## Heterogeneity and Unemployment Dynamics

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#### Abstract

This paper develops new estimates of inflows and outflows for unemployment with an approach that allows for unobserved heterogeneity across workers as well as direct effects of unemployment duration on unemployment-exit probabilities. We Önd that shocks to the number of newly unemployed account for half the variance of unemployment, and the most important factor in rising U.S. unemployment during recent recessions is newly unemployed individuals who are likely to experience significantly longer durations of unemployment. The evidence suggests that recessions are characterized by important changes in the circumstances under which people become unemployed and that unemployment insurance contributes to longer job search.

Keywords: business cycles, unemployment duration, unobserved heterogeneity, duration dependence, state space model, extended Kalman Ölter

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## Introduction

What accounts for the sharp spike in the unemployment rate during recessions? The answer traditionally given by macroeconomists was that falling product demand leads firms to lay off workers, with these job separations a key driver of economic downturns. That view has been challenged by Hall (2005) and Shimer (2012), among others, who argued that cyclical fluctuations in the unemployment rate are instead primarily driven by declines in the job-finding rates for unemployed workers.

This debate has become particularly important for understanding the Great Recession and its aftermath. In June 2011–two years into the recovery– the unemployment rate still stood at  $9.1\%$ . higher than the peak in any postwar recession other than 1982. Even more troubling, the average duration of those unemployed at that time was 40 weeks, about twice the highest value reached in any month over 1947-2005. Of those workers who had been unemployed for less than one month in June 2011, only 53% were still unemployed the next month. By contrast, of those who had been unemployed for more than 6 months as of June 2011, 93% were still unemployed the following month.

As seen in Figure  $1<sup>1</sup>$ , this phenomenon that the long-term unemployed find jobs or leave the labor force more slowly than others is a strikingly consistent feature in the postwar data, and could be fundamental for understanding the respective contributions of unemployment inflows and outflows during recessions. For example, workers who lose their jobs due to involuntary permanent separation may have a more difficult time finding new jobs than people who quit voluntarily (Bednarzik, 1983; Fujita and Moscarini, 2013). If the number of involuntary separations increases during a recession, it could show up as what other researchers have interpreted as a fall in the job-finding rate and increase in the duration of unemployment even if the key driver of the recession was the increase in involuntary separations.

The phenomenon that unemployment exit rates fall with the duration of unemployment has

$$
p_{t+1}^{4.6} = \frac{U_t^4 + U_t^5 + U_t^6 - U_{t+1}^5 - U_{t+1}^6 - U_{t+1}^7}{U_t^4 + U_t^5 + U_t^6}
$$

<sup>&</sup>lt;sup>1</sup>The plotted value for  $p_{t+1}^{4.6}$  was calculated from

for  $U_t^n$  the number unemployed with duration n months at t. Other magnitudes were constructed analogously from the raw data on  $U_t^n$ . See Appendix A for more details on data construction.

been widely studied, with explanations falling into two broad categories. One possibility is that the experience of being unemployed for a longer period of time directly changes the characteristics of a Öxed individual. Following van den Berg and van Ours (1996) we will refer to this possibility as "genuine duration dependence". For example, individuals lose more human capital the longer they are unemployed (Acemoglu, 1995; Ljungqvist and Sargent, 1998). Eriksson and Rooth (2014) and Kroft, Lange, and Notowidigdo (2012) reported controlled experiments suggesting that employers are more likely to choose to hire someone who has been looking for work for a shorter period of time, while Faberman and Kudlyak (2013) concluded from micro data on applications to job openings on a job search website that search intensity decreases as the duration of job search increases. We will refer to such negative genuine duration dependence, that is, a condition where a longer period spent in unemployment directly reduces the probability of finding a job, as "unemployment scarring." Another possibility is positive genuine duration dependence. For example, the longer a person has been unemployed, the more willing they may be to accept a low-paying job or simply to drop out of the labor force. Katz  $(1986)$  and Katz and Meyer  $(1990a,b)$  argued that these effects may become important as unemployment benefits become exhausted. We will refer to the possibility that the probability of exiting unemployment increases as a consequence of a longer duration of unemployment as "motivational" effects.

A quite different explanation for the differences in unemployment exit probabilities across the different duration categories in Figure 1 is that there are important differences across job-seekers from the very beginning, arising for example from differences in the reason the individuals left their previous job or in differences in ex ante abilities or motivation across workers. The longer an individual is observed to have been unemployed, the greater the chance that the individual is a member of a group whose unemployment exit probabilities were low to begin with. That such cross-sectional heterogeneity might be important for the question studied by Hall and Shimer was recognized as far back as Darby, Haltiwanger, and Plant (1986), who argued that heterogeneity accounted for falling job-finding rates during recessions in a manner consistent with the traditional macroeconomic interpretation of recessions. A number of researchers have tried to investigate this hypothesis by looking at differences across job seekers in observable characteristics such as demographics, education, industry, occupation, geographical region, and reason for unemployment. Baker (1992), Shimer (2012), and Kroft, Lange, Notowidigdo, and Katz (2013) found that such variables contributed little to variation over time in long-term unemployment rates, while Aaronson, Mazumder and Schechter (2010), Bachmann and Sinning (2012), Barnichon and Figura (2013), Hall  $(2014)$ , and Hall and Schulhofer-Wohl  $(2014)$  documented important differences across observable characteristics. Elsby, Michaels and Solon (2009) found that incorporating observable heterogeneity reduced the imputed role of cyclical variation in unemployment exit rates.

However, no two individuals with the same coarse observable characteristics are in fact identical. It seems undeniable that a given pool of unemployed individuals that conditions on any set of observed characteristics is likely to become increasingly represented by those with lower ex ante exit probabilities the longer the period of time for which the individuals have been unemployed. Most of the above studies assume that conditional on observable characteristics, unemployed individuals are identical in terms of their transition probabilities into and out of unemployment. The result is that the imputed exit probabilities are determined solely from the current month's labor force statistics as if every month was a new steady state of the economy, not taking into account the fact that each individual has a unique history of unemployment. This approach misses a key feature of economic recessions and unemployment dynamics. Once one acknowledges heterogeneity across workers, the pool of those looking for work at a given point in time- and therefore the exit rates for individuals in that group—depends on the specific history of conditions whereby those individuals came to be unemployed. This means that more information than the current month's labor force statistics is necessary to account for the different histories of unemployed individuals and thus to credibly analyze the contributions of the inflows and outflows.

A large literature has attempted to separate genuine duration dependence from cross-sectional heterogeneity based on observable covariates for unemployed workers (Heckman and Singer, 1984) and the difference between calendar time and individual duration (van den Berg and van Ours, 1996). Our approach is closest to that in Hornstein (2012) who used dynamic accounting identities to track directly the way the characteristics of the pool of unemployed workers with unobserved cross-sectional heterogeneity would depend on the previous history. Hornstein used a minimumdistance estimation with identification achieved by smoothing penalties and only considered negative genuine duration dependence. By contrast, our paper provides a completely specified dynamic model that allows for both time variation in unobserved cross-sectional differences in worker characteristics as well as nonmonotonic genuine duration dependence.

Our approach offers a number of other advantages over previous studies. We provide a statistical framework for generating variance decompositions as well as historical decompositions of observed changes in unemployment over any subsample. In doing so we resolve a key shortcoming in much of the previous literature. Most previous studies used correlations between unemployment and the steady-state unemployment rate predicted by either inflows or outflows to draw conclusions about how much of the variation in unemployment is due to each factor. However, the unemployment rate is highly serially correlated and possibly nonstationary. What do we even mean by its variance, and how do we distinguish between the contribution to this variance of short-term versus longterm ináuences? Previous studies often addressed these issues by using some kind of detrending procedures. By contrast, our paper develops a complete statistical model with nonstationary driving processes, which as a by-product generates a forecast of unemployment at any horizon in the future. Since the forecast error at any specified horizon has a stationary distribution and well defined mean squared error whether or not the underlying process is nonstationary, as in den Haan (2000) we can calculate the fraction of the variance in unanticipated changes in unemployment over any horizon that is attributable to the various shocks in the model. This allows us to measure the dynamic contributions of different factors to unemployment and allows us to make very clear statements about the importance for short-run, medium-run, and long-run dynamics as well as over specific historical episodes. This is one of the key innovations of our approach and is entirely new to this literature.

In Section 1 we introduce the data that we will use in this analysis based on the number of jobseekers each month who report they have been looking for work at various search durations. We describe the accounting identities that will later be used in our full dynamic model and use average values of observable variables over the sample to explain the intuition for how such duration data can be used to separately identify cross-sectional heterogeneity and genuine duration dependence. We also use these calculations to illustrate why cross-sectional heterogeneity appears to be more important than genuine duration dependence in terms of explaining the broad features of these data.

In Section 2 we extend this framework into a full dynamic model in which we postulate the existence of two types of workers at any given date. Type  $H$  workers have a higher ex ante probability of exiting unemployment than type  $L$  workers, and all workers are also subject to potential scarring or motivational effects. Our model postulates that the number of newly unemployed individuals of either type, as well as the probability for each type of exiting the pool of unemployed at each date, evolve over time according to unobserved random walks. We show how one can calculate the likelihood function for the observed unemployment data and an inference about each of the state variables at every date in the sample using an extended Kalman filter.

Empirical results are reported in Section 3. We find that variation over time in the inflows of the newly unemployed are equally important as outáows from unemployment in accounting for errors in predicting aggregate unemployment up to 6 months ahead, with inflows becoming slightly more important than outflows for longer-horizon forecast errors. Changes in inflow and outflow probabilities for type  $H$  workers are equally important as those for type  $L$  workers in accounting for unemployment forecast errors at a 3-month horizon, whereas shocks to entry and exit probabilities for type L individuals are the main source of errors in predicting unemployment when looking a year or more into the future. In most recent recessions, shocks to the inflows of type  $L$  workers were the most important cause of rising unemployment during the recession. We find a smaller and nonmonotonic contribution of genuine duration dependence, with scarring effects dominating up to 6 months but motivational effects apparent for those unemployed longer than a year.

We offer interpretations of our findings in Section 4 by relating our estimated series to those available from other sources. We conclude that a key difference between type  $L$  and type  $H$ workers is the circumstances under which they left their previous job. Our imputed series for newly unemployed type L workers behaves very similarly to separate measures of the number of new job-seekers who were involuntarily separated from their previous job for a reason other than what was described as a temporary layoff. We further demonstrate that the parameters that characterize genuine duration dependence in our framework may be related in part to the operation of unemployment insurance. We Önd that when we use known dates for changes in duration of eligibility for unemployment insurance as shift factors in our parameterization of genuine duration dependence, there is an improved fit to the data. In normal times, the onset of what we have termed a motivational effect (in other words, of positive duration dependence) does not set in until after 6 months, that is, after eligibility for unemployment insurance is exhausted. By contrast, in periods of extended eligibility these motivational effects do not become pronounced until after 12 months of unemployment.

