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Term Paper

THE TIME CONSISTENCY PROBLEM
MONETARY POLICY MODELS

Jan Störger
cand. rer. pol.

Mannheim University
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Dr. Alexander Ludwig
Mannheim Research Institute for the Economics of Aging, MEA
Prof. Axel Börsch-Supan

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Jan Störger,
js@schaurer-stoerger.de

1. INTRODUCTION

This paper is an introduction to the discussion about the time inconsistency of optimal policy, which arises in many dynamic problems if rational expectations are assumed.[1] The structure of this paper is as follows: First I introduce the general problem of time inconsistency mainly based on the arguments of Kydland and Prescott (1977).[23] Since this paper concentrates on monetary models, I then give an overview on the relevant literature. In the monetary context I explain the Barro and Gordon setup under perfect information as a single and also as a repeated game with two different reputation mechanisms. Further on I make use of the Backus and Driffill model to explain how the time inconsistency may not arise under asymmetric information about the policymaker's preferences. The models of Cukierman and Liviatan (1991) and Cukierman (2000a) show that time inconsistency arises for private information about the policymaker's ability to precommit and to control inflation. Furthermore there may be asymmetric information about economic shocks or, as shown in this paper, with a timing where the public has to built expectations about the shock prior to its realization and the policymaker's reaction to it. In this situation a fix policy rule is welfare-inferior to a flexible rule policy, which in addition is time consistent even for high uncertainty. Finally I give a short comment on the empirical evidence for some of the models.[2]

1.1. **The inconsistency of optimal plans.** As pioneers of time consistency research can be mentioned Strotz (1955-1956)[33], Friedman (1969)[17] and (1971)[18] and Hammond (1976)[19].

Following Barro and Gordon (1983)[5], and assuming rational expectations, time consistency is a property of a solution in which individuals have correct expectations about a future policy. Whenever it is desirable for a policymaker to renege from today's expectations by choosing another policy in the future, under rational expectations, such a deviation will be anticipated and therefore it is not credible that the policymaker will not renege. Due to this lack of credibility the time consistent solution is generally inferior to those where cheating would work or where no reneging from expectations would take place. Başar

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[1]Kydland and Prescott (1977)[23, p. 474] show that time inconsistency also arises under the assumption that agents cannot perfectly forecast future policy choices. *Rational expectations* mean that agents correctly anticipate the expected value given the value's distribution function.[25, p. 439]

[2]Although there is quite a lot of math used in this paper and there is no appendix, I find this structure necessary to support the main arguments. Whenever it was possible I modified the notation of the original models a bit in the hope to make it more convenient for readers to follow. At some points more or less extensive algebra is necessary to come to the solutions but I am able to give a full documentation to anyone who is interested. Finally I want to thank Alex Cukierman for the kind correspondence.

and Olsder (1999)[6, pp. 249-252] say that *for a solution to be time consistent, the players should have no rational reason, at any future stage of the game, to deviate from the adopted policies.* This characterisation of time consistency is quite intuitive unlike the one implied earlier in the work of Kydland and Prescott (1977)[23, pp. 475-476]:

Proposition 1. *A policy is consistent if it ignores the effect of (expected) current policy on decisions in the past and the effect of past decisions on the social objective function at all times.*

Proposition 2. *A consistent policy is suboptimal if both of these effects are non-zero.*

Kydland and Prescott (1977)[23, pp. 473-474] show that a discretionary policy does not generally lead to optimality. They define a *discretionary policy* as the policymaker's best action given the current situation. Unlike a *rule policy* here the action depends only on current influences and thus is generally time variant. Using policy rules may lead to Pareto-superior outcomes. Agents decide based on their expectations of future policy decisions. Therefore current decisions are not just affected by the present and the past but also by agents' predictions of the future. Agents' expectations are altered according to their experiences in the past and the current situation. Any unanticipated policy will alter these expectations in the future. Whenever the effect of current policy on agents' decisions in the past and the effect of past decisions on the social objective function at all times are both non-zero, a consistent policy, which by Kydland and Prescott's (1977)[23, p. 476] definition ignores both of these effects, is suboptimal.

They demonstrate the incompatibility of a consistent policy and optimality as follows[23, p. 476]:

There is a world with two players, a policymaker who chooses a policy π_t and an agent who makes a decision x_t simultaneously, in every period $t = 1, ..., T$. Both players agreed on a social objective function $S(x_1, ..., x_T, \pi_1, ..., \pi_T)$ prior to the game.

In a world that exists for two periods, in $t = 2$ the policymaker maximizes this social objective function by his policy choice π_2 as follows:

$$\max_{\pi_2} S(x_1, x_2, \pi_1, \pi_2) \tag{1.1}$$

subject to the agent's past ($t = 1$) and current ($t = 2$) choices:

$$x_1 = X_1(\pi_1, \pi_2) \tag{1.2}$$

and

$$x_2 = X_2(x_1, \pi_1, \pi_2) \tag{1.3}$$

Notice that the agent's first step in the world depends on the policy of both periods. It is assumend that the agent has some sense of the

policy choices for all periods.[3] The agent's second and last step in the world also depends on his first step.

Solving this maximization-problem we end up with the following first order condition (FOC):

$$\frac{\partial S}{\partial x_2}\frac{\partial X_2}{\partial \pi_2} + \frac{\partial S}{\partial \pi_2} + \frac{\partial X_1}{\partial \pi_2}\left[\frac{\partial S}{\partial x_1} + \frac{\partial S}{\partial x_2}\frac{\partial X_2}{\partial x_1}\right] = 0 \qquad (1.4)$$

A time consistent policy has no allowance for any effects of future policy on today's choices and therefore it ignores the effect of π_2 upon x_1, that is

$$\frac{\partial X_1}{\partial \pi_2} = 0 \qquad (1.5)$$

But if there is such a non-zero effect of future policy on today's decisions and we can assume the agent's past decision to have a non-zero effect on the social objective function, then the optimal policy rule is different from the reduced FOC of a time consistent policy rule, that is

$$\frac{\partial S}{\partial x_2}\frac{\partial X_2}{\partial \pi_2} + \frac{\partial S}{\partial \pi_2} = 0 \qquad (1.6)$$

This simple maximization problem of Kydland and Prescott (1977)[23, p. 476] depicts the suboptimality of time consistent policy assuming that the agent has some knowledge of how the policymaker decides, which means that equation 1.5 does not hold, and that past decisions of the agent matter for the social objective.

2. Monetary policy and time inconsistency

Auernheimer (1974)[1] may be among the first researchers who mention a time consistency problem in a monetary context, although Johnson (1969)[21, pp. 132-133] already doubts policymakers' ability to exploit the Phillips curve[4]. Phelps (1967)[27] and Friedman (1968)[16] argued that the Phillips trade-off does not persist in the long-run. Rather in the long-run there is a natural rate of unemployment, independent of the steady state inflation.[5] Under rational expectations surprise inflation cannot arise. Every rise in inflation would be anticipated and therefore would have no decreasing effect on unemployment.

