Speed of Aircraft

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Abstract

The velocity with which terrorists crashed the ill-fated planes onto the buildings on September 11, 2001 is an important parameter in any post-mortem analyses on the collapse of the buildings. As is well known, the kinetic energy carried by the planes changes with the square of the velocity, while their momentum grows in proportion to this velocity. Thus, an accurate determination of the speed is an essential datum in the estimation of the dynamic effects elicited by the collision and the initial damage to the structures.

Using various publicly available video recordings as described in this article, I have been able to obtain reasonably accurate estimates of the speed of flight of the planes that collided onto the Twin Towers. A summary of the results is as follows:

Target	Flight	Aircraft	Impact Time	**Velocity**	
				km/hr	**mph**
North Tower	AA-11	Boeing 767-200	8:46:20 AM	691	429
South Tower	UA-175	Boeing 767-200	9:02:48 AM	810	503
Pentagon	AA-77	Boeing 757-200	9:38 AM	555	345

The velocities listed in this table for the two WTC planes are in excellent agreement with flight data based on radar provided by the NTSC[1]. The radar speeds are basically 10% larger, a difference that could easily be explained by the higher altitude at which the aircraft may have remained visible to radar and the probable speedup caused by the descent. Indeed, during their final approach, the airplanes —whose transponders had been disabled— were flying as low as some 300m (1000 ft) above the ground (i.e. the height of impact), an altitude that is barely above the rooftops of the skyscrapers in lower Manhattan, so radar is likely to have been blind to them. By contrast, the estimates given herein are based on the last mile of flight prior to collision.

[1] E. Lipton and James Glanz, “First Tower to Fall Was Hit at Higher Speed, Study Finds”, *The New York Times*, February 23, 2002,

On the other hand, the velocity given for the plane that plunged into the Pentagon comes from information contained in the recovered flight data recorder[2]. The flight numbers and aircraft type listed are from a report by the *Washington Post* in the days following the attack. Finally, the impact times of the planes that crashed onto the WTC are from seismic records obtained at the Palisades N.Y. seismic station, Lamont-Doherty Earth Observatory, Columbia University[3]. Since the station is 34 km away from the WTC, in the table above I have subtracted 6 seconds from the reported times to account for the estimated travel time of the seismic waves from the WTC to Palisades.

The above data indicates that the terrorists flew towards the WTC close to the ground at nearly the full cruising speed of the planes, which is about 900 km/h (560 mph) at a normal altitude of 10km (33,000 ft). It is surprising that the inexperienced pilots that the terrorists were could still steer the planes at those speeds and hit their target head on. Also, consideering that the air at low altitudes is much denser than that at the normal cruising height, the pilots greatly exceeded VNE ("never exceed velocity") and thereby risked disintegration of the aircraft by air friction.

Pitfalls in determining the speed from videos

The velocity of the two Boeing 767-200 planes that were crashed onto the Twin Towers is not precisely known, especially the speed of the North Tower plane. The speed calculations are made more complicated by the following facts:

- The original format in which the videos were recorded is not only unknown to me, but they were also converted back and forth (once or twice) between the American NTSC format and the British PAL system. These two video standards differ in various aspects, which include the number of frames displayed each second and the screen resolution. In the NTSC system, there are 30 frames per second, while in the PAL system the number is 25. This affects the time estimation obtained by counting frames in slow motion. The hardware available uses various competing ways of converting from one to the other format, the more sophisticated and expensive of which is based on image interpolations in both space and time. Most conversions, however, are done by simply moving (or deleting) scanning lines and frames in one system to the closest position in space and time in the other, or by taking averages. These introduce artifacts and confounding ghosts in the video, particularly with moving objects and/or panning cameras. An excellent description of troubles with video conversions can be found at a web site in the U.K.[4].
- Some of the videos include running time counters or indices. In principle, these can also be used to determine elapsed times by subtraction of the indices. Care is required, however, because it is unknown if these counters were added in transcription, or were already contained in the initial recordings. Also, the fractions of second run from 0:24 or 0:29, depending on whether the index format was added in PAL or NTSC.
- Many of the videos have clearly been slowed down by a factor of perhaps two or three, in order to show in more impressive detail the incoming planes immediately before collision. Thus, I had to pay careful attention to detect slow motions and discard these videos (for example, speed of flames and smoke, etc.). I could not compensate for the slow motion

[2] "September Eleventh: The days After, The Days Ahead", *Civil Engineering*, ASCE, Vol. 71, No. 11 (November), page 48, 3rd paragraph, 1st line

[3] Won-Young et al, *EOS, Transactions, American Geophysical Union*, Vol. 82, No. 47, Nov. 20, 2001

[4] http://www.ee.surrey.ac.uk/Contrib/WorldTV/

effect, because the slowdown factors were not readily available to me or determinable from the videos alone.

