Abstract

Cost functions dual to stochastic production technologies are derived and their properties are discussed. These cost functions are shown to be consistent with expected-utility maximization without placing serious structural restrictions on the underlying technology.

Cost Functions and Duality for Stochastic Technologies

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1. Introduction

Perhaps the most singular aspect of agricultural production is its randomness. Certainly, the stochastic nature of agricultural production and the economic problems associated with adjusting to it have provided the most commonly accepted arguments for agriculture's 'special nature', and consequently for its frequently preferential treatment in the economy. A similar spirit seems to pervade the analytical thinking of agricultural economists: Because agricultural production is stochastic, and because stochastic production is inherently different from nonstochastic production, it is often thought that common concepts from economic theory no longer apply. Nowhere is this more apparent than in the confusion that has arisen in agricultural economics over the existence of cost functions for stochastic technologies. A succinct statement of the conventional thinking has been provided by Pope and Chavas (1994):"...if one restricts attention to cost functions that are independent of risk preferences,...consistency of cost minimization with expected utility maximization imposes some structure on production technology."

Even in the case of a nonstochastic technology, consistency of cost minimization imposes some structure on the production technology in the sense that the existence of cost functions requires some minimal regularity properties. Typically, these include that the output chosen is technically feasible and that input sets be closed. This paper shows that closedness and technical feasibility *are the only conditions that a stochastic technology must satisfy in order for well behaved cost functions, exhibiting all their usual properties in terms of input prices, to exist.* And under the presumption that individuals maximize the expected utility of net returns, these cost functions are independent of the producer's risk preferences. Moreover, if these conditions are satisfied, the cost function is dual to a technology exhibiting free disposal of inputs and convexity of input sets that is observationally equivalent to the original stochastic technology. In other words, duality theory applies exactly for stochastic technologies under the same assumptions required for it to apply to nonstochastic technologies.

In what follows, we first introduce our notation and our definition of the technology. For concreteness sake, we use a representation of the technology similar to that analyzed by Pope and Chavas (1994), but more general in that it applies to non-differentiable technologies. We show that well-behaved cost functions exist for this technology, develop the properties of these cost functions, state a duality result relating the cost function to stochastic technology, and then show that maximizers of the expected utility from net returns always minimize cost. After that we briefly consider extensions of our approach to the more general state-contingent formulation of production uncertainty found, for example, in Chambers and Quiggin (1992, 1996, 1997) and then, for the purposes of illustration, present a simple example of the formulation of an optimal multiple-peril crop insurance program using our methods.

2. The Model

Consider a firm whose attitudes toward risk are characterized by a von Neumann-Morgenstern utility function, $U(W)$, where W denotes terminal wealth. We presume that the utility function is strictly increasing and continuous. Terminal wealth is assumed to take the form: $W = w_0 + f(\mathbf{x}, \varepsilon) - \mathbf{w} \cdot \mathbf{x}$, where w_0 is initial wealth, $\mathbf{x}$ is an n-dimensional vector of nonnegative inputs committed prior to the resolution of uncertainty, $\mathbf{w}$ is an n-dimensional vector of strictly positive and nonstochastic input prices, f is a nonnegative function giving stochastic revenue resulting from the application of inputs $\mathbf{x}$ and the stochastic factor ε. The stochastic factor may be variously interpreted as a stochastic random input beyond the control of the producer and not known at the time that input allocation decisions are made, or as an indicator of the state of the world. Unlike most earlier studies, there is no need for us to assume that the technology is sufficiently smooth to be differentiable or even continuous. Instead, we only assume that f is upper semi-continuous[1] in $\mathbf{x}$. Because differentiable technologies are always upper semi-continuous, it follows, for example, that our results cover the entire range of technologies considered by Pope and Chavas (1994). However, our results also apply to an even broader class of technologies (specifically those with closed input requirement sets). An empirically important example of a stochastic technology that is not differentiable but which is upper semi-continuous is the class of Leontief technologies. ε is assumed to have a fixed support given

[1]A function $h(\mathbf{z})$ is upper semi-continuous in $\mathbf{z}$ if its upper contour sets $\{\mathbf{z} : h(\mathbf{z}) \geq h\}$ are closed sets for all h.

by the closed interval $\Xi \subset \Re$ with a monotone probability distribution function $G(\varepsilon)$ with $G'(\varepsilon) = g(\varepsilon) \geq 0$.

