pi_s-\pi_s^*\right)p_s\boldsymbol{I}_s\leq 0.$$

If the producer enrolls in an arbitrage-free area-yield insurance contract

$$\sum_{s\in\Omega}\left(\pi_s-\pi_s^*\right)p_s\boldsymbol{I}_s=0.$$

If the area-yield insurance contract is profitable to the insurer, Proposition 1 requires that the states with highly valued indemnities roughly match up with the states where the virtual probability is greater than the actual probability. States with low valued indemnities will tend to correspond to states where the virtual probability is less than the actual probability. When the contract is arbitrage-free, the divergences of the actual probabilities from the virtual probabilities must be orthogonal to the value of the indemnities.

Manipulating conditions (1) for an interior equilibrium yields.

Proposition 2 *Any interior solution to (1) satisfies*

$$\sum_{s\in\Omega}C_s\left(\mathbf{w},\mathbf{r},\mathbf{p}\right)=1,$$

$$\sum_{s\in\Omega}C_s\left(\mathbf{w},\mathbf{r},\mathbf{p}\right)p_s\boldsymbol{I}_s=q.$$

We conclude from the first condition in Proposition 2, which we refer to as the *arbitrage condition,* that the optimal state-contingent revenue vector must belong to what Chambers and Quiggin (2000a) have referred to as the *efficient frontier*, i.e., those state-contingent revenue vectors potentially capable of being profit maximizing for a risk-neutral individual. This also requires that the cost of risklessly increasing revenue by one unit also match the cost of doing so. If this condition failed to hold, then all individuals, be they risk-averse or not would have the incentive to risklessly increase state-contingent revenue. The second condition in Proposition 2, upon observing from (1) that in equilibrium $C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) = \pi_s^*$, just requires that for there to an exist an interior solution for the area-yield insurance coverage, the cost of an acre enrolled must equal its expected return as evaluated at the virtual probabilities.

Chambers and Quiggin (2000a, 2000b) show that a fundamental property of differentiable Schur-concave preferences is that

$$\left(\frac{W_s(\mathbf{y})}{\pi_s} - \frac{W_r(\mathbf{y})}{\pi_r}\right)(y_s - y_r) \leq 0,$$

for all s and r, with strict inequality if preferences are strictly generalized Schur-concave and $y_s \neq y_r$. Consequently, if preferences are differentiable and strictly generalized Schur concave, then for $\mathbf{y}$ stochastic

$$\sum_{s\in\Omega} W_s(\mathbf{y})\left(y_s - \sum_{s\in\Omega}\pi_s y_s\right) < 0. \tag{2}$$

Substituting componentwise allows us to rewrite this expression as

$$\sum_{s\in\Omega} W_s(\mathbf{y})\left(r_s - \sum_{t\in\Omega}\pi_t r_t + a\left(p_s I_s - \sum_{t\in\Omega}\pi_t p_t I_t\right)\right) < 0.$$

By complementary slackness and (1),

$$\sum_{s\in\Omega} W_s(\mathbf{y})\, r_s = \sum_{s\in\Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p})\, r_s \sum_{t\in\Omega} W_t(y),$$

and

$$aq\sum_{s\in\Omega} W_s(\mathbf{y}) = \sum_{s\in\Omega} W_s(\mathbf{y})\, p_s I_s.$$

Substituting these results into (2) and rewriting establishes

$$(3) \qquad \sum_{s\in\Omega}\pi_s r_s - \sum_{s\in\Omega} C_s(\mathbf{w},\mathbf{r},\mathbf{p})\, r_s + a\left(\sum_{s\in\Omega}\pi_s p_s I_s - q\right) > 0.$$

The left-hand side of expression (3) is the response of the producer's expected profit at the optimum to a radial expansion of the state-contingent revenue vector and the enrolment in the area-yield contract. By (3), it follows immediately that the producer must realize a gain in expected profit from radially expanding state-contingent revenues and enrolment. An immediate conclusion is:

Proposition 3 *If the area-yield insurance contract is commercially viable, the producer always realizes a gain in expected profit from radially expanding his optimal state-contingent revenue vector.*

To understand the economic intuition behind Proposition 3, notice that (3) implies that an individual with generalized Schur concave preferences always foregos some expected profit. Of course, this is the essence of risk aversion, the producer foregos expected profit in return for a reduction in the dispersion of his state-contingent returns. When the area-yield insurance contract is commercially viable, the individual can never make a positive expected profit from any positive level of enrolment. His reason for enrolling is not to raise his expected return but to mitigate the production and price risk that he faces through the use of the area-yield insurance contract. Hence, the requirement that a radial expansion in state-contingent revenues and enrolment leads to an increase in expected profit translates directly into a requirement that the producer forego some expected profit from production.[5]

Essentially, therefore, (3) requires that a producer with generalized Schur concave preferences operates on a smaller scale than a risk-neutral individual facing the same technology and a commercially viable area-yield insurance contract in the sense that the risk-averter can always profitably increase his scale of production and insurance enrolment, whereas a risk-neutral individual who is behaving optimally cannot. In particular, if the area-yield insurance contract is arbitrage free, then the scale of the production operation for the risk-averter must be smaller in this sense than that of the risk-neutral individual. Moreover, as Chambers and Quiggin (2000b) show, if one imposes constant absolute riskiness on the production technology, this finding can be made precise. It follows from their Result 3 that if the technology exhibits constant absolute riskiness (defined below), a risk-averse producer facing area-yield insurance will produce a lower expected output than a risk-neutral individual.

