Capital Precommitment and Competition in Supply Schedules.

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Abstract

Although the Cournot and Bertrand equilibrium concepts have dominated economic analysis of oligopoly problems, neither has a compelling theoretical rationale. However, notions of capacity commitment have been used to rationalise the Cournot equilibrium. At the same time, the idea of competition in supply schedules under uncertainty has been used by Klemperer and Meyer to derive an equilibrium concept intermediate between Cournot and Bertrand. In this paper, we combine these two approaches and show that under the assumptions of Cobb-Douglas technology and constant elasticity demand, an equilibrium in markups can result.
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1 Introduction.

Game-theoretic analysis of oligopoly problems has been extended in many directions over the last two decades. Among the most notable extensions have been the strategic trade theory literature and studies of research and development. In all of this literature analysis has been dominated by the Cournot-Nash solution concept, where the firms' strategic variable is quantity, with occasional attention being paid to the Bertrand solution, where the strategic variable is price. An intermediate case where the strategic variable is the markup over average cost is examined by Grant and Quiggin (1993).

As was first observed by Grossman (1981), it may be more natural to consider firms choosing strategies specified as supply schedules. The class of such strategies includes fixed quantity and fixed price schedules as special cases. For a market in which the demand is not subject to any shocks, it is easy to see that the Cournot-Nash equilibrium will also be a Nash equilibrium for the game

where both firms are free to choose any supply schedule. Assuming that one player has committed to a vertical supply schedule at the Cournot equilibrium quantity, any strategy for the other player which yields the Cournot equilibrium price-quantity pair will be optimal, and this set obviously includes the Cournot vertical supply schedule. The same reasoning shows that the Grant-Quiggin markup equilibrium is a Nash equilibrium for the general game in supply schedules. It may similarly be shown that a Bertrand equilibrium may be approached arbitrarily closely. All of this gives rise to the suspicion that almost any possible result may be derived as a Nash equilibrium for a game in supply schedules. This conjecture has been proved by Klemperer and Meyer [KM](1989), who show, for the duopoly problem, that any pair of quantities (along with the price determined by the market demand curve) where both firm earn non-negative profits can be supported as the Nash equilibrium of a game with strategies specified as supply schedules. Simple replication of plays does nothing to improve this situation, with the Folk Theorem reinforcing the complete agnosticism that follows from the KM result.[1]

KM (1989) seek to resolve the problem posed by their own result by introducing uncertainty. They show that if firms, faced with suitably defined uncertainty, compete by choosing over arbitrary price-quantity schedules, an equilibrium will exist and, given sufficient linearity, will be unique. In the KM approach, firms have complete freedom in the nature of the price-quantity schedule they adopt but are not able to make that schedule depend on the state of nature. The critical point is that, given that the supply-schedules are not state-contingent, each schedule in the equilibrium will be both ex ante optimal and ex post optimal in each state of the world. The KM solution concept has been applied to problems in strategic trade theory (see Laussel [1992], and Grant & Quiggin [1996]) and the behavior of electricity pricing pools (Green & Newbery [1990]).

A separate criticism of the Cournot solution has been advanced by writers from Bertrand onwards, who argue that the one-shot game equilibrium cannot be rationalized as the outcome of a process of dynamic adjustment by rational players. Kreps and Scheinkman (1983) address the lack of dynamic foundations for the Cournot solution. They show, for the special case of zero marginal cost up to capacity, that a two-stage game characterized by first-round precommitment of capacities (that is, maximum quantities) and second-round

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[1]See for instance Fudenberg and Maskin (1986) or more recently, Fudenberg, Levine and Maskin (1994) which provides a Folk theorem for repeated games in which the players observe a public outcome that only imperfectly signals the actions played.

Bertrand price competition yields the Cournot solution. It is natural to think of the Kreps-Scheinkman solution and the traditional Bertrand solution as polar cases. In the Kreps-Scheinkman case all costs are precommitted and in the Bertrand case, no costs are precommitted. Generalisations which may be regarded as including both special cases are offered by Dixon (1985, 1986) and Vives (1986). The basic approach is to consider two-input production functions where one input (capital) is committed in the first stage and the other input (labour) is committed in the second stage. The capital investment decision, made in the first stage, is strategic. In the second stage, however, firms behave competitively, that is, they choose the labour input, and therefore the output, so as to set marginal cost equal to the market price, which they treat as parametric. Although a number of technological specifications are considered, the most tractable is that of Cobb-Douglas technology, examined by Dixon (1985). In this case, as the technical parameter representing the capital share goes from one to zero, the solution goes from Cournot to Bertrand.

