Actuality and Knowability*

David J. Chalmers

It is widely believed that for all p, or at least for all entertainable p, it is knowable a priori that (p iff actually p). It is even more widely believed that for all such p, it is knowable that (p iff actually p). There is a simple argument against these claims from four antecedently plausible premises.

The argument is given below. Here 'A', 'E', 'K', '$\Box$', '$\Diamond$' stand for 'Actually' 'Someone entertains', 'Someone knows', 'Necessarily', and 'Possibly', while '$\rightarrow$' and '$\leftrightarrow$' are the material conditional and biconditional. In addition, q is any (entertainable and expressible) proposition that no-one actually entertains, while r is $\neg Eq$, the proposition that no-one entertains q.[1]

1. Ar
2. $Ar \rightarrow \Box Ar$
3. $\Box(K(r \leftrightarrow Ar) \rightarrow (r \leftrightarrow Ar))$
4. $\Box(r \rightarrow \neg K(r \leftrightarrow Ar))$

---

5. $\neg\Diamond K(r \leftrightarrow Ar)$

Premise (1) requires only the plausible claim that some proposition is not actually entertained. If one cavils by appealing to gods or an infinite future, we can simply restrict the E and K operators to humans and to the current epoch.

[0]Forthcoming in *Analysis*.

[1]In the argument schema below, 'r' stands in for a sentence R expressing r (I cannot express r directly, for obvious reasons), while 'Ar' stands in for ⌜Actually R⌝, and likewise for other constructions. Elsewhere in the article, quoted propositional expressions such as "'r'" and "'Ar'" should be understood as referring to the corresponding sentences, while unquoted propositional expressions such as 'Ar' should usually be understood as picking out the propositions that the corresponding sentences express. In a few contexts, such as when stating logical principles and inside 'that'-clauses, unquoted propositional expressions are used as stand-ins for the corresponding sentences.

Premises (2) and (3) appear to be instances of core principles of the logics governing K and A: $Ap \to \Box Ap$ and $\Box(Kp \to p)$. Perhaps one can deny that the English word 'actually' satisfies (2), but it is hard to deny that there is a technical term that works this way. Denying (3) appears to require allowing that there can be knowledge of false propositions (although more on this later).

Premise (4) follows from two principles governing entertaining: entertaining a proposition requires entertaining its constituents, and knowing a proposition requires entertaining that proposition. If r is true, so that no-one entertains q, then the first principle entails that no-one entertains $r \leftrightarrow Ar$ (of which q is a constituent), and the second principle then entails that no-one knows $r \leftrightarrow Ar$. Someone might object to these principles by holding that knowledge is dispositional while entertaining is occurrent, or that entertaining is not a subpropositional notion. One can straightforwardly get around these worries by reinterpreting 'Ep' as 'Someone entertains a proposition of which p is a proper or improper constituent', and reinterpreting 'Kp' as 'Someone occurrently knows p', or 'Someone knows p while entertaining p'.

The conclusion follows from the premises by classical logic and the weak modal logic **K** (in fact, by minimal logic and by the version of **K** on which theorems of minimal logic are necessary). From (3) and (4), one can derive $\Box(K(r \leftrightarrow Ar) \to ((r \leftrightarrow Ar)\&\neg r))$. From (1) and (2), one can derive $\Box(K(r \leftrightarrow Ar) \to Ar)$. From these two claims one can derive $\Box(K(r \leftrightarrow Ar) \to (r\&\neg r))$, from which the conclusion follows.

The original version of the conclusion is the negation of an instance of the key thesis that for all p, it is knowable that $p \leftrightarrow Ap$. Even if we make the modifications above, the modified conclusion that it is not occurrently knowable by humans that $r \leftrightarrow Ar$ is no less interesting, as the standard reasons for holding that it is knowable a priori that $p \leftrightarrow Ap$ are also reasons for holding that it is occurrently knowable by humans that $p \leftrightarrow Ap$ (at least when the proposition is entertainable by humans).