In Section 5 we investigate the robustness of our approach to various alternative specifications, including alternative methods to account for the change in the CPS questionnaire in 1994, various parameterizations for genuine duration dependence, allowing for correlation between the innovations of the underlying structural shocks in our model, and the possible effects of time aggregation. While such factors could produce changes in some of the details of our inference, our overall conclusions (summarized in Section 6) appear to be quite robust.

## 1 Observable implications of heterogeneity

The Bureau of Labor Statistics reports for each month  $t$  the number of Americans who have been unemployed for less than 5 weeks. Our baseline model is specified at the monthly frequency, leading us to use the notation  $U_t^1$  for the above BLS-reported magnitude, indicating these individuals have been unemployed for 1 month or less as of month t. BLS also reports the number who have been unemployed for between 5 and 14 weeks (or 2-3 months, denoted  $U_t^{2.3}$ ), 15-26 weeks  $(U_t^{4.6})$  and longer than 26 weeks  $(U_t^{\tau,+})$ . We also used the raw CPS micro data from which these aggregates were constructed to break down the last group further into those unemployed with duration 7-12 months  $(U_t^{7.12})$  and those with longer than 1 year  $(U_t^{13.+})$ .<sup>2</sup>

The data used in our analysis are graphed in Figure 2. Our purpose in this paper is to explore what variation in these duration-specific components  $U_t^x$  across time can tell us about unemployment dynamics. Our focus will be on the following question– of those individuals who are newly unemployed at time t, what fraction will still be unemployed at time  $t + k$ ? We presume that the answer to this question depends not just on aggregate economic conditions over the interval  $(t, t + k)$  but also on the particular characteristics of those individuals. Let  $w_{it}$  denote the number of people of type  $i$  who are newly unemployed at time  $t$ , where we interpret

$$
U_t^1 = \sum_{i=1}^I w_{it}.\tag{1}
$$

We define  $P_{it}(k)$  as the fraction of individuals of type i who were unemployed for one month or less as of date  $t - k$  and are still unemployed and looking for work at t. Note that in order for someone to have been unemployed for 2-3 months at time  $t$ , they either must have been newly unemployed

<sup>2</sup> See Appendix A for further details of data construction.

at  $t-1$  and still looking for a job at t, or they were newly unemployed at  $t-2$  and still looking at  $t-1$  and  $t$ :

$$
U_t^{2.3} = \sum_{i=1}^{I} \left[ w_{i,t-1} P_{i,t}(1) + w_{i,t-2} P_{i,t}(2) \right]. \tag{2}
$$

Likewise

$$
U_t^{4.6} = \sum_{i=1}^{I} \sum_{k=3}^{5} [w_{i,t-k} P_{i,t}(k)] \tag{3}
$$

$$
U_t^{7.12} = \sum_{i=1}^{I} \sum_{k=6}^{11} [w_{i,t-k} P_{i,t}(k)] \tag{4}
$$

$$
U_t^{13,+} = \sum_{i=1}^{I} \sum_{k=12}^{47} [w_{i,t-k} P_{i,t}(k)] \tag{5}
$$

where following Hornstein (2012) we terminate the calculations after 4 years of unemployment.

To get some intuition about what observation of the  $U_t^x$  aggregates can tell us about  $w_{it}$  and  $P_{it}(k)$ , we consider in this section some simple time-invariant examples. Suppose that none of the above magnitudes depended on time. How much could we learn from the average values of  $U^x$ ? As a first simplest case, suppose that everyone was identical before they became unemployed, and how long they have been unemployed had no consequences for the probability of finding a job next month. In other words, our first case assumes that the fraction of unemployed individuals at time  $t-1$  who are still unemployed at t is some constant p regardless of the date or the individual's circumstances. For this special case we would have

$$
P_{it}(k) = p^k \text{ for all } i, t. \tag{6}
$$

In this case, equations (1) and (2) imply

$$
U^{2.3} = U^1(p + p^2).
$$

Thus under the above assumptions, just by observing the average number of people newly unemployed and the average number unemployed of duration 2-3 months, we could obtain an estimate of p:

$$
U^{2.3}/U^1 = p + p^2. \tag{7}
$$

Table 1 reports that on average over 1976-2013, there were 3,160 thousand Americans who reported themselves to be newly unemployed and 2,377 thousand reporting an unemployment spell that so far had continued for 2 or 3 months. Equation (7) would then imply an estimate  $\hat{p} = 0.501$ for the average fraction of unemployed individuals who would still be unemployed one month later.

Table 1: Average number unemployed by duration of unemployment (in thousands, 1976-2013)

	T 72.3	174.6	$\frac{1}{7}7.12$	$17^{13.+}$
3,160	2,377	1.184	998	720

However, note that these same homogeneity assumptions would also imply

$$
U^{4.6}/U^1 = (p^3 + p^4 + p^5)
$$
\n(8)

If indeed  $\hat{p} = 0.501$ , equation (8) would predict a value for  $U^{4.6}$  of 697, whereas we see in Table 1 that the actual value is 1,184. If workers really were all identical, we would expect to see far fewer individuals whose unemployment spells lasted longer than 3 months than we do in the data. The indicated conclusion is that those individuals who have been unemployed for 3 months on average have different characteristics (and a lower probability of finding a job next month) than the typical worker who has only been unemployed for 1 month.

Consider next a generalization of the above special case in which there is still no heterogeneity across workers and no aggregate variation  $(w_{it} = w \text{ for all } i \text{ and } t)$ , but we do allow for genuine duration dependence arising from factors referred to in the introduction as scarring or motivational effects. Specifically, suppose that the fraction of unemployed individuals who had been unemployed for  $\tau$  months as of the previous month who are still unemployed in the current month is given by some function  $p(\tau)$ . Unemployment scarring would correspond to  $p(\tau)$  being an increasing function of  $\tau$ , while if the motivational effect dominates,  $p(\tau)$  would be a decreasing function of  $\tau$ . In this case (6) generalizes to

$$
P_{i,t}(k) = p(1)p(2)\cdots p(k),
$$

and (7) and (8) become

$$
U^{2.3}/U^1 = p(1) + p(1)p(2)
$$
\n(9)

$$
U^{4.6}/U^1 = p(1)p(2)p(3)[1 + p(4) + p(4)p(5)].
$$
\n(10)

Suppose we were willing to choose a parametric form for the function  $p(\tau)$  as in Katz and Meyer (1990b):

$$
p(\tau) = \exp\{-\exp[x + d(\tau - 1)]\} \quad \text{for} \quad \tau = 1, 2, 3, \dots \tag{11}
$$

One benefit of this functional form is that  $p(\tau)$  is guaranteed to be between 0 and 1 for any values of  $x, d$ , or  $\tau$ , a feature that will be helpful when we get to a generalization of this set-up in the following section in which we will allow for variation in  $x$  over time. A negative value for the parameter  $d$ would correspond to unemployment scarring whereas  $d > 0$  would represent a motivational effect. Substituting (11) into (9) and (10) produces a system of 2 equations which we can solve numerically for x and d as functions of the observed values for  $U^{2.3}/U^1$  and  $U^{4.6}/U^1$  given in Table 1. The solution turns out to be  $x = -0.316$  and  $d = -0.227$ . The negative value for d is supportive of the unemployment scarring hypothesis, consistent with the inference above that it is not possible to reconcile the relative values of  $U^1$ ,  $U^{2.3}$ , and  $U^{4.6}$  without some kind of heterogeneity.

The problem with relying purely on genuine duration dependence is seen if we try to use the inferred values for the function  $p(\tau)$  to estimate the value for  $U^{7.12}$  and  $U^{13.+}$ . These turn out to be 938 and 3,243, respectively. Note in particular that this predicted value for  $U^{13,+}$  is far larger than the observed value of 720. Any "unemployment scarring" that is operating on workers who have been unemployed for 6 months or longer seems to be very different from that experienced by those unemployed for only 2-5 months. One possibility is that the functional form  $(11)$  is misspecified. However, another possibility worth exploring is that there are important ex-ante differences between individuals, with some likely to get a job more quickly than others. As a result of these ex-ante differences, when one looks at a given pool of workers who have been unemployed for  $\tau$  months, a larger fraction of the pool is going to be accounted for by those with lower job-finding probabilities the larger the value of  $\tau$ .

To illustrate how this could work, suppose there are  $I = 2$  types of workers, which we will

label type  $H$  and type  $L$  in anticipation of the normalization that type  $L$  workers have a lower probability of exiting unemployment. With cross-sectional heterogeneity but no genuine duration dependence, equation (6) becomes

$$
P_{it}(k) = p_i^k \text{ for all } t. \tag{12}
$$

Substituting (12) into (1) through (4) gives a system of 4 equations which we can solve for the 4 unknowns  $(w_H, w_L, p_H, p_L)$  as functions of the observed averages  $(U^1, U^{2.3}, U^{4.6}, U^{7.12})$ . The solution turns out to be  $w_H = 2,647$ ,  $w_L = 513$ ,  $p_H = 0.412$  and  $p_L = 0.871$ . The type H workers comprise a very high fraction, 83.8%, of the initial pool of unemployed  $U^1$ . But because they are more likely to be the ones who find jobs quickly, there are fewer type  $H$  workers included in group  $U^{2.3}$  and even fewer in  $U^{4.6}$  and  $U^{7.12}$ . This changing composition can account for the feature of the data that a specification without cross-sectional heterogeneity would attribute to unemployment scarring.

We can also use these values for  $(w_H, w_L, p_H, p_L)$  in equation (5) to get a predicted value for  $U^{13,+}$  of 749, not far from the observed value of 720. These calculations suggest that cross-sectional heterogeneity is a more promising potential explanation of unemployment dynamics than genuine duration dependence.

Finally, we note that it is possible to estimate a model that allows for both cross-section heterogeneity and genuine duration dependence. Suppose we generalize (11) to

$$
p_i(\tau) = \exp\{-\exp[x_i + d(\tau - 1)]\} \quad \text{for} \quad \tau = 1, 2, 3, \dots \tag{13}
$$

for  $i = H$  or L. Equations (1)-(5) then give us a system of 5 equations in the 5 unknowns  $(w_H, w_L,$  $x_H, x_L, d$ . The solutions turn out to be  $w_H = 2,637, w_L = 513, p_H(1) = 0.412, p_L(1) = 0.872$ and  $d = 0.003$ . These estimates allow little role for genuine duration dependence, with the slightly positive value for  $d$  now implying that motivation may be more important than scarring. However, this effect is quite tiny: the probability of exiting unemployment goes up by around 0.001 as the duration of unemployment increases by 1 month.

The above calculations demonstrate that given parametric assumptions, it is possible to come up with estimates of the relative importance of cross-sectional heterogeneity and genuine duration dependence in explaining why some individuals remain unemployed for so long. However, the examples discussed so far were quite limited in that we assumed that all parameters were constant over time. More generally, the observed values of  $U_t^x$  for some particular t could tell us about the portions and probabilities for different types of workers at that date if we knew something about the prior history. By assuming that the magnitudes of  $w_{Ht}, w_{Lt}, x_{Ht}, x_{Lt}$  evolve gradually over time, we can use a nonlinear state-space model to form an inference about the changing values of these magnitudes and separately infer the contribution of time-invariant genuine duration dependence, as we demonstrate in the next section.