- The filming position was generally not known to me, a situation that introduced an unknown degree of geometric perspective or parallax effect. However, in most cases these recording positions appeared to have been sufficiently distant from the target that the parallax effect could safely be disregarded.
- In many videos, the camera either panned or zoomed into the target (or both), a situation that greatly complicates the determination of flight distances.

The details of these estimations are detailed in the sections that follow.

Velocity of North Tower plane

A dramatic video taken by French filmmaker Jules Naudet[5] from a distance of about one kilometer to the World Trade Center shows the crash of the first Boeing 767-200 against the North Tower, and appears to be the sole graphic documentation available of this grisly event. The initial footage of this video depicts fireman Chief Joe Pfeifer at the intersection of Lispenard and Church Streets checking out a gas leak below the northeast corner of that intersection. The initial scenes are shot along Lispenard, in an East to West direction. As a jet plane is heard, Chief Pfeifer turns up his head to the sky in reaction to the engine noise just as the plane races by overhead, but the plane can't yet be seen in the video. The camera then pannes immediately into a north to south direction as well as upwards, past and up the ATT building on Church Street between Lispenard and Walker Streets, and shows the plane in its last fractions of a second racing towards the tower and hitting it with devastating effect, at which time the camera zooms into the ensuing fireball.

Fig. 1: Two scenes from J. Naudet's video. Drawings by Cecilia Lewis Kausel

In this video, the plane can be seen only in its last second or so before impact. In the sketch above on the right, the arrow that follows the dotted line, which in turn shows the estimated flight path, indicates this. Despite the scant evidence contained in the seven or so seconds in this sequence, this video still provides enough useful information that permits estimating the speed of flight with reasonable accuracy. This is done as follows.

[5] Alan Feuer, "Ground Zero: The Images", *The New York Times*, January 12, 2002, Late Edition, Section A, Page 1

The noise of the jet engines —a whining sound whose pitch decreases steadily because of the Doppler effect— can be heard briefly during the time it rises above the rather high background noise in the video. The sound becomes discernible as Chief Pfeifer faces the camera and a pedestrian crossing the street just disappears behind his left elbow, an instant that we can designate as time t=0. At this moment, he starts turning his body counterclockwise and looking up. The sound then vanishes below the street noise some three seconds later just as he touches his helmet and begins to lower his head.

