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AN EXPERIMENTAL 4–PHASE D.P.S.K. STEREO SOUND SYSTEM: the effect of multipath propagation

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AN EXPERIMENTAL 4-PHASE D.P.S.K. STEREO SOUND SYSTEM: THE EFFECT OF MULTIPATH PROPAGATION

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Summary

This Report describes a theoretical and experimental study of the effects of multipath propagation ('echoes') on an experimental 4-phase d.p.s.k. stereo sound system. The system has already found experimental applications in outside broadcast radio links and could be used in satellite broadcasting.

It is shown in the Report that, for an echo to impair reception significantly when signal-strengths are low, its relative amplitude must be at least 15%. Also, the effect of an echo is largely independent of its delay. The results of the tests have also been used to calculate the effect of co-channel interference.

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1. Introduction

Increasing use is being made of digital techniques for the distribution and broadcasting of high-quality sound signals, because digital coding of an analogue signal greatly increases the immunity of the signal to transmission impairments.

At present, high-quality sound signals are distributed to the principal v.h.f./f.m. transmitters in digital form. Another application of digital methods is found in parts of the television network in which the sound signal is carried in digital form by modifying the waveform of the line synchronising pulses of the video signal (sound-in-syncs).

There are several other areas where digital techniques might be applied with advantage. An area in which experimental application has already been found is outside broadcast (OB) radio links. When a digital system has been used, it has been found possible to achieve stereo operation over long distances.* Other potential areas for digital techniques are direct satellite broadcasting and possible terrestrial broadcasting in part of Band I when that Band ceases to be used for 405-line television.

Primarily with the OB radio link application in mind, an experimental system for transmitting a digitally coded high-quality stereo sound signal has been developed, using 4-phase differential-phase-shift-keying (d.p.s.k.). A description of the experimental equipment and its noise performance is given in References 1 and 2.

* Two stereo Outside Broadcasts were carried out in December 1977 using 4-phase d.p.s.k. signals on radio links.

In practical applications of this system, the performance, in terms of maximum range for a given error-rate, will be limited by a number of factors. One such factor is noise and the effect of this has been described earlier.[1] The conclusion of this earlier work was that the system performance was very close to the theoretical limits. Other factors are co-channel and adjacent-channel interference and multipath propagation. For terrestrial applications such as OB radio links, multipath propagation is likely to be one of the most significant of these factors. In the case of CEEFAX transmissions, it has been shown that the combined effect of noise and multipath is critical in determining the limit of the service area.

This Report describes laboratory measurements which were made to study the effect of multipath propagation on the experimental 4-phase d.p.s.k. system. The measurements were made using a v.h.f. multipath simulator[3] which can simulate echoes over a wide range of delays and levels. The Report also compares the measured performance of the d.p.s.k. system with that predicted theoretically.

2. The 4-phase d.p.s.k. system

2.1. General

A brief description of the experimental 4-phase d.p.s.k. system is given in the following sections. A more complete description is given in References 1 and 2. Recently, some minor modifications have been made to the equipment so that it differs slightly from that described earlier and these changes are included in the description below.

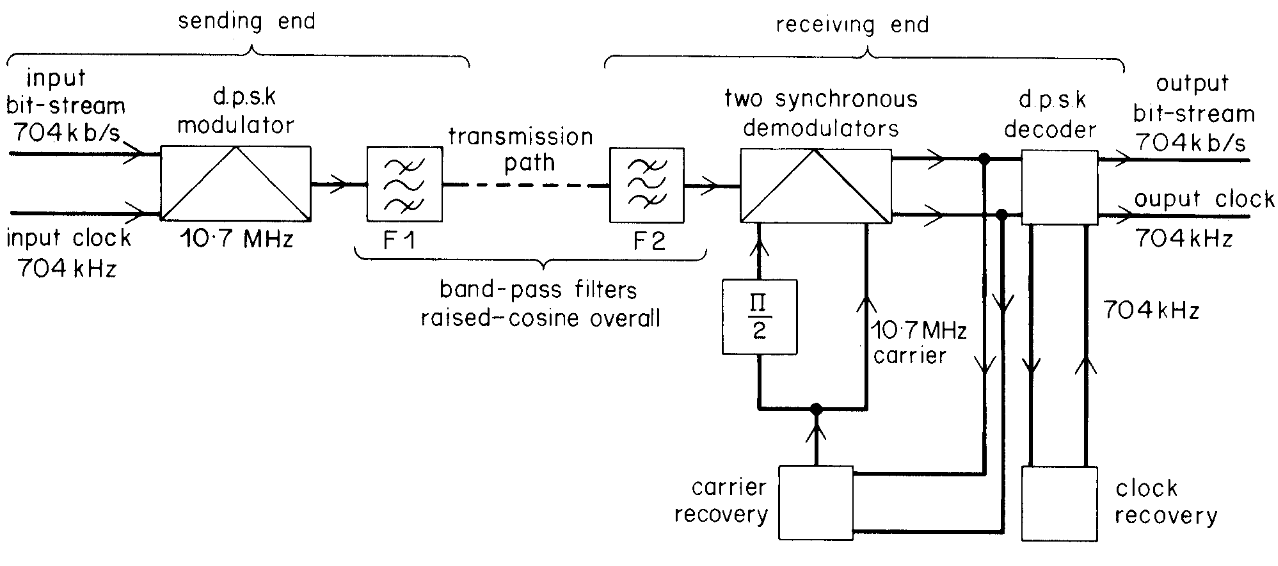


Fig. 1 - Block diagram of experimental modem

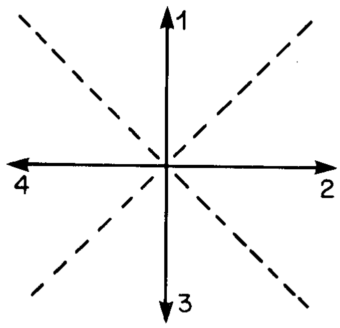


Fig. 2 - Phase states of d.p.s.k. signal

A block diagram of the experimental modem is shown in Fig. 1. It operates at a frequency of 10·7 MHz; this is a standard intermediate frequency and was chosen for convenience.

2.2. Modulation

The 4-phase d.p.s.k. system was chosen because it offers a good compromise between the conflicting requirements of low carrier-power (for a given range of radio link), narrow bandwidth and low cost.

In this system, the information is transmitted by changing the phase of the carrier in steps which are integral multiples of 90°. The phase of the transmitted signal has four rest-states numbered 1 to 4 in Fig. 2. The signal is thus made up of a set of symbols each of which has four possible values and therefore carries two bits of information. The rate at which the phase of the signal changes is known as the symbol-rate f_s which is equal to half the transmitted bit-rate f_b.

