A Proposed Synthesis of Classical and Keynesian Growth

I believe that one can combine the essential propositions of Keynesian-type (KT) and what I call Classical-Type (CT) growth theories in a simple way. All it takes are three relations, two of which are common to both traditions, and a third which is also implicit in both.

The motivation for this was the recognition that both CT and KT theories emphasize that accumulation is regulated by profitability. Yet KT theory finds that profit-driven accumulation leads to persistent differences between actual and normal (minimum cost) capacity utilization.  In CT theory, as in Harrodian-type (HT) theory, such an outcome is not plausible because accumulation is only sustainable when investment expectations are roughly correct in the long run, i.e. when the actual rate of capacity utilization gravitates around the normal rate[1]. But in the case of HT theory, the latter outcome requires that investment is governed by thrift (as in Solovian growth theory), rather than by profitability. CT theory presumes that it is possible to have profit-driven accumulation *and* self-consistent investment. But if the existing literature is correct, we must choose one or the other.

I would argue that there is simple way to make both outcomes possible. The secret lies in the treatment of business retention *ratio* (the business saving rate). To see this, consider an initial situation in which there is ongoing accumulation at normal capacity utilization with some initial private savings rate. In this situation, firms are financing their investments from retained earnings, from sales of new equities and bonds to households, and from bank loans corresponding to their desired debt ratios. Now if the desired rate of accumulation were to rise, desired accumulation would exceed normal accumulation, which would require firms to seek additional finance. While their actions may or may not induce households to raise their own savings rate (Kaldor, 1966, Appendix: A Neo-Pasinetti Theorem, pp. 316-319), firms can also fill part of this gap through a higher retention ratio[2] and the rest of it through a greater reliance on bank loans. The last would inject demand into the system and raise the level of output as in the traditional multiplier, while the first two would somewhat raise the overall savings rate. Given that accumulation responds to both normal profitability and the gap in capacity utilization, if this process is stable (which is demonstrated below) it will continue until actual accumulation gravitates around the new profit-driven desired rate.

It turns out that the character of the long run situation depends solely on whether the business savings rate responds *at all* to the gap between desired and normal accumulation. If it does even a bit, then the new long

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[1] As Garegnani (1992, p. 56) notes, investment is driven by its expected rate of return at expected normal utilization – i.e. by its expected normal rate of profit.

[2] Since the business savings rate is a component of the overall savings rate with a weight equal to the profit share, Eichner and Woods and others suppose that the profit-share, i.e. the markup, is raised at this point to raise the overall savings rate. But for this to work  over the long run, the real wage rate must be assumed to be entirely passive: i.e. a fixed money wage even in the long run, so that the real wage falls and the real profit share rises when markups rise. If workers fight to maintain a particular historically-determined standard of living, the money wage cannot be taken to be fixed in the longer run.

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run is characterized by profit-driven accumulation at roughly normal capacity utilization, which is the CT outcome. Conversely, the KT state (profit-driven accumulation with capacity utilization different from normal) obtains only if the retention ratio does not respond at all.

In what follows, the demonstration of these claims is cast at the most abstract level, because that is where the real difference arises. Let $I$ = investment, $S$ = savings, $s$ = the savings propensity, $Y$ = output, $Yn$ = normal capacity (at the lowest point on the cost curve), $Ymax$ = maximum technical capacity (engineering capacity), $K$ = capital stock, $g_K$ = the accumulation rate (growth rate of $K$), $u$ = capacity utilization = $Y/Ymax$, $u_n$ = normal capacity utilization (determined by the lowest cost point on the cost curve) = $Yn/Ymax$, $g_K(r_n)$ = the rate of accumulation at the normal rate of profit ($r_n$), and $R$ = $Ymax/K$.

The first two equations below are common to both CT and KT (particularly Post Keynesian, PK) theory: the short run equilibrium condition; and what Lavoie calls the standard PK accumulation function, written in such a way as to separate out accumulation at normal profitability and accumulation responding to changes in capacity utilization[3] through the parameter h, which is subject to the standard PK restrictions required for stability (see Appendix A).