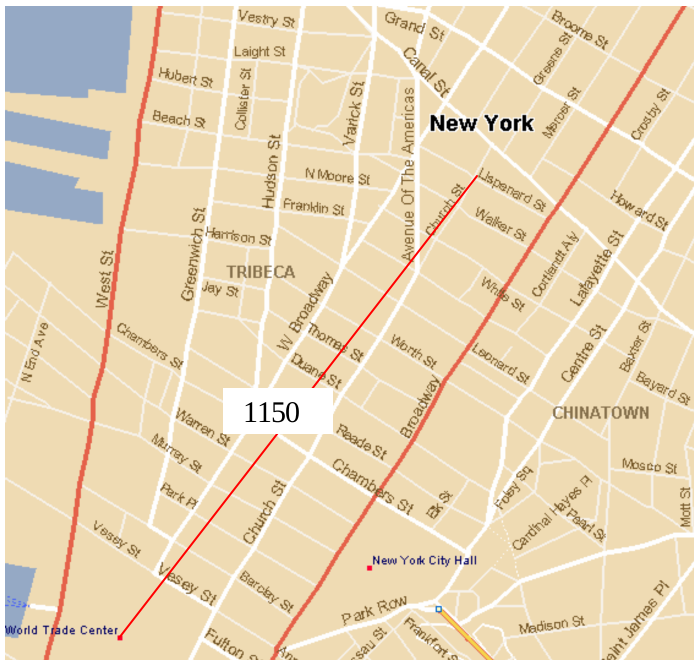

Fig. 2: Map of Lower Manhattan showing location of filming position

Now, the engine noise should be audible both before and after the passage of the plane, and in all likelihood for an equal duration before and after that fact. Thus, it is reasonable to assume that the plane flies by overhead at the center of the noise interval, that is, at time t=1.5 seconds. However, this sound must have been delayed by its travel time from the plane to the ground. Because of the direction in which the fireman looked up to the sky as well as the orientation of the towers, the likely trajectory must have been close to the arrow from the camera to the WTC on the map shown in Fig. 2, and not much further west. It is also known that the plane flew at an altitude of between 300 and 400 meters, because that is the height at which it collided with the North Tower, so that must have been the approximate distance to the ground. Considering that sound travels in air at some 340 m/s, it follows that the engine noise must have been delayed by about one second, so the plane actually flew by overhead somewhat earlier, namely at time t=1.5-1.0=0.5 s. The plane then plunged into the North Tower 194 frames after time zero, which corresponds to t=194/30 = 6.5 s. Thus, the estimated flight time from Lispenard to the WTC is T=6.5-0.50 = 6.0 s, give or take half a second or so.

On the other hand, using the *MS Streets-98* program, I determined the distance d from the video camera to the North Tower to be d=1150 m, to an accuracy of perhaps 40 m, and confirmed this distance by timing with a stopwatch the delay of the explosion boom, which is 3.4 seconds or 1156m. Hence, the estimated flight velocity is

$$v = (1150 \pm 40)/(6 \pm 0.5) = 192 \times (1 \pm 40/1150 \pm 0.5/6) = 192 \times (1 \pm 0.12) \text{ m/s}$$

that is, the speed of the North Tower plane is on the order of v=192 m/s = 691 km/hr = 429 mph, with a likely accuracy of 12%.

Velocity of South Tower plane

The speed of the plane that crashed onto the South Tower can be determined with greater confidence than that of the North Tower. This is because there are several videos taken from different angles available which show the last few seconds prior to the collision. In the pages that follow, I estimate this velocity using the following data:

- Video showing collision from a northerly view
- CNN Video showing collision from an easterly view
- Video showing collision from an easterly view
- Angle of flight inferred from the previous three videos
- Speed of plane inferred from Brooklyn Bridge video (best evidence!)

Velocity and trajectory of aircraft inferred from northerly view video

Consider the sketch of the video image together with its matching plan view shown in Fig. 3a (left side), and assume tentatively that the camera is infinitely far away so that all lines of sight are parallel to each other, i.e. neglect parallax. The angle of view can then be determined from the apparent widths a, b of the North Tower in the still images obtained from the video by relating these to the building's known width L=64m:

$$a = L\cos j\ , \qquad b = L\sin j \qquad \tan j = b/a$$

Also, let b be the angle between the plane's flight direction and the normal to the south face of the South Tower. The distance d traveled by the plane when its nose just emerges from the right edge of the image (i.e. screen, which is indicated by the vertical line) and t seconds later touches the right edge of the (visible) North Tower is

$$d = \frac{c}{\sin(j + b)} = \frac{c}{a}\,\frac{\cos j}{\sin(j + b)}\,L$$

from which the plane's speed $v = d/t$ can be determined. Now, the measured distances on the image are a=76 mm, b=45 mm, and c=205 mm, which would give for the viewing angle