Under these assumptions, an expected utility maximizer chooses the input allocation to solve the following problem:

$$\max_{\mathbf{x}} \left\{ \int_{\Xi} U(w_0 + f(\mathbf{x}, \varepsilon) - \mathbf{wx}) dG(\varepsilon) \right\}.$$

3. The Cost Function

The most modern approach to deriving a cost function for a nonstochastic technology is to specify the technology in terms of input sets or input correspondences which give the input combinations capable of producing a given bundle of outputs. Here we pursue a similar strategy except that we infer an input correspondence for a profile of stochastic revenues from the stochastic technology described in the previous section. By a profile or trajectory of stochastic revenues, we mean a relation which gives for every realization of the stochastic factor, $\varepsilon \in \Xi$, a level of revenue which we shall denote by $r(\varepsilon)$. Perhaps examples based on special cases of the technology detailed in the previous section best illustrate the concept of a trajectory. Consider the cases of multiplicative and additive uncertainty given by:

$$
\begin{aligned}
f(\mathbf{x},\varepsilon) &= f^m(\mathbf{x})\,\varepsilon, \\
f(\mathbf{x},\varepsilon) &= f^a(\mathbf{x}) + \varepsilon,
\end{aligned}
$$

where $\Xi = [0, Q]$. When the input bundle is fixed at any particular level, say $\mathbf{x}^*$, then in the multiplicative case the stochastic technology generates a trajectory of revenues that is depicted pictorially as a line with slope $f^m(\mathbf{x}^*)$ emanating from the origin and stopping at the point $f^m(\mathbf{x}^*)\,Q$, while in the additive case, the stochastic technology generates a profile of revenues that is depicted pictorially as a line with slope one and vertical intercept $f^a(\mathbf{x}^*)$ which ends at the point $f^a(\mathbf{x}^*) + Q$ (see Figure 1). Hence, choosing a particular input combination is equivalent to picking a profile of revenues when a stochastic technology prevails. Or alternatively choosing to produce a profile is equivalent to picking an input combination if the profile in question is technically feasible.

Now what does it take to be able to produce an arbitrary trajectory $R = \{r(\varepsilon) : \varepsilon \in \Xi\}$ of stochastic revenues using the stochastic technology developed above? Clearly, an input vector can produce a particular profile if and only if $\mathbf{x}$ satisfies: $f(\mathbf{x},\varepsilon) \geq r(\varepsilon)$ for all $\varepsilon \in \Xi$. So continuing to denote the profile by R, its input set is defined by the correspondence:

$$
\begin{aligned}
V(R) &= \{\mathbf{x} : f(\mathbf{x}, \varepsilon) \geq r(\varepsilon), \varepsilon \in \Xi\} \\
&= \cap_{\Xi}\{\mathbf{x} : f(\mathbf{x}, \varepsilon) \geq r(\varepsilon)\} \\
&= \cap_{\Xi} v(r(\varepsilon), \varepsilon),
\end{aligned}
$$

where $v(r(\varepsilon), \varepsilon)$ denotes the *ex post* input set associated with producing the single stochastic revenue $r(\varepsilon)$ given that ε actually occurs, i.e., $\{\mathbf{x} : f(\mathbf{x}, \varepsilon) \geq r(\varepsilon)\}$. Put another way, $v(r(\varepsilon), \varepsilon)$ is the collection of input combinations that will produce the *ex post* revenue, $r(\varepsilon)$, given that ε occurs. Figure 2 illustrates, for graphical simplicity, the case where $\Xi = \{1, 2\}$. When $\varepsilon = 1$, the isoquant for the level of revenue given by r_1 is illustrated as the lower boundary of the set $v(r_1, 1)$ under the presumption (made for purposes of illustration only) that inputs are freely disposable, and the isoquant for the level of stochastic revenue r_2 is given by the lower boundary of the set $v(r_2, 2)$. The intersection of these two input sets, $V(R)$, is given by all input combinations in the shaded area. Notice, in particular, that the input set for this stochastic technology will typically be kinked at points of intersection of the frontiers of the *ex post* input sets.

