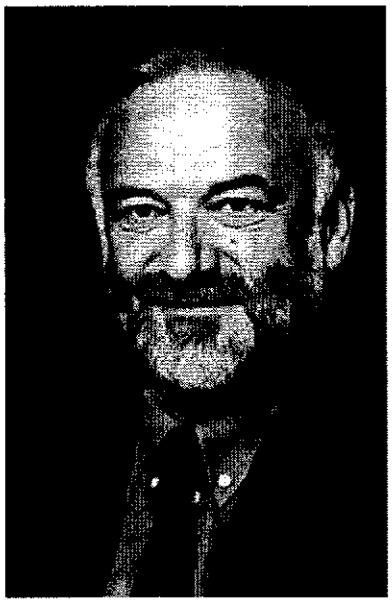

Classical Political Economy and Modern Theory

Essays in honour of Heinz Kurz

Edited by Christian Gehrke, Neri Salvadori, Ian Steedman and Richard Sturn

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his profit by making use of a monopoly position in circulation and cheating the workers' *(ivi)*, an idea which he can then reject as utterly alien to Marx for whom

> 'surplus value is created in production and not in circulation, [in accordance with] the specific feature of capitalism as a mode of production, not merely as a mode of distribution [in which] the labourer works under the control of the capitalist, who compels him to work . . . for longer than is necessary merely to replace the means of subsistence he consumes'
>
> (op. cit., 63).

What may, however, not be clear to a reader of Rowthorn's three passages above is how 'Ricardo and his modern followers' can ever have thought that the commodities making up the capitalists' net share of the social product might have come into existence unless 'the labourer works . . . for longer than is necessary merely to replace the means of subsistence he consumes'. Surely the commodities capitalists consume and accumulate require some labour to be produced? The question remains of course unanswered when Rowthorn goes on to explain what he means by the idea of the capitalists using 'a monopoly position in circulation' and 'cheating the workers' by such means. No references to 'neo-Ricardian' texts are on the other hand given there by which one could assess meaning and justification of this second allegation, which is also rather surprising though in any case quite different from that about a denial of surplus labour.

4 This could easily be done in terms of the 'integrated wage goods sector' (on that 'sector', cf. P. Garegnani, *Value and Distribution in the Classical Economists and Marx*, Oxford Economic Papers, June 1984, 13–20) or of Sraffa's own Standard System (*op. cit.*, Ch. IV).

6 The empirical linearity of Sraffa's critical output–capital ratios

Anwar Shaikh

Introduction

Heinz Kurz is a major figure in the development of classical economic theory, and a model of theoretical clarity and rigour. I offer this paper as a small token of my appreciation for his many writings on the classical and Sraffian theory of relative prices. As I argue at the end, my findings on the virtual linearity of output–capital ratios and the resulting near-linearity of standard prices and aggregate wage–profit curves leads us back to the price and distribution theories of Ricardo and Marx, not to those of neo-classical economics. In what follows, I will use the term 'prices of production' to signify prices which reflect a common rate of profit, and 'direct prices' to signify money prices which are proportional to total (direct and indirect) labor requirements, that is proportional to labor values.

Sraffa argues that relative prices of production can behave in very complicated ways as the distribution between wages and profits is changed. Since one source of this complexity may stem from the movements of the price of the numeraire commodity, Sraffa proposes to construct a numeraire which would itself be 'under no necessity, arising from the conditions of production of the industry itself, either to rise or fall in value relative to any other commodity when wages rose or fell'. He shows that this ideal numeraire, a composite which he calls the standard commodity (which may also be viewed as a particular 'average' sector), can always be constructed and that it gives rise to a linear relation between the wage rate in terms of this numeraire and the corresponding rate of profit. He also proves that imposing this linear wage–profit relation on the relative price system amounts to measuring all prices in terms of the standard commodity without having to actually calculate the latter (Sraffa, 1960, Chs III–IV, and quote from 16). Even so, he argues that prices expressed in these terms, that is standard prices, may still exhibit complicated movements. As the wage rate is successively raised, the corresponding rate of profit falls, and prices of production adjust to accommodate these changes. But changes in output prices affect the money value of the means of production, which requires output prices to change again to attain a common rate of profit, which feeds back on the prices of means of production, and so on. In the end, 'the price of a product . . . may rise or it may fall, or it may even alternate in rising and falling, relative to its means of production' (*op. cit.* 15).

The movements in the output–capital ratio are therefore the key to the complexity, or simplicity, of the theoretical movements of relative prices (*op. cit.*, 12–15). The question here is: How do they behave in practice?

Theoretical calculations

Let **p, l, A, D, K** be price and labor coefficients row vectors and square input-output, depreciation and capital coefficient matrices, respectively, for an n-sector system, and let w, r be the scalar wage and profit rates, respectively. Then the Sraffa price system for a pure circulating capital model has **K** = **A** and **D** = **0**, so that

$$\mathbf{p}(r) = w\mathbf{l} + \mathbf{p}(r)(\mathbf{A} + \mathbf{D}) + r\mathbf{p}(r)\mathbf{K} = w\mathbf{l} + (1 + r)\,\mathbf{p}(r)\mathbf{A} \qquad (6.1)$$

