

Development of an Energy-budget Snowmelt Updating Model for Incorporating Feedback from Snow Course Survey Measurements

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Abstract

A novel model for updating snowmelt estimates utilizing sparse, yet high quality snow course survey (SCS) measurements for the purpose of improving long term stream flow forecasts is presented. The paper provides details on the development of the mathematical scheme of the model from first principles of energy and mass balance of the snowpack. The development methodology represents a successful attempt at the challenging task of explicitly tying a complex daily highly nonlinear energy-budget snowmelt function to the irregular SCS measurements. The updating scheme facilitates the updating of sensitive snowmelt parameters that have a long-term seasonal impact on snowmelt runoff. The updating model is integrated within a river flow forecasting model, the UBC Watershed Model, for the purpose of improving seasonal stream runoffs. Preliminary application of the updating model shows significant improvements in both the snowpack and stream flow forecasts.

Key words: snowmelt; snow; modeling; snow water equivalent; runoff mechanisms; hydrology; snowpack; snow course survey; watershed model; water

1 Introduction

From the massive glacial fed rivers of the Indus in Pakistan and the Peace in Canada to the smaller rivers in many parts of the world, snowmelt constitutes a significant component of stream runoff both in quantity and time distribution. In fact, accumulations of snow known as snowpack act as a natural reservoir and regulator of stream runoffs. Due to its packing and reflective characteristics a snowpack builds up during the winter season trapping significant amounts of precipitation. With the advent of warmer temperatures in the spring that could continue well into the summer season, the snowpack releases its water content over a period of days to months depending on its size and prevalent meteorological, topographical and landscape conditions of temperature, wind, sun radiation, tree cover, aspect, etc.

In many parts of the world, the releases of snowmelt sustain large urban and agricultural activities during the drier and in many cases rainless seasons (see Flerchinger and Cooley (2000), Marks and Winstral (2001), and Archer (2003)). On the other hand, excessive snowmelt runoff due to prolonged rises in temperature and/or intense rain on snow events can pose threat to dams and other water works, spill over river banks, and potentially result in loss of life and property (see Marks et al. (1998)). It is therefore of utmost importance for an efficient, reliable and safe water resources management and planning to understand the snow accumulation and melt phenomenon and develop appropriate tools to assess its capacity and estimate its contribution to stream runoffs.

Typically, snowmelt contribution to runoff is computed within the larger framework of a streamflow forecasting (watershed) model, such as the UBC Watershed Model (UBCWMM), which incorporates and integrates the overall roles of the principal hydrometeorological processes including precipitation, evapotranspiration, snowmelt, surface, interflow and groundwater movement (Singh and Woolhiser (2002)). These models are essential tools in the practice of water resources planning and management for the long and short-term forecasting. The quality of runoff estimations generated by these models vary widely and is not easily associated with a particular hydrological component. However, in mountainous regions, specific consideration to snowmelt is critical for a successful application of a streamflow forecasting system.

Two main approaches are applied in these models to calculate snowmelt: the energy budget and temperature index methods (Ferguson (1999)). The former is based on applying basic heat transfer principles to simulate the heat-exchange across the snowpack interface and keep account of its energy balance to estimate potential releases of snowmelt. Due to its physically based approach, this method was considered more representative and reliable, yet arguably less practical due to the requirement of detailed and hard to obtain information. The latter was developed on the premise that snowmelt/accumulation process can be accurately captured in terms of indices based on daily temperatures. Although widely applied due to its simplicity and light data requirements, it has been criticized for oversimplification and relaxation of data requirements at the expense of the reliability of snowmelt estimates (Walter et al. (2005)).

Irrespective of the approach used in modelling the snowmelt process, reference measurements including snow course survey (SCS) measurements, satellite images of snowpack, snow pillow real-time readings are important feedback information for assessing the quality and potentially updating snowmelt estimates. Although each type of snowpack reference information has its merits and value in estimating snowmelt, SCS measurements are particularly superior due to the meticulous data gathering method used to ensure a good representation of snowpack water content in a given area.

Snow course surveys are based on averaging the snow water equivalent (SWE), defined as the volume of equivalent mass of water, of several snow core samples collected along a course selected to represent a given area and minimize the effect of wind redistribution of snowfall (Singh and Singh (2001)). These elaborate measurements are typically conducted every two to four weeks during periods of active snowpack accumulation and depletion. They therefore provide valuable information on the state of the snowpack and snowmelt potential (Winstral and Marks (2004)).

A computer model has been developed to update critical parameters of an energy-budget snowmelt model utilizing the irregularly paced SCS measurements. The updating model is based on a mathematical function formulated to calculate adjustments in these parameters required to match calculated SWEs with measured ones. The updating model is integrated into the UBCWM, which is used primarily in seasonal river flow forecasting for mountainous watersheds.

This paper presents the theoretical basis and mathematical development of the updating model. The paper also presents the results from a preliminary application of the snowpack updating model on the seasonal river flow forecasts of the Illecillewaet River basin in British Columbia, Canada.

2 Energy and mass balances of the snowpack

Although the process of the transformation of water from its solid state (snow or ice) into liquid is well understood and analytically solvable, the generation of snowmelt from a snowpack is rather complex due to the evolving nature of the snowpack which affects its heat transfer properties and the highly variable natural heat transfer mechanism that involves several interlinked meteorological, topographical and landscape conditions.

The snowpack heat transfer and mass balances can be conceptually analyzed based on the control volume approach, where the snowpack is considered as a free body encompassed by an imaginary control surface that delineates it from the surrounding environment (see Fig. 1). In the following sections, the heat and mass balance of the snowpack is assessed via analyzing the heat and mass exchange across the control surface.

2.1 Energy balance of the snowpack

The energy balance of the snowpack is dictated by several heat exchange processes. The snowpack absorbs solar shortwave radiation that is partially blocked by cloud cover and reflected by snow surface. A longwave heat exchange takes place between the snowpack and its surrounding environment that includes overlaying air mass, tree cover and clouds. Convective (sensible) heat exchange between the snowpack and the overlaying air mass is governed by the temperature gradient and wind speed. Moisture exchange between the snowpack and the overlaying air mass is accompanied with latent heat transfer that is influenced by vapor pressure gradient and air wind. Rain on snow

Snowpack Heat & Mass Balance

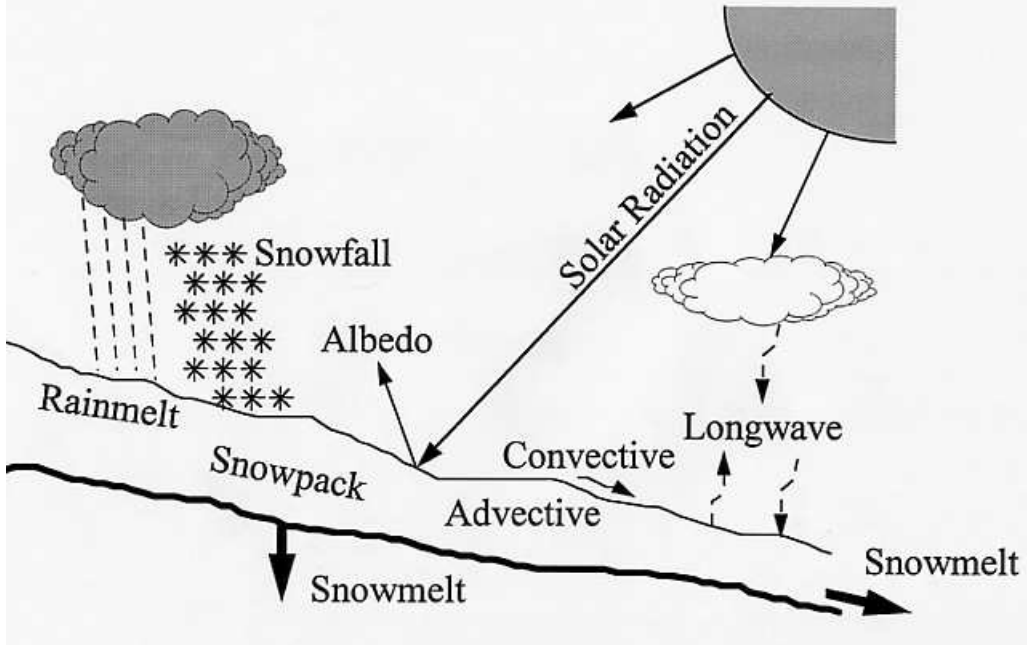


Fig. 1. Snowpack Heat and Mass Balance

could induce significant heat input to the snowpack. A generally insignificant conductive heat exchange takes place between the snowpack and the underlying ground.

Theoretically, a snowpack has to reach a $0\text{ }^{\circ}\text{C}$ isothermal condition before it starts to melt. At subfreezing temperatures, a snowpack has a negative heat storage or internal energy, U_i , defined as the amount of energy required to raise the snowpack temperature to $0\text{ }^{\circ}\text{C}$. With increasing heat input, U_i will increase to a maximum value of zero at temperature $0\text{ }^{\circ}\text{C}$. Any additional heat input will then go towards melting the snowpack. Taking into account the most significant heat exchange processes, the rate of change of U_i can be expressed as follows (Gray and Prowse (1992)):

$$\frac{\partial U_i}{\partial t} = Q_s + Q_l + Q_h + Q_e + Q_a + Q_g - Q_m \quad (1)$$

where

- Q_s is the net flux (energy per unit surface area per unit time) of incoming shortwave insolation;
- Q_l is the net flux of longwave heat exchange between snowpack and surrounding objects;
- Q_h is the flux of convective(sensible) heat exchange between snowpack and overlying air mass;
- Q_e is the flux of latent heat exchange via vapor exchange through condensation and sublimation;
- Q_a is the flux of advective energy induced by rain on snow;
- Q_g is the flux of conductive energy exchange between snowpack and underlying ground; and
- Q_m is the energy available for snowmelt, which becomes nonzero if the internal energy U reaches a value of zero (at temperature $0\text{ }^{\circ}\text{C}$).

