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## Managing Chaos

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*“Chaos is inevitable. In the sense that perturbation is evolutionary, it's also desirable.*

*But managing it is essential. It's no use for any of us to hope that someone else will do it.*

*Do you have your own personal strategies in place?” C. P. Brinkworth, 2006 [1]*

Chaos means that strategies go wildly astray. It is often associated with missed deadlines, understaffing, runaway costs, and similar situations generally considered negative. Under these circumstances “Chaos” describes a situation where the goals of a strategy are unachievable and therefore the outcomes become random, unpredictable and often undesirable. This is exemplified in a recent E-mail message by Bill Ford to all of Ford Motor’s employees saying: "The business model that sustained us for decades is no longer sufficient to sustain profitability." Geoffrey Colvin, senior editor at Fortune magazine, analyzes Ford’s problems in his article “Managing in Chaos” [2].

But what if the goals of a strategy are achievable, but a small deviation from the plan or regulation leads to very different outcomes? [3] This behavior is called deterministic

chaos [4]. Without management, deterministic chaos can produce arbitrary outcomes, some may be very positive some may be very negative. For instance, the evolution of an organization is deterministic chaos, if it encourages thinking out of the box and implements these new ideas rapidly, such as Google's "Chaos by design" strategy [5]. If the management does a good job in prioritizing ideas for implementation the overall outcome is positive. This appears to be a particularly good recipe for success at research facilities and educational institutions [6].

In the following we discuss the management of some very simple deterministic chaotic agents which are subject to a small amount of noise. The agents can be thought of as business units or other nonlinear dynamical systems. The chaotic agents are controlled by a control unit, which could be a manager or a computer algorithm. This is by no means a simulation of managing a real world social organization or business entity. Models where simple control units manage a set of simple deterministic chaotic agents may provide intuition or illustrate a paradigm for the management of entities which have realistic strategies, but where a small deviation from the plan or regulation leads to a very different outcome. Finally we discuss managing networks of deterministic chaotic agents.

**Predicting chaos is hard, controlling chaos is easy.** More precisely, long term predictions of deterministic chaos are hard, since even very small amounts of noise can change the motion significantly. Short term predictions and even medium term predictions of chaos are not that difficult, since the motion is governed by a deterministic equation, plus some small noise [8]. In contrast, controlling the chaotic motion of an

agent is often easy, both short term and long term. Just apply a control force which is equal to the difference between the next state of the agent and the target, and it will go to the target [8]. This requires predicting the next state, which is a short term prediction, and therefore possible for chaotic agents. This control algorithm would not work for a random motion, since random motion can not be predicted, not even for one time step. This chaos control algorithm was introduced by Hubler in 1989 [9] and since has been further developed and widely used [10, 11].

Fig. 1 shows open loop control of chaotic logistic map dynamics for three different targets [8]. Open loop control is not always stable, only if the target is in the convergent region of the state space [12]. In convergent region two neighboring states get even closer at the next time step [13]. The state space of a chaotic agent can be divided into two regions, the convergent region and the rest, the divergent region. The dashed area in Fig. 1 is the convergent region. **If the target is in the convergent region, then chaos control is stable.**

Even if the target dynamics is chaotic or random, the control is stable if the target dynamics is in the convergent region. Fig. 1c shows the conversion of an uncontrolled chaotic logistic map dynamics into a controlled chaotic logistic map dynamics. The control unit, and everyone who has access to the target dynamics, can make long term predictions of controlled chaos, whereas anyone else can only make short term predictions of controlled chaos. For the control unit, controlled chaos is predictable, and still has most of the benefits of chaos. Chaotic agents constantly explore the state- space

and have high potential for improving their performance, in particular in evolving environments [14].

What would happen if a control unit tries to control a set of slightly different chaotic agents, with a single control force? If a control force is designed for an agent, and then applied to agents with different parameter values the control may or may not work. For simple systems, such as logistic map chaos with parameter value  $a=3.8$ , controlling chaos works as long as the difference between the agents is less than 25%. This means a control unit can control a set of chaotic agents with a single control force if the difference between the agents is less than 25%. However, if the diversity of the agents greater, the control fails. In this numerical example we use a chaotic target, which is a rather sophisticated control. If the target is simpler the control works for groups of agents with a larger diversity.