In this paper, we seek to combine elements of the Kreps-Scheinkman and Klemperer-Meyer approaches. Following Kreps and Scheinkman we consider a two-stage game with capital committed in the first round and decisions on the variable input (and hence on production and pricing) being made in the second round. However, we model a (constant returns to scale) Cobb-Douglas technology of which the Kreps-Scheinkman model can be viewed as a special case. The first-round choice of capital stock determines the second-round variable cost function for each firm. In the second round, we derive the KM solution concept for competition in supply schedules. For the special case of constant-elasticity demand, we show that the resulting equilibrium solution is identical to the equilibrium in markups previously analyzed by Grant and Quiggin (1993). We show that depending on the technology, the equilibrium solution may range from the Bertrand solution (if all costs are variable and so constant returns to scale implies marginal cost is constant) to the Cournot solution (if all costs are fixed and so the marginal cost of producing more than the amount for which the committed capital stock allows is infinite - the case explicitly modelled by Kreps and Scheinkman). Although the range of solutions derived is the same as that obtained by Dixon and Vives and the limiting cases correspond to the same limiting technology, the strategies generating these solutions and the mapping from the technological parameters to the equilibrium outcome are quite different. In particular, firms in our analysis are strategic in both the capital commitment stage and the variable production stage.

Our analysis should also be distinguished from Kreps and Scheinkman's discussion of the extension of their model to allow both costly capacity and production. They note that provided both capacity and production have convex cost structures and that capacity is costly at the margin, the unique equilibrium outcome is the Cournot outcome computed by using the *sum* of the two cost functions. The difference between their results and ours stems from our alternative specification of technology for which the variable cost although convex is never infinite for finite production and the "noise" in the demand function. Kreps and Scheinkman anticipated that this "noise" factor might change their analysis dramatically. This conjecture was confirmed in a negative sense by Hviid (1991), who showed that under uncertainty no pure strategy Nash equilibrium will exist for the KS problem. Here we confirm the conjecture in a different way. Using the KM solution concept, we show the KS result holds for their original specification of technology, but for more general specifications the Cournot result will not obtain under uncertain demand.

The next section provides the formulation of our two-stage oligopoly model and characterizes the perfectly competitive case as a benchmark. As is standard for two-stage games we solve for the equilibrium working backwards from the second stage sub-games. In section 3 we show that beginning from the point where all firms have installed their capital stocks and this has become common knowledge, there is a unique equilibrium in mark-ups that constitutes a KM equilibrium in supply schedules. We have been unable to find any other non-markup equilibria in supply schedules for these sub-games. And although we have not been able to demonstrate so rigorously, we strongly suspect for our specification of firms' cost technology and the demand function that the mark-up equilibrium is the unique equilibrium for the second-stage competition in supply schedule game. Section 4 characterizes the unique symmetric outcome in the first stage capital stock choices given that firms anticipate they will be competing in mark-ups in the second stage. We conclude in section 5.

2   General Set-Up

We assume the market demand for a homogeneous product has constant price elasticity but that it is subject to a *log-linear* shock, that can be described by a scalar random variable θ. Following KM we assume that this random variable has positive density on the support $[0, \infty)$ and so the market demand function can be expressed as

$$D(p, \theta) = \theta p^{-\phi}, \quad \phi \geq 1 \tag{1}$$

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The $n \geq 2$ firms have access to a common (constant returns to scale) Cobb-Douglas technology described by the production function

$$q_i = K_i^{\frac{1}{1+\gamma}} L_i^{\frac{\gamma}{1+\gamma}}, \gamma \geq 0$$

where K_i (respectively, L_i) is the amount of capital (respectively, labor) used by the firm in its production. We choose the units of capital and labor so that their (certain) factor prices are $1/(1+\gamma)$ and $\gamma/(1+\gamma)$ respectively.

The two-stage competition runs as follows. In the first stage, firms simultaneously and independently sink their capital investments for subsequent competition. After this first state, each firm learns how much capital its competitors have installed. Then *before* the demand shock is realized, firms commit to a supply schedule.[2] That is, each firm i chooses a mapping from prices into output, denoted $\hat{S}_i$ which embodies its production (and hence, given its capital stock, its labor input demand) as a function of the market price. The profile of the firms' second stage supply schedules $\left(\hat{S}_1, \ldots, \hat{S}_n\right)$ will be denoted by $\hat{\mathbf{S}}$, while $\hat{\mathbf{S}}_{-i}$ will denote the supply function profile of all the firms except firm i. For a given realization of the demand shock θ, and a given strategy profile of supply schedules the market outcome is thus

$$(\hat{p}(\theta); \hat{q}_1(\theta), \hat{q}_2(\theta), ..., \hat{q}_n(\theta)) \text{ where } \hat{q}_i(\theta) = \hat{S}_i(\hat{p}(\theta)) \text{ for all } i$$

and $\hat{p}(\theta)$ is the unique market clearing price. As KM do, we assume that if a market-clearing price does not exist or is not unique, then no production takes place and firms' variable profits are zero.[3]

As a base case, we note that if all firms acted competitively in both stages (that is, given they offer their competitive supplies in the second stage each firm sinks a capital investment in the first stage that yields a zero expected profit), each firm would make a capital investment of

$$K^c = \frac{1}{n} \operatorname{E}\left[\theta^{\frac{1+\gamma}{\theta+\gamma}}\right]^{\frac{\theta+\gamma}{1+\gamma}} \tag{2}$$

and offer a supply schedule in the second stage of:

$$S_{K^c}^c = K^c p^\gamma \tag{3}$$

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[2]That is, firms cannot employ *state* contingent supply schedules.

