The Multi-Stage-Lambert Scheme for Steering a Satellite-Launch Vehicle (SLV)

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Abstract–The determination of an orbit, having a specified transfer time (time-of-flight) and connecting two position vectors, frequently referred to as the Lambert problem, is fundamental in astrodynamics. Of the many techniques existing for solving this two-body, two-point, time-constrained orbital boundary-value problem, Gauss' and Lagrange's methods were combined to obtain an elegant algorithm based on Battin's work. This algorithm included detection of cross-range error. A variable TYPE, introduced in the transfer-time equation, was flipped, to generate the inverse-Lambert scheme. In this paper, an innovative adaptive scheme was pre-sented, which was called "the Multi-Stage-Lambert Scheme". This scheme proposed a design of autopilot, which achieved the pre-decided destination position and velocity vectors for a multi-stage rocket, when each stage was detached from the main vehicle after it burned out, completely.

Keywords–Lambert scheme, inverse-Lambert scheme, multi-stage Lambert scheme, two-body problem, transfer-time equation, orbital boundary-value problem

NOMENCLATURE

A. Symbols (in alphabetical order)

a	Semi-major axis of the ellipse
a_m	Semi-major axis of the minimum-energy orbit
c	Length of cord of the arc connecting the points corresponding to radial
e	Eccentricity of the ellipse
E	Eccentric anomaly
E_1	Eccentric anomaly corresponding to the launch point
E_2	Eccentric anomaly corresponding to the final destination
f	True anomaly
G	Universal constant of gravitation
m	Mass of spacecraft
M	Mass of earth
$\mathbf{n}$	Unit vector, indicating normal to the trajectory plane
p	Parameter of the orbit
g	Flight-path angle
r	Radial coördinate
r_1	Radial coördinate of launch point
r_2	Radial coördinate of final destination
$\mathbf{r}$	Radius vector in the inertial coördinate system
$\mathbf{r}_2$	Radius vector of final destination
t	Universal time
t_1	Launch time
t_2	Time of reaching final destination
$TYPE$	Variable expressing direction of motion of spacecraft relative to earth's rotation
$\mathbf{v}$	Velocity vector in the inertial coördinate System
m	Product of universal constant of gravitation and sum of masses of spacecraft and earth
t	Time of pericenter passage
q	Transfer angle
f	Angle of inclination of velocity vector

Astrodynamical terminologies and relationships are given in Appendices A and B, respectively. Appendix C contains a proof of the relationship connecting true and eccentric anomalies, providing a justification of positive sign in front of radical.

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B. Compact Notations

In order to simplify the entries, $\hat{I} = \sqrt{1-e^2}$, $' = \sqrt{\frac{1-e}{1+e}}$, m = *G (m + M)*, are used in the expressions and equations.

I. INTRODUCTION

Determination of trajectory is an important problem in astrodynamics. For a spacecraft moving under the influence of gravitational field of earth in free space (no air drag) the trajectory is an ellipse with the center of earth lying at one of the foci of the ellipse. This constitutes a standard two-body-central-force problem, which has been treated, in detail, in many standard text-books [1, 2]. The trick is to first reduce the problem to two dimensions by showing that the trajectory always lies in a plane perpendicular to the angular-momentum vector. Then the problem is set up in plane-polar coördinates. Angular mo-mentum is conserved and the problem, effectively, reduces to one-dimensional problem involving only the variable ***r*** [3].

A problem famous in astrodynamics, called “the Lambert Problem”, is based on the Lambert theorem [4, 5]. According to this theorem the orbital-transfer time depends only upon the semi-major axis, the sum of the distances of the initial and the final points of the arc from the center of force as well as length of the line segment join-ing these points. Based on this theorem a problem called the Lambert problem is formulated. This problem deals with determination of an orbit having a specified flight-time and connecting the two position vectors. Battin [6, 7] has set up the Lambert problem involving computation of a single hypergeometric function. Since transfer-time (time-of-flight) computation is done on-board, it is desirable to use an algorithm employ-ing as few computation steps as possible. The use of polynomials instead of actual expression and reduction of the number of degrees of freedom contribute towards the same goal.