## 2 Dynamic formulation

The previous section discussed a static example in order to illustrate how cross-sectional heterogeneity and genuine duration dependence can be identified from observed reports of unemployment duration. However, our main interest lies in the contribution of the two types of heterogeneity to unemployment dynamics. Here we set up a state-space model where the dynamic behavior of the observed vector  $y_t = (U_t^1, U_t^{2.3}, U_t^{4.6}, U_t^{7.12}, U_t^{13.4})'$  is determined as a nonlinear function of latent dynamic variables- the inflows and outflow probabilities for unemployed individuals with unobserved heterogeneity. Due to the nonlinear nature of the resulting model, we draw inference on the latent variables using the extended Kalman filter.

#### 2.1 State-space representation

We assume smooth variation over time for the latent variables of interest,  $w_{Ht}, w_{Lt}, x_{Ht}, x_{Lt}$ with each assumed to follow an unobserved random walk, e.g.,

$$
w_{Ht} = w_{H,t-1} + \epsilon_{Ht}^{w}
$$

A random walk is a flexible and parsimonious way of modeling time-varying latent variables. As in the previous steady-state example, we consider 4 years to be the maximum duration.<sup>3</sup> Suppose that we observe the elements of  $y_t$  with measurement error  $r_t = (r_t^1, r_t^{2.3}, r_t^{4.6}, r_t^{7.12}, r_t^{13.4})'$ . The

 $3$ Allowing a different maximum duration of unemployment, for instance, 3 years, does not change the results significantly.

measurement equations are thus written as follows;

$$
U_t^1 = \sum_{i=H,L} w_{it} + r_t^1 \tag{14}
$$

$$
U_t^{2.3} = \sum_{i=H,L} [w_{i,t-1} P_{i,t}(1) + w_{i,t-2} P_{i,t}(2)] + r_t^{2.3}
$$
 (15)

$$
U_t^{4.6} = \sum_{i=H,L} \sum_{k=3}^{5} [w_{i,t-k} P_{i,t}(k)] + r_t^{4.6}
$$
 (16)

$$
U_t^{7.12} = \sum_{i=H,L} \sum_{k=6}^{11} [w_{i,t-k} P_{i,t}(k)] + r_t^{7.12}
$$
\n(17)

$$
U_t^{13,+} = \sum_{i=H,L} \sum_{k=12}^{47} [w_{i,t-k} P_{i,t}(k)] + r_t^{13,+}
$$
 (18)

where

$$
P_{i,t}(j) = p_{i,t-j+1}(1)p_{i,t-j+2}(2)...p_{i,t}(j).
$$
\n(19)

We assume that for type i workers who have already been unemployed for  $\tau$  months as of time  $t-1$ , the fraction who will still be unemployed at t is given by

$$
p_{i,t}(\tau) = \exp[-\exp(x_{i,t} + d_{\tau})] \quad \text{for} \quad \tau = 1, 2, 3, ... \tag{20}
$$

where  $d_{\tau}$  determines the nature of genuine duration dependence experienced by an unemployed individual with duration of unemployment  $\tau$  months and  $x_{it}$  is a time-varying magnitude influencing the unemployment exit probability for all workers of type  $i$  regardless of their duration. Like the inflows  $w_{LT}$  and  $w_{Ht}$ , we assume that the parameters  $x_{Lt}$  and  $x_{Ht}$  governing outflow probabilities also follow a random walk. Note that because we have assumed that the genuine-duration dependence effects as summarized by  $d<sub>\tau</sub>$  are time-invariant and that the type-specific effects  $x<sub>it</sub>$  evolve smoothly over time, it is possible to estimate a different value for the parameter  $d_{\tau}$  for each  $\tau$ . We investigated a number of different specifications for  $d<sub>\tau</sub>$  and found the best fit using linear splines at  $\tau = 6$  and  $\tau = 12$  which we use for the baseline analysis:

$$
d_{\tau} = \begin{cases} \n\delta_1(\tau - 1) & \text{for } \tau < 6 \\ \n\delta_1[(6 - 1) - 1] + \delta_2[\tau - (6 - 1))] & \text{for } 6 \le \tau < 12 \\ \n\delta_1[(6 - 1) - 1] + \delta_2[(12 - 1) - (6 - 1)] + \delta_3[\tau - (12 - 1)] & \text{for } 12 \le \tau. \n\end{cases} \tag{21}
$$

Positive  $\delta_j$  for  $j = 1, 2, 3$  imply motivational effects while negative values imply unemployment scarring over the relevant duration ranges.

We can arrive at the likelihood function for the observed data  $\{y_1, ..., y_T\}$  by assuming that the vector of measurement errors  $r_t$  are independent Normal, where  $R_1$ ,  $R_{2.3}$ ,  $R_{4.6}$ ,  $R_{7.12}$  and  $R_{13,+}$  are the standard deviations of  $r_t^1, r_t^{2.3}, r_t^{4.6}, r_t^{7.12}$  and  $r_t^{13.4}$  respectively:

$$
r_t \sim N(0, R)
$$

$$
\underbrace{R}_{5\times 5} = \left[\begin{array}{ccccc} R_1^2 & 0 & 0 & 0 & 0 \\ 0 & R_{2,3}^2 & 0 & 0 & 0 \\ 0 & 0 & R_{4,6}^2 & 0 & 0 \\ 0 & 0 & 0 & R_{7,12}^2 & 0 \\ 0 & 0 & 0 & 0 & R_{13,+}^2 \end{array}\right].
$$

Let  $\xi_t$  be the vector  $(w_{Lt}, w_{Ht}, x_{Lt}, x_{Ht})'$  and  $\epsilon_t = (\epsilon_{Lt}^w, \epsilon_{Ht}^w, \epsilon_{Lt}^x, \epsilon_{Ht}^z)'$ . Our assumption that the latent factors evolve as random walks would be written as

$$
\frac{\xi_t}{4 \times 1} = \xi_{t-1} + \frac{\epsilon_t}{4 \times 1}
$$
\n
$$
\frac{\epsilon_t}{4 \times 1} \sim N(\underbrace{0}_{4 \times 1}, \underbrace{\Sigma}_{4 \times 1})
$$
\n
$$
\frac{\xi_t}{4 \times 1} = \begin{bmatrix} (\sigma_L^w)^2 & 0 & 0 & 0 \\ 0 & (\sigma_H^w)^2 & 0 & 0 \\ 0 & 0 & (\sigma_L^x)^2 & 0 \\ 0 & 0 & (\sigma_L^x)^2 & 0 \\ 0 & 0 & 0 & (\sigma_H^x)^2 \end{bmatrix}.
$$
\n(22)

In Section 5 we will also report results for a specification in which the shocks are allowed to be contemporaneously correlated.

Since the measurement equations (14)-(18) are a function of  $\{\xi_t, \xi_{t-1}, ..., \xi_{t-47}\}$ , the state equation should describe the joint distribution of  $\xi_t$ 's from  $t - 47$  to t, where I and 0 denote a  $(4 \times 4)$  identity and zero matrix, respectively:

$$
\begin{bmatrix}\n\xi_t \\
\xi_{t-1} \\
\xi_{t-2} \\
\vdots \\
\xi_{t-46} \\
\xi_{t-47}\n\end{bmatrix} = \begin{bmatrix}\n\frac{I}{4 \times 4} & \frac{0}{4 \times 4} & 0 & 0 & \dots & 0 & 0 & 0 \\
I & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\
0 & I & 0 & 0 & \dots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \dots & I & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & I & 0 & 0\n\end{bmatrix} \begin{bmatrix}\n\xi_{t-1} \\
\xi_{t-2} \\
\xi_{t-3} \\
\vdots \\
\xi_{t-47} \\
\xi_{t-48}\n\end{bmatrix} + \begin{bmatrix}\n\frac{\epsilon_t}{4 \times 1} \\
0 \\
\frac{\epsilon_t}{4 \times 1} \\
0 \\
\vdots \\
0 \\
0\n\end{bmatrix}.
$$
\n(23)

#### 2.2 Estimation

Our system takes the form of a nonlinear state space model in which the state transition equation is given by (23) and observation equation by (14)-(18) where  $P_{i,t}(j)$  is given by (19) and  $p_{i,t}(\tau)$  by (20). Our baseline model has 12 parameters to estimate, namely the diagonal terms in the variance matrices  $\Sigma$  and R and the parameters governing genuine duration dependence,  $\delta_1$ ,  $\delta_2$ and  $\delta_3$ . Because the observation equation is nonlinear in  $x_{it}$ , the extended Kalman filter can be used to form the likelihood function for the observed data  $\{y_1, ..., y_T\}$  and form an inference about the unobserved latent variables  $\{\xi_1, ..., \xi_T\}$ , as detailed in Appendix B. Inference about historical values for  $\xi_t$  provided below correspond to full-sample smoothed inferences, denoted  $\hat{\xi}_{t|T}$ .

## 3 Results for the baseline specification

We estimated parameters for the above nonlinear state-space model using monthly data on  $y_t = (U_t^1, U_t^{2.3}, U_t^{4.6}, U_t^{7.12}, U_t^{13.4})'$  for  $t =$  June 1976 through June 2013. To deal with seasonality and measurement error the series represent 12-month moving averages of the raw data. Table 2 provides parameter estimates for our baseline model. We find a statistically significant negative value for  $\delta_1$ , the parameter that governs genuine duration dependence for unemployment durations less than 6 months. The negative coefficient is consistent with the scarring hypothesis—the longer someone from either group has been unemployed, provided the duration has been 5 months or less, the more likely it is that person will be unemployed next month. On the other hand, we

find an estimate for  $\delta_2$  statistically insignificant and near zero (applying to individuals unemployed for more than 5 months and less than 1 year), and a statistically significant positive value for  $\delta_3$ (unemployment lasting for a year and over): Once someone has been unemployed for more than a year, it becomes more likely as more months accumulate that they will either Önd a job or exit the labor force in any given month, consistent with what we have labeled motivational effects.

Although the values of  $\delta_1$  and  $\delta_3$  are statistically significant, they play a relatively minor role compared to ex ante heterogeneity in accounting for differences in exit probabilities by duration of unemployment. If we set  $\delta_1 = \delta_2 = \delta_3 = 0$ , our estimates imply that the number of people unemployed for  $4-6$  months' duration would have been  $16\%$  lower on average than the values implied by our baseline estimates which incorporate short-duration scarring effects. On the other hand, the number unemployed for longer than a year would have been 27% higher on average than the values implied by our baseline estimates as a consequence of shutting down the motivational effects captured by  $\delta_2$  and  $\delta_3$ . The average level of total unemployment coming from all groups would be about the same if we set  $\delta_1 = \delta_2 = \delta_3 = 0$  compared to the average value predicted by our baseline model.

