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## ECONOMIC POLICY IN A GROWTH CONTEXT: A CLASSICAL SYNTHESIS OF KEYNES AND HARROD

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### ABSTRACT

This paper clarifies key differences between Harrodian and Keynesian theories and policies, and develops a classical alternative to both. The stability of the Harrodian warranted path is proved, and the Keynesian paradox of thrift is shown to be transient. Distinct Harrodian fiscal policies are derived, and Post-Keynesian debates about Harrodian dynamics are addressed. Finally, it is argued that business and household savings are fundamentally different, and it is shown that if the business savings rate responds at all to the investment-savings gap, it becomes possible to have both profit-driven accumulation as in Keynes and normal capacity growth as in Harrod.

### 1. INTRODUCTION

Keynesian theories of effective demand are predicated on the notion that short-run equilibrium output is driven by exogenous demand, via the workings of the multiplier. In the basic case, exogenous demand consists solely of fixed investment, and the multiplier is the reciprocal of an exogenously given private savings rate:  $Y = I/s$ . With both investment<sup>1</sup> and the savings rate given in the short run, it is output that must adjust to make savings equal investment (supply equal demand) in the short run. Two central propositions follow. First, a rise in investment will raise equilibrium output by a multiplied level. Second, a fall in the savings rate will also raise output. The latter is the

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<sup>1</sup> Since investment is said to depend on long-term expectations of demand and profits, it is sometimes emphasized that investment can change in a volatile manner from one short run to another.

Keynesian *paradox of thrift*. The basic framework is easily extended to the incorporate government spending, exports, taxes and imports, the former two entering as further components of exogenous demand, and the latter two typically as components of the multiplier (via the tax rate and the import propensity). Then increases in government spending and exports will raise output and employment, as will reductions in tax rates and import propensities. Put differently, sustained increases in budget deficits and trade surpluses will always increase nominal output, and will increase real output whenever labor is not fully employed.

Harrod adopts the Keynesian framework for the analysis of the short run. But a crucial divergence emerges when he turns to the longer run. In the basic Keynesian case ( $Y = I/s$ ), there is no output growth unless exogenous investment just happens to be rising over time. A rise in the given savings rate ( $s$ ) will then lower the level of output, but other things being equal, it will leave its rate of growth unchanged. For Harrod, it is a fundamental mistake to treat investment as exogenous. Both he and Domar point out that investment not only creates demand but also simultaneously expands capacity. This means that the path of investment can be sustained only if the resulting growth in capacity is matched by a corresponding growth in demand. Otherwise the very expectations that motivated the investment path will be falsified, and investment will adjust accordingly. On this basis, Harrod shows that the *only self-consistent path* for investment is one that generates output (and investment) growth at a particular 'warranted' rate ( $s \cdot R_n$ ), where  $s$  is the private savings rate and  $R_n$  is the normal capacity/capital ratio (see section 2 for details). Hence if  $s$  and  $R_n$  are exogenous, output growth is exogenous precisely because investment growth is endogenous. This in turn implies that while a fall in the savings rate may raise the short-run level of output via the multiplier, it will also *lower* the trend rate of growth by lowering the warranted rate. *This is the Harrodian paradox of growth*. Third, and most famously, this trend rate of growth will generally differ from the 'natural rate of growth' needed to maintain a constant rate of employment.<sup>2</sup> This is Harrod's second great puzzle, the first one being the apparent instability of the warranted path.

The critical differences between the two approaches carry over when we incorporate government spending and export demand into the analysis (Moudud, 2002). In the Keynesian approach, growth can be government-led

<sup>2</sup> The traditional Keynesian solution to this problem has been to suppose that the aggregate savings rate also depends on the distribution of income between wages and profits, with the latter adjusting over the long run so as to make the warranted rate of growth equal to the natural rate (Shaikh, 2003, pp. 132–5).

or export-led, should investment growth be inadequate. Since export-led growth requires success in the world market, which may not be forthcoming, fiscal policy emerges as a central means of regulating the economy. But things turn out rather differently in the Harrodian story. Three results are striking. First, the previously mentioned paradox of growth applies also to the tax rate and the import propensity: a fall in either may initially stimulate the economy (the Keynesian paradox of thrift), but it will ultimately slow down output growth and hence undermine and even reverse the original stimulus. Second, if government spending happens to grow at an 'excessive' rate, so that the share of government spending in output rises, this will reduce the long-term growth of output, and produce rising budget deficits or falling surpluses. The paper develops the formal conditions that define an 'excessive' rate of growth. Third, the fiscal policy necessary to maintain a socially desired rate of employment is correspondingly more complicated, depending not only on the tax rate and the level of government spending, but also on the growth rate of the latter. Nonetheless, fiscal policy provides a means of adapting the warranted rate of growth to the 'natural rate', which addresses Harrod's second puzzle. So in the end we come back to the original Keynesian claim: although full employment is not automatic, the state might nonetheless be able to keep it in view.

These properties will be analyzed in more detail in the rest of the paper. We will see that what matters is the behavior of particular *ratios*: savings, import and tax rates on one side, and investment, government and export shares in output on the other. Changes in wage, interest and exchange rates can modify the basic patterns only insofar as they alter the foregoing ratios. The study of these latter effects is beyond the scope of the present work, whose principal aim is to clarify the essential differences between the Keynesian and Harrodian frameworks and to outline a synthesis of the two. A great virtue of Keynesian theory is that it emphasizes the regulation of accumulation by profitability, which is also an essential theme in classical and Marxian theory. An equally great virtue of Harrodian theory is its insistence that sustainable accumulation must gravitate around a normal rate of capacity utilization, the latter being defined by minimum costs. However, since both approaches rest on the exogeneity of savings propensities, it appears that one must choose either Keynesian profit-driven growth at arbitrary rates of capacity utilization, or Harrodian thrift-driven growth at normal rates of capacity utilization. Indeed, the Post-Keynesian (PK) literature has been greatly concerned with just this dichotomy. But this Delphic choice becomes unnecessary once one recognizes that Marx's theory of business savings provides a straightforward way to reconcile profit-driven growth with the requirement for normal capacity utilization.

## 2. THE BASIC BALANCES IN THE COMMODITY MARKET

The aggregate commodity market can be characterized by two basic balances. Aggregate excess demand ( $E$ ) measures the balance between aggregate demand and supply, and short-run equilibrium holds approximately if there is some (relatively fast) process that makes excess demand fluctuate around zero. On the other hand, the ratio of output to capacity measures the rate of capacity utilization ( $u$ ), and long-run equilibrium holds if there is some (relatively slow) process that makes the actual rate of capacity utilization fluctuate around the desired rate. Keynesian economics typically concentrates on short-run equilibrium, while Harroddian economics typically concentrates on long-run equilibrium.

At the aggregate level, let  $Z$  = demand,  $Y$  = supply,  $I$  = investment,  $S$  = private savings,  $G$  = government spending,  $T$  = taxes,  $X$  = exports and  $IM$  = imports. Then the level of excess demand  $E$  can be written as the sum of demand injections ( $I + G + X$ ) minus demand leakages ( $S + T + IM$ ).

$$\bar{E} \equiv Z - Y = (I + G + X) - (S + T + IM) = (I - S) + (G - T) + (X - IM) \quad (1)$$

Any given degree of excess demand will react back on the level of output and hence income, which in turn will change all endogenous variables. For the sake of simplicity in exposition, we will assume that the leakages are all proportional to income ( $S = s_p \cdot Y$ ,  $T = \theta \cdot Y$ ,  $IM = \mu \cdot Y$ ), and that the injections  $G$  and  $X$  are exogenous (but not necessarily constant). Thus in short-run equilibrium ( $E = 0$ ) we can write the familiar multiplier relation

$$Y = (I + G + X) / s \quad [\text{General Keynesian multiplier}] \quad (2)$$

where  $s = s_p + \theta(1 - s_p) + \mu$ ,  $s_p$  = the private savings rate out of total income,  $\theta$  = the tax rate,  $\mu$  = the import propensity. With a balanced budget ( $G = T$ ) and balanced trade ( $X = IM$ ),<sup>3</sup> short-run equilibrium implies  $I - S = 0$ , so

$$Y = I / s_p \cdot (1 - \theta) \quad [\text{Keynesian multiplier with balanced budget and trade}] \quad (2')$$

Equation (2') highlights the central role of investment as the fundamental driver of demand and output within standard Keynesian-type (KT) theories

<sup>3</sup> It is interesting to note that equation (2') would also obtain under the 'New Cambridge' hypothesis of 'Twin Deficits' which claimed on empirical grounds that private sector expenditures roughly equal (after tax) private sector receipts:  $(C + I) - (Y - T) = I - [(Y - T) - C] = I - S = 0$  (Fetherston and Godley, 1978; Godley, 2000). Then from equation (1), short-run equilibrium ( $E = 0$ ) implies that  $(G - T) + (X - M) = 0$ , which says that a government sector deficit ( $G - T > 0$ ) must be offset by a foreign sector deficit ( $X - M < 0$ ).

of effective demand.<sup>4</sup> In the Keynesian framework, investment is typically treated as exogenous, albeit volatile, in the short run. Harrod's central point is that such a treatment is inadequate. It is commonplace to note that investment is motivated by the profits expected on the new plant and equipment under consideration. But implicit in any such relation is a *desired rate of utilization* of the prospective new capital (Garegnani, 1992, pp. 150–1): given some expected growth in demand<sup>5</sup> and the capital intensity of new technology, the amount of investment that is justified depends on the economically desired rate of utilization.

Let us define normal capacity output  $Y_n$  as the normal (potential) output corresponding to the desired utilization rate of the existing capital stock, i.e. the utilization rate at which the operation of a given plant is most profitable in the long run. Three things should be noted. First, that normal capacity output is generally quite different from engineering capacity ( $Y_{max}$ ), which is the maximum sustainable output of a given structure of plant and equipment. Second, within the limits of engineering capacity, production can be expanded or contracted by varying the length of the working day (overtime, more shifts, etc.), and by varying its intensity (speed-up, etc.). Thus any given capital stock implies a variable range of utilizations and hence of outputs. Third, since unit costs will also vary with utilization, not all utilization levels will be equally profitable, and the lowest cost point of these defines economic capacity. It is *this* point that determines whether actual or prospective capacity is under- or over-utilized (Harrod, 1952, pp. 150–1; Foss, 1963, p. 25; Kurz, 1986, pp. 37–8, 43–4; Shapiro, 1989, p. 184). Hence it is this point that will serve as the crucial regulator of changes in capacity, i.e. of investment itself.

To illustrate these notions, consider an average plant with engineering capacity  $Y_{max} = 45$ , which can be potentially operated at up to two equal shifts, each shift producing up to 20 units of output. Moreover, in keeping with standard KT assumptions let us suppose that unit prime costs (average variable costs, AVC) are constant at all levels of output within a given shift (Andrews, 1949, p. 80; Lavoie *et al.*, 2004, p. 129). The first shift has overhead costs = 100 and unit prime costs = 20, while the second shift has a 30 per cent premium on both of these items. The resulting average fixed cost (AFC), AVC and average cost (AC) curves are depicted in figure 1 as solid lines. Under the assumed cost structures, the lowest average cost point is at the end of the first shift at  $Y = 20$ . This defines *long-run economic capacity*, i.e. the

<sup>4</sup> An exception to the primacy of investment as the driver of output in equation (2') is Thirwall's argument that exports drive output (Thirwall, 2002, chapter 4).

<sup>5</sup> New demand for a firm's products can come from its competitors and/or from an expansion of the market.

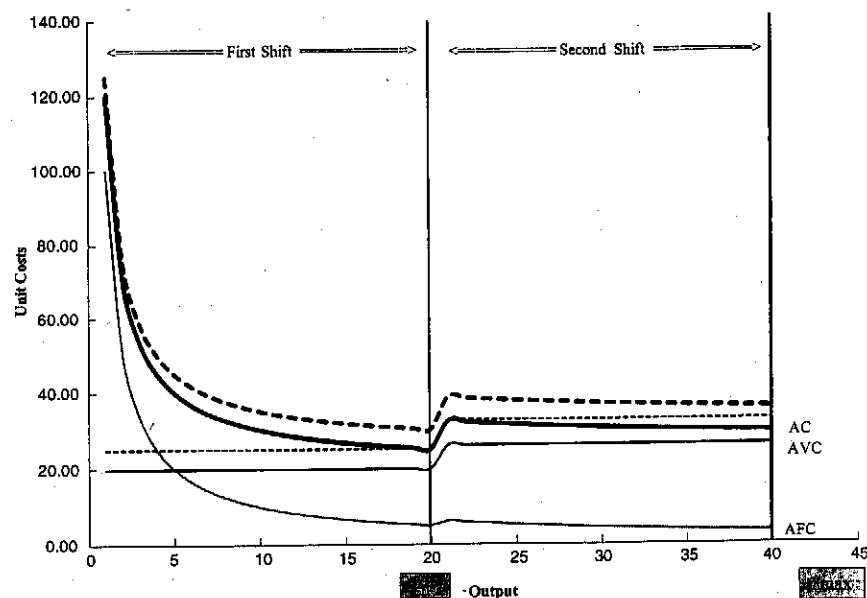


Figure 1. Output, costs and normal capacity utilization.

normal level of output ( $Y_n = 20$ ) and normal rate of capacity utilization ( $u_n = Y_n/Y_{max} = 44$  per cent).