For the experimental system the bit-rate was originally 652 kbit/s and the symbol-rate was 326 ksymbols/sec. Subsequently, these figures were increased to 704 kbit/s and 352 ksymbols/sec respectively, to match equipment that exists for coding a high-quality stereo sound signal into a digital bit-stream of 704 kbit/s.[4] Each sound channel has 15 kHz bandwidth. The sampling frequency is 32 kHz and the samples are first coded into 13-bit linear p.c.m. form[5] and then 'near-instantaneously' companded[6] into 10 bits per sample.

Referring to Fig. 1, the data to be transmitted plus an appropriate clocking signal are applied to the 4-phase d.p.s.k. modulator. The 4-phase d.p.s.k. signal is produced by rapidly switching the phase of the output from a 10·7 MHz crystal oscillator. The output signal is identical to that produced by amplitude modulation of in-phase and quadrature carriers by rectangular pulses. Digital circuitry within the modulator converts the incoming 704 kbit/s serial bit-stream into a form suitable for driving the switches that set the phase of the output d.p.s.k. signal.

2.3. Spectrum shaping filters

The spectrum of the d.p.s.k. signal is shaped by the filters shown as F_1 and F_2 in Fig. 1 so that the signal at the input to the demodulator has a 'raised-cosine' spectrum. Both filters are bandpass and operate at the intermediate frequency of 10·7 MHz.

The signal at the output of the modulator can be regarded as changing instantaneously between phase-states and it has a spectrum of the form (sin x)/x with zero energy at frequencies spaced from the carrier by multiples of the symbol-rate (352 kHz). The source filter F_1, has a rectangular frequency characteristic which restricts the bandwidth of the d.p.s.k. signal to ±350 kHz.

The receiver filter F_2, converts the truncated (sin x)/x spectrum at the output of the source filter F_1 into a 'raised-cosine' spectrum.

2.4. Demodulation

At the receiver, the d.p.s.k. signal is synchronously demodulated by two synchronous demodulators fed by in-phase and quadrature feeds of a locally-derived reference (Fig. 1). The locally-derived reference is recovered from the incoming d.p.s.k. signal using the 'baseband remodulation'[2] method of carrier recovery. The circuit resembles a rather complicated phase-locked-loop in which the conventional phase-detector is replaced by circuitry which processes the output of the two synchronous demodulators.

The phases of the two demodulating carriers, with respect to the incoming d.p.s.k. signal, are shown by the dashed lines in Fig. 2. By examining the polarity of the outputs from the two synchronous demodulators, the phase of the received signal can be placed in one of four quadrants. The phase-change between successive symbols of the d.p.s.k. signal can then be deduced by comparing the quadrants that the d.p.s.k. signal occupies from one symbol to the next. Once the phase-changes between successive symbols have been found, the transmitted message can be decoded.

An essential part of the decoding process is a clocking-signal which samples the outputs of the two synchronous demodulators at the symbol-rate. Each symbol of the signal must be sampled when its value has reached one of the discrete signal states. Sampling during transitions between states will increase the probability of error when noise is present.

The clocking signal is recovered from the outputs of the two synchronous demodulators. These outputs are two-level 'raised-cosine' pulse streams and the transitions between the two levels are separated by an integral number of symbol periods. The clock-recovery circuit detects these transitions and phase-locks an oscillator whose free-running frequency is close to the bit-rate frequency (704 kHz). The symbol-rate clocking signal (352 kHz) is derived by frequency-dividing the bit-rate clocking signal.

Originally, the oscillator within the clock-recovery circuit was a simple multivibrator. More recently, this has been replaced by a crystal oscillator which results in much less jitter on the recovered clocking-signals.

The received bit-rate and symbol-rate clocking-signals are passed on to the decoder and used to decode the demodulated data signal and to reform it into a serial bit-stream.

3. Multipath performance of experimental modem

The aim of the experiments described here was to investigate the effect on a d.p.s.k. signal of an echo having a wide range of delays and levels. In a practical application of the system, multipath propagation alone is very unlikely to cause errors in the demodulated bit-stream. If this were the case, then the level of the echo would have to be about 70% of the level of the direct signal. The main practical effect of multipath propagation is expected to be a general reduction in the ruggedness of the signal. For example, consider a receiving site where there is only a direct signal, i.e. no multipath effect. For any given error-rate of the demodulated d.p.s.k. signal, a certain signal-to-noise ratio (and hence received field-strength) will be required. When an echo is present, the field-strength will have to be greater to obtain the same error-rate.

From the above discussion, it is clear that a good way of expressing the effect of multipath is in terms of the increase in received field-strength required to compensate for an echo of particular level and delay. This approach forms the basis of the experimental measurements.

The experimental arrangement adopted for measuring the effect of multipath is shown in block form in Fig. 3. The digital signal used in the tests was a long pseudo-random sequence which repeated itself every $2^{15} - 1$ bits. The demodulated sequence was compared with a suitably delayed version of the source sequence and the detected errors were applied to a counter.

A single echo was added to the d.p.s.k. signal using a v.h.f. multipath simulator.[3] As shown in Fig. 3, this device fed the d.p.s.k. signal along a direct path, and a delayed path, whose delay and attenuation were adjustable. The relative delay between the two paths could thus be varied in 100 ns steps over the range 0 to about 100 μs. A phase-rotator was included in the delayed-signal path in order to allow fine adjustments to be made to the relative phase of the direct and delayed signals. The delayed signal, after passing through a variable attenuator, was combined with the direct signal in a hybrid. Using the variable attenuator, the relative level of the delayed signal could be varied in 0·1 dB steps over the range 0 to −25 dB.

Because the multipath simulator[3] was designed to operate within the v.h.f. Band, it was necessary for the 10·7 MHz signal at the output of the d.p.s.k. modulator to be converted to 90·7 MHz before being applied to the simulator. Similarly, after passing through the multipath simulator the signal was down-converted back to 10·7 MHz.

Wideband Gaussian noise from a zener-diode was added to the signal at the output of the down-converter. The level of the added noise was controlled by a variable attenuator so that the carrier-to-noise ratio at the input to the demodulator could be varied in 0·1 dB steps over the range 0 to 20 dB.