This is usefully expressed in terms of total (direct and indirect) labor requirement $\mathbf{v} = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$ and total capital coefficients $\mathbf{H} = \mathbf{K}(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{A}(\mathbf{I} - \mathbf{A})^{-1}$. Here, **v** is the vector of unit labour values, that is vertically integrated labour requirements per unit product (Pasinetti 1977, 76, 123, 141 fn 123).

$$\mathbf{p}(r) = w\mathbf{v} + r\mathbf{p}(r)\mathbf{H} \qquad (6.2)$$

The preceding system consists of n-equations in n+2 variables (n prices, w, r). When the wage is zero, this reduces to $\mathbf{p}(R) = R\mathbf{p}(R)\mathbf{H}$, where $\mathbf{p}(R)$ is the all-positive dominant left eigenvector of H and the maximum rate of profit R is the reciprocal of the dominant eigenvalue of **H**. When the wage (w) is positive, the relation between the wage rate, the profit rate and relative prices is complicated by the fact that a chosen numeraire may contribute its own variations to all price ratios. Sraffa shows that there is a standard commodity (sector) which need not vary in price as the wage changes, and that the wage in terms of the price of this standard commodity will then be a linear function of r/R for any given single-product technology (**l**, **A**). Since the standard commodity is unique for viable single-product systems, imposing this linear relation $w = 1 - r/R$ on the price system in equations (6.1) or (6.2) is equivalent to selecting the standard commodity as numeraire (Sraffa 1960, 30–2). Hence the system of standard prices (prices expressed in terms of the standard commodity) can be written as

$$\mathbf{p}(r) = (1 - r/R)\,\mathbf{v} + r\mathbf{p}(r)\,\mathbf{H} = \mathbf{v} + r\mathbf{v}\left(H - \frac{1}{R}I\right) + (\mathbf{p}(r) - \mathbf{v})\mathbf{H} \qquad (6.3)$$

Note that the Sraffa standard prices in equation (6.3) are implicitly in labour units, since $\mathbf{p}(0) = \mathbf{v}$. Then the three components on the right-hand side of equation (6.3) can be given familiar interpretations. The first component is the Ricardian term, the total labor requirements (labour value) vector $\mathbf{v} = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$. The first two components can in turn be viewed as the Marxian term (within a Sraffian context),[1] the vertically integrated equivalent of Marx's transformation procedure. This is the sum of labour values **v** and price-value deviations $r\mathbf{v}\left(\mathbf{H} - \frac{1}{R}\mathbf{I}\right)$, the size of the

latter being dependent on the rate of profit and on the degree to which industry vertically integrated organic compositions differ from the 'average' (that is standard) composition.[2] Finally, the third component is the Wicksell–Sraffa term $(\mathbf{p}(r) - \mathbf{v})\,\mathbf{H}$, which represents the feedback effects of standard price–value deviations on the prices of means of production. These feedback effects are central to Sraffa's analysis. In this regard, it is useful to express the j[th] price as

$$p(r)_j = w(r)v_j + p(r)_j\left(\frac{r}{VR(r)_j}\right) \qquad (6.4)$$

where $VR(r)_j \equiv \frac{p(r)_j}{\mathbf{p}(r)\mathbf{H}^{(j)}}$ is the j[th] vertically integrated output–capital ratio which is a function of r. Sraffa tells us that each output–capital ratio VR_j starts from a particular value which is specific to the industry at r = 0 and each then converges to the common ratio R at r = R (Sraffa, 1960, 17). This is evident from the price system in equation (6.3): at r = 0, $\mathbf{p}(r) = \mathbf{v}$ so the j[th] labour value of the vertically integrated output–capital ratio is $\frac{v_j}{\mathbf{v}\mathbf{H}^{(j)}}$, where $\mathbf{H}^{(j)}$ is the j[th] column of the total capital coefficients matrix **H** so that $\mathbf{v}\mathbf{H}^{(j)}$ represents the labour value of the total input requirement per unit output, that is vertically integrated constant capital per unit output; on the other hand, at w = 0, r = R, and the price system reduces to $\mathbf{p}(R) = R\mathbf{p}(R)\mathbf{H}$, so that the j[th] vertically integrated output–capital ratio is $\frac{p_j(R)}{\mathbf{p}(R)\mathbf{H}^{(j)}} = R$, which is the same for all industries and is also the labour value of net output–capital ratio, that is the living labour/dead labour ratio, in the standard sector.[3] The question then arises: How do these individual industry output–capital ratios move as they proceed from their individual labour value ratios to the common labour value ratio of the standard commodity? If they do it smoothly, then standard prices will deviate smoothly from values. But if, as Sraffa appears to suggest, they cross back and forth with R before arriving at their common limit, then the corresponding industry-standard prices will follow complicated paths.

Empirical evidence

Data for empirical matrices and vectors **A′**, **l′**, **X′** was derived from the US Bureau of Economic Analysis (BEA) and adjusted to remove a fictitious 'industry' which the BEA creates in order to treat private homeowners as businesses renting out their own homes to themselves. All empirical matrices and vectors are in terms of actual money flows, which means that observed total gross 'output' represents total money flows $X'_j \equiv pm_jX_j$ and calculated prices and direct and total labour requirements are ratios of the theoretical variables v_j, $p_j(r)$ to corresponding market prices pm_j: $v'_j = \frac{v_j}{pm_j}$, $p(r)'_j = \frac{p(r)_j}{pm_j}$ (see the Data Appendix).