The fact that the normal rate of capacity utilization can be well below the engineering rate tells us that firms typically have a great deal of short-term unused capacity. But this is hardly evidence of persistent excess capacity, for 'an optimal amount of [unused capacity] exists and depends on economic costs' (Winston, 1974, p. 1301). What matters from the point of view of capitalistic competition is the lowest cost point of utilization. For instance, if demand happens to be outrunning normal capacity, the average firm can always move to a second shift in the short run. But since competition among producers favors those with lower costs, it behooves firms to try to expand their capacity relative to demand so as to return to the lowest cost point of production. If the resulting macroeconomic process is stable (see section 5), normal capacity will grow faster than demand and the rate of capacity utilization will fall back towards normal. Thus over the long run, capacity utilization will continually fluctuate around the normal rate. At a more concrete level, the output corresponding to normal capacity utilization may include some desired level of normal capacity reserves needed to meet demand fluctuations and to survive against competitors, so that the normal competitive level of utilization may be somewhat below the exact 'ideal' point

(Winston, 1974; Kurz, 1986). As we can see from figure 1, it is better to be on the lower cost portion of the cost curve to the left of  $Y_n$  than on the higher cost portion to the right. Once again, the existence of positive reserve capacity does not necessarily imply excess capacity, just as the existence of positive money holdings does not necessarily imply an excess supply of money. True excess capacity arises only when the actual rate of capacity utilization is below the desired rate, i.e. when actual reserves are greater than desired reserves.

Several PK authors have assumed that the normal rate will itself vary with the actual rate of capacity utilization (see section 9). We have just seen that the normal rate is determined by the (real) cost structure of firms. So then the question becomes: can changes in the actual utilization rate change the normal rate, say because changes in labor market conditions affect real wages and hence (real) prime costs? This possibility is depicted in figure 1 by means of dotted lines: the new AVC is (say) 25 per cent higher in each shift (i.e. 25 in the first shift and 32.50 in the second), which in turn shifts the AC curve up. But as is evident from figure 1, *this has no effect whatsoever on the normal rate of capacity utilization*, because even with these changed costs the minimum cost point remains  $Y_n = 25$ . It should be added that this conclusion would hold equally well if it had turned out that the second shift output ( $Y = 40$ ) had been the lowest cost point instead, say because the cost premium on the second shift was small enough to make it so. In either case, the minimum cost rate of capacity utilization is largely immune to variations in the actual rate of capacity utilization, although it might change in the long run due to changes in capital intensity of production, changes in workweek, etc. In what follows we will therefore take  $u_n$  to be exogenously given.

Lastly, a gravitational balance between normal capacity and demand does not imply Say's Law. The latter amounts to the claim that any supply will generate a matching demand, so supply limits such as full employment are the only effective ones (Foley, 1985; Sowell, 1987). On the other hand, the classical and Harrodian notion of normal capacity utilization says that normal capacity and demand will *mutually* adjust to achieve some kind of balance. With this in mind, we define the actual and normal rates of capacity utilization as

$$u = Y/Y_{max} \quad (3)$$

$$u_n = Y_n/Y_{max} \quad (4)$$

where  $Y_{max}$  = the engineering level of capacity,  $Y_n$  = the normal capacity level of output,  $u$  = the actual rate of capacity utilization and  $u_n$  = the normal rate of capacity utilization. Thus the variable  $(u - u_n)$  is the key barometer of

investment prospects. It should be obvious that normal capacity utilization represents the point at which the actual capital stock is consistent with actual output. Equivalently, normal capacity utilization exists when the actual output–capital ratio is equal to the desired output–capital ratio. Hence the notion of *normal capacity utilization* is a particularly important form of *stock–flow consistency*.

### 3. TRADITIONAL KEYNESIAN ECONOMICS IN A GROWTH CONTEXT

In the traditional Keynesian story, investment is also exogenous in the short run, so that (as in equation (2)), it is the sum of exogenous demand elements ( $I + G + X$ ) that determines the short-run equilibrium level of output via the multiplier ( $1/s$ ), as in equation (2). Most often, these elements are treated as if they are also stationary in the short run, save for exogenous jumps in their levels (Michl, 2002, chapter 3, pp. 29–47).

Harrod's first point of departure was to insist that all variables be considered in terms of their *time paths* (Kregel, 1980, p. 100), stationary paths being a very special subset. This step in the Harroddian direction is relatively innocuous, because considering time-varying elements of exogenous demand merely extends the reach of the Keynesian model without changing any of its essential features. Thus the general multiplier relation of equation (2) becomes

$$Y_t = (I_t + G_t + X_t)/s \quad [\text{Dynamic multiplier}] \quad (5)$$

Now we can see that the time path of short-run equilibrium output is generally driven by the combined time paths of exogenous investment, government and export demands. A rise in the level (or growth rate) of any one element will raise the level (or growth rate) of short-run equilibrium output and employment. For instance, raising the rate of growth of government spending and thereby increasing the budget deficit will unambiguously raise the growth rate of output. Moreover, since  $s = s_p + \theta(1 - s_p) + \mu$ , reductions in the private savings rate ( $s_p$ ), the tax rate ( $\theta$ ) and/or the import propensity ( $\mu$ ) will raise the level of output *insofar* as they do not lower the growth rate of exogenous demand. This separation between the two sets of variables is the real secret behind the Keynesian paradox of thrift within a growing system (Michl, 2002, *op. cit.*). Since the level of productivity links employment to output, the Keynesian framework then implies that appropriate combinations of the tax rate ( $\theta$ ) and the level and growth rate of government spending ( $G_t$ ) can achieve any desired level or rate of growth of employment. This

promise has always been central to Keynesian policy (Asimakopulos, 1991, pp. 188–9).

### 4. THE IMPOSSIBILITY OF EXOGENOUS INVESTMENT: THE CRITICAL INSIGHT OF HARROD AND DOMAR

In the Keynesian formulation, all elements of exogenous demand ( $I, G, X$ ) are on an equal footing. But this is improper, because even though all three are co-equal on the side of demand, they are not so on the side of potential supply. Investment is 'more equal' than the others, since it also creates new capacity.

Because investment in the Keynesian system is merely an instrument for generating income, the system does not take into account the extremely essential, elementary and well-known fact that investment also increases capacity. This *dual* character of the investment process . . . provides us with both sides of the equation. (Domar, 1946, reprinted in Sen, 1970, *Growth Economics*, pp. 67–8)

The duality of investment is most evident in the basic case, in which the government budget and foreign trade are balanced, because then investment is the sole driver of both the level of aggregate demand and the change in capacity (equation (2') derived previously). If we define the engineering capacity–capital ratio as  $R = Y_{\max}/K$ , and use the notation  $Y'_{\max}$  and  $K'$  to designate first derivatives, then when  $R$  is constant over time (Harrod–Neutral technical change) we can also write  $R = Y'_{\max}/K' = Y'_{\max}/I = Y'_n/(u_n \cdot I)$ , where  $Y'_{\max}$  = the increment in engineering capacity,  $Y'_n = u_n \cdot Y_{\max}$  (equation (4)) and  $K'$  = the increment in capital = investment ( $I$ ). Then we have a system corresponding to Harrod's own starting point (Harrod, 1939).

$$Y_t = I_t/(s_p \cdot (1 - \theta)) \quad [\text{Dynamic multiplier effect of investment}] \quad (2')$$

$$Y'_{nt} = I_t \cdot u_n \cdot R \quad [\text{Dynamic capacity expansion effect of investment}] \quad (6)$$

The full consequences of dealing with time paths now come home to roost. For instance, under the traditional assumption that investment is stationary, output would also be stationary from equation (2'), while this same given level of investment would continuously expand desired capacity from equation (6). Thus any constant level of investment would imply a continuously falling level of capacity utilization. But any such outcome would be entirely inconsistent with the assumption of a constant ongoing level of investment, for investment is predicated on a need for capacity expansion, and

continuously falling capacity utilization contradicts that need. *The traditional Keynesian assumption of stationary variables is simply not viable because it is stock-flow inconsistent.*<sup>6</sup>

Because positive investment always increases capacity, only a rising path of exogenous demand is consistent with any path of positive investment. In the basic case, this means that investment must at least be growing in order to generate demand (via the multiplier) sufficient to match a growing level of capacity. Yet even this will not be sufficient. For instance, suppose that investment grew at some arbitrary rate  $\alpha$ , so that  $K'_t \equiv I_t = I_a \cdot e^{\alpha t}$ , where  $I_a$  represents its autonomous level. Then capital stock  $K_t = (1/\alpha) \cdot I_a \cdot e^{\alpha t} = (1/\alpha) \cdot I_t$  and  $Y_{\max t} = R \cdot K_t = R \cdot I_t / \alpha$  since  $R \equiv$  engineering capacity  $\equiv Y_{\max t} / K_t$ . Finally, from the multiplier,  $Y_t = I_t / (s_p \cdot (1 - \theta))$  (equation (2')). Hence from equation (3), the capacity utilization rate will be constant at some particular level:

$$u_t = Y_t / Y_{\max t} = [I_t / s_p \cdot (1 - \theta)] / (R \cdot I_t / \alpha) = \alpha / [s_p \cdot (1 - \theta) \cdot R] \quad (7)$$

But investment will be self-consistent *only* if the rate of growth of investment yields an actual capacity utilization rate equal to the desired rate ( $u_t = u_n$ ), i.e. only if investment grows at a special rate  $\alpha^* = s_p \cdot (1 - q) \cdot R \cdot u_n = s \cdot R_n$ , which is of course Harrod's warranted rate ( $g_w$ ). Along this particular path,  $Y = Y_n$ , where  $Y = I / s_p \cdot (1 - \theta)$  and  $Y_n \equiv u_n \cdot Y_{\max} \equiv u_n \cdot R \cdot K$ , so that for any given  $u_n$  and  $R$ , output, capital and investment all grow at the warranted rate.

$$g_Y = g_K = g_I = g_w \equiv s \cdot R \cdot u_n \quad (8)$$

## 5. THE STABILITY OF THE HARRODIAN WARRANTED PATH

A great deal has been written about the supposed instability of the Harrodian warranted path, beginning with the writings of Harrod and Domar themselves (Harrod, 1933; Domar, 1946; Sen, 1970, pp. 10-14). For the purposes of this paper, it is sufficient to demonstrate that a simple and sensible dynamic adjustment process renders the warranted path perfectly stable. The crucial point here is that the warranted path is a *moving target*. Experience teaches us that in order to hit a moving target, it is necessary to track its path and then adjust this tracking in light of past overshooting or undershooting errors. In the present case, the target is expected demand, which in a Harrodian framework is generally growing; and the error is  $(u - u_n)$ , the gap

<sup>6</sup> The stationary case would only be internally consistent if net investment were zero (so that capacity is fixed), and government spending and exports were just at the level that would utilize the capacity in place (so that there would be no incentive for net investment to change).

between actual and desired capacity utilization. Hence planned capacity must incorporate the expected path of demand and make its adjustments relative to that target path. This is a perfectly general principle, which applies equally well to the case where there is no tracking because the target is stationary.

One simple application of this principle comes from the formalization of Hicks' stock-flow adjustment principle (Hicks, 1985, chapter 10, pp. 97-107). Suppose that in the long run the rate of growth of capacity ( $Y_n$ ) adjusts *relative* to the rate of growth of expected demand ( $D^e$ ) in reaction to the utilization gap  $(u - u_n)$  with some reaction coefficient  $k > 0$ , and some zero-mean error ( $\varepsilon_1$ ).

$$g_{Y_n} = g_{D^e} + k \cdot (u - u_n) + \varepsilon_1 \quad (k > 0) \quad (9)$$

Since both Keynesian and Harrodian theories assume that demand equals supply over the short run (Asimakopulos, 1991, pp. 40-1), they implicitly assume that output *successfully* targets expected demand with some zero-mean error ( $\varepsilon_2$ ).

$$g_Y = g_{D^e} + \varepsilon_2 \quad (10)$$

Combining these two equations gives us an adjustment process with a zero-mean error  $\varepsilon_3 = \varepsilon_1 + \varepsilon_2$ .

$$g_{Y_n} = g_Y + k \cdot (u - u_n) + \varepsilon_3 \quad [\text{Hicks-Harrod (HH) dynamic adjustment principle}] \quad (11)$$

Equation (11) might be called a HH adjustment mechanism. It requires nothing more than that firms try to adjust capacity growth relative to output growth whenever they are facing capacity discrepancies, and it yields a completely stable adjustment of capacity utilization to a normal level independently of all other processes or other parameters (such as  $s_p$ ,  $q$ ,  $R$ , etc.). To prove its stability, note that since  $u = Y / Y_{\max} = (Y / Y_n) \cdot u_n$  (equation (3)),  $g_{Y_n} - g_Y = -u' / u$ , so that equation (11) is equivalent to the one below, where  $\varepsilon \equiv -\varepsilon_3$ :

$$u' = -k \cdot (u - u_n) \cdot u + \varepsilon \quad (12)$$

Since  $u$  is always positive and  $k > 0$ , the HH adjustment process is completely stable around  $u = u_n$  (the warranted path) for any positive value of the reaction coefficient  $k$ . In the presence of shocks given by the error term  $\varepsilon$ , capacity utilization will fluctuate around the normal level so that actual growth will fluctuate around the warranted rate. Note that this result obtains even though changes in the capacity occur through changes in investment, which also changes demand.