Figure 3 plots smoothed estimates for  $p_{i,t}(1)$ , the probability that a newly unemployed worker of type i at  $t-1$  will still be unemployed at t. These average 0.44 for type H individuals and 0.90 for type  $L$  individuals, with probabilities of remaining unemployed rising for both groups during recessions. The probabilities for both groups declined immediately after the end of the recessions of 1980 and 1982. However, they either continued to go up or stayed at a high level long after the ends of subsequent recessions as one feature of what has sometimes been described as jobless recoveries.

Figure 4 plots inflows of individuals of each type into the pool of newly unemployed. Type  $H$ workers constitute 91% on average of the newly unemployed. Inflows of both types increase during recessions. New inflows of both type  $H$  and  $L$  workers declined immediately after the end of the recessions of 1980 and 1982, but the pattern became different in subsequent recessions. While new inflows of type  $H$  workers declined immediately after the end of the 1990-91, 2001, and 2007-2009 recessions, the inflows of type  $L$  workers either continued to go up or were slow to return to normal. This changing behavior of type  $L$  workers' inflows appears to be another important characteristic of jobless recoveries.

The Great Recession is unique in that the continuation probabilities of both groups as well as the inflows of type  $L$  workers reached higher levels than any earlier dates in our data set. In addition, the failure of continutation probabilities to recover after the recession along with the sustained high level of type  $L$  workers' inflows also distinguish the Great Recession from previous recessions.

The combined implications of these cyclical patterns are summarized in Figure 5. Before the Great Recession, the share of type L workers fluctuated between  $20\%$  and  $40\%$ , falling during expansions and rising during and after recessions. But during the Great Recession, the share of type L workers skyrocketed over 60% and continued to rise for two more years after the end of the recession. The usual recovery pattern of a falling share of type  $L$  workers has since been observed, but there is a long way to go to return to levels within the historical norm.

#### 3.1 Variance decomposition

Many previous studies have tried to summarize the importance of different factors in determining unemployment by looking at correlations between the observed unemployment rate and the steadystate unemployment rate predicted by each factor of interest alone; see for example Fujita and Ramey (2009) and Shimer (2012). One major benefit of our framework is that it delivers a much cleaner answer to this question in the form of variance decompositions.

Variance decomposition is a familiar method in linear VARs for measuring how much each shock contributes to the mean squared error (MSE) of an s-period-ahead forecast of a magnitude of interest.<sup>4</sup> Here we focus on forecasts of the total number of people unemployed. In a linear VAR, both the MSE and the portion attributable to each component are functions of population parameters that depend on the horizon s but not the date, and the sum of the contributions of each of the factors exactly equals the overall MSE.

In our case we have the simple system for the latent  $(4 \times 1)$  vector

$$
\xi_{t+1} = \xi_t + \epsilon_{t+1}
$$

from which

<sup>4</sup> See for example Hamilton (1994a, Section 11.5).

$$
\xi_{t+s} = \xi_t + \epsilon_{t+1} + \epsilon_{t+2} + \epsilon_{t+3} + \dots + \epsilon_{t+s} \n= \xi_t + u_{t+s}.
$$

Letting  $y_t = (U_t^1, U_t^{2.3}, U_t^{4.6}, U_t^{7.12}, U_t^{13.4})'$  denote the  $(5 \times 1)$  vector of observations for date t, our model implies that  $y_t = h(\xi_t, \xi_{t-1}, \xi_{t-2}, ..., \xi_{t-47})$  where  $h(\cdot)$  is a known nonlinear function. Hence

$$
y_{t+s} = h(u_{t+s} + \xi_t, u_{t+s-1} + \xi_t, ..., u_{t+1} + \xi_t, \xi_t, \xi_{t-1}, ..., \xi_{t-47+s}).
$$

We can take a first-order Taylor expansion of this function around  $u_{t+j} = 0$  for  $j = 1, 2, ..., s$ ,

$$
y_{t+s} \simeq h(\xi_t, ..., \xi_t, \xi_t, \xi_{t-1}, ..., \xi_{t-47+s}) + \sum_{j=1}^s [H_j(\xi_t, \xi_t, ..., \xi_t, \xi_t, \xi_{t-1}, ..., \xi_{t-47+s})]u_{t+j}
$$

for  $H_j(\cdot)$  the  $(5 \times 4)$  matrix associated with the derivative of  $h(\cdot)$  with respect to its jth argument. Using the definition of  $u_{t+j}$ , this can be rewritten as

$$
y_{t+s} \simeq c_s(\xi_t, \xi_{t-1}, \dots, \xi_{t-47+s}) + \sum_{j=1}^s [\Psi_{s,j}(\xi_t, \xi_{t-1}, \dots, \xi_{t-47+s})] \epsilon_{t+j}
$$
(24)

for  $\Psi_{s,j}(\cdot)$  a known  $(5 \times 4)$ -valued function of  $\xi_t, \xi_{t-1}, ..., \xi_{t-47+s}$ . The MSE associated with an s-period-ahead forecast of  $y_{t+s}$  is then

$$
E(y_{t+s} - \hat{y}_{t+s|t})(y_{t+s} - \hat{y}_{t+s|t})' = \sum_{j=1}^{s} [\Psi_{s,j}(\xi_t, \xi_{t-1}, ..., \xi_{t-47+s})] \Sigma[\Psi_{s,j}(\xi_t, \xi_{t-1}, ..., \xi_{t-47+s})]' \tag{25}
$$

$$
= \sum_{j=1}^{s} \sum_{m=1}^{4} \Sigma_m[\Psi_{s,j}(\xi_t, \xi_{t-1}, ..., \xi_{t-47+s})e_m][\Psi_{s,j}(\xi_t, \xi_{t-1}, ..., \xi_{t-47+s})e_m]'
$$

for  $e_m$  column m of the  $(4\times4)$  identity matrix and  $\Sigma_m$  the row m, column m element of  $\Sigma$ . Thus the contribution of innovations of type L worker's inflows (the first element of  $\epsilon_t = (\epsilon_{L,t}^w, \epsilon_{H,t}^w, \epsilon_{L,t}^x, \epsilon_{H,t}^x)'$ ) to the MSE of the s-period-ahead linear forecast error of total unemployment,  $\mathbf{1}'y_t$ , is given by

$$
\mathbf{1}' \sum_{j=1}^{s} \Sigma_1[\Psi_{s,j}(\xi_t, \xi_{t-1}, ..., \xi_{t-47+s})e_1][\Psi_{s,j}(\xi_t, \xi_{t-1}, ..., \xi_{t-47+s})e_1]'\mathbf{1}
$$
(26)

where 1 denotes a  $(5 \times 1)$  vector of ones. Note that as in the constant-parameter linear case, the sum of the contributions of the 4 different structural shocks would be equal to the MSE of an s-period-ahead linear forecast of unemployment in the absence of measurement error. However, in our case the linearization is taken around time-varying values of  $\{\xi_t, \xi_{t-1}, ..., \xi_{t-47+s}\}.$  We can evaluate equation (26) at the smoothed inferences  $\{\hat{\xi}_{t|T}, \hat{\xi}_{t-1|T}, ..., \hat{\xi}_{t-47+s|T}\}$  and then take the average value across all dates  $t$  in the sample. This gives us an estimate of the contribution of the type  $L$  worker's inflows to unemployment fluctuations over a horizon of  $s$  months:

$$
q_{s,1} = T^{-1} \sum_{t=1}^{T} \mathbf{1}' \sum_{j=1}^{s} \Sigma_1 [\Psi_{s,j}(\hat{\xi}_{t|T}, \hat{\xi}_{t-1|T}, ..., \hat{\xi}_{t-47+s|T}) e_1] [\Psi_{s,j}(\hat{\xi}_{t|T}, \hat{\xi}_{t-1|T}, ..., \hat{\xi}_{t-47+s|T}) e_1]' \mathbf{1}.
$$

Consequently  $q_{s,1}/\sum_{ }^{4}$  $m=1$  $q_{s,m}$  would be the ratio of the first factor's contribution to unemployment volatility at horizon s.

Figure 6 shows the contribution of each factor to the mean squared error in predicting overall unemployment as a function of the forecasting horizon. If one is trying to forecast unemployment at a horizon of less than 3 months, the single most important source of uncertainty is the inflow of new type H workers into unemployment followed by uncertainty about the future exit probabilities for type H workers. However, the farther one is looking into the future, the more important uncertainty about what is going to happen to type  $L$  workers becomes. If one is trying to predict one or two years into the future, the single most important source of uncertainty is ináows of new type  $L$  workers, followed by uncertainty about their outflows. Much of the MSE associated with a 2-year-ahead forecast of unemployment comes from not knowing when the next recession will begin or the current recession will end. For this reason, the MSE associated with 2-year-ahead forecasts is closely related to what some researchers refer to as the "business cycle frequency" in a spectral decomposition. If we are interested in the key factors that change as the economy moves into and out of recessions, inflows and outflows for type  $L$  workers are most important. We will provide additional evidence on this point in Section 3.2.

The last panel of Figure 6 breaks these contributions separately into inflows and outflows.

Inflows and outflows are of roughly equal importance in accounting for the error we would make in predicting total unemployment 6 months or less into the future. As we try to forecast farther into the future, uncertainty about future inflows becomes modestly more important, with inflows accounting for 59% of the 2-year-ahead MSE.