Kydland and Prescott (1977)[23] demonstrate the inconsistency of optimal inflation policy and Barro and Gordon (1983)[5] refine their idea in their famous model which we will discuss in the following subsections.

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[3]Perfect foresight is not needed but rather there has to be some correlation between X_1 and π_2, specifically $\frac{\partial X_1}{\partial \pi_2} \neq 0$.

[4]See Fisher (1926)[15] and Phillips (1958)[28].

[5]That is a vertical Phillips curve in the long-run.

2.1. **Perfect information and single interaction.** Barro and Gordon (1983)[5] introduce a game theoretic framework with a monetary policymaker and the public playing a simultaneous prisoners' dilemma[6] against each other.

The policymaker's utility function is

$$U_t^{Pol} = \theta b(\pi_t - \pi_t^e) - \frac{a}{2}\pi_t^2 \tag{2.1}$$

π_t is the actual inflation rate chosen by the policymaker, whereas π_t^e represents public's expectations about the inflation rate. Whenever there is an unanticipated inflation, the policymaker can generate a decrease in unemployment according to the Phillips curve. The second term represents the negative impact of inflation, be it unanticipated or not. θ is a dummy variable being either 0 for a policymaker who does not care about the benefit from exploiting the Phillips curve at all and being 1 otherwise. Later we assume θ to be private information of the policymaker. For any positive b, assuming $\theta = 1$ any rise of π_t above π_t^e is beneficial for the policymaker.

By $\frac{a}{2}\pi_t^2$ they assume the costs of inflation to rise with inflation increasingly.

The public has the following utility function

$$U_t^{Pub} = -(\pi_t - \pi_t^e)^2 \tag{2.2}$$

such that every deviation of the actual inflation rate π_t from the expected inflation rate π_t^e means a quadratic and negative utility for the public.

In the following subsections we consider a one-stage prisoners' dilemma with perfect information and the assumption of a weak policymaker, that is $\theta = 1$, in which both players, the policymaker and the public, move simultaneously.

2.1.1. *Discretionary policy.* The unconstrained optimization problem and solution of the weak policymaker is

$$\max_{\pi_t} U_t^{Pol} = b(\pi_t - \pi_t^e) - \frac{a}{2}\pi_t^2 \tag{2.3}$$

FOC:

$$\hat{\pi}_t = \frac{b}{a} \tag{2.4}$$

where the weak policymaker does not take into account that agents have rational expectations. As we see later this leads to the discretionary and third best (from the weak policymaker's perspective) outcome with an anticipated inflation rate of $\hat{\pi}_t = \pi_t^e = \frac{b}{a}$ and payoffs $\hat{U}_t^{Pol} = -\frac{b^2}{2a} < \hat{U}_t^{Pub} = 0$. Any choice of a lower inflation rate would result in a negative effect on employment and is not in the interest of the weak policymaker.

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[6]See Rapoport and Chammah (1965)[29].

2.1.2. *Rational expectations.* By solving the unconstrained optimization problem of the public,

$$\max_{\pi_t^e} U_t^{Pub} = -(\pi_t - \pi_t^e)^2 \tag{2.5}$$

FOC:

$$\pi_t = \pi_t^e \tag{2.6}$$

rational expectations can be assumed. In the next subsection we will see that this leads to the inefficient but time consistent outcome because systematic cheating cannot take place under this assumption.

2.1.3. *Rule policy.* Now the policymaker maximizes his utility given that the public anticipates his choice of π_t correctly:

$$\max_{\pi_t} U_t^{Pol} = b(\pi_t - \pi_t^e) - \frac{a}{2}\pi_t^2 \tag{2.7}$$

s.t.

$$\pi_t = \pi_t^e \tag{2.8}$$

FOC:

$$\pi_t^* = 0 \tag{2.9}$$

The public's best response to this result is to choose $\pi_t^e = 0$, which is confirmed by the policymaker's action, such that $\pi_t = \pi_t^e = 0$.

Payoffs $U_t^{Pol*} = U_t^{Pub*} = 0$ are Pareto-optimal but time inconsistent and there is a profitable deviation for the policymaker: Given $\pi_t^e = 0$ the policymaker's best response is $\tilde{\pi}_t = \frac{b}{a}$ as we will see in the next subsection. We therefore call $\pi_t^* = 0$ to be the ideal rule, which leads to the second best solution, from the weak policymaker's perspective.[7]

2.1.4. *Cheating.* Assuming that the public expects the optimal rule zero inflation $\pi_t^e = \pi_t^* = 0$, the maximization problem of the weak policymaker becomes:

$$\max_{\pi_t} U_t^{Pol} = b(\pi_t - \pi_t^e) - \frac{a}{2}\pi_t^2 \tag{2.10}$$

s.t.

$$\pi_t^e = \pi_t^* = 0 \tag{2.11}$$

FOC:

$$\tilde{\pi}_t = \frac{b}{a} \tag{2.12}$$

Given that cheating was successful, the resulting payoffs $\tilde{U}_t^{Pol} = \frac{b^2}{2a} > \tilde{U}_t^{Pub} = -\left(\frac{b}{a}\right)^2$ are sub-optimal but time consistent and

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[7]In the literature it is generally assumed that the policymaker's objective function somehow represents welfare. From this perspective, the solution with the highest welfare (first best) is the one where cheating takes place, although looking at the payoffs in the game matrix reveals that Pareto-optimality is given in the solution under the ideal rule (here considered the second best solution).

considered the first best solution from the weak policymaker's perspective.

$\theta = 1$	Public $\pi_t^e = 0$	Public $\pi_t^e = \dfrac{b}{a}$
$\pi_t = 0$	0 / 0	$-\left(\dfrac{b}{a}\right)^2$ / $-\dfrac{b^2}{a}$
$\pi_t = \dfrac{b}{a}$	$-\left(\dfrac{b}{a}\right)^2$ / $\dfrac{b^2}{2a}$	0 / $-\dfrac{b^2}{2a}$

FIGURE 1. A weak policymaker

$\theta = 0$	Public $\pi_t^e = 0$	Public $\pi_t^e = \dfrac{b}{a}$
$\pi_t = 0$	0 / 0	$-\left(\dfrac{b}{a}\right)^2$ / 0
$\pi_t = \dfrac{b}{a}$	$-\left(\dfrac{b}{a}\right)^2$ / $-\dfrac{b^2}{2a}$	0 / $-\dfrac{b^2}{2a}$

FIGURE 2. A strong policymaker

2.1.5. *Weak versus strong policymaker.* Like Backus and Driffill (1985) [3], we consider now a so called strong government[8], which does not care about the benefit from surprise inflation at all, that is $\theta = 0$. For a