Equation (11) was derived from the sensible requirement that firms accelerate capacity growth *relative* to demand growth whenever they experience above-normal rates of capacity utilization. It is a dynamic accelerator in growth and utilization rates. But since the multiplier  $Y = I/s$  implies that  $g_Y = g_I$  and the definition  $g_K \equiv I/K$  implies that  $g_I = g_K/g_K + g_K$ , we see that the HH adjustment also implies a *stable non-linear multiplier-accelerator*.

$$g'_K = -g_K \cdot k \cdot (u - u_n) + \varepsilon \quad \text{[Dynamic non-linear-accelerator form of the HH adjustment principle]} \quad (13)$$

Finally, equation (11) also allows us to express the HH adjustment process as an accumulation (investment) function, which can then be compared with a general PK function. Consider the simple case in which the engineering capacity/capital ratio ( $R = Y_{\max}/K$ ) and the normal capacity utilization rate ( $u_n = Y_n/Y_{\max}$ ) are both given. Then the normal capacity/capital ratio  $R_n = R \cdot u_n = Y_n/K$  is constant, so that  $g_{Y_n} = g_K$ . The resulting HH accumulation function in equation (14) leads to a normal rate of capacity utilization in the long run. By way of contrast, the PK function in equation (15) in which  $g_K(r_n)$  represents the portion of accumulation that is driven by normal profitability (i.e. by the rate of profit at normal capacity utilization)<sup>7</sup> leads to a long-run rate of capacity utilization arbitrarily different from the normal rate (Dutt, 1997, pp. 245–6; Lavoie, 2003, p. 59; 1996, pp. 122–3). The two functions are algebraically similar, yet they give rise to fundamentally different long-run outcomes because they inscribe fundamentally different processes. We revisit this issue in section 9.

$$g_K = g_Y + k \cdot (u - u_n) + \varepsilon \quad \text{[The HH accumulation function]} \quad (14)$$

$$g_K = g_K(r_n) + h \cdot (u - u_n) + \varepsilon \quad \text{[A standard PK accumulation function]} \quad (15)$$

A dynamic adjustment process in terms of growth rates, as in the HH process, is only one way to approach the issue. We can also conceptualize the adjustment in terms of the change in the ratio of the desired variable to the target variable, this ratio being adjusted in light of past experience. For instance, we have already seen that the HH adjustment process in equation (11), expressed in growth rates  $g_{Y_n}$  and  $g_Y$ , is equivalent to the one in equation (12) expressed in terms of the rate of change of the ratio  $u = (Y/Y_n) \cdot u_n$ . More generally, we may also conceptualize dynamic adjustments in terms of changes in some relevant ratio, such as that of investment to output (or profit). In a series of

<sup>7</sup> A standard PK accumulation function can be written in the form  $g_K = F(g_K(r_n), u)$ . Lavoie (1996, p. 119) lists it as  $g_K = a + b \cdot u + c \cdot r_n$ , from which we can get  $g_K = g_K(r_n) + h \cdot (u - u_n)$ , where  $g_K(r_n) = c_0 + c \cdot r_n$ ,  $c_0 = (a + b \cdot u_n)$ ,  $h = b$ .

papers (Shaikh, 1989, 1991, 1992) I have shown that the stability of the warranted path can be easily established within a classical framework that distinguishes between investment in circulating capital (additional raw materials and labor power) and investment in fixed capital (additional plant and equipment). Circulating investment expands inputs, which expand *actual output* one production period later. This is a fundamental point in Marx's schemes of reproduction, for instance, and in input-output growth economics (Shaikh, 1992, pp. 323–8). On the other hand, fixed investment expands *potential output* (capacity), which is a fundamental point in Harrod–Domar models of growth. Both forms of investment raise effective demand, but circulating investment directly raises the capacity utilization rate while fixed investment directly lowers it. When the ratio of each component to output is allowed to operate independently, the warranted path is generally stable. In this regard, it is particularly ironic that in his seminal essay, Harrod himself does distinguish between fixed and circulating investment, but then immediately ignores their differences by assuming that 'circulating and fixed [elements] are lumped together' (Harrod, 1939, pp. 47–8).

## 6. HARRODIAN DYNAMICS: THE PARADOX OF THRIFT IS A TRANSIENT PHENOMENON

We now return to the central issue, which has to do with the further implications of endogenous investment. One striking consequence is that a fall in the savings rate has deleterious consequences in the long run. In the basic case being considered (equation (1) with  $G = T$  and  $X = IM$ ), excess demand  $E_t = I_t - s_p \cdot (1 - \theta) \cdot Y_t$ . Then a fall in  $s_p$  will initially give rise to positive excess demand, which will in turn stimulate production. From the multiplier relation (equation (2')) we know that if the growth rate of investment were to remain unchanged, output will eventually rise to a new higher level  $Y_t = I_t/s_p \cdot (1 - \theta)$  corresponding to the lower savings rate  $s_p$ , with no change in output growth. If capacity utilization was initially normal, it would now have risen above that, so investment would accelerate and further stimulate output. However, if the process is stable, the acceleration of investment would expand capacity more rapidly than output, so that capacity utilization would drift back down. At some point the system would go back to gravitating around the warranted path growing at the rate  $s_p \cdot (1 - \theta) \cdot R \cdot u_n$ , which will now be lower since  $s_p$  has fallen. Thus we have two results. The short-run equilibrium level of output and employment will have *risen*. But over time the corresponding short-run equilibrium path will decay because the long-run equilibrium rate of growth will have *fallen*. The short-run pumping arising from the paradox of

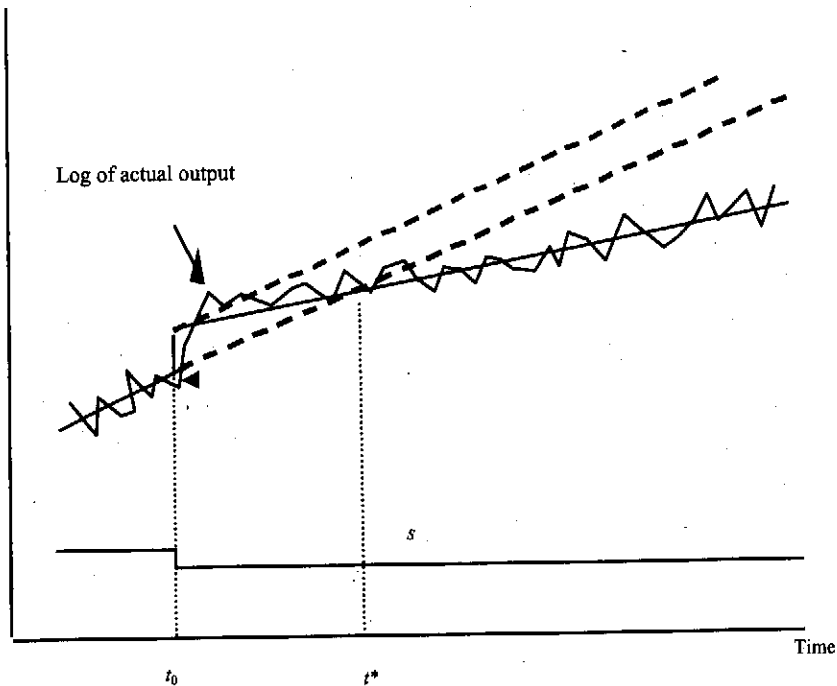
log  $Y(t)$ 

Figure 2. Effects of a fall in the savings rate ( $s$ ) on the path of output.

thrift will therefore give way to the longer-run slow-down arising from the Harrodian growth effect. It is important to note that this growth effect follows solely from the constancy of the savings rate and the stability of the warranted path. It does not require any financial mechanism (such as an increase in the interest rate) or input supply restrictions (such as full employment). Conversely, insofar as these factors do come into play, they can only modify the Harrodian pattern insofar as they modify  $s_p$ ,  $\theta$  or  $R$  (Harrod, 1973, chapter 7).

Figure 2 depicts the effects of this process on the log of output. Actual output fluctuates around its initial warranted path (heavy solid line) until time  $t_0$ , at which point the private savings rate ( $s_p$ ) falls to a new lower level. If investment were to remain unchanged, then in keeping with the traditional multiplier story the fall in the savings rate would simply shift the new short-run equilibrium path upwards parallel to the old one (both shown as dashed lines). But we know that investment must respond to the resultant changes in capacity utilization, so that the new warranted path (also shown as a heavy solid line) will now be rising more slowly. After time  $t^*$ , output will be actually lower

than it would have been otherwise. The short-run stimulatory effect of a fall in the savings rate is ultimately overcome by the corresponding growth slow-down. Conversely, raising the savings rate in order to raise the trend growth rate will initially reduce output and employment. Appendix A demonstrates these effects through a spreadsheet simulation of the HH adjustment process in equation (11), as shown in figures 5 and 6 and Table A1 of the Appendix.

#### 7. GOVERNMENT SPENDING, EXPORTS AND GROWTH: THE POSSIBILITY OF TOO MUCH OF A GOOD THING

A similar line of argument can be pursued for the more general case in which government budgets and foreign trade are not balanced, by moving to the general dynamic multiplier (equation (5)). Since  $Y_t = Y_{nt}$  along any self-consistent path, the requisite growth path for investment is one along which  $Y_t = (I_t + G_t + X_t)/s = Y_{nt} = u_n \cdot R \cdot K_t$ , or

$$I_t = (s \cdot u_n \cdot R) \cdot K_t - (G_t + X_t) \quad [\text{General warranted path for investment}]^8 \quad (16)$$

where now  $s = s_p + \theta(1 - s_p) + \mu$ ,  $s_p$  = the private savings rate,  $\theta$  = the tax rate,  $\mu$  = the import propensity,  $R = Y_{\max}/K$ ,  $u_n = Y_n/Y_{\max}$ . Since  $I/K \equiv g_K$ , and  $g_Y = g_{Y_n} = g_K$  along the warranted path (equation (8)), if we define  $g_t$ ,  $x_t$  as the government and export shares in income, respectively, we get two alternate expressions for the warranted rate of growth:<sup>9</sup>

$$g_Y = [s - (G_t + X_t)/Y_t] \cdot u_n \cdot R \quad (17)$$

$$g_Y = [s_p \cdot (1 - \theta) + (\theta - g_t) + (\mu - x_t)] \cdot u_n \cdot R \quad (17')$$

The preceding expressions reveal that growth in exogenous demand will enhance the rate of growth rate of output if it causes the share of total exogenous demand in output to fall, and will reduce the growth rate if it causes this share to rise. Since exogenous demand ( $G_t + X_t$ ) affects the level of output  $Y_t$ , the conditions under which growth in the former is 'modest' or 'excessive' are not obvious. The issue is addressed in Appendix B, which

<sup>8</sup> Equation (16) is also shown in Palumbo and Trezzini (2003, footnote 17, p. 129).

<sup>9</sup> Since  $Y_t = Y_{nt}$  and  $g_Y = g_{Y_n} = g_K \equiv I/K$  along the warranted path,  $g_Y = I/K_t = (s \cdot u_n \cdot R) - [(G_t + X_t)/Y_t] \cdot (Y_n/Y_{\max}) \cdot (Y_{\max}/K_t) = (s \cdot u_n \cdot R) - [(G_t + X_t)/Y_t] \cdot (u_n \cdot R) = [s - (G_t + X_t)/Y_t] \cdot (u_n \cdot R)$ , which is equation (17). Letting  $g_t \equiv G_t/Y_t$  and  $x_t \equiv X_t/Y_t$ , and noting that  $s \equiv s_p + \theta(1 - s_p) + \mu$ , we can rewrite this as  $g_Y = [s_p(1 - \theta) + (\theta - g_t) + (\mu - x_t)] \cdot u_n \cdot R$ , which is equation (17').



shows the pre-existing warranted rate of growth provides the line of demarcation: growth in exogenous demand is 'modest' if it remains below this critical rate, and 'excessive' if it rises above it. Growth in government spending and exports will enhance output growth ('crowding in') if it causes output to expand more rapidly than itself, so that the total share of exogenous demand in output ( $(G_t + X_t)/Y_t = g_t + x_t$ ) declines. This outcome is depicted in figure 3. Conversely, if exogenous demand does grow too rapidly, so that its share in output rises, then the warranted rate of growth will actually decline ('crowding out'), as depicted in figure 4.

