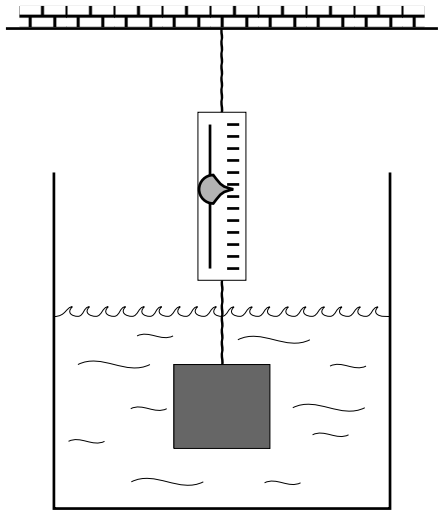
PHYSNET

MISN-0-48

INTRODUCTION TO FLUID STATICS AND DYNAMICS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

INTRODUCTION TO FLUID STATICS AND DYNAMICS

by

J. Kovacs, Michigan State University

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A. Resource Summary 2

Title: **Introduction to Fluid Statics and Dynamics**

Author: Jules Kovacs, Michigan State University

Version: 2/1/2000 Evaluation: Stage 2

Length: 1 hr; 16 pages

Input Skills:

1. Draw a one-body force diagram, given the environment of an object (MISN-0-10).
2. Set up and solve static equilibrium problems (MISN-0-6).
3. Use the work-energy principle (MISN-0-20).
4. Use the principle of conservation of energy for conservative forces (MISN-0-21).

Output Skills (Knowledge):

K1. State Archimedes' Principle, referring to an appropriate diagram.

K2. Apply the principle of energy conservation to derive Bernoulli's Theorem starting with a careful definition of each of the quantities in the expression of the theorem.

Output Skills (Problem Solving):

S1. Start from the one-body diagram and Archimedes' Principle and determine the density of objects immmersed in liquids, given informatin about the volume of the object, the depth of immersion and the apparent weight. Also be able to determine all of the forces on an immersed object.

S2. Use Bernoulli's Equation and the continuity equation to solve problems dealing with the flow of incompressible fluids.

External Resources (Required):

1. M. Alonso and E. Finn, *Physics*, Addison-Wesley (1970). For availability, see this module's *Local Guide*.

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Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

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1. Introduction

The general principles that have been developed and applied to discrete particles and rigid systems, consisting of distributions of particles, also apply to systems which may be considered as continuous distributions and which are not part of a rigid system. The motion of fluids and the static (stationary) behavior of rigid objects immersed in a fluid are examples of the application of the general principles to such systems.[1] Archimedes' Principle summarizes the effects expected when the principles associated with static equilibrium are applied to objects immersed in fluids. Bernoulli's Theorem results from the application of the principle of energy conservation to the steady state motion of a fluid.

2. Archimedes' Principle

Note: The designation AF below refers to M. Alonso and E. Finn, *Physics*, Addison-Wesley (1970). For availability, see this module's *Local Guide*.

AF: Archimedes' Principle relates to static equilibrium of objects immersed in fluids and is treated only very briefly in AF. It is stated on page 115 between equations 7.22 and 7.23 where it arises in conection with a discussion about the limiting speed of an object falling through a viscous fluid. Its derivation appears as an exercise.

3. Bernoulli's Theorem

AF: Section 10.13 (pp. 200-203). The basic equation of Bernoulli's Theorem is derived and applied to an example, *fluid flow* in a pipe.

[1] The fluid movement of masses of air around the surface of the earth as a consequence of the forces arising due to the earth's rotation is examined briefly in (MISN-0-18).

Acknowledgments

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A. Resource Summary

MOD: This module's Problem Supplement.

AF: M. Alonso and E. Finn, *Physics*, Addison-Wesley (1970). For availability, see this module's *Local Guide*.