An elegant Lambert algorithm, presented by Battin, was scrutinized and omissions/oversights in his calculations pointed out [8]. Battin’s for-mulation, which highlighted the main principles involved, was developed and expanded to a set of formulae suitable for coding in the assembly language. These formulae could be used as a practical scheme outside the atmosphere for steering a satellite-launch vehicle (SLV). This scheme computes velocity and flight-path angle required at any intermediate time to be compared with the actual velocity and the actual flight path angle of the spacecraft, as reported by the on-board computing system. A spacecraft cannot reach the desired location if cross-range error is present. Battin’s original work does not address this issue. A mathematical formulation was given by the author to detect cross-range error [8]. Algorithms were developed and tested, which indicated cross-range error. In order to correct cross-range error velocity vector should be perpendicular to normal to the desired trajectory plane (***i. e.***, the velocity must lie, entirely, in the desired trajectory plane).

A variable ***TYPE*** was introduced in the transfer-time equation to incorporate direction of motion of the spacecraft [9]. This variable can take on two values, +1 (for spacecrafts moving in the direction of rotation of earth) and –1 (for spacecrafts moving opposite to the direction of rotation of earth). In the inverse-Lambert scheme, *TYPE* was flipped, whereas all other parameters remained the same [8]. For an efficient trajectory choice, a transfer time close to minimum-energy orbital transfer time was selected. A procedure for finding the minimum-energy orbital transfer time is included in this work. Additionally, formulae are given to compute the orbital parameters in which the SLV must be locked in at a certain position, at a time, *t*, based on the Lambert scheme.

In this paper, “*the Multi-Stage-Lambert Scheme*” is presented. In this formulation, section-wise corrections are achieved, where destination point of the first stage is initiating point of the second stage and so on. In this way, position and rate satu-rations (out of range deviations) are avoided. A similar formulation was, earlier, given for the Q System [9].

II. STATEMENT OF THE PROBLEM

In order to choose a particular trajectory on which the spacecraft could be locked so as to reach a certain point one must select a certain parameter to fix this trajectory out of the many possible ones connecting the two points. One is, therefore, interested to put the Kepler equation in such a form so as to make it computationally efficient utilizing hypergeometric functions or quadratic functions instead of circular functions (sine or co-sine, *etc.*). This equation should express transfer time between these two points in terms of a series or a polynomial, and another formula should be available to compute flight-path angle, correspon-ding to this transfer time. Velocity desirable for a particular trajectory may, then, be computed on- board using this formula and compared with velocity of the spacecraft obtained from integration of acceleration information, which is available from on-board accelerometers and rate

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gyroscopes.

III. The Lambert Theorem

In 1761 Johann Heinrich Lambert, using a geometrical argument, demonstrated that the time taken to traverse any arc (now called transfer time), $t_2 - t_1$, is a function, only, of the major axis, a, the arc, $(r_1 + r_2)$, and length of chord of the arc, c, for elliptical orbits, *i. e.*,

$$t_2 - t_1 = f(r_1 + r_2, c, a)$$

The symbol, *f*, is used to express functional relationship in the above equation (do not confuse with true anomaly). Therefore, one notes that the transfer time does not depend on true or eccentric anomalies of the launch point or the final destination. Fig. 1 illustrates geometry of the problem.

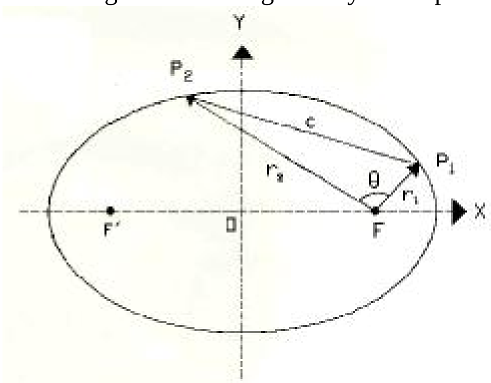

Fig. 1 Geometry of the boundary-value problem

Mathematically, transfer time for an elliptical trajectory may be shown to be [6]

$$\sqrt{\frac{m}{a^3}}(t_2 - t_1) = (a - \sin a) - (b - \sin b)$$

where, $\sin^2\frac{a}{2} = \frac{r_1 + r_2 - c}{4a}$, $\sin^2\frac{b}{2} = \frac{r_1 + r_2 + c}{4a}$.