Another interesting question is the control of simple agent networks, for instance the control of a chaotic leader-follower system. We consider the situation where the dynamics of a chaotic agent (leader) is imitated by a second agent (follower), and we assume there is some feedback from the follower to the leader. The Henon map is a simple model for such a system. We study the dynamics a chaotic leader-follower network which is controlled by a control unit [15]. In this case the convergent region depends only on the state of the leader. Therefore a **stable control of the leader-follower network can be easily achieved by controlling the leader**. Fig. 2 shows the uncontrolled and controlled chaos in a leader-follower network. The state of the leader

and the follower are plotted versus time. For the first 20 time steps there is no control, and the dynamics is Henon map chaos. Afterward a control force is applied, where the target dynamics is a chaotic logistic map dynamics inside the convergent region of the Henon map. The plot illustrates that the leader-follower system quickly approaches the target, and thus behaves like a chaotic logistic map. This controlled chaotic dynamics is predictable for the control unit, even for a long period of time.

Long term prediction of uncontrolled chaos is virtually impossible in large networks of chaotic agents. However it appears to be possible to switch such networks to controlled chaos, which makes them predictable, without losing the benefits of chaotic systems.

Even though the dynamics of social organizations are much more complicated than these simple chaotic models, it is conceivable that a similar approach can be used to predict and control them.