Substitute the second expression in Proposition 2, into (3) to obtain

$$\sum_{s\in\Omega}\left(\pi_s - C_s\left(\mathbf{w},\mathbf{r},\mathbf{p}\right)\right)\left(r_s + ap_sI_s\right) > 0.$$

Recognizing that $C_s\left(\mathbf{w},\mathbf{r},\mathbf{p}\right) = \pi_s^*$ while using Proposition 1 now yields

Proposition 4 *If the producer purchases a commercially viable area-yield insurance contract,*

$$\sum_{s\in\Omega}\left(\pi_s - C_s\left(\mathbf{w},\mathbf{r},\mathbf{p}\right)\right)r_s > 0.$$

Under risk neutrality, the producer would choose state-contingent revenues so that

$$\pi_s - C_s\left(\mathbf{w},\mathbf{r},\mathbf{p}\right) = 0, \quad s\in\Omega.$$

By Proposition 4, the divergences from risk-neutral behavior are positively correlated with state-contingent incomes. So in the states where state-contingent production revenue is relatively high, marginal cost of that state-contingent revenue will tend to be less than the subjective probability of that state occuring. Hence, at the margin, it would be desireable for a risk-neutral individual to expand that state-contingent revenue. Conversely, in the states where income is relatively low, at the margin it would be desireable for a risk-neutral individual to decrease state-contingent revenues. In essence, Proposition 4 implies that the producer compresses the dispersion of his state-contingent revenues relative to that of a risk-neutral individual even in the presence of area-yield insurance. Proposition 4, therefore, manifests the notion of *risk-averse efficiency* identified by Peleg and Yaari as a necessary condition for equilibrium for a risk-averter making choices over a convex set of state-contingent alternatives. Chambers and Quiggin (2000a) have confirmed this behavior for an individual with generalized Schur concave preferences in the absence of insurance markets, and Proposition 4 establishes that it continues to hold in the presence of commercially viable area-yield insurance. Figure 3 illustrates in the case of two states of nature. A risk-neutral individual produces where the *fair-odds line*, whose slope is given by the subjective probabilities of the producer, is tangent to his isocost curve at point A as illustrated. A strictly risk-averse individual produces instead at a point like C, where the fair-odds line cuts the isocost curve.

2.2 Individual producer responses to area-yield insurance

One of the primary advantages of a properly designed area-yield insurance contract is that it virtually eliminates the insurer's moral hazard. It does so because, given the assumption that the individual producer is so small that he cannot affect the risk pool's average yield, the individual's indemnity is independent of his or her production actions.[6] However, the essence of insurance (and moral hazard) is that provision of insurance alters behavior. Even apart from moral-hazard concerns, the way in which insurance provision alters producer behavior is of general interest. In recent years, for example, a lively debate has emerged on the impact that insurance provision has on input utilization with particular emphasis on chemical input use (Horowitz and Lichtenberg; Babcock and Hennessy; Smith and Goodwin).

Area-yield insurance, in fact, will alter risk-averse producer behavior. Generally, how it alters producer behavior is determined in a state-contingent framework by a complex interplay between the producer's state-contingent preferences and his state-contingent technology. As modelled here, both are more flexible than normally encountered in most studies of agricultural insurance. Therefore, to be more informative about the possibilities encountered, it is interesting to consider some special cases. In this section by examining some special preference structures and restrictions on the technology, we consider the roles that the producer's preferences and the technology play in determining his response to area-yield insurance.

To encompass the range of possible producer attitudes toward risk, we consider the polar cases of risk neutrality and complete aversion to risk. Risk-neutral preferences are trivial. If the insurance is commercially viable, then a risk-neutral individual is not directly affected by its provision. He will adopt the same production plan as in the absence of insurance and will only invest in insurance if it is arbitrage free. However, if it is arbitrage free, his expected return from investing in insurance is zero, so he has no strong incentive to invest.

Now consider a producer whose preferences over state-contingent incomes are given by

$$e(\mathbf{y}) = \min\{y_1, ..., y_S\}.$$

These preferences are generalized Schur concave and risk-averse for all possible probability vectors. These preferences are the most extreme version of risk aversion imaginable as they literally imply that the producer only cares about his income in the worst state of nature. Because these preferences cannot be made additively separable across states of nature, they cannot be represented by expected-utility preferences, but they are consistent with generalized expected utility models such as rank-dependent preferences.

To provide a point of comparison, we first examine how such an individual produces in the absence of area-yield insurance.[7] The producer's problem is now:

$$\begin{aligned} & \max_{\mathbf{r}} \{\min\{r_1 - C(\mathbf{w}, \mathbf{r}, \mathbf{p}), ..., r_S - C(\mathbf{w}, \mathbf{r}, \mathbf{p})\}\} \\ = & \max_{\mathbf{r}} \{\min\{r_1, ..., r_S\} - C(\mathbf{w}, \mathbf{r}, \mathbf{p})\}. \end{aligned}$$

The results one expects to emerge are transparent intuitively. The producer should produce where her indifference curve just 'sits' on one of her isocost curves. Because maximin preferences have indifference curves that are 'L-shaped' around the equal-revenue ray, we therefore expect her to locate at a point on the equal-revenue ray. In other words, the producer chooses a non-stochastic production pattern. In fact, we can conclude even more: The producer not only chooses a non-stochastic production pattern, but she chooses to produce where the efficient frontier, identified earlier, intersects the equal-revenue ray. Let $\mathbf{r}^*$ denote the producer's optimal state-contingent revenue vector. Now suppose, contrary to our assertion, that $\mathbf{r}^*$ does not lie on the equal-revenue vector, and consider perturbing any single element of $\mathbf{r}^*$, say r_s, by the small amount δr_s. The associated variation in the producer's objective function is

$$\left(\delta^{\min} - C_s(\mathbf{w}, \mathbf{r}, \mathbf{p})\right) \delta r_s$$

where $\delta^{\min} = 1$ if $r_s \in \min\{r_1, ..., r_S\}$ and 0 otherwise. So if $r_s \notin \min\{r_1, ..., r_S\}$, the variation in the producer's objective function is

$$-C_s(\mathbf{w}, \mathbf{r}, \mathbf{p})\delta r_s$$

which implies that the producer's welfare can be increased by decreasing this state-contingent revenue towards the equal-revenue vector. Hence, the optimal state-contingent revenue vector must involve no revenue uncertainty.

Because the optimal production pattern can involve no revenue uncertainty, the decisionmaker's problem then reduces to

$$\max_r \{r - C(\mathbf{w}, r\mathbf{1}^s, \mathbf{p})\}$$

with the associated first-order condition:

$$1 - \sum_{s \in \Omega} C_s(\mathbf{w}, r\mathbf{1}^S, \mathbf{p}), \quad r \geq 0$$

in the notation of complementary slackness. Therefore, if the producer chooses to produce, she will locate at the point where the equal-revenue ray, the bisector, intersects the efficient frontier.