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[8]The expressions government, policymaker and central bank are used here as synonyms.

policymaker of this type playing the discretionary inflation rate $\hat{\pi}_t = \frac{b}{a}$ is a strictly dominated strategy. Therefore he never reneges from the ideal rule $\pi_t^* = 0$ and thus no time inconsistency arises. On the opposite for a weak policymaker playing $\pi_t = 0$ is a strictly dominated strategy.[9]

2.2. **Perfect information and repeated interaction.**

2.2.1. *Finite time horizon - the chain store paradox.* Under a finite time horizon the appropriate solution concept is that of backward induction. By solving the game beginning in the final period and considering only such strategies for all earlier stages that are themselves Nash equilibria in the subgames of the later stages, strategies that are not played under discretion will not be equilibrium strategies. The resulting subgame perfect Nash equilibrium is by construction time consistent. In the current setup this means that the discretionary equilibrium from the final period will occur in all preceding subgames. This paradox becomes very clear in the well known chain-store game of Selten (1978)[32]. Under perfect information assuming an infinite horizon therefore becomes necessary to avoid this paradox.[8, p. 18]

2.2.2. *Infinite horizon - private sector plays tit for tat.* Barro and Gordon (1983)[5, p. 108] consider a "tit for tat" strategy[10] played by the public:

$$\pi_t^e = \pi_t^* = 0 \text{ if } \pi_{t-1} = \pi_{t-1}^e$$
$$\pi_t^e = \hat{\pi}_t = \frac{b}{a} \text{ if } \pi_{t-1} \neq \pi_{t-1}^e$$

The public expects zero inflation if the policymaker met peoples expectations in the preceding period. Otherwise the public expects the discretionary inflation rate. Since it takes the public only one period to fully update their expectations, the government gets punished for one period only and its credibility is completely restored thereafter.

A mechanism to enforce cooperation, that is here the ideal rule $\pi_t^* = 0$, has to satisfy the following incentive constraint:

$$\underbrace{U_{t+1}^{Pol*} - \hat{U}_{t+1}^{Pol}}_{Enforcement} \geq \underbrace{\tilde{U}_t^{Pol} - U_t^{Pol*}}_{Temptation} \tag{2.13}$$

$$\underbrace{\delta \frac{a}{2} \left[\left(\frac{b}{a} \right)^2 - \pi^2 \right]}_{Enforcement} \geq \underbrace{\frac{a}{2} \left[\frac{b}{a} - \pi \right]^2}_{Temptation} \tag{2.14}$$

δ is the discount factor, which determines the degree of discounting future payoffs.

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[9]See figures 1 and 2.

[10]See Axelrod (1984)[2] and Rogoff (1987)[30] for more about trigger mechanisms.

If π is assumed to be the ideal rule $\pi_t^* = 0$ the incentive constraint reduces to

$$\underbrace{\delta \frac{a}{2} \left[\frac{b}{a}\right]^2}_{Enforcement} \geq \underbrace{\frac{a}{2} \left[\frac{b}{a}\right]^2}_{Temptation} \tag{2.15}$$

or

$$\delta \geq 1 \tag{2.16}$$

which cannot be true for plausible values of δ satisfying $0 < \delta < 1$. Assuming $\delta = 1$, that is future payoffs are valued the same as current payoffs, as Backus and Driffill (1985) do, it is generally possible to enforce the ideal rule using the above mechanism. But whenever we take into account discounting of future payoffs it is impossible to enforce the ideal rule, that is zero inflation at all times, with the described mechanism.
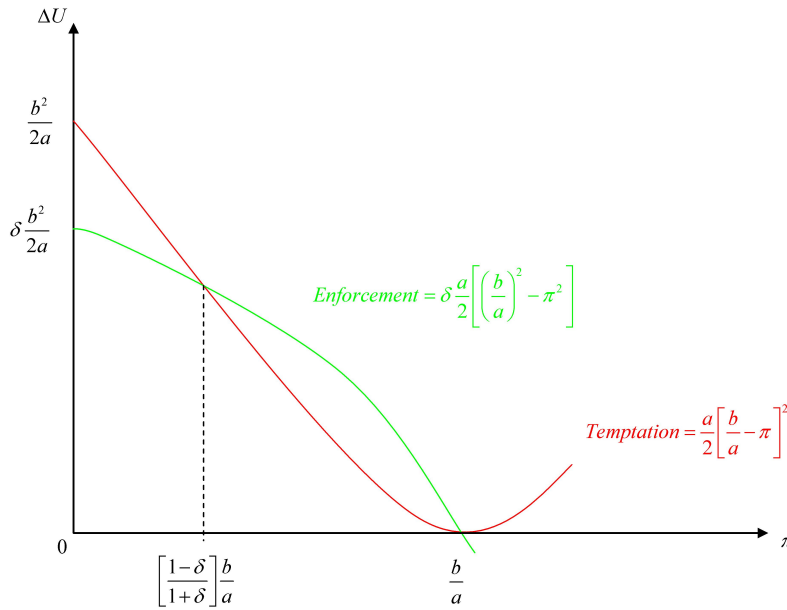

FIGURE 3. Enforcement vs. temptation utility surplus under a tit for tat mechanism[5, p. 112]

If not the ideal rule, what rule can be enforced here? Let π be some positive inflation rule. Then solving for π we get

$$\underbrace{\left[\frac{1-\delta}{1+\delta}\right]\frac{b}{a}}_{\pi_{BER}} \leq \pi \leq \underbrace{\frac{b}{a}}_{\hat{\pi}} \tag{2.17}$$

where π_{BER} is the Best Enforceable Rule (BER), that is the lowest enforceable inflation rate using the above mechanism. This BER is in fact a weighted average of $\pi^* = 0$ and $\hat{\pi} = \frac{b}{a}$. The weights are determined by the discount factor δ. For $\delta = 1$ the BER would be the ideal rule ($\pi_{BER} = \pi^* = 0$) and for $\delta = 0$ it would be the discretionary inflation rate ($\pi_{BER} = \hat{\pi} = \frac{b}{a}$).[11]

2.2.3. *Inifinite horizon - private sector plays grim trigger.* Public now plays the following punishment mechanism called "grim trigger"[12][24, pp. 67, 68]:

$\pi_t^e = \pi_t^* = 0$ if $\pi_s = \pi_2^e \ \forall s < t$
$\pi_t^e = \hat{\pi}_t = \frac{b}{a}$ otherwise