#### 3.2 Historical decomposition

A separate question of interest is how much of the realized variation over some historical episode came from particular structural shocks. In case of a linear VAR, we can decompose the historical time path for y between some date t and  $t + s$  into the component that would have been predicted at time t and the part that is due to innovations in each of the shocks. A similar approach can be adopted in our case. The smoothed inferences satisfy

$$
\hat{\xi}_{t+s|T} = \hat{\xi}_{t|T} + \hat{\epsilon}_{t+1|T} + \hat{\epsilon}_{t+2|T} + \hat{\epsilon}_{t+3|T} + \ldots + \hat{\epsilon}_{t+s|T}
$$

where  $\hat{\epsilon}_{t+s|T} = \hat{\xi}_{t+s|T} - \hat{\xi}_{t+s-1|T}$ . For any date  $t+s$  we then have the following model-inferred value for the number of people unemployed:

$$
\mathbf{1}'h(\hat{\xi}_{t+s|T},\hat{\xi}_{t+s-1|T},\hat{\xi}_{t+s-2|T},...,\hat{\xi}_{t+s-47|T}).
$$

For an episode starting at some date  $t$ , we can then calculate

$$
\mathbf{1}'h(\hat{\xi}_{t|T}, \hat{\xi}_{t|T}, \hat{\xi}_{t|T}, ..., \hat{\xi}_{t|T}, \hat{\xi}_{t-1|T}, ..., \hat{\xi}_{t+s-47|T}).
$$

This represents the path that unemployment followed between t and  $t + s$  as a result of initial conditions at time t and not any of the shocks between t and  $t+s$ . Given this path for unemployment that is implied by initial conditions, we can then isolate the contribution of each separate shock between t and  $t + s$ . Using the linearization in equation (24) allows us to represent the realized deviation from this path in terms of the contribution of individual historical shocks:

$$
y_{t+s} \simeq c_s(\hat{\xi}_{t|T}, \hat{\xi}_{t-1|T}, \dots, \hat{\xi}_{t-47+s|T}) + \sum_{j=1}^s [\Psi_{s,j}(\hat{\xi}_{t|T}, \hat{\xi}_{t-1|T}, \dots, \hat{\xi}_{t-47+s|T})] \hat{\epsilon}_{t+j|T}.
$$
 (27)

From the above equation, we get a contribution for example of  $\epsilon_{L,t+1}^w, \epsilon_{L,t+2}^w, ..., \epsilon_{L,t+s}^w$  (the shocks to  $w<sub>L</sub>$  between  $t + 1$  and  $t + s$ ) to the deviation between the level of unemployment at  $t + s$  from the value predicted on the basis of initial conditions at t:

$$
\mathbf{1'}\sum_{j=1}^s [\Psi_{s,j}(\hat{\xi}_{t|T},\hat{\xi}_{t-1|T},...,\hat{\xi}_{t-47+s|T})]e_1\hat{\epsilon}'_{t+j|T}e_1.
$$

Figure 7 shows the contribution of each component to the realized unemployment rate in the last four recessions. In each panel, the solid line (labeled  $U_{base}$ ) gives the change in the unemployment rate relative to the value at the start of the episode that would have been predicted on the basis of initial conditions. In each case this displays a very modest decline. Unanticipated shocks to all four of our factors contributed to the rise in unemployment in these recessions. Typically an increase in the inflow of type  $L$  workers is the single most important reason that unemployment rises during a recession. A continuing increase in these inflows even after the recession was over were important factors in the jobless recoveries from the last three recessions. Declines in the exit probabilities for both types of workers are also an important reason why unemployment does not fall more quickly once the recession is over.

Panel D of Figure 7 shows that a big inflow of type  $L$  workers was by far the most important factor in the Great Recession.<sup>5</sup> These results offer a new perspective on an emerging debate about the causes of the fall in average unemployment exit probabilities and increase in very long spells of unemployment observed in this recession. Hall (2014) argued that the explanation is a compositional shift of jobseekers toward types with low exit probabilities, for instance permanent job losers. By contrast, recent studies by Bachman and Sinning (2012) and Kroft, Lange, Notowidigdo and Katz (2013) concluded that compositional changes played little role. We add a new factor that none of these studies considered, which is the possibility of changes in the inflows of workers with unobserved heterogeneity, and find that it provides an additional reason to favor Hall's interpretation. Our estimates suggest that a growing ináow of type L workers gradually changed the composition of

<sup>&</sup>lt;sup>5</sup>Because of the length and severity of the recession of 2007-2009, the linearization (27) around the January 2007 values on which Panel D is based becomes poorer as we try to predict values for 2010. This is why the " $U_{\text{all}}$ " line in Panel D falls below the actual path of unemployment in the case of this recession. As a robustness check, we also calculated the exact nonlinear contribution of each component in isolation of the others to the actual observed unemployment rate and the picture is very similar. The advantage of the linear decomposition is that the sum of the individual contributions exactly equals the aggregate, whereas the same is not true in a nonlinear dynamic representation.

the pool of unemployed, and that this is the primary reason that the composite exit probabilities for the pool declined. The probability of exiting unemployment fell for both type L and type H workers during the Great Recession (Figure 3) and this also contributed to rising unemployment (see the blue circles and green dashes in Figure 7D). However, according to our estimates, the most important factor in rising unemployment duration was the increased share of type L workers in the pool of the unemployed, and this resulted primarily from changes in inflows rather than outflows.

## 4 Discussion

The above inferences were arrived at purely by attributing the time series plotted in Figure 2 to movements in unobserved latent factors. Here we discuss the relation between our findings and those emerging using additional data.

## 4.1 Who are the type L workers?

Shimer (2012) concluded that the most important potential source of heterogeneity across different workers could be differences in the reasons the individuals became unemployed. He found that the job-finding probability of job losers on temporary layoff is higher than that of other job losers and the fraction of unemployment represented by job losers not on temporary layoff exhibits clear counter-cyclicality. Darby, Haltiwanger and Plant (1986) argued that counter-cyclicality in the average unemployment duration mainly comes from the increased ináow of prime-age workers suffering permanent job loss who are likely to have low job-finding probabilities. Bednarzik (1983) also noted that permanently separated workers are more likely to experience a long duration of unemployment, while Fujita and Moscarini (2013) showed that the unemployed who are likely to experience long-term unemployment spells tend to be those who are not recalled to work by their previous employers.

Figure 8 breaks down people looking for work in terms of the reason they came to be unemployed. Dark bars describe the share of people who have been looking for work for less than one month by reason and white bars the share of those who have been looking for more than 6 months by reason. Both permanent job losers and job losers on temporary layoff account for about one fifth of new entrants into the pool of unemployed. By contrast, those on temporary layoff account for less than 3% of the unemployed with duration longer than 6 months, while around half of the long-term unemployed are accounted for by permanent job losers. This means that the unemployment exit probabilities of permanent job losers are much lower than those of job losers on temporary layoff.

Panel A of Figure 9 plots the inflows to unemployment by reason. Both the inflows of permanent job losers and those on temporary layoff exhibit counter-cyclicality. They rise as the recession begins and fall as the recession ends. Since permanent job losers tend to have a lower unemployment exit probability as shown in Figure 8, we compare the number of those newly unemployed who gave permanent separations from their previous job as the reason to our estimate of the number of newly unemployed type L workers in panel B of Figure  $9.6$  The two series were arrived at using different data and different methodologies but exhibit remarkably similar dynamics. By contrast, our series for newly unemployed type L workers does not look much like any of the other series in Panel A. Notwithstanding, note the difference in scale between the two series plotted in Panel B– our estimate of  $w_{Lt}$  is only about half as large as the number of workers included in the permanent separation category.

The striking feature of Figures 3 and 4 is that the overwhelming majority of newly unemployed individuals are able to find a new job quickly, and Figure 5 shows that the longer an expansion continues, the more the pool of unemployed individuals consists of those we have labeled as type H. These features seem related to the well-known observation that in normal times there is a tremendous amount of churning in the labor market, with millions of workers entering and exiting the unemployment pool every month even as the overall unemployment rate remains low— see for example, Davis, Faberman and Haltiwanger (2006). Lazear and Spletzer (2012) showed using micro data from JOLTS that churning is procyclical, with quits accounting for the major part of it. However, our measure of type  $H$  inflows rises during recessions. It is clear that in addition to normal churning arising from those who quit their job voluntarily, unemployment due to temporary layoffs is another important part of what we have characterized as type H unemployment. Temporary layoffs rise during recessions, but insofar as many of these individuals often return to their old jobs relatively quickly, our procedure is likely assigning most of those on temporary layoff to type  $H$ rather than type L: Indeed, Panel C of Figure 9 shows that the dynamics of our imputed type  $H$ 

 $6$ Permanent separations include permanent job losers and persons who completed temporary jobs. The separate series, permanent job losers and persons who completed temporary jobs, are publicly available from 1994, but their sum (permanent separations) is available back to 1976.

inflows are very similar to the sum of job quitters, those on temporary layoff, and entrants to the labor force.

Within any categorization based on observable characteristics there are still important differences across individuals. Our results imply that the great difficulties that some workers face in finding new jobs only characterizes a subset of those permanently separated and likely a smaller subset of those who quit their jobs voluntarily. Within the "permanently separated" category, many workers do end up being recalled to their old positions (Fujita and Moscarini, 2013), and such individuals are again likely be included in our type  $H$  designation. On the other hand, some of the individuals in every reported BLS category may have a history of low performance or poor interpersonal and communication skills<sup>7</sup> and would be categorized in our approach as type L. Although allowing for unobserved heterogeneity within any given group of common observed characteristics seems critical for this kind of study, our conclusion is that the single most important distinction between the latent classes of workers identified by our approach arises from the circumstances under which the individuals came to be unemployed, with permanently separated workers likely accounting for the majority of our type L workers. Normal churning of the labor market and temporary layoffs appear to be a big part of what we are capturing with our type  $H$  designation, with many permanently separated workers and labor force entrants who are hired as replacement workers likely also included in our H group.

A separate paper by Ahn (2014) provides further evidence in support of this interpretation. Ahn allows for both observed and unobserved heterogeneity by fitting models like the one developed here to subsets of workers sorted based on observable characteristics. She replaced our observation vector  $y_t$  based on aggregate unemployment numbers with  $y_{jt} = (U_{jt}^1, U_{jt}^{2.3}, U_{jt}^{4.6}, U_{jt}^{7.12}, U_{jt}^{13.4})'$  where  $U_{jt}^{2,3}$  for example denotes the number of workers with observed characteristic j who have been unemployed for 2-3 months, the idea being that within the group j there are new inflows  $(w_{iHt}$ and  $w_{jLt}$ ) and outflows  $(p_{jHt}$  and  $p_{jLt}$  of two unobserved types of workers. Of particular interest for the present discussion are the results when  $j$  corresponds to one of the 5 reasons for why the individual was looking for work. Panel A of Figure 10 displays Ahn's estimated values for new

 $^{7}$ ManpowerGroup's 2013 Talent Shortage Survey showed that there is growing shortage of interpersonal skills. Firms reported that a lack of interpersonal skills like communication, collaboration and creativity, and a disregard for punctuality, appearance and áexibility are important problems among the entry-level job candidates.

inflows of type L workers for each of the categories as well as the sum  $\sum_{j=1}^{5} \hat{w}_{jLt|T}$ . Our series  $\hat{w}_{Lt|T}$ inferred from aggregate data is also plotted again for comparison. The sum of micro estimates is very similar to our aggregate estimates, and the individual micro components reveal clearly that those we have described as type  $L$  workers primarily represent a subset of people who were either permanently separated from their previous job or are looking again for work after a period of having been out of the labor force.

Ahn  $(2014)$  also calculated the models' inferences about the total number of type L individuals in any given observable category  $j$  who were unemployed in month  $t$ . These are plotted in Panel B of Figure 10. Here the correspondence between the aggregate inference and the sum of the micro estimates is even more compelling, as is the conclusion that type  $L$  unemployed workers represent primarily a subset of those permanently separated from their old jobs or re-entering the labor force.