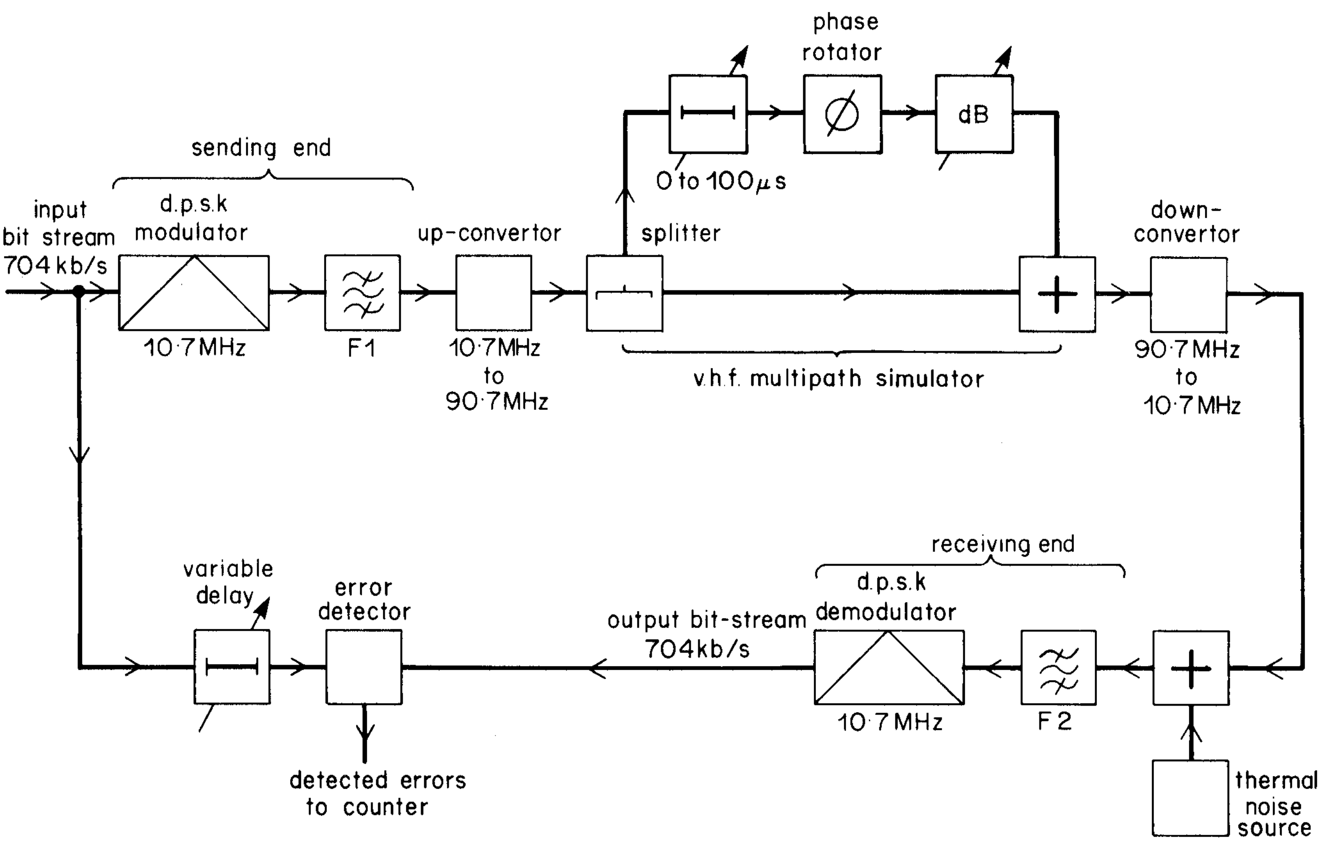
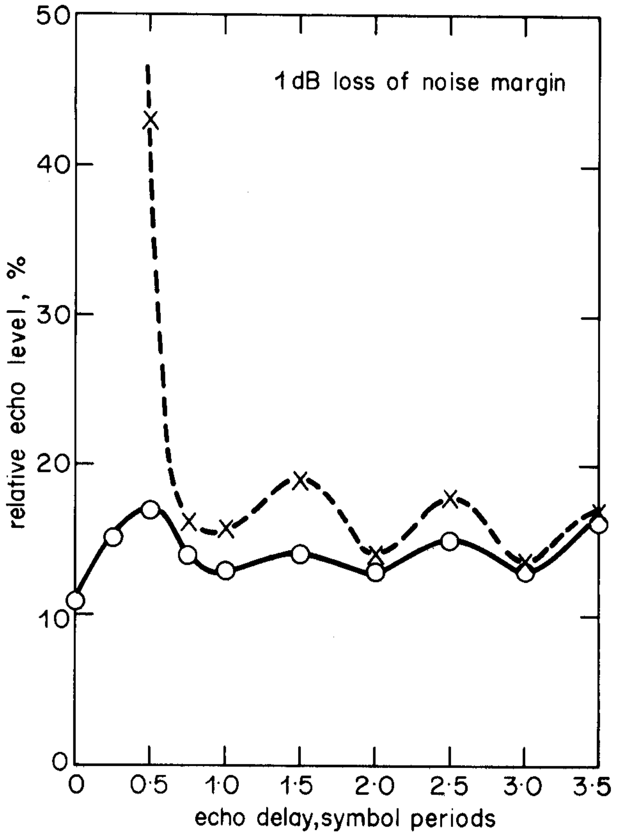
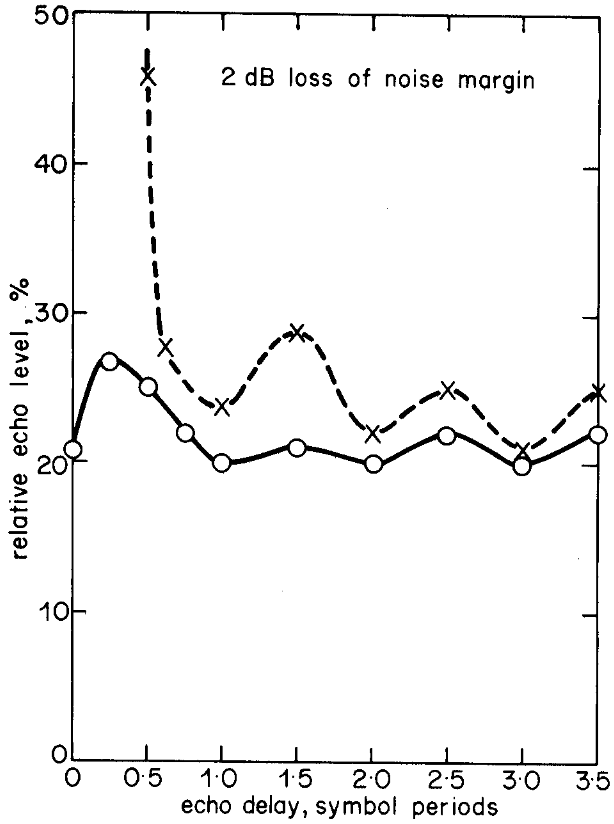