The present value of the policymaker's expected payoff if he always plays the ideal rule $\pi_t^* = 0$ is

$$PV_{Enforcement}^{Pol} = \sum_{t=0}^{T} \underbrace{\delta^t \cdot 0}_{\pi^*=0} = 0 \tag{2.18}$$

whereas his present value when surprising once in the first period and being punished forever afterwards is

$$PV_{Temptation}^{Pol} = \underbrace{\frac{b^2}{2a}}_{\tilde{\pi}_{t=0} > \pi_{t=0}^e = 0} + \underbrace{\sum_{t=1}^{T} \delta^t \left(-\frac{b^2}{2a}\right)}_{\hat{\pi}_{t>0} = \pi_{t>0}^e = \frac{b}{a}} \tag{2.19}$$

Assuming $T = \infty$ this becomes

$$PV_{Temptation}^{Pol} = \underbrace{\frac{b^2}{2a}}_{\tilde{\pi}_{t=0} > \pi_{t=0}^e = 0} + \underbrace{\frac{\delta}{1-\delta}\left(-\frac{b^2}{2a}\right)}_{\hat{\pi}_{t>0} = \pi_{t>0}^e = \frac{b}{a}} \tag{2.20}$$

The policymaker will comply with the announced zero inflation without an explicit agreement over a the ideal rule whenever

$$PV_{Enforcement}^{Pol} \geq PV_{Temptation}^{Pol} \tag{2.21}$$

which is

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[11]See figure 3, which depicts enforcement versus temptation utility surplus under a tit for tat mechanism.[5, p. 112]

[12]Also known as "Friedman"[2, p. 36].

$$\delta \geq \frac{1}{2} \tag{2.22}$$

For $\delta \geq \frac{1}{2}$ there is no time inconsistent behavior and the policymaker always follows the zero inflation rule. We therefore get a an optimal and time consistent solution here.

2.3. **Intrinsic uncertainty.**

2.3.1. *Different preferences over surprise inflation.* Backus and Driffill (1985)[3] introduced public uncertainty over the policymaker's preferences[13] to the Barro and Gordon (1983) setup. Public is uncertain about government's objective function[3, p. 532]. θ be the policymaker's private information and can be either 0 or 1. The game is now repeated finitely with $t = 0, ..., T$ periods. As for now we assume $T = 2$.

Let y_{T-1} be the probability that a government of type $\theta = 1$ plays zero inflation in the penultimate period $T - 1$, p_{T-1} be the probability (reputation) that government is of type $\theta = 0$ in $T - 1$ and p_T be the probability (reputation) that government is of type $\theta = 0$ in T given that it has played zero inflation in the preceding period $T - 1$.

Looking at the final period T we know that the policymaker reveals his true type θ because he cannot be punished. Therefore in T a type who cares about the benefits of surprise inflation will play $\hat{\pi}_T = \frac{b}{a}$.

In a hypothetical Separating Equilibrium (SEQ) a government of type $\theta = 1$ does not pretend to be of type $\theta = 0$ and plays $\hat{\pi}_{T-1} = \frac{b}{a}$. The expected inflation rate in $T - 1$ is

$$\pi^e_{T-1} = \underbrace{[p_{T-1} \cdot 1 + (1 - p_{T-1})y_{T-1}]}_{prob(\pi_{T-1}=0)} 0 + \underbrace{[(1 - p_{T-1})(1 - y_{T-1})]}_{prob(\pi_{T-1}=\frac{b}{a})} \frac{b}{a} \tag{2.23}$$

or

$$\pi^e_{T-1} = \underbrace{[(1 - p_{T-1})(1 - y_{T-1})]}_{prob(\pi_{T-1}=\frac{b}{a})} \frac{b}{a} \tag{2.24}$$

The policymaker's expected utility in $T - 1$ consequently is

$$U^{Pol}_{T-1} = \frac{b^2}{a} \left[\frac{1}{2} - (1 - p_{T-1})(1 - y_{T-1}) \right] \tag{2.25}$$

In a SEQ the policymaker of type $\theta = 1$ reveals his type in $T - 1$. Therefore $\pi^e_T = \pi_T = \frac{b}{a}$, that is the public knows the inflation rate in the final period T.

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[13]Public is uncertain whether the policymaker is of the strong type ($\theta = 0$) or the weak type ($\theta = 1$).

The present value (PV) of the policymaker's expected utility then is:

$$PV_{SEQ}^{Pol} = \underbrace{\frac{b^2}{a}\left[\frac{1}{2} - (1 - p_{T-1})(1 - y_{T-1})\right]}_{T-1} + \underbrace{\delta\left[-\frac{b^2}{2a}\right]}_{T} \qquad (2.26)$$

or

$$PV_{SEQ}^{Pol} = \frac{b^2}{a}\left[\frac{1}{2}(1 - \delta) - (1 - p_{T-1})(1 - y_{T-1})\right] \qquad (2.27)$$

In a hypothetical Pooling Equilibrium (PEQ) a policymaker of type $\theta = 1$ pretends to be of type $\theta = 0$ and plays $\pi_{T-1} = 0$ and the public builds its expectations π_T^e according to what it has seen in $T - 1$.

The policymaker's payoff in $T - 1$ is

$$U_{T-1}^{Pol} = -\frac{b^2}{a}(1 - p_{T-1})(1 - y_{T-1}) \qquad (2.28)$$

Using Bayes' rule the probability that government is of type $\theta = 1$ whenever it has played zero inflation in the preceding period $T - 1$ is:

$$prob(\theta = 1|\pi_{T-1} = 0) = \frac{prob(\pi_{T-1} = 0|\theta = 1) \cdot prob(\theta = 1)}{prob(\pi_{T-1} = 0)} \qquad (2.29)$$

or

$$1 - p_T = \frac{y_{T-1} \cdot (1 - p_{T-1})}{1 \cdot p_{T-1} + y_{T-1} \cdot (1 - p_{T-1})} \qquad (2.30)$$

The expected inflation rate in T is

$$\pi_T^e = (1 - p_T)\frac{b}{a} = \frac{y_{T-1}(1 - p_{T-1})}{1 \cdot p_{T-1} + y_{T-1}(1 - p_{T-1})}\frac{b}{a} \qquad (2.31)$$

Thus we see that the expected inflation rate in T is lower as in $T - 1$:

$$\underbrace{\frac{y_{T-1}(1 - p_{T-1})}{p_{T-1} + y_{T-1}(1 - p_{T-1})}\frac{b}{a}}_{\pi_T^e} < \underbrace{[(1 - p_{T-1})(1 - y_{T-1})]\frac{b}{a}}_{\pi_{T-1}^e} \qquad (2.32)$$

The present value of the policymaker's expected utility in a hypothetical PEQ is

$$PV_{PEQ}^{Pol} = \underbrace{-\frac{b^2}{a}(1 - p_{T-1})(1 - y_{T-1})}_{T-1} + \underbrace{\delta\left[b\left(\frac{b}{a} - \pi_T^e\right) - \frac{b^2}{2a}\right]}_{T} \qquad (2.33)$$

or

$$PV_{PEQ}^{Pol} = \frac{b^2}{a} \left[-(1 - p_{T-1})(1 - y_{T-1}) + \delta \left(\frac{1}{2} - (1 - p_T) \right) \right] \quad (2.34)$$

