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The inverse cube law for dipoles

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It is unfortunate that so many students are never introduced to the inverse cube law during their courses and are left puzzled the first time they experience it later on when practicing their profession. I have also read papers in which scientists have spent considerable time, effort, and funds, to explain experimentally confirmed inverse cube dependency for cases in which according to the simple laws based on point entities, should instead have given an inverse square dependency. The simple mathematical analyses included here will hopefully avoid such waste of resources in the future, and hopefully become part of ordinary level physics textbooks.

We learn that the force between two charges, two magnetic monopoles, or two masses all follow an inverse square law, however, most of the time, the scientific reader is not made aware of an important assumption, that of being able to model these entities as point objects. If the entities cannot be reduced to a point, then, the inverse square laws cannot be applied. As I shall mathematically show, the inverse square law changes into an inverse cube law approximation for the case of dipoles. In practice, a physicist finds that most of real life applications cannot be modelled by point entities, but only by dipoles. These dipoles are commonly met in dielectrics, magnets, and molecules. In magnetism, nobody has yet identified a magnetic particle which can be defined as a point monopole. All physical magnets to date are in fact known to consist of dipoles having a north and a south pole and their force field will therefore always follow the inverse cubed law for dipoles. Same applies to charges acting on electric dipoles, and one cannot exclude the theoretical possibility of the same applying to mass dipoles.

Mathematical derivation of the inverse cubed law

This derivation theoretically applies to all forces, which obey the inverse square law when applied to point entities.

Electrostatic Force $F_P = KQ_1Q_2/R^2$ .... $K=1/4\pi\ \varepsilon_o$, Q = charge, R = distance

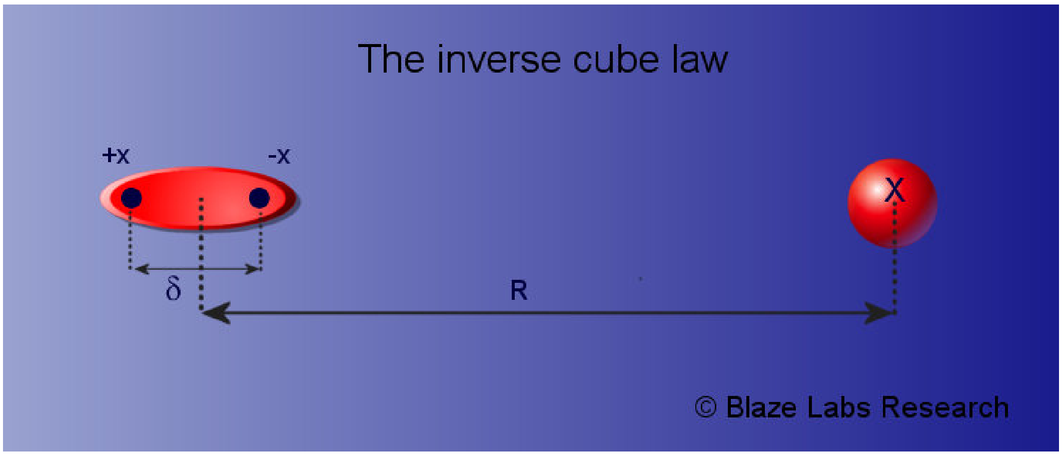
Magnetic Force $F_P = Um_1m_2/R^2$ .... $U = 1/\mu$, m = magnetic monopoles strength, R = distance

Gravitational Force $F_P = GM_1M_2/R^2$ .... G = gravitational constant, M= mass, R = distance

So, in general $F_P = k\ X_1\ X_2 / R^2$

where F_P = force magnitude for point entities, k = constant, X = entity unit, R= distance between entities.

We shall now define an additional parameter δ which in practice is a short distance between two point entities forming a single dipole. Distance R will therefore define the much longer distance between the centre of the dipole and another point entity X.



In the above diagram the dipole is made up of two opposite entities +x and –x separated by a distance δ, acted at a much larger distance R by the point entity +X. Since the negative part of the dipole is attracted to +X, the dipole will orientate itself with the negative side facing +X point entity. Thus if we measure distance R from the centre point of the dipole to point +X, we find that the distance from +X to +x is $R+\delta/2$ and that from +X to –x is $R-\delta/2$. Therefore since the distance between +X and –x is shorter than that between +X and +x, the force polarity between two opposite entities will govern the motion of the dipole with respect to the point entity. For opposite charges and magnetic poles, this means that a dipole will always move toward point +X, independently of the polarity of X.

The net force acting between the dipole and point entity X will be:

$$F_D = k\,X\,x / (R-\delta/2)^2 - k\,X\,x / (R+\delta/2)^2$$

we can rewrite the above in the form:

$$F_D = [kXx/R^2] / (1-\delta/2R)^2 - [kXx/R^2] / (1+\delta/2R)^2$$

For the condition $\delta \ll 2R$, which was set as one of our assumptions, we are justified to apply the binomial approximation $(1+x)^n \approx 1+nx$, or $1/(1+x)^n \approx 1-nx$, valid for $x \ll 1$. This reduces:

$1/(1-\delta/2R)^2$ to $1+\delta/R$, and $1/(1+\delta/2R)^2$ to $1-\delta/R$

The force field equation can therefore be approximated as:

$$F_D \approx [kXx/R^2](1+\delta/R) - [kXx/R^2](1-\delta/R)$$

$$F_D \approx [kXx/R^2](1+\delta/R - 1 + \delta/R)$$

$$\mathbf{F_D \approx 2kXx\delta/R^3} \text{ or simply } \mathbf{F_D \propto 1/R^3}$$

As is obvious from the above mathematical analysis, the simple inverse square law relation given for point charges, magnetic monopoles or point masses does NOT apply for the simple dipole case, for which the inverse cube law must be applied. It is also shown that the force vector between a dipole and a point entity is always the same polarity as that given for two opposite polarity point entities, which in general is defined as an attractive force.