$$j = \arctan\frac{b}{a} = \arctan\frac{45}{76} = 30.63^\circ$$

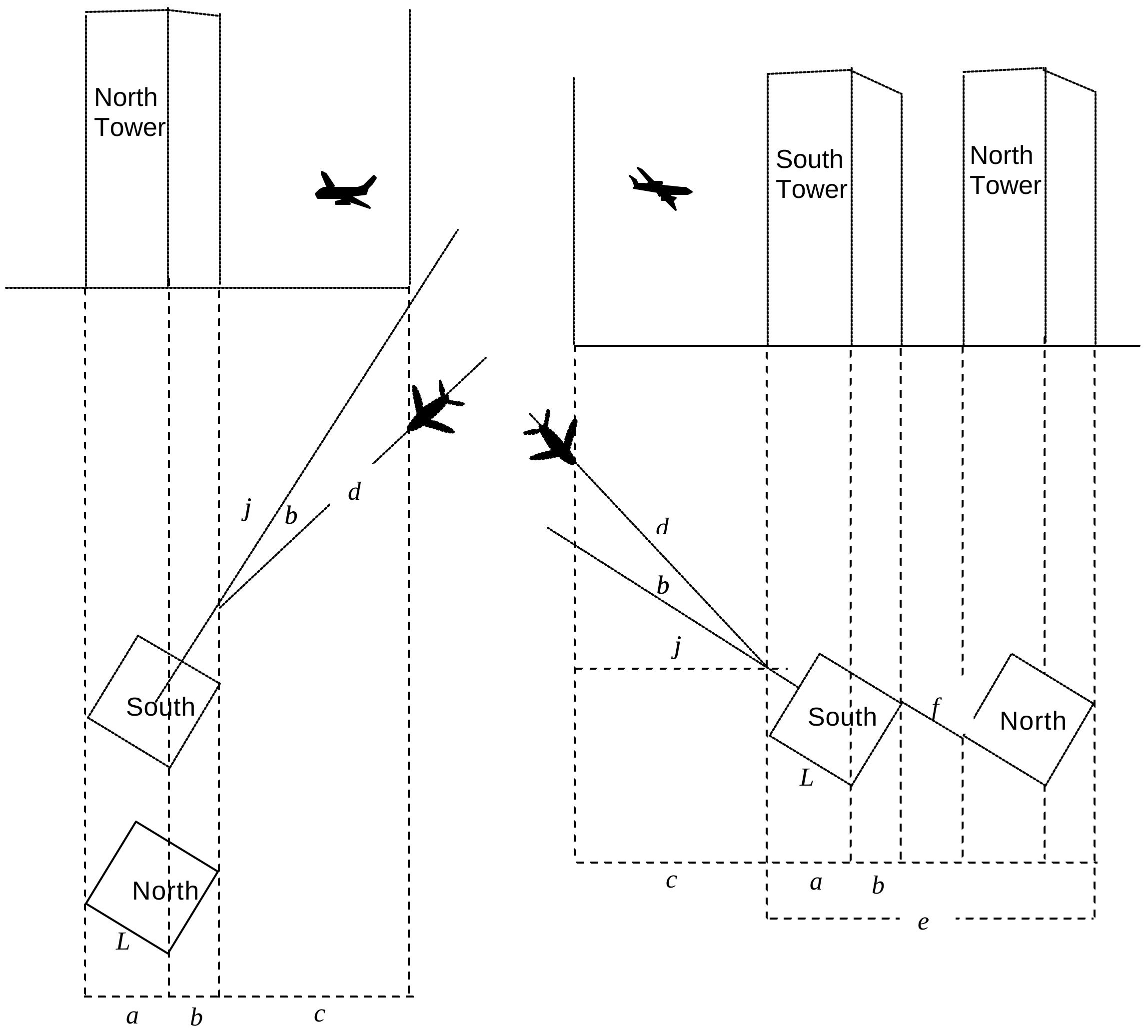

Fig. 3: Diagrams for northerly view (left) and easterly view (right) videos. (Unknown broadcaster).

Considering that the orientation of the WTC is some 27 degrees east of north, the above angle is thus only some four degrees west of north, so the camera's filming direction was nearly directly from north to south. The angle b can be found by combining the previous information with data from other images taken from an East-West direction. As will be seen, this angle is on the order of 15 degrees. The above values imply

$$d = \frac{205}{76}\,\frac{\cos 30.63}{\sin(30.63+15)}64 = 208\text{ m}$$

On the other hand, the time elapsed between the appearance of the plane on the right edge of the screen until its nose crosses the line of sight to the right of the North Tower is t = 1 sec. This time interval follows both from the time counter in the video (2:57:23 to 2:58:22), and by counting the number of frames in the video, which was shot at 30 frames per second.

While the plane traverses this path, the camera gradually zooms in and pans slightly to the left, but this motion has no effect on the measured time. Thus, the plane's flight speed is on the order of 208 m/s. The actual value may perhaps be somewhat larger on account of the fact that we have neglected the parallax.

While the camera position in the still image used here is unknown, the line of sight of 4 degrees west of north would place it somewhere on Chambers Street or the Hudson River waterfront North of there. If so, the camera distance may range anywhere from 600 m to perhaps 1 km.

Velocity and trajectory of aircraft inferred from easterly view video

Consider next the still image and matching diagram shown in Fig. 3b on the right. Neglecting the parallax as in the previous section, the angle of view is

$$j = \arctan b / a$$

with a=60 mm and b=20 mm on the image. Hence, j =18 degrees. Since the towers are aligned at 27 degrees east of north, i.e. the perpendicular is 27 degrees south of east, this implies that the eastern view is at 9 degrees south of east (=27-18).