It is worth noting that there are also versions of the argument that do not rely on the notions of constituency or of entertaining. The argument requires only a true proposition r that satisfies (4): that is, such that r necessitates $\neg K(r \leftrightarrow Ar)$. For example, if one rejects the notion of constituency because one takes propositions to be sets of possible worlds, one can simply take r to be any true proposition (a pair of worlds including the actual world and one other, for example) such that $\neg Kr$ holds in all r-worlds. Then assuming that Ar is the necessary proposition, r will satisfy (4), and the conclusion follows.

Another version of the argument, relevant for someone who accepts constituency but has doubts about entertaining, takes 'Eq' to mean 'Someone knows $(p \leftrightarrow Ap)$ for some p of which

q is a constituent.' Then it remains plausible that there is q such that $\neg Eq$. Then $\neg Eq$ satisfies (4), and the conclusion follows. This version of the argument has the advantage that one can understand 'K' in terms of knowledge rather than occurrent knowledge, yielding a conclusion that more directly negates the conclusion of the original thesis about knowability. An opponent might respond by denying that on this sense of 'E', there exists q such that $\neg Eq$. But for this denial to be reasonable, the opponent will almost certainly have to hold that $p \leftrightarrow Ap$ is not just knowable but known for all p. This claim is unattractive on most views of propositions (at least once omniscient beings are set aside), and is made more so once one notes that many of the known propositions cannot be occurrently known.

Still, the coherence of this form of resistance should be noted. More generally, if one accepts that for all p, $K(p \leftrightarrow Ap)$, then one will deny that there is any true proposition r that satisfies (4), and one will correspondingly deny (5). But likewise, if one denies that for all p, $K(p \leftrightarrow Ap)$, it is hard to deny that there is a true proposition r that satisfies (4) (while there is room in logical space to do so, such a position is hard to motivate), so it is hard to deny (5). We can take all this into account by putting the conclusions in an alternative conditional form: if there is p such that $p \leftrightarrow Ap$ is not known, then there is p such that $p \leftrightarrow Ap$ is not knowable; and if there is p such that $p \leftrightarrow Ap$ is not occurrently known, then there is p such that $p \leftrightarrow Ap$ is not occurrently knowable. The antecedents of these conditionals are highly plausible, so in what follows I will focus on the unconditional conclusions, but much of what I say also applies to the conditional versions.

It is hard to deny that there is r that satisfies (1) and (4), and many will likewise find (2) and (3) hard to reject. Given the strength of the premises, the most straightforward thing to do is to accept the conclusion, and to reject the view that it is always knowable a priori that $p \leftrightarrow Ap$.

If one accepts the conclusion, other surprising consequences follow. As well as rejecting the common view that it is always knowable a priori that $p \leftrightarrow Ap$, one may also have to accept that knowability is not closed under logical entailment. It is plausible that $\forall p(p \leftrightarrow Ap)$ is known and therefore knowable, and $r \leftrightarrow Ar$ is logically entailed by in some systems, but $r \leftrightarrow Ar$ is not knowable. In addition, one may also have to accept that provability (as usually understood) does not entail knowability. The sentence '$p \leftrightarrow Ap$' is a theorem in many systems of 'actually'-involving logic.[2] If so, then the proposition $r \leftrightarrow Ar$ is provable, in the standard sense that there exists a proof of it. Nevertheless, $r \leftrightarrow Ar$ is not knowable, and also is not provable in the sense that someone could prove it: the proof cannot be used to gain knowledge of $r \leftrightarrow Ar$.

[2]For example, '$p \leftrightarrow Ap$' is a theorem of Hazen's (1978) natural deduction system S5A. Of course such systems will not have an unrestricted principle of necessitation for provable sentences. The standard notion of provability applies to

Given the surprising consequences, one may want to examine the options for responding to the argument in more detail. A number of the available options are tied to different available views of the semantics of 'actually' and of the way it behaves in epistemic and modal contexts.