Although the shortwave and longwave radiation components tend to dominate in most cases, the relative importance of each heat transfer mechanism can vary considerably from one area to another and over different times of the day and year. The heat balance is dictated by several factors including the snowpack and site characteristics related to climate, topography, orientation, latitude, altitude, tree cover, as well as time of the day and year. A brief description of each heat transfer mechanism is presented below.

Shortwave radiation Although the intensity of solar radiation normal to the Earth's atmospheric perimeter is constant at approximately 1.35 kJ/m^2 per second (solar constant), the actual amount that arrives at any given point on Earth, is drastically reduced. Over 50% of radiation is reflected by cloud cover, scattered by air molecules and air-borne particles, and absorbed by air compounds of ozone, water vapor, carbon dioxide and nitrogen (Gray and Prowse (1992)). Incident solar radiation on the snowpack surface is further attenuated with respect to that on a horizontal surface as function of local factors including land slope, aspect, exposure, latitude, time of the year, and the ratio of diffuse to direct-beam radiation. Forest cover further blocks direct sun radiation.

A significant portion of shortwave radiation could be reflected by the snowpack surface. The reflectivity of snowpack is generally assessed by its albedo (A), defined as the percentage of reflected shortwave radiation. Albedo of fresh snow can reach as high as 90%, but can deteriorate to as low as 30% during melting season. Snow albedo can be significantly reduced by forest dirt, debris, sand, and other material.

Longwave radiation A portion of shortwave energy absorbed by the snowpack is emitted back to overlying air and surrounding as a longwave radiation. This outgoing longwave energy is balanced by a longwave radiation from cloud cover and canopy that originated as shortwave energy absorbed by these objects. The net longwave energy flux is significant in forested areas and during cloudy periods as cloud cover and canopy reflects back most of the energy absorbed earlier as shortwave. In open areas in contrast significant outgoing longwave radiation during nighttime results in the cooling of the snowpack and delaying the release of snowmelt.

The snowpack behaves as a near ideal radiator (black body) and accordingly, its longwave energy emission can be described by Stefan-Boltzmann equation:

$$Q_{ls} = \epsilon\sigma(T_s + 273)^4 \quad (2)$$

where

Q_{ls} is the longwave energy flux emitted by the snowpack in $\frac{\text{J}}{\text{m}^2\text{s}}$ (joules per square meter per second);
 ϵ is the emissivity of the snowpack, which ranges from 0.97 (dirty snow) to 0.99 (fresh snow) (Gray and Prowse (1992)) ($\epsilon = 1$ for a black body);
 σ is Stefan-Boltzmann constant ($5.735 \times 10^{-8} \frac{\text{J}}{\text{m}^2\text{sK}^4}$); and
 T_s is the snowpack surface temperature in Celsius degrees ($^{\circ}\text{C}$).

For cloudless conditions in open and unforested areas, air-borne particles and air constituents (within the first 100 meter layer) emit longwave energy downwards into the snowpack. The air mass can be treated empirically as a gray body with a portion of the longwave emission attributed to water vapor. Accordingly many of the open-sky longwave radiation equations reported in the literature incorporate parameters representing air humidity (Quick (1995)). For example, Anderson (1954) presents the following equation for open-sky longwave radiation as function of vapor pressure and temperature:

$$Q_{la} = \sigma(T_a + 273)^4(0.749 + 0.0049e_a) \quad (3)$$

where

Q_{la} is the longwave energy flux emitted by the open sky air downwards in $\frac{\text{J}}{\text{m}^2\text{s}}$;
 σ is the Stefan-Boltzmann constant;
 e_a is the vapor pressure (millibars); and
 T_a is the atmosphere temperature in Celsius degrees ($^{\circ}\text{C}$).

Due to the relatively small dependance on humidity, Quick (1995) suggests simplifying equation to the following:

$$Q_{la} = \sigma(T_a + 273)^4(0.757) \quad (4)$$

Clouds act as black bodies and their longwave energy emissions can therefore be represented by Stefan-Boltzmann equation as follows:

$$Q_{lc} = \epsilon\sigma(T_c + 273)^4 \quad (5)$$

where

Q_{lc} is the longwave energy flux emitted by cloud cover in $\frac{\text{J}}{\text{m}^2\text{s}}$; and
 T_c is the cloud temperature in Celsius degrees ($^{\circ}\text{C}$).

Convective and latent heat (turbulent) exchange A temperature gradient above the snowpack results in a convective heat exchange that could be greatly accelerated under high wind velocities. Moisture transfer either to the snowpack via condensation or out of the snowpack by sublimation results in a latent heat gain or loss, respectively. Latent heat transfer is also affected by wind conditions. The relative significance of convective and latent heat melt varies dramatically. The melt contribution of these processes is relatively low under clear warm weather due to the stability of the overlying warm air layer. In contrast, their contribution can be very significant under windy conditions or during winter rain on snow events. Quick (1995) presents the following equations for estimating the convective and latent heat fluxes:

$$Q_h = 0.438R_M \left(\frac{p}{101} \right) T_a v_w \quad (6)$$

$$Q_e = 1.706R_M T_a v_w \quad (7)$$

$$R_M = 1 - 7.7R_I \quad (8)$$

$$R_I = 0.095 \frac{T_a}{v_w^2} \quad (9)$$

where

Q_h is the convective heat flux in $\frac{J}{m^2s}$;

Q_e is the latent heat flux in $\frac{J}{m^2s}$;

R_M is a dimensionless reduction factor;

R_I is the bulk Richardson number which is a measure of stability of air mass;

p is the atmospheric pressure in kilopascals;

T_a is air mean temperature in $^{\circ}C$; and

v_w is the wind speed in m/s .

Rain melt For rain on a melting snowpack, the advective heat released by the rain can be estimated as follows (USACE (1998)):

$$Q_a = C_p P_r (T_r - T_s) / 1000 \quad (10)$$

where

Q_a is the advective heat flux released by rain in $\frac{J}{m^2s}$;

C_p is specific heat of rain ($4.29 \frac{J}{g^{\circ}C}$);

P_r is the rainfall in mm/s ;

T_r is the temperature of rain ($^{\circ}C$); and

T_s is the snow temperature ($^{\circ}C$).

If rain falls on a subfreezing snowpack, a portion of it is expected to freeze and release its heat of fusion, which is generally difficult to determine due to lack of measurements and consequently not accounted for in the overall snowpack heat exchange (Gray and Prowse (1992)).

Conductive exchange with ground Due to its low thermal conductivity, snow reduces significantly heat exchange between the underlying ground and atmosphere. In most situations, the ground heat flux is quite small and can therefore be dropped out of the snowpack heat exchange equation (Gray and Prowse (1992)).

2.2 Mass balance of the snowpack

The mass balance of a snowpack is governed by the following basic equation:

$$\frac{\partial S_w}{\partial t} = P_s + R_f - ES - M_s \quad (11)$$

where

S_w is the snow water equivalent (SWE) of the snowpack;

P_s is the snowfall, which adds volume to the snowpack during;

R_f is the rainfall on the snowpack that has frozen;

ES is evaporation and sublimation; and

M_s is the snowmelt.

Under ideal conditions, a mass of pure ice under positive heat influx starts to melt at a constant temperature of 0 °C based on the heat of fusion formula:

$$M_{ice} = \frac{E_{in}}{H_{fw}\rho_w} \quad (12)$$

where

M_{ice} is the mass of melted ice;

E_{in} is the heat (energy) absorbed by the ice;

H_{fw} is the heat fusion of ice (energy per unit mass) defined as the amount of energy required to convert a unit mass of ice into water at 0 °C; and

ρ_w is water density (mass per unit volume).

Based on equations 1 and 12 the potential snowmelt M_p , defined as the volume of snow per unit area per unit time that could be melted by Q_m given availability of snow mass, can be calculated as follows:

$$\begin{aligned} M_p &= \frac{Q_m}{H_{fw}\rho_w B} \\ &= \frac{Q_m}{334.9 \frac{J}{g} \times 1000 \frac{kg}{m^3} \times B} \times 1000 \frac{mm}{m} \\ &= 2.986 \times 10^{-6} \frac{Q_m}{B} \end{aligned} \quad (13)$$

where

M_p is the volume of potential snowmelt in mm/s (1 mm is equivalent to a volume of 1 mm over an area of m^2 , equivalent to 1 liter);

Q_m is the energy available for snowmelt as defined in equation 1 in $\frac{J}{m^2 s}$;

H_{fw} is the heat fusion of ice equal to 334.9 $\frac{J}{g}$ (Joule/gram);

ρ_w is water density equal to 1000 $\frac{kg}{m^3}$; and

B is the thermal quality of the snow defined as the ratio of heat required to melt a unit mass of snow to that of ice at 0 °C, which ranges in value from 0.95 to 0.97 for a melting snowpack (USACE (1998));

2.3 Numerical representation of the heat and mass balance of the snowpack

Based on equations 1, 11 and 13, the heat/mass balance of a snowpack can be expressed in a numerical form as follows:

From equation 1

$$\frac{U_i(t) - U_i(t-1)}{\Delta t} = Q_s(t) + Q_l(t) + Q_h(t) + Q_e(t) + Q_a(t) + Q_g(t) - Q_m(t) \quad (14)$$

where t represents a time index, and Δt represents time increment between $t - 1$ and t . Rearranging the terms results in:

$$U_i(t) = Q_x(t) - Q_m(t)\Delta t \quad (15)$$

where

$$Q_x(t) = U_i(t-1) + [Q_s(t) + Q_l(t) + Q_h(t) + Q_e(t) + Q_a(t) + Q_g(t)] \Delta t \quad (16)$$

A negative $Q_x(t)$ indicates that the snowpack is in a subfrozen state; while a positive value indicates that the snowpack is in a melting state. Therefore:

- For a frozen snowpack ($Q_x(t) < 0$):

$$Q_m(t) = 0 \quad (17)$$

$$U_i(t) = Q_x(t) \quad (18)$$

- For a melting snowpack ($Q_x(t) > 0$):

$$Q_m(t) = \frac{Q_x(t)}{\Delta t} \quad (19)$$

$$U_i(t) = 0 \quad (20)$$

The above equations and the snowpack mass balance equation can be formulated into a numerical algorithm to estimate the snowmelt, and the internal energy and mass of the snowpack as follows:

- $Q_x(t)$ is calculated from energy fluxes and prior internal energy using equation 16
- If $Q_x(t) < 0.0$ then the snowpack is in a frozen state and:

$$U_i(t) = Q_x(t) \quad (21)$$

$$Q_m(t) = 0 \rightarrow M_s(t) = 0 \quad (22)$$

$$S_w(t) = S_w(t-1) + [P_s(t) + R_f(t) - ES(t)] \Delta t \quad (23)$$

- If $Q_x(t) > 0.0$ then the snowpack is in a melting state and $Q_m(t) = \frac{Q_x(t)}{\Delta t}$ based on equation 19. $M_p(t)$ can be readily calculated from $Q_m(t)$ based on equation 13. The melting snowpack can be in one of two states:
 - Snowpack mass is larger than potential snowmelt, $M_p(t)\Delta t < [S_w(t-1) + (P_s(t) + R_f(t) - ES(t))\Delta t]$:

$$U_i(t) = 0 \quad (24)$$

$$M_s(t) = M_p(t) \quad (25)$$

$$S_w(t) = S_w(t-1) + (P_s(t) + R_f(t) - ES(t) - M_s(t))\Delta t \quad (26)$$

- Snowpack mass is less than potential snowmelt, $M_p(t)\Delta t > [S_w(t-1) + (P_s(t) + R_f(t) - ES(t))\Delta t]$:

$$U_i(t) = 0 \quad (27)$$

$$M_s(t) = \frac{1}{\Delta t} [S_w(t-1) + (P_s(t) + R_f(t) - ES(t))\Delta t] \quad (28)$$

$$S_w(t) = 0 \quad (29)$$

3 Snowmelt routine in the UBC watershed model

The UBC Watershed Model (UBCWMM) is a conceptual model designed to estimate river runoffs based on the simulation of the hydro-meteorological processes of precipitation, evapotranspiration, surface runoff, interflow, percolation, and groundwater movement. The application of UBCWMM has been successful in several mountainous watersheds across the world, most notably in Western Canada, Himalayas in India and Pakistan and South America (Quick (1995))

The UBCWMM is particularly suited for mountainous watersheds with minimal data. The model relies on dividing the watershed into several elevation zones (bands). Each band is assigned meteorological variables of precipitation and temperatures calculated from corresponding measurements based on gradients representing orographic temperature and precipitation effects.

Operating per elevation zone, the UBCWMM estimates rain and snow precipitations based on total precipitation and temperature. A snowmelt routine keeps account of the snowpack accumulation and depletion based on an energy budget approach as described in Section 2. The sum of rainfall and precipitation less evapotranspiration are processed through a priority allocation system to fast, interflow and groundwater runoff. Fast runoff is routed over

impervious areas, and after accounting for percolation into groundwater, the remaining flow is routed as a medium interflow runoff.

One of the main features that sets the UBCWM apart from many of its counterparts is the comprehensive and detailed energy-budget snowmelt routine, which was tested and calibrated against the elaborate snow hydrology study carried out by the US Army Corps of Engineers (USACE (1956)). The algorithm of the UBCWM snowmelt routine is described in the following sections.

3.1 Estimation of energy fluxes

Components of the snowpack heat exchange are expressed in a set of formulas based on the USACE "Snow Hydrology" research findings and studies (USACE (1956)). Assuming minimal availability of meteorological data, all equations are expressed in terms of one or more of the three commonly available meteorological data variables: precipitation and maximum and minimum temperatures. The model is based on daily calculation of snowmelt, assuming a heat fusion of ice equal to $334.9 \frac{J}{g}$ and a snowpack thermal quality, $B = 1$.

Shortwave radiation Considering blocking by cloud cover and reflection by snow surface, the net shortwave radiation absorbed by the snowpack is estimated as follows:

$$Q_s(t) = I_s(t) [1 - C(t)] [1 - A(t)] \quad (30)$$

where C is the fraction of cloud cover; A is the snow albedo; and I_s is the incident solar radiation.

Longwave radiation For an open sky day, the net longwave radiation Q_{lo} is the difference between longwave radiation by the snowpack Q_{ls} and that emitted downward into the snowpack by the overlying air mass, Q_{la} as represented by equations 2 and 4, respectively. The temperature term, $(T + 273)^4$ in these longwave emission equations can be expanded as follows:

$$(T + 273)^4 = 273^4 \left[1 + \frac{4T}{273} + \frac{6T^2}{273^2} + \frac{4T^3}{273^3} + \frac{T^4}{273^4} \right] \quad (31)$$

Since temperatures encountered in snowmelt setting are significantly less than the absolute temperature of 273 Kelvins, the last three terms on the righthand side of equation 31 have generally very small values and can be therefore left out without changing the estimate of longwave emission beyond the margin of error. Consequently:

$$\begin{aligned} (T + 273)^4 &\approx 273^4 \left[1 + \frac{4T}{273} \right] \\ &\approx 273^4 [1 + 0.015T] \end{aligned} \quad (32)$$

Based on equations 2, 4, and 32, and assuming $\epsilon = 1$, the net longwave radiation for an open sky day, Q_{lno} , can be estimated as follows:

$$\begin{aligned} Q_{lno}(t) &= Q_{la}(t) - Q_{ls}(t) \\ &= 0.757\sigma(T_a(t) + 273)^4 - \epsilon\sigma(T_s(t) + 273)^4 \\ &= 273^4\sigma [0.757(1 + 0.015T_a(t)) - (1 + 0.015T_s(t))] \\ &= 3.62T_a(t) - 4.78T_s(t) - 77.41 \quad \text{in } J/m^2s \end{aligned} \quad (33)$$

Based on equations 2, 5, and 32, and assuming $\epsilon = 1$, the net longwave radiation for a cloudy day, Q_{lnc} , can be estimated as follows:

$$\begin{aligned} Q_{lnc}(t) &= Q_{lno}(t) - Q_{ls}(t) \\ &= \epsilon\sigma(T_c(t) + 273)^4 - \epsilon\sigma(T_s(t) + 273)^4 \\ &= 273^4\sigma [(1 + 0.015T_c(t)) - (1 + 0.015T_s(t))] \\ &= 4.78(T_c(t) - T_s(t)) \quad \text{in } J/m^2s \end{aligned} \quad (34)$$

For a partially cloudy day equations 33 and 34 can be combined and weighted based on the cloud cover fraction C as follows:

$$\begin{aligned} Q_l(t) &= Q_{lno}(t)(1 - C(t)) + Q_{lnc}(t)C(t) \\ &= (3.62T_a(t) - 77.41)(1 - C(t)) + 4.78(T_c(t) - T_s(t))C(t) \end{aligned} \quad (35)$$

For a melting snowpack, $T_s = 0$, equation 35 can be expressed as

$$Q_l(t) = (3.62T_a(t) - 77.41)(1 - C(t)) + 4.78T_c(t)C(t) \quad (36)$$

Equation 36 can be expressed in terms of equivalent mm/day as follows:

$$Q_l(t) = (0.94T_a(t) - 20.10)(1 - C(t)) + 1.24T_c(t)C(t) \quad (37)$$

Convective and Latent heat exchange The convective and latent heat transfer are calculated based on equations 6 and 7, respectively. These equations can be expressed in terms of mm/day as follows:

$$Q_h(t) = 0.113R_M(t)\left(\frac{p}{101}\right)T_a(t)v_w(t) \quad (38)$$

$$Q_e(t) = 0.44R_M(t)T_a(t)v_w(t) \quad (39)$$

where R_M is as defined in equations 8 and 9.

Rain melt Melt due to rain on snow is calculated as shown in equation 10.

Conductive ground heat exchange due to its insignificant contribution to the overall heat exchange process, heat exchange with underlying ground is not considered in the UBCWM snowmelt routine.

3.2 Estimation of critical meteorological variables

As mentioned earlier, the main reason for the restricted practical application of the energy budget approach is the general lack information on critical meteorological variables especially in mountainous and rugged areas, where calculation of snowmelt is most important. These variables include cloud cover, snow albedo and snowfall. For sites where no information on these variables is available and only typical data on temperature and precipitation is available, the UBCWM snowmelt relies on a set of temperature-based equations for estimating these variables (Quick (1995)). These equations are described in the following sections.

Cloud Cover Due to the moderating effect of cloud cover on diurnal temperature range, cloud cover can be indexed by the difference between daily maximum and minimum temperatures. Cloud cover can be estimated as follows:

$$C(t) = 1 - \frac{T_x(t) - T_n(t)}{\Delta T_s} \quad (40)$$

where ΔT_s is the daily temperature range for open sky conditions at a given elevation.

Snow Albedo Snow albedo is estimated based on a recession model with the snowpack assigned a value of 0.90 immediately following a fresh snowfall with a volume higher than a given threshold (e.g. 15 mm). The albedo is then calculated on a daily basis using the following decay equation:

$$A(t) = 0.9A(t-1) \quad \text{for } A(t) > A_{sv} \quad (41)$$

Upon reaching the settled value A_{sv} , albedo is then calculated using the following exponential decay equation:

$$A(t) = A_{sv} \exp\left(-\frac{M_c}{K_L}\right) \quad \text{for } A(t) > 0.3 \quad (42)$$

where M_c is the cumulative seasonal snowmelt in millimeters, and K_L is a constant selected in the order of the total seasonal melt.