Now consider how the same producer would choose to produce in the presence of area yield crop insurance. By an exactly parallel argument, it follows that the producer will choose state-contingent revenues from production so that

$$r_s + ap_sI_s = r_k + ap_kI_k, \quad s, k \in \Omega,$$

or

$$r_s - r_k = a(p_kI_k - p_sI_s) \quad s, k \in \Omega.$$

In other words, the producer arranges his state-contingent revenue vector to effectively create full insurance for the indemnity risk that the introduction of area-yield crop insurance brings. If he enrolls no acreage, then he acts exactly as in the absence of the area-yield contract. If the producer did not fully insure in this fashion, it would always be possible to lower at least one state-contingent revenue, thus realizing a cost saving, without altering the producer's evaluation of the state-contingent gross revenue vector.

We conclude, therefore, that once differences in mean returns are corrected for, the producer's choice of state-contingent revenues in the presence of area-yield insurance will be riskier than in its absence. The introduction of area-yield insurance, therefore, clearly increases the riskiness of the production patterns for the most risk-averse individuals.

Letting ρ denote the stable gross income from farming and insurance operations, then optimal state-contingent revenues must satisfy

$$r_s = \rho - ap_s I_s, \quad s \in \Omega.$$

Hence, his production cum area-yield crop insurance problem is

$$Max_{a,\rho} \left\{ \rho - aq - C\left(\mathbf{w}, \rho \mathbf{1}^S - a\mathbf{p}\mathbf{I}, \mathbf{p}\right) \right\}$$

The first-order conditions are

$$\begin{aligned} 1 - \sum_{s \in \Omega} C_s(\mathbf{w}, \rho \mathbf{1}^S - a\mathbf{p}\mathbf{I}, \mathbf{p}) &\leq 0, \quad \rho \geq 0, \\ \sum_{s \in \Omega} C_s(\mathbf{w}, \rho \mathbf{1}^S - a\mathbf{p}\mathbf{I}, \mathbf{p}) p_s I_s - q &\leq 0, \quad a \geq 0. \end{aligned}$$

The completely risk-averse producer chooses his sure gross income to locate himself on the efficient frontier. If he does otherwise, as we have already seen above, he foregoes an opportunity to increase profit for sure by increasing or decreasing each state-contingent revenue. He also chooses his insurance position so that the insurance premium just equals the expected value of the indemnities as evaluated at the virtual probabilities that are defined by his location on the efficient frontier. In other words, apart from equating his marginal rate of substitution (which is zero) for state-contingent incomes to his marginal rate of transformation of state-contingent revenues, he behaves exactly in accordance with Proposition 2. Finally, the producer only chooses to participate in the area-yield insurance program if he can increase his sure income by doing so. Summarizing, we have

Proposition 5 *If the producer's evaluation of state-contingent incomes is given by*

$$e(\mathbf{y}) = \min\{y_1, ..., y_S\},$$

the introduction of area-yield insurance leads the producer to exactly balance his indemnity risk with his revenue risk so that

$$r_s - r_k = a\left(p_k I_k - p_s I_s\right).$$

Optimal state-contingent revenues in the presence of area-yield insurance are never less risky than in its absence, and the farmer only participates in the area-yield insurance program if he can raise his net income with certainty.

Thus, the general response of a risk-averse producer to the introduction of area-yield insurance seemingly lies somewhere between no adjustment and adopting a state-contingent revenue vector that is perfectly inversely correlated with the value of the indemnity vector. In the latter case, a producer who has originally arranged his production pattern so that idiosyncratic risk is eliminated now arranges production patterns so that his idiosyncratic risk exactly reflects the systemic risk. Loosely speaking, therefore, one expects area-yield insurance to have virtually no impact on the riskiness of production patterns for individuals whose aversion to risk is relatively low, but will encourage individuals with more extreme aversion to risk to modify their production patterns so that their idiosyncratic risk more closely tracks systemic risk. This represents an optimal adjustment to the opportunities for individual income smoothing that area-yield insurance offers. Moreover, it is precisely the more risk-averse individuals who would have done their utmost to self insure and, thus, to eliminate idiosyncratic risk prior to the introduction of area-yield insurance. Hence, it seems natural to speculate that the net result of introducing area-yield insurance would be an increase in systemic risk that results from producers increasing their idiosyncratic risk in an attempt to take advantage of the income smoothing properties of the area-yield insurance contract. This happens in spite of the fact that area-yield insurance is specifically designed to have no effect on idiosyncratic risk.

To refine our analysis, we now consider a convenient specification of the technology that allows us to make inferences about the effect of area-yield insurance on optimal state-contingent revenues. Suppose that the technology exhibits constant absolute riskiness in the sense of Chambers and Quiggin (2000a, 2000b) so that

$$C(\mathbf{w}, \mathbf{r}, \mathbf{p}) = \hat{C}\left(\mathbf{w}, \mathbf{T}\left(\mathbf{r}, \mathbf{p}, \mathbf{w}\right), \mathbf{p}\right)$$

where

$$T\left(\mathbf{r} + \delta \mathbf{1}^{S}, \mathbf{p}, \mathbf{w}\right) = T\left(\mathbf{r}, \mathbf{p}, \mathbf{w}\right) + \delta, \quad \delta \in \Re,$$

$$T(\lambda\mathbf{r},\lambda\mathbf{p},\mathbf{w}) = \lambda T(\mathbf{r},\mathbf{p},\mathbf{w}), \quad T(\mathbf{r},\mathbf{p},\lambda\mathbf{w}) = T(\mathbf{r},\mathbf{p},\mathbf{w}) \quad \lambda > 0,$$

and $\hat{C}(\mathbf{w},\mathbf{T}(\mathbf{r},\mathbf{p},\mathbf{w}),\mathbf{p})$ is positively linearly homogeneous in input prices, homogeneous of degree zero in $T(\mathbf{r},\mathbf{p},\mathbf{w})$ and $\mathbf{p}$, non-decreasing and convex in $T(\mathbf{r},\mathbf{p},\mathbf{w})$, non-increasing in $\mathbf{p}$. $T(\mathbf{r},\mathbf{p},\mathbf{w})$ is non-decreasing and convex in $\mathbf{r}$.