Because $f(\mathbf{x}, \varepsilon)$ is upper semi-continuous, each input set $v(r(\varepsilon), \varepsilon)$ is a closed set, and thus by a standard result, $V(R)$ must also be closed as it is formed by taking the intersection of an infinite number of closed sets. Having a clear notion of an input set it is now an easy matter to define a well-behaved cost function for the trajectory R. We have:

$$c(\mathbf{w},R) = \min_{\mathbf{x}} \{\mathbf{w} \cdot \mathbf{x} : \mathbf{x} \in V(R)\}$$

if $V(R)$ is nonempty and ∞ otherwise. Using well-known arguments one can establish that because $V(R)$ is a closed set, this cost function actually exists and possesses all the properties usually associated with cost functions in the vector of input prices (Chambers, 1988, Chapter 2). Hence, we state, without proof, the following obvious result:

Proposition 3.1. *: $c(\mathbf{w},R)$ satisfies: $c(\mathbf{w},R) \geq 0$; $c(\mu\mathbf{w},R) = \mu c(\mathbf{w},R), \mu > 0$; $\mathbf{w}' \geq \mathbf{w} \Rightarrow c(\mathbf{w}',R) \geq c(\mathbf{w},R)$; $c(\mathbf{w},R)$ is concave and continuous in $\mathbf{w}$.*

At this juncture, it is worth emphasizing that this fundamental result about the existence of a cost function for the stochastic technology only rests upon the single assumption that $f(\mathbf{x},\varepsilon)$ is upper semi-continuous in $\mathbf{x}$.

In addition to satisfying these usual properties of a cost function, $c(\mathbf{w},R)$ also satisfies Shephard's lemma. Namely, if there exists a unique solution to the cost minimization problem, then the cost function is differentiable in input prices, and its gradient in input prices is the vector of cost minimizing demands. And, if the cost function is differentiable in input prices, there exists a unique solution to the cost minimization problem which is equal to the gradient of the cost function in input prices (Färe, 1988).

Before turning to the possible dual relation between $c(\mathbf{w},R)$ and $V(R)$, it is worthwhile to divert our attention for a moment and illustrate how $c(\mathbf{w},R)$ relates to the cost functions for the *ex post* revenue functions, i.e., the cost functions associated with particular realizations of $f(\mathbf{x},\varepsilon)$. To that end, we define the *ex post* cost functions:

$$C(\mathbf{w},r,\varepsilon) = \min\{\mathbf{w} \cdot \mathbf{x} : \mathbf{x} \in v(r,\varepsilon)\}$$

if $v(r,\varepsilon)$ is nonempty and ∞ otherwise. Denote:

$$\mathbf{x}(\mathbf{w},R) \in \arg\min\{\mathbf{w} \cdot \mathbf{x} : \mathbf{x} \in V(R)\}.$$

By the definition of $V(R)$, it follows immediately that for all $r(\varepsilon) \in R$, $\mathbf{x}(\mathbf{w},R) \in v(r(\varepsilon),\varepsilon)$. Hence,

$$\mathbf{w} \cdot \mathbf{x}(\mathbf{w},R) \geq C(\mathbf{w},r(\varepsilon),\varepsilon), r(\varepsilon) \in R,$$

from which we immediately conclude:.

Proposition 3.2. *:* $c(\mathbf{w},R) \geq \max_{R} \{C(\mathbf{w},r(\varepsilon),\varepsilon)\}$.