Snow Formation Function In most cases, only the total volume of precipitation is measured with no specific reference to the form of precipitation of rainfall or snow. The UBCWM estimates that all precipitation, PP , is snow, P_s , if the average temperature, T_a is less than $0.0^\circ C$. If T_a is higher than a threshold temperature, T_r , then all precipitation is rain, P_r . If $(0.0 \leq T_a \leq T_r)$ then snowfall is calculated as follows:

$$P_s(t) = \left[1 - \frac{T_a(t)}{T_r} \right] PP(t) \quad (43)$$

3.3 The algorithm of the UBCWM snowmelt routine

The UBCWM snowmelt routine is based on an energy-budget approach, where snowpack internal energy and mass and snowmelt are estimated on a daily basis utilizing three basic measured meteorological variables: daily maximum temperature (T_x), minimum temperature (T_n) and total precipitation (PP). The algorithm of the UBCWM snowmelt is described in the following bullets:

- (1) The average temperature is calculated as the mean of the maximum and minimum temperatures:

$$T_a(t) = \frac{T_x(t) + T_n(t)}{2} \quad (44)$$

- (2) The snowfall, $P_s(t)$, is estimated from the total precipitation and average temperature using the snow formation function described in section 3.2.
- (3) The cloud cover fraction, $C(t)$, is calculated based on diurnal temperature range according to equation 40. The snow albedo, $A(t)$, is calculated according to the decay-based functions described in section 3.2.
- (4) The daily shortwave, longwave, convective, latent and rain melt heat fluxes are calculated as described in section 3.1.
- (5) The energy cumulative energy term, $Q_x(t)$, defined in section 2.3 is calculated based on the following modified version of equation 16:

$$Q_x(t) = \lambda U_i(t-1) + Q_n(t) \quad (45)$$

where λ , with a value ranging from 0 to 1, is introduced to represent unaccounted for warming of the snowpack during the day; and

$$Q_n(t) = Q_s(t) + Q_l(t) + Q_h(t) + Q_e(t) + Q_a(t) \quad (46)$$

Δt is dropped from equation 45 and further equations since all rate (per unit time) variables (heat fluxes, precipitation, melt) are expressed in terms of daily values.

- (6) Based on the value of $Q_x(t)$, the thermal state of snowpack is determined and its internal energy, $U_i(t)$, mass, $S_w(t)$, and snowmelt, $M_s(t)$ are estimated along the lines of the numerical formulation introduced in section 2.3 as follows:

- If $Q_x(t) > 0.0$, then heat exchange leads to a surplus that goes into melting snow ($Q_m(t) = Q_x(t)$). $Q_m(t)$ can be then converted into equivalent potential melt, $M_p(t)$.

$$\left. \begin{array}{l} U_i(t) = 0.0 \\ M_s(t) = M_p(t) \\ S_w(t) = S_w(t-1) + P_s(t) - M_s(t) \end{array} \right\} \text{if } M_p(t) < S_w(t-1) + P_s(t) \quad (47)$$

$$\left. \begin{array}{l} U_i(t) = 0.0 \\ M_s(t) = S_w(t-1) + P_s(t) \\ S_w(t) = 0.0 \end{array} \right\} \text{if } M_p(t) < S_w(t-1) + P_s(t) \quad (48)$$

- If $Q_x(t) > 0.0$, then the snowpack is still in frozen state, and:

$$U_i(t) = Q_x(t); M_s(t) = 0.0 \quad \text{and} \quad S_w(t) = S_w(t-1) + P_s(t) \quad (49)$$

- The frozen rain, $R_f(t)$ and the evaporation/sublimation rate, $ES(t)$ have been dropped from the above mass balance equations since they are accounted for in other modules of the UBCWM. The former is represented as rain in the UBCWM, while the latter is considered at a later stage in the model for the combined snowmelt and rainfall.

4 The snowpack updating approach

In general, watershed models rely on streamflow measurements for calibration, verification and updating. However, snowpack measurements are ideal for longer term calibration and updating due to the long lag time between accumulation of snowfall precipitation and snowmelt. Several types of measurements are available including satellite images, snow pillow real-time readings, and SCSs (Singh and Singh (2001)).

As mentioned in an earlier section, SCSs are generally representative of a certain area in the basin since they are conducted at several locations over an area, rather than a single point, to reduce the influence of wind redistribution of snowfall. Also, the course is selected in an open area away from large objects and canopy. However, the irregularity and low frequency of snow course measurements make their use in automated calibration and updating of regular (mostly daily) snowmelt routines quite challenging. In this paper, a mathematical routine has been developed to automatically update critical snowpack parameters utilizing SCS measurements as they become available.

4.1 The snowpack updating parameters

To facilitate the updating of daily snowpack calculations based on irregular snow course measurements, four parameters are introduced into the UBCWM snowmelt routine to modify the calculation of three of the most important variables: cloud cover fraction, snow albedo, and snowfall formation as follows:

Cloud Cover Fraction As discussed earlier, cloud cover is an important element in the shortwave and longwave heat fluxes. It tends to attenuate diurnal fluctuations in heat exchanges and has a longer term warming impact during cooler times of the year and a cooling effect during warmer times. Therefore, adjusting its value is expected to alter the timing and distribution of snowmelt. Two methods are proposed to modify the cloud cover:

- The daily temperature range for open sky conditions, ΔT_s , can be treated as a variable whose value can be adjusted to modify the estimate of cloud cover based on equation 40.
- A new parameter, η_c , is introduced into equation 40 to modify cloud cover as follows:

$$C(t) = \eta_c \left[1 - \frac{T_x(t) - T_n(t)}{\Delta T_s} \right] \quad (50)$$

Snow Albedo Snow albedo plays an important part in the deflection of shortwave heat radiation which is generally the major heat flux into the snowpack. A new parameter, η_a , is introduced to modify snow albedo in the shortwave heat flux equation (30) as follows:

$$Q_s(t) = I_s(t) [1 - C(t)] [1 - \eta_a A(t)] \quad (51)$$

Snowfall The hydrological response of a watershed system is dependant, among other factors, on the form of precipitation either as snowfall or rainfall. Higher snowfall contribution to the total precipitation results in a seasonal attenuation of stream runoffs. The contribution of snowfall in the UBCWM can be modified by treating the threshold temperature, T_r , in the snowfall formation equation (43) as a variable. Higher T_r results in higher snowfall contributions and vice versa.

4.2 The approach to developing the snowpack updating model

The objective of the proposed snowpack updating model is to update the values of the four aforementioned parameters (ΔT_s , η_c , η_a and T_r) to achieve a match between that SWE calculated by the UBCWM and that obtained from a SCS. However, the relationship between the two quantities is highly non-linear and not easily derivable due to the complexity of the snowpack model and the mismatch between the daily frequency of the UBCWM calculations and the variable multi-day span between SCS measurements.

This technical hurdle was overcome by developing explicit relationships between each of the snowpack updating parameters and consecutive SCS measurements, which have formed the basis for a snowpack updating model. The approach used in deriving these relationships can be outlined in the following three steps, which will be elaborated in later sections:

- (1) First, an explicit relationship is derived between the snowpack mass at a given time, t , on one side and the sum of net energy fluxes, $\sum_{i=t-k+1}^t Q_n(i)$, and the sum of snowfall, $\sum_{i=t-k+1}^t P_s(i)$, over a given period of k days and the snowpack mass at time, $t - k$, on the other side:

$$S_w(t) - S_w(t - k) = F\left(\sum_{i=t-k+1}^t Q_n(i), \sum_{i=t-k+1}^t P_s(i)\right) \quad (52)$$

- (2) Second, for each updating parameter (ΔT_s , η_c , η_a and T_r), a function is developed between a selected parameter on one side and $\sum Q_n$ and $\sum P_s$ on the other side.
- (3) Finally, for each updating parameter, the equation obtained from step(1) is combined with the one derived for the parameter through step(2) to obtain an explicit function of the parameter in terms of a given pair of two consecutive snowpack measurements.

4.3 Step 1: Deriving an explicit multi-day relationship between snowpack mass and heat fluxes and snowfall

The snowpack mass in the UBCWM snowmelt routine (section 3.3) is calculated based on a daily recursive function of previous mass, net energy fluxes and snowfall. The daily recursive approach is computationally convenient and efficient, since only previous day variables are only kept account of reducing the structural complexity of the model and computational overhead. However, it can not be directly associated with multi-day variables, which is a necessary requirement for devising an automated updating approach based on the sporadic SCS measurements.

As step 1 in the development of the proposed snowpack updating model (section 4.2), an explicit function has been derived between the snowpack mass at a given time, t , and sum of the accumulated net heat fluxes and snowfall over a period of k days and snowpack mass at time $t - k$. The mathematical development of this function is presented below:

- Let k be the measurement interval for two consecutive snowpack measurements $\tilde{S}_w(t)$ and $\tilde{S}_w(t_k)$, and let $\kappa_f(i)$, $\kappa_p(i)$, and $\kappa_a(i)$ for $t - k \leq i \leq t$ be the snowpack thermal condition tags defined as follows:
 - For a frozen snowpack, i.e., $Q_x(i) \leq 0.0$:
$$\kappa_f(i) = 1, \kappa_p(i) = \kappa_a(i) = 0.0$$
 - For a melting snowpack, $Q_x(i) > 0.0$:
$$\left. \begin{array}{l} \kappa_p(i) = 1.0 \\ \kappa_a(i) = 0.0 \end{array} \right\} \text{For } Q_x(i) < S_w(i-1) + P_s(i)$$

$$\left. \begin{array}{l} \kappa_p(i) = 0.0 \\ \kappa_a(i) = 1.0 \end{array} \right\} \text{For } Q_x(i) > S_w(i-1) + P_s(i)$$
 - Let $\kappa_{fp}(i) = \kappa_f(i) + \kappa_p(i)$, then $\kappa_{fp}(i) = 1.0$ indicates that the snowpack is either frozen or melting.