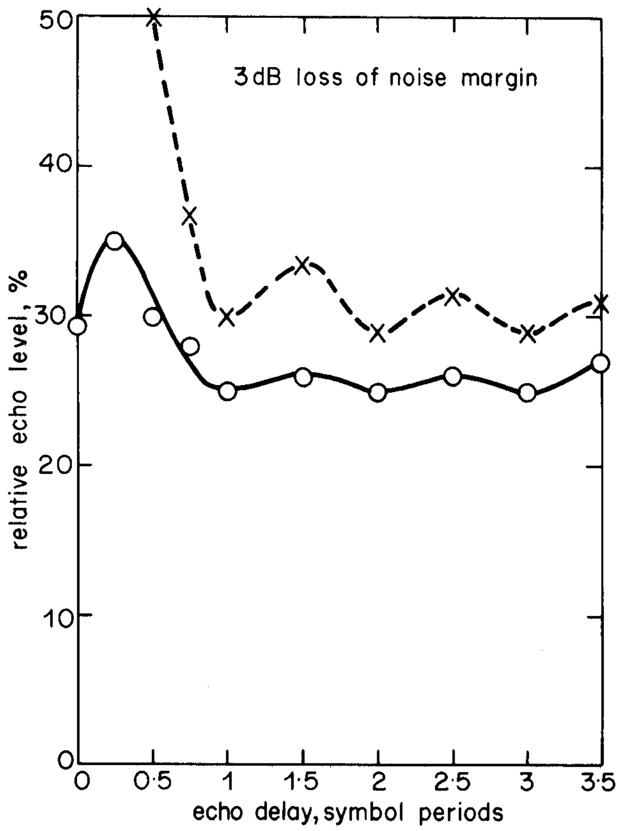
Fig. 3 - Experimental set-up for measuring multipath performance



(a)



(b)



(c)

Fig. 4 - Measured levels of echo as a function of echo delay for 1 dB, 2 dB and 3 dB loss of noise margin, measured at the limiting bit-error rate of 1 in 10^3

(*a*) 1 dB loss of noise margin (*b*) 2 dB loss of noise margin

(*c*) 3 dB loss of noise margin

O———O worst-case echo

X– – – –X best-case echo

The two bandpass filters F_1 and F_2 have been already described in Section 2.3 and were placed at each end of the simulated transmission path.

The procedure for the tests was as follows. In the absence of a delayed signal the level of the noise added to the d.p.s.k. signal was adjusted until the bit-error rate of the demodulated data signal (measured by counting the number of errors in one second) was 1 in 10^3. The carrier-to-noise power ratio for this error rate was measured as being 10·8 dB. This value of error rate was found to be convenient since the errors then occurred frequently enough for accurate measurement. It also represents the error rate at which it would be desirable to mute the received signal since the effect of the errors on the sound programme at that rate is generally graded as 'somewhat annoying'.[7]

A delayed signal with a chosen relative level and delay was then added keeping the noise level constant. This was so that the phase-rotator (see Fig. 3) could be adjusted to give a minimum and a maximum value of error rate. These settings correspond to the extremes between which the effect of a signal with this nominal delay could vary, depending upon the precise carrier phase at which the delayed signal combined with the direct signal at the receiver.

For each of the phase rotator settings in turn, the level of the added noise was then reduced, initially, by 1 dB and the relative level of the delayed signal that, under those circumstances, restored the bit-error rate to 1 in 10^3 was recorded. This level of delayed signal effectively degrades or reduces the noise margin by 1 dB; in order to compensate for the effect of a delayed signal of this form, the received field-strength would need to be increased by 1 dB. The above process was repeated for noise level reductions of 2 dB and 3 dB.

Measurements were made using echoes with delays up to 10 μs and the results are plotted in Fig. 4; three sets of graphs are shown, one for each level of noise added to the d.p.s.k. signal. The delay of the echo has been plotted in terms of numbers of symbol periods of the d.p.s.k. signal (the symbol-rate was 352 ksymbols/sec and a symbol period was 2·84 μs). For an echo with a one symbol period delay, the difference between direct and reflected signal paths would be about 0·8 km (0·5 miles).

It can be seen from the results shown in Fig. 4 that the effect of echoes with delays in the range 1 to 2 symbol periods is virtually the same as that of echoes in the range 2 to 3 symbol periods. For longer delays, the results are likely to repeat themselves in a similar manner. This is due to the fact that, for a random data signal, there is no correlation between symbols that are more than one symbol period apart. Digital sound signals should give the same effect as they are virtually random signals.

It is also apparent from the results that for delays of longer than one symbol period there is little difference between worst-case and best-case echoes. However, for shorter delays, there is a marked difference between the results for worst-case and best-case echoes. In fact, for delays less than about half a symbol period, a best-case echo can improve the performance of the system, as suggested in Fig. 4. This effect can be explained by considering the hypothetical case of an echo of zero delay whose phase relative to the direct signal can be varied. If, say a 30% echo is added in antiphase to the direct signal, the resultant signal would simply decrease in amplitude by 30% with a consequent loss of about 3 dB in noise margin. On the other hand if the echo is added in phase the amplitude of the resultant signal will increase by 30%. The effect of the echo is therefore beneficial and it will improve the performance of the system.

4. Theoretical effect of multipath propagation on a 4-phase d.p.s.k. system

The calculation of the effect of both multipath and noise on a 4-phase d.p.s.k. system is somewhat complicated, both because the d.p.s.k. signal has several states and because the noise has Gaussian statistics.

A reasonably exact analysis using computer simulation has been given elsewhere[8] but a somewhat simplified analysis that adopts several assumptions without sacrificing much accuracy is given in the Appendix. It is felt that the simplified analysis leads to a better understanding of the effects of multipath propagation than can be gained from very complicated mathematics.

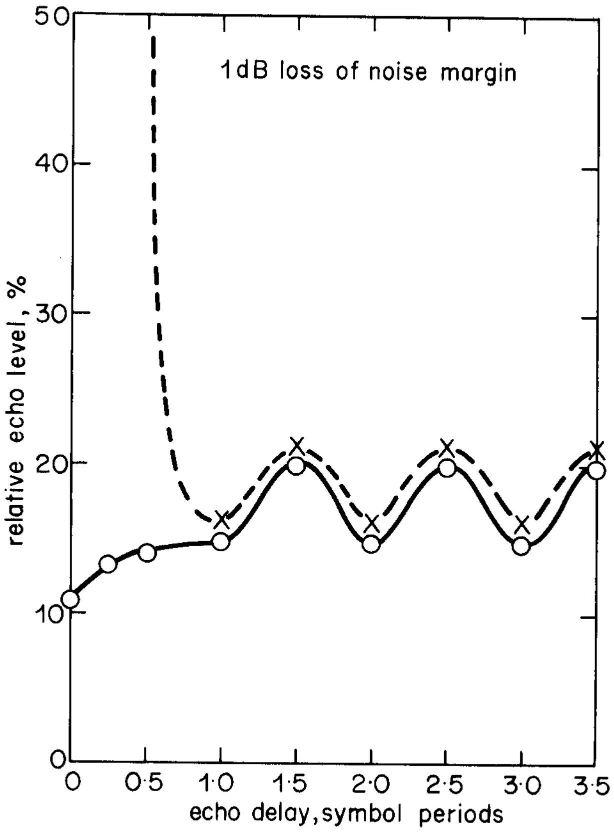
The assumptions that have been made for the simplified analysis are listed below:

(a) The instrumentation of the system was assumed to be perfect. The outputs from the two synchronous demodulators (see Fig. 1) were assumed to have 100% eye-height. The carrier- and clock-recovery circuits were assumed not to impair the performance of the system. In theory the system should give a bit-error rate of 1 in 10^3 when the received carrier-to-noise ratio falls to 10·3 dB (see Appendix Section 8.2). The experimental system gave this error rate for a carrier-to-noise ratio of 10·8 dB, as stated in Section 3.