[3]As KM observe, this does not constrain the firms' behavior in any important way but rather simply precludes the non-existence or the non-uniqueness of a market clearing outcome arising in equilibrium.

So, given the realization of the demand shock, θ, the total labor demanded and good supplied by the n firms (if they act competitively in the second stage) are

$$L_{K^c}(\theta) = \theta^{\frac{1+\gamma}{\phi+\gamma}} \left[\mathrm{E}\left[\theta^{\frac{1+\gamma}{\phi+\gamma}}\right]^{\frac{\phi-1}{1+\gamma}} \right] \text{ and } Q_{K^c}^c(\theta) = \theta^{\frac{\gamma}{\phi+\gamma}} \mathrm{E}\left[\theta^{\frac{1+\gamma}{\phi+\gamma}}\right]^{\frac{\phi}{1+\gamma}} \quad (4)$$

Hence the expected labor demand and total quantity supplied in the competitive case are

$$\mathrm{E}\left[L_{K^c}(\theta)\right] = \mathrm{E}\left[\theta^{\frac{1+\gamma}{\phi+\gamma}}\right]^{\frac{\phi+\gamma}{1+\gamma}} = K^c \quad (5)$$

$$\mathrm{E}\left[Q_{K^c}^c(\theta)\right] = \mathrm{E}\left[\theta^{\frac{\gamma}{\phi+\gamma}}\right] \mathrm{E}\left[\theta^{\frac{1+\gamma}{\phi+\gamma}}\right]^{\frac{\phi}{1+\gamma}} \leq \mathrm{E}\left[\theta\right], \text{ as } \phi \geq 1 \quad (6)$$

Thus the factor prices have been 'normalized' to impose an expected labor-capital ratio for the competitive case of 1.[4]

3 'Competition in Supply Schedules' Subgames.

Suppose that in the first stage the firms have installed capital levels $(K_1, \ldots, K_n)$. Beginning from the point where $(K_1, \ldots, K_n)$ becomes common knowledge consider the subgame where each firm's strategy is to choose a supply schedule $\hat{S}_i$. Recall from the definition of a Nash equilibrium that in order to support $(\hat{p}(\theta); \hat{q}_1(\theta), \hat{q}_2(\theta), \ldots, \hat{q}_n(\theta))$ as a Nash equilibrium market outcome resulting from the strategy profile $\hat{\mathbf{S}}$, we require that for each firm i, choosing the supply schedule $\hat{S}_i$ should be profit-maximizing for that firm, given the other firms are choosing the profile $\hat{\mathbf{S}}_{-i}$. As the capital costs are sunk, all this entails is that the *variable* profit maximizing price and output combination of firm i, $(\hat{p}(\theta); \hat{q}_i(\theta))$, given the residual demand that it faces for that realization of θ, be on the schedule $\hat{S}_i$.

If the other firms are choosing the supply schedule profile $\hat{\mathbf{S}}_{-i}$ then the residual demand facing firm i is:-

$$D_i^R(p, \theta) \equiv D(p, \theta) - \sum_{j \neq i} \hat{S}_j(p)$$

Hence the variable profit of firm i, if it chooses price p, is:

$$\pi_i(p, \theta) = pD(p, \theta) - p\sum_{j \neq i} \hat{S}_j(p) - C_i\left(D(p, \theta) - \sum_{j \neq i} \hat{S}_j(p)\right)$$

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[4]If demand has unity elasticity ($\phi = 1$), then the competitive: total captial stock, expected total labor demand and expected total market supply are all equal to $\mathrm{E}[\theta]$ and the expected market clearing price is 1. Moreover, the total labor demand for given θ is θ.