1) $I = S = s*Y$                              [Short run equilibrium]

2) $g_K = g_K(r_n) + h \cdot (u - u_n)$                      [CT and PK accumulation function]

The third equation rests on the notion that the business savings rate responds to the gap between desired and normal accumulation. For instance, Kurz (1992, p. 86) suggests that business "savings ratios may themselves depend on the rate of accumulation" so that "in times of relatively high accumulation firms [may] … increase the proportion of retained profits". The argument can also be derived from Marx's discussion of the difference between the circuits of revenue (in which household savings reside) and the circuit of capital (in which there is retained earnings, i.e. business savings). To highlight the issue at hand, both the household savings rate and the profit-share are taken to be constant, so that the private savings rate also responds to the accumulation gap when the business savings rate does[4], via some positive parameter $\sigma$[5].

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[3] The general PK accumulation function has the form $g_K = F(g_K(r_n), u)$. Lavoie (1996, p. 119) lists it as $g_K = a + b \cdot u + c \cdot r_n$ , which we can write as $g_K = g_K(r_n) + h \cdot (u - u_n)$, where $g_K(r_n) = c_0 + c \cdot r_n$ , $c_0 = (a + b \cdot u_n)$, and $h = b$.

[4] Aggregate output $Y = W + P = (W + DIV) + RE = Y_h + S_b$, where $W$ = the wage bill, $P$ = profits = $DIV + RE$ = dividends + retained earnings. Total savings is the sum of household savings out of household income ($S_h = s_h \cdot Y_h = s_h \cdot (Y - RE)$) and business savings RE out of profits ($RE = \rho \cdot P$), where $s_h$ and $\rho$ are the household and business savings (retention) rates, respectively. The aggregate savings rate $s \equiv S/Y = [s_h \cdot (Y -$

3) $s' = \sigma \cdot (g_K - g_K(r_n))$

The system formed by equations 1-3 gravitates around normal capacity utilization (the proof is in Appendix A). Figures 1-2 depict its characteristic response to a shock to the investment level at t = 10 and a shock to the accumulation rate at t = 30. Note that despite both types of shocks, the system converges to u = 1 (normal capacity utilization) because the savings rate is endogenous. Thus over the long run accumulation is driven by normal profitability and capacity utilization gravitates around its normal level. Conversely, if the savings rate reaction parameter is fixed at zero (i.e. the savings rate is assumed to be independent of the accumulation gap), the system reverts to equations 1-2 alone, and one gets all the standard PK results: the long run rate of capacity utilization is different from normal, and both shocks will raise the capacity utilization rate permanently because the savings rate is fixed (Figures 3-4).

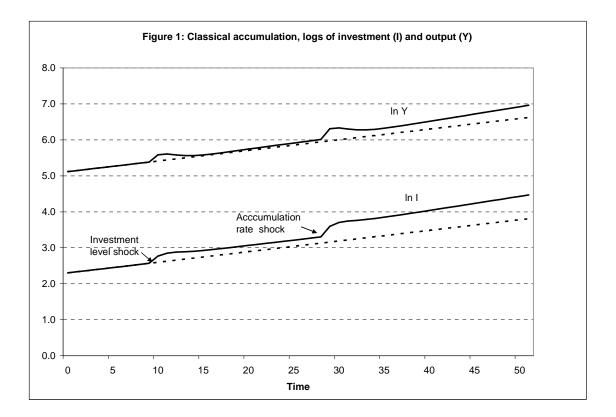Several things follow from the endogeneity of the business savings rate.