The cost function for the stochastic revenue profile thus provides an upper bound for all the *ex post* cost functions, and in particular always provides an upper bound for the *ex post* revenue that is the costliest to produce. Sometimes, the inequality in the proposition can be replaced by an equality. This is always true, for example, when $\mathbf{x}$ is a scalar. And in Figure 2 if the relative input prices are given by the dashed line segment ww', the cost function for the stochastic revenue profile is given by the cost function for the costliest *ex post* revenue. More generally, however, it is not. Suppose, for example, that relative input prices are given by the dashed line segment w^*w^*. For these relative prices, least cost over $V(R)$ is given by point A which is not cost minimizing for either *ex post* revenue.

This link between the *ex post* cost functions and $c(\mathbf{w}, R)$ also helps illustrate the role that technical feasibility of a revenue profile plays in determining $c(\mathbf{w}, R)$. As an example, consider the case of multiplicative production uncertainty discussed earlier and illustrated in Figure 1. If such a technology applies and one chooses a revenue profile with a positive intercept in Figure 1, no combination of inputs will be able to produce that profile because no combinations of inputs is capable of producing a strictly positive output in the worst case, $\varepsilon = 0$, under multiplicative uncertainty even if the revenue profile is achievable in all other states of nature. Hence, $C(\mathbf{w}, r(0), 0) = \infty$ and consequently $c(\mathbf{w}, R) = \infty$.

Another immediate implication of this Proposition is that the cost function dual to the stochastic production technology will not generally be smoothly differentiable in all the elements of R. The nondifferentiability of $c(\mathbf{w}, R)$ emerges from the fact that the output set dual to $V(R)$:

$$R(\mathbf{x}) = \{R : \mathbf{x} \in V(R)\}$$

is not strictly convex and its frontier possesses kinks (Chambers and Quiggin, 1998).

4. Duality for the Stochastic Technology

Arguably the single most important development in the theory of cost and production was Shephard's (1953, 1970) discovery of the dual correspondence between the production structure and the cost function. This discovery has had important consequences at both an empirical and theoretical level. In this section, we show that $c(\mathbf{w},R)$ is dual to a stochastic production structure characterized by an input set $V(R)$ that is closed, convex, and satisfies free disposability of inputs. We start by defining the *shadow input correspondence*:

$$V^*(R) = \cap_{\mathbf{w}>\mathbf{0}}\{\mathbf{x} : \mathbf{w} \cdot \mathbf{x} \geq c(\mathbf{w},R)\}.$$

Because $V^*(R)$ is defined by the intersection of closed half spaces, it must be closed and convex by standard results on convex sets (Rockafellar, 1970). Furthermore, it is also apparent that $\mathbf{x}' \geq \mathbf{x} \in V^*(R) \Rightarrow \mathbf{x}' \in V^*(R)$, i.e., the shadow input correspondence satisfies free disposability of inputs. By standard duality theorems (e.g., Färe, 1988), it follows immediately that:

Proposition 4.1. *: If $V(R)$ satisfies the following properties: $\mathbf{0}^n \notin V(R)$ for all $R = \{r(\varepsilon) > 0 : \varepsilon \in \Xi\}$, $V(R)$ is a convex set, $\mathbf{x}' \geq \mathbf{x} \in V(R) \Rightarrow \mathbf{x}' \in V(R)$,* *and $V(R)$ is closed, then $V(R) = V^*(R)$.*

An immediate implication of the proposition is that a cost function derived from a stochastic technology characterized by closed input sets (upper semi-continuity of $f(\mathbf{x},\varepsilon)$) is dual to a stochastic technology characterized by closed and convex input sets satisfying free disposability of inputs. Thus, even if the technology from which $c(\mathbf{w},R)$ is derived does not satisfy these properties[2], there will exist a stochastic technology satisfying these properties which is observationally equivalent to the original technology in the sense that a cost minimizer will make the same economic choices from this technology (the one corresponding to the shadow input correspondence) as he or she would from the original technology. Hence, if one can establish (as we do in the next section) expected-utility maximizers minimize cost, then it follows immediately that no true generality is lost from an economic perspective in operating with a technology satisfying the same properties as $V^*(R)$.