Differentiating this expression with respect to p we obtain the first order (necessary) condition for a variable profit-maximizing price.

$$D(p,\theta) - \sum_{j \neq i} \hat{S}_j(p) + [p - C_i'(D_i^R(p,\theta))] \left[\frac{\partial D(p,\theta)}{\partial p} - \sum_{j \neq i} \frac{\partial \hat{S}_j(p)}{\partial p} \right] = 0 \quad (7)$$

Hence for $\hat{S}_i$ to be a best supply schedule choice for firm i we require equation 7 to hold for $p = \hat{p}(\theta)$ for all θ in $[0, \infty)$ and

$$\hat{S}_i(\hat{p}(\theta)) = \hat{q}_i(\theta) \qquad (8)$$

Dividing equation 7 through by $\hat{Q}(\theta) \equiv \sum_j \hat{q}_j(\theta) \ (= \hat{S}(\hat{p}(\theta)) = D(\hat{p}(\theta), \theta))$ and substituting in equation 8 we have:-

$$\frac{\hat{q}_i(\theta)}{\hat{Q}(\theta)} + \left[\frac{\hat{p}(\theta) - C_i'}{\hat{p}(\theta)} \right] \left[\frac{\partial D(\hat{p}(\theta), \theta)}{\partial p} \left(\frac{\hat{p}(\theta)}{D(\hat{p}(\theta), \theta)} \right) - \sum_{j \neq i} \frac{\partial \hat{S}_j(\hat{p}(\theta))}{\partial p} \left(\frac{\hat{p}(\theta)}{\hat{S}_j(\hat{p}(\theta), \theta)} \right) \left(\frac{\hat{q}_j(\theta)}{\hat{Q}(\theta)} \right) \right] = 0$$

Let

$$\varepsilon(\theta) \equiv -\frac{\partial D(\hat{p}(\theta), \theta)}{\partial p} \left(\frac{\hat{p}(\theta)}{D(\hat{p}(\theta), \theta)} \right) \text{ and } \eta_j(\theta) \equiv \frac{\partial \hat{S}_j(\hat{p}(\theta))}{\partial p} \left(\frac{\hat{p}(\theta)}{\hat{S}_j(\hat{p}(\theta), \theta)} \right)$$

denote, respectively, the (price) elasticity of market demand and the (price) elasticity of the supply schedule $\hat{S}_j$ evaluated at price $\hat{p}(\theta)$. Let

$$s_j(\theta) = \frac{\hat{q}_j(\theta)}{\hat{Q}(\theta)}$$

denote the market share of firm j in the outcome $(\hat{p}; \hat{q}_1, \hat{q}_2, ..., \hat{q}_n)$.

Utilizing these definitions, equation 7 can be re-expressed more succinctly as:

$$\frac{\hat{p}(\theta) - C_i'(\hat{q}_i(\theta))}{\hat{p}(\theta)} = \frac{s_i(\theta)}{\left[\varepsilon(\theta) + \sum_{j \neq i} s_j(\theta) \eta_j(\theta) \right]}, \quad \text{for all } \theta \in [0, \infty) \qquad (9)$$

In words, the above condition states that a firm's price-cost margin should be equal to the inverse of the residual demand that it faces in equilibrium for each realization of θ. The price elasticity of the residual demand facing the firm is simply the sum of the price responsiveness of the market demand plus a (market share) weighted sum of the price responsiveness of the other firms given that they have committed to the profile of supply schedules $\hat{\mathbf{S}}_{-i}$, all divided by the market share of firm i.

Given its capital investment, K_i, made in the first stage, the variable cost function for firm i is thus

$$C_i(q; K_i) = \frac{\gamma}{1 + \gamma} L, \text{ where } K_i^{\frac{1}{1+\gamma}} L^{\frac{\gamma}{1+\gamma}} = q$$

that is,

$$C_i\left(q; K_i\right) = \frac{\gamma}{1+\gamma} K_i^{-\frac{1}{\gamma}} q^{1+\frac{1}{\gamma}} \tag{10}$$

from which we can derive marginal variable cost

$$C_i'\left(q; K_i\right) = K_i^{-\frac{1}{\gamma}} q^{\frac{1}{\gamma}} \tag{11}$$

So the "competitive" supply schedule for this firm, given its first stage capital investment is

$$S_i^c\left(p; K_i\right) = K_i p^\gamma \tag{12}$$

Intuitively, firm i's share of the market if all firms acted competitively would be $K_i / \left(\sum_j K_j\right)$. In a non-competitive equilibrium, one would expect that firms are committed to supply schedules that 'lie to the left' of their competitive counterparts. From (9), (11) and the demand specification, it seems natural to focus on a supply schedule that corresponds to a constant mark-up over marginal variable cost (which for this cost function also corresponds to a constant mark-up over *average* variable cost - see Grant and Quiggin, 1993). That is, consider the situation where each firm chooses a supply schedule such that C_i'/p is equal to a constant λ_i for some λ_i less than 1. Thus from (11) it follows that the supply schedule offered by firm i is

$$\hat{S}_i\left(p; K_i\right) = K_i\left(\lambda_i p\right)^\gamma \tag{13}$$

Notice that if all firms are choosing a 'constant mark-up over marginal variable cost' supply schedule the elasticity of each firm's offered supply is constant and equal to γ and firm i's equilibrium share is also constant and equal to

$$s_i = \frac{K_i \lambda_i^\gamma}{\sum_j K_j \lambda_j^\gamma} \tag{14}$$

In particular, these supply elasticities and equilibrium market shares are *independent* of the demand shock. Hence the supply schedule profile $\mathbf{S}^*$, corresponding to the profile of mark-ups $(1 - \lambda_1, \ldots, 1 - \lambda_n)$ is an equilibrium in supply schedules for the subgame if, for each i,

$$1 - \lambda_i = \frac{K_i \lambda_i^\gamma / \left(\sum_j K_j \lambda_j^\gamma\right)}{\phi + \gamma\left(1 - K_i \lambda_i^\gamma / \left(\sum_j K_j \lambda_j^\gamma\right)\right)} \tag{15}$$