The most important property of technologies exhibiting constant absolute riskiness is that the cost level corresponding to the efficient frontier is unique. Hence, in this special case, the efficient frontier corresponds exactly to an unique isocost contour. The easiest way to discern this property is to differentiate both sides of the expression

$$T\left(\mathbf{r}+\delta\mathbf{1}^S,\mathbf{p},\mathbf{w}\right) = T(\mathbf{r},\mathbf{p},\mathbf{w}) + \delta$$

with respect to δ and evaluate the resulting directional derivative at $\delta = 0$ to obtain

$$\sum_{s\in\Omega} T_s(\mathbf{r},\mathbf{p},\mathbf{w}) = 1.$$

Using this fact, the definition of constant absolute riskiness, and the arbitrage condition in Proposition 2 yields in producer equilibrium that

$$\hat{C}_T(\mathbf{w},\mathbf{T}(\mathbf{r},\mathbf{p},\mathbf{w}),\mathbf{p}) = 1.$$

Thus, assuming an interior solution, the arbitrage condition in Proposition 2 determines an unique level of T and thus of revenue cost. Because the arbitrage condition in Proposition 2 also applies in the absence of area-yield insurance (Chambers and Quiggin, 2000a),[8] we conclude:

Proposition 6 *If the technology exhibits constant absolute riskiness, the introduction of area-yield insurance does not affect the level of revenue cost incurred by a risk-averse producer.*

Accordingly, the only effect that the introduction of area-yield insurance has on the farmer's production activities is to alter the expected value and the riskiness of his state-contingent revenues. Thus, constant absolute riskiness offers a particularly tractable framework in which to study the effect of the provision of area-yield insurance on the producer's decisions. Denote the farmer's optimal state-contingent revenue in the absence of insurance by $\mathbf{r}^o$ and the farmer's optimal state-contingent revenue in the presence of insurance by $\mathbf{r}^i$. By Proposition 6,

$$C(\mathbf{w}, \mathbf{r}^o, \mathbf{p}) = C(\mathbf{w}, \mathbf{r}^i, \mathbf{p}).$$

Now suppose that $\sum_{s\in\Omega} \pi_s r_s^o \leq \sum_{s\in\Omega} \pi_s r_s^i$. It then follows by the properties of the revenue-cost function that

$$C(\mathbf{w}, \mathbf{r}^o, \mathbf{p}) \geq C\left(\mathbf{w}, \mathbf{r}^i + \left(\sum_{s\in\Omega} \pi_s r_s^o - \sum_{s\in\Omega} \pi_s r_s^i\right)\mathbf{1}, \mathbf{p}\right),$$

from which we conclude by revealed preference that

$$e(\mathbf{r}^o - C(\mathbf{w}, \mathbf{r}^o, \mathbf{p})\,\mathbf{1}) \geq e\left(\mathbf{r}^i + \left(\sum_{s\in\Omega} \pi_s r_s^o - \sum_{s\in\Omega} \pi_s r_s^i\right)\mathbf{1} - C(\mathbf{w}, \mathbf{r}^\mathbf{o}, \mathbf{p})\,\mathbf{1}\right).$$

This observation yields

Proposition 7 *If the producer's technology exhibits constant absolute riskiness and* $\sum_{s\in\Omega} \pi_s r_s^o \leq \sum_{s\in\Omega} \pi_s r_s^i$, *then*

$$\mathbf{r}^o \preceq_e \mathbf{r}^i + \left(\sum_{s\in\Omega} \pi_s r_s^o - \sum_{s\in\Omega} \pi_s r_s^i\right)\mathbf{1}.$$

Proposition 7 implies that, once mean effects are compensated for, the producer perceives the revenue mix produced in the presence of insurance as being more risky than the revenue mix he produces in the absence of area-yield insurance. Hence, as we observed in the case of maximin preferences, the introduction of area-yield insurance encourages the farmer to adopt a riskier production pattern if it also encourages him to expand the scale of his operation.

Turning to the effect of the provision of area-yield insurance on input utilization, we employ the method suggested by Chambers and Quiggin (2000b) of decomposing the adjustment in input usage associated with the introduction of area-yield insurance. This decomposition consists of a pure risk effect, which compares input utilization at the vectors with common means, $\mathbf{r}^o$ and $\mathbf{r}^i + \left(\sum_{s\in\Omega} \pi_s r_s^o - \sum_{s\in\Omega} \pi_s r_s^i\right)\mathbf{1}$, and an expansion effect corresponding to the translation movement from $\mathbf{r}^i + \left(\sum_{s\in\Omega} \pi_s r_s^o - \sum_{s\in\Omega} \pi_s r_s^i\right)\mathbf{1}$ to $\mathbf{r}^i$.[9]

Proposition 7, in conjunction with results reported in Chambers and Quiggin (2000b) now implies

Corollary 8 *If the producer's technology exhibits constant absolute riskiness and* $\sum_{s\in\Omega}\pi_s r_s^o \leq \sum_{s\in\Omega}\pi_s r_s^i$*, then the introduction of area-yield insurance has a pure risk effect on an input that is positive (negative) if the input is a risk complement (risk substitute). The expansion effect on the input is positive (negative) if the input is non-regressive (regressive) to expansions of the revenue vector in the direction of the equal-income vector.*

So, under the present assumptions, if area-yield insurance leads a producer to expand the scale of his operation, as measured by the size of expected revenue, it must also bring with it the introduction of a riskier mean-compensated revenue. This brings with it an expansion of the use of inputs that are risk complementary and non-regressive at the no-insurance equilibrium. Intuitively, one might think here in terms of inputs such as chemical fertilizers which are traditionally seen as risk complementary. If inputs are risk substituting, then the tendency of the area-yield insurance program to push the producer toward a riskier production pattern will lead to potentially offsetting effects on input usage. The pure risk effect tends to diminish the input's use, while presuming that the input is non-regressive, the expansion effect on revenues tends to expand its usage.

$>$From Proposition 7, it follows that a sufficient condition for the producer to increase the riskiness of his state-contingent revenues is that the introduction of area-yield insurance leads him to operate on a larger scale. Intuitively, it seems plausible that the provision of insurance by mitigating the adverse risk consequences from operating on the intensive margin would encourage the producer to operate on a larger scale. More generally, however, whether the producer expands expected revenue as a result of the introduction of area-yield insurance depends upon his preferences, his technology, and the opportunities for income smoothing that area-yield insurance provides. We saw earlier that differences in preferences could lead one producer to decline enrolment in area-yield insurance where another producer facing the same technology would seek positive enrolment.