- Desired investment is driven by its underlying normal profitability, and aggregate savings adapts itself to aggregate investment. These two propositions are completely consistent with a classical approach and also with the "the central thesis of the General Theory that firms are free, within wide limits, to accumulate as they please, and that the rate of saving of the economy as a whole accommodates itself to the rate of investment that they decree" (Robinson, 1962, pp. 82-83)[6].
- However, because the classical accommodation is partially achieved through a higher savings propensity, the overall multiplier effects will be smaller than those implied by traditional Keynesian theory.
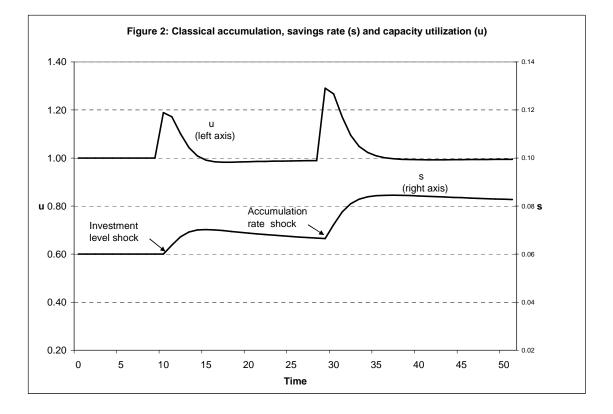
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RE) + $\rho \cdot RE$)/Y = [$s_h$ + $\rho \cdot (1 - s_h)$(P/Y)]. Then even if the household savings rate and the profit share are constant, the aggregate savings rate will respond to any finance gap if the retention rate does.

[5] Harrod suggests a savings rate response function very similar to that in equation 3. But his argument concerns the *deviation* of the short run savings rate (s) from the long run "normal" rate ($s_n$). The latter is determined by the normal savings propensities of households and businesses, and by the normal profit share, as shown in footnote 4. This normal savings rate, along with the normal capacity-capital ratio $R_n$, determines the long run warranted rate of growth ($s_n \cdot u_n \cdot R_n$). Harrod thinks of the actual rate of growth as gravitating around the long run normal warranted rate, so a certain range of differences between the two is normal. But he also makes "scattered" references to the possibility that household and business savings rates will deviate from their long run values when the actual rate of accumulation is "abnormally" different from the long run warranted rate. For instance, if the former is abnormally lower than the latter, savings rates will also fall, which will lower the short run ("special") warranted rate ($s \cdot u_n \cdot R$) and help narrow the gap (Pugno, 1998, pp. 152, 155-163). This is a stabilizing effect, and if it were large enough, it would ensure gravitation around the long run warranted path itself. But the centrality of the latter implies that in the long run accumulation is driven by thrift, not by profitability.

[6] It should be noted that when Robinson speaks of "rates" of savings and investment, she means their levels – i.e. their flows per unit time.
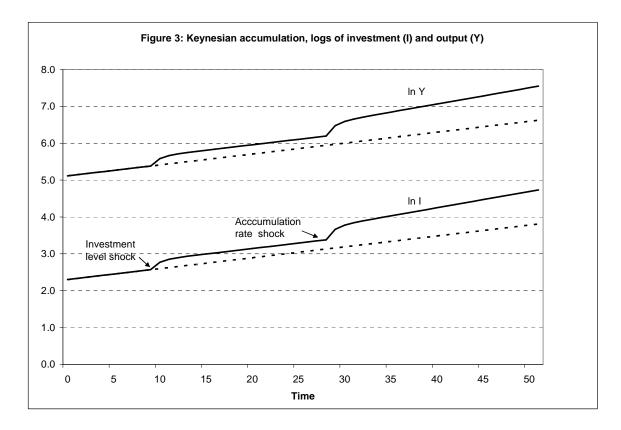
- As in Harrod and the classicals, the actual rate of capacity utilization will gravitate around the normal rate (Figure 2). This does not obtain in the Keynesian system (Figures 4)

- As in Keynesian theory, a rise in the autonomous level of investment with no change in its growth rate will raise the level of the long run output path without changing its growth rate (the investment shock in Figures 1 and 3). But because the savings rate also rises in the classical case, the corresponding effect on output is much smaller than in the Keynesian case.

- As in Keynesian theory, a rise in the normal rate of profit will raise the normal rate of accumulation (see equation 2), which will raise the long run rate of accumulation (as shown by the slopes of  lnY in Figures 1 and 3).