#### 4.2 Determinants of genuine duration dependence

Our results imply a nonmonotonic pattern for genuine duration dependence, with negative dependence (which we have referred to as scarring effects) dominating up to 6 months, modestly positive duration dependence (motivational effects) between 7-12 months, and much stronger motivational effects setting in after 12 months. It is interesting that different researchers have produced evidence of both negative and positive duration dependence using different methods and data sets. Kroft, Lange, and Notowidigido (2012), Kroft, Lange, Notowidigdo, and Katz (2013), Faberman and Kudlyak (2013), and Eriksson and Rooth (2014) all found evidence consistent with negative duration dependence, while Katz (1986), Katz and Meyer (1990a,b), Rothstein (2012), and Farber and Valletta (2013) found evidence of positive duration dependence attributable to eligibility for unemployment insurance. Kerchkoffs, De Neubourg and Palm (1994) and van den Berg and van Ours (1994) found nonmonotonic genuine duration dependence in data from the Netherlands.

It is interesting that our finding of positive duration dependence only begins to show up after 6-12 months, the times at which unemployment insurance (UI) is typically exhausted. It is possible to investigate further the role of UI by making use of the known time-variation in UI eligibility over our sample. An extension of eligibility would automatically be implemented (providing an additional 13 weeks of eligibility beyond the usual 26 weeks) in any state whose unemployment rate exceeds 6.5%, and an additional 20 weeks if the unemployment rate exceeds 8.0%.<sup>8</sup> Trying to perform a detailed analysis of all the state-by-state differences over time as well as additional changes in the legislation itself such as those enacted during the Great Recession would be extremely difficult. However, it is informative to conduct a quick test of whether UI eligibility may be a factor in our results by allowing the coefficients  $\delta_j$  that characterize genuine duration dependence to take on different values when the national unemployment rate is above  $6.5\%$ , times when it is likely that most workers automatically became eligible for extended UI benefits.

Let  $\delta_j^0$  be the coefficient on unemployment duration for months t in which the national unemployment rate  $(u_t)$  is 6.5% or below and  $\delta_j^E$  be the coefficient when  $u_t$  is greater than 6.5%. We re-estimated our state space model with (20) replaced by

$$
p_{i,t}(\tau) = \exp[-\exp(x_{i,t} + d_{\tau}^{j_t})]
$$

where  $j_t = 0$  if  $u_t \leq 6.5$  and  $j_t = E$  if  $u_t > 6.5$  with

$$
d_{\tau}^{j} = \begin{cases} \delta_{1}^{j}(\tau - 1) & \text{for } \tau < 6\\ \delta_{1}^{j}[(6 - 1) - 1] + \delta_{2}^{j}[\tau - (6 - 1))] & \text{for } 6 \leq \tau < 12\\ \delta_{1}^{j}[(6 - 1) - 1] + \delta_{2}^{j}[(12 - 1) - (6 - 1)] + \delta_{3}^{j}[\tau - (12 - 1)] & \text{for } 12 \leq \tau. \end{cases}
$$

Adding 3 new parameters  $(\delta_1^E, \delta_2^E, \delta_3^E)$  to the model results in an increase in the log likelihood of 44.7, leading to a rejection (p-value  $< 0.001$ ) of the null hypothesis that the values of  $\delta_j$  are constant over time in favor of the alternative that they vary over time depending on eligibility for unemployment benefits. The estimated values of  $\delta_i^j$  $i<sub>i</sub>$  are reported in Table 3. When the unemployment rate is below 6.5% and eligibility for UI ends after 6 months, the estimates imply that unemployment scarring operates up to 6 months  $(\delta_1^0 < 0)$  while motivational effects begin to

<sup>&</sup>lt;sup>8</sup>The extended benefit program is triggered when a state's insured unemployment rate (IUR), the ratio of insured unemployed workers to the total employment, or total unemployment rate (TUR) reach certain levels. All states must pay up to 13 weeks of extended benefits if the IUR for the previous 13 weeks is at least  $5\%$  and is  $120\%$  of the average of the rates for the same 13-week period in each of the two previous years. There are two other optional thresholds that states may choose (states may choose one, two, or none). If the state has chosen a given option, it would provide the following. Option 1: an additional 13 weeks of benefits if the state's IUR is at least  $6\%$ , regardless of previous years' averages. Option 2: an additional 13 weeks of benefits if the state's TUR is at least  $6.5\%$  and is at least 110% of the state's average TUR for the same 13 weeks in either of the previous two years; an additional 20 weeks of benefits if the state's TUR is at least  $8\%$  and is at least  $110\%$  of the state's average TUR for the same 13 weeks in either of the previous two years (Whittaker and Isaacs, 2014).

dominate after 6 months  $(\delta_2^0, \delta_3^0 > 0)$ . On the other hand, when UI eligibility is extended beyond 6 months, the scarring effect on those unemployed less than 6 months is similar to that seen in normal times  $(\delta_1^0 \simeq \delta_1^E)$  but motivational effects are not significant until after 12 months duration  $(\delta_2^E \simeq 0, \delta_3^E > 0)$ . This suggests that expiration of UI eligibility may be one factor in why we find positive duration dependence to set in after 6-12 months, reinforcing the conclusions of Katz (1986), Katz and Meyer (1990a,b), Rothstein (2012), and Farber and Valletta (2013).

## 5 Robustness checks

Here we examine how our conclusions would change under a number of alternative specifications, including changes in the unemployment measures used, alternative specifications of genuine duration dependence, possible correlations among the shocks, and reformulation of the model in terms of weekly rather than a monthly frequency. Further details for all of these alternative specifications are reported in the online appendix.

#### 5.1 Accounting for the structural break in the CPS survey

As noted in Appendix A, a redesign in the CPS survey in 1994 introduced a structural break with which any user of these data has to deal. Our baseline estimates reported in Section 3 use the adjustment suggested by Polivka and Miller (1998). Here we summarize how our results would change if we were to instead use the adjustment employed by Hornstein (2012).

Table 4 summarizes the implications of alternative specifications for what we see as the most important conclusions that emerge from our baseline analysis. The table breaks down the MSE of a forecast of the overall level of unemployment at 3-month, 1-year, and 2-year forecast horizons into the fraction of the forecast error that is attributable to various shocks. Column 1 gives the numbers implied by our baseline specification and highlights our key conclusion that inflows account for about half the variance at all horizons. Inflows and outflows for type  $L$  workers are of equal importance to those for type  $H$  workers at a 3-month horizon, but shocks to inflow and outflow probabilities for type L workers are the most important factors at a 1- or 2-year horizon.

Column 2 of Table 4 reports the analogous variance decompositions when we instead use Hornsteinís data adjustment. This produces very little change in these numbers. The most important difference we found between our results using the two data sets is that while the 7-12-month duration dependence parameter  $\delta_2$  is positive but statistically insignificant for our baseline specification, it is positive and statistically significantly greater than zero when Hornstein's adjustment is used.

Note that although we report Schwarzís (1978) Bayesian criterion in row 3 of Table 4, the value of this criterion for column 2 is not comparable with the others due to a different definition of the observable data vector  $y_t$ .

#### 5.2 Alternative specifications for genuine duration dependence

Our baseline specification assumed that a single parameter  $\delta_1$  described genuine duration dependence for any worker unemployed for less than 6 months. We also estimated a model in which each of the observed duration categories (2-3 months, 4-6 months, 7-12 months, and greater than 12 months) was characterized by a different genuine duration parameter, replacing (21) with

$$
d_{\tau} = \left\{ \begin{array}{ll} \delta_1^A(\tau-1) & \text{for } \tau < 3 \\[1mm] \delta_1^A(3-2) + \delta_1^B(\tau-2) & \text{for } 3 \leq \tau < 6 \\[1mm] \delta_1^A(3-2) + \delta_1^B(5-2) + \delta_2(\tau-5) & \text{for } 6 \leq \tau < 12 \\[1mm] \delta_1^A(3-2) + \delta_1^B(5-2) + \delta_2(11-5) + \delta_3(\tau-11) & \text{for } 12 \leq \tau. \end{array} \right.
$$

Adding this additional parameter  $\delta_1^B$  results in only a trivial improvement in the likelihood function and virtually no change in any of the variance decompositions, as seen in column 3 of Table 4.

Column 4 of Table 4 reports implications for variance decompositions from using the timevarying parameterization of genuine duration dependence whose parameter values were reported in Table 3. As noted in Section 4.2, allowing duration dependence to change with eligibility for extended unemployment insurance leads to a significant improvement in the log likelihood, and in fact is the specification among all those we considered that achieves the best value for the Schwarz criterion. Nevertheless, allowing for time-varying genuine duration dependence does not change any of our conclusions about the importance of different shocks in explaining unemployment dynamics, as seen in Table 4.

#### 5.3 Allowing for correlated shocks

Our baseline specification assumed that the shocks to  $w_{Lt}$ ,  $w_{Ht}$ ,  $p_{Lt}$  and  $p_{Ht}$  were mutually uncorrelated. It is possible to generalize this in a parsimonious way by allowing a factor structure to the innovations,  $\varepsilon_t = \lambda F_t + u_t$ , where  $F_t \sim N(0, 1)$ ,  $\lambda$  is a  $(4 \times 1)$  vector of factor loadings, and  $u_t$  is a  $(4 \times 1)$  vector of mutually uncorrelated idiosyncratic components with variance matrix  $E(u_tu_t')=Q$ :

$$
E(\varepsilon_t \varepsilon_t') = \lambda \lambda' + Q
$$
  

$$
Q = \begin{bmatrix} (q_H^w)^2 & 0 & 0 & 0 \\ 0 & (q_L^w)^2 & 0 & 0 \\ 0 & 0 & (q_H^x)^2 & 0 \\ 0 & 0 & 0 & (q_L^x)^2 \end{bmatrix}
$$

:

 $\overline{a}$ 

In this case the variance decomposition (25) becomes

$$
E(y_{t+s} - \hat{y}_{t+s|t})(y_{t+s} - \hat{y}_{t+s|t})' = \sum_{j=1}^{s} [\Psi_{s,j}(\xi_t, \xi_{t-1}, ..., \xi_{t-47+s})](\lambda \lambda' + Q)[\Psi_{s,j}(\xi_t, \xi_{t-1}, ..., \xi_{t-47+s})]'
$$
  

$$
= \sum_{j=1}^{s} [\Psi_{s,j}(\xi_t, \xi_{t-1}, ..., \xi_{t-47+s})] \lambda \lambda' [\Psi_{s,j}(\xi_t, \xi_{t-1}, ..., \xi_{t-47+s})]'
$$
  

$$
+ \sum_{j=1}^{s} \sum_{m=1}^{4} Q_m [\Psi_{s,j}(\xi_t, \xi_{t-1}, ..., \xi_{t-47+s}) e_m][\Psi_{s,j}(\xi_t, \xi_{t-1}, ..., \xi_{t-47+s}) e_m]
$$

for  $Q_m$  the row m, column m element of  $Q$ . Because the factor  $F_t$  has an effect on all four components, it is not possible to impute the term involving  $\lambda \lambda'$  to any one of the four shocks individually. However, we can calculate the portion of the MSE that is attributable to this aggregate factor along with those of each of the individual idiosyncratic shocks in  $u_t$ . This is reported in column 5 of Table 4, and variance decompositions are plotted in Figure 11. The aggregate factor by itself accounts for 36% of the MSE of a 3-month-ahead forecast of unemployment, and ináows and outflows of type  $H$  workers account for another 43%. The aggregate factor is strongly correlated with flows of type  $L$  workers. If we isolate the idiosyncratic component of each shock that is uncorrelated with the other three, shocks to outflows of type L workers account for only  $1/5$  of the 3-month-ahead forecast error and less than 1/3 of the 2-year-ahead forecast error. There is essentially no role for the idiosyncratic component of inflows of type L workers, since changes in these inflows are so highly correlated with the other three shocks. But these correlations are completely consistent with our interpretation that a key characteristic of economic recessions is a change in the reasons individuals become separated from their jobs. These changes affect all aspects of unemployment dynamics including a decrease in those individuals' subsequent success in exiting the pool of unemployed.