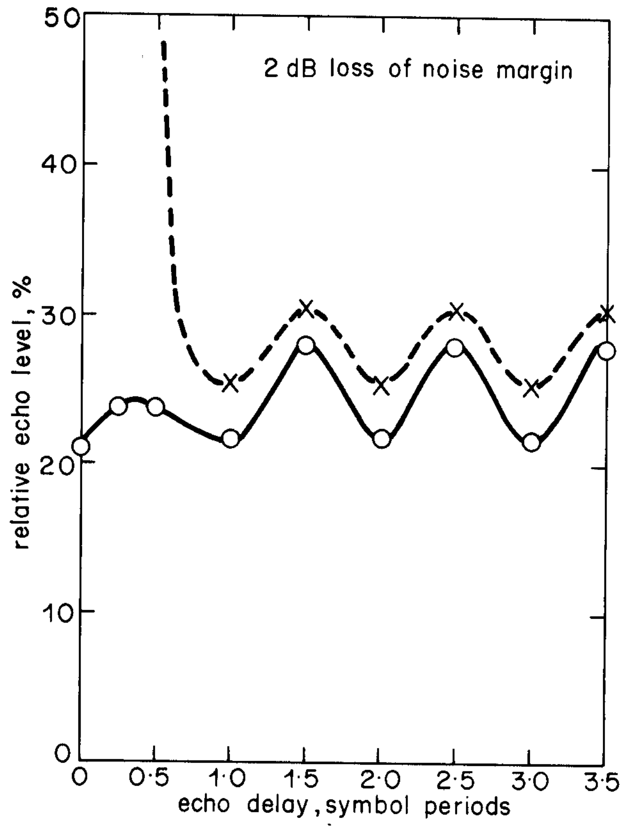
(b) In the presence of multipath propagation and noise, the carrier-recovery circuit was assumed to align itself so as to minimise the effect of the multipath. This assumption was verified by experiment on the practical 4-phase d.p.s.k. modem. The effect is significant for echo delays which are shorter than one symbol period.

(c) The influence of multipath propagation on clock-recovery was assumed to be small. Again, this was verified by experiment.

The calculated results are plotted in Fig. 5 using the same format as for the experimental results so that a direct comparison is possible.



(*a*)



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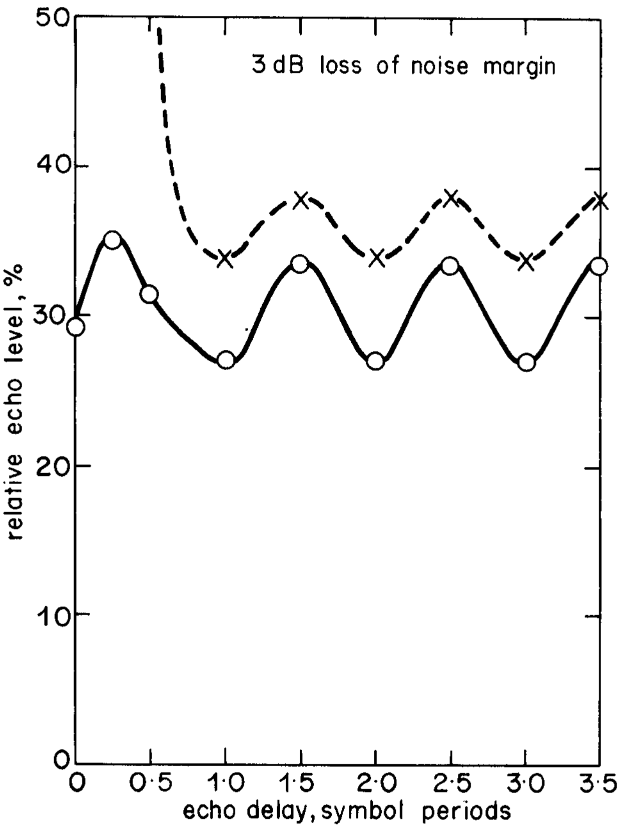

(*c*)

Fig. 5 - Calculated levels of echo as a function of echo delay for 1 dB, 2 dB, and 3 dB loss of noise margin, calculated at the limiting bit-error rate of 1 in 10^3

(*a*) 1 dB loss of noise margin (*b*) 2 dB loss of noise margin

(*c*) 3 dB loss of noise margin

O———O worst-case echo

X– – – –X best-case echo

5. Discussion

The experimental and calculated results shown in Figs. 4 and 5 are in broad agreement. In general, the calculated results indicate that the d.p.s.k. signal should be slightly more immune to multipath propagation than was found in the experiments. However, the shapes of the two sets of curves are very similar and any differences can be accounted for by over-simplifications in the theoretical analysis.

Both sets of results indicate that, under conditions of limiting field-strength an echo with a relative level of 15% will be equivalent to a 1 dB loss of signal-strength. Furthermore, for a 3 dB loss in 'noise margin', an echo of 25% to 30% would be necessary. These figures are largely independent of the echo delay if worst-case situations only are considered. Taking an average of the results for best- and worst-cases the signal is slightly more tolerant of echoes with short delays (less than one symbol period).

It is very difficult to interpret the results of this Report in terms of their practical implications in applications such as digital sound broadcasting in part of Band I. There is very little statistical information on the incidence of multipath at receiving sites. Furthermore, the incidence of multipath propagation will be affected by the transmission frequency and the directivity of transmitting and receiving aerials. Work is in hand within Research Department[9] using an existing 405-line television transmission to gather statistical information on the incidence of multipath signals. Once these statistics are available it should be possible to make an assessment of the ruggedness of the 4-phase d.p.s.k. system in practice.