Skill	Ref.	Items
K1	AF	p. 115; Problem 10.29
K2	AF	Sect. 10.13, Fig. 10.20

Skill	Ref.	Items
S1	MOD	Probl. Suppl., Problem 1
S2	MOD	Probl. Suppl., Problem 3
	AF	Sect. 10.13, Figs. 10.22, 10.23; Ex. 10.10; Question 14 (p. 204), Problems 10.26, 10.27, 10.28

LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as "The readings for CBI Unit 48." Do **not** ask for them by book title.

PROBLEM SUPPLEMENT

CAUTION: Carry along the dimensions of all of the quantites that go into your solution to assure yourself that you are using a consistent set of units. When you do the appropriate cancellations of units, each term in your equation will have the same dimensions.

1. A large container, open at the top, is half-filled with water (density $1.00\,\mathrm{gm/cm^3}$). On top of this, filling the container, floats a layer of oil (density $0.60\,\mathrm{gm/cm^3}$). Into this two-layer fluid is immersed a cube of side length L of wood whose density is $0.90\,\mathrm{gm/cm^3}$.

   a. Determine where the cube comes to rest when equilibrium is established. Is is at the bottom of the container? At the air-oil interface? At the oil-water interface? Wherever it is, determine the fraction of the cube is in each medium (air, oil, water).

   b. Determine the fraction of the cube that would be out of water if the oil were removed.

1. Brief Answers (detailed assistance is given in the *Spec. Ass. Suppl.*):

   a. One-fourth in oil, three-fourths in water, none in air. [S-1]

   b. One-tenth in air, nine-tenths in water. [S-2]

2. AF: Problem 10.29.

3. An open water tank has its upper surface of water 2.0×10^1 meters above an exit pipe. At the upper surface of the water, the pressure is 1.00 atm and the speed of the water is negligible.

   a. Determine the pressure at the exit pipe when no water is flowing.

   b. Determine how much the pressure at the exit pipe is diminished if water flows there at a speed of $8.0\,\mathrm{m/s}$.

   c. Determine the maximum permissible speed of flow through this pipe if the pressure at the exit pipe is to remain above 2.0 atm.

   d. Determine the minimum diameter of the exit pipe is needed to maintain a pressure of 2.0 atm or more and the need for water may be as great as 1.0×10^3 kg/sec. Numerical assistance: One atmosphere of pressure may be taken as $1.00 \times 10^5\,\mathrm{N/m^2}$, the density of water as $1.00 \times 10^3\,\mathrm{kg/m^3}$.

Answers (detailed assistance is given in the *Spec. Ass. Suppl.*):

a. 2.96 atm. *Help: [S-3]*

b. diminished by 0.32 atm. *Help: [S-3]*

c. 13.9 m/s *Help: [S-3]*

d. 0.30 m diameter

4. AF: Question 14 (p. 204), Problems 10.26, 10.27, and 10.28.

Answers:

10.26 a. 10 m/s, 2.38×10^5 N/m^2

b. 300 kg/min (or 0.3 m^3/min)

c. $P_1 + (1/2)\rho v_1^2$ is the energy per unit volume This divided by ρ is the energy per kilogram Answer is 250 joules per kilogram

10.27 a. 10 m/s, 2.57×10^5 N/m^2

b. 300 kg/min

c. 250 J/kg

10.28 b. 11.2 s

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS-problem 1a)*

Because this is a static equilibrium problem, the condition that must be satisfied is that the resultant force on the cube must be zero. That is, the weight of the cube, W, is balanced by the upward bouyant force B on the cube exerted by the fluids.

The upward force B, according to Archimedes, is equal in magnitude to the weight of the volume of fluid displaced by the object. If the cube were immersed completely in the oil only, the weight of oil displaced (the weight of an amount of oil whose volume equals the cube's volume) would be less than the cube's weight so B would be less than W and the cube would sink deeper. If the cube were completely immersed in water only, it would not sink so deep that it would rise because B would be larger than W and the net force on the cube would be upward, not zero. Hence, in the oil-water situation, equilibrium would be occur with the cube partially submerged in the oil and partially in the water.