#### 5.4 Time aggregation

Focusing on monthly transition probabilities understates flows into and out of unemployment since someone who loses their job in week 1 of a month but finds a new job in week 2 would never be counted as having been unemployed. Shimer (2012) argued that this time-aggregation bias would result in underestimating the importance of outflows in accounting for cyclical variation in unemployment, and Fujita and Ramey (2009), Shimer (2012) and Hornstein (2012) all formulated their models in continuous time.

On the other hand, Elsby, Michaels and Solon (2009) questioned the theoretical suitability of a continuous-time conception of unemployment dynamics, asking if it makes any sense to count a worker who loses a job at 5:00 p.m. one day and starts a new job at 9:00 a.m. the next as if they had been unemployed at all. We agree, and think that defining the central object of interest to be the fraction of those newly unemployed in month  $t$  who are still unemployed in month  $t + k$ , as in our baseline model, is the most useful way to pose questions about unemployment dynamics. Nevertheless, and following Kaitz (1970), Perry (1972), Sider (1985), Haltiwanger and Plant (1987), Baker (1992), and Elsby, Michaels and Solon (2009) we also estimated a version of our model formulated in terms of weekly frequencies as an additional check for robustness.

We can do so relatively easily if we make a few simplifying assumptions. We view each month t as consisting of 4 equally-spaced weeks and assume that in each of these weeks there is an inflow of  $w_{it}$  workers of type i, each of whom has a probability  $p_{it}(0) = \exp[-\exp(x_{it})]$  of exiting unemployment the following week. This means that for those type  $i$  individuals who were newly unemployed during the first week of month t,  $w_{it}[p_{it}(0)]^3$  are still unemployed as of the end of the month. Thus for the model interpreted in terms of weekly transitions, equation (14) would be replaced by

$$
U_t^1 = \sum_{i=H,L} \{w_{it} + w_{it}[p_{it}(0)] + w_{it}[p_{it}(0)]^2 + w_{it}[p_{it}(0)]^3\} + r_t^1.
$$

Likewise (15) becomes

$$
U_t^{2.3} = \sum_{i=H,L} \sum_{s=1}^4 \left\{ w_{i,t-1} [p_{i,t-1}(1)]^{8-s} + w_{i,t-2} [p_{i,t-2}(2)]^{12-s} \right\} + r_t^{2.3}
$$

for  $p_{it}(\tau)$  given by (20)-(21) for  $\tau = 1, 2$ . Note that although this formulation is conceptualized in terms of weekly inflow and outflows  $w_i$  and  $p_i$ , the observed data  $y_t$  are the same monthly series used in our other formulations, and the number of parameters is the same as for our baseline formulation.

The weekly formulation achieves a slightly higher value for the likelihood function but does not materially change our substantive conclusions (see column 6 of Table 4 and Figures 12 and 13). Inflows and outflows are still of roughly equal importance, though whereas in our baseline model inflows accounted for more than half of the variance at all horizons, in the weekly specificaton inflows account for  $40\%$  of the 3-month-ahead MSE and only exceed  $50\%$  when one looks more than  $2-1/2$  years ahead. Our results are thus consistent with Shimer's conclusion that if one thinks in terms of higher frequency flows into and out of unemployment, the role of outflows becomes a little more significant, but do not change our overall conclusion.

## 6 Conclusion

People who have been unemployed for longer periods than others have dramatically different probabilities of exiting unemployment, and these relative probabilities change significantly over the business cycle. Even when one conditions on observable characteristics, unobserved differences across people and the circumstances under which they came to be unemployed are crucial for understanding these features of the data.

We have shown how the time series of unemployment levels by different duration categories can be used to infer inflows and outflows from unemployment for workers characterized by unobserved heterogeneity. In contrast to other methods, our approach uses the full history of unemployment data to summarize inflows and outflows from unemployment and allows us to make formal statistical statements about how much of the variance of unemployment is attributable to different factors as well as identify the particular changes that characterized individual historical episodes.

In normal times, 90% of those who are newly unemployed find jobs quickly. But in contrast to the conclusions of Hall  $(2005)$  and Shimer  $(2012)$ , we find that half the variance in unemployment comes from shocks to the number of newly unemployed, and a key feature of economic recessions is newly unemployed individuals who have significantly lower job-finding probabilities. Our inferred values for the size of this group exhibits remarkably similar dynamics to separate measures of the number of people who permanently lose their jobs. We conclude that recessions are characterized by a change in the circumstances under which people become unemployed that makes it harder for them to find new jobs.

We also attribute a smaller part of the differences in unemployment exit probabilities to an effect of duration itself. We find that this effect is correlated with changes in eligibility for unemployment insurance, with unemployment exit probabilities starting to rise once eligibility is exhausted.



Figure 1. Unemployment exit probability by duration of unemployment, June 1976-June 2013.  $p_t^1$ : exit probability for those unemployed for only one month;  $p_t^{2.3}$ : those unemployed 2-3 months;  $p_t^{4.6}$ : 4-6 months;  $p_t^{7.+}$ : more than 6 months.



Panel B:  $U^{7.12}$  and  $U^{13.+}$ 

<sup>1980</sup> <sup>1985</sup> <sup>1990</sup> <sup>1995</sup> <sup>2000</sup> <sup>2005</sup> <sup>2010</sup> <sup>0</sup>

500

1000

1500

Figure 2. Number of unemployed individuals (in thousands) by duration of time they have already been unemployed as of the indicated date. Panel A: those unemployed 1 month, 2-3 months, and 4-6 months. Panel B: those unemployed 7-12 months and more than 12 months.



Figure 3. Probability that a newly unemployed worker of each type will still be unemployed the following month. Figure shows  $\hat{p}_{it|T} (1)$  for  $i = L, H.$ 



Figure 4. Number of newly unemployed workers of each type. Figure shows  $\hat{w}_{it|T}$  for  $i = L, H$ .



Figure 5. Share of total unemployment accounted for by each type of worker.



Figure 6. Fraction of variance of error in forecasting total unemployment at different horizons attributable to separate factors. Horizontal axis: number of months ahead s for which the forecast is formed. Panel A: contribution of each of the factors  $\{w_{Ht}, w_{Lt}, x_{Ht}, x_{Lt}\}$  separately. Panel B: combined contributions of  $\{w_{Ht}, x_{Ht}\}$  and  $\{w_{Lt}, x_{Lt}\}$ . Panel C: combined contributions of  $\{w_{Ht}, w_{Lt}\}$  and  $\{x_{Ht}, x_{Lt}\}.$ 



Figure 7. Historical decompositions of four U.S. recessions.



Figure 8. Share of unemployment by reason (1994-2013 average).



Figure 9. Panel A: Ináows to unemployment by reason for unemployment. Panel B: Ináows of type L workers compared with workers newly unemployed due to permanent job loss or end of a temporary job. Dotted line: number of individuals unemployed for less than 5 weeks Solid line:  $\hat{w}_{Lt|T}$ . Panel C: Inflows of type H workers compared with total workers newly unemployed due to temporary layoffs, quits and entrance to the labor force. Dotted line: number of individuals unemployed for less than 5 weeks  $\,$  Solid line:  $\hat{w}_{Ht|T} .$ 



Panel A



Panel B

Figure 10. Inflows and total numbers of type  $L$  workers by reason of unemployment. Panel A: number of type L individuals who are newly unemployed by reason of unemployment along with the sum across reasons (thick fuchsia) and inference based on uncategorized aggregate data (dashed black). Panel B: number of type L workers who have been unemployed for any duration by reason of unemployment along with the sum across reasons (thick fuchsia) and inference based on uncategorized aggregate data (dashed black). Source: Ahn (2014).



Figure 11. Fraction of variance of error in forecasting total unemployment at different horizons attributable to separate factors in the model with correlated errors. Horizontal axis: number of months ahead s for which the forecast is formed. Panel A: contribution of the aggregate factor  $F_t$  along with the idiosyncratic components of  $\{w_{Ht}, w_{Lt}, x_{Ht}, x_{Lt}\}$  separately. Panel B: combined contributions of idiosyncratic components of  $\{w_{Ht}, x_{Ht}\}$  and  $\{w_{Lt}, x_{Lt}\}$  along with aggregate factor  $F_t$ . Panel C: combined contributions of idiosyncratic components of  $\{w_{Ht}, w_{Lt}\}$  and  $\{x_{Ht}, x_{Lt}\}$ along with aggregate factor  $F_t$ .



Figure 12. Fraction of variance of error in forecasting total unemployment at different horizons attributable to separate factors for the weekly transition model. Horizontal axis: number of months ahead s for which the forecast is formed. Panel A: contribution of each of the factors  ${w_{Ht}, w_{Lt}, x_{Ht}, x_{Lt}}$  separately. Panel B: combined contributions of  ${w_{Ht}, x_{Ht}}$  and  ${w_{Lt}, x_{Lt}}$ . Panel C: combined contributions of  $\{w_{Ht}, w_{Lt}\}$  and  $\{x_{Ht}, x_{Lt}\}$ .



Figure 13. Historical decompositions of four recessions implied by the weekly model.

$\sigma_L^w$	$0.0131***$	$R_1$	$1.00e-5***$	$\delta_1$	$-0.0884***$	
	(0.0010)		$(1.95e-6)$		(0.0069)	
$\sigma_H^w$	$0.0202***$	$R_{2,3}$	$1.00e-5***$	$\delta_2$	0.0171	
	(0.0008)		$(3.71e-6)$		(0.0218)	
$\sigma_L^x$	$0.0463***$	$R_{4.6}$	$0.0592***$	$\delta_3$	$0.1393***$	
	(0.0033)		(0.0021)		(0.0234)	
$\sigma_H^x$	$0.0096***$	$R_{7,12}$	$0.0025***$			
	(0.0004)		(0.0003)			
		$R_{13+}$	$0.0012***$			
			(0.0002)			
No. of Obs.		445				
Log-likelihood		6027.30				

Table 2. Parameter estimates for the baseline model

Notes to Table 2. White (1982) quasi-maximum-likelihood standard errors in parentheses.