Hence within a Harrodian framework rising exogenous demand can be a good thing, but too much of it can be a bad thing. In both cases, the initial impact would be stimulatory. But in the 'modest' case the rate of growth would also rise, so there would be double benefits, whereas in the 'excessive' case the rate of growth would fall and hence eventually undermine the initial stimulus by time  $t^*$ , as depicted in figure 4. In this case, the overall pattern would be the same as those of a decline in the private savings rate shown previously in figure 2: the paradox of thrift is a transient phenomenon.

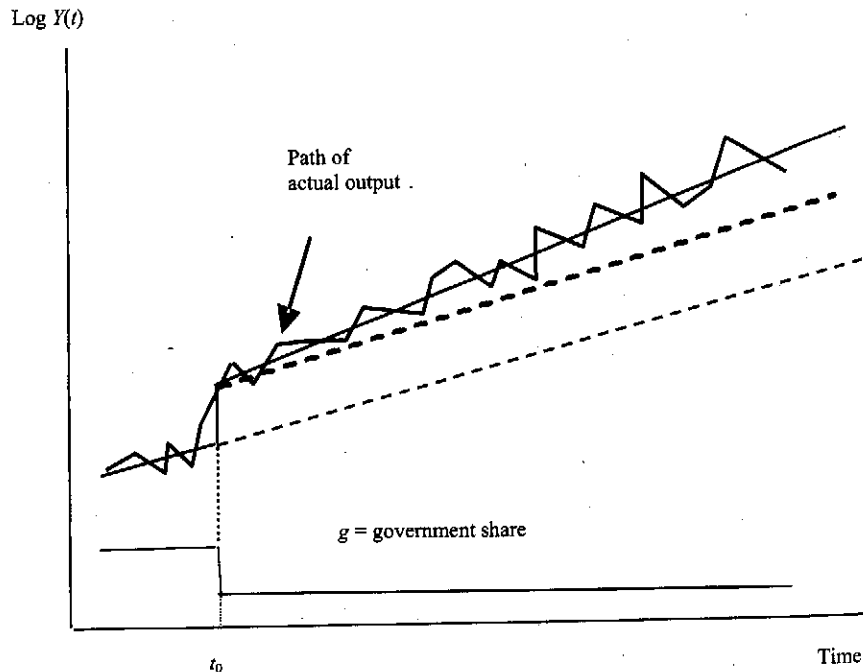


Figure 3. 'Modest' government spending raises the growth rate of output.

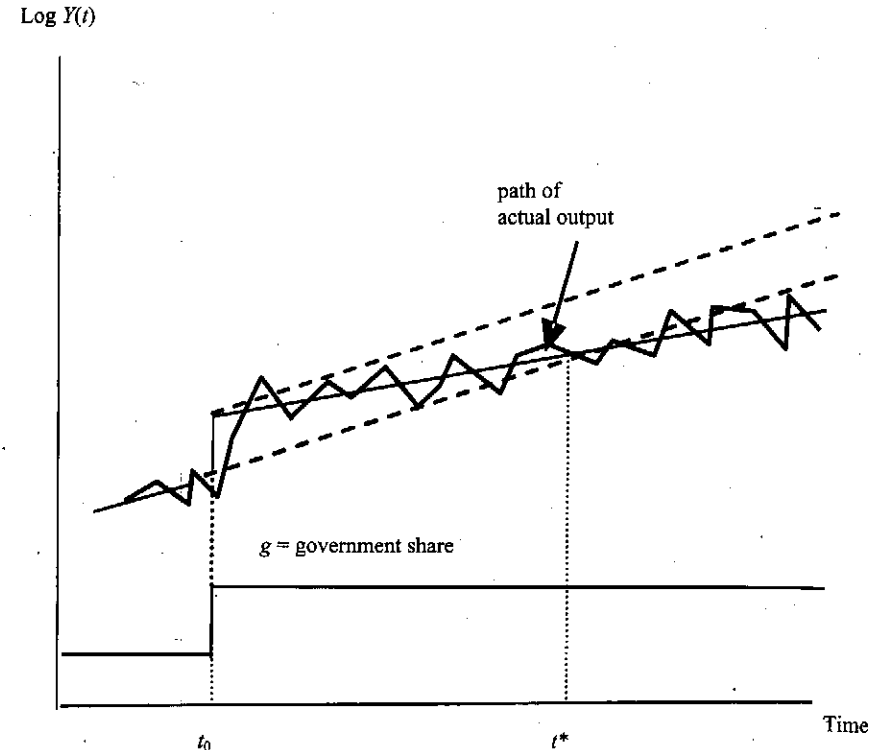


Figure 4. 'Excessive' government spending lowers the rate of growth of output.

## 8. EMPLOYMENT POLICY IN A GROWTH CONTEXT

One of the central implications of the preceding analysis is that Harrodian fiscal policy will have different policy prescriptions from standard Keynesian policy. Consider the implications for employment goals. The long-run rate of growth of the demand for labor is the difference between the warranted rate of growth of output and the rate of growth of productivity. If the demand for labor happens to be growing less rapidly than the supply of labor, then the unemployment rate will rise. To close this gap by means of fiscal policy, it is necessary to raise the warranted rate of growth, and Harrodian theory (equation (17')) tells us that we must then lower the government share and/or raise the tax rate—i.e. we must reduce the government deficit *relative* to income (Harrod, 1973, chapter 7, pp. 100–21). If we happen to start from a growth rate below the warranted rate of growth (a slump), there is space to raise the growth rate of government spending and also enhance long-term growth. But

if the existing growth rate is at or above the warranted rate, it might be necessary to reduce the rate of government spending because even though it will initially deflate the economy, in the long run it will raise its rate of growth.

An interesting exception to this general rule occurs under the 'New Cambridge' hypothesis previously put forward by Godley and his co-authors, in which government deficits are completely offset by trade deficits (see footnote 3). This hypothesis implies that  $(\theta - g_t) + (\mu - x_t) = 0$ , so that the expression for the warranted growth rate reduces to  $g_Y = s \cdot R_n \equiv s_p \cdot (1 - \theta) \cdot R$ . We know in general that an increase in the government deficit will pump up the economy in the short run. This is the Keynesian effect. Depending on the effect on output, the ratio of the government deficit to output will decline or rise. At any given private savings rate, the subsequent effect on the long-term growth rate will depend on what happens to the trade balance ratio. If the government and trade deficit ratios are relatively independent of each other, then a rise in the former will reduce long-term growth (Harrod's point). On the other hand, if the two ratios happen to be exact twins, a rise in the government deficit ratio will harm the foreign account but not long-term growth (Godley's point). Partially offsetting deficits will produce a mixture of the two effects. At any given tax rate, the rate of growth of government spending is the key. It can have positive effects in both the short and long term if the rate of growth of government spending remains below the pre-existing warranted rate of growth (Appendix B). But if government spending grows too rapidly, its longer-term effect will be generally negative, leading to some reduction in the long-term growth rate, and possibly some worsening of the trade balance. It is possible to have too much of a good thing. As always, these effects have nothing to do, *per se*, with changing interest rates or employment constraints. They arise from the properties of the path along which capacity utilization is normal.

Even though the Keynesian and Harrodian frameworks differ about the long-run effects of changes in savings, government spending and tax rates, they concur on one essential thing: fiscal policy has the capacity to adjust the rate of employment in a socially desired direction. In this sense, Harrodian economics attempts to complete the Keynesian project by extending it to the case of endogenously generated growth.

#### 9. POST-KEYNESIAN ATTEMPTS TO RETAIN KEYNESIAN PROPOSITIONS IN A GROWTH CONTEXT

Several Post-Keynesian authors concerned to defend KT results have objected to the Harrodian conclusions on a variety of grounds.

A first set of objections seems to arise because the authors in question associate Harrodian warranted growth with a steady-state path having fixed attributes.<sup>10</sup> But the regulation of actual growth by the warranted rate only requires that the ratio of capacity to demand fluctuate around a constant ratio (i.e. that it be *stationary*, in the statistical sense, around  $u_n$ ). Continuing shocks keep the fluctuations going, and since the adjustment process takes place through changes in investment, both capacity and demand vary in response (section 5 and Appendix A): warranted growth is a center of gravity, not as a steady state. Thus it is incorrect to say that the stability of the ratio of capacity to demand 'is inconsistent with the idea that capacity tends to adjust to demand' (Palumbo and Trezzini, 2003, p. 114). On the contrary, this stability is achieved precisely because capacity *is* able to adjust to demand (Garegnani, 1992, p. 62; Kurz, 1992, pp. 75–81).<sup>11</sup> Moreover, there are many levels of output consistent with any given warranted rate of growth, because a shock can raise or lower the path itself, as in the case of the various pairs of parallel dotted lines in figures 2–4. One way to see this is to note that when actual growth fluctuates around warranted growth with some random zero-mean error  $\varepsilon$ ,  $g_K = s \cdot R_n + \varepsilon$ . Then since  $g_{K_t} \equiv \Delta K_t / K_{t-1} = \Delta \log K_t$ , we have  $\log K_t = s \cdot R_n + \log K_{t-1} + \varepsilon$ . Thus the warranted paths of capital and output follow a unit root process (random walk with drift) whose trend growth rate is given by  $s \cdot R_n$  (the warranted rate of growth) but whose *level* depends on current and past shocks (history). Therefore it is not correct to say, as Garegnani (1992, p. 58) does, that if the 'average utilization of capacity over time were to be at the desired level, the path of future capital accumulation would be completely determined'.

A second set of objections stem from the claim that in Harrod's long-run equilibrium 'history plays no role in determining the final outcome' (Dutt, 1997, p. 239). This is a red herring, for we have already noted that the particular level of the output path is determined by past and current events,

<sup>10</sup> For instance, Palumbo and Trezzini say that the problem of relating actual and normal capacity utilization is one that 'stems from the steady-state assumption itself' (Palumbo and Trezzini, 2003, p. 114). Similarly, Kurz (1992, p. 85) cites Marglin (1984, pp. 474–5) to the effect that 'in the long run . . . there is no excess capacity to accommodate investment demand'.

<sup>11</sup> Palumbo and Trezzini (2003, p. 115) also raise the objection that the observed average rate of capacity utilization need not equal the normal rate. This is perfectly sensible, because shocks need not be symmetric (e.g. the positive effects of war spending and the negative effects of post-war recessions need not cancel out). But it only tells us that we cannot necessarily 'read back' the normal rate of capacity utilization from some arbitrary historical average. What they do not mention is that this conclusion applies equally well to excess demand, which means that would be equally incorrect to take actual output as a proxy for demand—i.e. to assume that realized quantities are proxies for equilibrium ones. Yet at an empirical level, this latter assumption is nearly universal in the KT tradition.

that government and export shares ( $g_t, x_t$ ) are time-varying, and that the private saving, tax and import rates ( $s_p, \theta, \mu$ ) need not be constant over time.

A third set of authors object to warranted growth because they claim that it 'amounts to denying aggregate demand any autonomous role in determining growth' (Palumbo and Trezzini, 2003, p. 112, italics added). At a level of abstraction in which there is only a private sector ( $Y_t = I_t/s_p$ ), it is true that the Harrodian framework implies that investment is endogenous so that there is no autonomous demand to regulate growth. But once we expand the framework to include government spending and exports, these autonomous injections and their associated autonomous tax and import leakages do affect growth. This is precisely why fiscal policy emerges as the central tool for growth and employments policy in Harrod, just as it does in Keynes.

The real objection of KT theorists lies elsewhere. KT theory assumes that lowering the private, public or foreign savings rates does not substantially harm the growth rate of exogenous demand. This is the real secret of the KT paradox of thrift, and it implies that fiscal policy has a substantial degree of freedom. In Harrodian theory under the same conditions, the pumping effect can be undermined by the corresponding reduction in the warranted rate of growth, and may be entirely reversed at some point (section 6). This is the Harrodian paradox of growth, and it implies particular limits to fiscal policy arising directly from its long-run effects on growth, independently of any feedback effects from wage and interest rates (sections 7–8).