Figure 6.1 presents the key evidence on the path of individual industry vertically integrated output–capital ratios (shown relative to R, the output–capital ratio of the standard industry). It is clear that individual output–capital ratios $VR(r)_j$ follow very smooth and *virtually linear* paths from their industry-specific initial labour value measures to the common labour value ratio of the standard industry R. Figure 6.2 focuses on the only four industries (oil and gas extraction; broadcasting and telecommunications; funds, trusts and other financial vehicles; and food services and drinking places) whose vertically integrated output–capital ratios exhibit any complexity at all. The first chart within Figure 6.2 covers the whole (0,1) range of r/R and we see that even in these four exceptional industries the move very smoothly for most part, with all turbulence confined to the range r/R > 0.75. The second chart zooms in on the upper range of r/R, and we see that these ratios can indeed switch directions as Sraffa implicitly suggests. But these switches are confined to the very small range of less than ±1 per cent of their final values (which is 1 for all industries).

If individual vertically integrated output-capital ratios $VR(r)_j$ were *exact* linear functions of r, the corresponding standard prices of individual commodities $p(r)_j$ would also be exact linear functions. The first proposition implies $VR(r)_j = VR_{0_j} + \frac{r}{R}\left(\frac{1}{R} - VR_{0_j}\right) = w(r)\,VR_{0_j} + r$, which is a linear function passing through the two requisite endpoints $VR(0)_j \equiv VR_{0j}$ and $VR(R)_j = R$. Substituting this into equation (6.4) and solving yields $p(r)_j = w(r)v_j + rVR_0 = {}_w(r)v_j + r\mathbf{v}\mathbf{H}^{(j)}$. But this is immediately recognizable as the sum of the first two parts of the right hand side of general price system in equation (6.3) – that is the Marx term in overall prices of production. Hence exactly linear $VR(r)_j$ would imply that the Wicksell–Sraffa component $(\mathbf{p}(r) - \mathbf{v})\mathbf{H} = \mathbf{0}$.

As shown in Figure 6.1, actual $VR(r)_j$ are very close to linear but not exactly so. Thus the corresponding actual standard prices turn out to be only near-linear, with a few close to quadratic. Still, of the sixty-five industry-standard prices only four exhibit a (single) reversal in direction of price–value deviations, and these are precisely the four industries whose output–capital ratios were shown in Figure 6.2 to reverse directions at very high standard profit shares r/R > 0.75 (by contrast with the actual profit share of 0.42 which obtains even at this high level of abstraction). Even here the reversals only occur for industries whose prices remain close to labour values throughout (within 10 per cent), and only at very high profit shares. The patterns in 1998 seem generic to US tables, both to newly available BEA tables covering 1997–2009 and earlier ones for 1947–73 (Shaikh, 1998).

The foregoing individual industry patterns immediately imply that the aggregate wage–profit curve will be a ratio of two linear functions of the profit rate, that is a rectangular hyperbola. To see this, multiply the price system in equation (6.2) by the net output vector **Y** to get

$$\mathbf{p}(r)\mathbf{Y} = w\mathbf{v}\mathbf{Y} + r\mathbf{p}(r)\mathbf{H}\mathbf{Y} \qquad (6.5)$$

The first term $\mathbf{p}(r)\mathbf{Y}$ is simply aggregate value added evaluated at prices of production. Given the definitions $\mathbf{v} = \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$, $\mathbf{Y} = (\mathbf{I} - \mathbf{A})\mathbf{X}$ and total