If area-yield insurance is to be useful in smoothing incomes for producers with such technologies, we expect intuitively that producer's optimal revenue should be positively correlated with revenue from the area yield, and therefore negatively correlated with the indemnity vector. In that case, we could reasonably say that the indemnity vector *insures* the producer's optimal revenue. We need to make this idea more precise in two ways. First, we need to specify the class of potentially optimal revenues that may be positively correlated with area yield in this sense. It is natural to focus on the efficient frontier because by Proposition 2 these are the state-contingent revenues that potentially can be optimal under area-yield insurance. Second, we need to make the intuitive concept of correlation used above more precise. Because, as illustrated by Figures 2a and 2b, the effectiveness of insurance ultimately depends on the producer's perception of risk, our definition will depend on the producer's risk ordering.

More formally, we will say that a random variable $\boldsymbol{\varepsilon}$ with zero mean *insures* cost function C if for all $\mathbf{r}$ on the efficient frontier for C, $\mathbf{r}+\boldsymbol{\varepsilon} \preceq_e \mathbf{r}$. Thus, our notion of $\boldsymbol{\varepsilon}$ insuring the cost function simply requires that adding it to all elements of the efficient frontier reduces the riskiness of the point on the efficient frontier.

With this definition and a further restriction on the certainty equivalent, we can prove that the producer always responds to the introduction of area-yield insurance by increasing his mean revenue and adopting a riskier state-contingent revenue vector. We say that the certainty equivalent, e, is *risk additive* on the set X if for any $\mathbf{y}, \mathbf{y}'$ in X such that $\mathbf{y}+ \left(E\left[\mathbf{y}'\right] - E\left[\mathbf{y}\right]\right) 1 \preceq_e \mathbf{y}'$ and $e\left(\mathbf{y}'\right) \leq e\left(\mathbf{y}\right)$ and for any $\boldsymbol{\varepsilon}$ with $E\left[\boldsymbol{\varepsilon}\right] = 0$ such that for all $\mathbf{y}$ in X, $\mathbf{y} + \boldsymbol{\varepsilon} \preceq_e \mathbf{y}$, $e\left(\mathbf{y}' + \boldsymbol{\varepsilon}\right) \leq e\left(\mathbf{y} + \boldsymbol{\varepsilon}\right)$. For expected-utility preferences, risk additivity corresponds to Kimball's notion of standard risk aversion. We have (a proof is in an appendix):

Proposition 9 *Assume preferences are risk-additive. If the producer's technology exhibits constant absolute riskiness and the net indemnity vector, $a\left(\mathbf{pI}-q\mathbf{1}\right)$, is arbitrage free and insures C, then*

$$\begin{aligned} E[\mathbf{r}^i] &\geq E[\mathbf{r}^o] \\ \mathbf{r}^o &\preceq \mathbf{r}^i + \left(E[\mathbf{r}^o]-E[\mathbf{r}^i]\right)\mathbf{1} \end{aligned}$$

2.3 Area-yield insurance and other contingent claims

Enrolling in an area-yield insurance program is equivalent to the purchase of a risky asset. That risky asset will only be purchased if it can be combined effectively with the producer's other risky asset, his state-contingent revenue from agricultural production. In principle, therefore, the introduction of area-yield insurance is exactly analogous from the producer's perspective to the introduction of any other contingent claim. Presuming that all contingent claims are arbitrage free,[10] then for area-yield insurance to be more attractive to the producer than other contingent claims, area-yield insurance must enable the producer to reduce the amount of idiosyncratic risk he faces more than these other contingent claims.

This observation lies behind the manner in which previous studies have chosen to analyze area-yield insurance (Miranda, Mahul, Wang et al.). In these studies, the producer's yield is separated into two components, the systemic risk associated with the risk faced by the risk pool, and the idiosyncratic risk specific to the producer. As modelled in these studies, the producer has no direct control over the idiosyncratic risk that he faces. This contrasts strongly with the present approach where the producer optimally chooses the amount of idiosyncratic risk which he will face.[11] It is the essence of area-yield insurance that it allows the producer to diversify away his systemic risk (in this sense).

More generally, it is also obviously true that the producer's random returns can generally be related in a similar manner to the 'market portfolio' even in the absence of area-yield insurance. The systemic risk would then be that associated with the market portfolio and the idiosyncratic risk would be the residual risk faced by the producer not accounted for by the market portfolio. This, of course, is the basic idea behind the capital asset pricing model and Ross's arbitrage pricing theorem. The financial relevance and with it the practical importance of area-yield insurance, therefore, hinges upon its ability to reduce the idiosyncratic risk the farmer faces beyond that which the market portfolio currently permits. In the language of the finance literature, area-yield insurance will be useful to the producer only if its state-contingent returns are not in the span of existing contingent claims. If area-yield insurance is in the span of existing state-contingent claims, its structure can be duplicated by creating an appropriate portfolio of these existing claims.

Area-yield insurance would then be redundant from the producer's perspective.

At present, there appears to exist no firm empirical evidence that confirms whether area-yield insurance is in the span of existing contingent claims. Because there exist a number of contingent claims market directly relevant to agriculture, including yield-based futures contracts, there exist some theoretical reasons to suspect that further investigation of this issue is merited. We address this issue by way of an illustrative example which shows that there exist circumstances, special circumstances to be sure, in which area-yield insurance will be redundant because of the prior existence of perhaps the most common form of an agricultural contingent claims markets, forward markets.