Again, let b be the angle between the plane's flight direction and the perpendicular to the south face of the South Tower. The distance d traveled by the plane when its nose just emerges from the left edge of the image (or screen) and t seconds later seems to touch the left edge of the South Tower is then

$$d = \frac{c}{\cos(j+b)} = \frac{c}{a}\,\frac{\cos j}{\cos(j+b)}L$$

Taking b=15 degrees and c=200 mm on the image, we obtain

$$d = \frac{200}{60}\,\frac{\cos(18.43)}{\cos(18.43+15)}64 = 242\,\mathrm{m}$$

While the plane covers the distance d=242 m from the edge of the screen to the edge of the South Tower, the time counter on the video changes from 15:07:07 to 15:08:07, which gives t=1 sec. Hence, the implied apparent flying speed is 242 m/s.

The camera position in the video image referred to previously above is unknown. The line of sight of 9 degrees south of east would place the camera somewhere in the vicinity of the Manhattan approach to the Brooklyn Bridge.

Velocity and trajectory of aircraft inferred from an easterly view CNN video

Consider now the still image and diagram in Fig. 4. The viewing angle is once more obtained as j = arctan b/a, with a=80 mm and b=30 mm on the screen image. Also, the actual length of the 767-200 seen in the image is 48.4m, while the building's width is L=64m. Thus, neglecting parallax, the viewing angle is j =21 degrees. Since the perpendicular to the towers' line of alignment is 27 degrees south of east, this implies an easterly view of the twin towers of 27+21=48 degrees south of east, which would place the camera roughly in the vicinity of Wall Street. Also, if (as will be shown) the aircraft travels at b=15 degrees from the alignment direction, then the aircraft in this video travels at 21-15=6 degrees from the image's plane (angle below horizontal in figure below).

The distances and lines shown on the sketch of the still image were measured on a flat screen while freezing the video. The left edge corresponds to the aircraft nose's position at 30 frames (i.e. 1 sec) before crossing the leftmost edge of the South Tower. The time counter at these two positions is 16:01:15 and 16:02:14. Hence, the apparent speed is

$$v = (140+175)\times 64\times \cos(21°) / 80\times \cos(9°) = 238 \text{ m/s}$$

which is consistent with the previously found values.