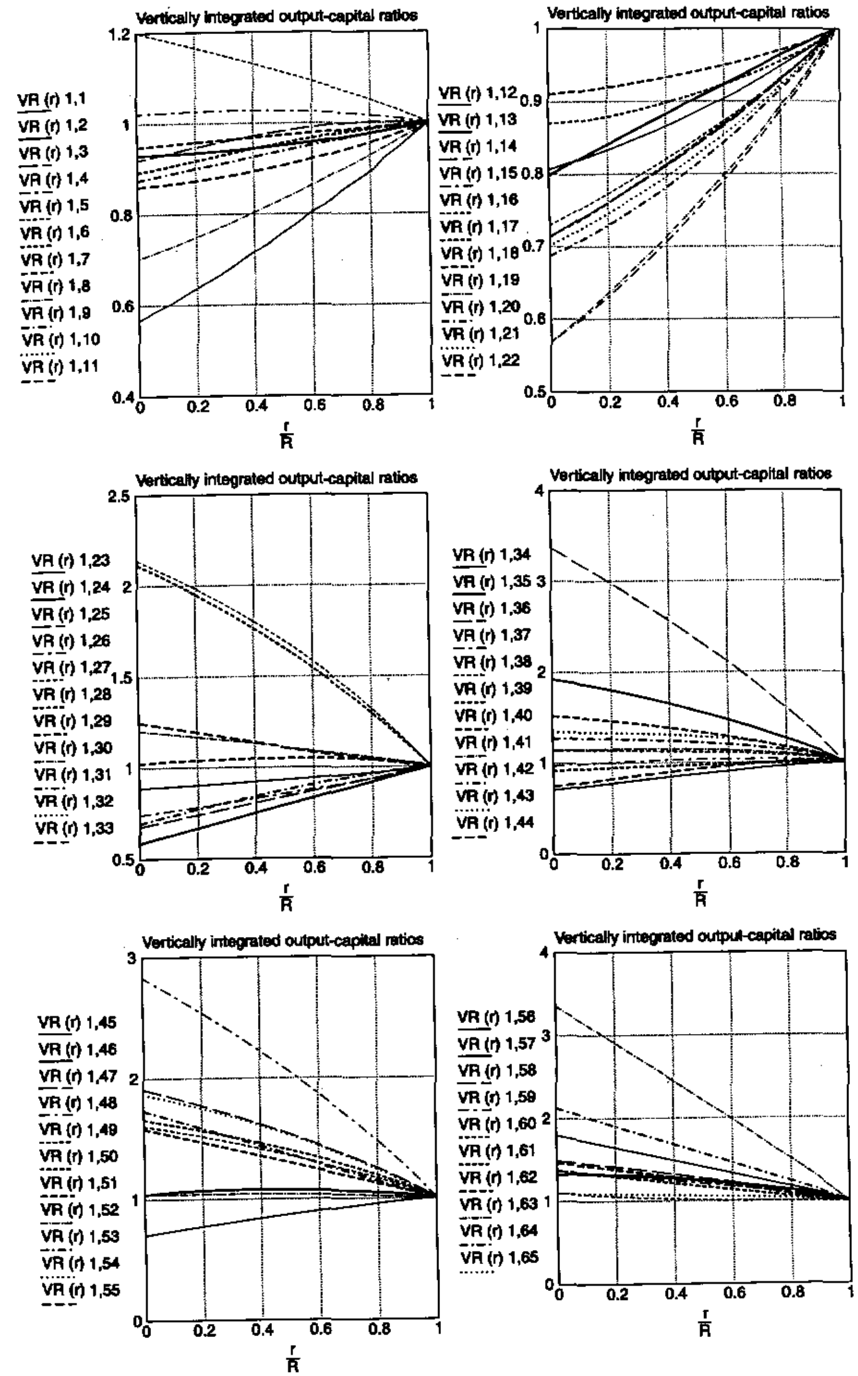

Figure 6.1 Vertically integrated output–capital ratios, 65 industries, US 1998.

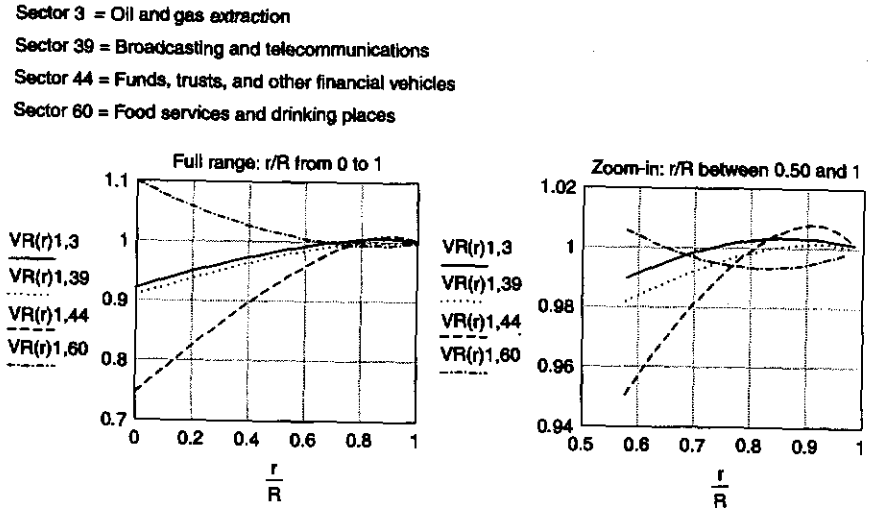

Figure 6.2 Vertically integrated output–capital ratios for four exceptional industries.

employment $L = \mathbf{l}\mathbf{X}$, the second term is simply the total wage bill wL. And the third term is aggregate profit, the product of the profit rate and the aggregate capital stock:

$$r\mathbf{p}(r)\mathbf{H}\mathbf{Y} = r\mathbf{p}(r)\,A(I-A)^{-1}\,(I-A)\,X = r\mathbf{p}(r)AX = rK(r),$$

where $K(r) = \mathbf{p}(r)\mathbf{H}\mathbf{Y} = \mathbf{p}(r)\mathbf{A}\mathbf{X}$ = the aggregate capital stock evaluated at prices of production. Then we can write the actual wage share $w(r)_a \equiv \frac{wL}{\mathbf{p}(r)\mathbf{Y}}$ as

$$w_a(r) = 1 - \frac{r}{R_a(r)} = \frac{R_a(r) - r}{R_a(r)} \qquad (6.6)$$

where $R_a(r)$ is a weighted average of individual vertically integrated output–capital ratios.

$$R_a(r) \equiv \frac{\mathbf{p}(r)\mathbf{Y}}{\mathbf{p}(r)\mathbf{H}\mathbf{Y}} = \frac{\sum_{j=1}^{n} p_j(r)Y_j}{\sum_{j=1}^{n} \mathbf{p}(r)\mathbf{H}^{(j)}Y_j} = \sum_{j=1}^{n}\left(\frac{p_j(r)}{\mathbf{p}(r)\mathbf{H}^{(j)}}\right)\left(\frac{\mathbf{p}(r)\mathbf{H}^{(j)}Y_j}{\sum_{j=1}^{n}\mathbf{p}(r)\mathbf{H}^{(j)}Y_j}\right) = \sum_{j=1}^{n} VR(r)_j\,\omega_j \qquad (6.7)$$

This is a convex combination of the weighted average of the vertically integrated output–capital ratios $VR(r)_j$, since the weights $w_j \equiv \left(\frac{\mathbf{p}(r)\mathbf{H}^{(j)}Y_j}{\sum_{j=1}^{n}\mathbf{p}(r)\mathbf{H}^{(j)}Y_j}\right)$ sum to one.