Let x be the height of block in the oil, so $L-x$ is the height of block in the water (draw a sketch). The total weight of fluid displaced is the weight of oil of volume $L^2\,x$ plus the weight of water of volume $L^2\,(L\,-\,x)$. Hence B is $B = \rho_{water} g L^2 (L - x) + \rho_{oil} g L^2 x$ and $W = \rho_{wood} g L^3$, the weight of the cube. Equating and solving for x,

$$x = \frac{(\rho_{water} - \rho_{oil})\,L}{\rho_{water} - \rho_{wood}} = 0.25\,L.$$

Thus one-fourth of the cube is in oil, three-fourths in water.

S-2 *(from PS-problem 1b)*

The bouyant force B is due only to the weight of that volume of water that's equal to the volume of the part of the cube that is submerged).

S-3 *(from PS-Problem 3)*

Application of Bernoulli's Theorem solves each of the parts of this problem.

$$\frac{1}{2}\rho v^2 + p + \rho g y = \text{ constant},$$

meaning that this quantity has the same value when evaluated at any point in the fluid. Using the subscript "1" for the top surface, and subscript "2" for a point in the fluid just at the exit pipe:

$$\frac{1}{2}\rho v_1^2 + p_1 + \rho g y_1 = \frac{1}{2}\rho v_2^2 + p_2 + \rho g y_2 \,.$$

Now try to solve each part of the whole problem without further assistance! If you find you just can't do it, help for part (a) is given in [S-6], for part (b) in [S-5], for part (c) in [S-4], and for part (d) in [S-7].

S-4 *(from PS-Problem 3c)*

If you want the pressure at the exit pipe to remain above 2 atmospheres, then the maximum speed must be such that

$(1/2)\,\rho v_{2,max}^2 = 0.96 \times 10^5\,\mathrm{N/m^2}$

$v_{2,max} = 13.9\,\mathrm{m/s}$

S-5 *(from PS-Problem 3b)*

Again $v_1 = 0$ (assume the tower is large enough so that the speed of a point on the surface at the top is negligible).

$p_1 = (1/2)\rho v_2^2 + p_2 + \rho g y_2$

$10^5\,\mathrm{N/m^2} = (1/2)(10^3\,\mathrm{kg/m^3})(8\,\mathrm{m/s})^2 + p_2 - 1.96 \times 10^5\,\mathrm{kg/(m\,s^2)}$

$p_2 = (10^5 + 1.96 \times 10^5 - 0.32 \times 10^5)\,\mathrm{N/m^2} = 2.64\,\mathrm{atmospheres},$

if water flows out of the exit pipe (diminished by 0.32 atmospheres with the water flowing).

S-6 *(from PS-Problem 3a)*

With the exit pipe closed v_1=0, v_2=0. Choosing the zero of altitude to be at the top surface in the tank (choosing the reference level for potential energy at the top surface), then

$y_1 = 0$, $y_2 = -20\,\mathrm{meters}$.

$p_1 = 1\,\mathrm{atmosphere} = 10^5\,\mathrm{newtons/m^2}$.

$p_2 = ?$, $\rho = 10^3\,\mathrm{kg/m^3}$

$10^5\,\mathrm{N/m^2} = p_2 + (10^3\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})(-20\,\mathrm{m})$

$p_2 = 10^5\,\mathrm{N/m^2} + 1.96 \times 10^5\,\mathrm{kg/(m\,s^2)}$

Note: $1\,\mathrm{kg/(m\,s^2)} = 1\,\mathrm{N/m^2}$

$p_2 = 2.96 \times 10^5\,\mathrm{N/m^2} = 2.96\,\mathrm{atmospheres}.$

S-7 *(from PS-Problem 3d)*

The rate of efflux of fluid (in kg/s) through opening of cross-sectional area A is $\rho\,dV/dt = dm/dt = \rho A v$, where dv/dt is the volume crossing area A per unit time and v is the velocity of efflux.

For a given rate, a large value of A requires a small value of v and vice-versa.

If $v_{max} = 13.9\,\mathrm{m/s}$ then, as seen above, the pressure is greater than 2 atmospheres. With the efflux rate fixed, this puts a minimum restriction on A.