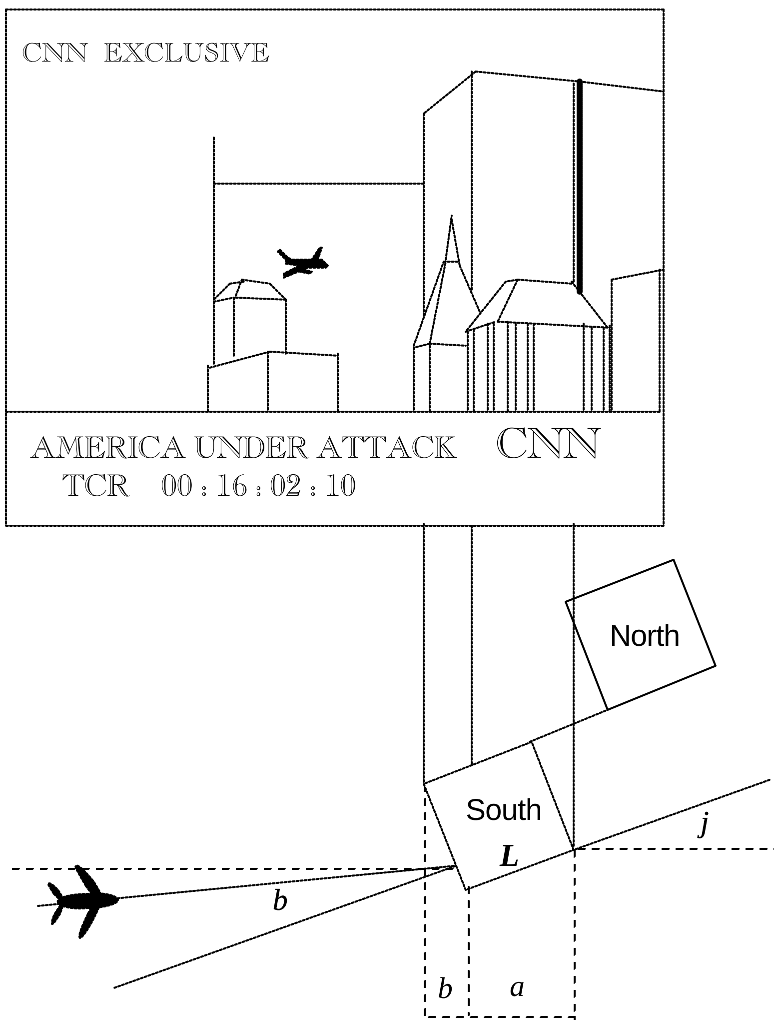

Fig. 4: Still image and diagram for easterly view CNN

Angle (azimuth) of flight

From the previous sections, the NS line of sight was 4 degrees west of north and the EW line of sight was 9 degrees south of east. These directions are indicated by the dashed arrows in the WTC neighborhood map shown in Fig. 5. Drawing parallels to the NS and EW lines of sight at the locations that match the right and left edges of the still images, respectively, which were both crossed by the aircraft at about 1 second prior to collision, we can estimate from their intersection the true location of the plane relative to the towers at this point in time. Drawing from this point the flight path to the South Tower, we obtain an angle of flight of about 15 degrees with respect to the alignment line of the two towers, which is 27 degrees east of north. Thus, this justifies the angle *b*=15 degrees we applied in the previous sections to estimate the flight velocity.

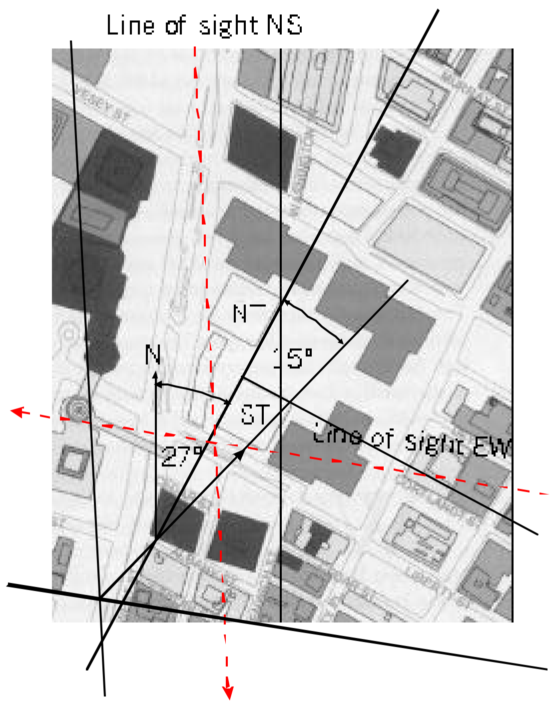

Fig. 5: Map of WTC neighborhood showing lines of sight. for northerly and easterly views

Speed of plane, as inferred from Brooklyn Bridge video

A very informative video showing the approach of the second plane to the South Tower was filmed from a position slightly to the North of the easternmost pier of the Brooklyn Bridge, almost immediately underneath the bridge. This places the filming position at about 1830 m from the World Trade Center, as determined by means of the *MS Streets-98* program. Fortuitously, the line of sight from this position to the World Trade Center is virtually perpendicular to the alignment line connecting the twin towers in the NE-SW direction (black and cyan lines shown in map below). This video, which was taken at a rate of 25 frames per second without zooming or panning, provides probably the best evidence available for determining the trajectory and speed of the plane.

