Humbug production function

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Neoclassical economics has always tried to portray wages and profits as mere technical variables. At an aggregate level, this is accomplished by connecting labour and capital to output through a 'well-behaved' aggregate production function, with the marginal products of labour and capital equal to the wage rate and profit rate, respectively. Thus in competitive equilibrium each social class is pictured as receiving the equivalent of the marginal product of the factor(s) it owns (Shaikh, 1980).

The original optimism that aggregate production functions and their corresponding marginal productivity rules could be derived from more detailed general equilibrium models eventually gave way to the sobering realization that the conditions for any such a derivation were 'far too stringent to be believable' (Fisher, 1971). Yet neoclassical economists continue to use aggregate production functions, apparently because they seem to fit the data well and their estimated marginal products closely approximate the observed wage and profit rates (so-called factor prices).

This apparent empirical strength of aggregate production functions is often interpreted as support for neoclassical theory. *But there is neither theoretical nor empirical basis for this conclusion.* We already know that such functions cannot be derived theoretically, except under conditions which neoclassical theory itself rejects (e.g. the simple labour theory of value) (Garegnani, 1970). Moreover, Fisher (1971) discovered through simulation studies that the aggregate data generated by microeconomic production functions were not generally well fitted by aggregate production functions; thar the functions which did best fit this data are not neoclassical in nature (this is a common finding, e.g. Walters, 1963); and that in simulation runs where the wage share happened to be roughly constant and aggregate Cobb–Douglas production functions happened to work well, this goodness of fit was puzzling because it held even when the theoretical conditions for aggregate production functions were flagrantly violated.

Shaikh (1974. 1980) has shown that this last result is simply an artifact of the constancy of the wage share. To see this, let r_t represent the rate of profit, and q_t, w_t, k_t the per worker net output, wages and capital, respectively, all at time t. Then the national accounting identity $q_t = w_t + r_t k_t$ can be differentiated to yield percentage rates of change q', w', etc., weighted by the profit share $s_t = r_t k_t / q_t$ and the labour share $1 - s_t = w_t / q_t$:

$$q_t' = B_t' + s_t k_t', \quad \text{where} \quad B_t' = (1 - s_t) w_t' + s_t r_t'. \tag{1}$$

The preceding relation says nothing about the nature of the underlying economic processes, since it is derived from an identity. But if social forces happen to produce a stable profit (and hence wage) share. so that $s_t = s$ (a constant), we can immediately integrate both sides of (1) to get

$$q_t = A_t k_t^s, \quad \text{where} \quad A_t = C\, e^{\int B' dt}, \; C = \text{a constants.} \tag{2}$$

Equation (2) looks like an aggregate Cobb Douglas production function with constant returns to scale, marginal products equal to factor prices, and a technical change shift parameter A_t. It will even seemingly reflect neutral technical change if the rate of change B_t' can be expressed as a function of time. And yet *it is not a production function at all, hut rather merely the algebraic expression of any social forces resulting in a constant share even when the underlying processes are definitely not neoclassical in nature.* To Illustrate this, we will now demonstrate that even a very simple 'anti neoclassical' (Robinsonian) economy will fit such a function.

Consider an economy at time t_0, in which all possible techniques of production are dominated by a *single* linear technique (linear because capital–labour ratios are equal across all sectors). With one dominant technique, there is no neoclassical substitutability among techniques, and the linear wage-profit curve of the dominant technique is also the wage-profit frontier for the whole economy (the line $q_0 R$ in Figure 1, for the given time period). Because q, k and R (net output/capital) are all *constant* along the wage-profit frontier, the marginal products of labour and capital therefore cannot even be defined. The determination of the so-called factor prices w and r cannot possibly be tied to some corresponding marginal products. Lastly, because q and k are constant for any given frontier, a frontier such as $q_0 R$ in Figure 1 contributes only a single point q_0, k_0 to the q_t, k_t space in Figure 2.

Now consider Harrod-neutral technical change, in which both output per worker q_t and the capital–labour ratio k_t rise at the same rate, so that the output&capital ratio R remains constant:

$$q_t / q_0 = k_t / k_0 = e^{at}, \quad \text{and since} \quad q_0 / k_0 = R, \; q_t / k_t = R \tag{3}$$

This is depicted in Figure 1 by the successive wage–profit frontiers and in Figure 2 by the corresponding (solid) straight line q_t of slope R.