Result 1 *Fix $(K_1, \ldots, K_n)$. For the second stage subgame there is a unique constant mark-up over marginal variable cost equilibrium and moreover this is an equilibrium in the general competition in supply schedule subgame.*

Proof. Existence: Consider the game where firms are restricted to choosing supply schedules of the form in (13). That is we can view the strategy space of this subgame as $[0,1]^n$. For a given strategy profile $(\lambda_1, \ldots, \lambda_n)$ and given realization of the demand shift parameter θ, straightforward calculation reveals that the market equilibrium price is

$$p = \left[\frac{\theta}{\sum_j K_j \lambda_j^\gamma}\right]^{\frac{1}{\phi + \gamma}}$$

and firm i's equilibrium quantity supplied to the market is

$$q_i = \left[\frac{\theta}{\sum_j K_j \lambda_j^\gamma}\right]^{\frac{\gamma}{\phi + \gamma}} K_i \lambda_i^\gamma$$

So firm i's equilibrium variable profit is

$$\pi_i(\lambda_1, \ldots, \lambda_n; K_1, \ldots, K_n) = \left[\frac{\theta}{\sum_j K_j \lambda_j^\gamma}\right]^{\frac{1+\gamma}{\phi+\gamma}} K_i \lambda_i^\gamma \left[1 - \frac{\gamma}{1+\gamma}\lambda_i\right] \qquad (16)$$

The subgame is *supermodular*[5] in $(\lambda_1, \ldots, \lambda_n)$ since for all i we have

$$\frac{\partial \ln \pi_i}{\partial \ln \lambda_l \partial \ln \lambda_k} = \frac{1+\gamma}{\phi+\gamma}\gamma^2 s_l s_k > 0 \text{ for all } l, k, l \neq k$$

Since π_i is continuous in λ_i it follows from Topkis (1979) that the set of pure Nash equilibria is non-empty and possesses greatest and least equilibrium points $\overline{\lambda}$ and $\underline{\lambda}$. That is, for any equilibrium strategy $\lambda = (\lambda_1, \ldots, \lambda_n)$ of the subgame, $\overline{\lambda} \geq \lambda \geq \underline{\lambda}$.

Moreover, each equilibrium of the mark-up game is also a equilibrium in the more general supply schedule game, since given all one's opponents are playing mark-ups, (9) and (15) imply that playing a constant mark-up is ex post optimal no matter what the actual realization of θ is.

Uniqueness: Suppose $\overline{\lambda} \neq \underline{\lambda}$, that is for all i, $\overline{\lambda}_i \geq \underline{\lambda}_i$ and there exists j such that $\overline{\lambda}_j > \underline{\lambda}_j$. Now both $\overline{\lambda}_i$ and $\underline{\lambda}_i$ satisfy (15) which can be expressed as

$$1 - \lambda_i = \frac{s_i}{\phi + \gamma(1 - s_i)} \qquad (17)$$

Now the LHS is obviously decreasing in λ_i, and the RHS is increasing in s_i. So going from $\underline{\lambda}$ to $\overline{\lambda}$ leads to the LHS of (17) weakly decreasing for all i and strictly decreasing for some j, and thus requires the RHS of (17) to be weakly decreasing for all i, and strictly decreasing for some j. But this in turn entails

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[5]See Fudenberg and Tirole (1991, s12.3 pp495-498) for definitions and discussion of supermodular games.

that s_i is weakly decreasing for all i, and strictly decreasing for some j, a contradiction. ■

We have been unable to show the existence of any other equilibrium in supply schedules for this second-stage game. Moreover, given that both demand and each firm's marginal cost schedule are log-linear and that demand is subject to a log-linear shock, one would have thought that one should be able to adapt KM's uniqueness result (1989, Proposition 4, p1261) to show that the mark-up equilibrium is indeed the unique equilibrium for the general competition in supply schedule subgame. Unfortunately, we have been unable to accomplish this but the intuition seems so strong, that we feel confident in focussing on the mark-up equilibrium for the rest of the analysis. At the very least we can say in our defence, that the mark-up equilibrium is 'salient' or 'focal' for both the analyst and the firms given the constant elasticity of demand and each firm's marginal cost schedule.