- Based on the tags defined above and the UBCWM snowmelt routine described in section (3.3), the snowpack internal energy and mass, and the snowmelt can be expressed as follows:

The snowpack internal energy

$$\begin{aligned} U_i(t) &= \kappa_f(t)Q_x(t) + \kappa_p(t)0.0 + \kappa_a(t)0.0 \\ &= \kappa_f(t)Q_x(t) \end{aligned} \quad (53)$$

The snowpack mass

$$\begin{aligned} S_w(t) &= \kappa_f(t)[S_w(t-1) + P_s(t)] + \kappa_p(t)[S_w(t-1) + P_s(t) \\ &\quad - Q_x(t)] + \kappa_a(t)0.0 \end{aligned}$$

which results in:

$$S_w(t) = \kappa_{fp}(t)P_s(t) - \kappa_p(t)Q_x(t) + \kappa_{fp}(t)S_w(t-1) \quad (54)$$

where $\kappa_{fp}(t) = \kappa_f(t) + \kappa_p(t)$

The snowmelt

$$M_s(t) = \kappa_f(t)0.0 + \kappa_p(t)Q_x(t) + \kappa_a(t)[P_s(t) + S_w(t-1)]$$

which results in:

$$M_s(t) = \kappa_p(t)Q_x(t) + \kappa_a(t)[P_s(t) + S_w(t-1)] \quad (55)$$

- A relationship defining snowpack mass at a given time in terms of internal energy, accumulated snowfall and net heat fluxes over a period of several past days is derived as follows:

From equation 54, for $k = 1$:

$$\begin{aligned} S_w(t) &= \kappa_{fp}(t)P_s(t) - \kappa_p(t)[\lambda\kappa_f(t-1)U_i(t-1) + Q_n(t)] \\ &\quad + \kappa_{fp}(t)S_w(t) \end{aligned} \quad (56)$$

Replacing the argument t with $t - 1$ in equation 56 results in:

$$\begin{aligned} S_w(t-1) &= \kappa_{fp}(t)P_s(t-1) - \kappa_p(t-1)[\lambda\kappa_f(t-2)U_i(t-2) \\ &\quad + Q_n(t-1)] + \kappa_{fp}(t-1)S_w(t-2) \end{aligned}$$

Substituting the above equation into equation 56 yields:

For $k = 2$

$$\begin{aligned} S_w(t) &= \kappa_{fp}(t)P_s(t) + \kappa_{fp}(t)\kappa_{fp}(t-1)P_s(t-1) - \kappa_p(t)Q_n(t) \\ &\quad - \kappa_{fp}(t)\kappa_p(t-1)Q_n(t-1) - \lambda\kappa_p(t)\kappa_f(t-1)U_i(t-1) \\ &\quad - \lambda\kappa_{fp}(t)\kappa_p(t-1)\kappa_f(t-2)U_i(t-2) \\ &\quad + \kappa_{fp}(t)\kappa_{fp}(t-1)S_w(t-2) \end{aligned} \quad (57)$$

Equation 57 can be generalized for $k \geq 2$:

$$\begin{aligned}
S_w(t) &= \sum_{i=t-k+1}^t \left(\prod_{j=i}^t \kappa_{fp}(j) \right) P_s(i) - \kappa_p(t) Q_n(t) \\
&\quad - \sum_{i=t-k+1}^{t-1} \left(\prod_{j=i+1}^t \kappa_{fp}(j) \right) \kappa_p(i) Q_n(i) \\
&\quad - \lambda \kappa_p(t) \kappa_f(t-1) U_i(t-1) \\
&\quad - \sum_{i=t-k}^{t-2} \lambda \left(\prod_{j=i+2}^t \kappa_{fp}(j) \right) \kappa_p(i+1) \kappa_f(i) U_i(i) \\
&\quad + \left(\prod_{i=t-k+1}^t \kappa_{fp}(i) \right) S_w(t-k)
\end{aligned} \tag{58}$$

- Expressing internal energy in terms of heat fluxes:

From equation 53:

$$\begin{aligned}
U_i(t) &= \kappa_f(t) Q_x(t) \\
&= \kappa_f(t) [\lambda \kappa_f(t-1) U_i(t-1) + Q_n(t)]
\end{aligned} \tag{59}$$

Replacing t with $t-1$ in equation 59 yields:

$$U_i(t-1) = \kappa_f(t-1) [\lambda \kappa_f(t-2) U_i(t-2) + Q_n(t-1)] \tag{60}$$

Substituting equation 60 into equation 59 results in:

$$\begin{aligned}
U_i(t) &= \lambda^2 \kappa_f(t) \kappa_f(t-1)^2 \kappa_f(t-2) U_i(t-2) + \kappa_f(t) Q_n(t) \\
&\quad + \lambda \kappa_f(t) \kappa_f(t-1)^2 Q_n(t-1)
\end{aligned} \tag{61}$$

For $t-k+1 \leq i \leq t$:

$$\begin{aligned}
U_i(i) &= \lambda^2 \kappa_f(i) \kappa_f(i-1)^2 \kappa_f(i-2) U_i(i-2) + \kappa_f(i) Q_n(i) \\
&\quad + \lambda \kappa_f(i) \kappa_f(i-1)^2 Q_n(i-1)
\end{aligned}$$

$U_i(i)$ can be expressed in terms of the snowpack variables from time i down to time $i-3$ as follows:

$$\begin{aligned}
U_i(i) &= \lambda^3 \kappa_f(i) \kappa_f(i-1)^2 \kappa_f(i-2)^2 \kappa_f(i-3) U_i(i-3) \\
&\quad + \kappa_f(i) Q_n(i) + \lambda \kappa_f(i) \kappa_f(i-1)^2 Q_n(i-1) \\
&\quad + \lambda^2 \kappa_f(i) \kappa_f(i-1)^2 \kappa_f(i-2)^2 Q_n(i-2)
\end{aligned}$$

In general, $U_i(i)$ can be expressed in terms of the snowpack variables from time i down to time $t-k$ as follows:

$$\begin{aligned}
U_i(i) &= \lambda^{i-t+k} \kappa_f(i) \kappa_f(t-k) \left(\prod_{j=t-k+1}^{i-1} \kappa_f(j)^2 \right) U_i(t-k) \\
&\quad + \kappa_f(i) [Q_n(i) + \sum_{j=t-k+1}^{i-1} \lambda^{i-j} \left(\prod_{l=j}^{i-1} \kappa_f(l)^2 \right) Q_n(j)]
\end{aligned}$$

$\kappa_f(i)^2 = \kappa_f(i)$ since $\kappa_f(i)$ is either 0.0 or 1.0. Therefore, the above equation can be written as follows:

$$\begin{aligned}
U_i(i) &= \lambda^{i-t+k} \left(\prod_{j=t-k}^i \kappa_f(j) \right) U_i(t-k) + \kappa_f(i) [Q_n(i)] \\
&+ \sum_{j=t-k+1}^{i-1} \lambda^{i-j} \left(\prod_{l=j}^{i-1} \kappa_f(l) \right) Q_n(j)
\end{aligned} \tag{62}$$

- Expressing snowpack mass at a given time in terms of internal energy, cumulative heat fluxes and snowfall over several days:

Substituting equation 62 into the fourth term of the right hand side of equation 58 (for $i = t - 1$) yields:

For $k > 2$

$$\begin{aligned}
\lambda \kappa_p(t) \kappa_f(t-1) U_i(t-1) &= \lambda^k \kappa_p(t) \left(\prod_{i=t-k}^{t-1} \kappa_f(i) \right) U_i(t-k) + \lambda \kappa_p(t) \kappa_f(t-1) [Q_n(t-1)] \\
&+ \sum_{i=t-k+1}^{t-2} \lambda^{t-i-1} \left(\prod_{j=i}^{t-2} \kappa_f(j) \right) Q_n(i)
\end{aligned} \tag{63}$$

For $k = 2$

$$\begin{aligned}
\lambda \kappa_p(t) \kappa_f(t-1) U_i(t-1) &= \lambda^2 \kappa_p(t) \kappa_f(t-1) \kappa_f(t-2) U_i(t-2) \\
&+ \lambda \kappa_p(t) \kappa_f(t-1) Q_n(t-1)
\end{aligned} \tag{64}$$

Substituting equation 62 into the fifth term of the right-hand side of equation 58 yields:

For $k > 3$

$$\begin{aligned}
&\sum_{i=t-k}^{t-2} \lambda \left(\prod_{j=i+2}^t \kappa_{fp}(j) \right) \kappa_p(i+1) \kappa_f(i) U_i(i) = \\
&+ [\lambda \left(\prod_{i=t-k+2}^t \kappa_{fp}(i) \right) \kappa_p(t-k+1) \kappa_f(t-k)] \\
&+ \lambda^2 \left(\prod_{i=t-k+3}^t \kappa_{fp}(i) \right) \kappa_p(t-k+2) \kappa_f(t-k+1) \kappa_f(t-k) \\
&+ \sum_{i=t-k+2}^{t-2} \lambda^{i-t+k+1} \left(\prod_{j=i+2}^t \kappa_{fp}(j) \right) \kappa_p(i+1) \left(\prod_{j=t-k}^i \kappa_f(j) \right) U_i(t-k) \\
&+ \lambda \left(\prod_{i=t-k+3}^t \kappa_{fp}(i) \right) \kappa_p(t-k+2) \kappa_f(t-k+1) Q_n(t-k+1) \\
&+ \sum_{i=t-k+2}^{t-2} \left\{ \lambda \left(\prod_{j=i+2}^t \kappa_{fp}(j) \right) \kappa_p(i+1) \kappa_f(i+1) [Q_n(i)] \right. \\
&+ \left. \sum_{j=t-k+1}^{i-1} \lambda^{i-j} \left(\prod_{l=j}^{i-1} \kappa_f(l) \right) Q_n(j) \right\}
\end{aligned} \tag{65}$$