In the case at hand, $m = G(m + M) \cong GM$, because $m << M$. Based on this theorem a formulation to calculate transfer time and velocity vector at any instant during the boost phase is developed. This formulation is termed as the Lambert scheme.

IV. The Lambert Scheme

Suppose a particular elliptical trajectory is connecting the points P_1 and P_2. Let t_1 and t_2 be the times, when the spacecraft passes the points P_1 and P_2, respectively, the radial coördinates being r_1 and r_2. The standard Kepler equation (t is time of pericenter passage)

$$\sqrt{m}(t - t) = a^{3/2}(E - e\sin E) \qquad (1)$$

may be expressed as

$$\sqrt{m}(t_2 - t_1) = 2a^{3/2}(y - \sin y \cos c) \qquad (2)$$

where $y = \frac{1}{2}(E_2 - E_1)$, $\cos c = e\cos\frac{1}{2}(E_2 + E_1)$.

This equation may be used to calculate transfer time between two points by iterative procedure. However, it is unsuitable for on-board compu-tation, because computing time is large owing to the presence of circular functions. In order to put this in a form involving power series, one intro-duces

$$2a_m = s = \frac{1}{2}(r_1 + r_2 + c) \qquad (3)$$

$$\Lambda s = \sqrt{r_1 r_2}\cos\frac{q}{2} \qquad (4)$$

where a_m is the semi-major axis of the minimum-energy orbit and q the transfer angle. From the geometry, one has

$$c = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2\cos q} \qquad (5)$$

$$y = \sqrt{1 - \Lambda^2(1 - x^2)} \qquad (6)$$

Where $x = \cos\frac{1}{2}(y + c)$. Further, introducing

$$h = y - \Lambda x \qquad (7)$$

$$S_1 = \frac{1}{2}(1 - \Lambda - xh) \qquad (8)$$

and a Q function

$$Q = \frac{4}{3}F(3,1;\frac{5}{2};S_1) \qquad (9)$$

expressible in terms of a hypergeometric function, $F(3,1;\frac{5}{2};S_1)$, instead of a circular function. This is needed to reduce onboard computation time. Transfer time may be expressed as

$$\sqrt{\frac{m}{a_m^3}}(t_2 - t_1) = h^3 Q + 4\Lambda h \qquad (10)$$

The magnitude of velocity, *v*, and flight-path angle, g, may be evaluated using the following expressions [4, 6]

$$v = \frac{1}{h}\sqrt{\frac{m}{a_m}}\sqrt{[\frac{2\Lambda a_m}{r_1} - (\Lambda + xh)]^2 + \frac{r_2}{r_1}\sin^2\frac{q}{2}} \qquad (11a)$$

$$\cos^2 g = \frac{r_2(1 - \cos q)}{2r_1 v^2} \qquad (11b)$$

The hypergeometric function in (9) is given by the continued-fraction expression (this expression is needed to reduce on-board computing time)

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$$F(3,1;\frac{5}{2};S_1)=\cfrac{3}{3-\cfrac{18S_1}{5+\cfrac{6S_1}{7-\cfrac{40S_1}{9-\cfrac{4S_1}{11-\cfrac{70S_1}{13-\cfrac{18S_1}{15-\cfrac{108S_1}{17-....}}}}}}}}$$

About 100 terms are needed to get an accuracy of 10^{-4}.

To compute the transfer time corresponding to the minimum energy orbit connecting the current position and the final destination one uses the transfer-time equation (11), with $x=0$ substituted in (8), corresponding to $a=a_m$ and solves it using Newton-Raphson method. Transfer time in the Lambert algorithm must be set close to this time. Fig. 2 shows the flow chart of Lambert algorithm.