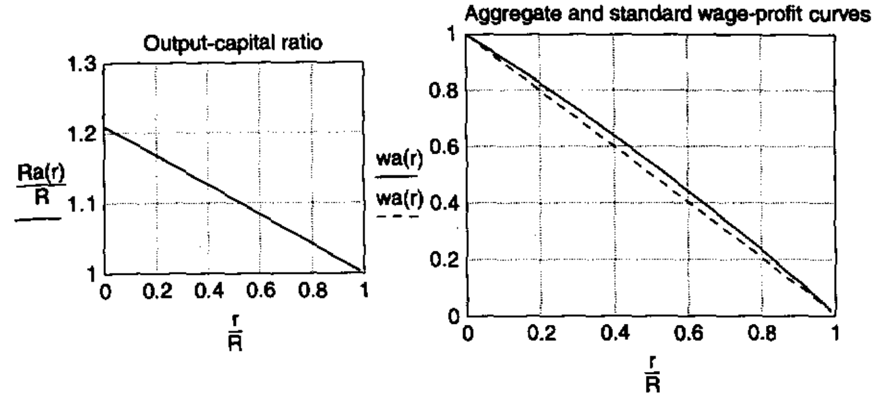

Figure 6.3 The aggregated vertically integrated output–capital ratio and wage–profit curve.

The empirical finding that industry vertically integrated output capital ratios are virtually linear implies their weighted average $R_a(r)$ will be almost exactly linear. In that case both the numerator and denominator in equation (6.6) will be linear functions of r, so that the actual wage–profit curve will be a rectangular hyperbola. The closer is the initial (labour) value of the aggregate output–capital ratio to (labour) value of the standard industry ratio (R), the less it will vary and the more linear will be the aggregate wage share curve.[4]

Figure 6.3 shows that all these expectations are fully realized: the ratio $\frac{R_a(r)}{R}$ turns out to be virtually linear, and the actual wage share is indeed near-linear.

Summary and implication

Classical theory maintains that market prices gravitate around prices of production. A vertically integrated Sraffian decomposition of the latter can be expressed as the sum of three components: labour values, deviations which increase linearly with the profit rate and a potentially complex set of Wicksell–Sraffa feedback effects of price changes on the means of production. Ricardo believed that the first component was dominant, and famously estimated that the remaining two would account for less than 7 per cent of the total. The size of the estimated profit share plays an important role in the observed size of the price–value deviations. In this paper, profit was defined abstractly as the difference between value added and employee compensation, and even so the distance between labour values account and prices of production is less than 20 per cent (distance being defined here by unit-free and scale-free measures such Euclidean distance or the coefficient of variation as in Mariolis and Soklis (2010) and Steedman and Tomkins (1998)). When profit is defined more concretely (net of indirect business taxes and depreciation) and fixed capital and inventories are taken into account, then this distance drops to about 10 per cent (see the Data Appendix). Ricardo rules.

Even though Marx initially develops his analysis in Volumes I–II under the assumption that prices are proportional to labour values, he is adamant that the two must be systematically different. In his famous (and incomplete) transformation procedure in Volume III, he derives prices of production as linear functions whose deviations from values increased with the rate of profit. The first two components of the Sraffian decomposition can be therefore viewed as the vertically integrated equivalent of Marx's procedure. The data clearly support Marx's general hypothesis that prices of production deviate smoothly and near-linearly from values.

Sraffa's elegant and elliptical text suggests that prices of production are likely to exhibit more complex patterns. He specifically cites the potentially complex behavior of individual sectoral output–capital ratios as being the source of complicated price movements. But at an empirical level, individual output–capital ratios turn out to be virtually linear functions of the rate of profit, so that individual prices of production and the aggregate wage–profit curve are near-linear.

Such findings clearly support the structural price theories of Ricardo and Marx. While they do not completely exclude reswitching, they certainly relegate it to a secondary role. This does not mean that they rehabilitate neoclassical economics. First of all, the structural determination of relative prices in equation is a far cry from the neoclassical theory of marginal cost pricing. Second, the difference between classical and neo-classical theories of profit is most evident *precisely* when prices are equal to labour values. This is the condition under which profit is exactly equal to the surplus value created in production. Even if we further posit an infinite number of co-existing techniques, timeless technical change, and a host of other non-classical assumptions, then equality of standard prices and values is also the condition under which an aggregate pseudo (surrogate) production function obtains, in the sense that frontier techniques corresponding to lower rates of profit will have higher (constant) capital–labour ratios.[5] But correlation is not causation: both profit-as-surplus value and the profit-rate-as-scarcity-price co-exist in this abstract space because their real theoretical differences lie elsewhere.[6]

Data appendix

Data sources and list of industries

The following data was taken from the US Bureau of Economics http://www.bea.gov/industry/iotables: industry-by-industry 65-order total requirements input–output tables **B**, after redefinitions designed to match commodity flows to industries; the vector of direct sectoral wage bills **w** constructed from the Employee Compensation portions of value-added flows in the use tables, after redefinitions; and the market values of industry gross outputs **X**′ from the same use tables. All data is available for 1997–2009, but here we use 1998 to illustrate the general patterns.