It is also interesting to notice the relationship between the competitive equilibrium market shares and the mark-up equilibrium market shares (that is, the s_is). For this asymmetric equilibrium (9) can be re-expressed as:-

$$\lambda_i \equiv \frac{C_i'(q_i(\theta))}{p(\theta)} = \frac{(s_i Q(\theta)/K_i)^{1/\gamma}}{p(\theta)} = \frac{\phi + \gamma - (1+\gamma)s_i}{\phi + \gamma - \gamma s_i} \tag{18}$$

Hence for any i and j we have:-

$$\left[\frac{s_i K_j}{s_j K_i}\right]^{1/\gamma} = \frac{[\phi + \gamma - (1+\gamma)s_i]}{[\phi + \gamma - \gamma s_i]} \frac{[\phi + \gamma - \gamma s_j]}{[\phi + \gamma - (1+\gamma)s_j]} \tag{19}$$

Result 2 *In the asymmetric mark-up equilibrium in supply schedules above,* $K_i > K_j$ *implies* $K_i/K_j > s_i/s_j > 1$.

Proof. Consider RHS of (19), as all four bracketed terms are positive by the assumptions on the parameters, simple algebraic manipulation reveals that this expression is less than one if and only if $s_i > s_j$. So we can rule out $s_j > s_i$ as this would imply LHS < 1 and RHS > 1. So $s_i > s_j$ and (19) imply LHS < 1 which can only hold if $s_i/s_j < K_i/K_j$. ■

Also notice that from (17) the mark-up is increasing in market share. Therefore, in an asymmetric equilibrium the bigger firms choose a larger mark-up for their pricing policy and enjoy a relatively (compared to the perfectly competitive outcome) smaller market share.

4 The Symmetric Equilibrium of Complete Game

Following the analysis of the previous subsection, given the first stage profile of capital investments $(K_1, \ldots, K_n)$, the second stage subgame equilibrium profile of mark-ups $(1 - \lambda_1, \ldots, 1 - \lambda_n)$ and the realization of the demand shock, θ, firm i's variable profit is given by (16). As firms all have access to the same technology, we shall focus only on symmetric equilibrium for the complete game . In the symmetric equilibrium case where each firm has in the first period sunk capital investment $\hat{K}$, for all i, $s_i = 1/n$, $\lambda_i = 1 - (n\phi - (n-1)\gamma)^{-1} = \lambda$ and

$$\pi_i = \theta^{\frac{1+\gamma}{\phi+\gamma}} \left(\frac{1}{n} \right) \left(1 - \frac{\gamma}{1+\gamma} \lambda \right) \left(n\hat{K}\lambda^\gamma \right)^{\frac{\phi-1}{\phi+\gamma}}$$

For $\hat{K}$ to be the (symmetric) equilibrium capital investment for each firm in the first period, it must be the case that given that all the other firms are choosing $\hat{K}$, the marginal expected variable profit from increasing the capital investment at $\hat{K}$ is equal to its factor price, that is $\mathrm{E}\left[\partial \pi_i \left(K_i = K_j = \hat{K} \right) / \partial K_i \right] = 1/(1+\gamma)$.

Differentiating (16)

$$\frac{d\pi_i}{dK_i}_{\rfloor K_i = K_j = \hat{K}} = \frac{\partial \pi_i}{\partial K_i} + \frac{\partial \pi_i}{\partial \lambda_i} \frac{d\lambda_i}{dK_i} + \sum_{j \neq i} \frac{\partial \pi_i}{\partial \lambda_j} \frac{d\lambda_j}{dK_i} \tag{20}$$

As (20) is evaluated at a Nash equilibrium the second term is zero and with simple algebraic manipulation we obtain:

$$\frac{d\pi_i}{dK_i}_{\rfloor K_i = K_j = \hat{K}} = \frac{\pi_i}{K_i} \left(1 - s_i \left[\frac{1+\gamma}{\phi+\gamma} \right] \right) + 0 - \sum_{j \neq i} \frac{\pi_i}{K_i} \frac{\gamma(1+\gamma)}{\phi+\gamma} s_j \frac{\partial \ln \lambda_j}{\partial \ln K_i} \tag{21}$$

In the appendix we show that at the symmetric equilibrium:

$$\frac{\partial \ln \lambda_j}{\partial \ln K_i} = \frac{\phi + \gamma}{n(\phi+\gamma) + \lambda [n(\phi+\gamma) - \gamma]^2} \tag{22}$$

Substituting this last expression into (21) yields:

$$\frac{d\pi_i}{dK_i}_{\rfloor K_i = K_j = \hat{K}} = \frac{\pi_i}{K_i} \left[1 - \frac{1+\gamma}{n(\phi+\gamma)} - \frac{\phi+\gamma}{n(\phi+\gamma) + \lambda [n(\phi+\gamma) - \gamma]^2} \right] > 0$$