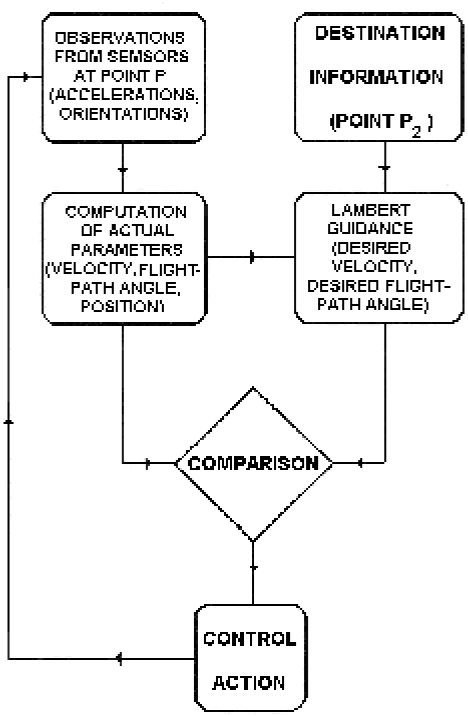

Fig. 2 Flow chart of the Lambert scheme

V. CROSS-RANGE-ERROR DETECTION (MATHEMATICAL MODEL)

For no cross-range error, velocity of the spacecraft must lie in the plane containing $\boldsymbol{r}\times\boldsymbol{r}_2$ (the trajectory plane). In other words, the velocity vector must make an angle of 90^0 with normal to the trajectory plane. This is equivalent to [8]

$$\boldsymbol{v}.(\boldsymbol{r}\times\boldsymbol{r}_2)=0 \qquad (12)$$

which says that $\boldsymbol{v}$ (current velocity), $\boldsymbol{r}$ (current position) and $\boldsymbol{r}_2$ (position of destination) are co-planar. For no cross-range error, the angle of inclination, f, between the velocity vector, $\boldsymbol{v}$, and normal to the trajectory, $\boldsymbol{n}$, must be 90^0. In the Lambert scheme a subroutine computes devi-ation of f from 90^0. Extended-cross-product steering [10], dot-product steering [11] and ellipse-orientation steering [12] could be used to eliminate cross-range error.

VI. THE INVERSE-LAMBERT SCHEME

Transfer-time equation between two points having eccentric anomalies E_1 and E_2, (corres-ponding to times t_1 and t_2, respectively) may be expressed as [8]

$$t_2-t_1=\sqrt{\frac{a^3}{m}}[(E_2-e\sin E_2)-(E_1-e\sin E_1)][TYPE] \qquad (13)$$

The variable $TYPE$ has to be introduced because the Kepler equation is derived on the assumption that *t* increases with the increase in *f*. Therefore, the difference

$$[(E_2-e\sin E_2)-(E_1-e\sin E_1)]$$

shall come out to be negative for spacecrafts orbiting in a sense opposite to rotation of earth. The variable $TYPE$ ensures that the transfer time (which is the physical time) remains positive in all situations by adapting the convention that $TYPE=+1$ for spacecrafts moving in the direc-tion of earth's rotation, whereas, $TYPE=-1$ for spacecrafts moving opposite to the direction of earth's rotation. This becomes important in computing correct flight-path angles in the Lambert scheme.

If one wants to put another spacecraft in the same orbit as the original spacecraft, but moving in the opposite direction, one can use the inverse- Lambert scheme. This can be accomplished by flipping sign of the variable, $TYPE$, whereas all the other parameters remain the same. Burns and Sherock [13] try to accomplish the same objective by a three-degree-of-freedom interceptor simulation designed to rendez-vous with a ballistic target: position and velocity matching, no flipping of *TYPE*. The strategy put forward by this author was simpler [8]: orbit matching, flipping of ***TYPE***. The inverse-Q system has, also, been proposed by the author to accomplish the same objective [9]. Do not confuse Q system with the symbol Q introduced in (9).