It is in this context that a set of authors have attempted to restore traditional KT theory by making accumulation autonomous enough to retain the paradox of thrift even in the long run (Lavoie, 2003, p. 59). To see how, it is useful to reformulate the short-run equilibrium condition as a relationship between growth and capacity utilization. Beginning from  $I = S = s \cdot Y$ , where in this case  $s = s_p \cdot (1 - \theta)$ , we can divide both sides by capital stock  $K$ , recall the definitions  $u = Y/Y_{max}$ ,  $R = Y_{max}/K$  and  $g_K = I/K$ , to arrive at the short-run equilibrium condition in growth form:

$$g_K = s \cdot R \cdot u \quad [\text{Growth form of short-run equilibrium: } I = S] \quad (18)$$

Then if  $g_K^*$  is some exogenously given long-run accumulation rate, the result is a unique long-run capacity utilization rate  $u^* = g_K^*/s \cdot R$ . In this case, the paradox of thrift obtains because a decrease in thrift (a fall in the savings rate) raises  $u^*$ —i.e. output rises relative to capacity. A similar result may hold even when accumulation is partially endogenous in the long run, provided that the curve specified by accumulation as a function of  $u$  (e.g. the standard PK accumulation function of equation (14)) intersects the curve specified by  $s \cdot R \cdot u$  at some positive (and plausible)  $u$  (Dutt, 1997, pp. 245–6; Lavoie, 1996, pp. 118–22). In both cases, the accumulation function determines the

long-run rate of capacity utilization, which may in principle take on any positive value. But then the rate of capacity utilization must be a 'free variable', in the sense that it can always remain different from the normal rate and always adapt as needed to some change in autonomous demand. The trouble, of course, is that a persistent difference between actual and normal capacity utilization implies persistent long-run disequilibrium, which is difficult to justify on economic grounds. In the face of stringent criticism on this point (Kurz, 1986; Auerbach and Skott, 1988; Palumbo and Trezzini, 2003, p. 112) the KT 'free variable' camp has turned in another direction, by positing that in the long run the 'desired' rate would itself adapt to the level of the actual rate (Amadeo, 1986; Lavoie, 1995; Dutt, 1997).<sup>12</sup> Since the desired rate of capacity utilization corresponds to a desired output–capital ratio, this is equivalent to *assuming an adaptive stock–flow norm*. With this, actual capacity utilization remains free enough to generate the paradox of thrift, while at the same time the actual rate and the desired rate come to correspond (thus retaining long-run equilibrium). However, this latter result derives from a crucial change in the definition of the 'desired' rate of capacity utilization. We saw in section 2 that the classical and Harrodian definitions of normal capacity, which refers to the point of lowest unit costs, need not change at all as actual capacity utilization changes (i.e. over the business cycle). We can label this 'desired-as-normal' capacity utilization ( $u_n$ ). On the other hand, the PK writers substitute a different notion, 'desired-as-situational' capacity utilization ( $u_s$ ), to which they append the further claim that  $u_s$  adjusts to  $u$ .<sup>13</sup> From classical and Harrodian points of view, the claim that the operating rate that firms come to 'desire' depends on what they happen to get<sup>14</sup> does nothing to address the claim that actual capacity utilization can be at levels arbitrarily different from the lowest cost point (which includes economically necessary reserves). Not surprisingly, many find this displacement of the issue

<sup>12</sup> Lavoie (2003, equation (8), p. 60) makes a similar assumption for the 'target rate of return', which he also calls the normal rate.

<sup>13</sup> All three arguments assume that actual capacity utilization adjusts to the normal rate. The dividing line is whether or not the latter also adjusts to the former.

<sup>14</sup> Lavoie (1996, pp. 120, 127–8) observes that firms hold varying degrees of excess capacity, from which he concludes the desired utilization rate (called  $u_s$  here to distinguish it from the classical and Harrodian normal rate  $u_n$ ) is 'subjectively normal' and 'conventional'. He further proposes that this desired rate will rise when the actual rate is above it. Dutt (1997, p. 247) arrives at the same pattern by assuming that incumbent firms reduce their desired rate of capacity utilization in order to increase their defensive reserves whenever they expect rates of entry greater than the present (Lavoie, 1996, footnote 25, p. 139). Entry rates are assumed to be proportional to accumulation rates, and since  $g_K(r_n)$  is taken to represent the expected rate of accumulation, this implies that firms reduce  $u_s$  when  $g_K(r_n) > g_K$ . Given the standard PK accumulation function  $g_K = g_K(r_n) + h \cdot (u - u_s)$ , this amounts to saying that  $u_s$  rises when  $u > u_s$ , just as in Lavoie.

to be highly problematic (Palumbo and Trezzini, 2003, p. 114; Flaschel and Skott, 2006, footnote 13, p. 318).

#### 10. SUMMARY, CONCLUSIONS AND A PROPOSED CLASSICAL SYNTHESIS

This paper has focused on the relation between KT and Harrodian-type (HT) theory. The concern here is with the fundamental structure of the arguments, rather than with specific models. At the most abstract level in which there is no state or foreign sector, the short-run equilibrium condition ( $I = S$ ) can also be expressed in growth form as a relation between the rate of accumulation ( $g_K$ ), the engineering capacity/capital ratio ( $R$ ) and the rate of capacity utilization ( $u$ ):  $g_K = s \cdot R \cdot u$  (equation (18)). Since both types of theory tend to treat the aggregate savings rate as exogenous, their basic differences stem from their respective treatments of  $g_K$  and  $u$ .

Harrodian-type theory argues that the accumulation rate will adjust to keep capacity utilization around some normal level ( $u_n$ ), the latter being determined by a plant's lowest cost point after allowing for expected fluctuations in demand (sections 2–3). This point is shown to be insensitive to fluctuations in relative costs induced by fluctuations in actual capacity utilization (i.e. over the business cycle). A Hicksian proof of the stability of this adjustment process is provided in this paper (section 5 and Appendix A) and simulated in Appendix A. With  $u = u_n$  in the long run, the self-consistent (warranted) rate of accumulation is endogenously determined:  $g_K^* = s \cdot R \cdot u_n$ . Although technical change may slowly modify  $R$ , and changes in the wage rate and the (socially determined) length and intensity of the working day may somewhat alter the normal utilization rate, in HT theory it is the savings rate (thrift) that principally drives growth. A decline in thrift will at first stimulate the economy via the multiplier (the short-run paradox of thrift), but since it will also lower the warranted rate of growth (the classical growth response), it will erode and eventually overturn the initial stimulus (section 6 and Appendix A). This result also holds when the analysis is extended to incorporate the government and foreign sectors, except that now the relevant savings rate is the sum of private, public and foreign rates. It then turns out that growth in government spending can accelerate the economy if it remains below a certain critical rate, but will decelerate it otherwise. As far as fiscal policy is concerned, it is therefore possible to have too much of a good thing (sections 7–8).

Keynesian-type theory begins from the notion that accumulation is fundamentally driven by expectations of an exogenous long-run profit rate ( $r^e$ ).

When the savings rate and the capacity–capital ratio are also independently given, this implies a utilization rate  $u_{KT} = g_K(r^e)/s \cdot R$ , which is generally different from the normal capacity utilization rate  $u_n$ . Allowing for some feedback of the rate of capacity utilization on accumulation does not change the nature of this outcome (see the end of section 9). Thus a fall in thrift will raise long-run capacity utilization—i.e. raise output relative to capacity, so that short-run paradox of thrift is carried over to the long run. The trouble is that a persistent difference between  $u_{KT}$  and  $u_n$  implies persistent long-run disequilibrium. In an effort to avoid this unacceptable implication, several proponents of KT theory have argued that firms actually base their investment decisions on some situational rate of capacity utilization ( $u_s$ ) rather than on the lowest cost rate of capacity utilization ( $u_n$ ). They further posit that this situational utilization rate will adapt to the actual rate over time. Since a desired rate of capacity utilization corresponds to a desired output–capital ratio, this last step is equivalent to positing an adaptive stock–flow norm. Under these conditions it becomes possible to have accumulation driven by long-term profit expectations and also claim the existence of long-run equilibrium ( $u = u_s$ ). But of course this requires abandoning the notion that the level of cost is central to the operation of firms.

The emphasis of KT theory on the regulation of accumulation by profitability ( $g_K \approx F[g_K(r^e)]$ ) is of great importance. But so too is the HT insistence that accumulation will only be sustainable if the actual rate of capacity utilization gravitates around the normal rate ( $u \approx u_n$ ), the latter being defined by the point of minimum average costs. In classical-type (CT) theory, both propositions must hold. Then the long-run expected rate of profit is equal to the normal rate of profit ( $r^e = r_n$ ), and accumulation is driven by normal profitability.<sup>15</sup> But with  $R$  given by technical conditions and  $u_n$  by cost considerations, the savings–investment equilibrium condition (equation (18)) implies a particular normal savings rate  $s_n$  (equation (19)) determined by the normal accumulation rate and hence by the normal investment share (since  $g_K(r_n) = I(r_n)/K$  and  $R \cdot u_n = (Y_{max}/K) \cdot (Y_n/Y_{max}) = Y_n/K$ ).

$$s_n = g_K(r_n)/(R \cdot u_n) = I(r_n)/Y_n \quad (19)$$

The question then becomes: how and why might the actual savings rate adapt to the needs of accumulation? What follows is a sketch of a much more detailed argument, which is to appear in a forthcoming book.

<sup>15</sup> As Garegnani (1992, p. 56) notes, investment is driven by its expected rate of return at expected normal utilization—i.e. by its expected normal rate of profit.

The root of the answer lies in Marx's distinction between the circuit of revenue  $C-M-C$  and the circuit of capital  $M-C-M'$ , where  $C$  refers here to commodities and  $M$  to money. In the circuit of revenue, which encompasses both wage/salary income and dividend income, money is *spent* on consumption goods and on financial assets (personal savings), the purpose of the latter being to create reserves for future consumption and various contingencies. But in the circuit of capital, money is an end-in-itself: it is *invested* ( $M$ ) with the aim of making profit ( $M' - M$ ), which in turn becomes the foundation for further accumulation (Marx, 1977, chapter 4).<sup>16</sup> Profit is then used to either pay dividends and interest costs or help finance current investment through retained earnings. From this point of view, it makes sense that the proportion of profit that is retained (the business savings rate or retention ratio) would depend on the relative need for investment finance. Indeed, this is precisely how Marx characterizes the matter. At one point in his treatment of accumulation in the schemes of reproduction, he illustrates the transition from a stationary economy (simple reproduction) to a growing one (expanded reproduction). He is scrupulously abstracting from credit at this point in his analysis in order to demonstrate that growth can be internally financed, so he assumes that the necessary increase in aggregate investment is financed entirely by an increase in aggregate retained earnings—i.e. that *the business savings rate* rises in order to provide the requisite finance (Marx, 1967, pp. 506–7; Shaikh, 1988, footnote 2, p. 85). Joan Robinson says exactly the same thing when she notes that the business savings rate is completely endogenous if 'the capitalists and managers retain as much profit as they need for investment' (Robinson, 1965, p. 178). More recently several authors have argued that the business savings rate need not be independent of business investment (Ruggles and Ruggles, 1992, pp. 119, 157–62; Blecker, 1997, pp. 187–8, 223–4; Gordon, 1997, pp. 97, 107–8; Pollin, 1997). As Blecker (1997, p. 188) notes, if business savings rates were indeed linked to their investment decisions, it 'would radically change the policy implications of the [empirically observed] saving–investment correlation'. My own argument is very much in this old-new tradition, although in this paper I am primarily concerned with the fundamental theoretical implications.

Although KT and HT theorists also sometimes stress the importance of the distinction between household and business income (Robinson, 1962, p. 62;

<sup>16</sup> The distinction between the circuits of revenue and capital, which is roughly the same as that between households and businesses, implies that household and business savings are fundamentally different in purpose. Thus it is not a matter of beginning with households at the highest level of abstraction as neoclassical theory typically does and then 'adding institutional reality [by] introducing corporations . . . as institutions with a role going beyond the preferences of their shareowners' (Lavoie, 1998, p. 419).

Kaldor, 1966, p. 310), their models traditionally assume that the business savings rate is independent of the needs for investment finance. Since the aggregate savings rate is a weighted average of fixed household and business savings rates,<sup>17</sup> aggregate savings can catch up to aggregate investment only if output adjusts via the traditional multiplier, or through adjustments in the wage–profit ratio that change the weights of the individual sectoral savings rates (Garegnani, 1992, pp. 47–8; Lavoie, 1998, pp. 419–23).

In the Marx–Robinson pure-internal-finance case there is no multiplier, while in the Keynesian pure-fixed-savings-rate case there is a full multiplier. Between the two lies the general case, and hence the general multiplier. All three cases are consistent with the 'Keynesian Hypothesis' that 'an independently determined level of investment . . . generates the corresponding amount of savings' (Garegnani, 1992, p. 47). But the general case is fundamentally different from the standard Keynesian one, because it turns out that as long as the business savings rate reacts *at all* to an increased finance gap, the actual savings rate adapts to the normal one and it becomes possible to have both profit-driven accumulation and normal capacity utilization. The antinomy is dissolved. To see this, consider an initial situation in which there is ongoing accumulation at normal capacity and the actual savings rate initially equals the normal savings rate ( $s_n$ ) defined in equation (19). In this situation, firms are financing their investments from retained earnings, from sales of new equities and bonds, and from bank loans corresponding to their desired debt ratios. Now if the rate of accumulation were to rise, desired investment would exceed existing savings, which would require firms to seek additional finance. While their actions may or may not induce households to raise their own savings rate (Kaldor, 1966, Appendix: A Neo-Pasinetti Theorem, pp. 316–19), firms can always fill part of this gap through a higher retention ratio and the rest of it through a greater reliance on bank loans. The last would inject demand into the system and raise the level of output as in the traditional multiplier, while the first two would somewhat raise the overall savings rate. Given that accumulation responds to both normal profitability and the gap in capacity utilization, this process would continue until the actual savings rate would equal the normal rate.