Industry list: 1 Farms; 2 Forestry fishing and related activities; 3 Oil and gas extraction; 4 Mining, except oil and gas; 5 Support activities for mining; 6 Utilities; 7 Construction; 8 Wood products; 9 Non-metallic mineral products; 10 Primary metals; 11 Fabricated metal products; 12 Machinery; 13 Computer and electronic products; 14 Electrical equipment, appliances, and components; 15 Motor vehicles, bodies and trailers, and parts; 16 Other transportation equipment; 17 Furniture and related products; 18 Miscellaneous manufacturing; 19 Food and beverage and tobacco products; 20 Textile mills and textile product mills; 21 Apparel and leather and allied products; 22 Paper products; 23 Printing and related support activities; 24 Petroleum and coal products; 25 Chemical products; 26 Plastics and rubber products; 27 Wholesale trade; 28 Retail trade; 29 Air transportation; 30 Rail transportation; 31 Water transportation; 32 Truck transportation; 33 Transit and ground passenger transportation; 34 Pipeline transportation; 35 Other transportation and support activities; 36 Warehousing and storage; 37 Publishing industries (includes software); 38 Motion picture and sound recording industries; 39 Broadcasting and telecommunications; 40 Information and data processing services; 41 Federal Reserve banks, credit intermediation, and related activities; 42 Securities, commodity contracts and investments; 43 Insurance carriers and related activities; 44 Funds, trusts, and other financial vehicles; 45 Real estate; 46 Rental and leasing services and lessors of intangible assets; 47 Legal services; 48 Computer systems design and related services; 49 Miscellaneous professional, scientific and technical services; 50 Management of companies and enterprises; 51 Administrative and support services; 52 Waste management and remediation services; 53 Educational services; 54 Ambulatory health care services; 55 Hospitals and nursing and residential care facilities; 56 Social assistance; 57 Performing arts, spectator sports, museums and related activities; 58 Amusements, gambling and recreation industries; 59 Accommodation; 60 Food services and drinking places; 61 Other services, except government; 62 Federal general government; 63 Federal government enterprises; 64 State and local general government; 65 State and local government enterprises.

Correction for owner-occupied housing (OOH)

The input–output matrix **A**′ and the gross output vector **X**′ incorporate entries for a fictitious real estate sub-industry because the BEA treats private homeowners as 'businesses' renting out their own homes to themselves (Mayerhauser and Reinsdorf, 2007). The BEA's addition of the imputed rental value of owner-occupied housing doubles the listed gross output of the real estate sector, just as its addition of the imputed maintenance and repair costs of owner-occupied housing raises the listed intermediate inputs of the real estate sector by 50 per cent. On the other hand, no addition is made to employee compensation because homeowners are not considered to pay wages to themselves.[7] These imputations raise total real estate market price and intermediate input but not the corresponding labour requirements, thereby greatly enhancing the deviation between this industry's market price and its corresponding labour values and prices of production. Removing the imputations brings us back to a more representative picture of actual real estate transactions. Two corrections are necessary. First, we reduce real estate gross output by the imputed gross output of owner-occupied housing, which is equivalent to dividing the original input–output coefficients in the real estate sector column by by the ratio (x) of non-imputed gross output to originally listed gross output. Second, in order to remove home maintenance and repair expenditures from this column we multiply its coefficients by the aggregate ratio (a) of non-imputed intermediate input total to the originally listed intermediate total. The aggregate ratio is used, since we have no information on the detailed distribution of these imputed expenditures. The combination of the two steps amounts to multiplying the whole real estate sector column by a/x. These corrections reduce the deviation between the real estate market price and its labour values from 250 per cent to 113 per cent. Even so, the latter is still by far the largest single industry deviation.

Calculation of values, prices of production and vertically integrated output–capital ratios and the aggregate wage–profit curve

The total requirements matrix published by the US BEA is $\mathbf{B}' \equiv (\mathbf{I} - \mathbf{A}')^{-1}$ where $\mathbf{I}$ is a 65-order identity matrix, from which we can derive the direct requirements input–output matrix $\mathbf{A}' = \mathbf{I} - (\mathbf{B}')^{-1}$. The j[th] component of the wage bill vector $\mathbf{W}$ is $W_j \equiv w_j L_j$ where w_j = the average wage in the j[th] sector and L_j = the total employment in the j[th] sector. This is used to derive the j[th] component of the labour coefficients vectors $\mathbf{l}'$ as $l'_j \equiv \dfrac{\left(\dfrac{W_j}{w}\right)}{X'_j} = \left(\dfrac{w_j}{w}\right)\left(\dfrac{L_j}{X'_j}\right) = s_j\left(\dfrac{L_j}{X'_j}\right)$, where $\left(\dfrac{L_j}{X'_j}\right)$ = employment per unit gross output, and $s_j \equiv \left(\dfrac{w_j}{w}\right)$ = the j[th] sector wage rate relative to the economy-wide wage rate w. The variable s_j is treated as a rough index of relative skills, so that l'_j may be considered the skill-adjusted labour coefficient of the j[th] sector. The economy-wide wage rate for 1998 was derived from NIPA tables as the ratio of aggregate employee compensation (Table 1.10, line 2) and aggregate employment, full- and part-time (Table 6.4D).[8]