The results of the multipath experiments described in this Report can also be applied to co-channel interference. Consider the case of an interfering d.p.s.k. signal that has exactly the same symbol rate and carrier frequency as the wanted d.p.s.k. signal. It is impossible then to distinguish between a long delayed echo (greater than one symbol period) and the co-channel interference. In both cases, for a random digital sequence, there is no correlation between the wanted and interfering signals. In practice, however, the carrier frequencies and the symbol rates of the two co-channel signals will be different from each other so that both the relative phase and the relative timing of the two signals will vary in a manner similar to that for a single echo with delay greater than one symbol period. As can be seen from Fig. 4, the effects of these variations in relative phase and delay would be small and, interpolating from the results of Fig. 4, a co-channel signal at a relative level of −20 dB (10%) would reduce the noise margin of the signal by about 0·7 dB. A co-channel signal more than 30 dB down on the wanted signal would produce an insignificant effect.

The results quoted in this Report can also be applied to 4-phase p.s.k. or d.p.s.k. systems in general, but certain conditions must be met before the results can be applied directly. The systems considered must have a performance that is close to the theoretical limit; this implies well-instrumented carrier- and clock-recovery circuits and a received eye-height of 80% or greater in the absence of multipath effects. For a system with poorer performance, the effect of multipath propagation will be greater than indicated by the results of Fig. 4. As a rule of thumb, an echo with a relative level of 10%, will produce the same effect on a system with 50% received eye-height as an echo of 20% relative level produces on a system with 100% received eye-height.

6. Conclusions

Measurements have been made of the effect of multipath propagation, involving a single reflected path, on the experimental 4-phase d.p.s.k. system. This case has also been analysed theoretically by making appropriate assumptions and approximations. Good agreement between the theoretical and practical results has been obtained. The results of the multipath measurements have also been used to calculate the effect of co-channel interference from an identical signal.

The experimental work has shown that near the limit of performance a single echo with a relative level of 15% is equivalent to a reduction of 1 dB in signal level, or signal-to-noise ratio. For a 3 dB loss in noise margin an echo level of about 25% to 30% is necessary. These results are largely independent of the echo delay; on average, the signal is slightly more immune to echoes with short delays (less than one symbol period delay) than those with long delays. For the case of co-channel interference, a protection ratio of greater than 20 dB will ensure that the noise margin of the wanted signal is degraded by less than 1 dB. In areas where the signal strength is greater, the system will have a greater immunity to multipath and co-channel interference.

The practical implications of the experimental results are difficult to assess, particularly for example in the case of possible digital sound broadcasting in part of Band I, because there is little knowledge about the incidence of multipath effects. Without suitable statistical evidence, it is not possible to draw any definite conclusion about the effect of multipath propagation on the service area for such a system. Nevertheless, it is expected that the effects of multipath propagation will not be too serious.

7. References

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8. Appendix

Calculation of the effect of multipath propagation

8.1. General

In this Appendix, the effect of multipath on the noise performance of the 4-phase d.p.s.k. system is calculated. As mentioned earlier (Section 4) the analysis is fairly complicated. Every attempt has been made to simplify the problem by making suitable approximations. Even so, the procedure to arrive at a single result is fairly involved and takes many individual steps.

In the following Sections, the effect of noise alone is first considered and then the joint effect of multipath and noise together. The first Section may appear unnecessary but it shows, in as simple a way as possible, how the analytical method is developed. It also introduces the notation used throughout the rest of the Appendix. Extensive use is made of vector diagrams as an aid to the analysis.

8.2. Bit-error rate due to noise alone

In this Section the carrier-to-noise ratio required for a bit-error rate of 1 in 10^3 is calculated from first principles.

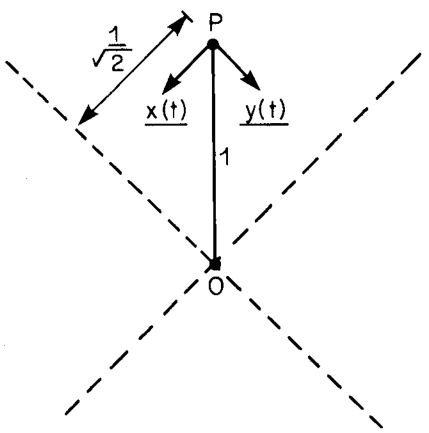

Fig. 6 - Vector diagram to illustrate the effect of noise in the absence of multipath

Referring to Fig. 6, the vector $\overline{OP}$ of unit amplitude represents the received d.p.s.k. signal occupying one of the four possible phase-states. The dashed lines represent the phases of the quadrature carriers used for synchronous demodulation. By examining the polarities of the outputs from the demodulators, the signal can be placed in one of four quadrants. The dashed lines in Fig. 6 can therefore be regarded as decision axes.

If the noise $\nu(t)$ added to the signal has an r.m.s. value σ, then the carrier-to-noise ratio =

$$20 \log_{10}\left(\frac{1}{\sigma\sqrt{2}}\right) \text{dB} \qquad (1)$$

The added noise $\nu(t)$ can be resolved into two quadrature components such that:

$$\nu(t) = x(t) \cos \omega_c t + y(t) \sin \omega_c t \qquad (2)$$

where ω_c = the carrier frequency in rads/s, or $\omega_c/2\pi$ is the carrier frequency.

The quantities $x(t)$ and $y(t)$ are uncorrelated Gaussian variables with the same r.m.s. value, σ, as $\nu(t)$.[10] The quadrature noise components are shown as $\underline{x(t)}$ and $\underline{y(t)}$ on the vector diagram and they too have the r.m.s. value σ.

The received phase-state or symbol will be placed in the wrong quadrant if either $\underline{x(t)}$ or $\underline{y(t)}$ crosses one of the decision axes. This will happen if the peak magnitude of $\underline{x(t)}$ or $\underline{y(t)}$ exceeds $1/\sqrt{2}$, with the right polarity. Whenever one symbol is incorrectly detected, two bits will be decoded in error.[1] However, since there are two bits for every symbol, the probability of a symbol-error is the same as the probability of a bit-error.

The probability of a bit-error can be written as

bit-error probability =

$$\tfrac{1}{2}\left(\text{probability that } x(t) \text{ has a magnitude greater than } \frac{1}{\sqrt{2}}\right)$$

$$+ \tfrac{1}{2}\left(\text{probability that } y(t) \text{ has a magnitude greater than } \frac{1}{\sqrt{2}}\right) \qquad (3)$$

The factors of a half arise since the probability of $x(t)$ or $y(t)$ having the right polarity to cause an error is a half.