Thus $\hat{K}$ satisfies:

$$\mathrm{E}_\theta \left[\frac{d\pi_i}{dK_i}_{\rfloor K_i = K_j = \hat{K}} \right] = \frac{1}{1+\gamma} \tag{23}$$

The $\hat{K}$ that satisfies (23) is unique since:

$$\frac{d^2 \pi_i}{dK_i^2}_{\rfloor K_i = K_j = \hat{K}} = \left(\frac{d\pi_i}{dK_i}_{\rfloor K_i = K_j = \hat{K}} \right)^2 / \pi_i - \left(\frac{d\pi_i}{dK_i}_{\rfloor K_i = K_j = \hat{K}} \right) / K_i$$

$$
\begin{aligned}
&= \left(\frac{d\pi_i}{dK_i}_{\lrcorner K_i = K_j = \hat{K}} \right) \left[\frac{d\pi_i/dK_i}{\pi_i} - \frac{1}{K_i} \right] \\
&= -\left(\frac{d\pi_i}{dK_i}_{\lrcorner K_i = K_j = \hat{K}} \right) \left[\frac{1+\gamma}{n\left(\phi+\gamma\right)} + \frac{\phi+\gamma}{n\left(\phi+\gamma\right) + \lambda\left[n\left(\phi+\gamma\right) - \gamma\right]^2} \right] < 0
\end{aligned}
$$

That is, the expected marginal increase in variable profits schedule cuts the (constant) marginal cost of capital schedule from above. Collecting these results together we have:

Result 3 *Given that firms anticipate that their opponents will be selecting mark-up strategies in the second stage competition in supply schedules, there is a* unique *symmetric equilibrium outcome of the two-stage game. In the first stage firms sink the capital investment $\hat{K}$, implicitly defined in (23), and in the second stage commit to the supply schedule that corresponds to the constant price-cost margin*

$$
1 - \lambda = \frac{1}{n\phi + (n-1)\gamma}
$$

Notice that for a particular firm, say i, that believes all other firms are committed to the same level of capital investment, increasing its capital investment, K_i, above theirs has a direct effect and a *strategic* effect on its variable profit in the second round. The direct effect is clearly to increase its variable profit since for a fixed profile of markups, a larger capital stock lowers the firm's variable costs. But by making itself larger than its competitors through raising K_i, provides firm i with an incentive to raise its markup and for the other firms to reduce theirs. This is simply an illustration of the 'compression' effect outlined in Result 2 . Hence, at the symmetric equilibrium, the strategic effect of investment is negative. So like Dixon (1985), the firms' strategic interaction in the first stage results in under-capitalisation relative to the perfectly competitive outcome. However, unlike Dixon (1985), as the firms in our analysis also interact strategically in the second-stage, their competition in supply schedules results in a *lower* labour demand than would result if they acted perfectly competitively.

The parameter γ is a measure of the relative flexibility of the firms' production in the second stage given the capital stocks chosen in the first stage. It is immediate from the analysis of the previous section and the expressions derived for the equilibrium mark-ups that for any configuration of capital stocks the firms' equilibrium mark-ups in the second stage are decreasing in γ. Moreover it is fairly straightforward (albeit, algebraically cumbersome) to show (and we trust, intuitive to see) that the overall profits in the symmetric equilibrium of

the complete two-stage game are also decreasing in γ. That is, the more flexible are the firms in the second period (and hence the less constrained they are by their choice of capital stocks in the first period) the closer the competition in supply schedules corresponds to the perfectly competitive outcome.

The two polar cases, $\gamma = 0$ and $\gamma = \infty$, correspond respectively, to the situation of no flexibility and complete flexibility of production in the second stage. The equilibrium outcomes are thus quite naturally identified as the Cournot and Betrand outcomes, respectively. With $\gamma = 0$,

$$\hat{K} = \frac{1}{n} \left[\frac{n\phi - 1}{n\phi} \right]^{\phi} \mathrm{E} \left[\theta^{\frac{1}{\phi}} \right]^{\phi}$$

and for the given realization of the demand shock, θ, the total market traded is

$$Q(\theta) = n\hat{K} = \left[\frac{n\phi - 1}{n\phi} \right]^{\phi} \mathrm{E} \left[\theta^{\frac{1}{\phi}} \right]^{\phi}$$

at the market clearing price of

$$p(\theta) = \left(\frac{\theta}{nK} \right)^{\frac{1}{\phi}} = \frac{\theta^{\frac{1}{\phi}}}{\mathrm{E} \left[\theta^{\frac{1}{\phi}} \right]} \left[\frac{n\phi}{n\phi - 1} \right]$$

That is, the quantity traded is invariant to the demand shock but the equilibrium price is not and firms earn non-zero expected profits.[6] With $\gamma = \infty$, the full flexibility (with constant marginal variable costs) leads to the perfectly competitive outcome, that is $p(\theta) = 1$, and $Q(\theta) = \theta$ (and each firm employing θ/n units of labor to produce their $1/n$ th share of the market).