It is important to note that, while theoretical matrices **A,l,X** are in terms of physical quantities, empirical matrices $\mathbf{A}'$, $\mathbf{l}'$, $\mathbf{X}'$ involve market prices. If we designate p_{mj} as the market price of a unit of output of the j[th] sector, then $\mathbf{A} = [a_{ij}] = \left[\dfrac{X_{ij}}{X_j}\right], \mathbf{l} = [l_j] = \left[\dfrac{L_j}{X_j}\right], \mathbf{X} = [X_j]$, whereas $\mathbf{A}' = \left[\dfrac{p_{m_i} a_{ij}}{p_{m_j}}\right] = \left[\dfrac{p_{m_i} X_{ij}}{p_{m_j} X_j}\right], \mathbf{l}' = [l'_j] = \left[\dfrac{L_j}{p_{m_j} X_j}\right], \mathbf{X}' = [p_{m_j} X_j]$. These two sets are easily related through the diagonal matrix of market prices $\langle p_m \rangle$. Then we can show that the empirical equivalents of the theoretical variables are the ratios of these variables to unit market prices (Shaikh, 1984, Appendix B, 82–4).

$$\mathbf{A}' = \langle p_m \rangle \mathbf{A} = \langle p_m \rangle^{-1}, \mathbf{l}' = \mathbf{l}\langle p_m \rangle^{-1}, \mathbf{X}' = \mathbf{X}\langle p_m \rangle^{-1} \tag{6.8}$$

$$\mathbf{v}' = \mathbf{l}'(\mathbf{I} - \mathbf{A}')^{-1} = \mathbf{l}\langle p_m \rangle^{-1}(\mathbf{I} - \langle p_m \rangle \mathbf{A} \langle p_m \rangle^{-1})^{-1} = \mathbf{v}\langle p_m \rangle^{-1} = \left[\frac{v_j}{p_{m_j}}\right] \tag{6.9}$$

$$\begin{aligned} \mathbf{H}' &= \mathbf{A}'\left(\mathbf{I} - \mathbf{A}'\right)^{-1} = \langle p_m \rangle \mathbf{A} \langle p_m \rangle^{-1}\left(\mathbf{I} - \langle p_m \rangle \mathbf{A} \langle p_m \rangle^{-1}\right)^{-1} \\ &= \langle p_m \rangle \mathbf{A}(\mathbf{I} - \mathbf{A})^{-1}\langle p_m \rangle^{-1} = \langle p_m \rangle \mathbf{H} \langle p_m \rangle^{-1} = \left[\frac{p_{m_i} H_{ij}}{p_{m_j}}\right] \end{aligned} \tag{6.10}$$

$$\begin{aligned} \mathbf{p}(r)' &= \left(1 - \frac{r}{R}\right)\mathbf{v}'\left(\mathbf{I} - r\mathbf{H}'\right)^{-1} = \left(1 - \frac{r}{R}\right)\mathbf{v}\langle p_m \rangle^{-1}\left(\mathbf{I} - \langle p_m \rangle r\mathbf{H}\langle p_m \rangle^{-1}\right)^{-1} \\ &= \left(1 - \frac{r}{R}\right)\mathbf{v}\left(\mathbf{I} - r\mathbf{H}\right)^{-1}\langle p_m \rangle^{-1} = \mathbf{p}(r)\langle p_m \rangle^{-1} = \left[\frac{p_j(r)}{p_{mj}}\right] \end{aligned} \tag{6.11}$$

As noted in the discussion of equation (6.2), the maximum rate of profit R is the reciprocal of the dominant eigenvalue of H. VR_j, $R_a(r)$, $w_a(r)$ were calculated as in equations (6.4), (6.7) and (6.6), respectively. All of these being ratios of price terms, the market price elements in **p** (*r*), **Y** cancel out.

Finally, the data which was developed here is for a circulating capital model in which profit is defined as the difference between value added (VA) and employee compensation (EC). A more concrete definition would exclude indirect business taxes (IBT) and depreciation (D) from profit. Second, in a pure circulating model the stock of capital is assumed to be equal to the flow of material costs and depreciation is assumed to be zero (see equation (6.1)). At a more concrete level we need to allow for total capital advanced (K), which is the sum of fixed capital and inventory stocks, and actual depreciation.

The abstract measure of the profit rate derived yields r = 49.34 per cent and R = 103.92 per cent, so that r/R = .475. In 1998, (EC + IBT + D)/EC = 1.323 (US BEA, NIPA Table 1.10, lines 9–10, 23). And from data for 1972 in Shaikh (1998), we find that K = 1.92A. Applying both of these ratios to the 1998 data reduces the profit rate to 15.2 per cent and the maximum profit rate to 54.15 per cent, so that r/R falls to 28.1 per cent.[9] This in turn reduces the distance between prices of production and labour values from our first estimate of 19–20 per cent to 10–11 per cent, which is consistent with Ricardo's hypothesis.[10]