If we were simply concerned with the best relation between inputs and output, then the *true* relation $q_t = Rk_t$ would be the correct one. But within neoclassical theory, such a fitted function would imply a constant marginal product of capital,

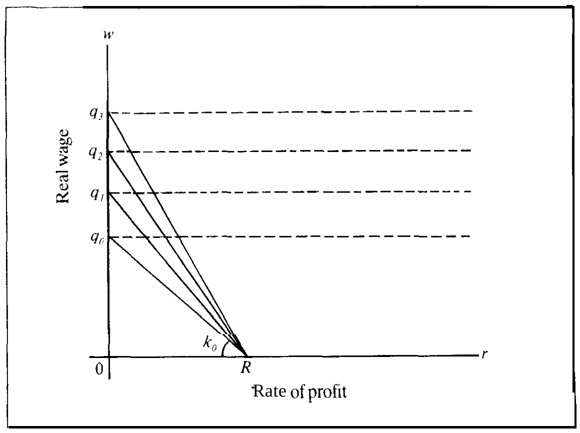

Figure 1

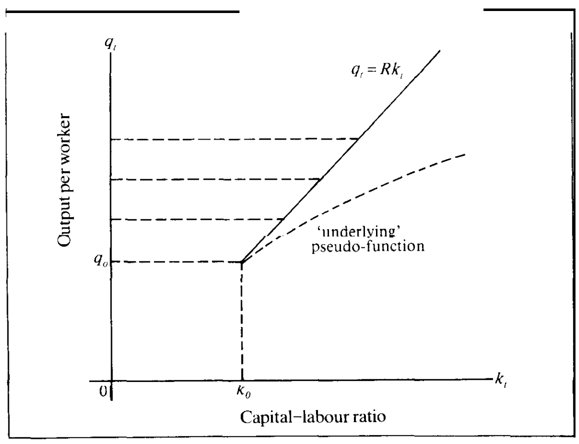

Figure 2

a zero marginal product of labour (Allen, 1967, pp. 45–6), and no technical change (since the 'shift parameter' R is constant). A good neoclassical would therefore have to reject this best (and true) fitted function in favour of some more 'appropriate' functional form (Fisher, 1971, pp. 312-13). How then might an aggregate production function fare in our anti-neoclassical world?

We have already assumed a constant profit share $r_t k_t/q_t = s$, and since the output-capital ratio $q_t/k_t = R$ is constant (equation (3)), it follows that the rate of profit $r_t = sR$ is constant. Similarly, the assumption of a constant wage share $w_t/q_t = 1 - s$ and a steadily growing output per worker $q_t = q_0 e^{at}$ (equation (3)), implies a steadily growing real wage $w_t = (1-s)q_0 e^{at}$. All this allows us to solve explicitly for B_t' and A, in equations (1)–(2):

$$B_t' = (1-s)w_t' + sr_t' = (1-s)a \tag{4}$$

$$q_t = C\ e^{(1-s)at} k_t^s, \quad \text{since} \quad A_t = C\ e^{(1-s)at} \tag{5}$$

Thus when the wage share is constant, *even a fixed proportion technology undergoing Harrod-neutral technical change is perfectly consistent with an aggregate pseudo-productionfunction* (equation (5)). This is, however, a law of algebra, not a law of production. The above reasoning has been shown to have grave implications for production function studies (Shaikh, 1980). For instance, Solow's (1957) so-called seminal technique for assessing technical change amounts to decomposing the true production relation into an 'underlying' pseudo-production function and a residual A, whose rate of change is then taken to measure technical progress (Figure 2). But this measures nothing more than distributional changes, since B_t is simply the weighted average of the rates of change of observed wage and profit rates (equations (l)-(2)). Similarly, Fisher's previously mentioned puzzle concerning the empirical strength of aggregate Cobb-Douglas production functions can be shown to be an artifact of the stability of the wage share over those particular simulation runs. Last, and perhaps most strikingly, it is interesting to note that even data points which spell out the word 'HUMBUG' can be well fitted by a Cobb-Douglas production function apparently undergoing neutral technical change and possessing marginal products equal to the corresponding 'factor prices'! Surely there is a message in this somewhere?

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