We have already established that $V(R)$ satisfies one of the properties in the proposition (closedness). We now briefly discuss conditions on $f(\mathbf{x},\varepsilon)$ which guarantee the existence of this duality. Free disposability of inputs is guaran-

[2]As a reviewer points out, the empirically appealing Just-Pope technology can violate free disposability of inputs.

teed by assuming that $f(\mathbf{x},\varepsilon)$ is nondecreasing in $\mathbf{x}$, while convexity of $V(R)$ is ensured by assuming that $f(\mathbf{x},\varepsilon)$ is quasi-concave in inputs. (Quasi-concavity of $f(\mathbf{x},\varepsilon)$ implies that each $v(r,\varepsilon)$ is convex, and standard results on convex sets then implies that $V(R)$ is convex (Rockafellar, 1970).) The final property in the proposition we might refer to as 'no free lunch' in accordance with standard terminology in the nonstochastic production literature. A sufficient condition for the technology to satisfy this property is that $f(\mathbf{0}^n,\varepsilon)$ not be capable of producing a positive revenue, i.e., some inputs must be committed if a positive revenue is to be had in any state.

Corollary 4.2. *If $f(\mathbf{x},\boldsymbol{\varepsilon})$ is a nondecreasing, upper semi-continuous, and quasi-concave function of the inputs that satisfies 'no free lunch'then* $V(R) = V^*(R)$.

5. Expected Utility Maximizers Do Minimize Cost

Now that we have derived a cost function for the stochastic technology that is dual to a stochastic technology possessing closed and convex input sets exhibiting free disposability of inputs, we shall demonstrate that the expected-utility maximizaton problem can be broken down into two stages. In the first stage, the producer acts to minimize cost of a revenue trajectory or profile, and in the second stage the producer picks the utility maximizing revenue profile. Define:

$$\mathbf{x}(\mathbf{w},w_0) \in \arg\max \left\{ \int_{\Xi} U(w_0 + f(\mathbf{x},\varepsilon) - \mathbf{wx}) dG(\varepsilon) \right\},$$

and let

$$r(\mathbf{w},w_0,\varepsilon) = f(\mathbf{x}(\mathbf{w},w_0),\varepsilon)$$

denote the stochastic revenue that would occur if ε is the realization of the stochastic factor and inputs are evaluated at their expected-utility maximizing levels. Put another way, $r(\mathbf{w},w_0,\varepsilon)$ is the optimal *ex post* revenue contingent upon the realization of ε. In this sense, it can be interpreted as ε-contingent or state-contingent revenue. With these definitions in hand, it is now easy to establish that the expected utility maximizing producer, in fact, acts to minimize cost. In particular, our claim is that *the expected utility maximizing producer chooses the input bundle so as to minimize the cost of producing the profile of stochastic revenues given by:*

$$R(\mathbf{w},w_0) = \{r(\mathbf{w},w_0,\varepsilon) : \varepsilon \in \Xi\}.$$

The easiest way to see that this must be true is to suppose the contrary and assume that there exists a bundle of inputs cheaper than $\mathbf{x}(\mathbf{w},w_0)$, which when combined with the stochastic factor ε is capable of producing $R(\mathbf{w},w_0)$. Call this bundle of inputs $\mathbf{x}'$. Now the fact that U is strictly increasing in W and $\mathbf{x}'$ produces $R(\mathbf{w},w_0)$ implies

$$\int_{\Xi} U(w_0 + f(\mathbf{x}', \varepsilon) - \mathbf{w} \cdot \mathbf{x}')dG(\varepsilon) > \int_{\Xi} U(w_0 + r(\mathbf{w}, w_0, \varepsilon) - \mathbf{w} \cdot \mathbf{x}(\mathbf{w},\mathbf{w_0}))dG(\varepsilon)$$

thus violating the definition of $\mathbf{x}(\mathbf{w},w_0)$ as the expected-utility maximizing input choice. This argument establishes that:

Proposition 5.1. *:* $\mathbf{w} \cdot \mathbf{x}(\mathbf{w},\mathbf{w_0}) \leq \mathbf{w} \cdot \mathbf{x}$ *for all* $\mathbf{x}$ *capable of producing the trajectory of optimal stochastic revenues* $R(\mathbf{w},w_0)$*.*