Notes

1 The classical economists and Marx treat wages as a fundamental part of capital invested, so that wages appear not only in costs but also in the stock of capital advanced. Sraffa chooses to treat wages as being paid at the end of the production period. Within this framework, Marx's (first approximation) of prices of production expressed in

standard Marxian notation would be $p'_j = C_j + V_j + \rho(C_j)$, where $\rho = \frac{S}{C} = \frac{S/V}{C/V}$ is the aggregate rate of profit in labour value terms and $S/V = S_j/V_j$ is the common rate of exploitation deriving from a common real wage and length/ intensity of the working day in each industry. Total labour value is $v_j \equiv C_j + V_j + S_j$ and labour value added is living labour $L_j \equiv V_j + S_j = v_j(1 + S_j/V_j) = v_j(1 + S/V)$ so that $p'_j = v_j + \left(\frac{S/V}{C/V}\right)C_j - \left(\frac{S_j/V_j}{C/V}\right)\left(\frac{C/V}{C_j/V_j}\right)C_j = v_j + \rho\left(1 - \frac{C/L}{C_j/L_j}\right)C_j$. It follows that these prices are linear functions of the general value rate of profit ρ whose paths depend on the extent to which any given industry's value added–capital ratio (L_j/C_j) differs from that of the average industry (L/C). A similar result can be derived when wages are treated as part of capital advanced, and the basic empirical patterns are the same as those shown here (Shaikh, 1998).

2 The j[th] element of the deviations vector is $\mathbf{vH}^{(j)} - \mathbf{v}\frac{1}{R} = v_j\left(\frac{\mathbf{vH}^{(j)}}{v_j} - \frac{1}{R}\right) = v_j\left(\frac{1}{VR_{0j}} - \frac{1}{R}\right)$ where VR_{0j} is the labour value of the vertically integrated output-capital ratio in the j[th] industry and R is the output-capital ratio in the standard industry (commodity), which by construction is the same at all prices, including those equal to labour values (Sraffa 1960, 16–17). The term in brackets in the j[th] element of the deviations vector therefore represents the deviations of individual industry vertically integrated constant capitals per unit output from those in the standard sector.

3 Sraffa's standard commodity of net outputs is the right-hand side eigenvector corresponding to $\mathbf{p}(R)$, so that it satisfies $\mathbf{Y}_s = R\mathbf{HY}_s$ and hence $\mathbf{vY}_s = R\mathbf{vHY}_s$. It follows that $R = \frac{\mathbf{vY}_s}{\mathbf{vHY}_s} = \frac{\mathbf{vY}_s}{\mathbf{vK(I-A)}^{-1}\mathbf{Y}_s} = \frac{\mathbf{vY}_s}{\mathbf{vKX}_s}$ is the net output–capital ratio, that is the living labour/dead labour ratio in the standard sector.

4 The same result obtains if we instead define the wage relative to the price of some basket of consumption goods (Ochoa 1984, 231). Then, given that individual commodity prices are near-linear, the price of the consumption goods basket will inevitably be linear, so that the real wage $w(r)/p_c(r)$ will be the ratio of two linear terms.

5 The money value of output per worker (y) is linked to that of capital per worker (k) through the identity $y \equiv w + rk$. At any switch point between two adjacent techniques the wage rate and the profit rate are the same for each technique, so that $\Delta y/\Delta k = r$. Then if there are an infinite number of switch points *and no reswitching*, one can use this relation to generate a pseudo production function to describe the wage–profit frontier. Samuelson's surrogate production function is the special case in which each wage–profit curve on the frontier is linear. None of this implies that $\Delta y/\Delta k$, or in the limiting case dy/dk, *causes* r. Baldone (1984) and Salvadori and Steedman (1988) make the interesting point that, if all techniques are linear, then one of them will necessarily dominate all the others, in which case the frontier is a single linear system with relative prices equal to relative labour values.

6 The apparent empirical fit of aggregate production functions is a separate matter, since it is rooted in particular operations which attempt to extract something that looks like an aggregate production function from the same accounting identity $y \equiv w + rk$ (Felipe and Mccombie 2005; Fisher 2005; Shaikh 2005).

7 The gross output of the real estate industry is directly available in $\mathbf{X}'$ while that of owner-occupied housing (OOH) is from NIPA Table 7.12, line 133. In 1998, the latter imputed figure of $681.10 billion was added to the $607.35 billion of the gross revenue of the actual real estate business sector. The total imputed intermediate inputs of the fictitious real estate sub-industry in NIPA Table 7.12, line 134 (imputed homeowner repair and maintenance expenditures) was $114.4 billion, in comparison to the total $342.23 billion intermediate input of the actual real estate sector. Nothing is added to employee compensation of the overall real estate sector, since homeowners are not assumed to pay themselves wages.

8 This procedure differs somewhat from Ochoa (1984, 225), who uses the lowest sectoral wage as the deflator.

9 Accounting for IBT and D does not change aggregate value added $VA \equiv \mathbf{p}(r)\mathbf{X} - \mathbf{AX}$, so the matrix $\mathbf{A}$ need not be changed. This in turn leaves $\mathbf{v} \equiv \mathbf{l}(\mathbf{I}-\mathbf{A})^{-1}$ unchanged. Thus the new estimates were derived solely through a new profit share $= [VA - (EC + IBT + D)]/VA$ and a new capital stock matrix $\mathbf{K} = 1.92\mathbf{A}$.

10 A more detailed derivation of individual industry capital stocks and depreciation flows has the additional important effect that it makes individual standard prices and output–capital ratios into completely linear functions of the profit rate *throughout their whole ranges* (Shaikh, 1998, 238–41).

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