What we might call the *face-value view* of 'actually' holds that there is a proposition expressed by 'Ap' such that 'KAp' and '$\Box Ap$' are true iff this proposition is known or necessary (and likewise for other 'A'-involving sentences). Given the face-value view, the conclusion (5) follows directly from the standard principles above ($Ap \rightarrow \Box Ap$, there can be no knowledge of false propositions) and the existence of a true proposition that satisfies (4).

A particularly clear illustration is provided by the *Russellian face-value view*, which combines the face-value view with the claims that 'p' expresses a Russellian proposition p and that 'Ap' expresses the Russellian proposition $p(@)$ holding that p is true in the actual world-state @.[3] If r is the Russellian proposition $\neg Eq$, where q is a Russellian proposition that is not actually entertained, then it is easy to see that if the Russellian proposition $r \leftrightarrow r(@)$ is entertained in a world, it is false in that world. So the proposition cannot be (occurrently) known, and given the face-value view, (5) follows.

The Russellian face-value view allows a diagnosis of the surprising consequences above. If this view is correct, it is possible to use the *sentence* '$r \leftrightarrow Ar$' to express knowledge. We might say that someone knows a sentence when they know the proposition it expresses in the world of knowledge (perhaps as presented under the guise of the sentence). In this sense, it is possible to know '$r \leftrightarrow Ar$'. But if one did so, in a possible world w, the sentence would express a proposition $r \leftrightarrow r(w)$ distinct from the proposition $r \leftrightarrow r(@)$ that it actually expresses.

One might say that the sentence '$r \leftrightarrow Ar$' is *semantically fragile*: the proposition it expresses depends on whether speakers attempt to know whether the sentence is true.[4] Semantic fragility yields a natural explanation of the unknowability of the proposition in question: one could know the sentence, but if one did, it would express a different proposition (that is, a proposition distinct from the proposition it actually expresses). It can also explain the failure of closure of knowability under logical entailment: one could use the the entailment to derive the sentence '$p \leftrightarrow Ap$', but

sentences, but we can extend it to propositions by saying that a proof of a proposition p in a system L is an abstract sequence of interpreted sentences such that the sequence is a proof in L (in virtue of the logical form of the sentences) of a sentence that expresses p.

[3]It is not obvious just what it is to know a proposition about the actual world @ in another possible world. Williamson (1987) raises questions about this notion in responding to Edgington (1985), who invokes the notion in addressing Fitch's paradox of knowability. Soames (2007) gives an account on which such knowledge involves a sort of complete descriptive specification of @.

if one did, the sentence would express a different proposition. Finally, it might explain the gap between knowability and provability: if anyone were to use the proof of the sentence to gain knowledge, the sentence would express a different proposition.[5]

Just as one can distinguish sentential and propositional notions of knowability, one can also distinguish sentential and propositional notions of apriority. It is plausible that the sentence '$r \leftrightarrow Ar$' is knowable a priori, in the sense that it is possible to know the proposition expressed by the sentence in the world of knowledge (perhaps as presented under the guise of the sentence). But in cases of semantic fragility, apriority of a sentence comes apart from apriority of the proposition it actually expresses. One might also distinguish two sorts of propositional justifiability, analogous to the two sorts of provability distinguished above. Just as in these cases there exists a proof of the proposition that cannot be used to prove the proposition, one might say that there exists an a priori justification of the proposition (perhaps deriving from the proof itself) that cannot be used to justify the proposition.

Semantic fragility is easy to overlook. A case study is provided by Soames (2007), who endorses the Russellian face-value view while holding that $p \leftrightarrow Ap$ is always contingent a priori when p is contingent ("There is an instance of the contingent a priori for each contingent truth"). He also gives an argument that the Russellian proposition in question is always knowable a priori. The demonstration above strongly suggests that Soames is incorrect. What has gone wrong?

Soames argues that even when one does not know the Russellian proposition $p \leftrightarrow p(@)$, one can come to know it a priori by a process that involves demonstrating the actual world-state @ as "This very world-state". But if one does not actually undergo the process, one cannot demonstrate

[4] There are other cases of semantic fragility. For example, one might introduce 'Knum' as a name for the number of propositions one will ever know. Then sentences involving 'Knum' will be semantically fragile, and some of them will introduce an analogous gap between sentential and propositional knowability. An example is 'Knum is the number of propositions that I will ever know', uttered by someone who never knows any propositions due to irrationality.