5 Conclusion

The representation of oligopolistic strategies in terms of markups over average cost has a long history, going back to the work of Hall and Hitch (1939). Markup models frequently perform well in empirical work (eg Thompson and Lyon, 1989), and are extensively used in macroeconomic analysis and in areas such as the literature on exchange rate pass-through (eg Hooper and Mann, 1989).

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[6]With $\gamma = 0$, a mark-up strategy is equivalent to setting a perfectly inelastic demand, that is choosing a quantity. Given the quantity profile $(q_1, \ldots, q_n)$ firm i's variable profit is

$$\pi_i = \left(\frac{\theta}{\sum_j q} \right)^{\frac{1}{\phi}} q_i$$

so

$$\frac{\partial \pi_i}{\partial q_i} = \pi_i \left[\frac{1}{q_i} - \frac{1}{\phi \sum_j q} \right] > 0, \text{ for all } q_i \geq 0$$

Hence firm i's best response is to set $q_i = \hat{K}$

However, markup models have generally lacked a microeconomic basis. They have been defended in pragmatic terms as being simple and realistic model of the way in which prices are set.

The aim of our analysis has *not* been to provide such a microeconomic basis, although the results presented above show that markup equilibria have at least as strong an *a priori* claim to be considered as reasonable a game-theoretic solution concept for the oligopoly problem as do the traditional Bertrand and Cournot concepts. Our extension of the Kreps-Scheinkman analysis shows that the existence of capital precommitment does not, in itself, make the Cournot equilibrium the natural solution concept for oligopoly models. With appropriate specifications of the technology, demand conditions and information sets that firms face, a range of outcomes, including equilibrium in markups may arise.

Our results might be seen as a confirmation of the observation of Sutton (1990) that, for any conceivable form of oligopolistic behavior, a game-theoretic 'explanation' may be found. As Sutton observes, to explain everything is to explain nothing. The positive side of Sutton's argument, however, is that given sufficient knowledge of the economic structure of the industry concerned and of the information flows to which institutional arrangements give rise, it may be possible to generate robust predictions of behavior. The work of Kreps and Scheinkman, in which capital investment decisions are separated from the strategic interactions determining prices and outputs, is an important step in this direction. Our results give emphasis to the need for detailed analysis of institutional arrangements and the information flows they generate as a basis for generating strong predictions. As Kreps and Scheinkman observe, the specification of the strategy space is critical in any analysis of economic problems using game theory. As they emphasize:

> "[S]olutions to oligopoly games depend on both the strategic variables that firms are assumed to employ and the context (game form) in which those variables are employed. The timing of decisions and information reception are as important as the nature of the decisions." (p327)

Unlike games in the ordinary sense, economic problems considered as games do not normally possess a unique specification of the strategy space imposed by the nature of the game itself. Rather, restrictions on the strategy space sufficient to yield sharp predictions concerning equilibrium outcomes can be derived only on the basis of extensive economic information.

Appendix

Proof of (22)

By differentiating (14), the expression for firm i's market-share in a mark-up equilibrium, and by differentiating (15), the first order condition that defines firm i's equilibrium markup, we obtain:

$$\frac{ds_i}{dK_i} = \frac{n-1}{n^2 \hat{K}} \left[1 + \gamma \left(\frac{\partial \ln \lambda_i}{\partial \ln K_i} - \frac{\partial \ln \lambda_j}{\partial \ln K_j} \right) \right] \tag{24}$$

$$\frac{ds_j}{dK_i} = -\frac{1}{n^2 \hat{K}} \left[1 + \gamma \left(\frac{\partial \ln \lambda_i}{\partial \ln K_i} - \frac{\partial \ln \lambda_j}{\partial \ln K_j} \right) \right] \tag{25}$$

and

$$\frac{d\lambda_i}{dK_i} \frac{K_i}{\lambda_i} \equiv \frac{d \ln \lambda_i}{d \ln K_i} = -\frac{ds_i}{dK_i} \frac{K_i}{\lambda_i} \left[\frac{\phi + \gamma}{(\phi + \gamma \left[1 - 1/n \right])^2} \right] \tag{26}$$

$$\frac{d\lambda_j}{dK_i} \frac{K_i}{\lambda_j} \equiv \frac{d \ln \lambda_j}{d \ln K_i} = -\frac{ds_j}{dK_i} \frac{K_i}{\lambda_j} \left[\frac{\phi + \gamma}{(\phi + \gamma \left[1 - 1/n \right])^2} \right] \tag{27}$$

Using (24) and (25) to substitute out $\frac{ds_i}{dK_i}$ and $\frac{ds_j}{dK_i}$ in (26) and (27) gives us the required expression for $\frac{d \ln \lambda_j}{d \ln K_i}$. $\blacksquare$

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