- Finally, a rise in the household savings rate will have no permanent impact on the rate of accumulation unless it happens happen to alter the profit rate (or the interest rate, when the normal profit is more concretely specified as net of interest), because otherwise it will simply reduce the retention ratio needed to support a given rate of accumulation. This might be viewed as an alternate path to Kaldor's neo-Pasinetti theorem (Lavoie 1998, p. 421).
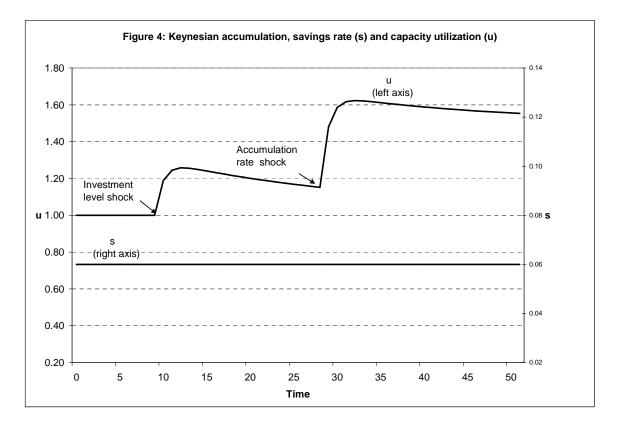
The important point is that this classical synthesis allows us to preserve central Keynesian arguments such as the dependence of savings on investment and the regulation of investment by expected profitability, without having to claim that actual capacity utilization will persistently differ from the rate desired by firms. It is true that the scope of the multiplier is somewhat reduced because part of the adjustment takes place through a variation in the savings rate. But given that this same mechanism leads to the gravitation of capacity utilization around the normal level, I would argue that the overall benefit is substantial. The effects of technical change, wage rates and interest rates, international factors, and fiscal and monetary policy, can then proceed from this foundation.

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Figure 1: Classical accumulation, logs of investment (I) and output (Y)



Figure 2: Classical accumulation, savings rate (s) and capacity utilization (u)

**Figure 3: Keynesian accumulation, logs of investment (I) and output (Y)**

In Y

In I

Acccumulation
rate  shock

Investment
level shock

Time

**Figure 4: Keynesian accumulation, savings rate (s) and capacity utilization (u)**

u
(left axis)

Accumulation
rate  shock

Investment
level shock

u

s

s
(right axis)

Time

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Appendix B: Stability of profit-driven accumulation with an endogenous business savings rate

Two basic relations are common to the PK and CT dynamical systems: short run equilibrium (I = S), which implies a relation between the rate of accumulation, the savings rate and the rate of capacity utilization; and an accumulation function such as the standard PK function. The relevant equations are reproduced below as they appear in the text.

<div align="center">Common Relations</div>

1)  $g_K = s \cdot R \cdot u$                                     [Short run equilibrium: I = S]

2) $g_K = g_K(r_n) + h \cdot (u - u_n)$                [the standard PK accumulation function]

      Classical                              Keynesian and PK

3)  $s' = \sigma \cdot (g_K - g_K(r_n))$                     3a) $s' = 0$

The sole difference between the two dynamical systems then arises from their treatment of the business savings rate. If we take the household savings rate and the distribution of income as given, so as to isolate the issue at hand, the difference between the two approaches is clear: the PK system appears as a special case of the CT system when we assume a fixed business savings rate. Thus the former can therefore be derived from the latter by setting the savings rate adjustment coefficient $\sigma = 0$ in equation 3, in which case profit-driven accumulation yields the standard PK result that capacity utilization remains persistently different from the normal rate. Conversely, any responsiveness at all on the part of the business savings rate ($\sigma > 0$) converts the PK system into a CT one, and it becomes possible to have both profit-driven accumulation and normal capacity utilization over the long run.

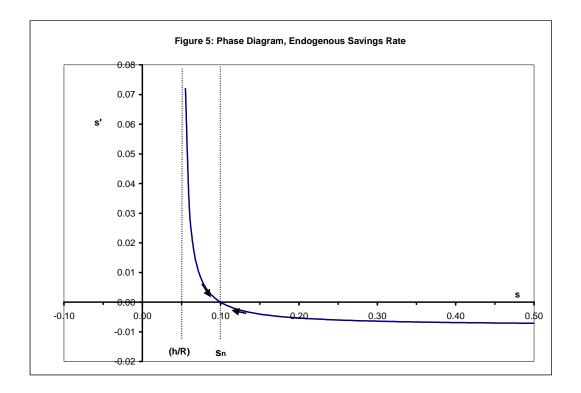The general case ($\sigma > 0$) yields a nonlinear differential equation in the endogenous savings rate[7].