VII. THE MULTI-STAGE-LAMBERT SCHEME

Let us consider a three-stage rocket. Destination point of the first (second) stage is the initiating point of the second (third) stage. Mathematically,

$$v_{1,final} \rightarrow v_{2,initial}; v_{2,final} \rightarrow v_{3,initial} \quad (14a)$$

$$\vartheta_{1 final} \rightarrow \vartheta_{2,initial}; \vartheta_{2,final} \rightarrow \vartheta_{3,initial} \quad (14b)$$

Equations (11a, b) can be used to compute velocities and flight-path angles at the initiating and destination points of various stages. By using these section-wise corrections, one can avoid position saturation and rate saturation.

VIII. DISCUSSION AND CONCLUSIONS

The Lambert problem is a fixed-transfer-time-boundary-value problem. The Lambert-scheme formulations available in literature run into problems in terms of computing correct flight-path angles and velocities, in particular, for spacecrafts moving opposite to earth rotation because of definition of time in the Kepler equation. This problem was resolved by introducing a variable, ***TYPE***, in the transfer-time equation. This, also, led to a natural formulation of the inverse-Lambert scheme. The Lambert scheme is applicable in free space, in the absence of atmospheric drags, for burnout times large as compared to on-board computation time (for example, if the burnout time for a given flight is 18 second and the computation time is 1 second, there may not be enough time to utilize this scheme). This is needed to allow sufficient time for the control decisions to be taken and implemented before the rocket runs out of fuel. Detection of cross-range error is incorporated in this formulation. It is assumed that rocket is fired in the vertical position so as to get out of the atmosphere with minimum expenditure of fuel. Later, in free space this scheme is applied to correct the path of rocket. Since the rocket remains in free space for most of the time, this method may be useful in calculating the desired trajectory.

The Lambert scheme is an explicit scheme, which generates a suitable trajectory under the influence of an inverse-square-central-force law (gravitational field of earth) provided one knows the latitudes and the longitudes of launch point and of destination as well as the transfer time (time spent by the spacecraft to reach destination).

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APPENDIX A: ASTRODYNAMICAL TERMINOLOGIES

Down-range error is the error in the range assuming that the vehicle is in the correct plane; cross-range error is the offset of the trajectory from the desired plane. An unwanted pitch movement shall produce down-range error; an unwanted yaw movement shall produce cross-range error.

For an elliptical orbit true anomaly, f, is the polar angle measured from the major axis ($\angle PFX$ in Fig. 3). Through the point P (current

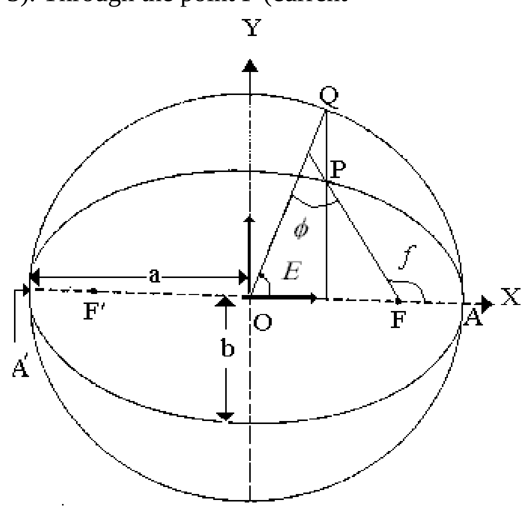

Fig. 3 Justification of the positive sign in ϶

position of spacecraft, $m\overline{PF} = r$, the radial coördinate) erect a perpendicular on the major axis. Q is the intersection of this perpendicular with a circumscribed auxiliary circle about the orbital path. The angle, $\angle QOF$ (cf. Fig.3) is called the eccentric anomaly, E.

For this orbit, pericenter, the point on the major axis, which is closest to the force center (point A in Fig. 3), is chosen as the point at which f = 0. Apocenter is the opposite point on the major axis, which is farthest from the force center (point A in Fig. 3). The line joining the pericenter and the apocenter is called the line of apsides.