Corollary 5.2. *:An expected-utility maximizer solves:*

$$\max_{R} \left\{ \int_{\Xi} U(w_0 + r\,(\varepsilon) - c(\mathbf{w},R))dG(\varepsilon) \right\}$$

In evaluating this result, it is important to recognize several things. Most importantly, the only restriction that is placed upon the producer's preferences (apart from those imposed by the expected-utility model) for the cost function to exist is that his or her utility be increasing in *ex post* wealth[3], and the only condition that consistency of cost minimization with expected-utility maximization imposes on the stochastic technology is that its input sets be closed. There is no need to make any presumption about the individual's degree of risk aversion, any degree of differentiability of the technology or the preference structure, or any other common measure of his or her attitudes toward risk. Thus, the proposition is more general than, say, those presented by Pope and Chavas (1994) which impose more structure upon both producer preferences and the technology. Second, by our duality results, the cost function that results from choosing inputs so as to minimize the cost of producing the stochastic revenue trajectory can be used to exhaustively characterize the economically relevant technology. Next this decomposition of the expected-utility maximization problem is distinct from those presented by Pope and Chavas (1994)[4], and hence our Proposition does not invalidate their claims about their cost functions. However,

[3]The Proposition is valid for even more general preference structures than expected utility. All that is required is that the producer's preferences be nondecreasing in $\mathbf{w} \cdot \mathbf{x}$.

[4]We would also argue that ours is the more natural decomposition of the producer maximization problem in the context of stochastic production

it does invalidate the naive conclusion that one might draw from their analysis, and that is, that expected utility maximizers do not minimize cost. *Rational expected utility maximizers always minimize cost.* Finally, and most importantly, the Proposition suggests a natural two-stage procedure to pursue in analyzing decisionmaking for a risk-averse producer facing a stochastic technology: First, find the minimum cost of producing all feasible revenue trajectories, and then choose the expected utility maximizing trajectory (trajectories).

6. Extensions and Applications

To keep our arguments as close as possible to the model of producer decisionmaking under uncertainty most familiar to agricultural economists, we have assumed that the individual producer maximizes expected utility. However, the only property of expected-utility maximization that we have explicitly employed in our arguments is the monotonicity of the von Neumann-Morgenstern utility function. It turns out that all our arguments continue to apply under even more general preference structures such as rank-dependent expected utility (Quiggin, 1993) or general smooth preferences (Machina, 1982) so long as producer preferences are at least weakly decreasing in producer input cost in the case of linear input prices or weakly decreasing in a separable function of inputs under more general preference structures, for example, generalized Schur-concave (Marshall and Olkin, 1979; Chambers and Quiggin, 1998) preference functions.

A more significant generalization is to proceed along the lines investigated by Chambers and Quiggin (1992, 1996, 1997, 1998) and extend the analysis beyond the case of stochastic revenue or production functions as considered in the present paper (as well as in most applied work on production under uncertainty) to the Arrow-Debreu model in which production possibilities for state-contingent commodities are described by technology sets. In this more general framework, the only restriction on the technology required for the existence of a cost function is that its input sets be closed[5]. The key advantage of this extension is that if $R(\mathbf{x})$ is strictly convex, its dual cost function will be smoothly differentiable avoiding the potential nondifferentiability of $c(\mathbf{w},R)$ that plagues the stochastic production function technology[6]. (As Chambers and Quiggin (1992, 1997) have pointed out the stochastic production function technology is degenerate in the sense that it leads to state-contingent output sets which are characterized by fixed coefficients.) Chambers and Quiggin (1997) use such a state-contingent

[5]Technically, this is an extremely mild restriction because it can never be contradicted empirically.

[6]Alternatively, nondifferentiability of the cost function can be avoided by placing enough structure upon the stochastic production function or revenue function to ensure that $R(\mathbf{x})$ is strictly convex.

production model in the finite-state case to analyze producer decision making in the presence of forward and futures markets exploiting the smoothness of the associated cost function to derive a range of arbitrage conditions and new results on hedging.