We start by narrowing our focus to the case where $S = 2$. Then Proposition 2 implies producer equilibrium is characterized by

$$\begin{aligned} C_1(\mathbf{w}, \mathbf{r}, \mathbf{p}) + C_2(\mathbf{w}, \mathbf{r}, \mathbf{p}) &= 1, \\ C_1(\mathbf{w}, \mathbf{r}, \mathbf{p})\, p_1 I_1 + C_2(\mathbf{w}, \mathbf{r}, \mathbf{p})\, p_2 I_2 &= q. \end{aligned}$$

So long as there is indemnity risk, that is $p_1 I_1 \neq p_2 I_2$, these equations can be solved to obtain

$$\begin{aligned} C_1(\mathbf{w}, \mathbf{r}, \mathbf{p}) &= \frac{p_2 I_2 - q}{p_2 I_2 - p_1 I_1}, \\ C_2(\mathbf{w}, \mathbf{r}, \mathbf{p}) &= \frac{q - p_1 I_1}{p_2 I_2 - p_1 I_1}. \end{aligned} \tag{4}$$

Notice, in particular, that (4) implies that the producer's optimal marginal rate of transformation between state-contingent revenues only depends upon the structure of the area-yield insurance contract. Hence, regardless of the producer's risk preferences, he will choose the same production equilibrium as all other producers who use the same technology and who participate in the area-yield insurance contract. Therefore, we conclude

Proposition 10 *If $S = 2$, and there is indemnity risk, then any interior production equilibrium will be independent of the producer's attitudes toward risk and identical for all producers facing the same technology and the same area-yield insurance policy.*

Proposition 10 is an example of a separation result for state-contingent technologies of the type isolated by Chambers and Quiggin (1997, 2000a). The reason it emerges in this special case is that the producer's arbitrage condition in Proposition 2 and the area-yield insurance indemnity structure combine to span the possible space of state-contingent outcomes. Since there exists a spanning portfolio, it follows from basic results in portfolio analysis that all producers facing the same stochastic technology make the same choices (Hirshleifer and Riley; Milne).

Proposition 10 can be explained in the following way. When there are only two states of nature, the existence of an area-yield insurance contract allows the producer to trade off state-contingent incomes at the rate of $\frac{p_2 I_2 - q}{p_1 I_1 - q}$, which is illustrated in Figure 1 by slope of the ray through A in the direction of $(p_1 I_1 - q, -q)$.[12] The individual will make these trades, by varying the size of his enrolment, until he equalizes his marginal rate of subsitution between state-contingent incomes to this ratio. But recall from (1) that the producer also equates his marginal rate of substitution to his marginal rate of transformation. Hence, the arbitrage opportunity offered by the presence of area-yield insurance and the producer's optimal arbitrage across states of nature leads the producer to equate his marginal rate of substitution to something that is independent of his risk preferences. Visually, this can be depicted by considering Figure 2a. The producer locates his production point, which is tangent to the dotted line in Figure 2a, giving the area-yield insurance opportunities, and then trades along the area-yield insurance ray until he encounters a point of tangency with his indifference curve.

These results are very similar to separation results contained in the context of forward markets by Chambers and Quiggin (1997, 2000a). There, too, it was found that in the two-state case, the producer facing an active forward market would produce in a fashion that is independent of his attitudes toward risk. The similarity of these findings, of course, manifests a more general phenomenon. Area-yield insurance always reduces to offering the producer a risky asset which he may or may not be able to combine with his production behavior to reduce the riskiness of his overall portfolio.

To illustrate, consider the case where there exists a forward price of $\tilde{q}$ at which the producer is free to execute forward contracts. This forward contract can be for *any commodity.* It need not be for any of the commodities that the producer producers. In what follows, to conserve on notation, we take the forward contract to be for the same commodity that the producer is producing, but we emphasize that the results are perfectly general. His state-contingent return from one unit of the commodity sold forward is $(\tilde{q}-p_1, \tilde{q}-p_2)$. If

$$\frac{\tilde{q}-p_1}{\tilde{q}-p_2}=\frac{p_2 I_2-q}{p_1 I_1-q}, \tag{5}$$

the ray defining the vector of state-contingent returns from the forward contract coincides with the ray defining the vector of state-contingent returns from the area-yield insurance contract. Visually, this ray lies over the one illustrated in Figure 1. Hence, by varying the size of his hedge the producer can exactly replicate any level of enrolment in the area-yield insurance contract. Area-yield insurance would be redundant in this case. More generally, if there are *any* two pre-exisiting forward markets whose state-contingent returns are linearly independent (regardless of which commodities are involved), the producer can replicate any level of enrolment in the area-yield insurance contract by taking appropriate positions in the two forward markets. Or, if there exists a pre-existing forward market for any commodity and a yield-based futures contract, which are independent, area-yield insurance will also be redundant. Essentially, the existence of the alternative markets allows the producer to design his own area-yield insurance contract. This individualized area-yield insurance contract always at least weakly dominates any that is introduced in the span of the forward contracts.

Returning to the case where there exists only a single forward contract, one might argue that there is no reason for a forward contract and an area-yield insurance contract to satisfy (5), but even in this instance, there exists at least one important case in which they always will. Notice that if the forward market is unbiased in the sense that

$$\tilde{q}=\pi_1 p_1+\pi_2 p_2,$$

then the slope of the state-contingent return from the forward contract is equal to the slope of the fair-odds line. In that case, Chambers and Quiggin (1997, 2000a) have shown that the producer operating in the forward market produces exactly as would a risk neutral individual and completely stabilizes his state-contingent returns at the level of maximum expected revenue.

Suppose also that the area-yield insurance contract is arbitrage free, then manipulating

(4) yields (this also follows from Proposition 1)

$$C_1(\mathbf{w}, \mathbf{r}, \mathbf{p}) = \pi_1,$$
$$C_2(\mathbf{w}, \mathbf{r}, \mathbf{p}) = \pi_2,$$

which in combination with the generalized Schur concavity of the certainty equivalent and (1) gives

Proposition 11 *If $S = 2$, there is indemnity risk, and the area-yield insurance contract is arbitrage free, then producers produce as though they were risk neutral and completely stabilize their income at maximal expected revenue.*

Proposition 11, when combined with the results of Chambers and Quiggin (1997, 2000a), reveals that arbitrage-free, area-yield insurance is redundant in the presence of an unbiased forward market for *any commodity* in the two-state case. Following an exactly analogous reasoning, one can also show that area-yield insurance is redundant in the presence of an unbiased yield-based futures contract. As an aside, we also note that Proposition 11 can be combined with the results of Chambers and Quiggin (2000b) to ascertain that under the assumption of constant absolute riskiness, area-yield insurance leads the producer to produce a higher expected revenue and to adopt a more risky production pattern in this case. This result can then be coupled with other results in Chambers and Quiggin (2000b) to determine the effect of area-yield insurance upon input use.