For $k = 3$

$$\begin{aligned}
& \sum_{i=t-k}^{t-2} \lambda \left(\prod_{j=i+2}^t \kappa_{fp}(j) \right) \kappa_p(i+1) \kappa_f(i) U_i(i) = \\
& [\lambda \left(\prod_{i=t-1}^t \kappa_{fp}(i) \right) \kappa_p(t-2) \kappa_f(t-3) \\
& + \lambda^2 \kappa_{fp}(t) \kappa_p(t-1) \kappa_f(t-2) \kappa_f(t-3)] U_i(t-3) \\
& + \lambda \kappa_{fp}(t) \kappa_p(t-1) \kappa_f(t-2) Q_n(t-2)
\end{aligned} \tag{66}$$

For $k = 2$

$$\sum_{i=t-k}^{t-2} \lambda \left(\prod_{j=i+2}^t \kappa_{fp}(j) \right) \kappa_p(i+1) \kappa_f(i) U_i(i) = \lambda \kappa_{fp}(t) \kappa_p(t-1) \kappa_f(t-2) U_i(t-2) \tag{67}$$

Substituting equations 64 and 67 into equation 58 yields for $k = 2$:

$$\begin{aligned}
S_w(t) &= \sum_{i=t-1}^t \left(\prod_{j=i}^t \kappa_{fp}(j) \right) P_s(i) - \kappa_p(t) Q_n(t) - [\kappa_{fp}(t) \kappa_p(t-1) \\
&+ \lambda \kappa_f(t-1) \kappa_p(t)] Q_n(t-1) - \lambda \kappa_f(t-2) [\lambda \kappa_p(t) \kappa_f(t-1) \\
&+ \kappa_{fp}(t) \kappa_p(t-1)] U_i(t-2) + \left(\prod_{i=t-1}^t \kappa_{fp}(i) \right) S_w(t-2)
\end{aligned}$$

Substituting equations 63 and 66 into equation 58 yields for $k = 3$:

$$\begin{aligned}
S_w(t) &= \sum_{i=t-2}^t \left(\prod_{j=i}^t \kappa_{fp}(j) \right) P_s(i) - \kappa_p(t) Q_n(t) - [\lambda \kappa_f(t-1) \kappa_p(t) \\
&+ \kappa_{fp}(t) \kappa_p(t-1)] Q_n(t-1) - [\lambda^2 \left(\prod_{i=t-2}^{t-1} \kappa_f(i) \right) \kappa_p(t) \\
&+ \lambda \kappa_{fp}(t) \kappa_f(t-2) \kappa_p(t-1) + \left(\prod_{i=t-1}^t \kappa_{fp}(i) \right) \kappa_p(t-2)] Q_n(t-2) \\
&- \lambda \kappa_f(t-3) [\lambda^2 \left(\prod_{i=t-2}^{t-1} \kappa_f(i) \right) \kappa_p(t) + \lambda \kappa_{fp}(t) \kappa_f(t-2) \kappa_p(t-1) \\
&+ \left(\prod_{i=t-1}^t \kappa_{fp}(i) \right) \kappa_p(t-2)] U_i(t-3) \\
&+ \left(\prod_{i=t-2}^t \kappa_{fp}(i) \right) S_w(t-3)
\end{aligned} \tag{68}$$

Substituting equations 63 and 65 into equation 58 and rearranging yields for $k \geq 4$:

$$\begin{aligned}
S_w(t) = & \sum_{i=t-k+1}^t \left(\prod_{i=j}^t \kappa_{fp}(j) \right) P_s(i) - \kappa_p(t) Q_n(t) - [\kappa_{fp}(t) \kappa_p(t-1) \\
& + \lambda \kappa_p(t) \kappa_f(t-1)] Q_n(t-1) - \left[\left(\prod_{i=t-1}^t \kappa_{fp}(i) \right) \kappa_p(t-2) \right. \\
& + \lambda^2 \kappa_p(t) \left(\prod_{i=t-2}^{t-1} \kappa_f(i) \right) + \lambda \kappa_{fp}(t) \kappa_p(t-1) \kappa_f(t-2) \left. \right] Q_n(t-2) \\
& - \left[\left(\prod_{i=t-2}^t \kappa_{fp}(i) \right) \kappa_p(t-3) + \lambda^3 \kappa_p(t) \left(\prod_{i=t-3}^{t-1} \kappa_f(i) \right) \right. \\
& + \lambda \kappa_{fp}(t) \kappa_{fp}(t-1) \kappa_p(t-2) \kappa_f(t-2) \\
& + \lambda^2 \kappa_{fp}(t) \kappa_p(t-1) \left(\prod_{i=t-3}^{t-2} \kappa_f(i) \right) \left. \right] Q_n(t-3) \\
& \dots \\
& \dots \\
& - \left[\left(\prod_{j=i+1}^t \kappa_{fp}(j) \right) \kappa_p(i) + \lambda^{t-i} \kappa_p(t) \left(\prod_{j=i}^{t-1} \kappa_f(j) \right) \right. \\
& + \lambda \left(\prod_{j=i+2}^t \kappa_{fp}(j) \right) \kappa_p(i+1) \kappa_f(i) \\
& + \sum_{j=1}^{t-2-i} \lambda^{t-j-i} \left(\prod_{l=t-j+1}^t \kappa_{fp}(l) \right) \kappa_p(t-j) \left(\prod_{l=j}^{t-j-1} \kappa_f(l) \right) \left. \right] Q_n(i) \\
& \dots \\
& \dots \\
& - \left[\left(\prod_{i=t-k+2}^t \kappa_{fp}(i) \right) \kappa_p(t-k+1) + \lambda^{k-1} \kappa_p(t) \left(\prod_{i=t-k+1}^{t-1} \kappa_f(i) \right) \right. \\
& + \lambda \left(\prod_{i=t-k+3}^t \kappa_{fp}(i) \right) \kappa_p(t-k+2) \kappa_f(t-k+1) \\
& + \sum_{i=1}^{k-3} \lambda^{k-i-1} \left(\prod_{j=t-k+1}^t \kappa_{fp}(j) \right) \kappa_p(t-i) \left(\prod_{j=t-k+1}^{t-i-1} \kappa_f(j) \right) \left. \right] Q_n(t-k+1) \\
& - \lambda \kappa_f(t-k) \left[\left(\prod_{i=t-k+1}^t \kappa_{fp}(i) \right) \kappa_p(t-k+1) + \lambda^{k-1} \kappa_p(t) \prod_{i=t-k+1}^{t-1} \kappa_f(i) \right. \\
& + \lambda \left(\prod_{i=t-k+3}^t \kappa_{fp}(i) \right) \kappa_p(t-k+2) \kappa_f(t-k+1) \\
& + \sum_{i=t-k+2}^{t-2} \lambda^{i-t+k} \left(\prod_{j=i+2}^t \kappa_{fp}(j) \right) \kappa_p(i+1) \left(\prod_{j=t-k+1}^i \kappa_f(j) \right) \left. \right] U_i(t-k) \\
& + \left(\prod_{i=t-k+1}^T \kappa_{fp}(i) \right) S_w(t-k)
\end{aligned} \tag{69}$$

- The snowpack mass equations derived above can be expressed in a single more concise form as follows:

$$S_w(t) = \sum_{i=t-k+1}^t \theta_1(i)P_s(i) - \sum_{i=t-k+1}^t \theta_6(i)Q_n(i) - \theta_7(t)U_i(t-k) + \theta_8(t)S_w(t-k) \quad (70)$$

where $\theta_1(i), \theta_2(i), \theta_3(i), \theta_4(i), \theta_5(i), \theta_6(i), \theta_7(t)$ and $\theta_8(t)$ are defined as follows:

$$\theta_1(i) = \begin{cases} \kappa_{fp}(i) & \text{for } i = t \\ \kappa_{fp}(i)\theta_1(i+1) & \text{for } t-k+1 \leq i \leq t-1 \end{cases}$$

$$\theta_2(i) = \begin{cases} \kappa_p(i) & \text{for } i = t \\ \theta_1(i+1)\kappa_p(i) & \text{for } t-k+1 \leq i \leq t-1 \end{cases}$$

$$\theta_3(i) = \begin{cases} 0.0 & \text{for } i = t \\ \lambda\kappa_f(i-1)\kappa_p(i+1) & \text{for } i = t-1 \\ \lambda\kappa_f(i)\theta_3(i+1) & \text{for } t-k+1 \leq i \leq t-2 \end{cases}$$

$$\theta_4(i) = \begin{cases} 0.0 & \text{for } i = t \& t-1 \\ \lambda\theta_1(i+2)\kappa_p(i+1)\kappa_f(i) & \text{for } t-k+1 \leq i \leq t-2 \end{cases}$$

$$\theta_5(i) = \begin{cases} 0.0 & \text{for } i = t, t-1 \& t-2 \\ \lambda\kappa_f(i)[\theta_5(i+1) + \lambda\kappa_f(i+1)\kappa_p(i+2)\theta_1(i+3)] & \text{for } t-k+1 \leq i \leq t-3 \end{cases}$$

$$\theta_6(i) = \theta_2(i) + \theta_3(i) + \theta_4(i) + \theta_5(i) \text{ for } t-k+1 \leq i \leq t$$

$$\theta_7(t) = \lambda\kappa_f(t-k)\theta_6(t-k+1)$$

$$\theta_8(t) = \theta_1(t-k+1)$$

4.4 Steps 2 and 3: Expressing snowpack updating parameters in terms of estimated and measured snowpack

In this section the snowpack parameters $\eta_c, \Delta T_s, \eta_a$ and T_r are expressed individually as functions of the snowpack.