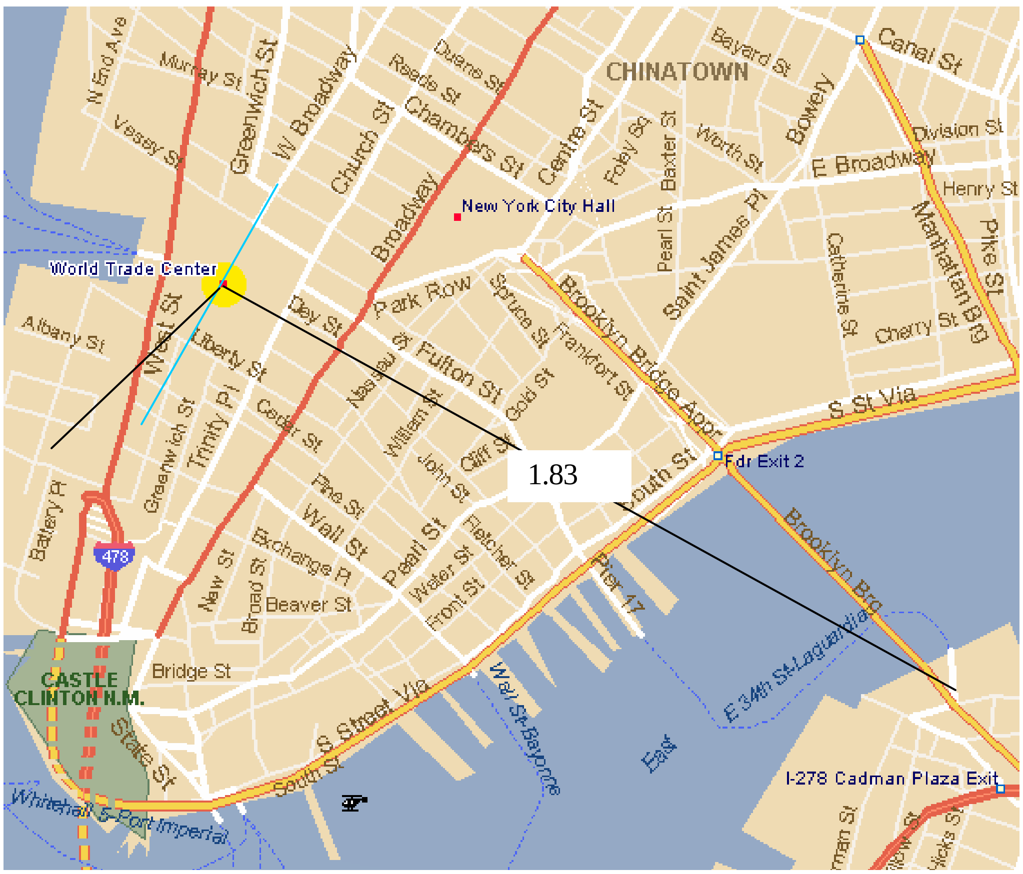

Fig. 6: Map of WTC neighborhood showing lines of sight for Brooklyn Bridge video

a) Apparent position of plane

A sequence of seven still images depicting the last four seconds of the plane's seemingly level flight toward the South Tower was used to track its position. Fig. 7 on the next page shows a sketch of the first of these images. The stills provide only the apparent position and distance of the plane to its collision point, because the plane is not traveling fully aligned with the twin towers, but at an angle of about 15 degrees further west of this direction (arrow in map).

Hence, the plane's apparent position must be corrected for parallax, which in this case can be carried out inasmuch as the filming position is known. The distance *d* between the apparent position of the plane and the South Tower can be obtained by measuring on the image the apparent position of the plane, comparing it against the known dimensions of the towers, and scaling this distance accordingly. The distance between the south face of the South Tower and the north face of the North Tower is 164m (my estimation), a reference distance that should be measured on the image at the height of flight, to compensate for the slight upwards perspective of the camera (arrows shown in the sketch below). The width itself need not be corrected for horizontal angle, because the view from the Brooklyn Bridge is virtually head on, and the difference in distance (depth) between the viewing point and the two towers (64m) is negligible compared to the camera distance (1830m). The results are as follows

Image distance [mm]	Time counter [sec]	Apparent position *d* [m]	Time to impact *t* [s]	Apparent velocity *v*=*d*/*t* [m/s]
64	15:36:18.80	750	4.12	182
57	15:36:19.40	668	3.52	189
50	15:36:19.96	586	2.96	197
41	15:36:20.60	480	2.32	206
21	15:36:21.80	246	1.12	219
12	15:36:22.25	141	0.67	210
0	15:36:22.92	0	0.00	-

Note: In the table above, we have converted the 0:24 frame index of the videos into decimal fractions of sec.