These rather stark examples are derived specifically for the two state case, which, of course, is quite unrealistic. However, they were chosen precisely to illustrate two essential points. Area-yield insurance can be redundant if it lies in the span of pre-existing contingent claims. And even if area-yield insurance does not lie in the span of pre-existing contingent claims, its financial and empirical relevance depends on the extent to which it can combine with the existing span of contingent claims to further reduce the idiosyncratic risk that the producer faces. Realistically, one might be hard-pressed to believe that a county-based area-yield insurance contract would lie in the span of existing contingent claims. But the real issue is not whether existing contingent claims can be improved upon, but whether the extra reduction in idiosyncratic risk brings with it gains large enough to cover the cost of introducing the area-yield insurance program.

Before leaving this discussion, we also point out that one can combine the arguments made in this section with those made in Chambers and Quiggin (1997, 2000a) to elaborate a complete theory of producer behavior in the presence of area-yield insurance and multiple futures and forward markets.

3 Selecting the Coverage Level

The GRP, as actually implemented, allows the producer to elect the level of coverage for which he wishes to enroll. The premium paid adjusts in accordance with the level of coverage chosen.[13] In this section, we modify our analysis to account for the possibility that the producer effectively can alter the trigger level by varying a parameter c, which we refer to as the coverage level. For analytic simplicity, we assume that both v and q are smooth functions of the parameter c.[14]

In analyzing the choice of the optimal coverage level, several observations should be made. First, the first-order conditions presented in (1) continue to apply. Thus, all of the results that are predicated upon them continue to apply when they are appropriately re-interpreted in terms of the optimal coverage level. We leave that reinterpretation to the reader. Second, if the area-yield insurance contract is to be commercially viable or arbitrage-free, the manner in which the trigger level and the enrolment price can vary in response to the coverage level must ensure the continued commercial viability of the area-yield contract. And, finally, even given the assumption that v and q are smooth functions of c, the indemnity vector will generally not be smoothly differentiable in c. This latter point can be illustrated by recalling that the indemnity in state s is

$$I_s = \max \left\{ v - \frac{1}{N} \sum_{n=1}^{N} v_{sn}, 0 \right\}.$$

Let v' denote the derivative of the trigger level with respect to c, which we shall assume to be positive. The variation in I_s associated with a small positive change in c is v' if $I_s > 0$ or if $v = \frac{1}{N} \sum_{n=1}^{N} v_{sn}$ and zero otherwise. On the other hand the variation in I_s for a small negative change in c is v' if $I_s > 0$ and zero otherwise. Hence, the right-hand and left-hand derivatives may not agree. To keep the analysis as simple as possible, we shall put such subtleties to the side and write the variation in the indemnity associated with a change in c as $v'\delta_s$ where δ_s is the appropriate subdifferential of I_s.

Our first observation is that the insurance contract is arbitrage-free only if it is also arbitrage-free at the margin, i.e.,

$$q' = v' \sum_{s \in \Omega} \pi_s p_s \delta_s. \tag{6}$$

The producer's first-order condition for the optimal coverage level is

$$a \sum_{s \in \Omega} e_s(\mathbf{y}) (p_s v' \delta_s - q') \leq 0, \quad c \geq 0, \tag{7}$$

which can be re-written as

$$a \left(v' \sum_{s \in \Omega} \pi_s^* p_s \delta_s - q' \right) \leq 0, \quad c \geq 0.$$

This condition requires that the producer adopt the coverage level that makes the insurance contract arbitrage-free at the margin when evaluated in terms of the virtual probabilities. Notice, in particular, that this just requires that a risk-neutral individual facing these virtual probabilities would have no strong-incentive to alter his or her coverage level. Thus, this condition can be recognized as yet another arbitrage result that requires producers to systematically exhaust any intrastate opportunities for gain from altering the coverage level. Although we do not state it directly, an obvious observation to draw from these results, by analogy with Proposition 1, is that the divergences of the actual probabilities from the virtual probabilities must be orthogonal to the marginal values of the indemnities associated with changing the coverage.

It follows immediately from these arguments and Proposition 2 that

Proposition 12 *Any interior solution to (1) and (7) satisfies*

$$\sum_{s \in \Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) = 1,$$

$$\sum_{s \in \Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) p_s I_s = q,$$

$$\sum_{s \in \Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) p_s \delta_s = \frac{q'}{v'}.$$

Perhaps the most important implication of Proposition 12 is that area-yield insurance contracts with a variable trigger level increase the producer's ability to take actions which potentially span the state space. We saw in Proposition 10 that the presence of indemnity risk and the producer's optimal reaction to price and production uncertainty allow the producer to create a portfolio that spans 2 space. Thus, in that case, all individuals facing the same technology and the same contract structure would take the same production actions regardless of their risk preferences. Here a similar result would emerge for the $S = 3$ case if the matrix

$$\begin{matrix} 1 & 1 & 1 \\ p_1 I_1 & p_2 I_2 & p_3 I_3 \\ p_1 \delta_1 & p_2 \delta_2 & p_3 \delta_3 \end{matrix}$$

is nonsingular. Production patterns would be independent of individual's attitudes toward risk. The explanation lies in the recognition that giving the producer the chance to select his trigger level effectively creates yet another financial asset in which the producer can take a position. Because all individuals face the same financial asset, in equilibrium, their marginal evaluations of the asset must agree. If they face enough such markets to span the state space, the conditions for a production equilibrium guarantee that their virtual probabilities must also agree regardless of their attitudes toward risk.

4 Conclusion

This paper has studied optimal producer behavior in the presence of area-yield insurance. The producer's optimal allocations of state-contingent revenues and insurance coverage have been characterized. The effect of the provision of area-yield insurance on production patterns and input use has been studied. Sufficient conditions for the provision of area-yield insurance to increase the riskiness of the individual producer's decisions have been derived and discussed. These results have been used to characterize how the provision of insurance impinges upon the producer's allocation of inputs. Attention has also been directed to cases where area-yield insurance may be redundant because of the prior existence of other financial instruments closely related to agriculture. A separation result has been derived for area-yield insurance contracts and stochastic technologies.