4.4.1 The Cloud Cover Correction Factor η_c

Substituting equations 30 and 37 into equation 46 yields:

$$Q_n(t) = I_s(t)[1 - C(t)][1 - A(t)] + [-20 + 0.94T_a(t)][1 - C(t)] + 1.24T_n(t)C(t) + Q_h(t) + Q_e(t) + Q_a(t) \quad (71)$$

Rearranging the above equation to obtain Q_n in terms of the cloud cover, C , yields:

$$Q_n(t) = \alpha(t)C(t) + \beta(t) \quad (72)$$

where $\alpha(t) = 1.24T_c(t)(t) + 20 - 0.94T_a(t) - I_s(t)[1 - A(t)]$; and

$$\beta(t) = I_s(t)[1 - A(t)] - 20 + 0.94T_a(t) + Q_h(t) + Q_e(t) + Q_a(t)$$

Modifying the cloud cover by the correction factor η_c results in:

$$Q_n(t) = \eta_c \alpha(t) C(t) + \beta(t) \quad (73)$$

Substituting equation 73 into equation 70 yields:

$$\begin{aligned} S_w(t) = & \sum_{i=t-k+1}^t \theta_1(i) P_s(i) - \left[\sum_{i=t-k+1}^t \theta_6(i) \alpha(i) C(i) \right] \eta_c \\ & - \sum_{i=t-k+1}^t \theta_6(i) \beta(i) - \theta_7(t) U_i(t-k) + \theta_8(t) S_w(t-k) \end{aligned}$$

Rearranging the above equation yields:

$$\begin{aligned} \eta_c = & \frac{1}{\sum_{i=t-k+1}^t \theta_6(i) \alpha(i) C(i)} [\theta_8(t) S_w(t-k) - S_w(t) \\ & - \theta_7(t) U_i(t-k) + \sum_{i=t-k+1}^t \theta_1(i) P_s(i) \\ & - \sum_{i=t-k+1}^t \theta_6(i) \beta(i)] \end{aligned} \quad (74)$$

By replacing $S_w(t)$ and $S_w(t-k)$ in equation 74 with corresponding measured values $\tilde{S}_w(t)$ and $\tilde{S}_w(t_k)$, an updated value of η_c can be calculated which would produce a match between measured and estimated SWE. However, the snowpack tags (κ_f , κ_p and κ_a) are also dependant on η_c , making it necessary to calculate their values through an iterative procedure. Fortunately, the iterative procedure is usually quite short, less than three iterations, and is not needed for many situations, as is the case for a melting snowpack, where $\kappa_p = 1$ and $\kappa_f = \kappa_a = 0$.

It should be noted that under certain conditions adjusting the value of η_c alone may not be sufficient to achieve the required match between calculated and measured SWEs. This happens, for example, if the snowpack is frozen and the calculated SWE is found to be less than the measured one. Under these conditions, the snowpack mass can be enhanced by updating the snow formation parameter, T_r , as described in section 4.4.4.

4.4.2 The Cloud Cover Parameter ΔT_s

Substituting equation 40 into equation 72 yields:

$$\begin{aligned} Q_n(t) = & \alpha(t) \left(1 - \frac{T_x(t) - T_n(t)}{\Delta T_s} \right) + \beta(t) \\ = & -\alpha(t) (T_x(t) - T_n(t)) \frac{1}{\Delta T_s} + \alpha(t) + \beta(t) \\ = & \alpha^*(t) \left(\frac{1}{\Delta T_s} \right) + \beta^*(t) \end{aligned} \quad (75)$$

where

$$\begin{aligned} \alpha^*(t) = & -\alpha(t) (T_x(t) - T_n(t)); \text{ and} \\ \beta^*(t) = & \alpha(t) + \beta(t) \end{aligned}$$

Substituting equation 75 into equation 70 yields:

$$S_w(t) = \sum_{i=t-k+1}^t \theta_1(i)P_s(i) - \left[\sum_{i=t-k+1}^t \theta_6(i)\alpha^*(i) \right] \frac{1}{\Delta T_s} \\ - \sum_{i=t-k+1}^t \theta_6(i)\beta^*(i) - \theta_7(t)U_i(t-k) + \theta_8(t)S_w(t-k)$$

Rearranging the above equation yields:

$$\frac{1}{\Delta T_s} = \frac{1}{\sum_{i=t-k+1}^t \theta_6(i)\alpha^*(i)} [\theta_8(t)S_w(t-k) - S_w(t) \\ - \theta_7(t)U_i(t-k) + \sum_{i=t-k+1}^t \theta_1(i)P_s(i) \\ - \sum_{i=t-k+1}^t \theta_6(i)\beta^*(i)] \quad (76)$$

The discussion on the application of equation 74 applies also to equation 76.

4.4.3 The Snow Albedo Correction Factor η_a

Equation 71 can be expressed as:

$$Q_n(t) = \alpha^o(t)A(t) + \beta^o(t) \quad (77)$$

where

$$\alpha^o(t) = -I_s(t)[1 - C(t)]; \text{ and} \\ \beta^o(t) = (I_s(t) - 20 + 0.94T_a(t))(1 - C(t)) + 1.24T_c(t)C(t) \\ + H_h(t) + H_e(t) + H_a(t)$$

Modifying the snow albedo by the correction factor η_a results in:

$$Q_n(t) = \eta_a \alpha^o(t)A(t) + \beta^o(t) \quad (78)$$

Substituting equation 78 into equation 70 results in:

$$S_w(t) = \sum_{i=t-k+1}^t \theta_1(i)P_s(i) - \sum_{i=t-k+1}^t \theta_6(i)\alpha^o(i)A(i)\eta_a \\ - \sum_{i=t-k+1}^t \theta_6(i)\beta^o(i) - \theta_7(t)U_i(t-k) + \theta_8(t)S_w(t-k)$$

Rearranging the above equation yields:

$$\begin{aligned}
\eta_a = & \frac{1}{\sum_{i=t-k+1}^t \theta_6(i)\alpha^o(i)A(i)} [\theta_8(t)S_w(t-k) - S_w(t) \\
& - \theta_7(t)U_i(t-k) + \sum_{i=t-k+1}^t \theta_1(i)P_s(i) \\
& - \sum_{i=t-k+1}^t \theta_6(i)\beta^o(i)] \tag{79}
\end{aligned}$$

Equation 79 can be used to update the snow albedo correction factor, η_a , utilizing snowpack measurements as suggested for η_c in section 4.4.1.

4.4.4 The Snow Formation Parameter T_r

Substituting equation 43 into equation 70 results in:

$$\begin{aligned}
S_w(t) = & \sum_{i=t-k+1}^t \theta_1(i)[1 - \frac{T_a(i)}{T_r}]PP(i) - \sum_{i=t-k+1}^t \theta_6(i)Q_n(i) \\
& - \theta_7(t)U_i(t-k) + \theta_8(t)S_w(t-k)
\end{aligned}$$

Rearranging the above equation results in:

$$\begin{aligned}
\frac{1}{T_r} = & \frac{1}{\sum_{i=t-k+1}^t \theta_1(i)T_a(i)PP(i)} [\theta_8(t)S_w(t-k) - S_w(t) \\
& - \theta_7(t)U_i(t-k) - \sum_{i=t-k+1}^t \theta_6(i)Q_n(i) \\
& + \sum_{i=t-k+1}^t \theta_1(i)PP(i)] \tag{80}
\end{aligned}$$

Equation 80 has more limitations than equations 74, 76 and 79, since it can only be used when $0.0 < T_a < T_r$. This problem can be overcome by using an iterative method.

4.5 Algorithm of the Snowpack Updating Model

Based on the set of relationships defined in the previous sections, a procedure has been developed to update a given snowpack parameter utilizing SCS measurements as they become available. The algorithm of this procedure is outlined below:

- (1) The snowpack thermal conditions tags (κ_f , κ_p , and κ_a) are calculated as described in section 4.3 in parallel with the daily calculation of snowpack mass using the UBCWM.
- (2) Upon availability of a new snowpack measurement (at time t), the variables $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7$ and θ_8 are back calculated from time t to the time, $t - k$, of the previous measurement based on the set of equations derived in section 4.3.
- (3) One of the snowpack parameters, $\eta_c, \Delta T_s, \eta_a$, or T_r can be then updated based on equations 74, 76, 79 or 80, respectively.

- (4) The newly updated value of the snowpack parameter can be then used in recalculating the snowpack starting from $t - k$ to t and step 1 is repeated. If the new set of tags values matches the previous one, the next step is then executed; otherwise steps 2, 3 and 4 are repeated.
- (5) The updated snowpack parameter is then used in the UBCWM to calculate the flow forecasts.
- (6) The previous steps are repeated whenever a new snowpack measurement becomes available.

5 Preliminary application of the snowpack updating model

To test its value in seasonal river flow forecasts, the snowpack updating model was used to update flow forecasts of the Illecillewaet River, a tributary of the Upper Columbia River in British Columbia, Canada. The Illecillewaet river basin is $1,150 \text{ km}^2$, with elevation ranging from 1,000 to 2,250 meters. Annual precipitation increases from 950 mm (47% snowfall) at elevation 443 meters to 2,160 mm (70% snowfall) at elevation 1,875 meters. These figures indicate heavy accumulation of snowpack during the winter, which generates significant snowmelt during the melting season that starts as early as March and extends well into August.

The UBCWM was used to generate daily river flow forecasts for two consecutive hydrological years (October 1st, 1974 - September 30, 1976). These forecasts were first generated without using the snowpack updating model. Utilizing SCS measurements at Mount Fidelity station at elevation 1,870 meters, the UBCWM was run in conjunction with the snowpack updating model. Only the cloud cover correction factor, η_c , was updated.

SCS measurements, snowpack SWE generated by the UBCWM only and those generated using the snowpack model are plotted in Figure 2. The corresponding values of the updating parameter η_c are shown in Figure 3. The results show consistently great improvement of estimated snowpack SWE. Figure 3 shows how the snowpack updating model adjust the estimate of cloud cover necessary to bring the estimation of snowpack mass in line with measurements. It can be noted from Figure 2 that the updating model was irresponsive to the first four measurements, since they were all recorded during the accumulation stage and thus were not suitable to monitor the snowmelt process. The updating process kicked in at the first SCS measurement during the melting season, which was quite valuable in improving snowpack calculation.