[5] Semantic fragility can also be used to help analyze the relation between the current puzzle and the paradox of knowability (Fitch 1963), which tells us that from the premise that some true proposition p is not known it follows that some proposition (the Fitch proposition $p \& \neg Kp$) is not knowable. The Fitch proposition is *alethically fragile*: one could plausibly investigate its truth-value and in doing so come to know whether it is true (at least if there are no other sources of unknowability associated with p), but if one did so, the proposition would have a truth-value different from its actual truth-value. The current proposition $r \leftrightarrow Ar$ is also unknowable, and is arguably also alethically fragile. But where the Fitch proposition is expressed by an alethically fragile sentence that is not semantically fragile, $r \leftrightarrow Ar$ is expressed by a semantically fragile sentence that is not alethically fragile. There may also be propositions of the form $p \leftrightarrow Ap$ that are not alethically fragile but that are still not knowable a priori because of semantic fragility. The key role of semantic fragility differentiates the current case from standard Fitch-style cases.

@ in this way. If one were to undergo the process one would demonstrate not @ but a different world-state w. One might thereby come to know the proposition $p \leftrightarrow p(w)$ a priori, but one would not thereby come to know the proposition $p \leftrightarrow p(@)$ a priori. So Soames' argument fails. Assuming that this process is the only way to come to know such propositions a priori (when p is not itself knowable a priori), then the natural conclusion is that (for such p), the Russellian proposition $p \leftrightarrow p(@)$ is knowable a priori iff it is known a priori.

Semantic fragility is not limited to a Russellian view of propositions. If one holds an object-involving Fregean view, on which the proposition expressed by a sentence is a Fregean proposition with the referents of simple expressions in the sentence as constituents, one will also be confronted with the issue of semantic fragility. If this view is combined with the view that the actual world-state is part of the extension of 'actually', then sentences such as '$r \leftrightarrow Ar$' will be semantically fragile, just as on the Russellian view. If this view is combined with a face-value semantics for 'actually', then it will yield consequences analogous to those of the Russellian face-value view.[6]

Of course, the fact that semantic fragility provides a diagnosis of the surprising consequences need not make those consequences easier to accept. One might still want to hold that $p \leftrightarrow Ap$ is always knowable a priori, or at least that it is always knowable. To do so, one needs an alternative view of the semantics of 'actually'.

One class of alternative views holds that the sentences in question are not semantically fragile, so that '$r \leftrightarrow Ar$' expresses the same proposition in all worlds. For example, one might hold that 'actually' is a primitive operator, or that the sentences in question express their primary intensions, or that 'Ap' expresses the same proposition as 'p'. But to avoid the consequences, one will still be left with a difficult choice between denying (2) and denying (3), and given the standard principles mentioned above, one will still be left denying the face-value view.[7] The natural upshot is that to avoid the conclusion, rejecting semantic fragility is less important than rejecting the face-value view.

[6] All this applies to the object-involving Fregean view in Chalmers (forthcoming), on which sentences express enriched propositions: structured two-dimensional entities involving both primary intensions and extensions as constituents. Given that '$r \leftrightarrow Ar$' expresses an enriched proposition with the actual world @ as a constituent and given face-value semantics for modal and epistemic contexts, (5) follows. To preserve standard two-dimensional claims about the apriority of '$p \leftrightarrow Ap$' (e.g. Davies and Humberstone 1980), one needs to invoke notions of sentential and propositional apriority that come apart from a priori knowability of a proposition (see Chalmers forthcoming, notes 24 and 25).