5 Appendix: Proof of Proposition 9

Suppose that preferences are risk-additive and that the net indemnity from insurance $a\,(\mathbf{pI}-q\mathbf{1})$ is arbitrage-free and insures C. Consider the optimum revenue in the presence of insurance $\mathbf{r}^i$ and $\mathbf{r}^o$. Under constant absolute riskiness, both are on the efficient frontier by Proposition 6, which corresponds to a unique isocost contour for C. Call that cost level $\hat{C}$. Now suppose that

$$\mathbf{r}^i + \left(\mathbf{E}[\mathbf{r^o}]-E[\mathbf{r}^i]\right)\mathbf{1} + a\,(\mathbf{pI}-\mathbf{q}) \preceq_e \mathbf{r}^o + a\,(\mathbf{pI}-\mathbf{q})\,.$$

Then the fact that

$$e(\mathbf{r}^o + a\,(\mathbf{pI}-q\mathbf{1}) - \hat{C}\mathbf{1}) \leq e(\mathbf{r}^i + a\,(\mathbf{pI}-q\mathbf{1}) - \hat{C}\mathbf{1})$$

implies under risk additivity that

$$e(\mathbf{r}^o-\hat{C}\mathbf{1}) \leq e(\mathbf{r}^i-\hat{C}\mathbf{1}).$$

This contradicts the optimality of $\mathbf{r}^o$ in the absence of area-yield insurance. Hence, it must be true that

$$\mathbf{r}^o + a\,(\mathbf{pI}-\mathbf{q}) \preceq_e \mathbf{r}^i + a\,(\mathbf{pI}-\mathbf{q}) + \left(\mathbf{E}[\mathbf{r^o}]-E[\mathbf{r}^i]\right)\mathbf{1},$$

or equivalently

$$e\left(\mathbf{r}^o + a\,(\mathbf{pI}-\mathbf{q}) - \hat{C}\mathbf{1}\right) \geq e\left(\mathbf{r}^i + a\,(\mathbf{pI}-\mathbf{q}) + \left(\mathbf{E}[\mathbf{r^o}]-E[\mathbf{r}^i]\right)\mathbf{1}-\hat{C}\mathbf{1}\right)$$

Now suppose $\mathbf{E}[\mathbf{r^o}]-E[\mathbf{r}^i] > \mathbf{0}$. The monotonicity of the certainty equivalent then implies

$$e\left(\mathbf{r}^i + a\,(\mathbf{pI}-\mathbf{q}) + \left(\mathbf{E}[\mathbf{r^o}]-E[\mathbf{r}^i]\right)\mathbf{1}-\hat{C}\mathbf{1}\right) > e\left(\mathbf{r}^i + a\,(\mathbf{pI}-\mathbf{q}) - \hat{C}\mathbf{1}\right)$$

which leads to another contradiction. Hence,

$$E[\mathbf{r}^i] \geq E[\mathbf{r}^o].$$

Now apply Proposition 7.

Notes

[1]Because we ultimately operate in terms of state-contingent revenues, all results generalize to the case of multiple outputs.

[2]This indemnity specification may appear to differ from that postulated in Miranda and in Skees, Black, and Barnett because the indemnity does not depend upon the 'scale' of enrollment. Let the scale of enrollment be denoted by i . Then in our notation, their indemnity structures, which differ from one another, are both special cases of

$$\begin{aligned} I_s &= \max\left\{ i\left(v_T - \frac{1}{N}\sum_n v_{sn}\right), 0\right\} \\ &= i\max\left\{ v_T - \frac{1}{N}\sum_n v_{sn}, 0\right\}. \end{aligned}$$

The individual enrolling a^* acres in such a program receives the total indemnity equalling

$$a^* i\max\left\{ v_T - \frac{1}{N}\sum_n v_{sn}, 0\right\}.$$

This is equivalent to our model upon setting $a = a^* i$. In the GRP, both a^* and i are subject to constraint. To preserve as much analytic simplicity as possible, we treat a as unconstrained except that we do not allow the individual to short the area-yield market. Thus, we consider 'optimal coverage' in the sense defined by Miranda. Placing restrictions on enrollment levels would change the analysis mainly by requiring us to examine a variety of corner solutions.

[3]Chambers and Quiggin (2000, Chapter 4) contains a complete discussion of the restrictions on $X(\mathbf{z})$ that are required for the revenue-cost function to satisfy these properties. Basically, they are that the input correspondence be closed and convex and not admit the possibility of either a free lunch or fixed costs.

[4]Of course, as is traditional in the net-returns model, we are ignoring the possible presence of wealth constraints. Such constraints would obviously place a bound on the long trades that would be feasible.

[5]This result parallels a similar result isolated by Chambers and Quiggin (1997) in their analysis of producer hedging behavior with forward and futures contracts. It generalizes to the case of stochastic technologies and general risk averse preference structure Sandmo's demonstration that an expected-utility risk averter facing price uncertainty produces less than a risk-neutral individual.

[6]More generally, however, area-yield insurance programs don't completely eliminate moral hazard. Unless the risk pool's population is very large in an economic sense, by affecting the risk pool's average yield, the individual does affect his indemnity as well as that of all others. This opens the possibility for strategic interaction in a symmetric game that is more commonly known as a team problem.

[7]These results closely follow the discussion in Chambers and Quiggin (2000, Chapter 5).

[8]We also note that it applies for risk-neutral producers.

[9]Chambers and Quiggin (2000b) define their pure risk effect for radially-corrected, as opposed to translation-corrected, revenue vectors and their expansion effect for radial movements of the mean-adjusted revenue vectors.

[10]Otherwise, income effects could lead an individual to adopt the other contingent claims.

[11]Of course, the standard approach is taken to abstract from the production side of the problem. But this abstraction also comes at the cost of denying the producer's ability to adjust his production risk optimally to the systemic risk associated with area-yield insurance. In short, it abstracts away from the very aspect of the production insurance problem that makes area-yield insurance potentially unique.

[12]Recall that Figure 1 is drawn under the presumption that the indemnity in state 2 is zero.

[13]See Skees, Black, and Barnett for the actual details.

[14]In the GRP, the producer is free to choose between several discrete alternatives.