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8. The dynamics of an agent or some other dynamical system is modeled with a map with additive forcing,  $x_{n+1} = f(x_n, a) + F_n$ ,  $n=0,1,2,\dots,N$ , where  $x_n$  is the state at time step  $n$ .  $a$  is the agent parameter. For the logistic map the mapping function is  $f(x_n, a) = a x_n (1 - x_n)$ , where  $0 < x_n < 1$ ,  $0 < a < 4$ . The chaotic logistic map is used to describe growth with limited resources. For  $a=3.8$  the dynamics is chaotic and this value is used in the figures.  $F_n = X_{n+1} - f(X_n, b)$  is a control force, where  $X_n$  is the target dynamics, and  $b$  is an estimate of the agent parameter  $a$ .  $f(X_n, b)$  is a short term prediction of the chaotic system. In Fig. 1a the target is  $X_n = 0.6$  and in Fig. 1b the target is  $X_n = 0.65$ . In Fig. 1c the target  $X_{n+1} = -4.4 + 20X_n - 20X_n^2$  is chaotic, where the distance between neighboring trajectories increases by a factor of two each time step. The distance between the trajectories is  $d = \sqrt{(x_N - X_N)^2 + (y_N - Y_N)^2}$ .
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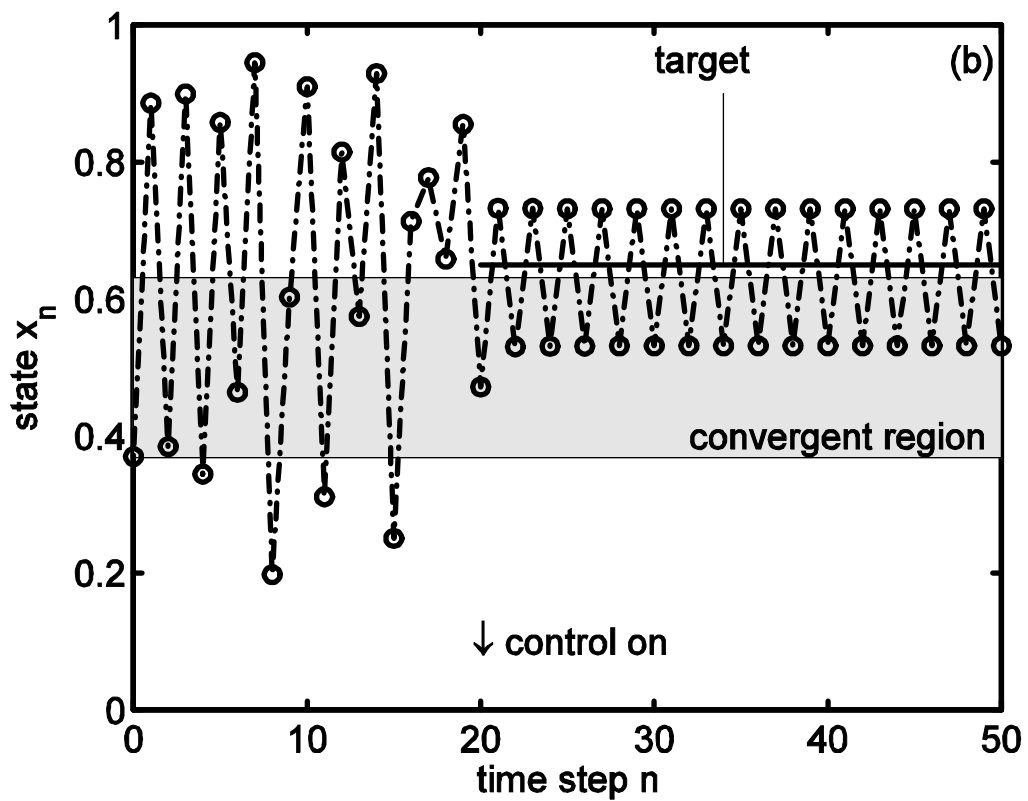
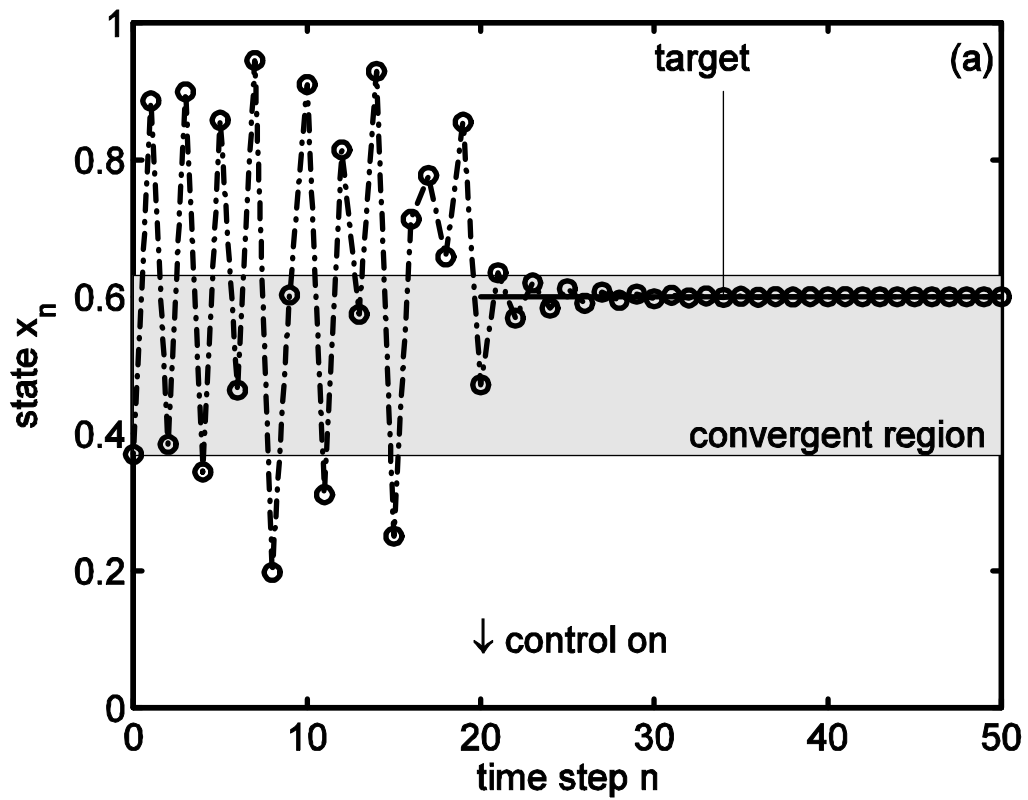
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12. Jackson, E.A. Controls of dynamic flows with attractors. Phys. Rev. A 1991, 44, 4839–4853.