[7] A view that rejects semantic fragility and holds that the sentence '$Ap \leftrightarrow p$' is always true when uttered will probably hold that that Ap is true at a world iff p is true there, and will almost certainly hold this biconditional across

To reject the conclusion, the most obvious strategy is to reject (3) and to reject the principle $\Box(KAp \rightarrow Ap)$. To do so without allowing that there can be knowledge of false propositions, one must also reject the face-value view: that is, one must deny that 'Ap' expresses a proposition such that 'KAp' and '$\Box Ap$' say that this proposition is known or necessary. It will be then be natural (although not compulsory) to hold $\Box(KAp \rightarrow Kp)$ and $\Box(KAp \rightarrow p)$.

There are a few ways to implement such a view. One might give a quotational analysis, on which epistemic contexts involve relations to sentences ('Ap') rather than propositions. On a quotational analysis, one might reject propositions entirely, or one might hold as above that knowing a sentence requires knowing the proposition it expresses (in the world of knowledge), perhaps as presented under the guise of the sentence. One might also give an ambiguity analysis, on which 'Ap' expresses one proposition in modal contexts (perhaps $p(@)$) and another in epistemic contexts (perhaps p). On these views, one can reject (3) without admitting that false propositions can be known.

Perhaps the most promising way to reject the conclusion is to give a scope analysis, on which 'Ap' says something like 'In the world, p' (where relative to any world w 'the world' denotes w), and in which the description here takes wide scope over modal but not epistemic operators. Then 'KAp' is read as 'K(in the world, p)', while '$\Box Ap$' is read as 'The world w is such that $\Box$(in w, p)'. In effect, the proposition expressed by 'Ap' is necessarily and a priori equivalent to the proposition expressed by 'p', 'KAp' is analyzed in terms of the truth of this proposition, but '$\Box Ap$' is not analyzed in terms of the necessity of this proposition. Again, the analysis allows one to deny (3) without admitting knowledge of false propositions.[8]

One might also adopt a pluralist view on which there are different readings of 'actually' that work in different ways. Perhaps there is one reading on which (2) is false, one on which (3) is false, and one on which the conclusion is true, for example. It is very plausible that there is an ordinary-English reading of 'Actually p' on which (2) and (5) are false. It is also plausible that there is an available philosophical reading (one that we might express by saying 'In this very world-state, p'), for which (2), (3), and (5) are all true. And it is not out of the question to hold

all worlds in which '$Ap \leftrightarrow p$' is uttered. For such a view to preserve (2), it must deny the face-value view of 'actually' in modal contexts, perhaps instead embracing a version of the scope analysis or the quotational analysis below.

[8]The scope analysis faces difficult problems when it is combined with quantification (see Humberstone 1982), but solutions to these problems have been proposed (see Forbes 1989). It is also worth noting that the scope analysis holds that the logical forms of (2) and (3) differ from their surface forms. I am taking it that (2) and (3) are specifications of surface form, so that the scope analysis rejects (3). If one instead took (2) and (3) to be specifications of logical form, the scope analysis would reject (2).

that there is a third reading of ‘actually’—perhaps the canonical philosophical reading – on which (2) is true and (5) is false, so that (3) is false also.

I am sympathetic to the pluralist view myself, but even on the pluralist view, the existence of a reading of ‘actually’ on which the argument is sound is enough to raise many of the original issues. For example, there arguably remains an intuition that for all p, the proposition expressed by ‘p iff in this very world-state p’ is trivial, knowable, and knowable a priori. If so, there remains the question of how to reconcile this intuition with the argument against it.

An alternative way of responding to the argument is to accept its conclusion while holding on to a version of the original thesis that $p \leftrightarrow Ap$ is always knowable, by arguing that there is a sense of “knowable” in which a proposition may be knowable even though it is not metaphysically possible that it be known. One version of this strategy appeals to agentive possibility (what is possible for an agent, where this might be understood as something in the vicinity of what the agent has the capacity to do), holding that a proposition is knowable when it is agentively possible for someone to know it, while denying that agentive possibility entails metaphysical possibility.[9] This position faces an obvious challenge, however, in that the original argument might be reformulated in terms of agentive possibility, and the case for the four key premises remains strong when they are read this way.