Therefore,

$10^3\,\mathrm{kg/s} = (10\,\mathrm{kg/m^3})(A)(13.9\,\mathrm{m/s})$

$A = (1/13.9)\,\mathrm{m^2}$

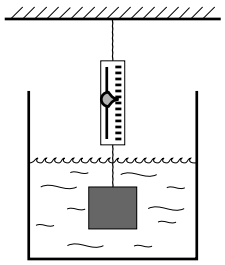
$\pi R^2 = (1/13.9)\,\mathrm{m^2}$

$R = 0.15\,\mathrm{meters}$, radius of outlet.

MODEL EXAM

1. See Knowledge Skills K1-K2.

2. A 2.00 cubic centimeter cube of gold, whose density is 19.3 times that of water is suspended from a spring scale and totally immersed in kerosene (the cube hangs inside the fluid touching neither sides nor bottom of the container). Kerosene has a density 0.80 times that of water. The spring scale is calibrated to read, in newtons, the value of the tension in the cord connecting the scale to whatever is suspended below it.



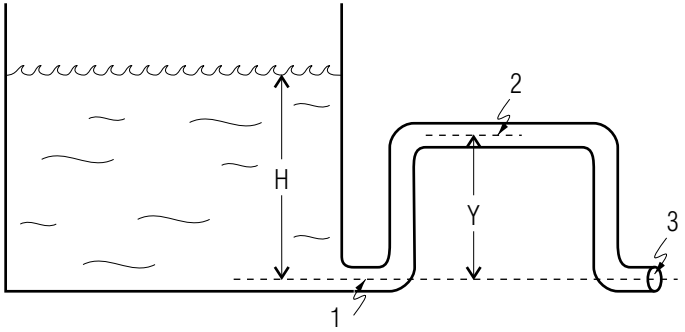
   a. Draw a one-body force diagram showing all of the forces acting on the cube. [O]

   b. Evaluate the magnitude of the bouyant force on the cube. [K]

   c. What would the scale read if the cube were hanging in the air? [C]

   d. What does the scale read when the cube is immersed in the gasoline? [H]

3.



A very large reservoir is filled to a height H above a discharge outlet which sends water through a vertical loop before discharging it at point (3) into the atmosphere. Water weighs 62.4 lbf per cubic foot. At point (1), the pipe has a cross-sectional area $A_1 = 1.00\,ft^2$, at point (2) $A_2 = 1.00\,ft^2$, while at (3), $A_3 = 0.75\,ft^2$. $H = 1.00 \times 10^3$ ft and

$Y = 7.0 \times 10^1$ ft. The outside pressure is 1.000 atmosphere (which is equal to 2116 lbf ft^{-2}).

   a. What is the pressure at the outlet (3)? [A]

   b. What is the pressure at the top of the reservoir? [N]

   c. What is the flow velocity of the fluid at the top of the reservoir? [G]

   d. What is the flow velocity at outlet (3)? [E]

   e. What volume of water discharges from the pipe per second? [B]

   f. What is the rate (volume per second) at which the water flows past a cross-sction of pipe at points (1) and (2)? [M]

   g. What is the flow velocity at (1)? [J]

   h. What is the flow velocity at (2)? [D]

   i. Find the pressure at (1). [L]

   j. Find the pressure at (2). [F]

   k. How high above the outlet can the level of point (2) be before the pressure at (2) drops below zero? (If the pressure gets negative, the water must support a tension, which it cannot. Hence the countinuous flow breaks up and the flow subsequently ceases.) [I]

Brief Answers:

A. 1 atm.

B. 60 m^3/s

C. 0.388 newtons

D. 60 m/s

E. 80 ft/sec

F. 0.226 atmospheres

G. zero

H. 0.372 newtons

I. 77.7 ft

J. 60 m/s

K. 0.0157 newtons

L. 2.29 atmospheres

M. 60 m^3/s at both sections.

N. 1 atm.

O. Your diagram should show three forces. Two upward, due to the bouyant force of the liquid and the tension in the cord, and one downward, due to the gravity pull of the earth.