A SEQ exists if the policymaker of type $\theta = 1$ has no incentive to pretend to be of type $\theta = 0$, that is

$$\underbrace{\frac{b^2}{a} \left[-(1 - p_{T-1})(1 - y_{T-1}) + \delta \left(\frac{1}{2} - (1 - p_T) \right) \right]}_{PV_{PEQ}^{Pol}}$$

$$\leq \underbrace{\frac{b^2}{a} \left[\frac{1}{2}(1 - \delta) - (1 - p_{T-1})(1 - y_{T-1}) \right]}_{PV_{SEQ}^{Pol}} \quad (2.35)$$

This implies $\delta \leq \frac{1}{2}$ given the pure strategy $y_{T-1} = 0$ (revelation) is the best response. For $\delta > \frac{1}{2}\frac{1}{p_{T-1}}$ given the pure strategy $y_{T-1} = 1$ (pretending) is the best response and players end up in a perfect Bayesian PEQ!

For $\frac{1}{2} < \delta \leq \frac{1}{2}\frac{1}{p_{T-1}}$ there is a mixed perfect Bayesian equilibrium (PBEQ) in which the policymaker plays the mixed strategy $y_{T-1} = \frac{p_{T-1}}{1 - p_{T-1}}(2\delta - 1)$ and the public has beliefs about θ according to Bayes' rule.

A reputational equilibrium where we see zero inflation from a policymaker of type $\theta = 1$ in $T - 1$ exists for a low degree of discounting (high δ) and high initial reputation p_{T-1}. Intuitively the more important future payoffs are, the more likely is it to play zero inflation and to foster reputation, to transfer payoffs from today into the future. For lower values of these variable the policymaker does either play no zero inflation at all or as a mixed strategy $0 < y_{T-1} < 1$.

For $\frac{1}{2} < \delta \leq \frac{1}{2}\frac{1}{p_{T-1}}$ the expected rate of inflation in $T - 1$ then is

$$\pi_{T-1}^e = (1 - 2\delta p_{T-1})\frac{b}{a} \quad (2.36)$$

In the mixed PBEQ inflation declines with a rise in δ or p_{T-1}.

Backus and Driffill originally ignored discounting in their setup.[14] For $\delta = 1$ the mixed equilibrium expected inflation becomes

$$\pi_{T-1}^e = (1 - 2p_{T-1})\frac{b}{a} \quad (2.37)$$

which implies $\pi_{T-1}^e = 0$ for $p_{T-1} = \frac{1}{2}$ and $\pi_{T-1}^e = \frac{b}{a}$ for $p_{T-1} = 0$. Further assuming $\pi_t^e \in \left[0, \frac{b}{a}\right] \forall t$, the public expects $\pi_{T-1}^e = 0$ for $p_{T-1} \geq \frac{1}{2}$.

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[14]They assume $\delta = 1$.

These results may be applied to finite and infinite horizons. As Blackburn and Christensen (1989) point out, subgame perfection is a sufficient but not a necessary condition for time consistency.[8, p. 7] Thus ensuring subgame perfection leads Backus and Driffill (1985) to a dynamically consistent equilibrium behavior for a finite time horizon [15]:[3, p. 534]

The public expects zero inflation with probability

$z_t = 1$ if $p_t > \left(\frac{1}{2}\right)^{T-t+1}$,
$z_t = \frac{1}{2}$ if $p_t = \left(\frac{1}{2}\right)^{T-t+1}$ and
$z_t = 0$ if $p_t < \left(\frac{1}{2}\right)^{T-t+1}$.

A government of type $\theta = 1$ chooses zero inflation with probability

$y_t = 1$ if $p_t > \left(\frac{1}{2}\right)^{T-t}$,
$y_t = \frac{p_t}{1-p_t} \frac{1-\left(\frac{1}{2}\right)^{T-1}}{\left(\frac{1}{2}\right)^{T-1}}$ if $0 < p_t \le \left(\frac{1}{2}\right)^{T-t}$ and
$y_t = 0$ if $p_t = 0$.

For a sufficiently high current reputation zero inflation will be played in equilibrium. The more time is left until the final period, that is the higher $T - t$, the lower is the minimum current reputation required for zero inflation being played and believed. As we can see, in this setup, the problem of time inconsistency of the optimal rule of zero inflation may disappear for sufficient initial reputation if the policymaker's interest in surprise inflation is not known to the public but can be signalled according to Bayes' rule.

Here the presence of incomplete information has positive effects on welfare because it enables strong types to signal their type by keeping initial inflation low. This being anticipated by the public reduces inflationary expectations as well.[34, pp. 444-454]

2.3.2. *Different abilities to precommit and to control inflation.* Barro (1986) shows that a commitment to zero inflation is optimal under full credibility but recognizes that zero inflation need not be the optimal value to commit when there is uncertainty about the ability of commitment.[4] Cukierman and Liviatan (1991) show that zero inflation is only optimal under full credibility because otherwise a strong government has an incentive to announce and deliver a positive inflation rate.[13]

A government is considered strong if it adheres to the announced policy, while a weak government does so only if it finds it optimal at every point in time. When these two types are not observable, it

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[15]Ignoring discounting again.

is the weak type of government that brings in uncertainty about a government's credibility. A weak government may pretend to be of the strong type without being bound by its announcemt. Thus even a strong government finds it optimal to deliver a positive inflation rate, because otherwise it would generate surprise deflation and an increase in unemployment. Since a strong government can only deliver what it has announced before, it announces a positive inflation rate, which is below the discretionary level however.[13, pp. 101, 105]

The timing here makes the difference: First the government makes its announcement which influences the public's expectations. This enables the strong government to signal its type prior to the formation of expectations. [13, p. 102]

We now have identical preferences for both types:

$$U^{Pol}(\pi, \pi^e) = b(\pi - \pi^e) - \frac{a}{2}\pi^2 \qquad (2.38)$$

Expectations are formed as follows:

$$\pi^e = \alpha\pi^a + (1 - \alpha)\frac{b}{a} \qquad (2.39)$$

The expected inflation rate is a convex combination of the announced and the discretionary value. α is the probability that the policymaker is strong.

A strong policymaker solves the following maximization problem:

$$\max_{\pi} b(\pi - \alpha\pi^a - (1 - \alpha)\frac{b}{a}) - \frac{a}{2}\pi^2 \qquad (2.40)$$

s.t.

$$\pi = \pi^a \qquad (2.41)$$

FOC:

$$\pi^a = \frac{b}{a}(1 - \alpha) \equiv\equiv \pi^* \qquad (2.42)$$

As we see now, the optimal inflation rate for a strong policymaker depends on the distribution of its type. Under certainty it would be either $\frac{b}{a}$ for $\alpha = 0$ or 0 for $\alpha = 1$.