Another attempt to find a different sense of “knowable” is inspired by the two senses of “provable” discussed earlier. According to this proposal, a proposition is knowable (a priori) iff there exists a conclusive (a priori) justification for it. It would take some work to make this proposal precise, but a natural thought is that a justification is roughly analogous to a proof: an abstract structure of propositions and evidence, standing in relations of support, grounded in evidence or at least in known or justified propositions. One could then say that an a priori justification is one grounded only in non-experiential evidence or in propositions that are known/justified a priori, and a conclusive justification is a justification appropriate for knowledge.

In effect, this analysis yields nonmodal notions of justifiability, knowability, and apriority that are distinct from the more familiar modal notions, and that are instead analogous to the standard

[9]Fara (2010) develops a strategy of this sort in responding to Fitch’s paradox. In a similar spirit, one might also use the proposal in the next paragraph to respond to Fitch’s paradox. One could consistently hold that there exists a conclusive justification for the Fitch proposition $p\&\neg Kp$, in virtue of there existing separate conclusive justifications for p and $\neg Kp$, although it is impossible to use this justification to know the proposition. Insofar as this notion yields a nonmodal notion of knowability analogous to the standard nonmodal notion of provability, it might also yield a sense in which the Fitch proposition is knowable.

nonmodal notion of provability. In cases of semantic fragility, all of these nonmodal notions can come apart from their modal counterparts. I think that it is not especially plausible that the nonmodal analysis provides a natural sense of "knowable" in English or that this is the sense that proponents of the original thesis intended. Still, the underlying thought that there are nonmodal notions in the vicinity of (a priori) knowability that work this way is well worth pursuing.

A related and perhaps preferable option is to accept the conclusion of the argument and to reject the original thesis, while finding claims nearby to the original thesis that are true, thereby explaining away the intuition that the original thesis is true. One version of this strategy appeals to theses using alternative readings of 'actually', while another invokes a thesis cast in terms of sentential apriority. A third version invokes the nonmodal notion of propositional apriority in this role, holding that any proposition of the form $p \leftrightarrow Ap$ is a priori in the nonmodal sense that there exists a conclusive a priori justification for it. In this way we might explain away the intuition that all such propositions are knowable a priori. Likewise, the intuition that $p \leftrightarrow Ap$ is always knowable might be explained in terms of sentential knowability or in terms of the existence of a conclusive justification for the proposition. I take this fragmentation of sentential apriority and knowability and different notions of propositional apriority and knowability to be perhaps the most interesting upshot of the original puzzle.

In conclusion: If one accepts an orthodox semantics for 'actually' and an orthodox understanding of apriority, one must reject the orthodox view that $p \leftrightarrow Ap$ is always a priori. Likewise, if one accepts the orthodox view that $p \leftrightarrow Ap$ is always a priori, one must adopt an unorthodox semantics for 'actually' or an unorthodox understanding of apriority.[10]

Bibliography

Chalmers, D.J. forthcoming. Propositions and attitude ascriptions: A Fregean account. *Nous*.

Davies, M. and Humberstone, L. 1982. Two notions of necessity. *Philosophical Studies* 38:1-30.

Edgington, D. 1985. The paradox of knowability. *Mind* 94:557-68.

Fara, M. 2010. Knowability and the capacity to know. *Synthese* 173:53-73.

Fitch, F. 1963. A logical analysis of some value concepts. *Journal of Symbolic Logic* 28: 135142.

Forbes, G. 1989. *The Languages of Possibility*. Blackwell.

[10]Thanks to Berit Brogaard, Kit Fine, John Hawthorne, Lloyd Humberstone, Joe Salerno, Jonathan Schaffer, Robbie Williams, and the Corridor discussion group.

Hazen, A. 1978. The eliminability of the actuality operator in propositional modal logic. *Notre Dame Journal of Formal Logic* 19:617-22.

Humberstone, I.L. 1982. Scope and subjunctivity. *Philosophia* 12:99-126.

Soames, S. 2007. Actually. *Proceedings of the Aristotelian Society*, Supplementary Volume 81: 251-77.

Williamson, T. 1987. On the paradox of knowability. *Mind* 96:256-61.