13. The convergent region is that part of the state space where neighboring trajectories get closer within one time step, i.e.  $|\partial f / \partial x| < 1$ . The convergent region for the logistic map is  $0.5 - 1/(2a) < x_n < 0.5 + 1/(2a)$ .

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15. The Henon map has two variables,  $x_n$  the state of the leader at time step  $n=0,1,2,\dots, N$ , and  $y_n$ , the state of the follower. The follower imitates the leader, i.e.  $y_{n+1} = b x_n + F_{y,n}$ . The dynamics of the leader is modeled by a logistic map, plus some feedback from the follower,  $x_{n+1} = 1 - a x_n^2 + y_n + F_{x,n}$ . Typical values are  $a=1.1$ , and  $b=0.3$ . The convergent region is  $(b-1)/(2a) < x_n < (1-b)/(2a)$ . The control forces are  $F_{x,n} = X_{n+1} - (1 - a X_n^2 + Y_n)$  and  $F_{y,n} = Y_{n+1} - b X_n$ , where  $X_n$  the target state of the leader and  $Y_n$ , the target state of the follower. Controlling the leader means, that  $X_n$  can be any dynamics, stationary, periodic, chaotic, or random within the convergent region, and there is no control of the follower, i.e.  $Y_{n+1} = b X_n$ .