Cukierman and Liviatan (1991)[13, p. 103] conclude that $\pi^e = \frac{b}{a}$ if there is no announcement at all. In such a case the policymaker's NEQ utility will be the well known outcome under discretion
$\hat{U}_{Pol} = -\frac{b^2}{2a} < U^*_{Pol} = -(1 - \alpha^2)\frac{b^2}{2a}$ for $\alpha > 0$. Obviously a strong type is better off announcing π^*.

A weak policymaker could simply announce the same inflation rate $\pi^a = \frac{b}{a}(1 - \alpha)$ but is, in contrary to the strong policymaker, not bound by this announcement and would therefore consequently choose an unanticipated $\hat{\pi} = \frac{b}{a}$ which is larger than the convex combination of itself and the strong type's optimal rate of inflation for positive values

of α. And indeed it is optimal for a weak type to announce π^* because any other announcement would reveal his true type. This is because the public knows that the strong type will always choose the optimal value. Public's expectations are then $\pi^e = \alpha\pi^* + (1 - \alpha)\frac{b}{a} = (1 - \alpha^2)\frac{b}{a}$. The weak policymaker's payoff from cheating $\tilde{U}_{Pol} = \frac{b^2}{a}\left(\alpha^2 - \frac{1}{2}\right)$ is strictly larger than that from adhering to his anouncement $U^*_{Pol} = -(1 - \alpha^2)\frac{b^2}{2a}$ for positive values of α.

In the resulting NEQ both types play the same announcement $\pi^* = \pi^a$ but different inflation rates, although both choose positive inflation rates under uncertainty. This result contradicts the results of Backus and Driffill (1985)[3] and Vickers (1986)[34].

The lower the reputation α of a strong policymaker is, the more it adjusts his behavior to that of the weak one.

Cukierman (2000a)[10] refines this work by adding a control error to the model.

He considers again the two types of government[16] as in his co-work with Liviatan from 1991 with different abilities to precommit. In addition to this now the two types also differ in their precision of controlling inflation, represented by an individual control error ϵ_i, $i = s, w$ being uniformely distributed in the range $(-r_i, r_i)$. By assuming $0 < r_s < r_w$, that is the strong type has better control over inflaton as the weak type, he follows the evidence for a positive correlation of the average level of inflation and its variance.[17]

Both types have the same two-period present value function:

$$b(\hat{\pi}_{T-1,i} + \epsilon_{T-1,i} - \pi^e_{T-1}) - \frac{a}{2}\left(\hat{\pi}_{T-1,i} + \epsilon_{T-1,i}\right)^2$$

$$+ \delta\left[b(\hat{\pi}_{T,i} + \epsilon_{T,i} - \pi^e_T) - \frac{a}{2}\left(\hat{\pi}_{T,i} + \epsilon_{T,i}\right)^2\right] \tag{2.43}$$

$\hat{\pi}_{t,i}$ is the type's planned inflation rate in period t which will only partly determine the actual inflation rate $\pi_{t,i} = \hat{\pi}_{t,i} + \epsilon_{t,i}$.

The timing now is as follows: First the policymaker announces the inflation rate $\pi^A_{t,i}$ he promises to choose in the current period and the public builds its expectation about the actual inflation rate that will be chosen according to $\pi^e_t = \beta_t\pi^A_{t,i} + (1 - \beta_t)\hat{\pi}_{t,i}$. The policymaker then choses the actual inflation rate π_t and finally nature chooses $\epsilon_{t,i}$.

Following again the method of backward induction to exclude such strategies that are not credible, Cukierman (2000a)[10] then looks at the subgame in T, where a weak type has the following maximization problem:

$$\max_{\hat{\pi}_{T,w}} b\left(\hat{\pi}_{T,w} + E\left[\epsilon_{T,w}\right] - \pi^e_T\right) - \frac{a}{2}\left(\pi^A_{T,w} + E\left[\epsilon_{T,w}\right]\right)^2 \tag{2.44}$$

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[16]Strong (s) and weak (w).

[17]See Devereux (1989)[14].

s.t.

$$E\left[\epsilon_{t,i}\right] = 0 \forall t, i \tag{2.45}$$

FOC:

$$\hat{\pi}_{T,w} = \frac{b}{a} \tag{2.46}$$

The resulting FOC again is the well known discretionary inflation rate from above. Since T is the last period and no punishment may follow, a weak type always wants to imitate the strong type by announcing $\pi_{T,w}^A = \pi_{T,s}^A$. The public expectations are $\pi_T^e = \beta_T \pi_{T,s}^A + (1 - \beta_T) \frac{b}{a}$.

For the weak type the second-period payoffs under no separation (NS) and separation (SEP) are[18]

$$E\left[U_{T,w}^{NS}\right] = \frac{b^2}{a}\beta_T^2 - \frac{1}{2}\left(\frac{b^2}{a} - a\sigma_{\epsilon,T,w}^2\right) \tag{2.47}$$

$$E\left[U_{T,w}^{SEP}\right] = -\frac{1}{2}\left(\frac{b^2}{a} - a\sigma_{\epsilon,T,w}^2\right) \tag{2.48}$$

A weak type does not want separation because his second period payoff would be smaller under separation, that is $E\left[U_{T,w}^{SEP}\right] < E\left[U_{T,w}^{NS}\right]$.

The strong type is dependable and chooses always $\hat{\pi}_{T,s} = \pi_{T,s}^A$ and public's expectations about his choice again are $\pi_T^e = \beta_T \pi_{T,s}^A + (1 - \beta_T) \frac{b}{a}$. His maximization problem then becomes:

$$\max_{\hat{\pi}_{T,s}} b\left(\hat{\pi}_{T,s} + E\left[\epsilon_{T,s}\right] - \pi_T^e\right) - \frac{a}{2}\left(\pi_{T,s}^A + E\left[\epsilon_{T,s}\right]\right)^2 \tag{2.49}$$

s.t.

$$E\left[\epsilon_{t,i}\right] = 0 \forall t, i \tag{2.50}$$

$$\hat{\pi}_{T,s} = \pi_{T,s}^A \tag{2.51}$$

$$\pi_T^e = \beta_T \pi_{T,s}^A + (1 - \beta_T)\frac{b}{a} \tag{2.52}$$

FOC:

$$\hat{\pi}_{T,s} = \pi_{T,s}^A = \frac{b}{a}(1 - \beta_T) \tag{2.53}$$

Obviously the strong type partially accomodates public suspicions. Again his choice of inflation is higher for lower reputation. Considering the maximization processes of both types in the final stage public expectations become:

$$\pi_T^e = \beta_T \frac{b}{a}(1 - \beta_T) + (1 - \beta_T)\frac{b}{a} = \frac{b}{a}\left(1 - \beta_T^2\right) \tag{2.54}$$

The strong type's second period payoffs are either

$$E\left[U_{T,s}^{NS}\right] = -\frac{1}{2}\frac{b^2}{a}\left(1 - \beta_T^2\right) - \frac{a}{2}\sigma_{\epsilon,T,s}^2 \tag{2.55}$$

or

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[18]Assuming that $E\left[\epsilon_{t,i}^2\right] = \sigma_{\epsilon,t,i}^2$ is the control error's variance.

$$E\left[U_{T,s}^{SEP}\right] = -\frac{a}{2}\sigma_{\epsilon,T,s}^2 \tag{2.56}$$

Contrary to a weak type, a strong type does want separation because his second period payoff would be higher under separation, that is $E\left[U_{T,s}^{NS}\right] < E\left[U_{T,s}^{SEP}\right]$.