<sup>17</sup> Aggregate output  $Y = W + P = (W + DIV) + RE = Y_h + S_b$ , where  $W$  = the wage bill,  $P$  = profits =  $DIV + RE$  = dividends + retained earnings, and  $Y_h = (W + DIV) = (Y - RE)$ . Total savings is the sum of household savings out of household income ( $S_h = s_h Y_h = s_h (Y - RE)$ ) and business savings out of profits ( $RE = \rho P$ ), where  $s_h$  and  $\rho$  are the household and business savings (retention) rates, respectively. The aggregate savings rate  $s \equiv SY/Y = [s_h (Y - RE) + \rho RE]/Y = [s_h + \rho(1 - s_h)(P/Y)]$ . Then even if the household savings rate and the profit share are constant, the aggregate savings rate will adapt to any finance gap if the retention rate changes in the presence of such a gap.

What matters here is that the business savings rate responds *at all* to the gap between desired investment and actual savings (which is also the gap between the normal and actual savings rates), because then the overall savings rate will adjust even if the household savings rate does not. Suppose that the business savings rate and hence the overall savings rate were to rise in the face of such a gap, as in equation (20). Then, since  $s_n$  is determined by the technology and the normal rates of profit and capacity utilization (equation (19)), the adjustment process described above is completely stable for any positive reaction coefficient  $\sigma$ , independently of any other processes or parameters in the system<sup>18</sup> (Appendix C).

$$s' = \sigma \cdot (s_n - s) \quad (20)$$

Other specifications are also possible. For instance, suppose that the savings rate were to rise whenever the actual rate of accumulation was above the normal rate (Kurz, 1992, p. 86)<sup>19</sup> as in equation (20).

$$s' = \sigma \cdot [g_K - g_K(r_n)] \quad (21)$$

Then even with the standard PK accumulation function and the very same stability conditions required in PK models, profit-driven accumulation will be stable around at normal rates of capacity utilization (Appendix C). Figure 5 illustrates the resulting paths of output and investment and figure 6 the corresponding paths of the savings rate and the capacity utilization rate (where  $u_n = 1$ ). Also shown are their characteristic responses to shocks. For instance, with a given normal rate of accumulation a permanent fixed addition to the level of investment at  $t = 10$  has only a modest effect on the output path level because the savings rate also rise, and of course has no effect on the long-term growth rate. On the other hand, a rise in the normal accumulation rate at  $t = 30$  raises both the relative level and growth rate of long-term output, as well as the savings rate (see Appendix C). In all cases there is a positive correlation between the growth rate and the savings rate, but the causation runs from the former to the latter.

<sup>18</sup> This is similar to the HH stock-flow adjustment process in equations (9)–(12).

<sup>19</sup> Kurz (1992, p. 86) raises the possibility that 'the savings ratios may themselves depend on the rate of accumulation' so that 'in times of relatively high accumulation firms [may] ... increase the proportion of retained profits'. However this leads him to conclude that 'other things being equal, the rise in profits realized per unit of capital will correspondingly be smaller' because 'there is no presumption that the actual trend of capital accumulation follows the warranted path'. But as proved in Appendix C, the responsiveness of business savings to the needs of accumulation is sufficient to ensure that actual accumulation will in fact gravitate around the warranted path.

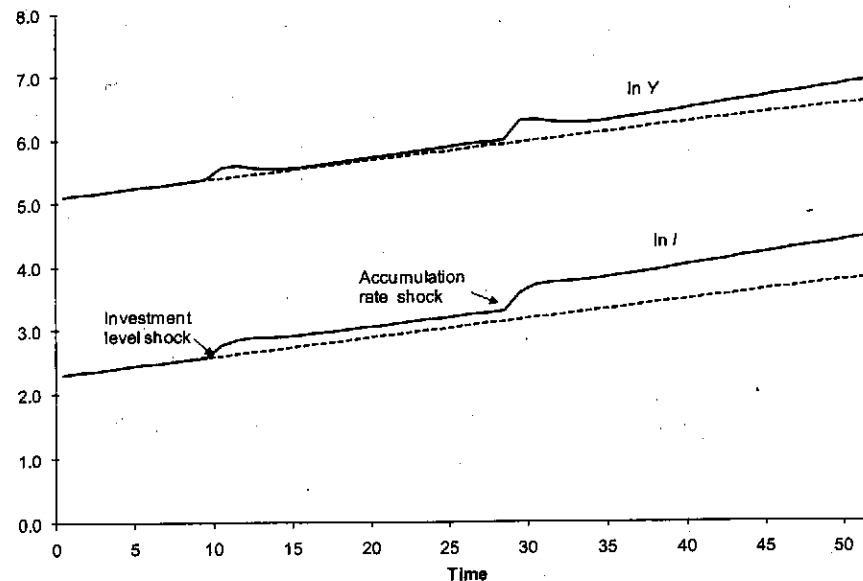


Figure 5. Classical accumulation, logs of investment ( $I$ ) and output ( $Y$ ).

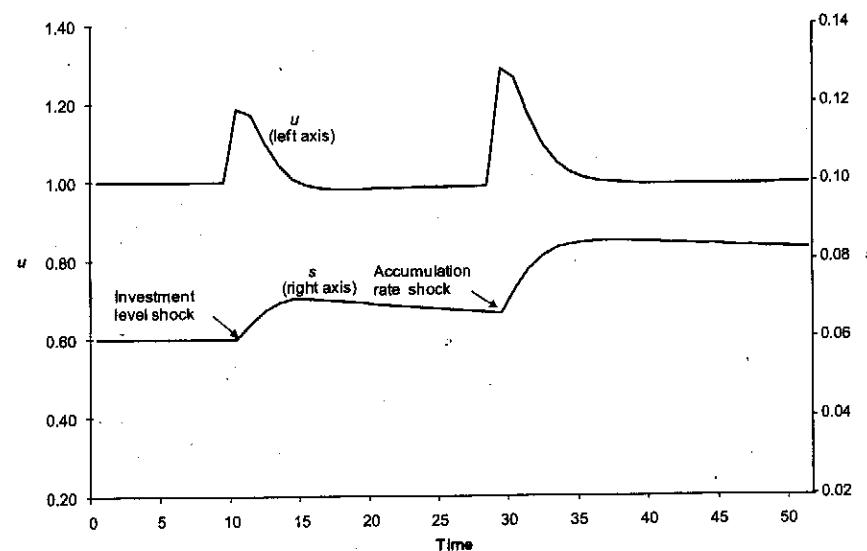


Figure 6. Classical accumulation, saving rate ( $s$ ) and capacity utilization ( $u$ ).

Several things follow from the endogeneity of the business savings rate. Desired investment is driven by its underlying normal profitability, and aggregate savings adapts itself to aggregate investment. These two propositions are completely consistent with a classical approach and also with the 'the central thesis of the General Theory that firms are free, within wide limits, to accumulate as they please, and that the rate of saving of the economy as a whole accommodates itself to the rate of investment that they decree' (Robinson, 1962, pp. 82–3).<sup>20</sup> However, because the classical accommodation is achieved partly through a higher savings propensity, the overall multiplier effects will be smaller than those implied by traditional Keynesian theory. As in Harrod and the classical, the actual rate of capacity utilization will gravitate around the normal rate. Nonetheless, a pure rise in the level of investment can raise the level of the long-run output path without changing its growth rate. On the other hand, a rise in the household savings rate will have no permanent impact on the rate of accumulation unless it happens to alter the profit rate or the interest rate, because otherwise it will simply reduce the retention ratio needed to support a given rate of accumulation. This might be viewed as an alternate derivation of Kaldor's neo-Pasinetti theorem (Lavoie, 1998, p. 421). Although the analysis of employment is outside the scope of this paper, it can be also shown that with the profit rate determined by a socially determined wage share, profit-driven accumulation can be nonetheless consistent with a persistent rate of unemployment (Shaikh, 2003). Finally, it should be clear from what has been said that the PK model is a special case of CT theory, which arises from the assumption of a fixed business savings rate—e.g. when the reaction coefficient  $\sigma$  is set equal to zero in equation (20) or (21) (see also Appendix C). All of these results provide the foundation for the main issue at hand, which is an alternate approach to fiscal policy.

## APPENDIX A

### *Dynamic stability of the warranted path in difference equation form*

Let  $K_t$  be the end of the period capital stock, so that flow of engineering capacity–output relative to the beginning of period capital stock is  $R \equiv Y_{\max t} / K_{t-1}$ . If  $R$  and  $u_0 \equiv Y_n / Y_{\max t}$  are both constant, then  $g_{Y_t} \equiv \Delta Y_n / Y_{n(t-1)} = g_{K(t-1)} \equiv \Delta K_{n(t-1)} / K_{n(t-2)}$ , so that equation (11) in the text can be written in a difference-form suitable for simulation as  $g_{Y_t} = g_{K(t-1)} - k \cdot (u_{t-1} - u_n) + \varepsilon$ ,

<sup>20</sup> It should be noted that when Robinson speaks of 'rates' of savings and investment, she means their levels—i.e. their flows per unit time.

where  $k > 0$  and  $\varepsilon =$  a zero-mean random variable. From the multiplier relation  $sY_t = I_t$  we get  $(1 + g_s) \cdot (1 + g_{Y_t}) = (1 + g_{I_t})$ , where the term  $(1 + g_s)$  allows for the effect of a potential shift in the savings rate (these effects being at the heart of the controversy between KT and Harrodian theory concerning the so-called paradox of thrift). Substituting the latter relation into the former gives us the difference form of the HH adjustment process in the face of a change in the savings rate:

$$g_{I_t} = [1 + g_{K(t-1)} - k \cdot (u_{t-1} - u_n)] \cdot (1 + g_s) - 1 \quad (A1)$$

At the most abstract level, the multiplier relation  $Y_t = I_t/s$  implies that a fall in the savings rate ( $s$ ) will raise the path of output relative to the path of investment. But this will only ensure a higher path of output if the rate of growth of investment is not reduced. This is where Keynesian and Harrodian theories diverge, because the latter argues that investment and output will mutually adjust until capacity utilization once again gravitates around a normal level. Equation (A1) represents such an adjustment process. If the growth rate of investment reacts 'slowly', the initial impact of a drop in the savings rate will be a rise in the level of the output path and in the rate of capacity utilization, just as KT theory proposes. Because the multiplier  $Y_t = I_t/s$  is assumed to hold in each period, a single period implicitly represents 'short-run time' (e.g. one quarter). Then one way to capture the notion of investment responding 'slowly' is to suppose that investment decisions only change over some appropriate period of 'long-run time' (e.g. one year).

The spreadsheet simulation shown in Table A1 embodies this distinction by having the growth rate of investment respond to the four-period average of variables such as capacity utilization ( $u$ ) and the change in the savings rate ( $g_s$ ). Zero-mean random noise is generated by using the Excel function combination (NORMSINV(RAND()\*sd) + m), where m = the mean (1 in this case) and sd = the standard deviation (2 in this run), and this noise is modified by the gain parameter  $\mu$ . The basic parameter values and the formulas for each of the variables are displayed in the sheet, along with two charts of the key variables. The run begins at  $t = 1$  from the long-run equilibrium situation  $g_Y = g_K = g_I = g_W$ , with subsequent fluctuations until  $t = 9$  due solely to random shocks. But in period 10 the savings rate ( $s$ ) is gradually reduced over four quarters from 0.20 to 0.12, to which investment growth responds 'slowly', i.e. in annual terms. The capacity utilization rate initially rises (the paradox of thrift) but then returns to gravitating around normal rate (figure A1). Output also rises faster than normal capacity but then falls below it before returning to a *now lower trend rate of growth* (figure A2). The paradox of thrift is thus completely overturned in the long run.