To illustrate this point, consider the problem of multiple-peril crop insurance as studied by (among others) Nelson and Loehman (1987) and Chambers (1989) under the assumption that $c(\mathbf{w},R)$ is smoothly differentiable. Assuming that the stochastic factor, ε, is contractible[7], an insurance company can write an insurance contract in which the *ex post* net indemnity depends upon the realization of the stochastic factor. Denote the net indemnity associated with the realization of the stochastic factor ε as $I(\varepsilon)$. Then assuming that the insurance company is risk-neutral, the socially optimal insurance contract solves the following maximization problem:

$$\underset{I,R}{Max}\left\{\int_{\Xi} U(w_0 + r(e) + I(\varepsilon) - c(\mathbf{w},R))dG(\varepsilon) - \int_{\Xi} I(\varepsilon)\, dG(\varepsilon)\right\}.$$

Letting $\pi(\varepsilon) = w_0 + r(e) + I(\varepsilon) - c(\mathbf{w},R)$, we obtain the following first-order

[7]This is equivalent to assuming that there is no problem of moral hazard or adverse selection.

conditions for the socially optimal multiple-peril crop insurance problem:

$$\begin{aligned} \left(U'\left(\pi\left(\varepsilon\right)\right)-1\right)g\left(\varepsilon\right) &= 0, \\ U'\left(\pi\left(\varepsilon\right)\right)g\left(\varepsilon\right)-\frac{\partial c(\mathbf{w},R)}{\partial r\left(\varepsilon\right)}\int_{\Xi}U'\left(\pi\left(\varepsilon\right)\right)dG(\varepsilon) &\leq 0, \quad r\left(\varepsilon\right)\geq 0, \quad \text{for all } \varepsilon\in\Xi, \end{aligned}$$

in the notation of complementary slackness[8].

Assuming that the farmer is strictly risk-averse, i.e., U is strictly concave, the above equality implies that a socially optimal multiple-peril crop insurance policy stabilizes farmer income at π^* which is determined as the implicit solution to $U'\left(\pi^*\right)=1$. Substituting this result into the second expression then yields that the optimal production pattern is determined by:

$$g\left(\varepsilon\right)-\frac{\partial c(\mathbf{w},R)}{\partial r\left(\varepsilon\right)}\leq 0, \quad r\left(\varepsilon\right)\geq 0,$$

which is the production pattern which maximizes expected profit from farming. Hence, in a very simple and straightforward fashion we are able to reconfirm the Nelson and Loehman (1987) result that socially optimal crop insurance in

[8]The notation, $\frac{\partial c(\mathbf{w},R)}{\partial r(\varepsilon)}$, is exact in the case where the state space is discrete. For the continuous state-space case, this derivative should be interpreted as the Fréchet derivative of $c(\mathbf{w},R)$ evaluated at ε.

the absence of moral hazard and adverse selection involves full insurance for the farmer[9] while having the farmer produce at the point which maximizes expected profit from farming. Other generalizations are straightforward and are left to the interested reader.

7. Concluding Comments

State-contingent production under uncertainty, like production of commodities differentiated in time and space, is merely a special case of a general multiple-input, multiple-output technology. Hence, as we demonstrate above the duality tools developed for the latter automatically apply to the former. This proposition stated in this way appears self-evident, but the issue of whether duality methods are applicable under uncertainty has remained shrouded in confusion and conflicting claims. In this paper, it has been shown that provided input sets are closed and nonempty, a well-behaved cost function can be derived from any stochastic production or revenue function. The resulting cost function, in turn, is always dual to a stochastic production structure exhibiting convexity of input sets and free disposability of inputs. Hence, any stochastic production structure possessing closed and nonempty input sets will be observationally

[9] This manifests Borch's (1962) well-known rule for optimal risk sharing.

equivalent for cost minimizers (and hence maximizers of the expected utility of net returns) to a stochastic production structure possessing closed, convex, and input disposable input sets.

Historically, the dual approach to economic analysis has proven a powerful and tractable tool in the analysis of non-stochastic production problems yielding many new insights and analytical results. The results of this paper suggest that similar progress in the analysis of problems involving production under uncertainty is possible.