APPENDIX B: ASTRODYNAMICAL RELATIONHIPS

Some useful relationships among radial coördinate, eccentric anomaly, eccentricity and semi-major axis for an elliptical orbit are listed below:

$$r = a(1-e\cos E) \qquad \text{(B1}a\text{)}$$

$$r\cos f = a(\cos E - e) \qquad \text{(B1}b\text{)}$$

$$r\sin f = a \in \sin E \qquad \text{(B1}c\text{)}$$

$$\sqrt{r}\cos\frac{f}{2} = \sqrt{a(1-e)}\cos\frac{E}{2} \qquad \text{(B1}d\text{)}$$

$$\sqrt{r}\sin\frac{f}{2} = \sqrt{a(1+e)}\sin\frac{E}{2} \qquad \text{(B1}e\text{)}$$

The following may be useful in converting circular functions involving true anomalies to those involving eccentric anomalies and vice versa.

$$\cos f = \frac{\cos E - e}{1-e\cos E} \qquad \text{(B2}a\text{)}$$

$$\cos E = \frac{\cos f + e}{1+e\cos f} \qquad \text{(B2}b\text{)}$$

$$\sin f = \frac{\in \sin E}{1-e\cos E} \qquad \text{(B2}c\text{)}$$

$$\sin E = \frac{\in \sin f}{1+e\cos f} \qquad \text{(B2}d\text{)}$$

$$϶ \tan\frac{f}{2} = \tan\frac{E}{2} \qquad \text{(B2}e\text{)}$$

In Appendix C, the last relation is proved and a justi-fication is given for the positive sign taken in front of the square root appearing in the expression for ϶.

APPENDIX C: RELATION CONNECTING ECCENTRIC ANOMALY TO TRUE ANOMALY

In Fig. 3, semi-minor axis of the ellipse, b, is related to a by $b = a\in$. Do not confuse the point Q in Fig. 3 with the quantity Q defined in (9). Using the relations $r = \dfrac{p}{1+e\cos f}$ and $p = a(1-e^2)$, one may write, $r(1+e\cos f) = a(1-e^2)$. Rearranging

$$er\cos f = a(1-e^2) - r$$

Adding er to both sides and using (B1a) on the right-hand side, one gets

$$r(1+\cos f) = a(1-e)(1+\cos E) \qquad \text{(C1)}$$

Subtracting er from both sides and using (B1a) on the right-hand side, one gets

$$r(1-\cos f) = a(1+e)(1-\cos E) \qquad \text{(C2)}$$

Dividing (C2) by (C1)

$$\frac{1-\cos f}{1+\cos f} = \frac{(1+e)(1-\cos E)}{(1-e)(1+\cos E)}$$

Using the identities

$$1-\cos f = 2\sin^2\frac{f}{2}$$

$$1+\cos f = 2\cos^2\frac{f}{2}$$

with similar results for the expressions $(1-\cos E)$ and $(1+\cos E)$, one sees that the above equation reduces to

$$\tan^2\frac{f}{2} = \frac{1+e}{1-e}\tan^2\frac{E}{2}$$

which implies

$$\tan\frac{f}{2} = \pm\sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$$

Below, it is justified that only positive sign with the radical gives the correct answer. Consider Δ ORF (cf. Fig. 3). One notes that,

$$-p \le E \le p \Rightarrow -\frac{p}{2} \le \frac{E}{2} \le \frac{p}{2}$$

Further, $E \ge 0 \Rightarrow f \ge 0$; $E < 0 \Rightarrow f < 0$.

Therefore, $\dfrac{f}{2}$ and $\dfrac{E}{2}$ have the same sign. When $-\dfrac{p}{2} \le \dfrac{E}{2} < 0$, $\tan\dfrac{E}{2} < 0$, $\tan\dfrac{f}{2} < 0$, which implies that positive sign with the radical should be chosen. Similarly,

$$0 \le \frac{E}{2} \le \frac{p}{2},\ \tan\frac{E}{2} \ge 0,\ \tan\frac{f}{2} \ge 0$$

and, hence, positive sign with the radical is the correct choice.

Two-body problem can be handled elegantly by using the elliptic-astrodynamical-coördinate mesh [14, 15]. One discovers additional cons-tants of motion if the problem is set up using this formulation [16].

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