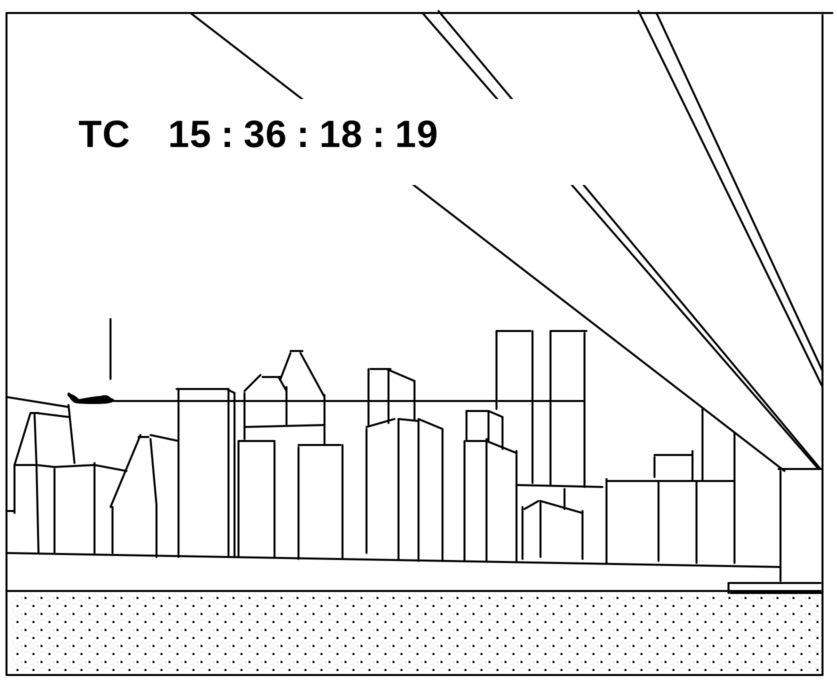

Fig. 7 a-g: Final approach, as seen from Brooklyn Bridge *b) Actual position of plane*

After measuring in the image the position *d* of the plane with respect to the South Tower, and considering the angle of flight *b* with respect to the apparent flight direction —which in the image is perpendicular to the Brooklyn Bridge line of sight— we can determine the actual position of the plane in terms of *b*, and thus the actual speed of flight. From the other videos of the WTC taken from a northern and eastern filming position, we know that the angle *b* is about 15 degrees. Thus, we can use this fact to determine the speed of flight.

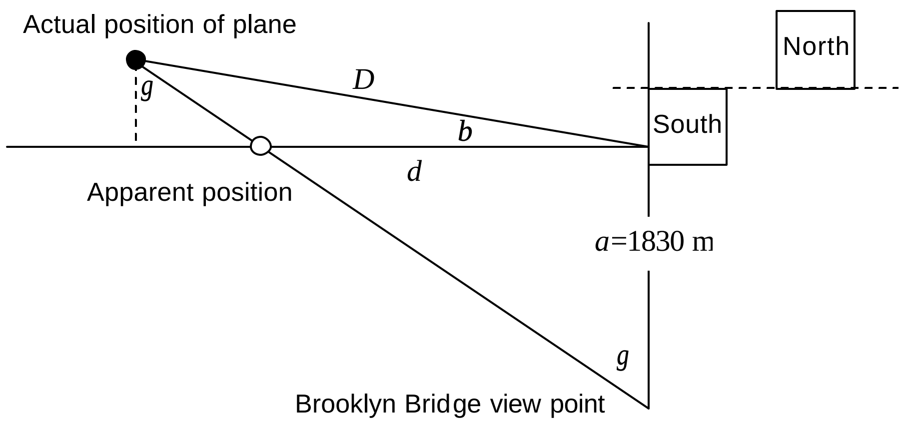

Fig. 8: Plan view of approach to South Tower, as seen from Brooklyn Bridge

From the triangles in the figure above, we can establish the following identity:

$$y = D \sin b = \left(D \cos b - d\right) / \tan g$$

Solving for D, we obtain

$$D = \frac{\cos g}{\cos(b + g)} d = m d$$

with

$$\tan g = d / a$$

and

$$m = \frac{\cos g}{\cos(b + g)}$$

with m being the magnification factor for both distance and velocity.

The local coordinates of the plane relative to the impact point are then

$$x = D\cos b$$

and

$$y = D\sin b$$

Combining these formulas with the data in the previous table, we obtain the following results:

d	*g*	*b*=15°		*b*=20°	
[m]	degrees	*m*	*v*	*m*	*v*
			[m/s]		[m/s]
750	22.29	1.163	**212**	1.251	228
668	20.05	1.147	**217**	1.231	233
586	17.76	1.132	**223**	1.205	237
480	14.70	1.114	**229**	1.177	242
246	7.66	1.074	**235**	1.119	245
141	4.41	1.057	**222**	1.095	230

The above table includes a computation for an angle of 20 degrees to estimate the effect on the speed of an uncertainty in the value of the approach angle. In the light of the above results, and considering also the velocities estimated from the previous NS and EW directions, we conclude that a best estimate for the speed of approach is 225 m/s (i.e. 810 km/hr, or 503 mph). This speed is in excellent agreement with information from air traffic controllers, who reported that "Flight 175 had screamed south over the Hudson Valley at about 500 miles per hour, more than double the legal speed"[6].

[6] M. L. Wald and K. Sack, "A Nation Challenged: The Tapes", *The New York Times*, October 16, 2001, Section A, Page 1