The second period payoffs of both types depend on the variance of the control error $\sigma_{\epsilon,T,i}^2$ and on the reputation β_T which is, under separation, either equal to 0 or 1. For $0 < \beta_T < 1$ there is no separation and reputation is intertemporally determined by Bayes' rule[19]:

$$\beta_T = \frac{\beta_{T-1}}{\beta_{T-1} + \frac{r_s}{r_w}\left(1 - \beta_{T-1}\right)} \tag{2.57}$$

From $0 < \frac{r_s}{r_w} < 1$ it follows that $\beta_T > \beta_{T-1}$, that is reputation grows over time with speed determined by $\frac{r_s}{r_w}$.

We now turn to the first stage equilibrium where we only consider equilibrium strategies from the second stage. The weak type faces the following maximization problem:

$$\max_{\hat{\pi}_{T-1,w}} b\left(\hat{\pi}_{T-1,w} + E\left[\epsilon_{T-1,w}\right] - \pi_{T-1}^e\right) - \frac{a}{2}\left(\pi_{T-1,w}^A + E\left[\epsilon_{T-1,w}\right]\right)^2$$

$$+ \delta\left[-\frac{1}{2}\left(\frac{b^2}{a} - a\sigma_{\epsilon,T,w}^2\right) + prob\left(NS|w\right)\frac{b^2}{a}\beta_T^2\right] \tag{2.58}$$

s.t.

$$E\left[\epsilon_{t,i}\right] = 0 \ \forall t, i \tag{2.59}$$

$$prob\left(NS|w\right) = \frac{1}{2r_w}\left(\hat{\pi}_{T-1,s} - \hat{\pi}_{T-1,w} + r_s + r_w\right) \tag{2.60}$$

FOC:

$$\hat{\pi}_{T-1,w} = \frac{b}{a} - \frac{\delta\left(\frac{b}{a}\right)^2\beta_T^2}{2r_w} \tag{2.61}$$

And for the strong type:

$$\max_{\hat{\pi}_{T-1,s}} b\left(\hat{\pi}_{T-1,s} + E\left[\epsilon_{T-1,s}\right] - \pi_{T-1}^e\right) - \frac{a}{2}\left(\pi_{T-1,s}^A + E\left[\epsilon_{T-1,s}\right]\right)^2$$

$$+ \delta\left[-\frac{a}{2}\sigma_{\epsilon,T,s}^2 - prob\left(NS|s\right)\frac{1}{2}\frac{b^2}{a}\left(1 - \beta_T^2\right)\right] \tag{2.62}$$

s.t.

$$E\left[\epsilon_{t,i}\right] = 0 \ \forall t, i \tag{2.63}$$

$$prob\left(NS|s\right) = \frac{1}{2r_s}\left(\hat{\pi}_{T-1,s} - \hat{\pi}_{T-1,w} + r_s + r_w\right) \tag{2.64}$$

$$\hat{\pi}_{T-1,s} = \pi_{T-1,s}^A \tag{2.65}$$

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[19]With $\beta_{T-1} = 1 - prob(w) = prob(s)$ and $prob\left(\pi_{T-1}|i\right) = \frac{1}{2r_i}$.

$$\pi^e_{T-1} = \beta_{T-1}\pi^A_{T-1,s} + (1 - \beta_{T-1})\frac{b}{a} \qquad (2.66)$$

FOC:

$$\hat{\pi}_{T-1,s} = \frac{b}{a}(1 - \beta_{T-1}) - \frac{\delta}{4r_s}\left(1 - \beta^2_T\right)\frac{b^2}{a} \qquad (2.67)$$

Cukierman's (2000a)[10, p. 65] conclusions are:

If there is no separation, reputation grows over time. β_T and β_{T-1} are positively correlated. A higher initial reputation β_{T-1} therefore leads to lower planned inflation of the weak type in the first period. Reputation grows with speed determined by $\frac{r_s}{r_w}$. The lower this ratio, the higher second period reputation.

He further shows that the planned inflation of a strong policymaker decreases with initial reputation if and only if the discount factor δ is sufficiently low.[10, p. 67] Note that in the models of Barro and Gordon (1983)[5] and Backus and Driffill (1985)[3] the discount factor had to be high enough for the time inconsistency to disappear. In Cukierman's (2000a) model under a high enough discount factor the marginal benefit of a higher planned inflation rises with initial reputation more as the losses of such an expansion. The threshold value of δ decreases with policymakers' relative preferences over unemployment versus price stability. So the more weight is given to unemployment relative to price stability, the closer is the range of discount factors that are associated with a positive correlation of initial reputation and planned inflation of the strong type.

Planned inflation of both types is lower for better control precision of the strong type, that is for lower values of r_s. This effect stems from the increase in second period reputation, which is connected with increased losses under separation for both types. Also the more precise the weak type's control of inflation, the lower is the strong type's choice of inflation. The second effect is due to an increase in incentive for the strong type to be even more conservative aiming to reveal his dependability to the public. In general more transparency, that is lower values of r_i, induces all policymakers to be less inflationary. This means that here intrinsic uncertainty about the inflation control precision leads to higher inflation.

2.4. **Extrinsic uncertainty.** There is also of course the possibility of extrinsic supply or demand shocks over which players may be informed asymmetrically. The optimal rule of zero inflation may then not be optimal. Instead a flexible rule can allow reactions to the shocks without raising inflation expectations above zero. Such a model can be found in Persson and Tabellini (2002)[26, pp. 19 et sqq.] and in Bofinger et al. (1996)[9, pp. 163-170]. This is basically a modified version of the Barro and Gordon model from 1983[5] with a stochastic Phillips curve of the

form $u = u_n - (\pi - \pi^e + \epsilon)$[20], where u is the rate of unemployment and u_n is the natural rate of unemployment.

Timing is as follows: First the public builds expectations π^e, then the shock ϵ is realized and finally the policymaker chooses the actual inflation rate π.