4) $s' = \sigma \cdot g_K(r_n) \cdot [(1 - (s/s_n)]/[((s/s_n) \cdot (s_n \cdot R/h)) - 1]$

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[7] Since $(g_K - g_K(r_n)) = h \cdot (u - u_n)$ from equation 2, and $u = g_K/s \cdot R$ and $u_n = g_K(r_n)/(s_n \cdot R)$ from equation 1, $(g_K - g_K(r_n)) = h \cdot [(g_K/s \cdot R) - u_n] = h \cdot [(g_K/s \cdot R) - (g_K(r_n)/s \cdot R) + (g_K(r_n)/s \cdot R) - u_n] = (h/s \cdot R) \cdot (g_K - g_K(r_n)) + h \cdot (g_K(r_n)/R) \cdot [(1/s) - (1/s_n)]$, since $(R \cdot u_n)/g_K(r_n) = 1/s_n$. Collecting terms gives $(g_K - g_K(r_n)) = h \cdot (g_K(r_n)/R) \cdot [(1/s) - (1/s_n)]/[1 - h/(s \cdot R)] = h \cdot (g_K(r_n) \cdot [(1 - (s/s_n)]/(s \cdot R - h) = g_K(r_n) \cdot [(1 - (s/s_n)]/((s/s_n) (s_n \cdot R/h) - 1)$. Substituting this into the savings rate adjustment equation 3 gives us equation 4, a nonlinear differential equation in the endogenous savings rate s.

Suppose that $h < s_n \cdot R$, so that $[((s/s_n) \cdot (s_n \cdot R/h)) - 1] > 0$ when $s \geq s_n$. Then in equation 21 as $s \rightarrow s_n$ from above, the denominator of the expression in square brackets remains positive and the numerator goes from negative to zero, so that s' goes from negative to zero also. Finally, for $s < s_n$, as $s \rightarrow h/R < s_n$, s' approaches infinity because the numerator is positive while denominator is also positive and approaches zero. The phase diagram corresponding to equation 4 (Figure 5) therefore has a single stable positive equilibrium at $s = s_n$, as long as $h < s_n \cdot R$. It is useful to note that $h < s_n \cdot R = g_K(r_n)/u_n$ is also the stability condition for the PK model (Dutt, 1997, p. 246;Lavoie, 1996, p. 122).

Finally, for the simulation results which were displayed in Figures 4-5 in the text, equations 2-3 were written as the difference equations $g_K = g_K(r_n) + h \cdot (u(-1) - u_n)$ and $s = s(-1) + \sigma \cdot h \cdot h \cdot (u(-1) - u_n)$, since $(g_K - g_K(r_n)) = h \cdot (u(-1) - u_n)$, with parameter values $R = 0.5$, $g_K(r_n) = 0.03$, $h = 0.01$, $\sigma = 2$, $u_n = 1$, and initial values at equilibrium levels $g_K(0) = g_K(r_n)$, $u(0) = u_n$, and $s(0) = s_n \equiv g_K(r_n)/R \cdot u_n$. Investment was derived as $I = g_K \cdot K(-1)$, output as $Y = I/s$, and capital as $K = I + K(-1)$, with initial values $I(0) = 10$, $K(0) = 343.333$. The initial equilibrium run is broken at $t = 10$ by a permanent addition to investment of 2.54 (which is 10 percent of investment in the prior period), and at $t = 30$ by a rise in the profit-driven component of accumulation $g_K(r_n)$ from 0.03 to 0.04. Figure 4 displays the paths of ln I and ln Y, while Figure 5 those of s and u.



Figure 5: Phase Diagram, Endogenous Savings Rate

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