The quantities $x(t)$ and $y(t)$ are Gaussian with r.m.s. value σ and it can be shown that the probability that one of them has a magnitude greater than $1/\sqrt{2}$ is

$$= 2\left(1 - P\left(\frac{1}{\sigma\sqrt{2}}\right)\right) \qquad (4)$$

where $$P(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp[-z^2/2]\, dz \qquad (5)$$

$P(z)$ is the normal or Gaussian distribution function and is tabulated in Reference 11. Hence from Equation (3), the bit-error probability

$$= 2\left(1 - P\left(\frac{1}{\sigma\sqrt{2}}\right)\right) \qquad (6)$$

For a bit-error probability of 1 in 10^3

$$P\left(\frac{1}{\sigma\sqrt{2}}\right) = 0{\cdot}9995 \qquad (7)$$

From the tables in Reference 11, Equation (7) is satisfied if

$$\frac{1}{\sigma\sqrt{2}} = 3{\cdot}29 \qquad (8)$$

Hence for a bit-error rate of 1 in 10^3 the carrier-to-noise ratio is given by:

$$\text{carrier-to-noise ratio} = 20 \log_{10}(3{\cdot}29)\ \text{dB} = 10{\cdot}3\ \text{dB} \qquad (9)$$

8.3. Effect of a single echo with n symbol periods delay

If the d.p.s.k. signal is modulated by random data, there is no correlation between one symbol of the message and the next. Hence, the effect of an echo with a one symbol period delay is exactly the same as one with any integral number of symbol periods delay.

The calculation of the effect of such an echo follows the same procedure as for the case with noise only, the only difference being that the vector diagram of Fig. 6 is complicated by the addition of the echo signal.

Referring to Fig. 7, the vector <u>OP</u> of unit amplitude represents the wanted signal occupying one of the four phase-states. Radiating from P, four other vectors of length α are shown representing an echo signal of relative level α. Four vectors are shown since the echo signal of one symbol period delay can have any one of four values, all of which are equally probable. At the sampling instant, the received d.p.s.k. signal will be the vector sum of the wanted and echo signals and will be represented in Fig. 7 by a vector from 0 to one of the tips of the four radials centred on P. The probability of the signal occupying any one of the four possible positions is ¼. As an inset to Fig. 7, the orientation of the noise vectors <u>x(t)</u> and <u>y(t)</u> are also shown (see previous Section).

In Fig. 7, the phase of the echo signal shown is a worst-case and will cause the greatest degradation to the d.p.s.k. signal. The reasons for this can be seen by considering the magnitudes required for the noise vectors <u>x(t)</u> and <u>y(t)</u> to cause the received d.p.s.k. signal to be placed in the wrong quadrant. For the case shown in Fig. 7, the noise vectors have to exceed a magnitude of $1/\sqrt{2} - \alpha$ before an error can be caused. The vector diagram for the best-case phase of echo is shown in Fig. 8. In Fig. 8 the noise vectors would have to exceed a greater magnitude, $1/\sqrt{2} - \alpha/\sqrt{2}$ before an error can be caused.

For the worst-case echo the probability of a bit-error is given below. Probability of a bit-error for worst-case echo phase

$$= \tfrac{1}{2} \times \tfrac{1}{4}\left[\text{probability that } x(t) \text{ has magnitude greater than } \frac{1}{\sqrt{2}} - \alpha\right]$$

$$+ \tfrac{1}{2} \times \tfrac{1}{2}\left[\text{probability that } x(t) \text{ has magnitude greater than } \frac{1}{\sqrt{2}}\right]$$

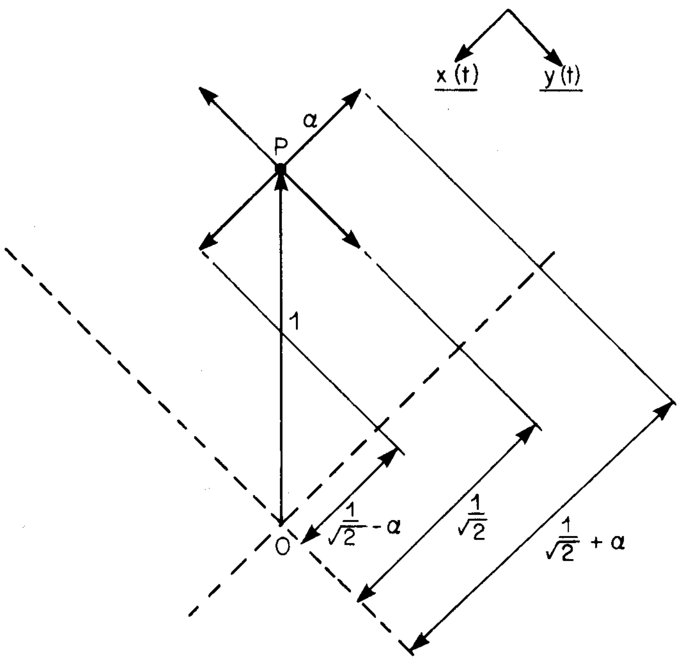

Fig. 7 - Vector diagram to illustrate the effect of a worst-case echo of n symbol periods delay

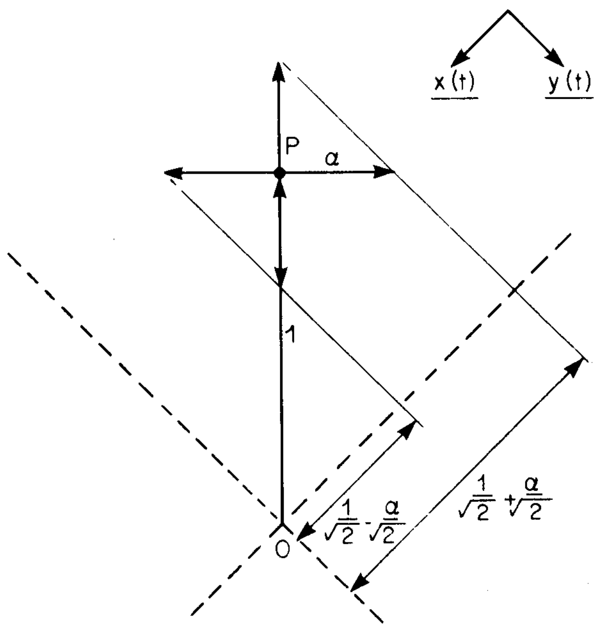

Fig. 8 - Vector diagram to illustrate the effect of a best-case echo of n symbol periods delay

$$+ \tfrac{1}{2} \times \tfrac{1}{4}\left[\text{probability that } x(t) \text{ has magnitude greater than } \frac{1}{\sqrt{2}} + \alpha\right]$$

+ the same three terms with $y(t)$ instead of $x(t)$ (10)

Using the results of the previous Section, Equation (10) can be re-written as

Probability of bit-error for worst-case echo phase

$$= \tfrac{1}{2}\left[1 - P\left(\frac{\frac{1}{\sqrt{2}} - \alpha}{\sigma}\right)\right] + \left[1 - P\left(\frac{1}{\sqrt{2}\sigma}\right)\right] + \tfrac{1}{2}\left[1 - P\left[\frac{\frac{1}{\sqrt{2}} + \alpha}{\sigma}\right]\right] \quad (11)$$

If α, the echo amplitude, is made equal to zero, Equation (11) reduces to Equation (6) which applies when there is no echo.