$\sigma_L^w$	$0.0129***$	$R_1$	$1.00e-5***$	$\delta_1^0$	$-0.0914***$
	(0.0010)		$(2.24e-6)$		(0.0080)
$\sigma_H^w$	$0.0200***$	$R_{2.3}$	$1.00e-5***$	$\delta_2^0$	$0.0939***$
	(0.0008)		$(3.80e-6)$		(0.0341)
$\sigma_L^x$	$0.0446***$	$R_{4.6}$	$0.0484***$	$\delta_3^0$	$0.0591*$
	(0.0031)		(0.0043)		(0.0312)
$\sigma_H^x$	$0.0097***$	$R_{7.12}$	$0.0024***$	$\delta_1^E$	$-0.0730***$
	(0.0004)		(0.0003)		(0.0080)
		$R_{13+}$	$0.0018***$	$\delta_2^E$	0.0332
			(0.0004)		(0.0251)
				$\delta_3^E$	$0.1372***$
					(0.0271)
No. of Obs.		445			
Log-Likelihood		6071.98			

Table 3. Parameter estimates for model with time-varying genuine duration dependence.

	Source	<b>Baseline</b>	Alternative	Unconstrained	Time-varying	Correlated	Weekly
	of shocks	model	data <sub>set</sub>	$\operatorname{GDD}$	$\operatorname{GDD}$	shocks	frequency
		(1)	(2)	(3)	(4)	(5)	(6)
No. of param.		$12\,$	$12\,$	$13\,$	15	$16\,$	$12\,$
Log-Likelihood		6027.30	5738.84	6027.60	6071.98	6049.03	6028.91
$\rm SIC$		$-11,981.41$	$-11,404.51$	$-11,975.92$	$-12,052.49$	$-12,000.49$	$-11,984.64$
$3\ \mathrm{month}$	$\cal F$	$\overline{\phantom{a}}$	$\overline{\phantom{a}}$		$\overline{\phantom{a}}$	$0.359\,$	
	$w_{\cal L}$	0.234	0.267	$0.234\,$	$0.239\,$	0.001	0.184
	$w_H$	$0.272\,$	0.240	$0.272\,$	$0.275\,$	$0.234\,$	0.217
	$p_{\scriptstyle L}$	0.237	$0.263\,$	$0.237\,$	0.224	0.211	0.205
	$p_H$	$0.257\,$	$0.231\,$	$0.257\,$	0.262	$0.194\,$	$0.394\,$
	Inflows	0.506	$0.507\,$	$0.506\,$	$0.514\,$	$0.235\,$	$0.401\,$
	L group	0.471	0.529	$0.471\,$	0.463	$0.212\,$	0.389
1 year	$\cal F$	$\overline{\phantom{a}}$				$0.509\,$	$\overline{a}$
	$w_{\cal L}$	0.424	0.420	0.428	0.447	$0.002\,$	0.326
	$w_{\cal H}$	0.112	0.092	$0.113\,$	$0.113\,$	0.098	$\,0.093\,$
	$p_{\scriptstyle L}$	0.347	$\!.391$	$\,0.343\,$	0.325	0.301	0.373
	$p_H$	$0.116\,$	$0.097\,$	$0.116\,$	$0.115\,$	$0.090\,$	0.209
	Inflows	$0.537\,$	$0.512\,$	$0.541\,$	0.560	$0.100\,$	0.419
	L group	$0.772\,$	0.811	0.771	0.772	$0.303\,$	0.699
$2$ year	$\cal F$	$\overline{\phantom{a}}$	$\overline{\phantom{a}}$		$\overline{\phantom{a}}$	0.585	$\overline{a}$
	$w_{\cal L}$	$0.516\,$	$0.472\,$	$0.522\,$	0.543	$0.003\,$	0.409
	$w_H$	$0.071\,$	$0.059\,$	$0.072\,$	$0.071\,$	0.062	$0.062\,$
	$p_{\scriptstyle L}$	$0.338\,$	0.407	0.331	$0.312\,$	0.291	$0.385\,$
	$p_H$	$0.075\,$	0.063	$0.075\,$	0.074	$0.059\,$	0.144
	<b>Inflows</b>	0.587	$0.530\,$	0.594	$0.614\,$	$0.065\,$	0.471
	L group	$0.854\,$	$0.878\,$	0.854	$0.855\,$	$0.294\,$	0.794

Table 4. Comparison of variance decomposition across different models

Notes to Table 4. SIC calculated as minus twice the log likelihood plus number of parameters k times log of sample size  $(T = 445)$ . Note that SIC for column (3) is not comparable with the others because the data on  $y_t$  are different.  $\,F$  denotes the aggregate factor.

# Appendix

A. Measurement issues and seasonal adjustment

The number unemployed for less than 5 weeks, for between 5 and 14 weeks, 15 and 26 weeks and for longer than 26 weeks are published by the Bureau of Labor Statistics. To decompose the number unemployed for longer than 26 weeks into that with duration between 27 and 52 weeks and with longer than 52 weeks, we used CPS microdata publicly available at the NBER website (http://www.nber.org/data/cps\_basic.html). An individual in the sample reports his or her duration of unemployment, if the person's labor force status is unemployment. Since the CPS is a probability sample, each individual is assigned a unique weight which is used to produce the aggregate data. From the CPS microdata, we first compute the shares out of the unemployed with duration longer than 26 weeks of unemployed individuals whose duration of unemployment is between 27 and 52 weeks and is longer than 52 weeks. Next, we multiplied the share of each group by the published number unemployed with duration longer than 26 weeks to calculate the number unemployed for between 27 and 52 weeks and for longer than 52 weeks. We take this step because the published number unemployed with duration longer than 26 weeks is different from that directly computed from the CPS microdata, although the difference is subtle. The difference arises because the BLS imputes the numbers unemployed with different durations to various factors, e.g., correction of missing observations.

An important issue in using these data is the redesign of the CPS survey in 1994. Before 1994, individuals were always asked how long they had been unemployed. After the redesign, if an individual is unemployed for two consecutive months, then her duration is recorded automatically as the sum of her duration last month and the number of weeks between the two months' survey reference periods. Note that if an individual was unemployed during each of the two weeks surveyed, but worked at a job in between, that individual would likely report duration of unemployment to be less than 5 weeks before the redesign, but the duration would be imputed to be a number greater than 5 weeks after the redesign.

As a result of this, many economists assume that the number unemployed less than 5 weeks is understated in the post-1994 data (Polivka and Miller,1998; Abraham and Shimer, 2002; Elsby, Michaels and Solon, 2009; Shimer, 2012; Hornstein, 2012). A common solution is to multiply the number of people unemployed less than 5 weeks by some factor for the post-1994 data. Polivka and Miller (1998) suggested a factor of 1.205. Elsby, Michaels and Solon (2009) suggested 1.154 and Shimer (2012) suggested 1.106. Hornstein (2012) increased the number unemployed with duration less than 5 weeks in the month  $t$  by  $10\%$  and subtracted the increase from the number unemployed with duration between 2 and 3 months in the month  $t$ . Following Polivka and Miller (1998), in our baseline analysis we multiply the reported number of unemployed with 1 month duration by 1.205. This increases the total number unemployed by 20.5% of  $U^1$ . To maintain the total number unemployed, we multiply  $\frac{U_t}{U_t+0.205U_t^1}$  by the number of each duration group, where  $U_t$  denotes the total number unemployed.

A final issue is how to handle seasonality in the raw data. In our study we have used 12-month moving averages of the raw data which not only de-seasonalizes in a model-free and parsimonious way but also helps control for measurement error in the original data.

## B. Estimation algorithm

The system (23) and (14)-(18) can be written as

$$
x_t = F x_{t-1} + v_t
$$

$$
y_t = h(x_t) + r_t
$$

for  $x_t = (\xi'_t, \xi'_{t-1}, ..., \xi'_{t-47})', E(v_tv'_t) = Q, \text{ and } E(r_tr'_t)$ The function  $h(.)$  as well as elements of the variance matrices R and Q depend on the parameter vector  $\theta =$  $(\delta_1, \delta_2, \delta_3, R_1, R_{2.3}, R_{4.6}, R_{7.12}, R_{13+}, \sigma_L^w, \sigma_H^w, \sigma_L^x, \sigma_H^x)'$ . The extended Kalman filter (e.g., Hamilton, 1994b) can be viewed as an iterative algorithm to calculate a forecast  $\hat{x}_{t+1|t}$  of the state vector conditioned on knowledge of  $\theta$  and observation of  $Y_t = (y'_t, y'_{t-1}, ..., y'_1)'$  with  $P_{t+1|t}$  the MSE of this forecast. With these we can approximate the distribution of  $y_t$  conditioned on  $Y_{t-1}$  as  $N(h(\hat{x}_{t|t-1}), H_t'P_{t|t-1}H_t + R)$  for  $H_t = \partial h(x_t)/\partial x_t'|x_t = \hat{x}_{t|t-1}$  from which the likelihood function associated with that  $\theta$  can be calculated and maximized numerically. The forecast of the state vector can be updated using

$$
\hat{x}_{t+1|t} = F\hat{x}_{t|t-1} + FK_t(y_t - h(\hat{x}_{t|t-1}))
$$

$$
K_t = P_{t|t-1}H_t(H_t'P_{t|t-1}H_t + R)^{-1}
$$
  

$$
P_{t+1|t} = F(P_{t|t-1} - K_tH_t'P_{t|t-1})F' + Q.
$$

A similar recursion can be used to form an inference about  $x_t$  using the full sample of available data,  $\hat{x}_{t|T} = E(x_t|y_T, ..., y_1)$  and these smoothed inferences are what are reported in any graphs in this paper; see our online appendix for further details.

Prior to the starting date June 1976 for our sample, BLS aggregates are available but not the micro data that we used to construct  $U_t^{13,+}$ . For the initial value for the extended Kalman filter, we estimate  $\hat{x}_{1|0}$  from pre-sample values for aggregates as described in the online appendix. By setting large diagonal elements of  $P_{1|0}$ , the particular value of  $\hat{x}_{1|0}$  has little influence on any of the results.

Maximization of the likelihood function  $\sum_{t=1}^{T} \log f(y_t|Y_{t-1})$  is made difficult by non-convexity and multimodality of the likelihood surface. We developed a new algorithm, which we call a PZ algorithm, which helped considerably in the estimation. The parameters in  $\theta$  are divided into several sets (e.g.,  $\theta^A$  and  $\theta^B$ ) and estimated by alternating between estimating one set while holding the others constant. Newton-Raphson was used to obtain a starting value for  $\theta^A$  given  $\theta^B$ and then pattern search (a derivative-free algorithm) was used to find a maximum with respect to  $\theta^A$ . Given an estimate for  $\theta^A$ , we then estimate  $\theta^B$  given  $\theta^A$  and iterate. This algorithm performs better than other algorithms in that the estimated parameters do not depend on starting values and the likelihood value found by the algorithm is greater than those found by other algorithms. In simulation exercises, our algorithm found the true global optimum in every case that we consider while other search algorithms often fail to find one given the same set of starting values. Further details on the algorithm are provided in the online appendix.

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