Table A1. Simulation of the HH adjustment

Time	s	g <sub>s</sub>	g <sub>t</sub>	I(-1)*(1 + g <sub>k</sub> (-1)K)	I	K	g <sub>k</sub>	Y	g <sub>y</sub>	g <sub>y</sub> <sup>W</sup>	Y <sub>max</sub>	Y <sub>n</sub>	Y/Y <sub>max</sub>	u	u <sub>0</sub>
1	0.20	0	0.100	1.000	1.000	11.000	0.100	5.000	0.100	0.100	10.000	5.000	0.500	0.500	0.500
2	0.20	0	0.028	1.028	1.028	12.028	0.093	5.140	0.028	0.100	11.000	5.500	0.467	0.467	0.500
3	0.20	0	0.016	1.045	1.045	13.073	0.087	5.223	0.016	0.100	12.028	6.014	0.434	0.434	0.500
4	0.20	0	0.093	1.142	1.142	14.215	0.087	5.710	0.093	0.100	13.073	6.536	0.437	0.437	0.500
5	0.20	0	0.086	1.241	1.241	15.455	0.087	6.203	0.086	0.100	14.215	7.107	0.436	0.436	0.500
6	0.20	0	0.142	1.417	1.417	16.872	0.092	7.084	0.142	0.100	15.455	7.728	0.458	0.458	0.500
7	0.20	0	0.125	1.593	1.593	18.465	0.094	7.966	0.125	0.100	16.872	8.436	0.472	0.472	0.500
8	0.20	0	0.014	1.616	1.616	20.081	0.087	8.078	0.014	0.100	18.465	9.233	0.437	0.437	0.500
9	0.20	0	0.084	1.751	1.751	21.832	0.087	8.754	0.084	0.100	20.081	10.041	0.436	0.436	0.500
10	0.18	-0.08	0.116	1.953	1.953	23.785	0.089	10.615	0.213	0.092	21.832	10.916	0.486	0.486	0.500
11	0.17	-0.09	0.067	2.083	2.083	25.868	0.088	12.401	0.168	0.084	23.785	11.893	0.521	0.521	0.500
12	0.15	-0.10	0.094	2.280	2.280	28.148	0.088	15.000	0.210	0.076	25.868	12.934	0.580	0.580	0.500
13	0.14	-0.11	0.058	2.413	2.413	30.561	0.086	17.741	0.183	0.068	28.148	14.074	0.630	0.630	0.500
14	0.12	-0.12	0.003	2.421	2.421	32.982	0.079	20.174	0.137	0.060	30.561	15.281	0.660	0.660	0.500
15	0.12	0.00	0.079	2.613	2.613	35.595	0.079	21.777	0.079	0.060	32.982	16.491	0.660	0.660	0.500
16	0.12	0.00	-0.068	2.437	2.437	38.032	0.068	20.305	-0.068	0.060	35.595	17.798	0.570	0.570	0.500
17	0.12	0.00	-0.137	2.104	2.104	40.136	0.055	17.531	-0.137	0.060	38.032	19.016	0.461	0.461	0.500
18	0.12	0.00	-0.052	1.995	1.995	42.131	0.050	16.627	-0.052	0.060	40.136	20.068	0.414	0.414	0.500
19	0.12	0.00	0.006	2.007	2.007	44.138	0.048	16.722	0.006	0.060	42.131	21.065	0.397	0.397	0.500

20	0.12	0.00	-0.037	1.933	1.933	46.070	0.044	16.107	-0.037	0.060	44.138	22.069	0.365	0.365	0.500
21	0.12	0.00	0.012	1.956	1.956	48.027	0.042	16.304	0.012	0.060	46.070	23.035	0.354	0.354	0.500
22	0.12	0.00	0.079	2.111	2.111	50.138	0.044	17.594	0.079	0.060	48.027	24.013	0.366	0.366	0.500
23	0.12	0.00	0.128	2.383	2.383	52.521	0.048	19.855	0.128	0.060	50.138	25.069	0.396	0.396	0.500
24	0.12	0.00	0.054	2.512	2.512	55.033	0.048	20.935	0.054	0.060	52.521	26.260	0.399	0.399	0.500
25	0.12	0.00	0.057	2.656	2.656	57.689	0.048	22.132	0.057	0.060	55.033	27.516	0.402	0.402	0.500
26	0.12	0.00	0.090	2.894	2.894	60.583	0.050	24.115	0.090	0.060	57.689	28.844	0.418	0.418	0.500
27	0.12	0.00	0.064	3.080	3.080	63.662	0.051	25.664	0.064	0.060	60.583	30.291	0.424	0.424	0.500
28	0.12	0.00	0.085	3.342	3.342	67.005	0.053	27.853	0.085	0.060	63.662	31.831	0.438	0.438	0.500
29	0.12	0.00	0.098	3.671	3.671	70.676	0.055	30.595	0.098	0.060	67.005	33.502	0.457	0.457	0.500
30	0.12	0.00	0.098	4.033	4.033	74.709	0.057	33.606	0.098	0.060	70.676	35.338	0.475	0.475	0.500
31	0.12	0.00	0.105	4.457	4.457	79.166	0.060	37.143	0.105	0.060	74.709	37.354	0.497	0.497	0.500
32	0.12	0.00	0.083	4.826	4.826	83.992	0.061	40.215	0.083	0.060	79.166	39.583	0.508	0.508	0.500
33	0.12	0.00	0.088	5.253	5.253	89.245	0.063	43.772	0.088	0.060	83.992	41.996	0.521	0.521	0.500
34	0.12	0.00	0.072	5.632	5.632	94.876	0.063	46.931	0.072	0.060	89.245	44.622	0.526	0.526	0.500
35	0.12	0.00	0.075	6.051	6.051	100.928	0.064	50.429	0.075	0.060	94.876	47.438	0.532	0.532	0.500
36	0.12	0.00	0.072	6.486	6.486	107.414	0.064	54.049	0.072	0.060	100.928	50.464	0.536	0.536	0.500
37	0.12	0.00	0.030	6.679	6.679	114.092	0.062	55.654	0.030	0.060	107.414	53.707	0.518	0.518	0.500

g<sub>t</sub> = {[1 + g<sub>k</sub>(-1) - k{avg(t - 4) - u<sub>t</sub>}] + [1 + g<sub>y</sub>avg(t - 4)]} - 1 + ε, where k > 0, ε = zero-mean random number, parameter values are: k = 0.3, R = 1, and the random shocks are modified by a gain parameter = 0.02, and 'avg(t - 4)' refers to the average of the preceding four values of the variable in question.



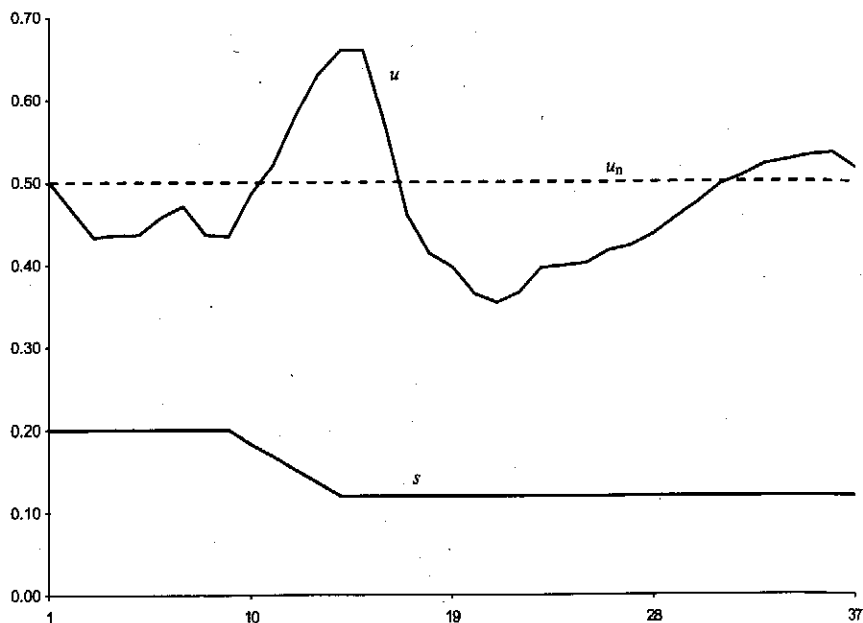


Figure A1. Warranted path stability: response of capacity utilization to a drop in the savings rate.

APPENDIX B

Fiscal policy and Harrodian warranted growth

The stability of the Harrodian warranted path implies that over the long run the growth rate of output is given by  $g_Y = g_{Yn} = s \cdot R$ , where  $R$  = the engineering capacity-capital ratio ( $Y_{max}/K$ ) and  $s$  = the social savings rate =  $s - g - x = s_p \cdot (1 - \theta) + (\theta - g) + (\mu - x)$  = the private savings rate out of pre-tax income + the net government savings rate + the net foreign savings rate.

In order to concentrate on the efficacy of fiscal policy, we will focus on the path of government spending ( $G_t$ ), keeping all other variables, including the tax rate ( $\theta$ ), constant by assumption. With this in mind, we write the warranted path, where  $s_1 = s_p + \theta(1 - s_p) + (\mu - x)$ , as

$$g_Y = (s_1 - g) \cdot R \tag{B1}$$

Note that if the government share ( $g$ ) rises (falls), then the warranted rate of growth of output falls (rises). The question we now consider is: how do

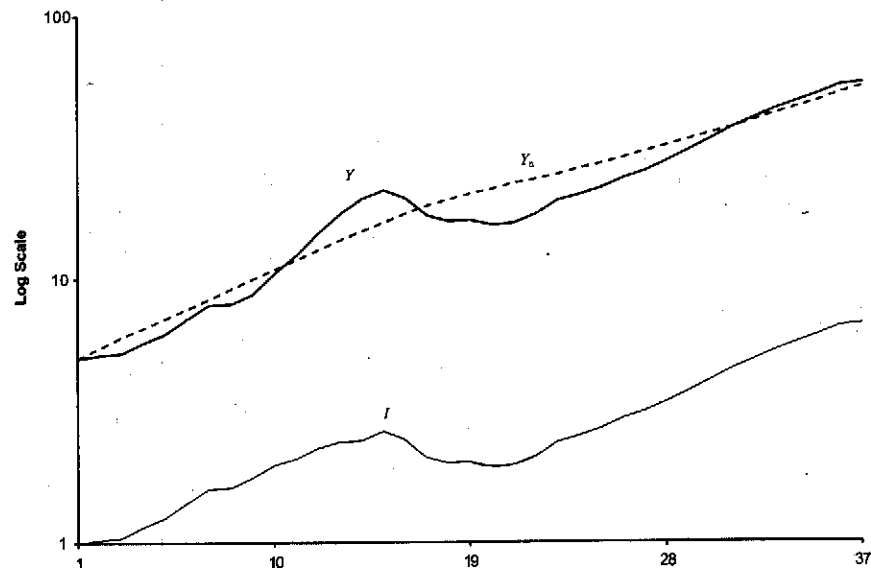


Figure A2. Warranted path stability: response of investment, capacity and output to a drop in the savings rate.

various paths of the level of government spending ( $G$ ) affect its share? Two obvious paths for government spending come to mind: it grows at some autonomous rate  $\gamma$ , or it adjusts in some manner in order to maintain some desired or at-least tolerable deficit share  $d^* = g^* - \theta$ . We will see that the first type of path inevitably leads to the second type.

Suppose that government spending grows at some constant rate  $\gamma$ .

$$G_t = G_A \cdot e^{\gamma t} \tag{B2}$$

where  $G_A$  = its 'scale' (shift parameter).<sup>21</sup> Since  $g \equiv G/Y$ ,  $g' = G'/Y - (Y'/Y) \cdot (G/Y) = [(G'/G) - g_Y] \cdot (G/Y)$ , which from equation (B2) gives

$$g' = (\gamma - g_Y) \cdot g \tag{B3}$$

Combining equations (B1) and (B3) yields the first-order non-linear differential equation

$$g' = (\gamma - s_1 \cdot R) \cdot g + g^2 \cdot R \tag{B4}$$

<sup>21</sup> In log terms,  $\ln G_t = \ln G_A + \gamma \cdot t$ , so that a change in  $G_A$  shifts the curve, and a change in  $\gamma$  alters its slope.

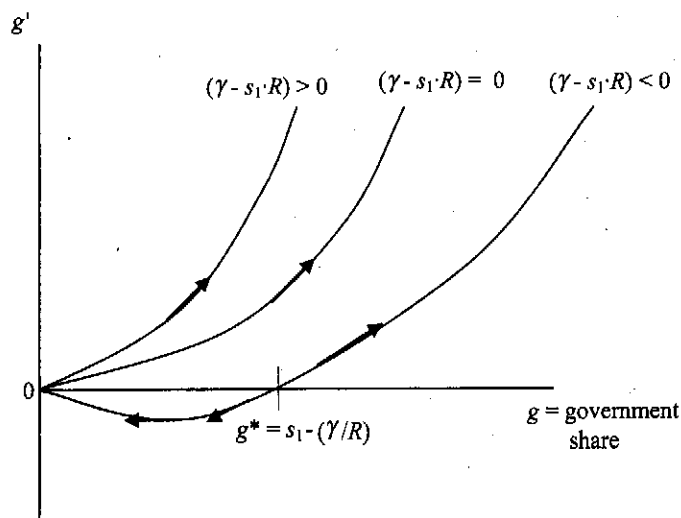


Figure B1. Instability of exogenously growing government spending.

Since  $g \geq 0$ , the phase diagram for this equation expresses three cases shown below:<sup>22</sup>  $(\gamma - s_1 \cdot R) > 0$ ;  $(\gamma - s_1 \cdot R) = 0$ ; and  $(\gamma - s_1 \cdot R) < 0$  (figure B1).

The first two curves in figure B1 represent the situations in which the rate of growth of government spending  $\gamma \geq s_1 \cdot R$ , in which case the government share ( $g$ ), and hence the budget deficit ( $g - \theta$ ), rises without limit while the rate of growth of output falls steadily.<sup>23</sup> For the third curve, when the growth in government spending is  $\gamma < s_1 \cdot R$ , there is a point at which the government share is constant at  $g^* = s_1 - (\gamma/R)$ , and the corresponding rate of growth of output is  $g_Y^* = \gamma$  (from equation (B1)). This seems to suggest that any autonomous rate of growth of government spending ( $\gamma$ ) will eventually induce output to grow at this same rate, provided the former is below some critical threshold rate ( $\gamma^* \equiv s_1 \cdot R$ ). But this quintessential Keynesian outcome

<sup>22</sup> When  $(\gamma - s_1 \cdot R) > 0$ ,  $g'$  consists of the sum of a positively sloped straight line and an exponential curve, and rises steadily; when  $(\gamma - s_1 \cdot R) = 0$ ,  $g'$  is just the exponential curve, which also rises steadily; and when  $(\gamma - s_1 \cdot R) < 0$ ,  $g'$  is the sum of a negatively sloped line and an exponential curve, which dips below the  $g$ -axis, crosses at  $g = s_1 - (\gamma/R)$ , and then rises.