In Equation (11), the third term can be neglected for echo amplitudes, α, greater than about 10%. Equation (11) then reduces to:

Probability of bit-error for worst-case echo

$$= \tfrac{1}{2}\left[1 - P\left(\frac{\frac{1}{\sqrt{2}} - \alpha}{\sigma}\right)\right] + \left[1 - P\left(\frac{1}{\sqrt{2}\sigma}\right)\right] \quad (12)$$

In the experiments described in Section 3, the bit-error rate was kept constant by balancing the levels of the noise and multipath. For each echo delay, the level of the added noise was adjusted to maintain a 1 in 10^3 bit-error rate. The noise level was then reduced initially by 1 dB and the amplitude of the echo, α, was then increased gradually to restore the bit-error rate to 1 in 10^3. This process was repeated for reductions in the noise level of 2 dB and 3 dB. Using Equation (12), the amplitude of the echo, α, in each case can be calculated.

In the absence of multipath, a 1 in 10^3 bit-error rate is reached (see Equation (8)) when $1/\sigma\sqrt{2} = 3{\cdot}29$. If the noise level is reduced by 1 dB, then $1/\sigma\sqrt{2} = 3{\cdot}69$. Hence for the case of a 1 dB reduction in noise level Equation (12) can be re-written as

$$1 \times 10^{-3} = \tfrac{1}{2}\left[1 - P\left(\frac{\frac{1}{\sqrt{2}} - \alpha}{\sigma}\right)\right] + \left[1 - P(3{\cdot}69)\right] \quad (13)$$

This Equation can be solved using the tables in Reference 11, to yield the result that

$$\frac{\frac{1}{\sqrt{2}} - \alpha}{\sigma} = 2{\cdot}92 \quad (14)$$

Since it is known that $\frac{1}{\sigma\sqrt{2}} = 3{\cdot}69$, then

$$\alpha = 14{\cdot}7\% \quad (15)$$

The above analysis has shown that a worst-case echo with an integral number of symbol periods delay with a relative amplitude of 14·7% will reduce the noise margin of the d.p.s.k. signal by 1 dB.

The process can be repeated for reductions in noise levels of 2 dB and 3 dB. An exactly similar process can be used for the best-case echo represented in Fig. 8. The results of the complete analysis are plotted in Fig. 5.

8.4. Effect of a single echo with (n + ½) symbol periods delay

The analytical procedure in this case is very similar to that followed in the previous Section. The vector diagram representing the direct signal and the echo is more complicated than either of those shown in Fig. 7 or Fig. 8. This is because for an echo delay midway between symbol periods, the echo signal at the sampling instant can occupy any one of nine positions.

Fig. 9 is a vector diagram showing the positions that the d.p.s.k. signal can occupy when it is midway between two symbols; the probability of each of the positions being occupied is also shown. The four corners of the square in Fig. 9 represent the normal four phase-states of the d.p.s.k. signal. The probability of each of these states being occupied is 1/16. The other positions shown are those that the signal occupies when making transitions between states. Using the vector diagram of Fig. 9 to represent the echo the vector diagrams representing the sum of the wanted signal and worst-case and best-case echoes can be derived. Following the same procedure as in the previous Section the effect of the echoes was then calculated and the results are plotted in Fig. 5.

8.5. Effect of a single echo with less than one symbol-period delay

In the previous two Sections, the effect of echoes with delays longer than one symbol period has been con-

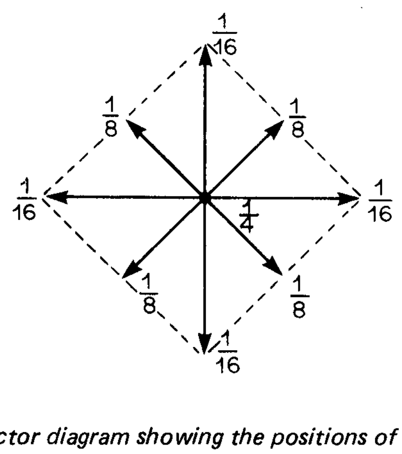

Fig. 9 - Vector diagram showing the positions of the d.p.s.k. signal and their probabilities of occupation midway between two symbols

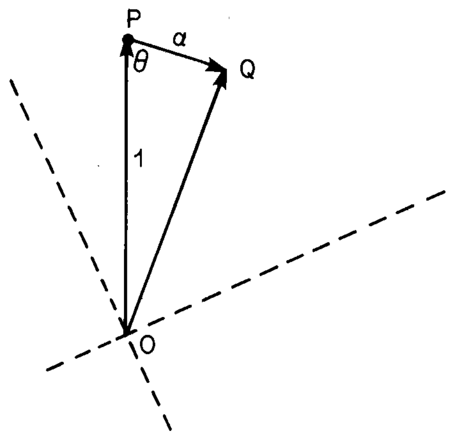

Fig. 10 - Vector diagram to illustrate the effect on the phase of the demodulating carriers of short-delayed echoes

sidered. During the analysis, it was assumed that the addition of the echo signal did not affect the phase of the demodulating carrier recovered from the incoming d.p.s.k. signal. For example, comparing the vector diagrams shown in Figs. 6, 7 and 8 the orientation of the dotted lines representing the phases of the quadrature demodulating carriers are all the same. This assumption was verified experimentally, and it is also not surprising since the addition of the echo signal does not disturb the mean phase of each state of the d.p.s.k. signal.

However, for the case of echoes with less than one symbol period delay, the above assumption is no longer valid. The addition of the echo disturbs or rotates the mean phase of each state of the signal and therefore the phase of the demodulating carrier is changed. As an example, consider the hypothetical case of an echo of level, α, with zero delay but with a phase different from that of the direct signal. Fig. 10 is a vector diagram representing this case. As before, the vector $\underline{OP}$ represents the direct signal occupying one of the four phase-states, the delayed signal with relative phase θ is shown by the vector $\underline{PQ}$ of length α. As can be seen, the resultant vector $\underline{OQ}$ has the same orientation as the direct signal only when θ = 0 or 180°. The carrier recovery circuit within the d.p.s.k. receiver will tend to align itself so that the resultant vector OQ is at 45° to each of the quadrature demodulating carriers as shown in Fig. 10. Hence, the phase of the demodulating carrier varies depending on the relative phase of the echo signal.

When calculating the effect of echoes with short delays, the change in the phase of the demodulating carrier must be accounted for. Experiments have confirmed that the carrier recovery circuit tends to align itself so as to minimise the effect of the echo.

The calculation of the effect of echoes with short delays can be made by following the same procedure as before. The effect of any changes in the phase of the demodulating carrier can be allowed for by applying a correction factor to the results. The results plotted in Fig. 5 contain an appropriate correction factor.

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