The policymaker has the following maximization problem:[21]

$$\max_{\pi} -c \left(u_n - \pi + \pi^e + \epsilon\right)^2 - \pi^2 \tag{2.68}$$

s.t.

$$E\left[\epsilon\right] = 0 \tag{2.69}$$

FOC:

$$\hat{\pi} = \frac{c}{1+c} \left(u_n + \pi^e + \epsilon\right) \tag{2.70}$$

The public has rational expectations:

$$\pi^e = E\left[\pi\right] = cu_n \tag{2.71}$$

A lot of algebra using the above conditions as well as the assumption that $E\left[\epsilon^2\right] = \sigma_\epsilon^2$, leads to the expected welfare[22] under discretion:

$$E\left[\hat{W}\right] = -\left(1+c\right)cu_n^2 - \frac{c}{1+c}\sigma_\epsilon^2 \tag{2.72}$$

Under the optimal rule policy of the Barro and Gordon model, that is $\pi^* = \pi^e = 0$, expected welfare becomes:

$$E\left[W^*\right] = -cu_n^2 - c\sigma_\epsilon^2 \tag{2.73}$$

Assuming now that $E\left[\hat{W}\right] < E\left[W^*\right]$ and solving for σ_ϵ^2, which represents the level of ex-ante uncertainty about the shock ϵ, leads to the result that the shock variance has to be small enough for a rule policy being better than a discretionary policy:

$$\sigma_\epsilon^2 < \left(1+c\right)u_n^2 \tag{2.74}$$

This shows that if the public is uncertain about a future shock and has to build expectations about it, a rule policy will only be efficient for sufficiently low uncertainty. More uncertainty requires flexibility by the policymaker to react appropriately to the extrinsic shock.

A possible solution is a flexible rule policy of the form $\pi_{flex} = 0 + k\epsilon$, which allows a proportional reaction on the shock. Rational expectations are then $\pi^e = E\left[\pi_{flex}\right] = kE\left[\epsilon\right] = 0$. k determines the degree of flexibility in this alternative policy rule. The optimal degree of flexibility k^* can be gained by maximizing as follows:

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[20]$E\left[\epsilon\right] = 0$

[21]The policymaker's objective function is now assuming quadratic benefits from exploiting the Phillips curve.

[22]Assuming that the policymaker's objective function represents welfare.

$$\max_{k} -c \left(u_n - \pi_{flex} + \pi^e + \epsilon \right)^2 - \pi_{flex}^2 \tag{2.75}$$

s.t.

$$\pi_{flex} = k\epsilon \tag{2.76}$$

$$E\left[\epsilon\right] = 0 \tag{2.77}$$

$$E\left[\epsilon^2\right] = \sigma_\epsilon^2 \tag{2.78}$$

FOC:

$$k^* = \frac{c}{1+c} \tag{2.79}$$

The resulting expected welfare under a flexible rule policy

$$E\left[W_{flex}\right] = -\frac{c}{1+c}\left[(1+c)\,u_n^2 + \sigma_\epsilon^2\right] \tag{2.80}$$

Comparing with the expected welfare under a fix policy rule and a discretionary policy, it follows that

$$E\left[W_{flex}\right] > E\left[\hat{W}\right] \tag{2.81}$$

and

$$E\left[W_{flex}\right] > E\left[W^*\right] \tag{2.82}$$

The flexible rule policy, by construction, welfare-dominates both other alternative policies. The reason intuitively is that with such a flexible rule an appropriate reaction on the shock is possible, without raising inflation expectations above zero. While the fixed zero inflation rule is only time consistent for sufficiently low uncertainty about the shock, there is a flexible rule which does even better in terms of welfare and is also time consistent.

2.5. **Empirical Evidence.** Ireland (1998) tests whether the Barro and Gordon model from 1983 explains the behavior of inflation in the United states.[20] Short-run dynamics of inflation and unemployment must be rejected while the long-run dynamics of a linear and positive cointegrated relationship are supported.

He uses a bivariate time-series model for inflation and unemployment with US quaterly data from 1960 to 1997. The finding is that the model does indeed serve as a positive theory for the inflation behavior during this period.

The data shows a rise in inflation between 1960 and 1980 and a subsequent fall. The unemployment rate moves somewhat correspondingly, not behaving consistent with a Philipps curve, showing a positive correlation instead.[20, p. 14] Unemployment is highest when inflation also has its peak.[20, pp. 16-17] According to the model, in the long-run inflation and unemployment should be nonstationary and cointegrated,

that is they move together somehow.[5][20, p. 5][23] This is confirmed by Ireland's results.[20, pp. 13-14]

Therefore these empirical results support the hypothesis that long-run trends of unemployment are coupled with similiar trends in the long-run behavior of inflation when the central bank cannot commit to a policy of price stability. Thus the time consistency problem may indeed underlie the behavior of inflation in the United States.[20, p. 9]

Ruge-Murcia (2002)[31] constructs a general model with asymmetric preferences in which the Barro and Gordon model from 1983 and another model of Cukierman (2000b)[11] are treated as special cases. The results basically confirm Ireland's results from 1998 but Likelihood Ratio (LR) tests suggest that US inflation can be explained better by Cukierman's model (2000b), where a central banker targets the expected natural rate of unemployment and weighs more severely the losses from positive than from negative deviations from the expected natural rate.[11, p. 1][31, pp. 4,12] This leads to an inflation bias even if the targeted employment is the normal level because of precautionary monetary expansion. Cukierman's data supports this source of inflation bias for the United States at least until 1985. Thereafter central banks became increasingly independent but precautionary motives may still be relevant for individual central banks.[12, pp. 560-561]

3. Final conclusions

Time inconsistent monetary planning may lead to positive inflation bias. Barro and Gordon (1983) show that zero inflation will be chosen by a weak policymaker under perfect information in a single stage game, due to rational expectations of the public. In the infinitely repeated game the tit for tat strategy played by the public will not lead to a zero inflation equilibrium for plausible discount factors but the unforgiving strategy grim trigger will.

In a repeated signalling game policymakers' different and invisible preferences over surprise inflation may lead to a zero inflation equilibrium for sufficiently high initial reputation and low degree of discounting.[3] Committing to zero inflation is only optimal under full credibility.[4] When there is private information about the ability to commit, even a strong government announces and delivers a positive inflation rate.[13] Uncertainty over policymakers' ability to control inflation also leads to higher inflation bias.[10]

Excessive public uncertainty about shocks also leads to time inconsistency of the zero inflation policy. Instead there is an optimal and time consistent flexible policy which does not raise inflation expectations above zero.[9][26]

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[23]Technically: The correlation coefficient between the inflation rate and the unemployment rate is positive, in contradiction to the Phillips curve.

There is evidence for time inconsistency underlying US inflation behavior.[20][31]

LIST OF FIGURES

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