<sup>23</sup> Since  $g \geq 0$ , we are confined to the positive orthant in  $g$ . Along the first two curves, this space implies that any positive  $g$  is associated with a positive  $g'$ , which means the point  $g = 0$  is unstable. Thus the government share  $g$ , and hence the budget deficit ( $g - \theta$ ), grows without limit, and from equation (20), this implies a continually falling warranted rate of growth.

is unstable.<sup>24</sup> when the initial government share  $g_0 > g^*$ , rising government deficits reappear; conversely, when  $g_0 < g^*$ , the government share shrinks towards zero, in effect because the rate of growth of government spending is less than the corresponding rate of growth of output. The point  $g_0 = g^*$  demarcates the two regimes.

The meaning of the instability around  $g = g^*$  becomes clearer if we begin from an initial situation in which government spending is not growing exogenously, but has rather adapted itself to the rate of growth of output in such a way as to maintain a given government share  $g_0$ , in a manner to be formalized shortly. From equation (B1) we know that the corresponding rate of growth of output is  $g_Y = (s_1 - g_0) \cdot R$ . But since the government share is being maintained at  $g_0$ , this means that government spending is also initially growing at same rate as output, i.e. at the rate  $\gamma = (s_1 - g_0) \cdot R$ . This implies the initial government share and the initial growth rate of government spending are related:  $g_0 = s_1 - (\gamma/R)$ . Note that this is the same relation as that corresponding to the point  $g^*$  in figure A1, except that in this case it is achieved through an adaptive rate of government spending. Now suppose that government spending switches to some exogenous rate  $\gamma \neq \gamma_0$ . Then our previous analysis tells us that if the new exogenous rate of government spending is  $\gamma \geq \gamma_0 = g_Y = (s_1 - g_0) \cdot R$ , the government share will rise steadily and output growth will fall steadily. Only when the new exogenous rate of growth of government spending  $\gamma < \gamma_0 = g_Y = (s_1 - g_0) \cdot R$  will the government share fall and the warranted rate of growth rise. Thus the key is the relation between the exogenous growth rate of government spending ( $\gamma$ ) and the pre-existing warranted rate of growth  $g_Y = (s_1 - g_0) \cdot R$ . As long as the rate of growth of government spending is kept below the pre-existing warranted rate (which can itself change over time), it can provide a short-run stimulus and also enhance long-run growth. This follows from the properties of the path along which capacity utilization is normal on average, without any recourse to interest rate effects or employment constraints.

In any case, whether or not exogenous government spending is growth-enhancing, it leads to continuously changing government shares and budget deficits, other things being equal. This implies that government spending must inevitably adapt to some target (or at least tolerable) budget deficit. Suppose that this deficit target was  $d^*$ . At any given tax rate ( $\theta$ ), the corresponding target government share  $g^* = d^* + \theta$  could be achieved through a

<sup>24</sup> In the third case, for any  $g_0 > g^*$ , we are on a curve at which  $g > 0$  and  $g' > 0$ , so  $g$  is rising. Conversely, for any positive  $g_0 < g^*$ , we are on a curve with  $g > 0$  and  $g' < 0$ , so that  $g$  is falling. In this last instance, as  $g$  approaches zero, so does its time rate of change ( $g'$ ), so that  $g$  never arrives at zero.

variable growth rate of government spending that rose above or fell below the growth rate of output according to whether the desired government share ( $g^*$ ) was above or below the actual (equation (B5), where  $\beta$  is a reaction coefficient). Since  $g = G/Y$ , this would imply a stable adjustment around the desired share,<sup>25</sup> as is evident in equation (B6). Then from the basic Harrodian policy relation (equation (17) in the text) a rise in the desired government share, or a fall in the tax rate, or an overall rise in the desired government deficit relative to income, will lower the warranted rate of growth.

$$G'/G = g_Y + \beta \cdot (g^* - g) \quad (\text{B5})$$

$$g' = \beta \cdot (g^* - g) \cdot g \quad (\text{B6})$$

## APPENDIX C

### Stability of profit-driven accumulation with an endogenous business savings rate

Two basic relations are common to the PK and CT dynamical systems: short-run equilibrium ( $I = S$ ), which implies a relation between the rate of accumulation, the savings rate and the rate of capacity utilization; and an accumulation function of a standard PK form (without the error term). The relevant common equations are reproduced below as they appear in the text.

$$g_K = s \cdot R \cdot u \quad [\text{Short-run equilibrium: } I = S] \quad (18)$$

$$g_K = g_K(r_n) + h \cdot (u - u_n) \quad [\text{A standard PK accumulation function}] \quad (15)$$

The subsequent difference between the PK and CT dynamical systems then arises *solely* from their respective treatment of the business savings rate. If we take the household savings rate and the distribution of income as given, so as to isolate the issue at hand, the difference between the two approaches is clear: the PK system appears as a special case of the CT system when we assume a fixed business savings rate. Thus the former can be derived from the latter by setting the savings rate adjustment coefficient  $\sigma = 0$  in equation (C1) or equation (C2), in which case profit-driven accumulation yields the standard PK result that capacity utilization remains persistently different from the normal rate. Conversely, *any responsiveness at all* on the part of the

<sup>25</sup> This latter adjustment is of the same form as the Hicksian stock-flow adjustment principle in equation (11).

business savings rate ( $\sigma > 0$ ) converts the PK system into a CT one, and it becomes possible to have both profit-driven accumulation and normal capacity utilization over the long run.

$$s' = \sigma \cdot (s_n - s), \quad \text{where } s_n = g_K(r_n)/(R \cdot u_n) = \text{the normal savings rate} \quad (\text{C1})$$

(equation (19))

or

$$s' = \sigma \cdot [g_K - g_K(r_n)] \quad (\text{C2})$$

If equation (C1) represents the adjustment process,  $s$  converges to  $s_n$  independently of any other processes or parameters in the system. Since  $s_n$  only obtains at  $u = u_n$ , this implies  $g_K = g_K(r_n)$  from equation (15). Thus CT accumulation is profit-driven as in Keynesian theory and also gravitates around a normal capacity utilization path as in Harrodian theory.

On the other hand, if equation (C2) represents the adjustment process, then we end up with a non-linear differential equation in the endogenous savings rate.<sup>26</sup>

$$s' = \sigma \cdot g_K(r_n) \cdot [(1 - (s/s_n))/\{(s/s_n) \cdot (s_n \cdot R/h) - 1\}] \quad (\text{C3})$$

Suppose that  $h < s_n \cdot R$ , so that  $\{(s/s_n) \cdot (s_n \cdot R/h) - 1\} > 0$  when  $s \geq s_n$ . Then in equation (C3) as  $s \rightarrow s_n$  from above, the denominator of the expression in square brackets remains positive and the numerator goes from negative to zero, so that  $s'$  goes from negative to zero also. Finally, for  $s < s_n$ , as  $s \rightarrow h/R < s_n$ ,  $s'$  approaches infinity because the numerator is positive while denominator is also positive and approaches zero. The phase diagram corresponding to equation (C2) (figure C1) therefore has a single stable positive equilibrium at  $s = s_n$ , as long as  $h < s_n \cdot R$ . It is useful to note that  $h < s_n \cdot R = g_K(r_n)/u_n$  is also the stability condition for the PK model (Lavoie, 1996, p. 122; Dutt, 1997, p. 246).

Finally, for the simulation results of this classical system, which were displayed in figures 5 and 6 in the text, equations (15) and (21) were written as the difference equations  $g_K = g_K(r_n) + h \cdot [u(-1) - u_n]$  and  $s = s(-1) + \sigma \cdot h \cdot [u(-1) - u_n]$ , since  $[g_K - g_K(r_n)] = h \cdot [u(-1) - u_n]$ , with parameter values  $R = 0.5$ ,  $g_K(r_n) = 0.03$ ,  $h = 0.01$ ,  $\sigma = 2$ ,  $u_n = 1$ , and initial values at

<sup>26</sup> Since  $[g_K - g_K(r_n)] = h \cdot (u - u_n)$  from equation (15), and  $u = g_K/s \cdot R$  and  $u_n = g_K(r_n)/(s_n \cdot R)$  from equation (18),  $[g_K - g_K(r_n)] = h \cdot \{(g_K/s \cdot R) - u_n\} = h \cdot \{(g_K/s \cdot R) - [g_K(r_n)/s \cdot R] + [g_K(r_n)/s \cdot R] - u_n\} = (h/s \cdot R) \cdot [g_K - g_K(r_n)] + h \cdot [g_K(r_n)/R] \cdot [(1/s) - (1/s_n)]$ , since  $(R \cdot u_n)/g_K(r_n) = 1/s_n$ . Collecting terms gives  $[g_K - g_K(r_n)] = h \cdot [g_K(r_n)/R] \cdot [(1/s) - (1/s_n)] \cdot [1 - h/(s \cdot R)] = h \cdot [g_K(r_n)] \cdot [1 - (s/s_n)]/(s \cdot R - h) = g_K(r_n) \cdot [1 - (s/s_n)]/[(s/s_n) \cdot (s_n \cdot R/h) - 1]$ . Substituting this into the adjustment equation (C2) gives us equation (C3), a non-linear differential equation in the endogenous savings rate  $s$ .

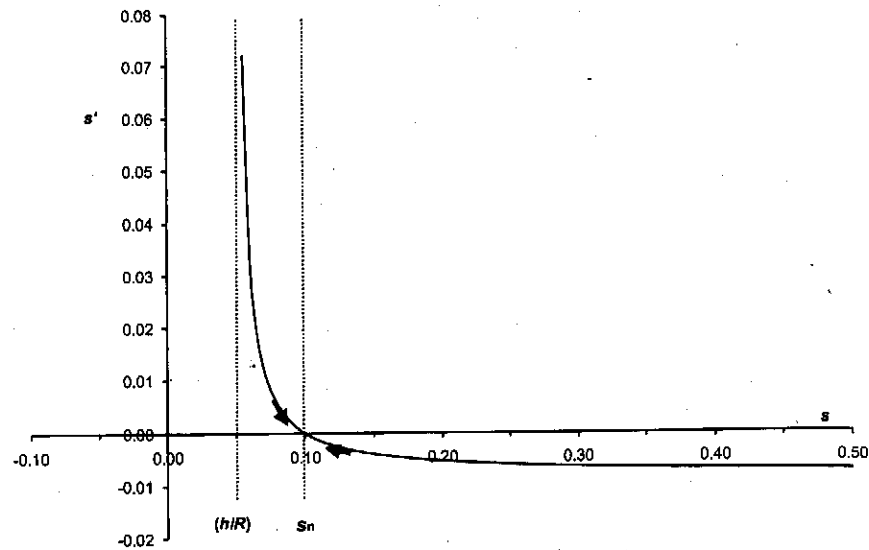


Figure C1. Phase diagram, endogenous savings rate.

equilibrium levels  $g_K(0) = g_K(r_n)$ ,  $u(0) = u_n$  and  $s(0) = s_n \equiv g_K(r_n)/R \cdot u_n$ . Investment was derived as  $I = g_K \cdot K(-1)$ , output as  $Y = I/s$ , and capital as  $K = I + K(-1)$ , with initial values  $I(0) = 10$ ,  $K(0) = 343.333$ . The equilibrium run is broken at  $t = 10$  by a permanent addition to investment of 2.54 (which is 10 per cent of investment in the prior period), and at  $t = 30$  by a rise in the profit-driven component of accumulation  $g_K(r_n)$  from 0.03 to 0.04. Figure 5 in the text displayed the paths of  $\ln I$  and  $\ln Y$ , while figure 6 displayed those of  $s$  and  $u$ .

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## RE-EXAMINING THE IMPLICATIONS OF THE NEW CONSENSUS: ENDOGENOUS MONEY AND TAYLOR RULES IN A SIMPLE NEOCLASSICAL MACRO MODEL

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### ABSTRACT

This paper re-examines the impact of endogenous money in a neoclassical model with interest-sensitive expenditures. It first outlines a benchmark model with exogenous money and the usual full employment and money growth-determined inflation results. It then replaces exogenous money with endogenous money, which is shown to generate model indeterminacy. Two methods of resolving this indeterminacy are then explored: money illusion and a Taylor rule for monetary policy, a key feature of new consensus models. The paper concludes that endogenous money has negative implications for the behaviour and interpretation of neoclassical and new consensus models.

### 1. INTRODUCTION

Much attention has recently been focused on the concept of a 'new consensus' in monetary theory. The idea of this consensus was initially applied to orthodox economists across the monetarist–Keynesian divide regarding the best framework for thinking about the conduct and effectiveness of monetary policy (Allsopp and Vines, 2000, p. 2). But Post Keynesian economists have shown considerable interest in the idea because one of its dimensions is explicit recognition that monetary policy is conducted using a short-term

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