To assess the impact of using the snowpack updating model on the performance of the UBCW, the daily river flow forecasts using the UBCWM alone and those generated using the snowpack updating model are compared against daily streamflows recorded at Environment Canada's Illecillewaet River gaging station at Greeley. The results are shown in Figures 4 and 5 for the active river periods April 15 to September 30 for the years 1975 and 1976, respectively. The results show a highly significant and relatively consistent improvement as a result of using the snowpack updating model, especially during the snowmelt seasons. In particular, the periods from mid April to mid July for both years show a dramatic improvement as the updating model adjusted the cloud cover correction factor upward to closely match snowpack measurements.

6 Summary and conclusion

This paper presents the details of developing an energy-budget based snowpack updating model, designed to make use of SCS measurements. The paper gives a theoretical overview of the energy and mass balance of the snowpack with emphasis on the most significant heat transfer processes including shortwave and longwave radiation, and convective, latent and rain melt heat exchange. The theoretical relationships were used to formulate a numerical algorithm for calculating snowpack internal energy and mass and snowmelt.

The snowmelt routine in the UBCWM was presented with emphasis on representing the energy budget and mass balance of the snowpack in terms of prevalent meteorological measurements of precipitation and temperature.

Clarifying the critical roles of cloud cover and snow albedo in the heat and mass balance of the snowpack, several parameters were introduced to modify these variables utilizing SCS measurements. The mathematical development and the algorithm of a novel snowpack updating routine were presented in detail.

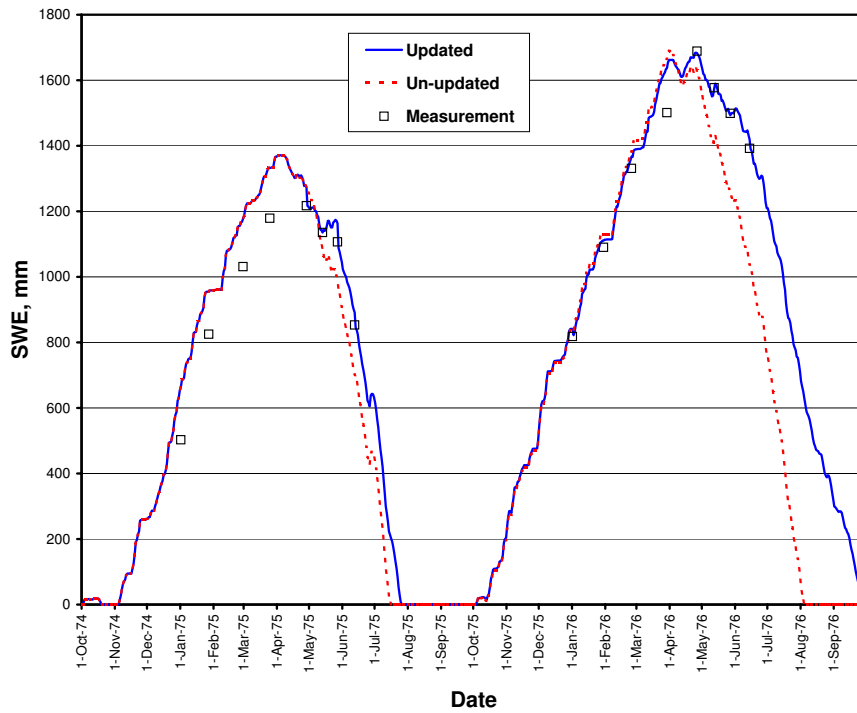


Fig. 2. Measured, updated and un-updated SWEs for Oct. 1, 1974- Sep. 30, 1976

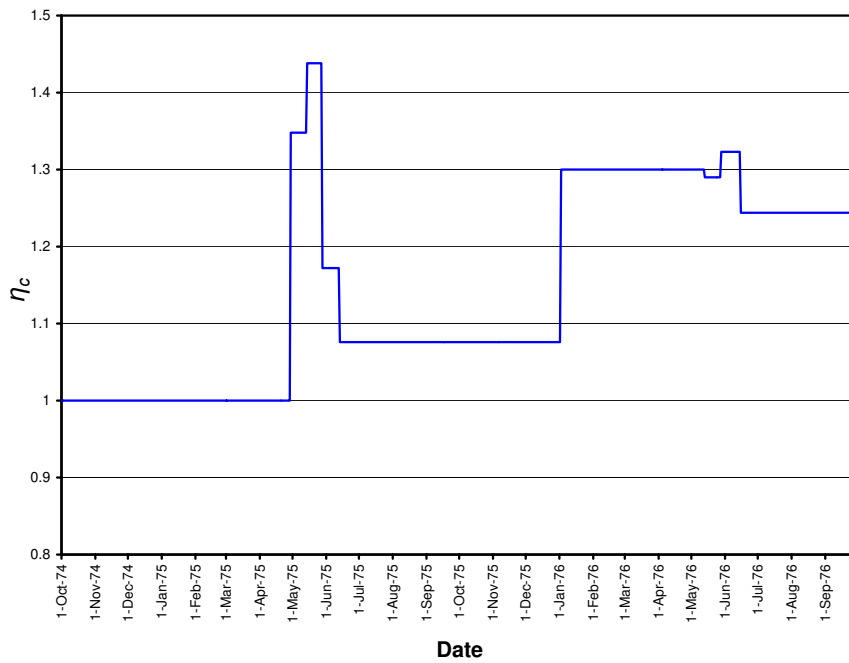


Fig. 3. Updated values of the cloud cover parameter η_c for Oct. 1, 1974- Sep. 30, 1976

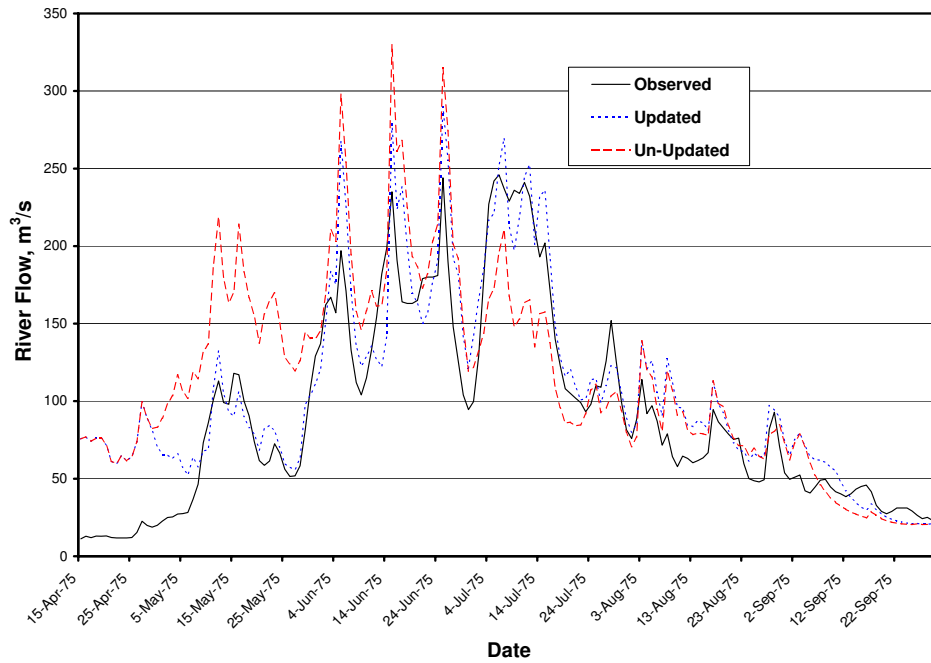


Fig. 4. Observed, updated and un-updated Illecillewaet River flows for Apr. 15 to Sep. 30, 1975

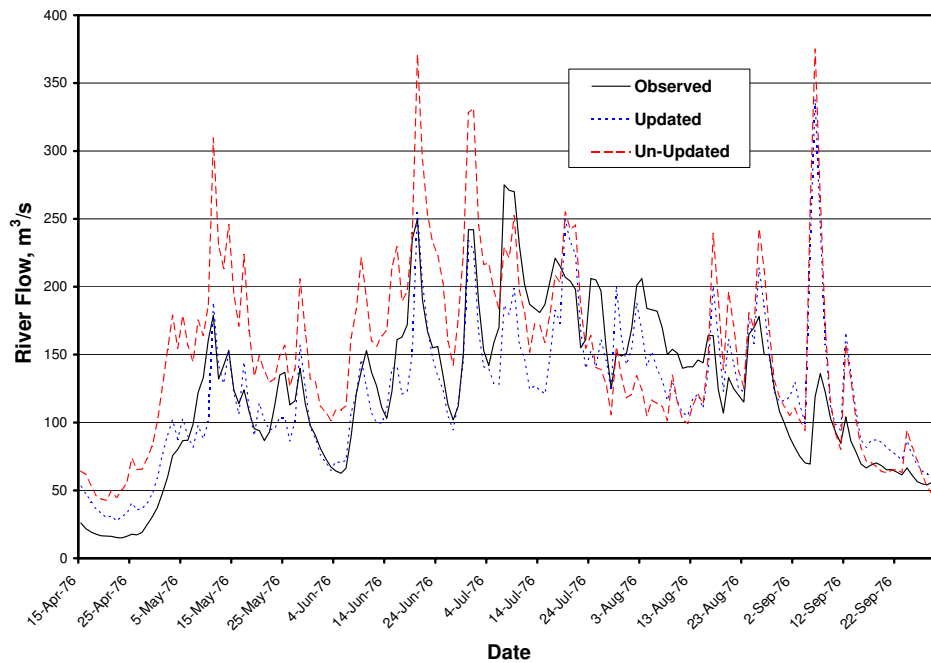


Fig. 5. Observed, updated and un-updated Illecillewaet River flows for Apr. 15 to Sep. 30, 1976

The proposed updating procedure was tested on generating snowpack and stream flow forecasts for the Illecillewaet River basin in British Columbia Canada. The results show significant improvement in both forecasts, despite the use of few SCS measurements.

The preliminary promising results of the presented snowpack updating model show its great potential for use in seasonal river flow forecasts for mountainous watersheds.

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