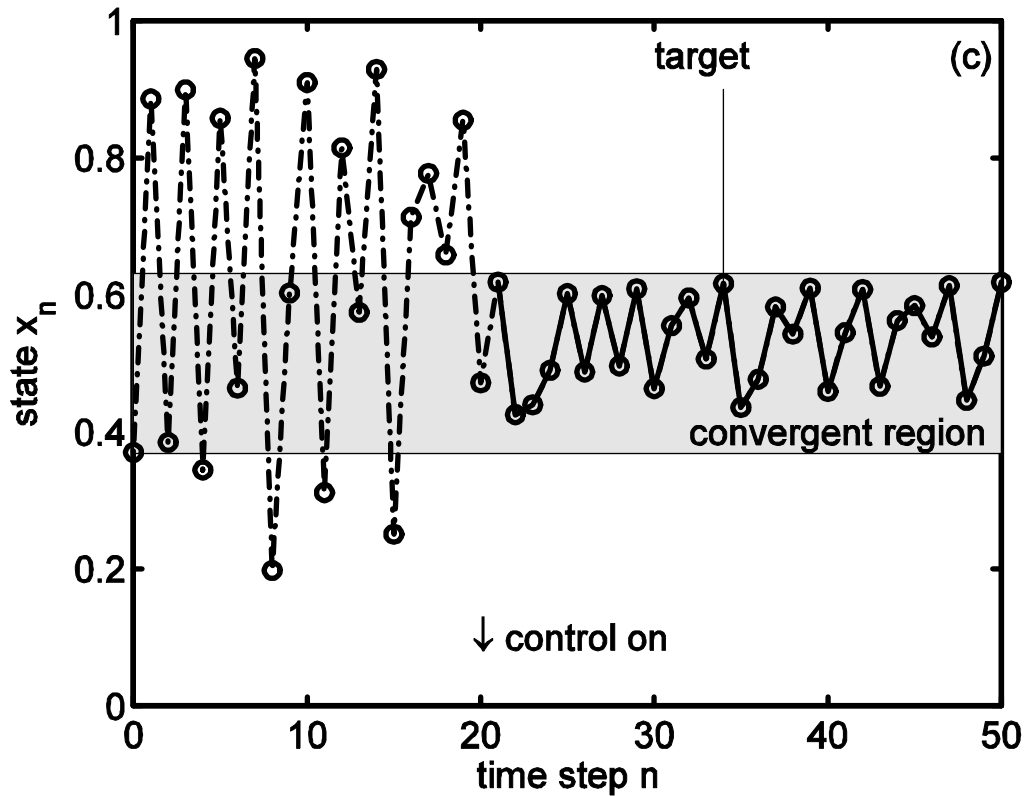


Figure 1. The state of a chaotic agent versus time [8]. The control starts at time step 20. The continuous line is the target. In plot (a) the target is inside the convergent region (gray area) and the control is stable. In plot (b), the target is outside the convergent region, and the control is unstable. In this case the dynamics does not get closer and closer to the target. In plot (c) the target is chaotic. Since the target is inside the convergent region, the control is stable, even if the target is chaotic.

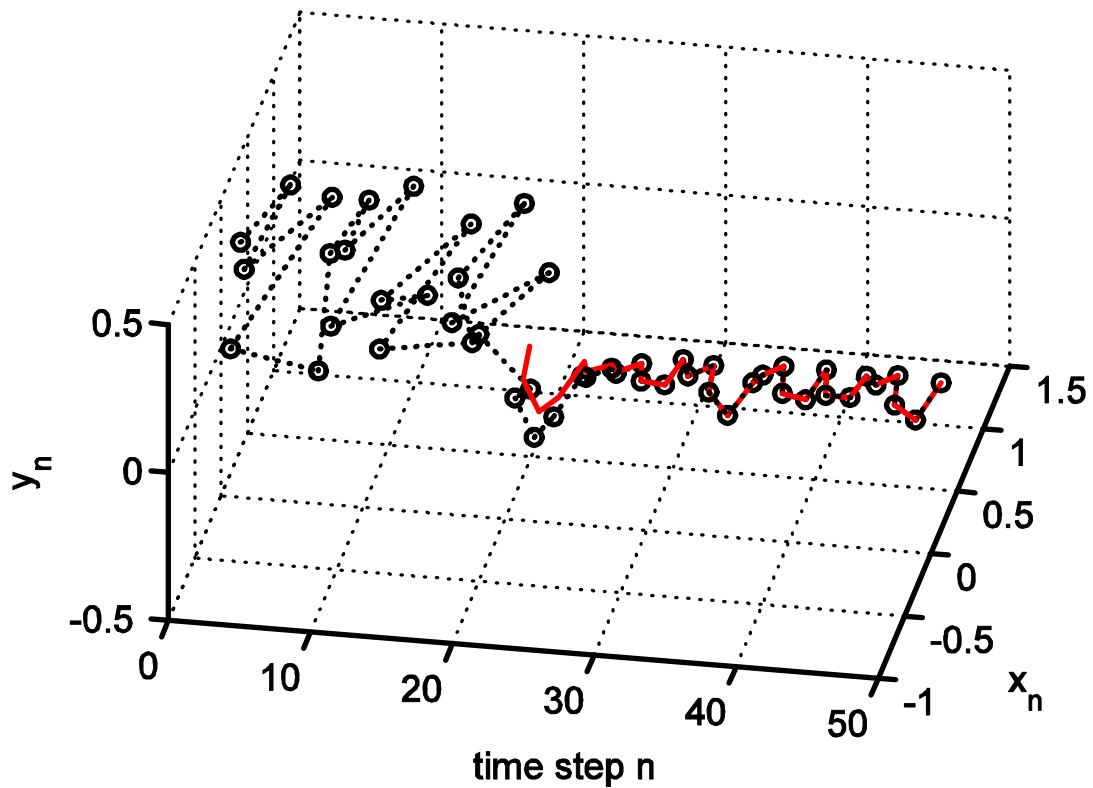


Figure 2. Controlling a chaotic leader-follower network [14]. This plot shows the state of the leader  $x_n$  and the state of the follower  $y_n$  versus time step  $n$ . Before time step  $n=20$  the chaotic network is uncontrolled and hard to predict, after time step  $n=20$  the chaotic network is controlled and predictable for the control unit. The red line is the chaotic target dynamics. In this computer simulation the chaotic network dynamics is very close to the target.